

Towards a parameter-free calculation of the nuclear matrix element for the $0\nu\beta\beta$ decay

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Acknowledgements

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Summary

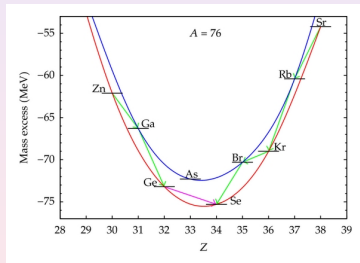
- The neutrinoless double- β decay
- The problematics of the calculation of the nuclear matrix element (NME) of $0\nu\beta\beta$ decay
- Nuclear structure calculations of the NME
- The realistic nuclear shell model
- Testing the theoretical framework: calculation of the GT strengths and the nuclear matrix element of $2\nu\beta\beta$ decay
- Conclusions and outlook

The detection of the $0\nu\beta\beta$ decay is nowadays one of the main targets in many laboratories all around the world, triggered by the search of "new physics" beyond the Standard Model.

- Its detection
 - would correspond to a violation of the conservation of the **leptonic number**,
 - may provide more informations on the nature of the neutrinos (the neutrino as a **Majorana particle**, determination of its **effective mass**, ..).

The double β -decay

The semiempirical mass formula provides two different parabolas for even-mass isobars:

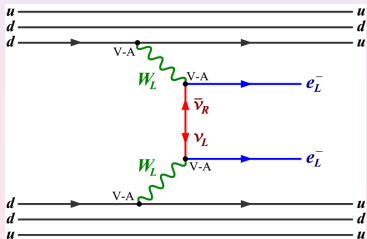


- Maria Goeppert-Mayer (1935) suggested the possibility to detect $(A, Z) \rightarrow (A, Z+2) + e^- + e^- + \bar{\nu}_e + \bar{\nu}_e$
- Historically, Giulio Racah was the first one, to test the neutrino as a Majorana particle, to consider the process:
 $(A, Z) \rightarrow (A, Z+2) + e^- + e^-$

The neutrinoless double β -decay

The inverse of the $0\nu\beta\beta$ -decay half-life is proportional to the squared nuclear matrix element (NME).

This evidences the relevance to calculate the NME



$$\left[T_{1/2}^{0\nu} \right]^{-1} = G^{0\nu} \left| M^{0\nu} \right|^2 \langle m_\nu \rangle^2 ,$$

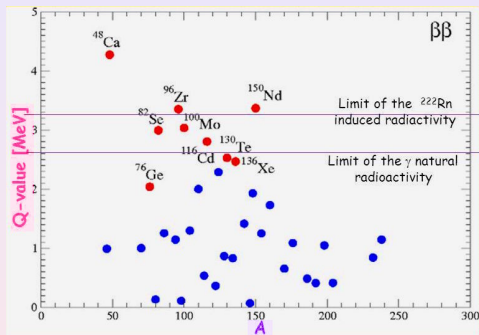
- $G^{0\nu}$ is the so-called phase-space factor, obtained by integrating over the single electron energies and angles, and summing over the final-state spins;
- $\langle m_\nu \rangle = \left| \sum_k m_k U_{ek}^2 \right|$ effective mass of the Majorana neutrino, U_{ek} being the lepton mixing matrix.

The detection of the $0\nu\beta\beta$ -decay

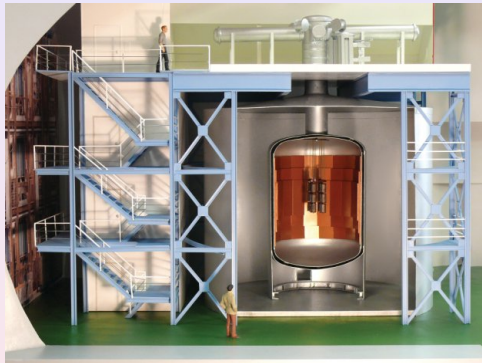
It is necessary to locate the nuclei that are the best candidates to detect the $0\nu\beta\beta$ -decay

- The main factors to be taken into account are:
 - the Q -value of the reaction;
 - the phase-space factor $G^{0\nu}$;
 - the isotopic abundance

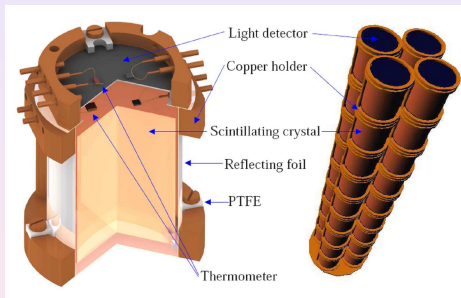
The detection of the $0\nu\beta\beta$ -decay



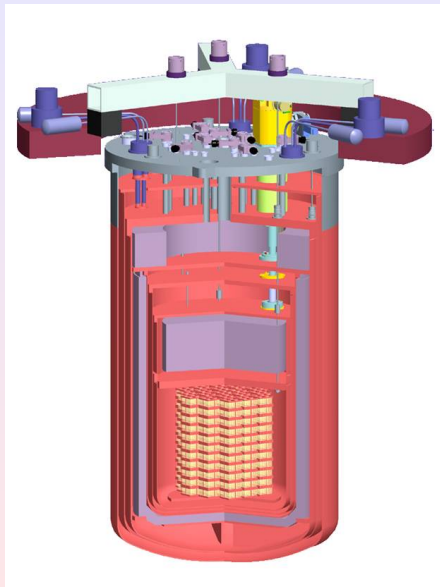
- First group: ^{76}Ge , ^{130}Te , and ^{136}Xe .
- Second group: ^{82}Se , ^{100}Mo , and ^{116}Cd .
- Third group: ^{48}Ca , ^{96}Zr , and ^{150}Nd .



- High purity enriched Ge crystal diodes (HPGe) as a beta decay source and particle detector.
- The detector array is suspended in a liquid argon cryostat lined with copper and surrounded by an ultra-pure water tank.
- The collaboration predicts less than one event each year per kilogram of material. A narrow spike around the $0\nu\beta\beta$ Q-value= 2039 keV is expected.



- Scintillating bolometers of ZnSe are the baseline choice of the LUCIFER experiment, whose aim is to observe the neutrinoless double beta decay of ^{82}Se .
- The bolometer is a 431 g ZnSe crystal with cylindrical shape (height 44.3 mm and diameter 48.5 mm).
- This experiment aims at a background lower than 10^{-3} counts/keV/kg/y in the energy region of the $0\nu\beta\beta$ -decay of ^{82}Se .



- TeO₂ crystals used as low heat capacity bolometers, arranged into towers and cooled in a large cryostat to approximately 10 m°K with a dilution refrigerator.
- The detectors are isolated from backgrounds by ultrapure low-radioactivity shielding.
- Temperature spikes from electrons emitted in Te $0\beta\beta$ are collected for spectrum analysis.

The calculation of the NME

The **NME** is given by

$$M^{0\nu} = M_{GT}^{0\nu} - \left(\frac{g_V}{g_A} \right)^2 M_F^{0\nu} - M_T^{0\nu} ,$$

where the matrix elements are defined as follows:

$$M_{\alpha}^{0\nu} = \sum_{m,n} \langle 0_f^+ | \tau_m^- \tau_n^- O_{mn}^{\alpha} | 0_i^+ \rangle ,$$

with $\alpha = (GT, F, T)$.

Since the transition operator is a two-body one, we may write it as:

$$M_{\alpha}^{0\nu} = \sum_{j_p j_{p'} j_n j_{n'} J_{\pi}} TBTD(j_p j_{p'}, j_n j_{n'}; J_{\pi}) \langle j_p j_{p'}; J^{\pi} T | \tau_1^- \tau_2^- O_{12}^{\alpha} | j_n j_{n'}; J^{\pi} T \rangle_a$$

The calculation of the NME

$$M_{\alpha}^{0\nu} = \sum_{j_p j_{p'} j_n j_{n'} J_{\pi}} TBTD(j_p j_{p'}, j_n j_{n'}; J_{\pi}) \langle j_p j_{p'}; J^{\pi} T | \tau_1^{-} \tau_2^{-} O_{12}^{\alpha} | j_n j_{n'}; J^{\pi} T \rangle$$

where the **two-body transition-density** matrix elements are defined as

$$TBTD(j_p j_{p'}, j_n j_{n'}; J_{\pi}) = \langle 0_f^{+} | (a_{j_p}^{\dagger} a_{j_{p'}}^{\dagger})^{J_{\pi}} (a_{j_n'} a_{j_n})^{J_{\pi}} | 0_i^{+} \rangle$$

and the Gamow-Teller (**GT**), Fermi (**F**), and tensor (**T**) operators as

$$\begin{aligned} O_{12}^{GT} &= \vec{\sigma}_1 \cdot \vec{\sigma}_2 H_{GT}(r) , \\ O_{12}^F &= H_F(r) , \\ O_{12}^T &= [3(\vec{\sigma}_1 \cdot \hat{r})(\vec{\sigma}_1 \cdot \hat{r}) - \vec{\sigma}_1 \cdot \vec{\sigma}_2] H_T(r) . \end{aligned}$$

The calculation of the NME

To describe the nuclear properties detected in the experiments, one needs to resort to nuclear structure models.

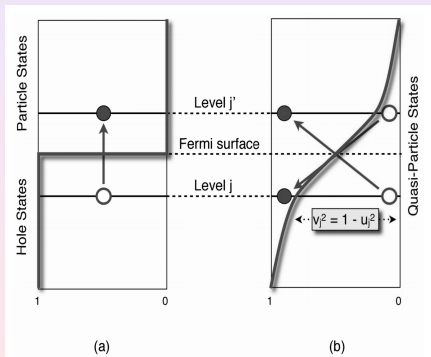
- Every model is characterized by a certain number of parameters.
- The calculated value of the **NME** may depend upon the chosen nuclear structure model.

All models may present advantages and/or shortcomings to calculate the **NME**

The QRPA

The quasiparticle random-phase approximation is based on the concept of “pairing” among the nucleons.

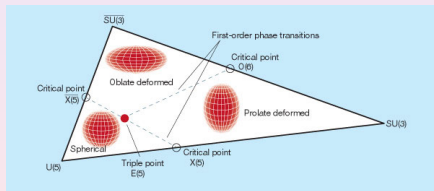
Particles are substituted with “quasiparticles”.



- **Advantage** → The dimension of the hamiltonian does not scale rapidly with the mass number A as with the shell model.
- **Shortcoming** → Results are strongly dependent on the choice of the free renormalization-parameter g_{pp} (g_{ph} is determined from experiment)

In the interacting boson model identical nucleons are paired so to generate bosons:

- $L = 0 \rightarrow s$ -boson
- $L = 2 \rightarrow d$ -boson

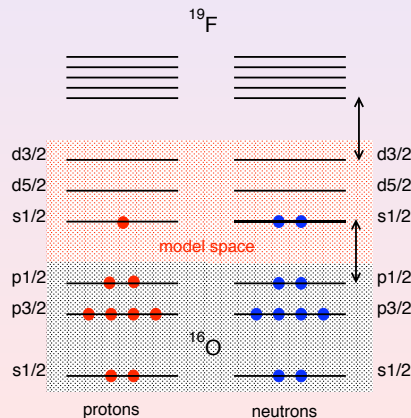


- **Advantage** $\rightarrow$ The computational complexity is drastically simplified
- **Shortcoming** $\rightarrow$ The configuration space is strongly reduced

The nuclear shell model

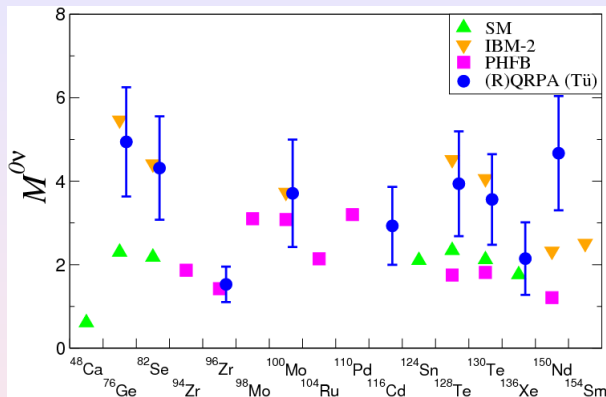
The nucleons are subjected to the action of a mean field, that takes into account most of the interaction of the nucleus constituents.

Only valence nucleons interact by way of a residual two-body potential, within a reduced model space.



- **Advantage** → It is a microscopic model, the degrees of freedom of the valence nucleons are explicitly taken into account.
- **Shortcoming** → High-degree computational complexity.

Nuclear structure calculations

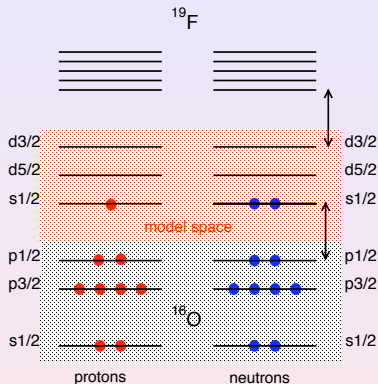


- The spread of nuclear structure calculations evidences inconsistencies among results obtained with different models

The realistic shell model

- The derivation of the **shell-model hamiltonian** using the many-body theory may provide a reliable approach
- The model space may be “shaped” according to the computational needs of the diagonalization of the **shell-model hamiltonian**
- In such a case, the effects of the **neglected degrees of freedom** are taken into account by the effective hamiltonian H_{eff} theoretically

An example: ^{19}F



- 9 protons & 10 neutrons interacting
- spherically symmetric mean field (e.g. harmonic oscillator)
- 1 valence proton & 2 valence neutrons interacting in a truncated model space

The degrees of freedom of the core nucleons and the excitations of the valence ones above the model space are not considered explicitly.

Effective shell-model hamiltonian

The shell-model hamiltonian has to take into account in an effective way all the degrees of freedom not explicitly considered

Two alternative approaches

- phenomenological
- microscopic

$$V_{NN} (+ V_{NNN}) \Rightarrow \text{many-body theory} \Rightarrow H_{\text{eff}}$$

Definition

The eigenvalues of H_{eff} belong to the set of eigenvalues of the full nuclear hamiltonian

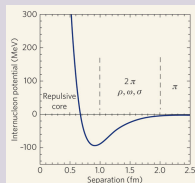
Workflow for a realistic shell-model calculation

- 1 Choose a realistic NN potential (NNN)
- 2 Determine the model space better tailored to study the system under investigation
- 3 Derive the effective shell-model hamiltonian by way of the many-body theory
- 4 Calculate the physical observables (energies, e.m. transition probabilities, ...)

Realistic nucleon-nucleon potential: V_{NN}

Several realistic potentials $\chi^2/datum \simeq 1$:
CD-Bonn, Argonne V18, Nijmegen, ...

Strong short-range repulsion



How to handle the short-range repulsion ?

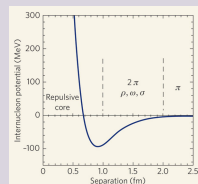
- Brueckner G matrix
- EFT inspired approaches

$$V_{NN} \rightarrow G$$

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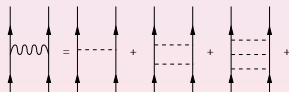
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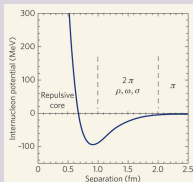
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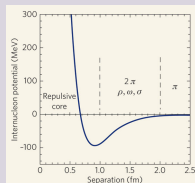
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 - $V_{\text{low-}k}$, SRG
 - chiral potentials

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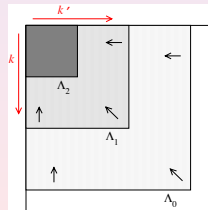
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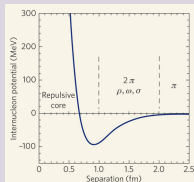
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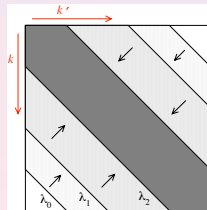
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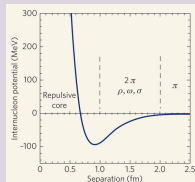
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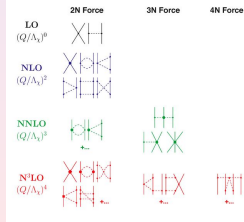
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Strong short-range repulsion



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The shell-model effective hamiltonian

A-nucleon system Schrödinger equation

$$H|\Psi_\nu\rangle = E_\nu|\Psi_\nu\rangle$$

with

$$H = H_0 + H_1 = \sum_{i=1}^A (T_i + U_i) + \sum_{i<j} (V_{ij}^{NN} - U_{ij})$$

Model space

$$|\Phi_i\rangle = [a_1^\dagger a_2^\dagger \dots a_n^\dagger]_i |c\rangle \Rightarrow P = \sum_{i=1}^d |\Phi_i\rangle \langle \Phi_i|$$

Model-space eigenvalue problem

$$H_{\text{eff}} P |\Psi_\alpha\rangle = E_\alpha P |\Psi_\alpha\rangle$$

The shell-model effective hamiltonian

$$\left(\begin{array}{c|c} PHP & PHQ \\ \hline QHP & QHQ \end{array} \right) \xRightarrow{QH P = 0} \mathcal{H} = X^{-1} H X \left(\begin{array}{c|c} P\mathcal{H}P & P\mathcal{H}Q \\ \hline 0 & Q\mathcal{H}Q \end{array} \right)$$

$$H_{\text{eff}} = P\mathcal{H}P$$

$$\text{Suzuki \& Lee} \Rightarrow X = e^{\omega} \text{ with } \omega = \left(\begin{array}{c|c} 0 & 0 \\ \hline Q\omega P & 0 \end{array} \right)$$

$$H_1^{\text{eff}}(\omega) = PH_1P + PH_1Q \frac{1}{\epsilon - QHQ} QH_1P - \\ - PH_1Q \frac{1}{\epsilon - QHQ} \omega H_1^{\text{eff}}(\omega)$$

The shell-model effective hamiltonian

Folded-diagram expansion

$\hat{Q}$ -box vertex function

$$\hat{Q}(\epsilon) = PH_1P + PH_1Q \frac{1}{\epsilon - QHQ} QH_1P$$

$\Rightarrow$ Recursive equation for $H_{\text{eff}} \Rightarrow$ iterative techniques
(Krenciglowa-Kuo, Lee-Suzuki, ...)

$$H_{\text{eff}} = \hat{Q} - \hat{Q}' \int \hat{Q} + \hat{Q}' \int \hat{Q} \int \hat{Q} - \hat{Q}' \int \hat{Q} \int \hat{Q} \int \hat{Q} \dots,$$

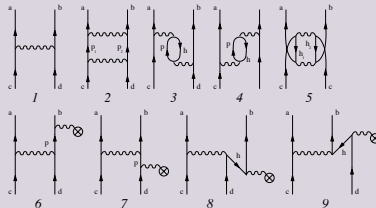
The perturbative approach to the shell-model H^{eff}

$$\hat{Q}(\epsilon) = PH_1P + PH_1Q \frac{1}{\epsilon - QH_0Q} QH_1P$$

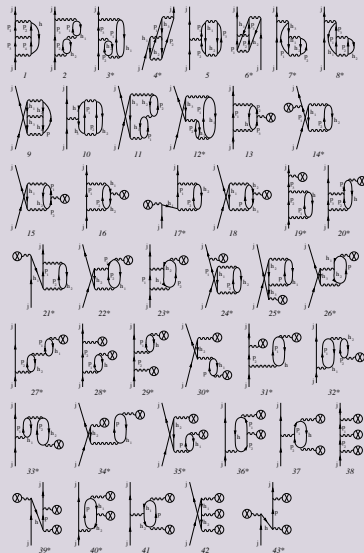
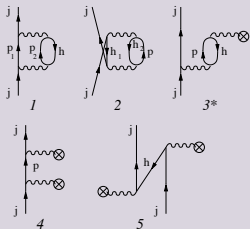
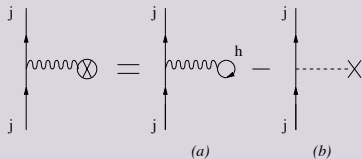
The $\hat{Q}$ -box can be calculated perturbatively

$$\frac{1}{\epsilon - QH_0Q} = \sum_{n=0}^{\infty} \frac{(QH_1Q)^n}{(\epsilon - QH_0Q)^{n+1}}$$

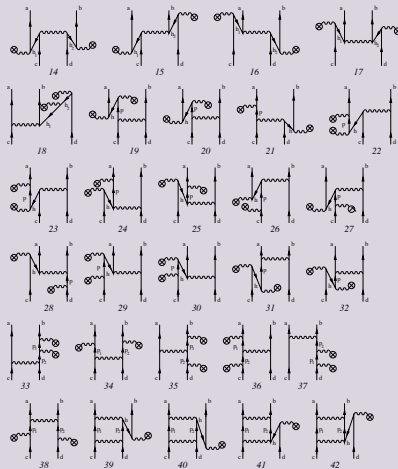
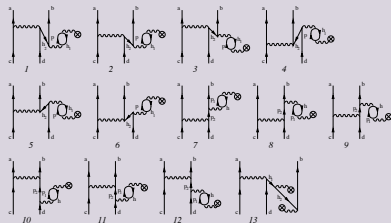
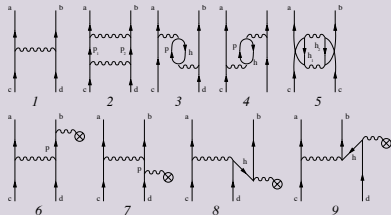
The diagrammatic expansion of the $\hat{Q}$ -box



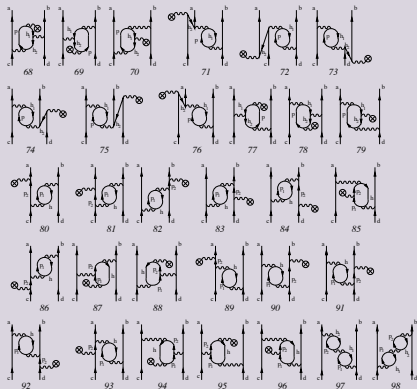
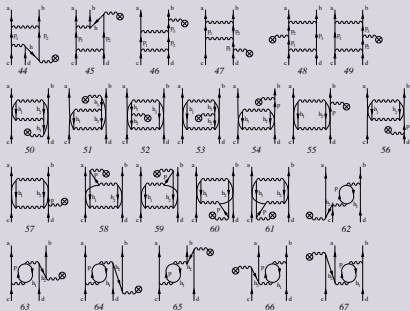
$\hat{Q}$ -box perturbative expansion: 1-body diagrams



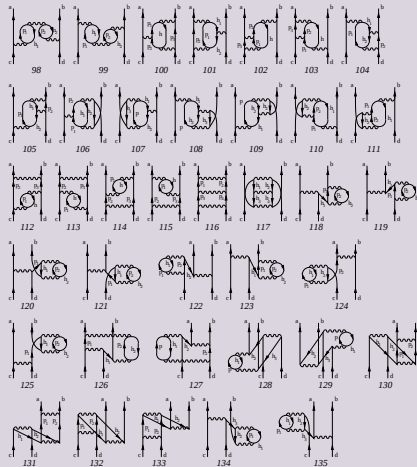
$\hat{Q}$ -box perturbative expansion: 2-body diagrams



$\hat{Q}$ -box perturbative expansion: 2-body diagrams



$\hat{Q}$ -box perturbative expansion: 2-body diagrams



The shell-model effective operators

Consistently, any shell-model effective operator may be calculated

It has been demonstrated that, for any bare operator Θ , a non-Hermitian effective operator Θ_{eff} can be written in the following form:

$$\Theta_{\text{eff}} = (P + \hat{Q}_1 + \hat{Q}_1 \hat{Q}_1 + \hat{Q}_2 \hat{Q}_1 + \hat{Q}_1 \hat{Q}_2 + \cdots)(\chi_0 + \chi_1 + \chi_2 + \cdots),$$

where

$$\hat{Q}_m = \frac{1}{m!} \left. \frac{d^m \hat{Q}(\epsilon)}{d\epsilon^m} \right|_{\epsilon=\epsilon_0},$$

ϵ_0 being the model-space eigenvalue of the unperturbed hamiltonian H_0

The shell-model effective operators

The χ_n operators are defined as follows:

$$\begin{aligned}\chi_0 &= (\hat{\Theta}_0 + h.c.) + \Theta_{00} , \\ \chi_1 &= (\hat{\Theta}_1 \hat{Q} + h.c.) + (\hat{\Theta}_{01} \hat{Q} + h.c.) , \\ \chi_2 &= (\hat{\Theta}_1 \hat{Q}_1 \hat{Q} + h.c.) + (\hat{\Theta}_2 \hat{Q} \hat{Q} + h.c.) + \\ &\quad (\hat{\Theta}_{02} \hat{Q} \hat{Q} + h.c.) + \hat{Q} \hat{\Theta}_{11} \hat{Q} , \\ &\dots\end{aligned}$$

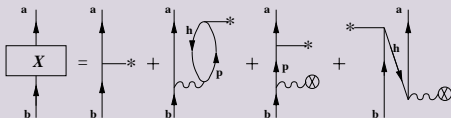
and

$$\begin{aligned}\hat{\Theta}(\epsilon) &= P\Theta P + P\Theta Q \frac{1}{\epsilon - QHQ} QH_1 P , \\ \hat{\Theta}(\epsilon_1; \epsilon_2) &= P\Theta P + PH_1 Q \frac{1}{\epsilon_1 - QHQ} \times \\ &\quad Q\Theta Q \frac{1}{\epsilon_2 - QHQ} QH_1 P , \\ \hat{\Theta}_m &= \frac{1}{m!} \left. \frac{d^m \hat{\Theta}(\epsilon)}{d\epsilon^m} \right|_{\epsilon=\epsilon_0} , \quad \hat{\Theta}_{nm} = \frac{1}{n!m!} \left. \frac{d^n}{d\epsilon_1^n} \frac{d^m}{d\epsilon_2^m} \hat{\Theta}(\epsilon_1; \epsilon_2) \right|_{\epsilon_1=\epsilon_0, \epsilon_2=\epsilon_0}\end{aligned}$$

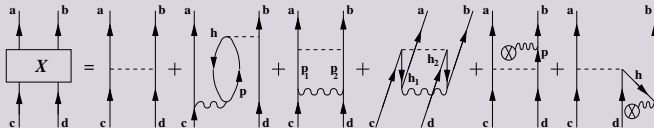
The shell-model effective operators

We arrest the χ series at χ_0 , and expand it perturbatively:

One-body operator

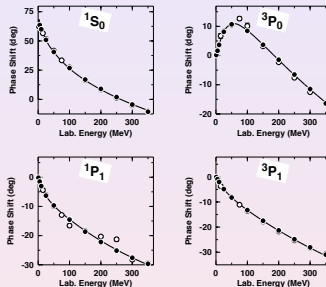


Two-body operator



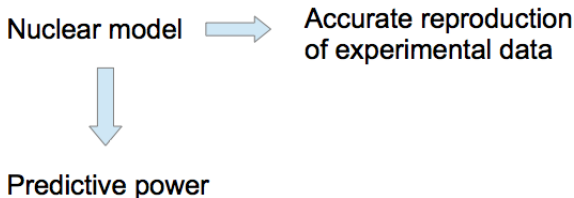
Our recipe for realistic shell model

- Input V_{NN} : $V_{\text{low}-k}$ derived from the high-precision NN CD-Bonn potential with a cutoff: $\Lambda = 2.6 \text{ fm}^{-1}$.



- H_{eff} obtained calculating the Q -box up to the 3rd order in perturbation theory.
- Effective operators are consistently derived by way of the the MBPT

Nuclear models and predictive power



Realistic shell-model calculations for ^{130}Te and ^{136}Xe



Check this approach calculating observables related to the GT strengths and $2\nu\beta\beta$ decay and compare the results with data.

$$\left[T_{1/2}^{2\nu}\right]^{-1} = G^{2\nu} |M_{2\nu}^{\text{GT}}|^2$$

Shell-model calculations for ^{130}Te , ^{136}Xe

PRL 115, 102502 (2015)

PHYSICAL REVIEW LETTERS

week ending
4 SEPTEMBER 2015

Search for Neutrinoless Double-Beta Decay of ^{130}Te with CUORE-0

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(CUORE Collaboration)

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20 JULY 2012

Search for Neutrinoless Double-Beta Decay in ^{136}Xe with EXO-200

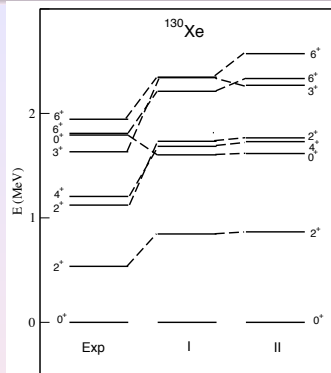
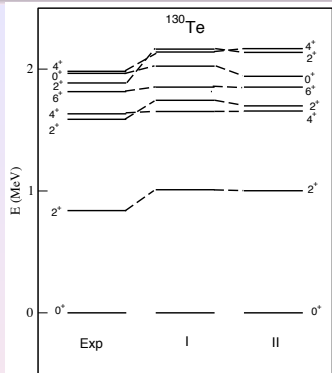
M. Auger,¹ D. J. Auty,² P. S. Barbeau,^{3,4} E. Beauchamp,⁴ V. Belov,⁵ C. Benitez-Medina,⁶ M. Breidenbach,⁷ T. Brunner,³ A. Burenkov,⁵ B. Cleveland,^{4,7} S. Cook,⁸ T. Daniels,⁸ M. Danilov,⁵ C. G. Davis,⁹ S. Delaquis,¹ R. deVoe,¹ A. Dobi,⁹ M. J. Dolinski,¹ A. Dolgolenko,⁵ M. Dunford,¹⁰ W. Fairbank, Jr.,⁴ J. Farine,⁴ W. Feldmeier,¹¹ P. Fierlinger,¹¹ D. Franco,¹ G. Giroux,¹ R. Gornea,¹ K. Graham,¹⁰ G. Gratta,¹ C. Hall,¹ K. Hall,¹ C. Hargrove,¹⁰ S. Herin,¹ M. Hughes,² A. Johnson,⁷ T. N. Johnson,¹² A. Karelin,¹ L. J. Kaufman,¹² A. Kuchenchikov,⁵ K. S. Kumar,¹³ D. S. Leonard,¹³ F. Leonard,¹³ D. Mackay,^{7,14} R. MacLellan,¹ M. Marino,¹⁵ B. Mong,¹ M. Montero Diez,¹ A. R. Müller,¹⁵ R. Neilson,¹⁴ R. Nelson,¹⁴ A. Odian,¹ I. Ostrovskiy,¹ K. O'Sullivan,¹ C. Ouellet,¹⁰ A. Piepkor,¹ A. Pocar,¹ C. Y. Prescott,¹ K. Pushkin,² P. C. Rowson,⁷ J. J. Russell,¹ A. Sabourov,¹ D. Sinclair,^{10,11} S. Slusky,¹ V. Sokhanov,⁵ T. Tait,¹ D. Tosi,¹ K. Twieker,¹ P. Vogel,¹⁵ J.-L. Vuilleumier,¹ A. Waite,¹ T. Walton,⁶ M. Weber,¹ U. Wichowski,¹ J. Wodin,¹ J. D. Wright,¹ L. Yang,¹⁶ Y.-R. Yen,¹⁰ and O. Ya. Zeldovich⁵

(EXO Collaboration)

- five proton and neutron orbitals outside double-closed ^{100}Sn
 $0g_{7/2}, 1d_{5/2}, 1d_{3/2}, 2s_{1/2}, 0h_{11/2}$
- 1292 two-body matrix elements and 10 SP energies: (I) theoretical SP energies, (II) empirical SP energies fitted to the observed low-lying states in ^{133}Sb and ^{131}Sn

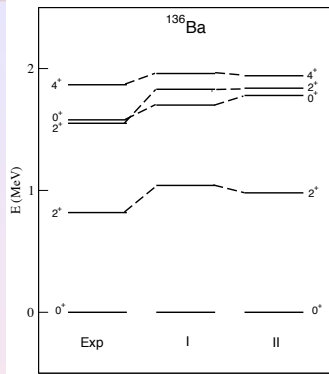
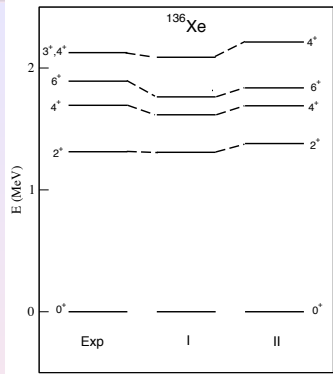
	Proton SP spacings		Neutron SP spacings	
	I	II	I	II
$0g_{7/2}$	0.0	0.0	0.0	0.0
$1d_{5/2}$	0.3	0.4	0.6	0.7
$1d_{3/2}$	1.2	1.4	1.5	2.1
$2s_{1/2}$	1.1	1.3	1.2	1.9
$0h_{11/2}$	1.9	1.6	2.7	3.0

Spectroscopy of ^{130}Te and ^{130}Xe



Nucleus	$J_i \rightarrow J_f$	$B(E2)_{\text{Expt}}$	I	II
^{130}Te	$2^+ \rightarrow 0^+$	580 ± 20	430	420
	$6^+ \rightarrow 4^+$	240 ± 10	220	200
^{130}Xe	$2^+ \rightarrow 0^+$	1170^{+20}_{-10}	954	876

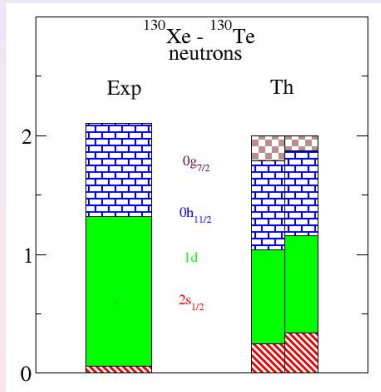
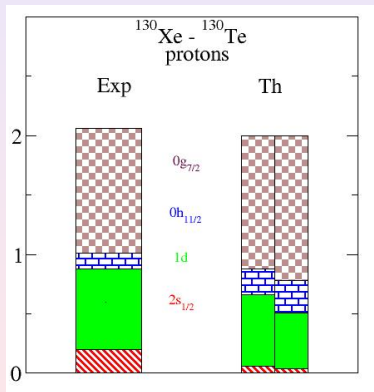
Spectroscopy of ^{136}Xe and ^{136}Ba



Nucleus	$J_i \rightarrow J_f$	$B(E2)_{\text{Expt}}$	I	II
^{136}Xe	$2^+ \rightarrow 0^+$	420 ± 20	300	300
	$4^+ \rightarrow 2^+$	53 ± 1	9	11
	$6^+ \rightarrow 4^+$	0.55 ± 0.02	1.58	2.42
^{136}Ba	$2^+ \rightarrow 0^+$	800^{+80}_{-40}	590	520

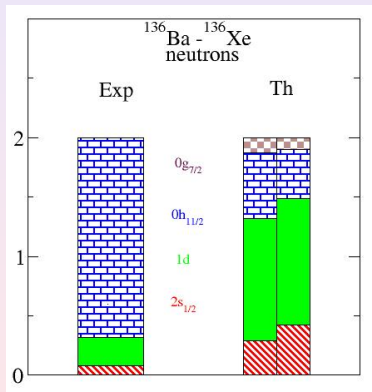
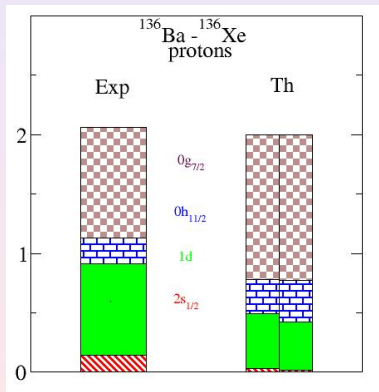
Proton/neutron occupancies/vacancies for $^{130}\text{Te} \rightarrow ^{130}\text{Xe}$

Data from the cross sections of the $(d, ^3\text{He})$ and $(\alpha, ^3\text{He})$

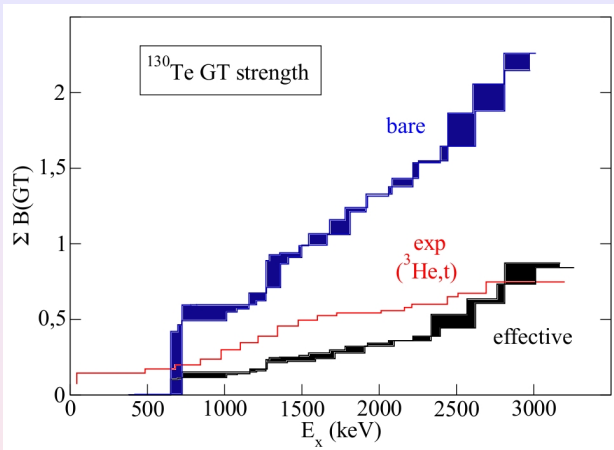


Proton/neutron occupancies/vacancies for $^{136}\text{Xe} \rightarrow ^{136}\text{Ba}$

Data from the cross sections of the $(d, ^3\text{He})$ and $(\alpha, ^3\text{He})$

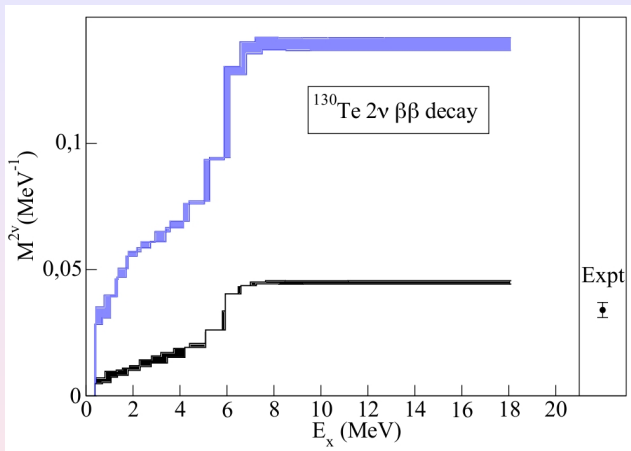


$^{130}\text{Te} \rightarrow ^{130}\text{Xe}$ GT- running sums



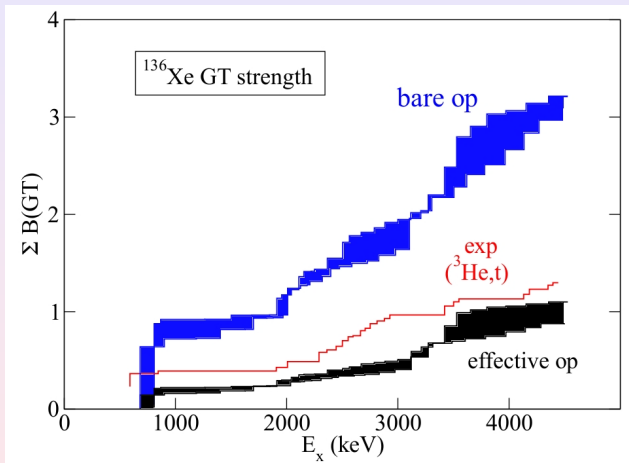
$$B(\text{GT}) = \frac{|\langle \Phi_f | \sum_j \vec{\sigma}_j \vec{\tau}_j | \Phi_i \rangle|^2}{2J_i + 1}$$

$^{130}\text{Te} \rightarrow ^{130}\text{Xe}$ nuclear matrix element

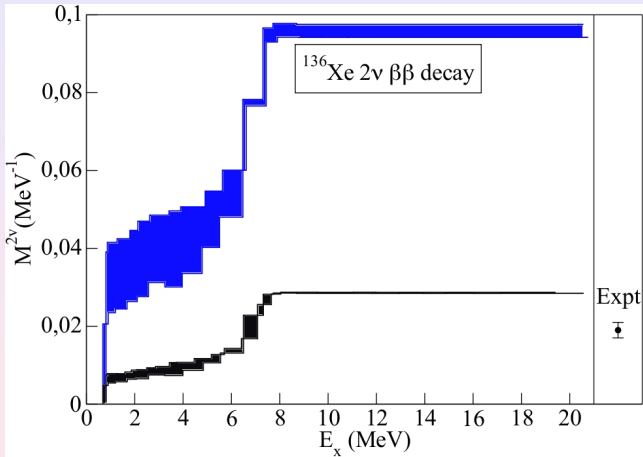


$$M_{2\nu}^{\text{GT}} = \sum_n \frac{\langle 0_f^+ || \vec{\sigma} \tau^- || 1_n^+ \rangle \langle 1_n^+ || \vec{\sigma} \tau^- || 0_i^+ \rangle}{E_n + E_0}$$

$^{136}\text{Xe} \rightarrow ^{136}\text{Ba}$ GT- running sums



$^{136}\text{Xe} \rightarrow ^{136}\text{Ba}$ nuclear matrix element



Conclusions and outlook

- The agreement of our results with the experimental data of GT strengths and $2\nu\beta\beta$ NME testifies the reliability of a microscopic shell-model calculation.
- Calculations (I) (all theoretical SM parameters) represents a fully microscopic SM calculation
- We have good prospects of the predictive power of realistic shell model to calculate $0\nu\beta\beta$
- Role of real three-body forces and three-body correlations (blocking effect) should be also investigated.