DoublePDFs and parton correlations
+
focus on same sign WW production

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In collaboration with :

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Outlook

- Introduction:
  - The 3D proton structure in single & double parton scatterings (DPS)
  - Double parton scattering and double parton distribution functions (dPDFs)
  - Double parton correlations (DPCs) in double parton distribution functions
- dPDFs in constituent quark models, a proton "imaging" via DPS?

  M.R., S. Scopetta and V. Vento, PRD 87, 114021 (2013)
  M. R., S. Scopetta, M. Traini and V.Vento, JHEP 12, 028 (2014)
- Calculation of the "effective X-section"

  M. R., S. Scopetta, M. Traini and V.Vento, PLB 752, 40 (2016)
  M. Traini, S. Scopetta, M. R. and V. Vento, PLB 768, 270 (2017)
- Analysis of correlations in dPDFs

  M. R., S. Scopetta, M. Traini and V.Vento, JHEP 10, 063 (2016)
  M. R., F. A. Ceccopieri, PRD 95, no. 3, 034040 (2017)
- Analysis of same-sign W pair production via double parton scattering

  F. A. Ceccopieri, M. R., S. Scopetta, arXiv:1702.05363
- Conclusions

How 3-Dimensional structure of a hadron can be investigated?

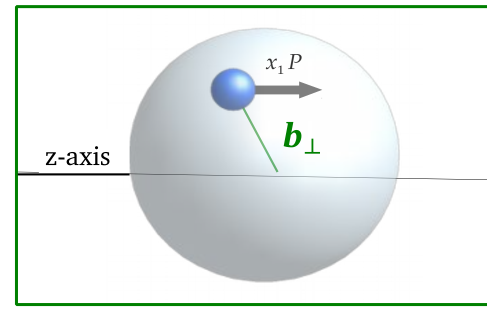
The 3D structure of a strongly interacting system (e.g. nucleon, nucleus..) could be accessed through different processes (e.g. SIDIS, DVCS, double parton scattering …), measuring different kind of parton distributions, providing different kind of information:

DVCS — ***Generalized Parton Distributions in impact parameter space***

$$\mathcal{H}(x_1, \boldsymbol{b}_\perp) \quad \mathcal{E}(x_1, \boldsymbol{b}_\perp) \ldots$$

longitudinal momentum fraction carried by the parton

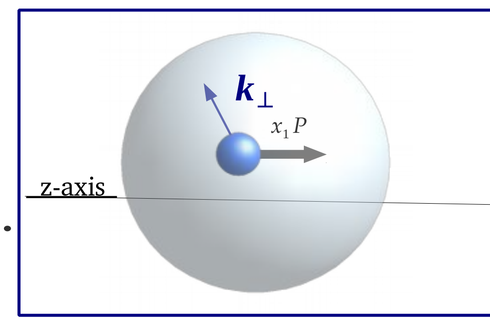
transverse distance between the parton and center of proton



SIDIS.. — ***Transverse Momentum Dependent parton distribution functions***

$$f_1(x_1, \boldsymbol{k}_\perp) \quad g_{1L}(x_1, \boldsymbol{k}_\perp) \; h_1(x_1, \boldsymbol{k}_\perp) \; f_{1T}^{\perp}(x_1, \boldsymbol{k}_\perp) \ldots$$

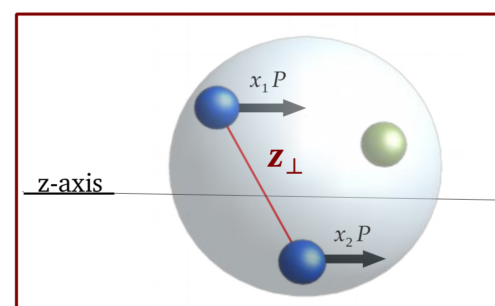
transverse component of the parton momentum



1 BODY

DPS — ***Double Parton Distribution Functions***

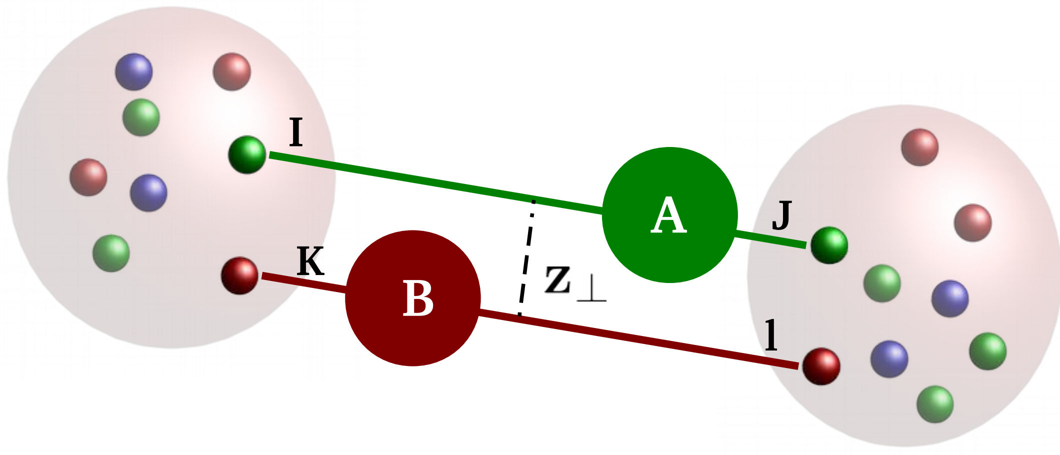
$$F_{UU}(x_1, x_2, \mathbf{z}_\perp) \quad F_{LL}(x_1, x_2, \mathbf{z}_\perp) \ldots$$



2 BODY

DPS and dPDFs from multi parton interactions

Multi parton interaction (MPI) can contribute to the, *pp* and *pA,* cross section @ the LHC:



The cross section for a DPS event can be written in the following way:
(N. Paver, D. Treleani, Nuovo Cimento 70A, 215 (1982))

dPDF

Momentum scale

$$d\sigma = \frac{1}{S}\sum_{i,j,k,l}\hat{\sigma}_{ij}(x_1,x_3,\mu_A)\hat{\sigma}_{kl}(x_2,x_4,\mu_B)\int d\tilde{z}_\perp F_{ik}(x_1,x_2,z_\perp,\mu_A,\mu_B)F_{jl}(x_3,x_4,\boxed{z_\perp},\mu_A,\mu_B)$$

Momentum fraction carried by the parton inside the hadron

Transverse distance between the two partons

DPS processes are important for fundamental studies, *e.g.* the background for the research of new physics and to grasp information on the 3D PARTONIC STRUCTURE OF THE PROTON

Parton correlations and dPDFs

@ LHC kinematics it is often used a factorized form of the dPDFs: $(\mathbf{x_1}, \mathbf{x_2}) - \mathbf{z}_\perp$ factorization:

$$F_{ij}(x_1, x_2, \vec{z}_\perp, \mu) = F_{ij}(x_1, x_2, \mu) T(\vec{z}_\perp, \mu)$$

* Here and in the following: $\mu = \mu_A = \mu_B$

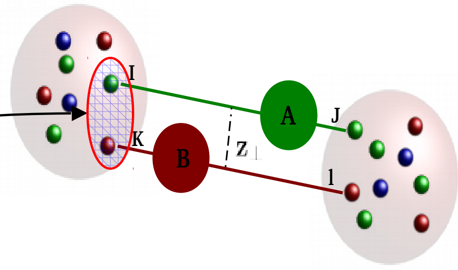
and $\mathbf{x_1}, \mathbf{x_2}$ factorization:

$$\overbrace{F_{ij}(x_1, x_2, \mu)}^{\text{dPDF (2-Body)}} = \underbrace{\overbrace{q_i(x_1, \mu)}^{\text{PDF (1-Body)}} \boxed{q_j(x_2, \mu)}} \theta(1 - x_1 - x_2)(1 - x_1 - x_2)^n$$

Unknown

Data available

NO CORRELATION ANSATZ



In this scenario, parton correlations inside the proton are neglected.
NO NEW INFORMATION!

- In principle, correlations are present!
- dPDFs are non-perturbative quantities ⟺ DPCs not calculated directly from QCD

HOW CAN WE BE SURE OF THE ACCURACY OF SUCH APPROXIMATION?

WHAT CAN WE LEARN ABOUT dPDFs AND THE PROTON STRUCTURE?

DPCs in constituent quark models (CQM)

- Main features:
  - potential model
  - effective particles
  - particles are strongly bound and correlated
- CQM are a proper framework to describe DPCs, but their predictions are reliable ONLY in the valence quark region at low energy scale, while LHC data are available at small x
- At very low x, due to the large population of partons, the role of correlations may be less relevant BUT theoretical microscopic estimates are necessary

pQCD evolution of the calculated dPDFs is necessary to move towards the experimental kinematics:

i) dPDF evaluated at the initial scale of the model → pQCD → *ii)* dPDF evaluated at high generic scale

CQM calculations are able to reproduce the gross-feature of experimental PDFs in the valence region. CQM calculations are useful tools for the interpretation of data and for the planning of measurements of unknown quantities (e.g., TMDs in SiDIS, GPDs in DVCS...)

Similar expectations motivate the present investigation of dPDFs

The Light-Front approach

Relativity can be implemented, for a CQM, by using a Light-Front (LF) approach yielding, among other good features, the correct support. In the Relativistic Hamiltonian Dynamics (RHD) of an interacting system, introduced by Dirac (1949), one has:

$$\boxed{a^{\pm} = a_0 \pm a_3}$$

RHD

- Instant Form: $t_0 = 0$
  Evolution Operator: $P^0 = E$
- **Front Form (LF):**
  $x^+ = t_0 + z = 0$
  Evolution Operator: P^-

- Full Poincaré covariance
- fixed number of on-mass-shell particles

Among the 3 possibles forms of RHD we have chosen the LF one since there are several advantages. The most relevant are the following:

- ✓ 7 Kinematical generators (maximum number): i) three LF boosts (at variance with the dynamical nature of the Instant-form boosts), ii) $\mathbf{P}^+, \ \mathbf{P}_\perp$, iii) Rotation around z.
- ✓ The LF boosts have a subgroup structure, then one gets a trivial separation of the intrinsic motion from the global one (as in the non relativistic (NR) case).
- ✓ In a peculiar construction of the Poincaré generators (Bakamjian-Thomas) it is possible to obtain a Mass equation, Schrödinger-like. A clear connection to NR.
- ✓ The IMF (Infinite Momentum Frame) description of DIS is easily included.

The LF approach is extensively used for hadronic studies (e.m. form factors, PDFs, GPDs, TMDs........)

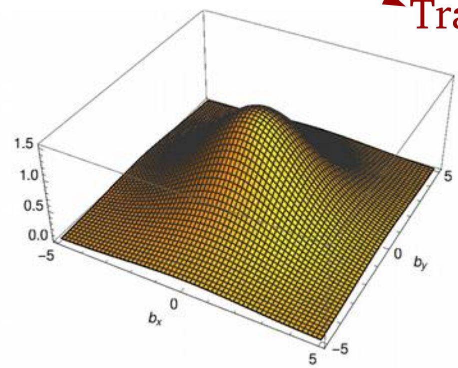
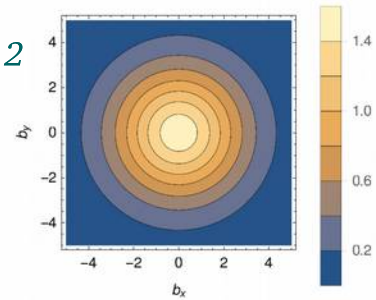
What we would like to learn: A first look at two partons inside the proton

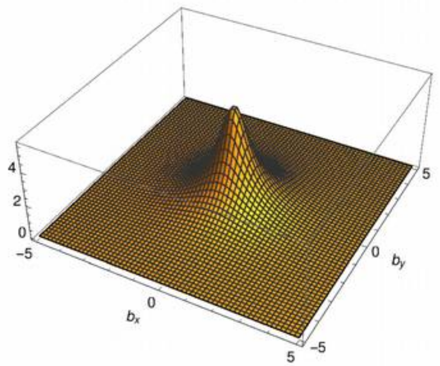
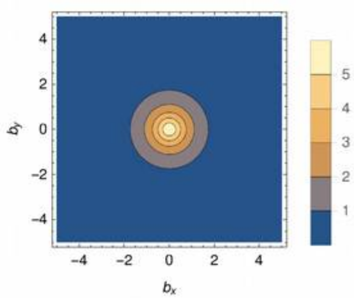
M.R., F. A. Ceccopieri, PRD 95, no. 3, 034040 (2017)

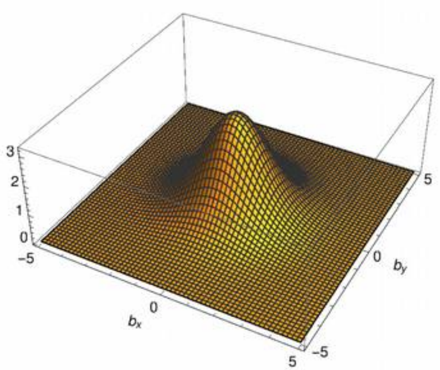
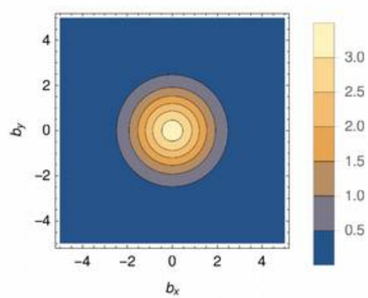
$$F_{u_v d_v}(x_1, x_2, \vec{b}_\perp, \mu_0^2) = \int d\vec{k}_\perp e^{i\vec{k}_\perp \cdot \vec{b}_\perp} F_{u_v d_v}(x_1, x_2, \vec{k}_\perp, \mu_0^2)$$

Transverse distance

$x_1 = 0.3 \quad x_2 = 0.2$

1)  

2)  

3)  

DIFFERENT MODELS

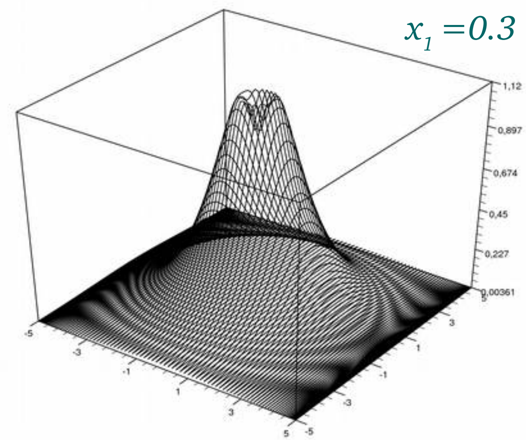
The distribution has been calculated within different CQM models:

1) M. Traini *et al*, Nucl. Phys. A 656, 400-420 (1999), non relativistic Hyper-Central CQM (potential by M. Ferraris *et al,* PLB 364 (1995))

2) M. Traini *et al*, Nucl. Phys. A 656, 400-420 (1999) (relativistic Hyper-Central CQM)

3) The harmonic oscillator

$x_1 = 0.3 \quad x_2 = 0.04$



E.g., in our model, quarks with similar longitudinal momentum fraction “prefer” to be close to each other!

Results on distributions with longitudinally and transversely polarized quarks are coming!

What we would like to learned: A link between dPDFs and GPDs?

The dPDF is formally defined through the Light-cone correlator:

$$F_{12}(x_1, x_2, \vec{z}_\perp) \propto \sum_X \int dz^- \left[\prod_{i=1}^{2} dl_i^- e^{i x_i l_i^- p^+}\right] \langle p|O(z, l_1)|X\rangle\langle X|O(0, l_2)|p\rangle \Big|_{l_1^+ = l_2^+ = z^+ = 0}^{\vec{l}_{1\perp} = \vec{l}_{2\perp} = 0}$$

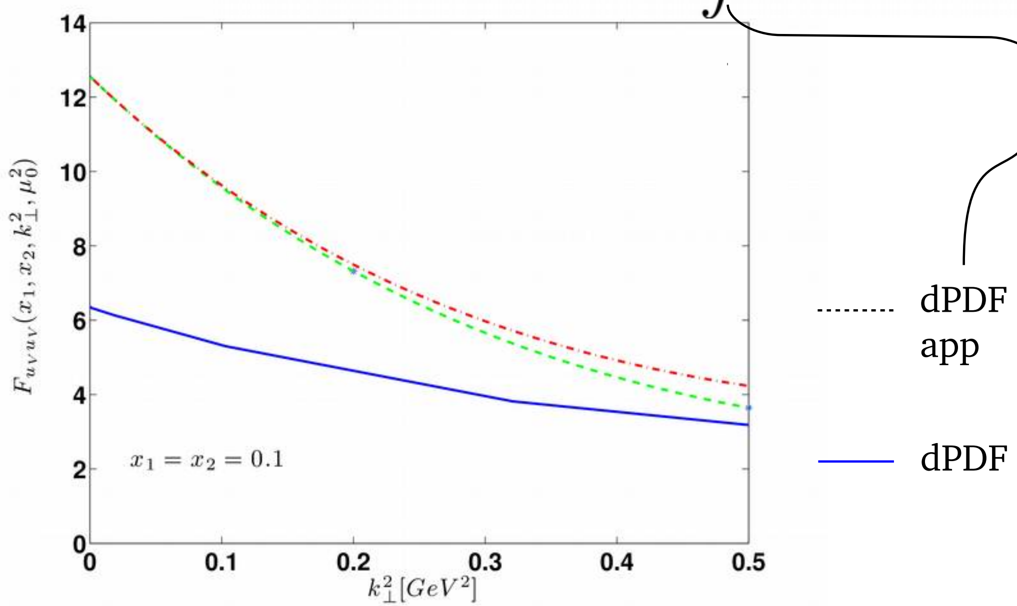
M. Diehl, D. Ostermeier, A. Schafer, JHEP 03 (2012) 089

Approximated by the proton state!

$$\int \frac{dp'^+ d\vec{p}'_\perp}{p'^+} |p'\rangle\langle p'|$$

$$F_{12}(x_1, x_2, \vec{z}_\perp) \sim \int d\vec{b} f(x_1, 0, \vec{b} + \vec{z}_\perp) \boxed{f(x_2, 0, \vec{b})}$$

GPDs depending on the impact parameter



In GPDs, the variables $\vec{b}$ and x are correlated!

Correlations between $\vec{z}_\perp$ and x_1, x_2 could be present in dPDFs !

What we learned:

M. R., S. Scopetta, M. Traini and V.Vento, JHEP 12, 028 (2014)

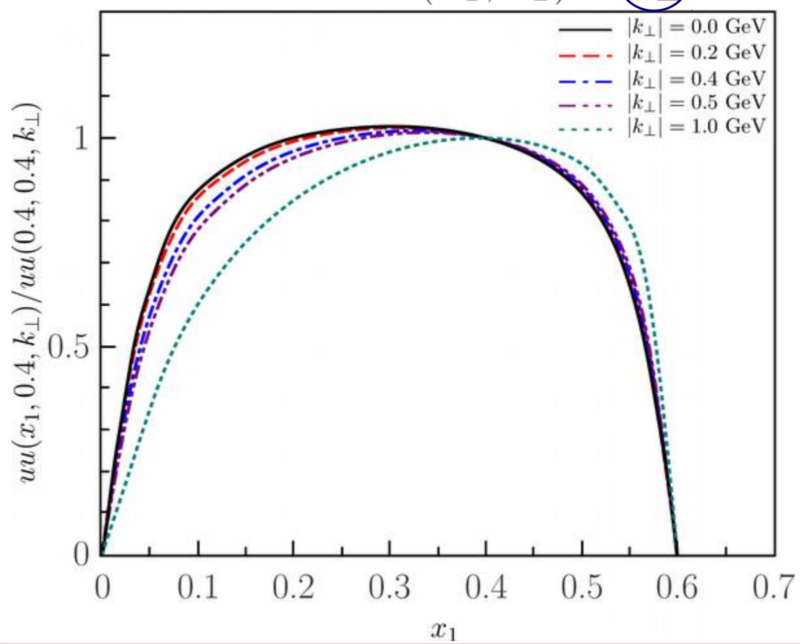
Here, ratios, sensitive to correlations, are shown in order to test the factorization ansatz!

Use has been made of relativistic Hyper-Central CQM.

$$\frac{F_{uu}(x_1, 0.4, k_\perp)}{F_{uu}(0.4, 0.4, k_\perp)}$$

Conjugate to $z_\perp$

Factorization: $(x_1, x_2) - k_\perp$

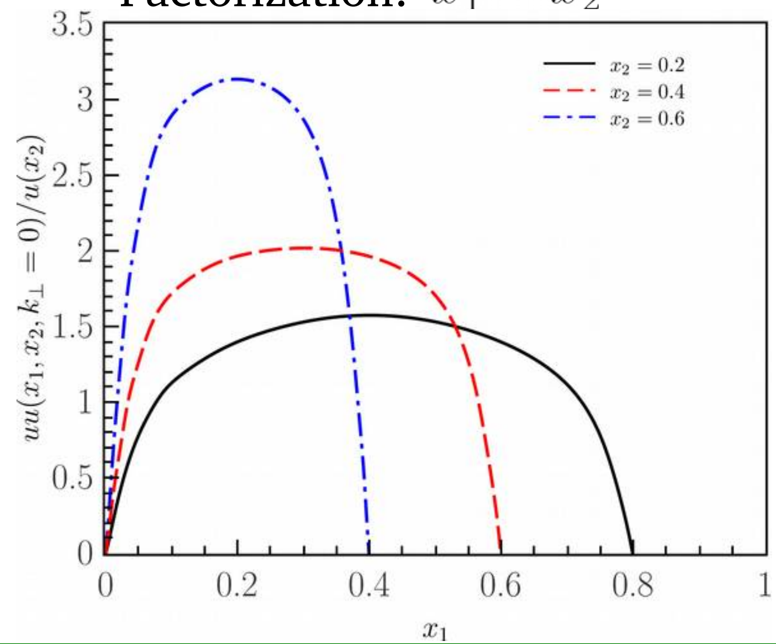


${}_2$GPD

$$\frac{F_{uu}(x_1, x_2, k_\perp = 0)}{F_u(x_2)}$$

PDF

Factorization: $x_1 - x_2$



The $(x_1, x_2) - k_\perp$ and $x_1 - x_2$ factorizations are violated!

The factorization ansatz is basically violated in all quark model analyses!

M.R., S. Scopetta and V. Vento, PRD 87, 114021 (2013)

H.-M. Chang, A.V. Manohar, and W.J. Waalewijn, PRD 87, 034009 (2013)

LF RELATIVISTIC EFFECTS I

M.R., F. A. Ceccopieri, PRD 95, no. 3, 034040 (2017)

The expressions of dPDF in the canonical (e.g. NR limit) and LF forms are quite similar for small values of x:

$$F_{[NR]}(x_1, x_2, k_\perp) = \int d\vec{k}_1 d\vec{k}_2 f(\vec{k}_1, \vec{k}_2, k_\perp) \delta\left(x_1 - \frac{k_1^+}{M_P}\right) \delta\left(x_2 - \frac{k_2^+}{M_P}\right)$$

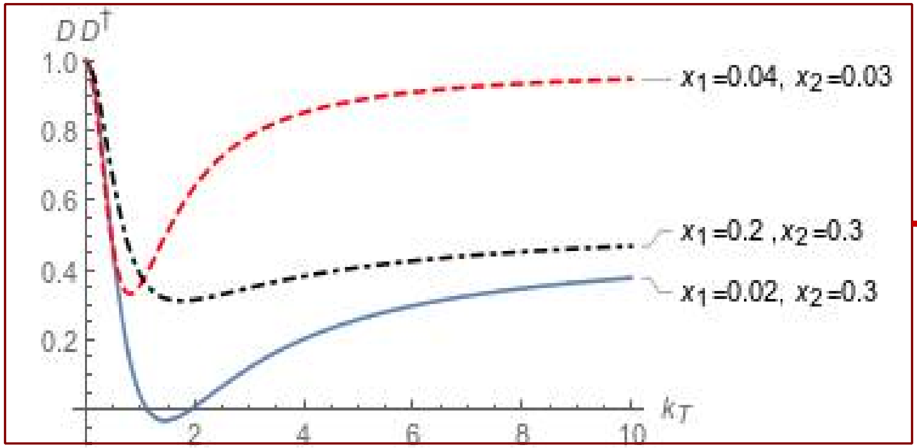
$$F_{[L]}(x_1, x_2, k_\perp) = \int d\vec{k}_1 d\vec{k}_2 f(\vec{k}_1, \vec{k}_2, k_\perp) \langle SPIN| \boxed{O_1(\vec{k}_1, \vec{k}_2, k_\perp) O_2(\vec{k}_1, \vec{k}_2, k_\perp)} |SPIN\rangle \times \delta\left(x_1 - \frac{k_1^+}{M_0}\right) \delta\left(x_2 - \frac{k_2^+}{M_0}\right)$$

Melosh Operators!

No constant quantities. They depend on momentum of partons!

$f(\vec{k}_1, \vec{k}_2, k_\perp)$ = product of the canonical proton wave-functions

For very small values of x_1 and x_2, the main difference in the two approaches, in the calculation of dPDF, is due to Melosh operators!



$$DD^\dagger = \langle SU(6)| O_1(\vec{k}_1, \vec{k}_2, k_\perp) O_2(\vec{k}_1, \vec{k}_2, k_\perp) |SU(6)\rangle$$

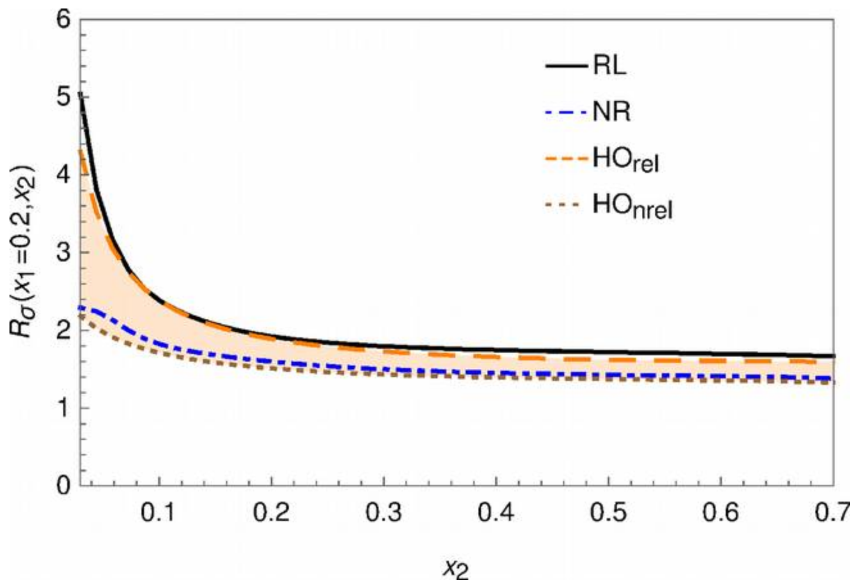
Correlations between x_i and $k_\perp$

LF RELATIVISTIC EFFECTS II

M.R., F. A. Ceccopieri, PRD 95, no. 3, 034040 (2017)

Melosh effects are studied in a quantity which simulates a ratio of DPS cross-sections:

$$R_\sigma(x_1,x_2) = \frac{\int d\vec{b}_\perp F_{[L]}(x_1,x_2,b_\perp)^2}{\int d\vec{b}_\perp F_{[NR]}(x_1,x_2,b_\perp)^2}$$



Relativistic Hyper central Model

NR Hyper central Model

Relativistic Harmonic Oscillator (HO) model $\alpha^2_{rel} = 25\ \text{fm}^{-2}$

$\alpha^2_{nrel} < \alpha^2 < \alpha^2_{rel}$

NR Harmonic Oscillator model $\alpha^2_{nrel} = 6\ \text{fm}^{-2}$

Important effect due to Melosh's operators found in this observable!!

Now the same analysis must be done at high energy scales and considering also gluons and sea quarks!

Introduction of non perturbative sea quarks

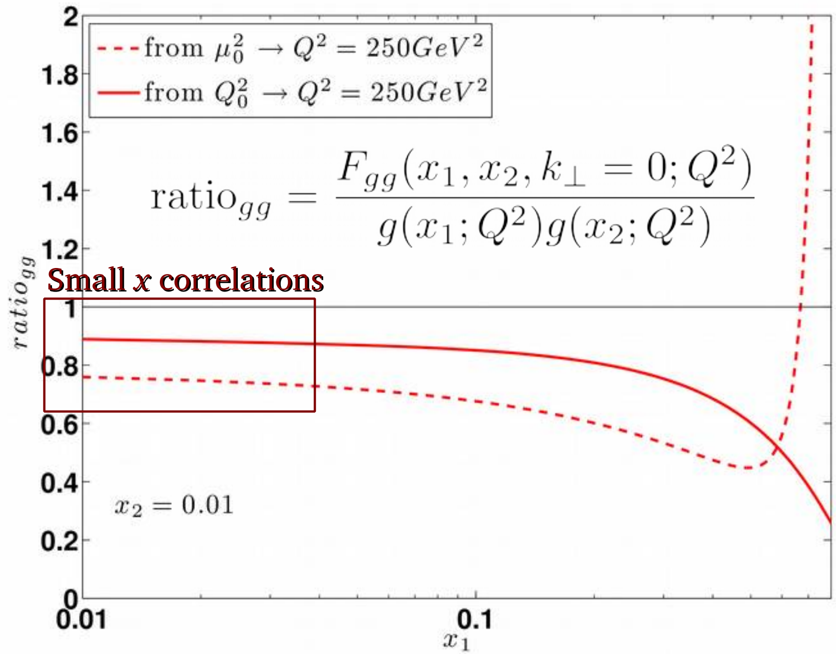
M. R., S. Scopetta, M. Traini and V.Vento, JHEP 10, 063 (2016)

From PDF analyses it is clear the necessity of including non perturbative sea quarks and gluons at the initial scale of the model. In order to face this problem, a simplified approach has been used:

$$F_{uu}(x_1, x_2, k_\perp = 0; Q_0^2) \sim \underbrace{F_{u_v u_v}(x_1, x_2, k_\perp = 0; Q_0^2)} + \boxed{\begin{aligned}&(1 - x_1 - x_2)^n \theta(1 - x_1 - x_2)\\ +\ & u_v(x_1; Q_0^2)\bar{u}(x_2; Q_0^2)\\ +\ & \bar{u}(x_1; Q_0^2) u_v(x_2; Q_0^2)\end{aligned}}$$

- Pure valence contribution obtained evolving in pQCD the model calculation of dPDF from the initial scale μ_0^2 to the scale Q_0^2

- Non perturbative sea quark contributions (effective high Fock states) *n=0.2*



Only sea quarks and gluons perturbatively generated

Sea quarks and gluons perturbatively and non perturbatively generated

PDF LO MSTW2008

$\bar{u}(x; Q_0^2)$ $u_v(x; Q_0^2)$ $Q_0^2 = 1\ \mathrm{GeV}^2$

$ratio_{gg} \neq 1$

CORRELATIONS

The Effective X-section

A fundamental tool for the comprehension of the role of DPS in hadron-hadron collisions is the so called "effective X-section" (see talk of **P. Bartalini**) σ_{eff} :

This object can be defined through a "pocket formula":

Sensitive to correlations ?

$$\sigma_{eff} = \boxed{\frac{m}{2}} \frac{\sigma_A^{pp'} \boxed{\sigma_B^{pp'}}}{\sigma_{double}^{pp}}$$

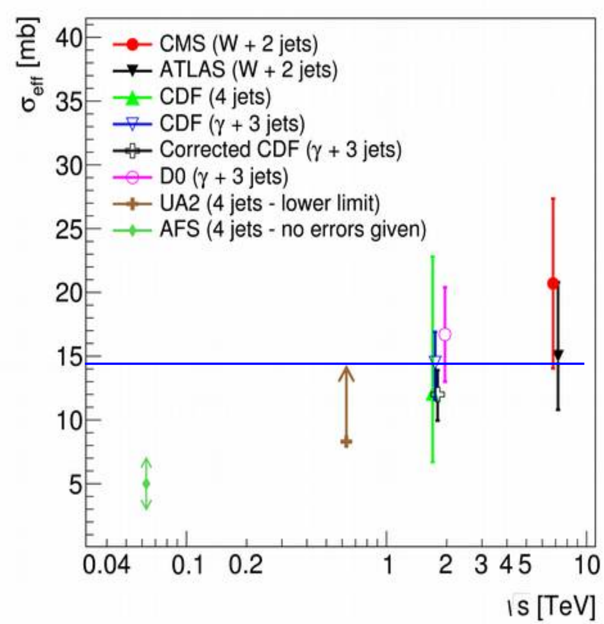
Differential cross section for the process: $\mathrm{pp'} \to A(B) + X$

Combinatorial factor

Differential cross section for a DPS event: $\mathrm{pp'} \to A + B + X$

....EXPERIMENTAL STATUS:

- Difficult extraction, approved analysis for the same sign *W's* production @LHC (RUN 2, see talk of **A. Rossi**)
- the model dependent extraction of σ_{eff} from data is consistent with a "constant" , nevertheless there are large errorbars (uncorrelated ansatz assumed!)
- different ranges in x_i accessed in different experiments.



High x for hard jets (heavy particles detected, large partonic s):

AFS ⟶ $y \sim 0;\ x_1 \sim x_2;\ 0.2 < x_{1,2} < 0.4$

CDF ⟶ $0.02 < x_{1,2,3,4} < 0.4$

valence region included

The Effective X-section calculation

M. R., S. Scopetta, M. Traini and V.Vento, PLB 752, 40 (2016)

$$\sigma_{eff} = \frac{m}{2}\frac{\sigma_A^{pp'}\ \sigma_B^{pp'}}{\sigma_{double}^{pp}}$$

This quantity can be written in terms of PDFs and dPDFs ($_2$GPDs)

Here the scale is omitted

Colour coefficient

$$\sigma_{\mathbf{eff}}(\mathbf{x_1},\mathbf{x_1'},\mathbf{x_2},\mathbf{x_2'}) = \frac{\sum_{\mathbf{i,k,j,l}} \mathbf{F_i(x_1)F_k(x_1')F_j(x_2)F_l(x_2')C_{ik}}\boxed{\mathbf{C_{jl}}}}{\sum_{\mathbf{i,j,k,l}} \mathbf{C_{ik}C_{jl}} \int \mathbf{F_{ij}(x_1,x_2;k_\perp)F_{kl}(x_1',x_2';-k_\perp)}\frac{\mathbf{dk_\perp}}{(\mathbf{2\pi})^\mathbf{2}}}$$

Non trivial x-dependence

If factorization between dPDF and PDFs held:

$$F_{ab}(x_1,x_2,\vec{k}_\perp) = F_a(x_1)F_b(x_2)\tilde{T}(\vec{k}_\perp)$$

$$\sigma_{eff}(x_1,x_1',x_2,x_2') \to \sigma_{eff} = \left[\int \frac{d\vec{k}_\perp}{(2\pi)^2}\tilde{T}(\vec{k}_\perp)T(-\vec{k}_\perp)\right]^{-1} = \left[\int d\vec{b}_\perp (T(\vec{b}_\perp))^2\right]^{-1}$$

Constant value w.r.t. x_i

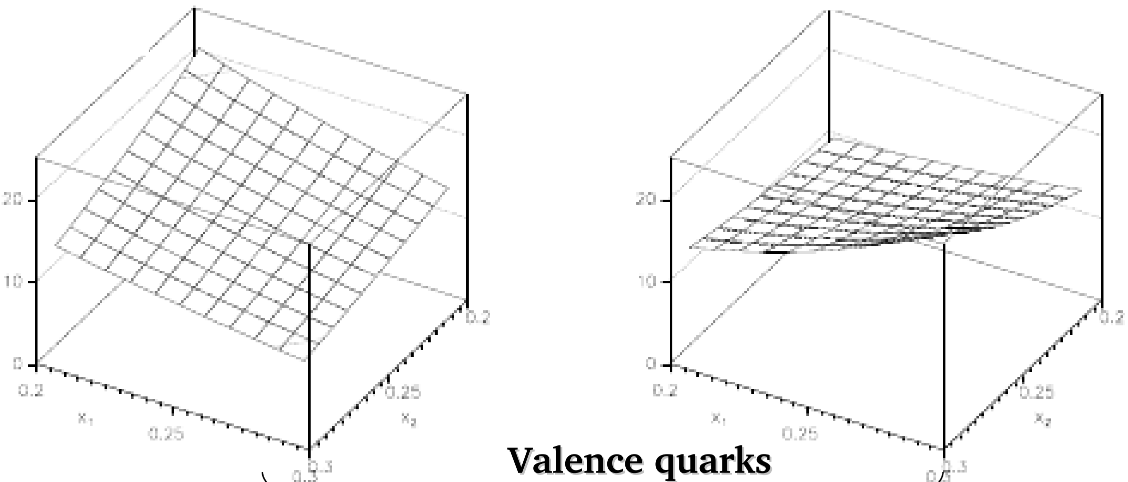
NO CORRELATIONS!

Conjugated variable to $\vec{k}_\perp$

M. R., S. Scopetta, M. Traini and V.Vento, PLB 752, 40 (2016)

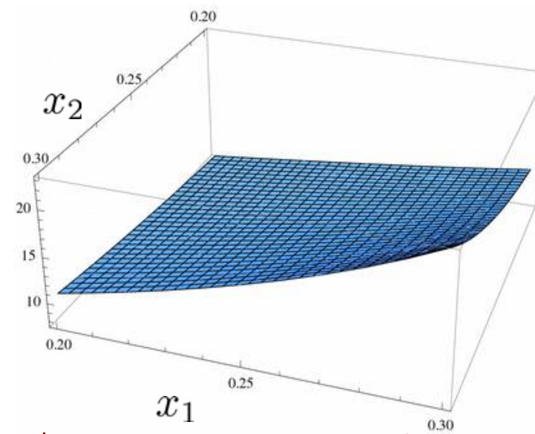
Our predictions of σ_{eff} , **without any approximation**, in the valence region at different energy scales:

$$\sigma_{eff}(x_1, x_2, \mu_0^2) \xrightarrow[\text{pQCD evolution of dPDFs}]{\text{pQCD evolution of PDFs}} \sigma_{eff}(x_1, x_2, Q^2 = 250\ \text{GeV}^2)$$



Valence quarks

$$\boxed{\overline{\sigma_{eff}} \sim 11\ \text{mb}}$$



Valence quark ⊗ Sea quark
Partons involved in, e.g., same sign WW production.

Similar results obtained with dPDFs calculated within AdS/QCD correspondence
M. Traini, M. R., S. Scopetta and V.Vento, PLB 768, 270 (2017)

- **x_i dependence of σ_{eff} may be model independent feature**
- **Absolute value of σ_{eff} is a model dependent result**

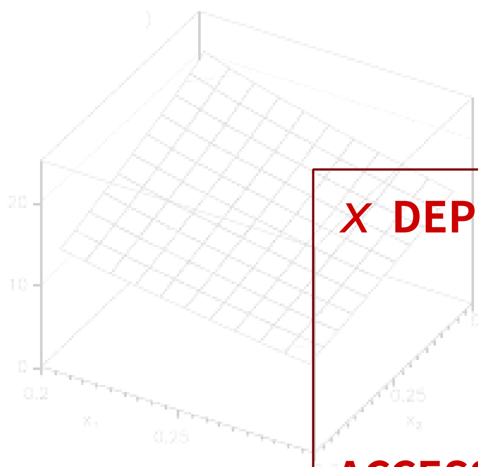
The old data lie in the obtained range of σ_{eff}

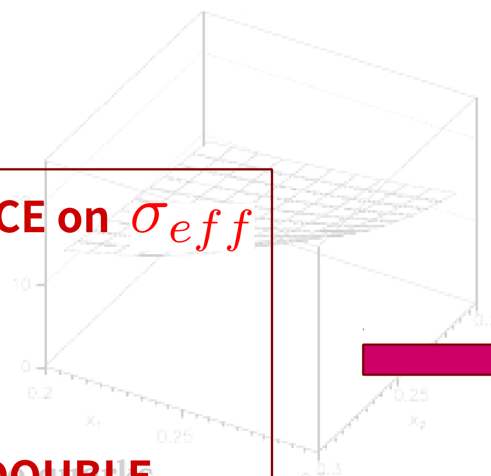
Numerical results

M. R., S. Scopetta, M. Traini and V.Vento, PLB 752, 40 (2016)

Our predictions of σ_{eff} in the valence region at different energy scales:

$\sigma_{eff}(x_1, x_2, \mu_0^2)$ $\sigma_{eff}(x_1, x_2, Q^2 = 250\ \mathrm{GeV}^2)$ $\sigma_{eff}(x_1, x_2, Q^2 = 250\ \mathrm{GeV}^2)$

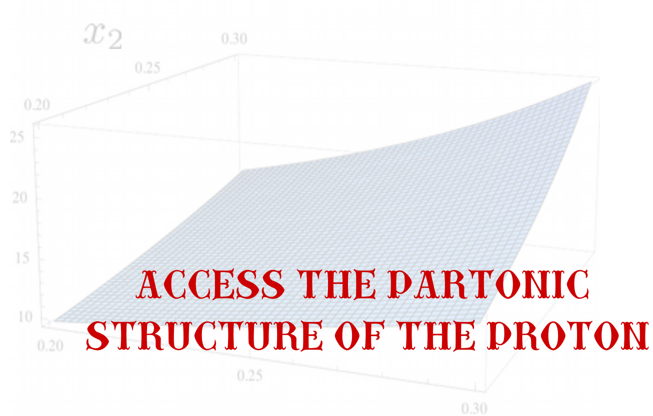




X DEPENDENCE on σ_{eff}

ACCESS THE DOUBLE PARTON CORRELATIONS

$\overline{\sigma_{eff}} \sim 11$ mb



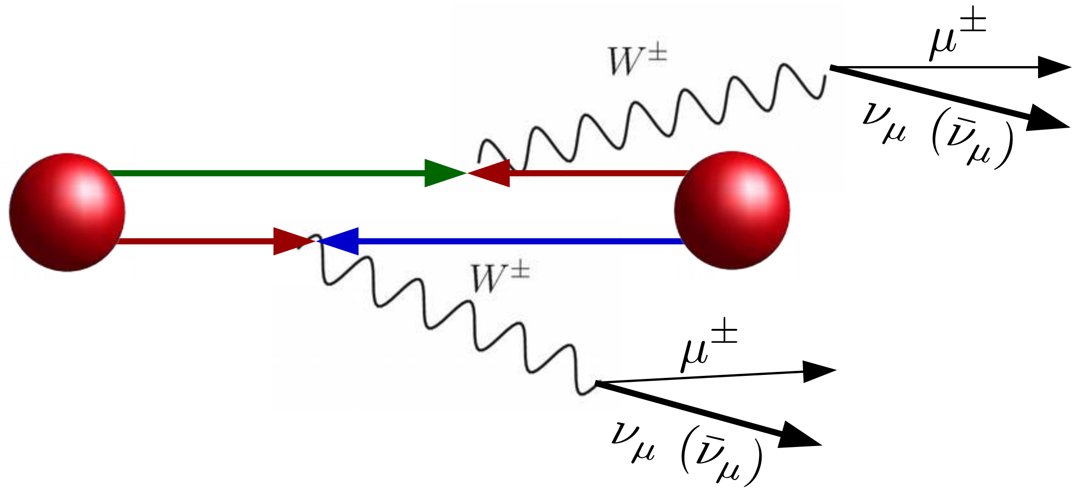
ACCESS THE PARTONIC STRUCTURE OF THE PROTON

Valence quark ⊗ Gluon

Valence quark ⊗ Sea quark
Partons involved in, e.g., same sign WW production.

Same sign W's in pp collisions at $\sqrt{s} = 13$ TeV at the LHC

F. A. Ceccopieri, M. R., S. Scopetta, arXiv:1702.05363



In this channel, the single parton scattering (usually dominant w.r.t to the double one) starts to contribute to higher order in strong coupling constant.

⬇

"Same-sign W boson pairs production is a promising channel to look for signature of double Parton interactions at the LHC."

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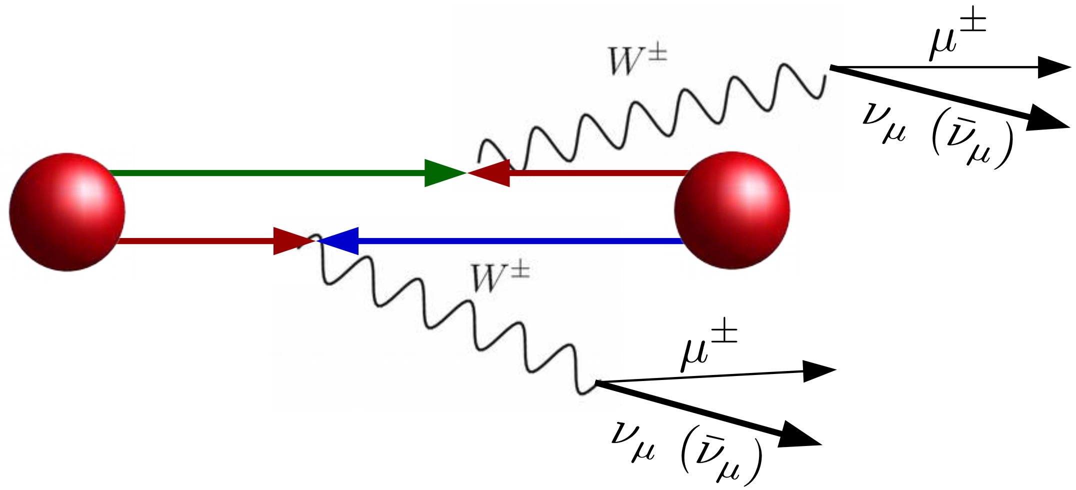
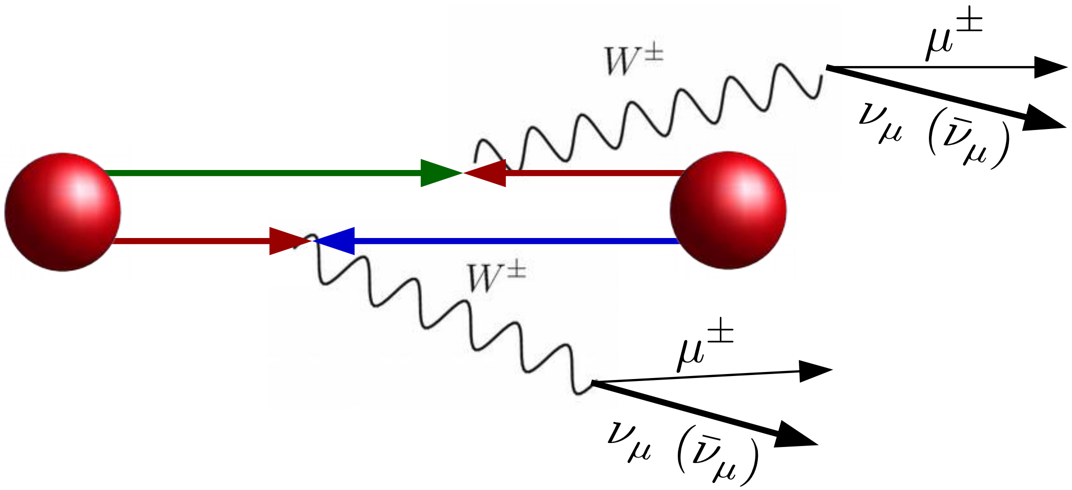


In this channel, the single parton scattering (usually dominant w.r.t to the double one) starts to contribute to higher order in strong coupling constant.

Can double parton correlations be observed for the first time in the next LHC run ?

Same sign W's in pp collisions at $\sqrt{s} = 13$ TeV at the LHC

F. A. Ceccopieri, M. R., S. Scopetta, arXiv:1702.05363



Kinematical cuts

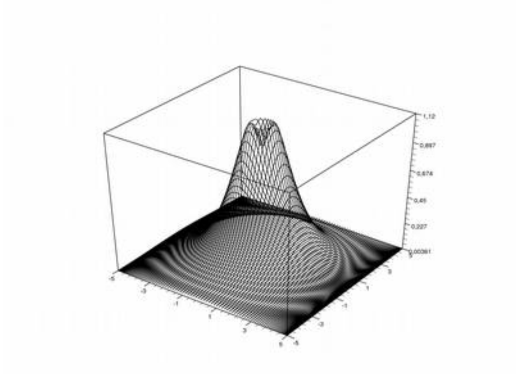
$$pp,\ \sqrt{s} = 13\ \text{TeV}$$
$$p_{T,\mu}^{leading} > 20\ \text{GeV},\quad p_{T,\mu}^{subleading} > 10\ \text{GeV}$$
$$|p_{T,\mu}^{leading}| + |p_{T,\mu}^{subleading}| > 45\ \text{GeV}$$
$$|\eta_\mu| < 2.4$$
$$20\ \text{GeV} < M_{inv} < 75\ \text{GeV or } M_{inv} > 105\ \text{GeV}$$

DPS cross section:

$$\frac{d^4\sigma^{pp\to\mu^\pm\mu^\pm X}}{d\eta_1 dp_{T,1} d\eta_2 dp_{T,2}} = \sum_{i,k,j,l} \frac{1}{2} \int d^2\vec{b}_\perp \boxed{F_{ij}(x_1, x_2, \vec{b}_\perp, M_W) F_{kl}(x_3, x_4, \vec{b}_\perp, M_W)} \frac{d^2\sigma_{ik}^{pp\to\mu^\pm X}}{d\eta_1 dp_{T,1}} \frac{d^2\sigma_{jl}^{pp\to\mu^\pm X}}{d\eta_2 dp_{T,2}} \boxed{\mathcal{I}(\eta_i, p_{T,i})}$$

$M_W \longrightarrow$ Momentum scale

We have used as input 3 models of dPDFs:



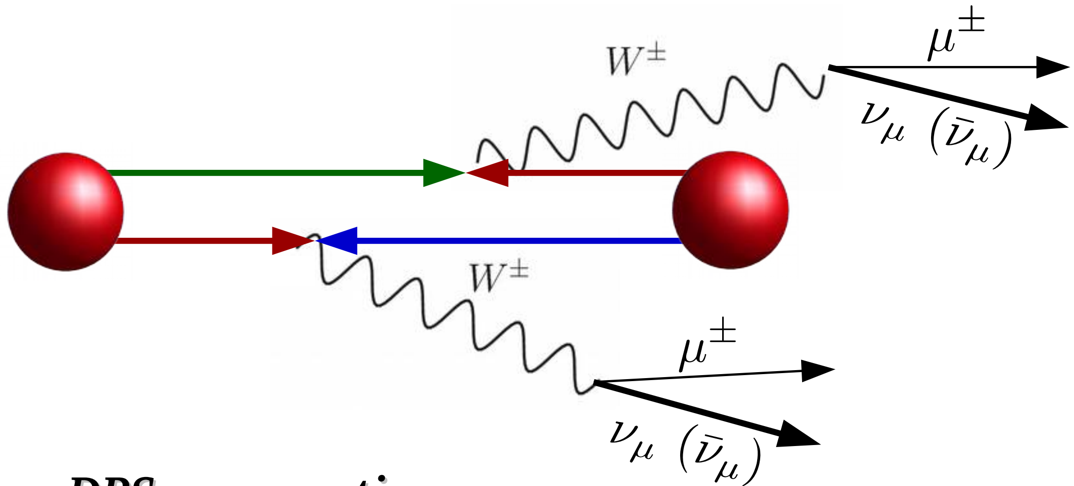
1)Model: **QM** M. R., S. Scopetta, M. Traini and V.Vento, JHEP 12, 028 (2014)

Longitudinal and transverse correlations arise from the relativistic CQM model describing three valence quarks

These correlations propagate to sea quarks and gluons through pQCD evolution

Same sign W's in pp collisions at $\sqrt{s} = 13$ TeV at the LHC

F. A. Ceccopieri, M. R., S. Scopetta, arXiv:1702.05363



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$M_W \longrightarrow$ Momentum scale

We have used as input 3 models of dPDFs:

2)Model: **MSTW**

$$F_{ab}(x_1, x_2, \vec{b}_\perp, M_W) = \boxed{a(x_1, M_W)} b(x_2, M_W) \boxed{T(\vec{b}_\perp)}$$

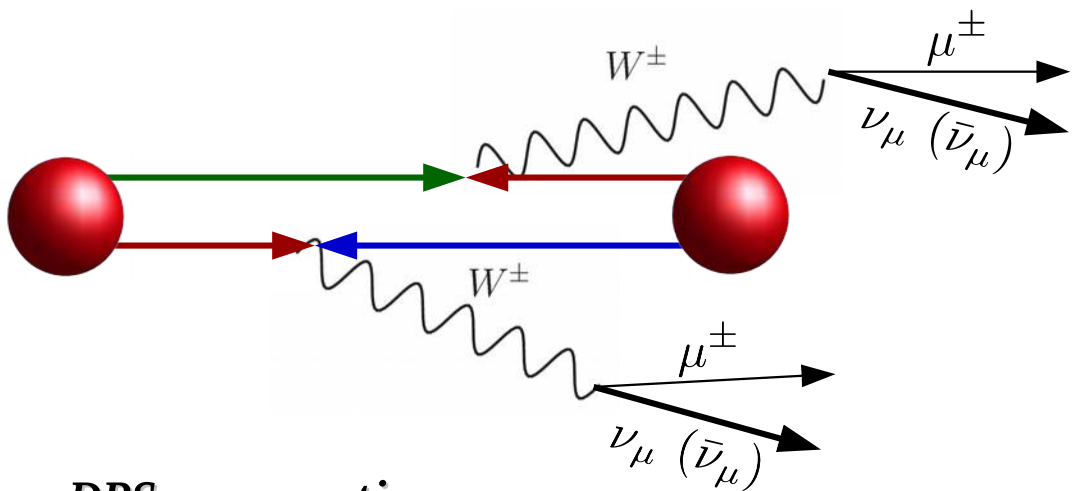
PDFs of the parametrization:
A.D. Martin *et al*. Eur. Phys. J. **C63,** 189 (2009)

Fixed by: $\bar{\sigma}_{eff} = \dfrac{1}{\int d\vec{b}_\perp [T(\vec{b}_\perp)]^2}$

$$\bar{\sigma}_{\text{eff}} = 17.8 \pm 4.2 \text{ mb}$$

Average of CMS and ATLAS extractions from the analysis of W+dijet.
New results on same sign W's discussed by **ALESSANDRO ROSSI**

F. A. Ceccopieri, M. R., S. Scopetta, arXiv:1702.05363



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$M_W \longrightarrow$ Momentum scale

We have used as input 3 models of dPDFs:

3)Model: **GS09** J.R. Gaunt and W.J. Stirling, JHEP **03,** 005 (2010)

$$F_{ab}(x_1,x_2,\vec{b}_\perp,\bar{Q}_0) = \boxed{a(x_1,\bar{Q}_0)b(x_2,\bar{Q}_0)}\boxed{T(\vec{b}_\perp)}\boxed{\rho_{ab}(x_1,x_2)} \xrightarrow{\text{pQCD evolution}} F_{ab}(x_1,x_2,\vec{b}_\perp,M_W)$$

Perturbative Correlations and Inhomogeneus term included

$\bar{Q}_0 =$ Initial scale

PDFs of the parametrization:
A.D. Martin *et al*. Eur. Phys. J. **C63,** 189 (2009)

Fixed by: $\bar{\sigma}_{eff} = \dfrac{1}{\int d\vec{b}_\perp [T(\vec{b}_\perp)]^2}$

$\bar{\sigma}_{\mathbf{eff}} = \mathbf{17.8 \pm 4.2}$ **mb**

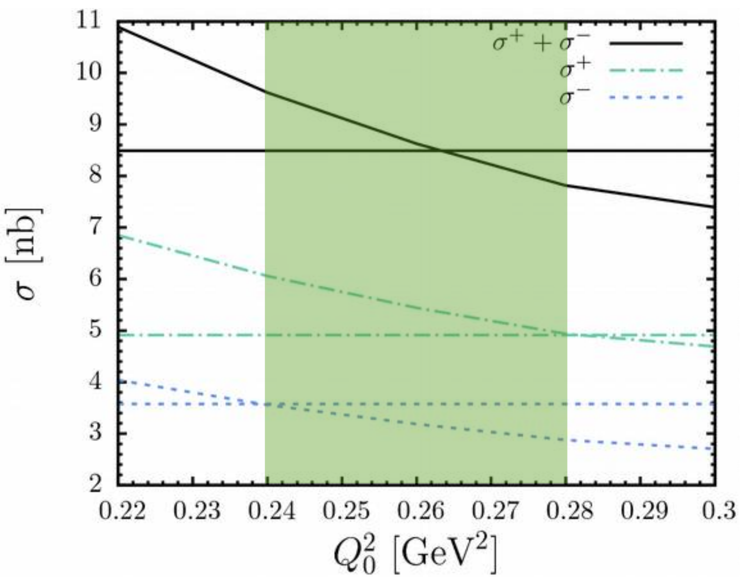
Added to fulfill dPDFs sum rules

Same sign W's in pp collisions at $\sqrt{s} = 13$ TeV at the LHC

F. A. Ceccopieri, M. R., S. Scopetta, arXiv:1702.05363

Fixing the initial scale Q_0^2 of dPDFs evaluated within the QM model:

- Since in this model the initial scale is originally located in the infrared regime, pQCD evolution and related observables, calculated by means of this model, are very sensitive to value of the initial scale Q_0.
- In order to fix Q_0 in this analysis use has been made of results on single parton scattering for $pp \to W^+ \to (\mu^+ \bar{\nu}_\mu)\ X;\ pp \to W^- \to (\mu^- \nu_\mu)\ X$



THE STRATEGY:

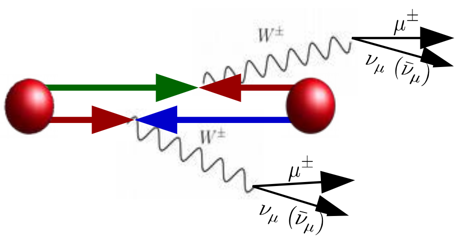
✔ σ^+, σ^- have been evaluated through DYNNLO [1] code by using PDFs of MSTW08 parametrization [2] (straight lines)

[1] S. Catani et. al., PRL 103, 082001 (2009); S. Catani et al., PRL 98, 222002 (2007)
[2] A.D. Martin *et al*. Eur. Phys. J. C63, 189 (2009)

✔ σ^+, σ^- have been evaluated through the PDFs calculated by means of the QM model starting from different values of Q_0

RESULT:

We found a range of values of Q_0 where the calculations within the LF approach get close to DYNNLO results



We associate a theoretical error to Q_0 :

$$\boxed{\delta Q_0^2 \Longrightarrow 0.24 < Q_0^2 < 0.28 \text{ GeV}^2}$$

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✔ The uncertainty due to neglected higher order perturbative corrections has been simulated by varying the final momentum scale:

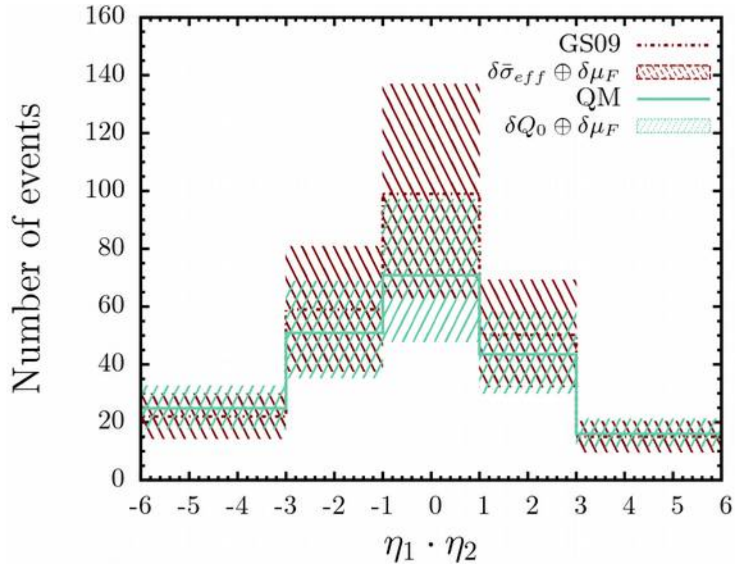
$$\delta\mu_F \Longrightarrow 0.5M_W < \mu_F < 2.0M_W$$

✔ The total cross section has been evaluated within the three models

✔ The differential cross section, converted in numbers of events, has been calculated w.r.t.: $\eta_1 \cdot \eta_2 \simeq \frac{1}{4} ln\frac{x_1}{x_3} ln\frac{x_2}{x_4}$

dPDFs	$\sigma^{++} + \sigma^{--}$ [fb]
MSTW	$0.77\ ^{+0.23}_{-0.21}\ (\delta\mu_F)\ ^{+0.18}_{-0.18}\ (\delta\bar{\sigma}_{eff})$
GS09	$0.82\ ^{+0.24}_{-0.26}\ (\delta\mu_F)\ ^{+0.19}_{-0.19}\ (\delta\bar{\sigma}_{eff})$
QM	$0.69\ ^{+0.18}_{-0.18}\ (\delta\mu_F)\ ^{+0.12}_{-0.16}\ (\delta Q_0)$

dPDFs	σ^{++} [fb]	σ^{--} [fb]	σ^{++}/σ^{--}
GS09	0.54	0.28	1.9
QM	0.53	0.16	3.4
GS09/QM	1.01	1.78	-



RESULTS:

- results of the three models are comparable within the errors;
- with $\mathcal{L} = 300$ fb^{-1} the central value of the predictions of the three models can experimentally discriminated;
- for the expected number of events:

✗ The maximum is found for $\eta_1 \cdot \eta_2 \sim 0$ where interacting partons share same momentum;

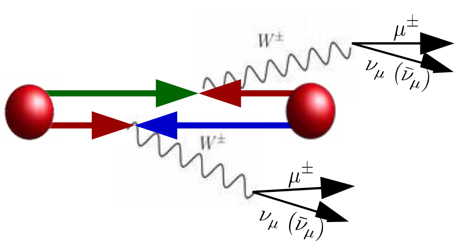
✗ For large $\eta_1 \cdot \eta_2$ the decreasing of the cross section is related to the decreasing behaviour of the dPDFs in the high x_i region

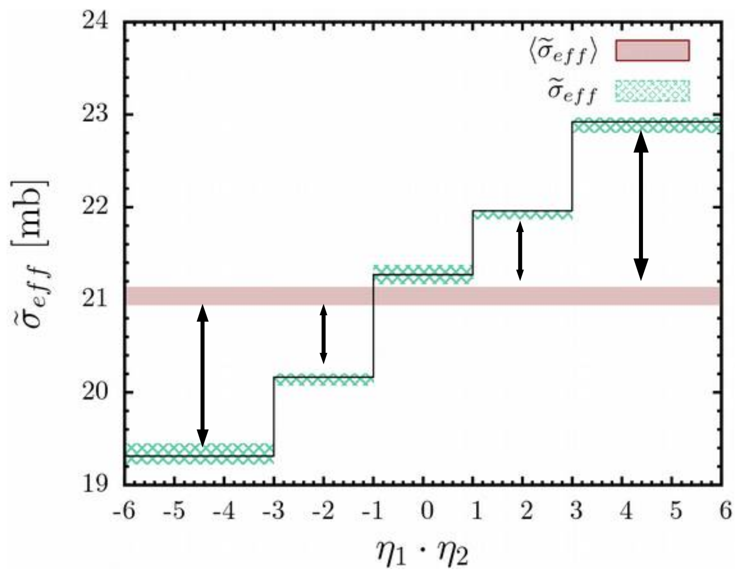
Same sign W's in pp collisions at $\sqrt{s} = 13$ TeV at the LHC

F. A. Ceccopieri, M. R., S. Scopetta, arXiv:1702.05363

In order to understand whether correlations can be accessed in experimental observations, using dPDF evaluated within the QM model, the effective cross section has been calculated for this process and compared with its mean value:

$$\langle \tilde{\sigma}_{eff} \rangle = 21.04 \,{}^{+0.07}_{-0.07}\,(\delta Q_0)\,{}^{+0.06}_{-0.07}(\delta \mu_F)\ \text{mb}\ .$$





$$\tilde{\sigma}_{eff} = \frac{m}{2} \frac{\sigma_A^{pp'}\ \sigma_B^{pp'}}{\sigma_{double}^{pp}}$$

Difference between green and red line is due to correlations effects

"Assuming that the results of the first and the last bins can be distinguished if they differ by 1 sigma, we estimated that

$$\boxed{\mathcal{L} = 1000\ \text{fb}^{-1}}$$

is necessary to observe correlations"

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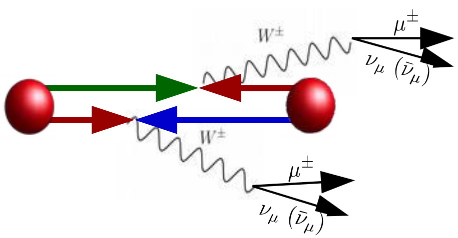

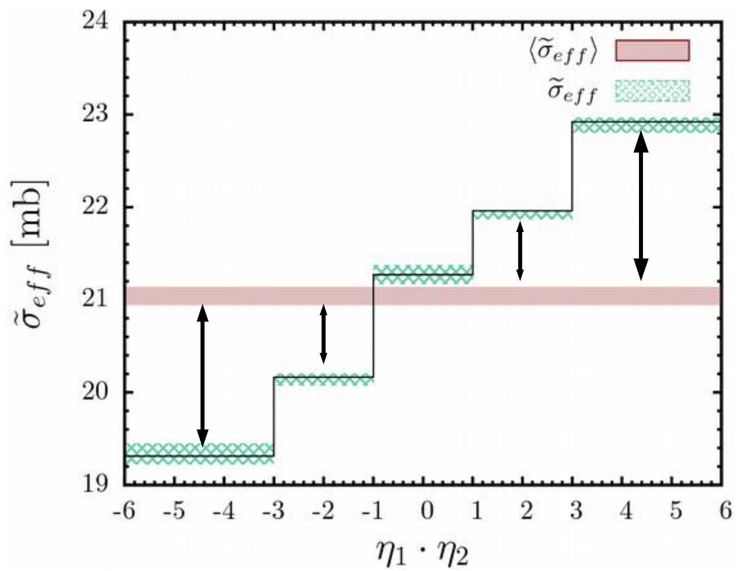

Difference between green and red line is due to correlations effects

To observe correlations, $\boxed{\mathcal{L} = 1000\ \mathrm{fb}^{-1}}$ is needed!

REACHABLE IN THE PLANNED LHC RUN

Same sign W's in pp collisions at $\sqrt{s} = 13$ TeV at the LHC

F. A. Ceccopieri, M. R., S. Scopetta, arXiv:1702.05363

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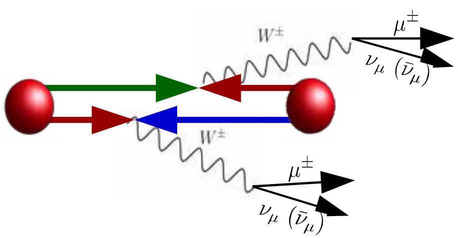
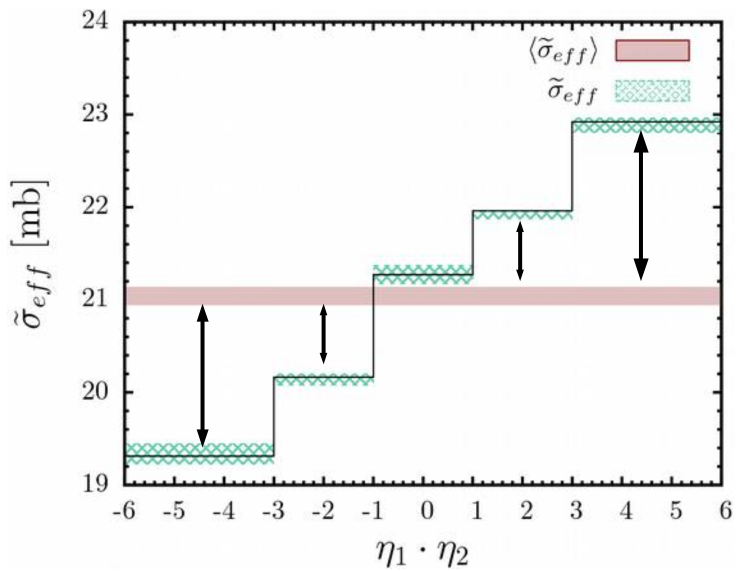




Difference between green and red line is due to correlations effects

IN THIS CHANNEL, THANKS TO THIS ANALYSIS, THE POSSIBILITY TO OBSERVE FOR THE FIRST TIME TWO-PARTON CORRELATIONS, IN THE NEXT LHC RUN, HAS BEEN ESTABLISHED

Conclusions

- **A CQM calculation of the dPDFs with a fully covariant approach**
  - ✓ **longitudinal and transverse correlations are found**;
  - ✓ deep study on relativistic effects: **transverse and longitudinal model independent correlations have been found**;
  - ✓ pQCD evolution of dPDFs, including non perturbative degrees of freedom into the scheme: **correlations are present at high energy scales and in the low *x* region**;
  - ✓ calculation of the effective X-section within different models in the valence region: **x-dependent quantity obtained!**
- **Study of DPS in same sign WW production at the LHC**
  - ✓ Calculations of the DPS cross section of same sign WW production
  - ✓ **dynamical correlations are found to be measurable in the next run at the LHC**

A proton imagining (complementary to the one investigated by means of electromagnetic probes) can/will be obtained in the next LHC runs!

What next?

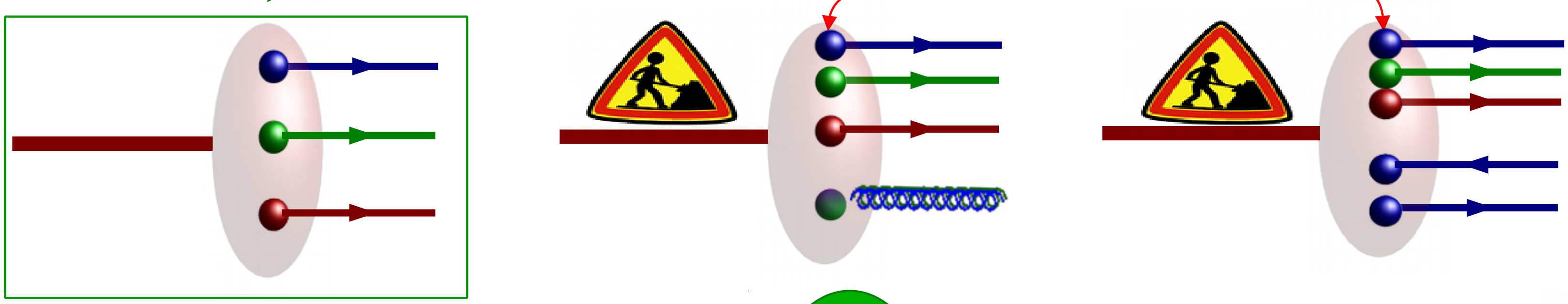
- A proton imagining (complementary to the one investigated by means of electromagnetic probes) can/will be obtaiend in the next LHC runs

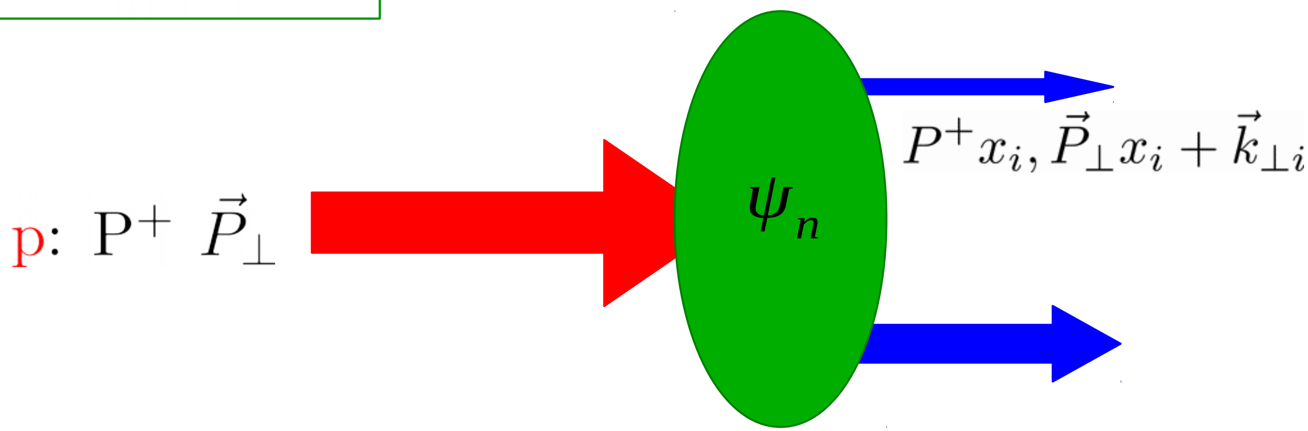
A Light-Front wave function representation

The proton wave function can be represented in the following way:

see *e.g.*: **S. J. Brodsky, H. -C. Pauli, S. S. Pinsky, Phys.Rept. 301, 299 (1998)**

$$|p, P^+ \ \vec{P}_\perp\rangle = \psi_{qqq}|qqq\rangle + \psi_{qqq\ g}|qqq\ g\rangle + \psi_{qqq\ q\bar{q}}|qqq\ q\bar{q}\rangle$$





$$\begin{cases} x_i = \dfrac{k_i^+}{P^+} \\ \sum_i^n x_i = 1 \\ \sum_i^n \vec{k}_{\perp i} = 0 \end{cases}$$

$\psi_n^{[l]}(x_i, \vec{k}_{\perp i}, \lambda_i) \longleftrightarrow$ Invariant under LF boosts!

dPDFs in a Light-Front approach

M. R., S. Scopetta, M. Traini and V.Vento, JHEP 12, 028 (2014)

Extending the procedure developed in **S. Boffi, B. Pasquini and M. Traini, Nucl. Phys. B 649, 243 (2003)** for GPDs, we obtained the following expression of the dPDF in momentum space, often called ${}_2$GPDs from the Light-Front description of quantum states in the intrinsic system:

$$F_{ij}(x_1,x_2,k_\perp) = 3(\sqrt{3})^3 \int \prod_{i=1}^{3} d\vec{k}_i \delta\left(\sum_{i=1}^{3}\vec{k}_i\right) \Phi^*(\{\vec{k}_i\},k_\perp)\Phi(\{\vec{k}_i\},-k_\perp)$$

Conjugate to $z_\perp$

$$\times \ \delta\left(x_1 - \frac{k_1^+}{M_0}\right)\delta\left(x_2 - \frac{k_2^+}{M_0}\right)$$

$$M_0 = \sum_i \sqrt{\vec{k}_i^2 + m^2}$$

GOOD SUPPORT

$$\mathrm{x}_1 + x_2 > 1 \Rightarrow F_{ij}(x_1,x_2,k_\perp) = 0$$

$$\Phi(\{\vec{k}_i\},\pm k_\perp) = \Phi\left(\vec{k}_1 \pm \frac{\vec{k}_\perp}{2}, \vec{k}_2 \mp \frac{\vec{k}_\perp}{2}, \vec{k}_3\right)$$

Now we need a model to properly describe the hadron wave function in order to estimate the LF ${}_2$GPDs

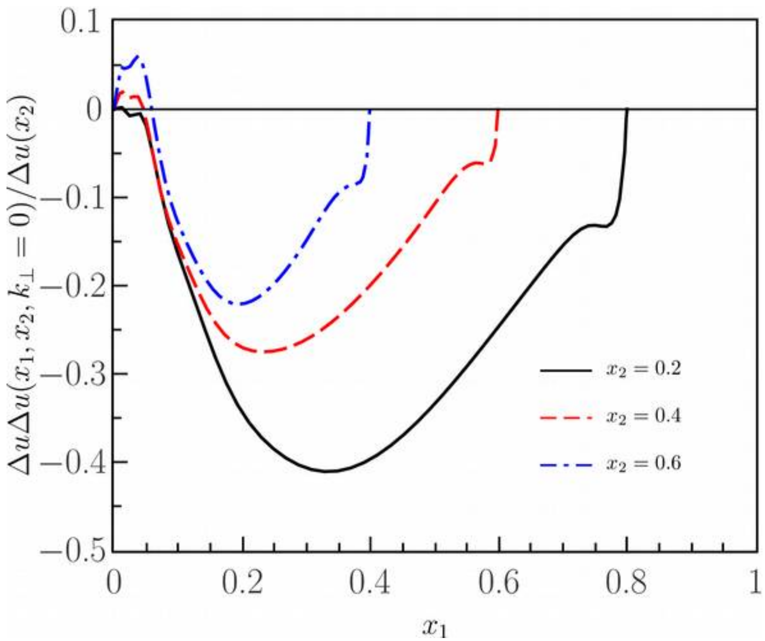
$$\Phi(\vec{k}_1,\vec{k}_2,\vec{k}_3) = D^{\dagger 1/2}(R_{il}(\vec{k}_1))D^{\dagger 1/2}(R_{il}(\vec{k}_2))D^{\dagger 1/2}(R_{il}(\vec{k}_3))\boxed{\psi^{[i]}(\vec{k}_1,\vec{k}_2,\vec{k}_3)}$$

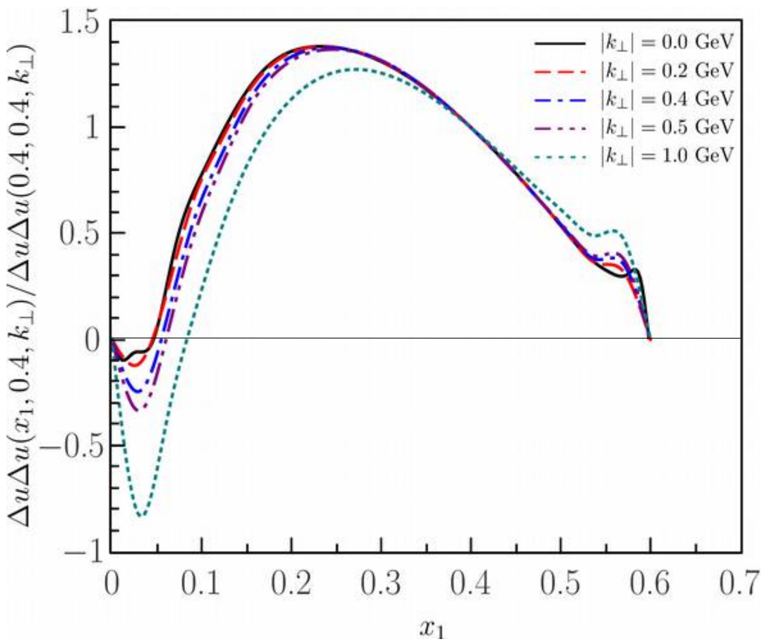
Melosh rotation

Instant form proton w.f.
<u>We need a CQM</u>!

Results for spin correlations

M. R., S. Scopetta, M. Traini and V.Vento, JHEP 12, 028 (2014)





$$u_{\uparrow(\downarrow)} u_{\uparrow(\downarrow)}(x_1, x_2, k_\perp) 3(\sqrt{3})^3 \int \prod_{i=1}^{3} d\vec{k}_i \delta\left(\sum_{i=1}^{3} \vec{k}_i\right) \delta\left(x_1 - \frac{k_1^+}{M_0}\right) \delta\left(x_2 - \frac{k_2^+}{M_0}\right) \Phi^*(\{\vec{k}_i\}, k_\perp) \frac{1 \pm \sigma_3(1)}{2} \frac{1 \pm \sigma_3(2)}{2} \Phi(\{\vec{k}_i\}, -k_\perp)$$

Here we have calculated: $\Delta u \Delta u(x_1, x_2, k_\perp) = \sum_{i=\uparrow,\downarrow} u_i u_i - \sum_{i \neq j = \uparrow,\downarrow} u_i u_j;$ $\boxed{|\Delta u \Delta u| \leq uu}$

(defined in **M. Diehl et Al, JHEP 03, 089 (2012),**
M. Diehl and T. Kasemets, JHEP 05, 150 (2013))

Positivity bound

This particular distribution, different from zero also in an unpolarized proton, contains more information on spin correlations, which could be important at small x and large t (LHC) !

Also in this case, both factorizations, $x_1 - x_2$ and $(x_1, x_2) - k_\perp$ are strongly violated!

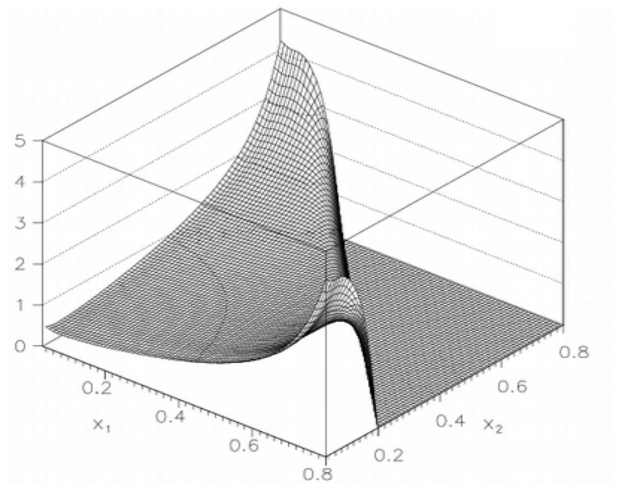
A pQCD evolution results: the non-singlet sector

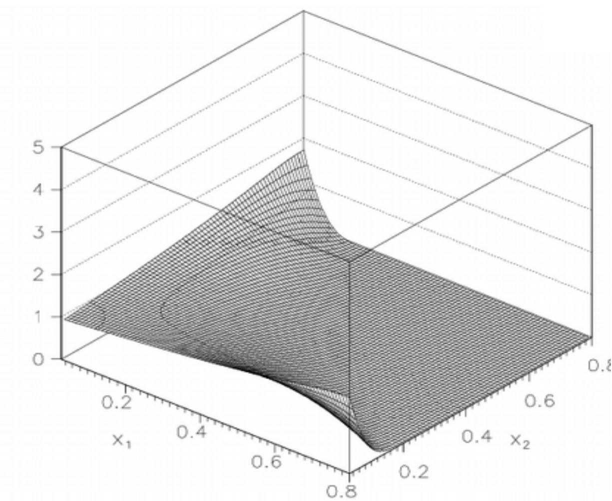
M. R., S. Scopetta, M. Traini and V.Vento, JHEP 12, 028 (2014)

*Here ${}_2$GPDs at $k_\perp = 0$

$Q^2 = \mu_0^2 \simeq 0.1$ GeV^2 | $Q^2 = 10$ GeV^2

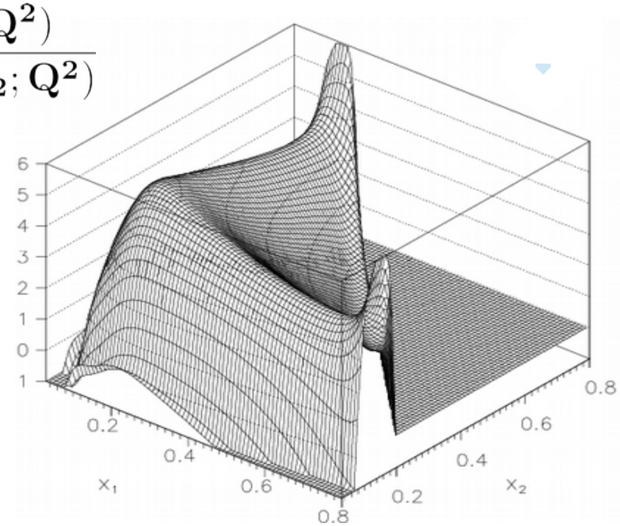
$$\mathbf{r_u(x_1, x_2; Q^2) = C_u \frac{uu(x_1, x_2; Q^2)}{u(x_1; Q^2)u(x_2; Q^2)}}$$

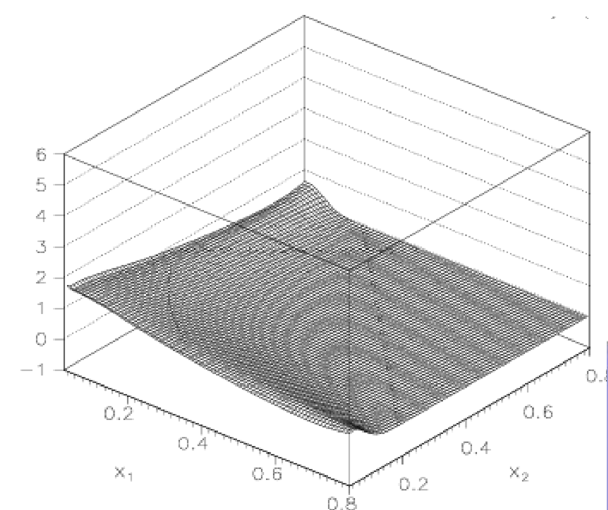




$$\mathbf{r_{\Delta u}(x_1, x_2; Q^2) = C_{\Delta u} \frac{\Delta u \Delta u(x_1, x_2; Q^2)}{\Delta u(x_1; Q^2)\Delta u(x_2; Q^2)}}$$

$$C_i = \frac{[\int dx F_i]^2}{\int dx_1 dx_2 F_{ii}(x_1, x_2, k_\perp = 0)}$$





All these ratios would be 1 if there were no correlations!

pQCD evolution of dPDFs calculations

The evolution equations for the dPDFs are based on a generalization of the DGLAP equations used, *e.g.*, for the single PDFs (**Kirschner 1979, Shelest, Snigirev, Zinovev 1982).**
Introducing the Mellin moments:

$$\langle x_1 x_2 F_{q_1,q_2}(Q^2)\rangle_{n_1,n_2} = \int_0^1 dx_1 \int_0^{1-x_1} dx_2\, x_1^{n_1-2}\, x_2^{n_2-2}\, x_1 x_2 F_{q_1,q_2}(x_1,x_2,Q^2) \;,$$

defining the moments of the quark-quark NS splitting functions at LO as follows:

$$P_{NS}^{(0)}(n_1) = \int dx\; x^{n_1} P_{NS}^{(0)}(x) \;,$$

using the modified DGLAP evolution equations, without the inhomogeneous term, since we are evaluating the valence dPDFs, one gets

$$\langle x_1 x_2 F_{q_1,q_2}(Q^2)\rangle_{n_1,n_2} = \left(\frac{\alpha(Q^2)}{\alpha(\mu_0^2)}\right)^{\frac{-P_{NS}^{(0)}(n_1) - P_{NS}^{(0)}(n_2)}{\beta_0}} \langle x_1 x_2 F_{q_1,q_2}(\mu_0^2)\rangle_{n_1,n_2}$$

The dPDF at any high energy scale is obtain by inverting the Mellin transformation:

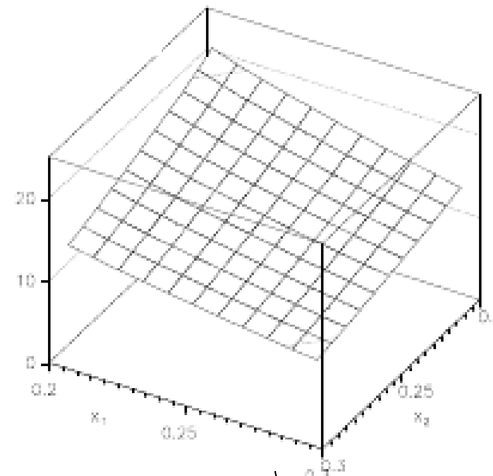
$$\begin{aligned} x_1 x_2 F_{q_1,q_2}(x_1,x_2,Q^2) &= \frac{1}{2\pi i}\oint_C dn_1\, \frac{1}{2\pi i}\oint_C dn_2 \\ &\times\; x_1^{(1-n_1)}\, x_2^{(1-n_2)}\, \langle x_1 x_2 F_{q_1,q_2}(Q^2)\rangle_{n_1,n_2} \end{aligned}$$

Numerical results I

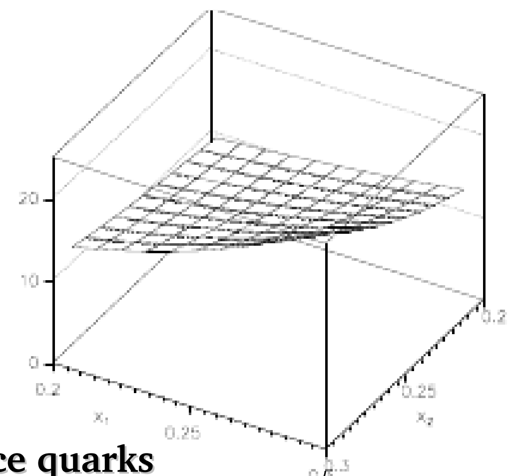
M. R., S. Scopetta, M. Traini and V.Vento, PLB 752, 40 (2016)

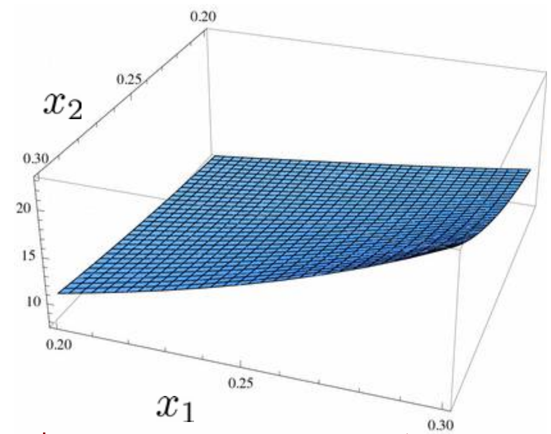
Our predictions of σ_{eff} in the valence region at different energy scales:

$\sigma_{eff}(x_1, x_2, \mu_0^2)$



$\sigma_{eff}(x_1, x_2, Q^2 = 250 \text{ GeV}^2)$





Valence quarks

$$\boxed{\overline{\sigma_{eff}} \sim 11 \text{ mb}}$$

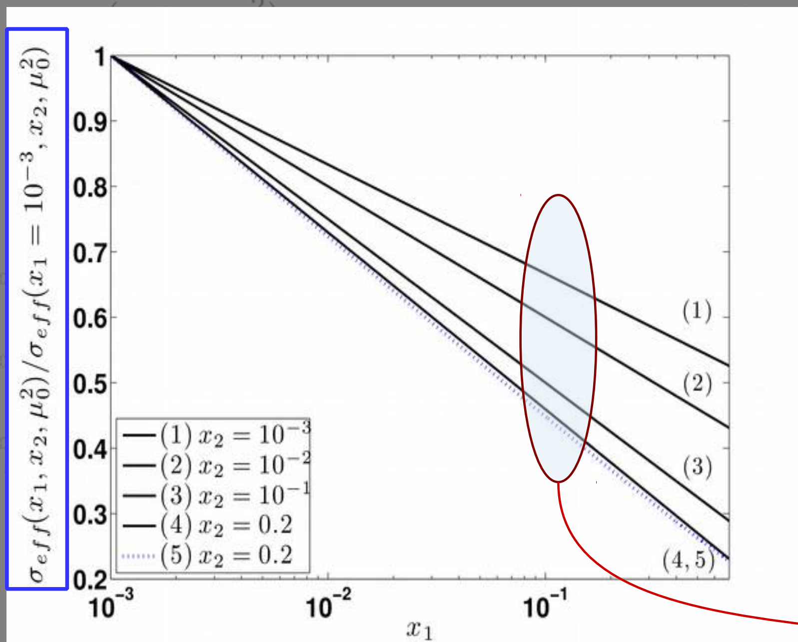
Valence quark ⊗ Sea quark
Partons involved in, e.g., same sign WW production.

The old data lie in the obtained range of σ_{eff}

Numerical results II

M. R., S. Scopetta, M. Traini and V.Vento, PLB 752, 40 (2016)
M. R., S. Scopetta, M. Traini and V.Vento, PLB 768, 270 (2017)

Our predictions of σ_{eff} in the valence region at different energy scales:



Ratio of σ_{eff} calculated by using:

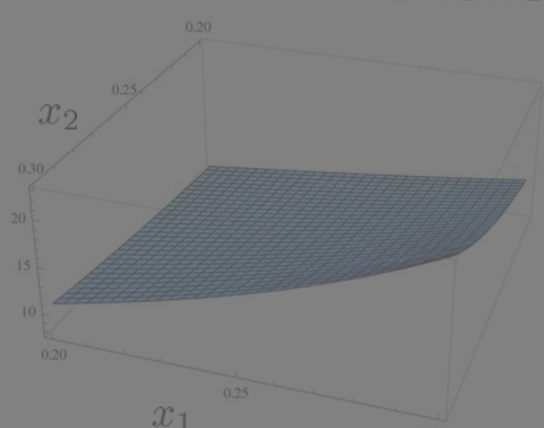
$$F_{12}(x_1, x_2, \vec{z}_\perp) \sim \int d\vec{b} f(x_1, 0, \vec{b} + \vec{z}_\perp) \boxed{f(x_2, 0, \vec{b})}$$

GPDs calculated within ADS/QCD soft wall model

$\sigma_{eff} \sim 11$ mb

Valence quark ⊗ Gluon

Also in this case, a strong *x* dependence is found!



Valence quark ⊗ Sea quark
Partons involved in, e.g., same sign WW production.

The old data lie in the obtained range of σ_{eff}

Effects of evolution and correlations I

M. R., S. Scopetta, M. Traini and V.Vento, 10, 063 (2016)

In the analysis of σ_{eff} , the factorized ansatz for dPDF in terms of PDFs, is commonly used. This is consistent with $\dfrac{F_{ab}(x_1, x_2, k_\perp = 0; Q^2)}{a(x_1; Q^2) b(x_2; Q^2)} \sim 1$

It is worth to notice that dPDFs and PDFs obey to different pQCD evolution scheme:

In order to distinguish effects of dynamical correlations, from those arising from the pQCD evolution, we have studied different ratios:

$$r_{ab}^{[1]} = \frac{F_{ab}(x_1, x_2, k_\perp = 0; Q^2)}{a(x_1; Q^2) b(x_2; Q^2)}$$

The numerator, being a dPDF, evolves with the usual pQCD evolution of dPDFs

The denominator evolves as the product of evolved single PDFs

Full Correlations

$$r_{ab}^{[2]} = \frac{[a(x_1; Q^2) b(x_2; Q^2)]^{dPDF}}{a(x_1; Q^2) b(x_2; Q^2)}$$

The numerator, product of PDFs, evolves with the pQCD evolution equations of dPDFs (PDF x PDF = dPDF)!

The denominator evolves as the product of evolved single PDFs

Perturbative Correlations

$$r_{ab}^{[3]} = \frac{F_{ab}(x_1, x_2, k_\perp = 0, Q^2)}{[a(x_1; Q^2) b(x_2; Q^2)]^{dPDF}}$$

The numerator, being a dPDF, evolves with the usual pQCD evolution of dPDFs

The numerator, product of PDFs, evolves with the pQCD evolution equation of dPDFs (PDF x PDF = dPDF)!

Non-Perturbative Correlations

Effects of evolution and correlations II

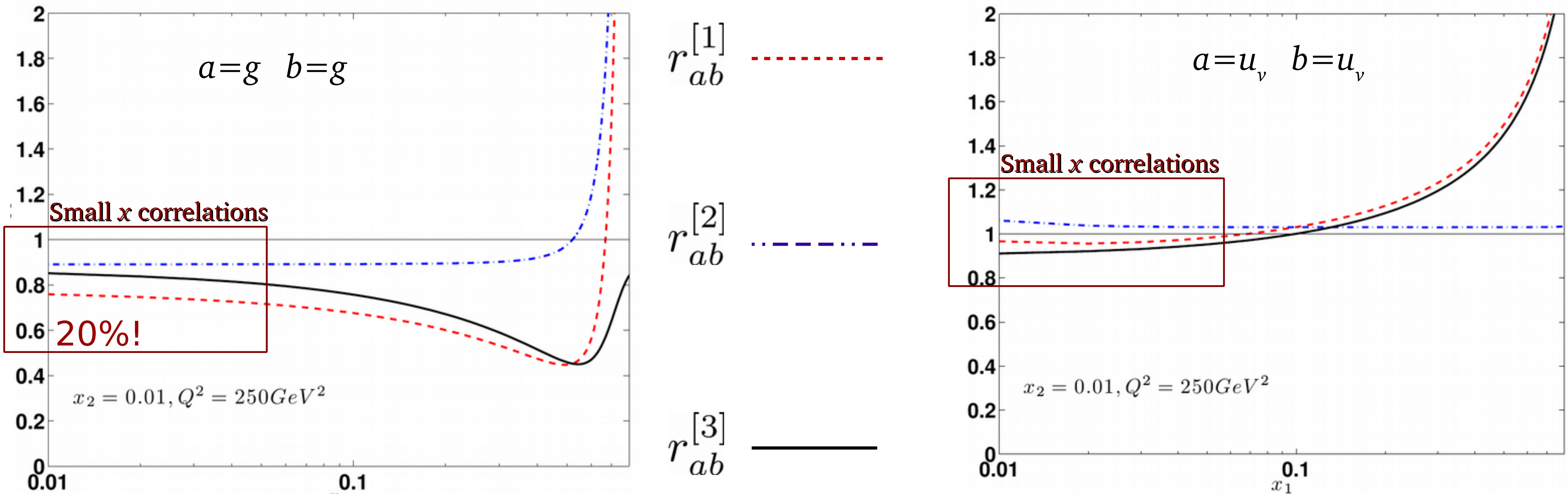
M. R., S. Scopetta, M. Traini and V.Vento, 10, 063 (2016)

Ratios previously shown are calculated for the following partonic spaces:
$a=u_v\ \ b=u_v$ and $a=b=g$

$$r^{[1]}_{ab} = \frac{F_{ab}(x_1,x_2,k_\perp=0;Q^2)}{a(x_1;Q^2)b(x_2;Q^2)}$$

$$r^{[2]}_{ab} = \frac{[a(x_1;Q^2)b(x_2;Q^2)]^{dPDF}}{a(x_1;Q^2)b(x_2;Q^2)}$$

$$r^{[3]}_{ab} = \frac{F_{ab}(x_1,x_2,k_\perp=0,Q^2)}{[a(x_1;Q^2)b(x_2;Q^2)]^{dPDF}}$$



Let us remark that usually in MC analyses, the effective X-section is estimated consistently with:

$$r^{[1]}_{ab} = r^{[2]}_{ab}\ r^{[3]}_{ab} \sim 1$$

$$r^{[1,2,3]}_{ab} \neq 1$$

CORRELATIONS

For $a=u_v\ \ b=u_v$, perturbative correlations compensate the non perturbative ones!

For $a=b=g$, perturbative and non-perturbative correlations coherently interfere.

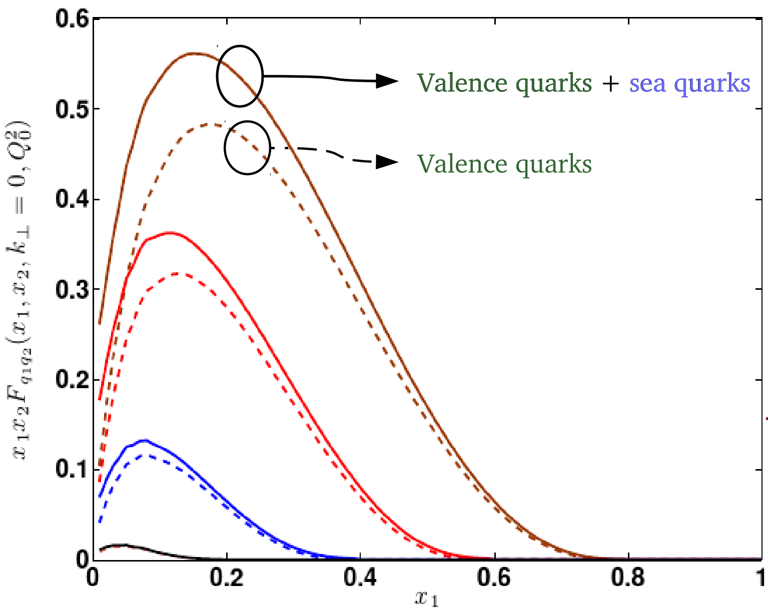
M. R., S. Scopetta, M. Traini and V.Vento, 10, 063 (2016)

From PDF analyses it is clear the necessity of including non perturbative sea quarks and gluons at the initial scale of the model. In order to face this problem, a simplified approach has been used:

$$F_{uu}(x_1,x_2,k_\perp=0;Q_0^2) \sim \underbrace{F_{u_v u_v}(x_1,x_2,k_\perp=0;Q_0^2)} + \boxed{(1-x_1-x_2)^n\theta(1-x_1-x_2) + u_v(x_1;Q_0^2)\bar{u}(x_2;Q_0^2) + \bar{u}(x_1;Q_0^2)u_v(x_2;Q_0^2)}$$

- Pure valence contribution obtained evolving in pQCD the model calculation of dPDF from the initial scale μ_0^2 to the scale Q_0^2

- Non perturbative sea quark contributions (effective high Fock states) *n=0.2*



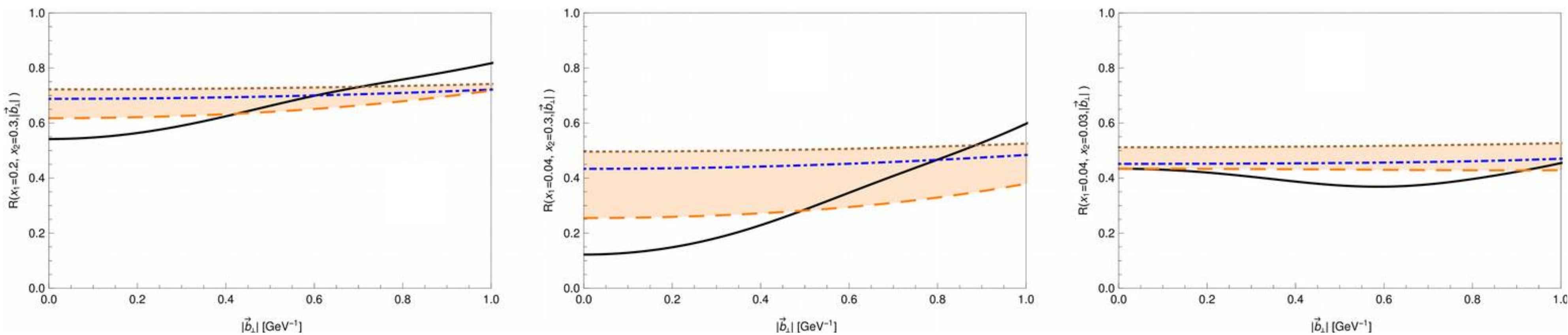
PDF LO MSTW2008
$\bar{u}(x;Q_0^2)$ $u_v(x;Q_0^2)$ $Q_0^2 = 1\ \text{GeV}^2$

$x_2=0.8$; $x_2=0.6$; $x_2=0.4$; $x_2=0.2$;

LF RELATIVISTIC EFFECTS II

M.R., F. A. Ceccopieri, PRD 95, no. 3, 034040 (2017)

We can estimate Melosh effetcs in dPDF studying this ratio: $\boxed{R(x_1, x_2, b_\perp) = \frac{F_{[L]}(x_1, x_2, b_\perp)}{F_{[NR]}(x_1, x_2, b_\perp)}}$



In these plots we can still appreciate correlations between x and $b_\perp$. Moreover the calculation has been performed using different quark models in order to show model independent effects!

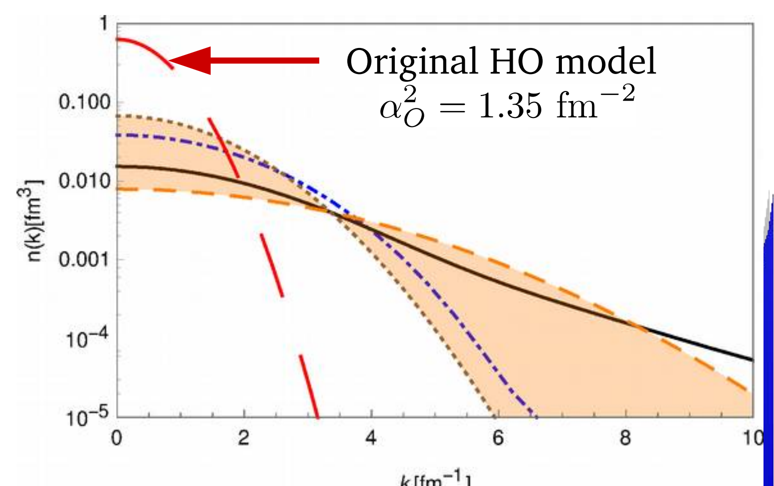
Relativistic Hyper central Model

NR Hyper central Model

Relativistic Harmonic Oscillator (HO) model $\alpha^2_{rel} = 25 \text{ fm}^{-2}$

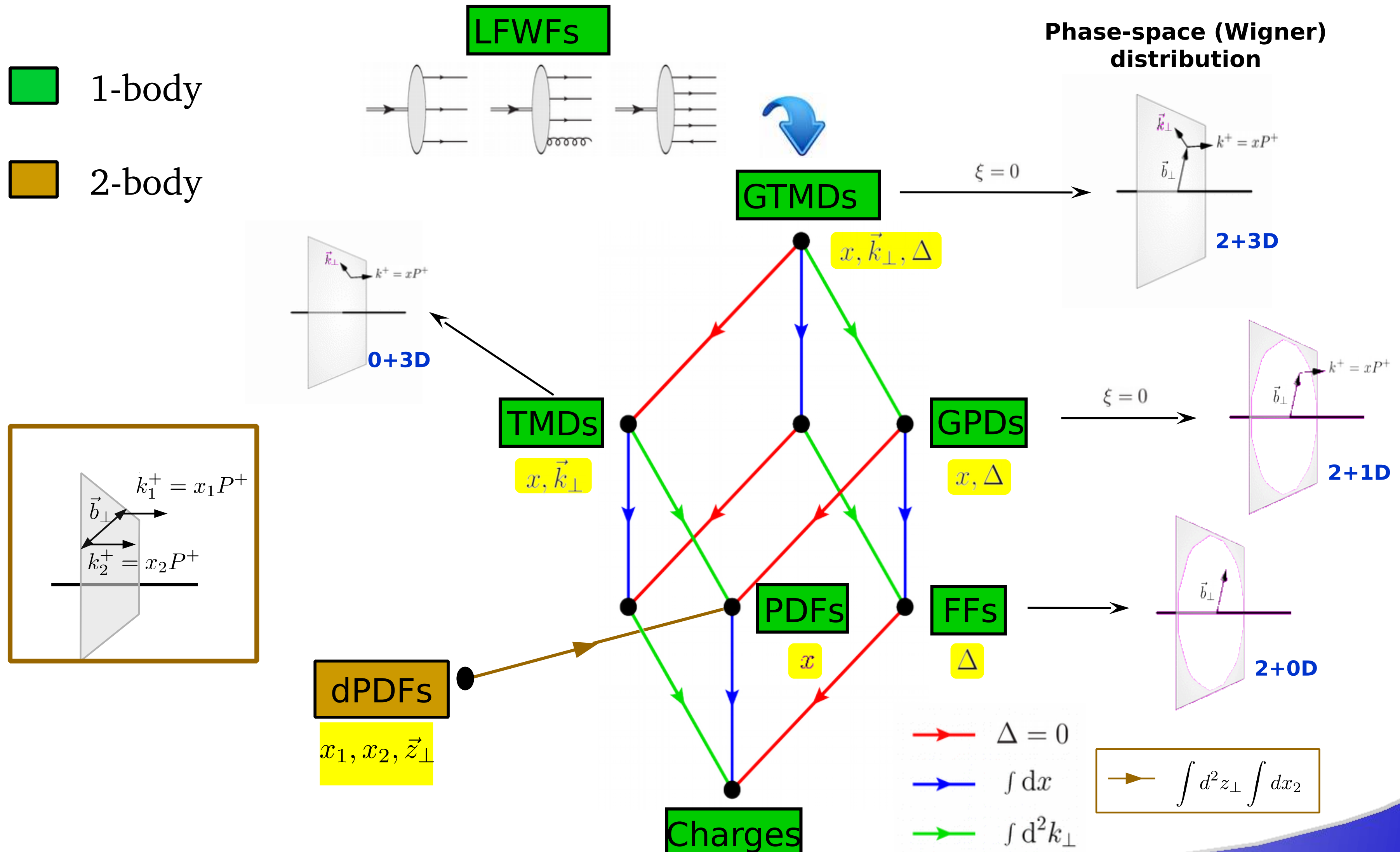
$\alpha^2_{nrel} < \alpha^2 < \alpha^2_{rel}$

NR Harmonic Oscillator model $\alpha^2_{nrel} = 6 \text{ fm}^{-2}$



How 3-Dimensional structure of a hadron can be investigated?

The 3D structure of a strongly interacting system (e.g. nucleon, nucleus..) could be accessed through different processes (e.g. SIDIS, DVCS, double parton sattering …), measuring different kind of parton distributions, providing different kind of information. The parton distribution puzzle is:



The Effective X-section calculation

M. R., S. Scopetta, M. Traini and V.Vento, PLB 752, 40 (2016)

$$\sigma_{eff} = \frac{m}{2}\frac{\sigma_A^{pp'}\ \sigma_B^{pp'}}{\sigma_{double}^{pp}}$$

This quantity can be written in terms of PDFs and dPDFs ($_2$GPDs)

In terms of PARTON DISTRIBUTIONS, $\sigma_{A(B)}^{pp'}$ and σ_{double}^{pp} can be written as follows:

$$\sigma_{A(B)}^{pp'}(x_1, x_1', \mu_1) = \sum_{\boxed{i,k}} F_i^p(x_1,\mu_1)\boxed{F_k^{p'}(x_{1'},\mu_1)}\hat{\sigma}_{ik}^{A(B)}(x_1, x_{1'}, \mu_1)$$

$$\boxed{i,k = \{q, \bar{q}, g\}}$$

Standard PDF

Proportional to colour coefficient and universal function: $C_{ij}\bar{\sigma}(x,x')$

$$\sigma_{double}^{pp}(x_1, x_1', x_2, x_2', \mu) = \frac{m}{2}\sum_{i,j,k,l}\hat{\sigma}_{ik}^{A}(x_1, x_1', \mu)\hat{\sigma}_{jl}^{B}(x_2, x_2', \mu) \times \int \frac{d\vec{k}_\perp}{(2\pi)^2} F_{ij}(x_1, x_2, k_\perp, \mu)\boxed{F_{kl}(x_1', x_2', -k_\perp, \mu)}$$

$_2$GPDs

Finally, combining the previous equations in the “pocket formula”, one obtains:

Here the scale is omitted

$$\boxed{\sigma_{\mathbf{eff}}(\mathbf{x_1}, \mathbf{x_1'}, \mathbf{x_2}, \mathbf{x_2'}) = \frac{\sum_{\mathbf{i,k,j,l}} \mathbf{F_i(x_1)F_k(x_1')F_j(x_2)F_l(x_2')C_{ik}C_{jl}}}{\sum_{\mathbf{i,j,k,l}} \mathbf{C_{ik}C_{jl}} \int \mathbf{F_{ij}(x_1, x_2; k_\perp)F_{kl}(x_1', x_2'; -k_\perp)\frac{dk_\perp}{(2\pi)^2}}}}$$

Non trivial x-dependence

What we would like to learn: A first look at two partons inside the proton

M.R., F. A. Ceccopieri, PRD 95, no. 3, 034040 (2017)

$$F_{u_v d_v}(x_1, x_2, \vec{b}_\perp, \mu_0^2) = \int d\vec{k}_\perp e^{i\vec{k}_\perp \cdot \vec{b}_\perp} \, F_{u_v d_v}(x_1, x_2, \vec{k}_\perp, \mu_0^2)$$

Transverse distance

The distribution has been calculated within different CQM models:

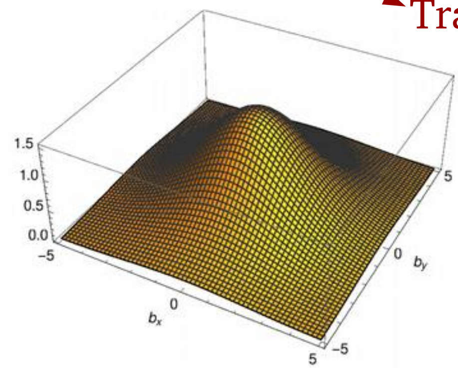
1) **M. Traini private communication, non relativistic Hyper-Central CQM (potential by E. Santopinto *et al*, PLB 364 (1995))**

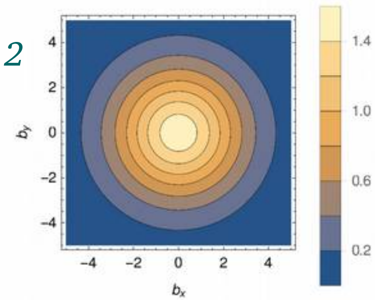
2) **M. Traini private communication** (relativistic Hyper-Central CQM)

3) **The harmonic oscillator**

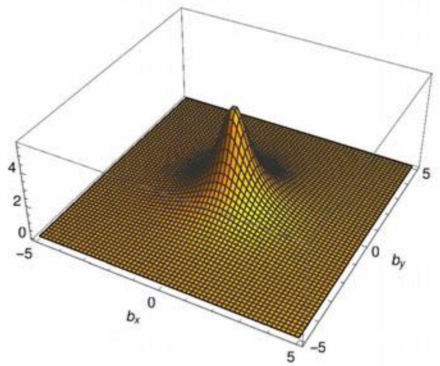
$x_1 = 0.3 \quad x_2 = 0.2$

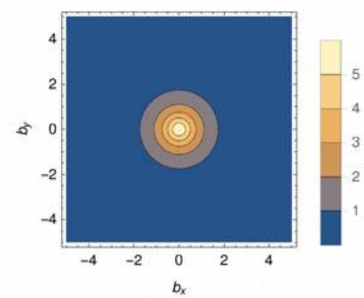
1)



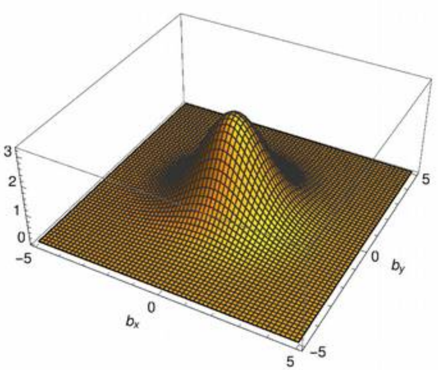


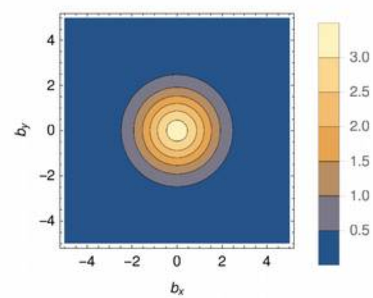
2)





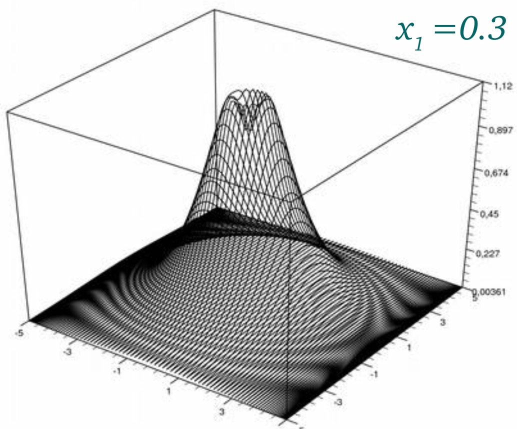
3)





D I F F E R E N T M O D E L S

$x_1 = 0.3 \quad x_2 = 0.04$



E.g., in our model, quarks with similar longitudinal momentum fraction "prefer" to be close to each other!

Results on distributions with longitudinally and transversely polarized quarks are coming!

The Effective X-section calculation

M. R., S. Scopetta, M. Traini and V.Vento, PLB 752, 40 (2016)

$$\sigma_{eff} = \frac{m}{2}\frac{\sigma_A^{pp'}\ \sigma_B^{pp'}}{\sigma_{double}^{pp}}$$

If factorization between dPDF and PDFs held:

$$F_{ab}(x_1,x_2,\vec{k}_\perp) = F_a(x_1)F_b(x_2)\tilde{T}(\vec{k}_\perp)$$

$$\sigma_{eff}(x_1,x_1',x_2,x_2') \to \underline{\sigma_{eff}} = \left[\int \frac{d\vec{k}_\perp}{(2\pi)^2}\tilde{T}(\vec{k}_\perp)T(-\vec{k}_\perp)\right]^{-1} = \left[\int d\vec{b}_\perp (T(\vec{b}_\perp))^2\right]^{-1}$$

Constant value w.r.t. x_i

Conjugated variable to $\vec{k}_\perp$

Here the scale is omitted

$$\sigma_{\mathrm{eff}}(\mathbf{x_1},\mathbf{x_1'},\mathbf{x_2},\mathbf{x_2'}) = \frac{\sum_{\mathrm{i,k,j,l}}\mathbf{F_i}(\mathbf{x_1})\mathbf{F_k}(\mathbf{x_1'})\mathbf{F_j}(\mathbf{x_2})\mathbf{F_l}(\mathbf{x_2'})\mathbf{C_{ik}}\mathbf{C_{jl}}}{\sum_{\mathrm{i,j,k,l}}\mathbf{C_{ik}}\mathbf{C_{jl}}\int \mathbf{F_{ij}}(\mathbf{x_1},\mathbf{x_2};\mathbf{k_\perp})\mathbf{F_{kl}}(\mathbf{x_1'},\mathbf{x_2'};-\mathbf{k_\perp})\frac{\mathbf{dk_\perp}}{(\mathbf{2\pi})^2}}$$

Non trivial x-dependence