Relativistic Heavy-Ion collisions: theory overview

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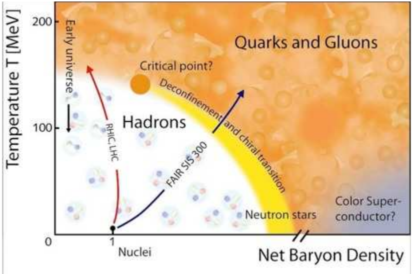
Heavy-ion collisions: exploring the QCD phase-diagram



QCD phases identified through the *order parameters*

- Polyakov loop $\langle L \rangle \sim e^{-\beta \Delta F_Q}$ energy cost to add an isolated color charge
- Chiral condensate $\langle \overline{q} q \rangle \sim$ effective mass of a "dressed" quark in a hadron

Region explored at LHC: *high-T/low-density* (early universe, $n_B/n_\gamma \approx 0.6 \cdot 10^{-9}$)

- From QGP (color deconfinement, chiral symmetry restored)
- to hadronic phase (confined, chiral symmetry breaking[1])

NB $\langle \overline{q} q \rangle \neq 0$ responsible for most of the baryonic mass of the universe: *only* $\sim$*35 MeV of the proton mass from* $m_{u/d} \neq 0$

[1] V. Koch, *Aspects of chiral symmetry*, Int.J.Mod.Phys. E6 (1997)

Virtual experiments: lattice-QCD simulations

- The best (unique?) tool to study QCD in the non-perturbative regime
- Limited to the study of equilibrium quantities

QCD on the lattice

Expectation values of operators are evaluated on a discretized euclidean lattice ($1/T = N_\tau a$) starting from the QCD partition function

$$\mathcal{Z} = \int [dU] \underbrace{\exp\left[-\beta S_g(U)\right] \prod_q \det\left[M(U, m_q)\right]}_{\text{MC weight}}$$

through a MC sampling of the field configurations, where

- $\beta = 6/g^2$
- S_g is the euclidean action of the gauge field;
- $U \in SU(3)$ is the link variable connecting two lattice sites;
- $M \equiv \not{D} + m_q$ is the Dirac operator

QCD at high-temperature: expectations

Based on *asymptotic freedom*, for $T \gg \Lambda_{QCD}$ hot-QCD matter should behave like a weakly-interacting ($g \ll 1$) plasma ($\neq$ gas, quarks and gluons are charged!) of massless quarks ($m_q \ll T$) and gluons. In such a regime T is the only available scale μ to evaluate the gauge coupling, for which one has

$$\lim_{T/\Lambda_{QCD}\to\infty} g(\mu \sim T) = 0$$

Hence one expects the asymptotic *Stefan-Boltzmann* behaviour

$$\epsilon = \frac{\pi^2}{30}\left[g_{\text{gluon}} + \frac{7}{8}g_{\text{quark}}\right] T^4,$$

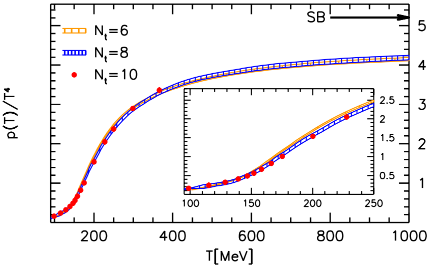
where

$$g_{\text{gluon}} = \underbrace{2 \times (N_c^2 - 1)}_{\text{pol.} \times \text{col.}} \quad \text{and} \quad g_{\text{quark}} = \underbrace{2 \times 2 \times N_c \times N_f}_{q/\overline{q} \times \text{spin} \times \text{col.} \times \text{flav.}}$$

count the number of bosonic and fermionic degrees of freedom

QCD on the lattice: results

In the last few years finite temperature continuum extrapolated ($a \to 0$) lattice results with realistic quark masses got available



Data by the W.B. Collaboration [JHEP 1011 (2010) 077]

- Pressure: $P = (T/V) \ln \mathcal{Z}$;

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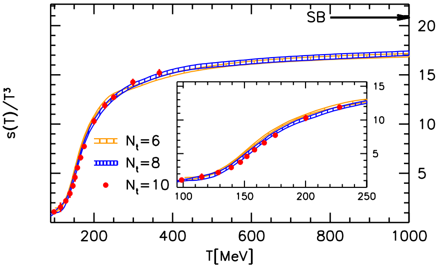

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- Entropy density: $s = \partial P / \partial T$;

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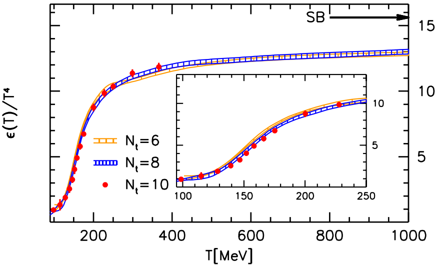

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- Pressure: $P = (T/V) \ln \mathcal{Z}$;
- Entropy density: $s = \partial P / \partial T$;
- Energy density: $\epsilon = Ts - P$;

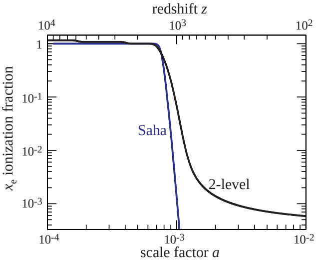
- Rapid rise in thermodynamical quantities suggesting a change in the *number of active degrees of freedom* (hadrons → partons): the most dramatic drop experienced by the early universe in which

$$H^2 = \frac{8\pi G}{3}\epsilon_{\rm rel} = \frac{8\pi G}{3}\frac{\pi^2}{30} g_* T^4$$

- One observes a systematic $\sim$20% deviation from the Stephan-Boltzmann limit even at large T: how to interpret it?

The QCD crossover: hadron vs atom formation

In the $\mu_B \to 0$ region the QCD transition is actually a *crossover*, i.e. a rapid but smooth change in the nature of the dominant charge (baryon, electric...) carriers, in analogy with the $e + p \leftrightarrow H + \gamma$ recombination in cosmology.



$$\frac{n_H}{n_p n_e} = \left(\frac{m_H}{m_p m_e}\frac{2\pi}{T}\right)^{3/2} \exp\left[\frac{m_p + m_e - m_H}{T}\right]$$

$$\approx \left(\frac{2\pi}{m_e T}\right)^{3/2} \exp\left[\frac{Q}{T}\right], \quad (Q = 13.6\ \mathrm{eV})$$

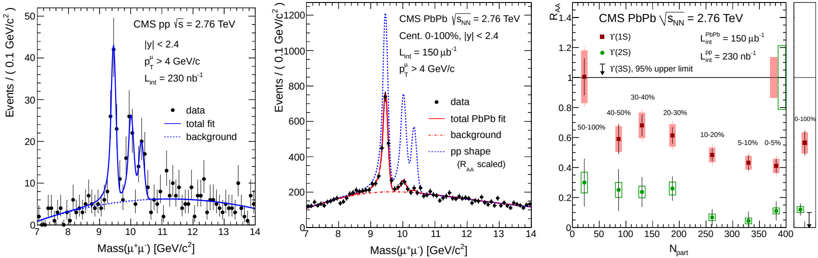
$X \equiv \frac{n_p}{n_p + n_H}$: ionization fraction (NB: $n_p = n_e$)

However they occur in *very different regimes*:

- One has $X = 0.5$ for $T_{\rm rec} = 0.323$ eV with $n_e^{\rm rec} \approx 0.122 (n_B/n_\gamma) T_{\rm rec}^3$. This corresponds to a *Debye screening radius* of the electric interaction $r_D \equiv (T/n_e e^2)^{1/2} \approx 24\,\mathrm{cm} \gg a_0 \sim 10^{-10}\mathrm{m}$: atomic properties unaffected! Crossover occurs in a *dilute regime*
- In the QGP $m_D \equiv r_D^{-1} = gT(N_c/3 + N_f/6)^{1/2}$. At $T = 0.2$ GeV, for $\alpha_s = 0.3$, one has $r_D \approx 0.4\,\mathrm{fm} \sim r_h$: color interaction strongly modified! Crossover occurs in a *strongly interacting regime*

The suppression of quarkonium

CMS results on the suppression of the excited states of Υ in Pb-Pb collisions



can be qualitatively explained as arising from the Debye-screening of the $Q\overline{Q}$ interaction in the QGP

$$V(r) = -C_F \frac{\alpha_s}{r} \quad \longrightarrow \quad -C_F \frac{\alpha_s}{r} e^{-m_D r},$$

as first proposed by Matsui and Satz in 1986 (PLB 178, 416-422).

Active degrees of freedom around the QCD crossover

Lattice-QCD calculations (nowadays with *realistic quark masses*) allows one to calculate the cumulants of conserved charges (baryon number, electric charge, strangeness) as well as of their product[2]

$$\langle X^m Y^n \rangle_c = \frac{\partial^{(m+n)}(\ln Z_{\rm QCD})}{\partial \hat{\mu}_X^m \partial \hat{\mu}_Y^n} \quad \text{with} \quad \hat{\mu}_i \equiv \mu_i / T,$$

where, considering the lowest orders, one has

$$\langle X^2 \rangle_c \equiv \langle \delta X^2 \rangle, \;\; \langle X^3 \rangle_c \equiv \langle \delta X^3 \rangle, \;\; \langle X^4 \rangle_c \equiv \langle \delta X^4 \rangle - 3 \langle \delta X^2 \rangle^2, \;\; \langle XY \rangle_c \equiv \langle \delta X \delta Y \rangle$$

Exploiting the fact that, at variance with hadrons, *all* quarks carry *fractional* baryon-number and electric charge, from the fluctuations of conserved charges and their correlations one can get information on the active degrees of freedom at a given temperature, i.e. whether they are hadrons (mesons and baryons) or deconfined quarks

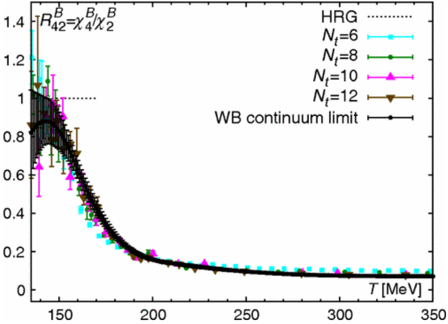
[2] M. Asakawa and M. Kitazawa, Prog.Part.Nucl.Phys. 90 (2016) 299

Active degrees of freedom around the QCD crossover

Fluctuations of *net* particle number (particles minus antiparticles) follow a Skellam distribution (difference of two Poissonian variables!). This provides a definite prediction for their cumulants:

$$\langle N^n \rangle_c = \langle N_{\rm part} \rangle + (-1)^n \langle N_{\rm antipart} \rangle \quad \longrightarrow \quad \frac{\langle N^{n+2m} \rangle_c}{\langle N^n \rangle_c} = 1$$

Having quarks baryon-number $1/3$, while hadrons 0 or 1...



...in the hadron-gas phase

$$\frac{\langle B^{n+2m} \rangle_c}{\langle B^n \rangle_c} = 1$$

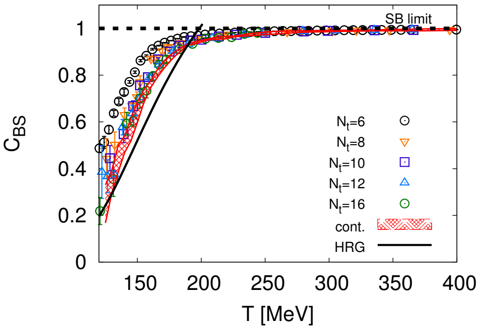
...in the QGP phase

$$\frac{\langle B^{n+2m} \rangle_c}{\langle B^n \rangle_c} = \frac{1}{9}$$

S. Borsanyi et al. PRL 113, 052301 (2014)

Strangeness around the QCD crossover

In the QGP phase strangeness is carried by s quarks, carrying also baryon number $B = 1/3$. In a HRG most of the strangeness is carried by kaons, for which $B = 0$; the lightest strange particle carrying baryon number $B = 1$ is the Λ. Correlation between strangeness and baryon-number fluctuations is a diagnostic tool of the active degrees of freedom!



S. Borsanyi et al., JHEP 1201 (2012) 138

One evaluates the quantity $(\langle S\rangle = 0)$

$$C_{BS} \equiv -3\frac{\langle BS\rangle_c}{\langle S^2\rangle_c} = -3\frac{\langle BS\rangle}{\langle S^2\rangle}$$

In the QGP phase

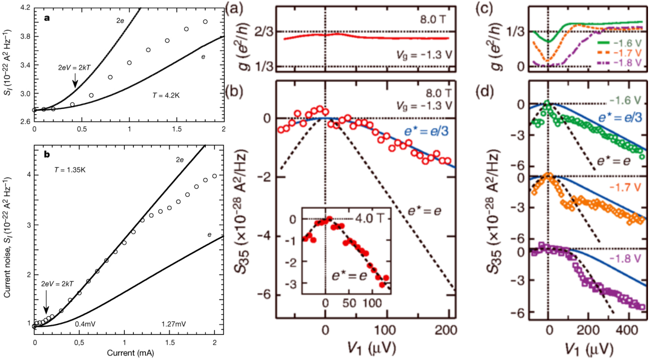
$$B = -(1/3)S \quad \longrightarrow \quad C_{BS} = 1$$

In the hadron-gas phase

$$C_{BS} = 3\frac{\langle\Lambda\rangle + \langle\overline{\Lambda}\rangle + \ldots + 3\langle\Omega^-\rangle + 3\langle\overline{\Omega}^+\rangle}{\langle K^0\rangle + \langle\overline{K}^0\rangle + \ldots + 9\langle\Omega^-\rangle + 9\langle\overline{\Omega}^+\rangle}$$

strongly dependent on temperature and very small at small temperature

Fluctuations are a very general tool to point-out the nature of *quasiparticle* excitations of a system. As an example, shot-noise measurement allows one to identify $e^* = 2e$ and $e^* = e/3$ charge-carriers in superconductivity and fractional quantum-Hall effect.

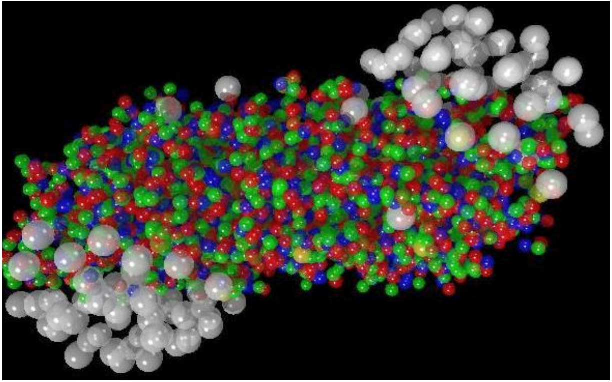


Charges passing through a potential barrier in the time-interval Δt follow a Poisson distribution, so that

$$\langle N^n \rangle_c = \langle N \rangle \quad \longrightarrow \quad \frac{\langle Q^2 \rangle_c}{\langle Q \rangle} = \frac{q^2 \langle N^2 \rangle_c}{q \langle N \rangle} = q$$

Real experiments: heavy-ion collisions

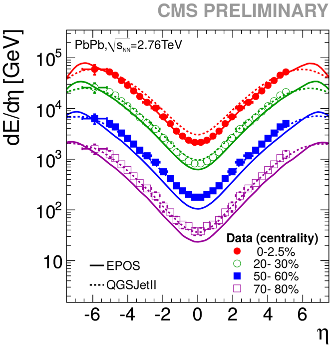
Heavy-ion collisions: a typical event

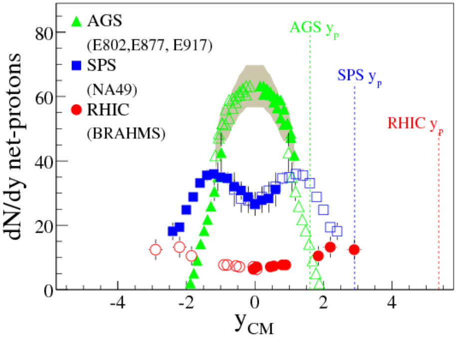


- Valence quarks of participant nucleons act as sources of strong color fields giving rise to *particle production*
- Spectator nucleons don't participate to the collision;

Almost all the energy and baryon number carried away by the remnants

Heavy-ion collisions: a typical event





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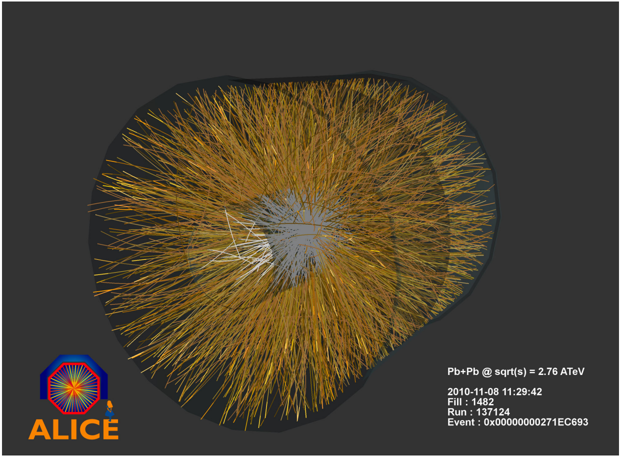
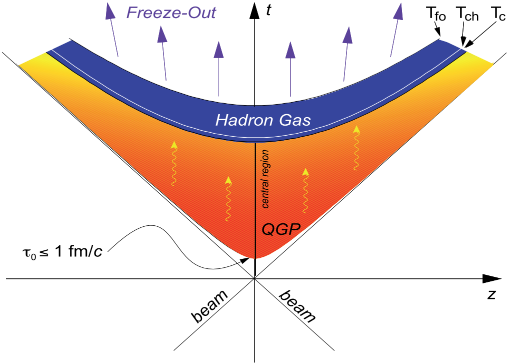


An example of Pb-Pb collision at the LHC, with thousands of track of charged hadrons reconstructed by the ALICE Time-Projection Chamber

Heavy-ion collisions: a cartoon of space-time evolution



- Soft probes (low-p_T hadrons): collective behavior of the *medium*;
- Hard probes (high-p_T particles, heavy quarks, quarkonia): produced in *hard pQCD processes* in the initial stage, allow to perform a tomography of the medium

Hadron yields at chemical freeze-out: statistical hadronization model and l-QCD

Statistical Hadronization Model in HIC's

The different hadron yields in HIC's turns out to be nicely described by a Hadron-Resonance Gas (an interacting system of hadrons is equivalent to a non-interacting system of hadrons and resonances)

$$Z_{\rm HRG} = \prod_{m_k < 2\,{\rm GeV}} Z_k(T, \mu_k)$$

with the chemical-freeze-out parameters (T, μ_B) fixed to reproduce to experimental measurements

$$n_h^{\rm exp} = n_h^{\rm th} + \sum_r n_r^{\rm th} \langle N_h^{(r)} \rangle.$$

In the above

$$n_k^{\rm th} = g_k \int \frac{d\mathbf{p}}{(2\pi)^3} \frac{1}{e^{(\epsilon_p - \mu_k)/T} \mp 1}$$

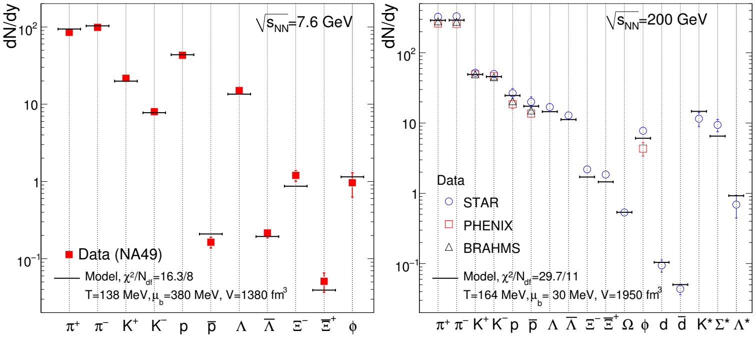
and $\mu_k = \mu_B B_k + \mu_S S_k + \mu_Q Q_k$, with μ_S and μ_Q fixed by the conditions

$$\langle S \rangle = 0 \quad \text{and} \quad \langle Q \rangle = (Z/A)\langle B \rangle$$

NB One uses GC ensemble, because charges are conserved exactly, but detectors cover a finite kinematic window $\longrightarrow$ EBE fluctuations

SHM and hadron yields in HIC's

SHM provides a satisfactory description of the hadron yields covering several orders of magnitude.



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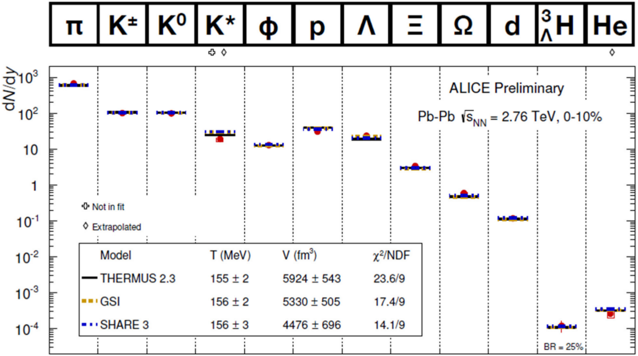

- As $\sqrt{s_{NN}}$ increases $\mu_B \to 0$

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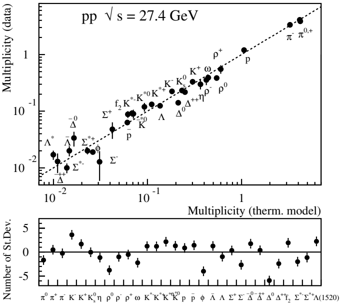
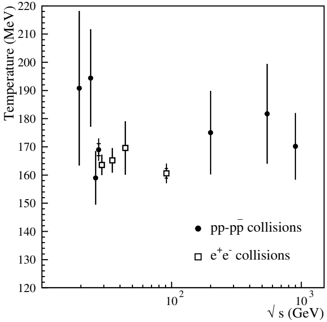




- As $\sqrt{s_{NN}}$ increases $\mu_B \to 0$
- Are hadrons directly produced at equilibrium in going from QGP to HG? See studies of hadronization in e^+e^- and $p\bar{p}$ collisions (F. Becattini, Z.Phys. C69 485-492 and Z.Phys. C76 269-286)

From l-QCD susceptibilities to freeze-out parameters

If the experimental fluctuations of conserved charges (baryonic and electric) are of thermal origin, assuming that one is able to correct for non-thermal effects (efficiency, kinematic cuts, neutral particle...), by connecting the cumulants of their distributions with lattice-QCD results for generalized susceptibilities one should be able to estimate the chemical freeze-out parameters $T_{\rm fo}$ and $\mu_{\rm fo}$ (see F. Karsch, Central Eur. J. Phys. 10, 1234 (2012)).. In fact, although l-QCD results are available only for zero chemical potential, one can perform a Taylor expansion of the susceptibilities around $\mu_B = 0$, e.g.

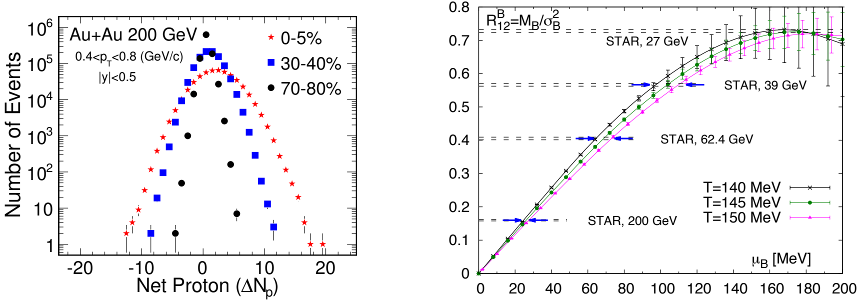
$$\chi^B_{2,\mu_B} = \chi^B_2 + \frac{1}{2}\chi^B_4\left(\frac{\mu_B}{T}\right)^2 + \dots \quad \chi^B_{1,\mu_B} = \chi^B_2\left(\frac{\mu_B}{T}\right) + \frac{1}{6}\chi^B_4\left(\frac{\mu_B}{T}\right)^3 + \dots$$

Considering the variance of the experimental baryon-number distribution one gets for instance

$$\frac{\langle B^2\rangle_c}{\langle B\rangle} = \frac{\chi^B_{2,\mu_B}}{\chi^B_{1,\mu_B}} = \frac{T}{\mu_B}\left[\frac{1+\frac{1}{2}(\chi^B_4/\chi^B_2)\left(\frac{\mu_B}{T}\right)^2+\dots}{1+\frac{1}{6}(\chi^B_4/\chi^B_2)\left(\frac{\mu_B}{T}\right)^2+\dots}\right],$$

allowing one to estimate μ_B/T.

From l-QCD susceptibilities to freeze-out parameters



- FO parameters fixed to reproduce higher-order cumulants of electric-charge and baryon number (exp: net proton!) distributions (P. Alba et al., PLB 738 (2014) 305-310)

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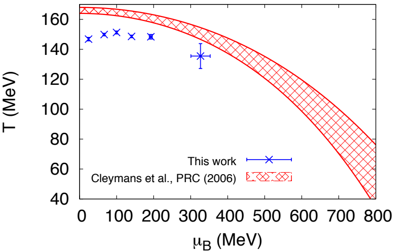
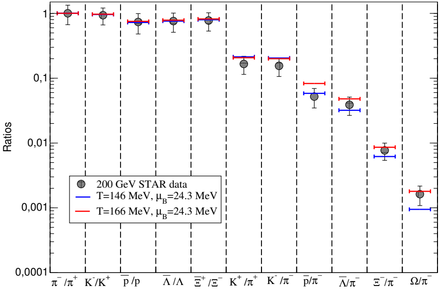


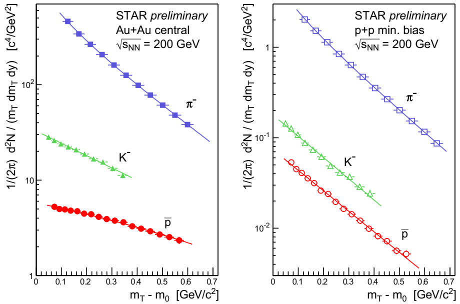


- FO parameters fixed to reproduce higher-order cumulants of electric-charge and baryon number (exp: net proton!) distributions (P. Alba et al., PLB 738 (2014) 305-310)
- Resulting FO temperature smaller than the one fixed by relative hadron yields (S. Borsanyi et al., PRL 113 (2014) 052301)
- Tension between proton and strange baryons: different freeze-out temperatures (R. Bellwied et al., PRL 111 (2013) 202302)? Missing states in the PDG?

Relativistic Hydrodynamics and HIC's: theoretical achievements and phenomenological successes

- Hadron momentum and azimuthal distributions consistent with the picture of the expansion of an *almost ideal fluid*
- Development of a consistent relativistic formulation of hydrodynamic equations in the presence of *dissipative effects*; derivation of the universal lower bound $\eta/s = 1/4\pi$ for the viscosity to entropy-density ratio, in rough agreement with the data
- Study of higher flow-harmonics and event-by-event fluctuations
- Discovery of collective effects in small systems, such as high-multiplicity p-Pb and d-Au collisions (also p-p?)

Hydro predictions: radial flow (I)



$$\frac{dN}{m_T dm_T} \sim e^{-m_T/\mathrm{T}_{\mathrm{slope}}} \equiv e^{-\sqrt{p_T^2+m^2}/\mathrm{T}_{\mathrm{slope}}}$$

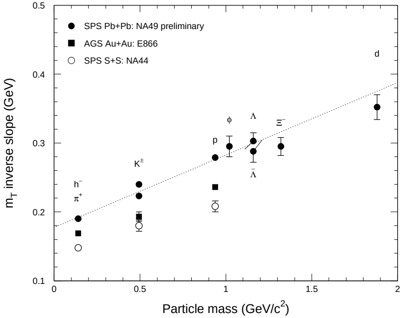
where $m_T \equiv \sqrt{p_T^2 + m^2}$

- $T_{\mathrm{slope}}(\sim 167\,\mathrm{MeV})$ *universal* for all hadrons in pp collisions (consistent with the *thermal* nature of particle production already suggested by hadron yields);
- T_{slope} *growing with m* in AA collisions: spectrum gets harder!

Hydro predictions: radial flow (II)

Physical interpretation:

Thermal emission on top of a collective flow



$$\frac{1}{2} m \langle \mathbf{v}_\perp^2 \rangle = \frac{1}{2} m \left\langle \left(\mathbf{v}_{\perp th} + \mathbf{v}_{\perp flow} \right)^2 \right\rangle = \frac{1}{2} m \langle \mathbf{v}_{\perp th}^2 \rangle + \frac{1}{2} m \mathbf{v}_{\perp flow}^2$$

$$\Longrightarrow \quad T_{\rm slope} = T_{\rm fo} + \frac{1}{2} m \mathbf{v}_{\perp flow}^2$$

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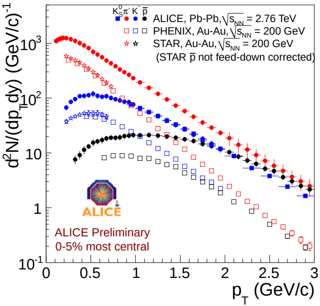


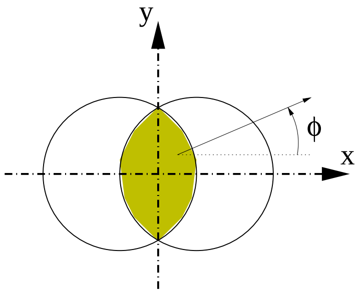
$$\frac{1}{2}m\langle \mathbf{v}_\perp^2 \rangle = \frac{1}{2}m\left\langle (\mathbf{v}_{\perp th} + \mathbf{v}_{\perp flow})^2 \right\rangle$$

$$= \frac{1}{2}m\langle \mathbf{v}_{\perp th}^2 \rangle + \frac{1}{2}m\mathbf{v}_{\perp flow}^2$$

$$\Longrightarrow \quad T_{\text{slope}} = T_{\text{fo}} + \frac{1}{2}m\mathbf{v}_{\perp flow}^2$$

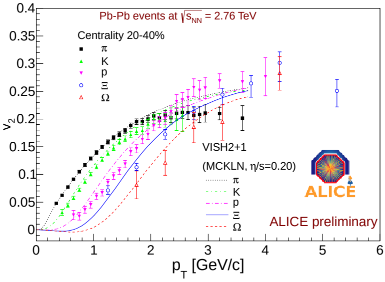
Radial flow gets larger going from RHIC to LHC!

Hydrodynamic behavior: elliptic flow



- In *non-central collisions* particle emission is not azimuthally-symmetric!

Hydrodynamic behavior: elliptic flow



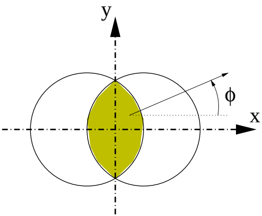
- In *non-central collisions* particle emission is not azimuthally-symmetric!
- The effect can be quantified through the *Fourier coefficient v_2*

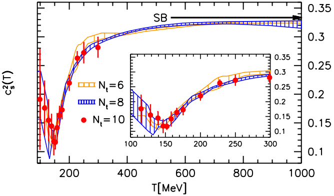
$$\frac{dN}{d\phi} = \frac{N_0}{2\pi}\left(1 + 2v_2\cos[2(\phi - \psi_{RP})] + \ldots\right)$$

$$v_2 \equiv \langle \cos[2(\phi - \psi_{RP})]\rangle$$

- $v_2(p_T) \sim 0.2$ gives a modulation 1.4 vs 0.6 for in-plane vs out-of-plane particle emission!

Elliptic flow: physical interpretation





- Matter behaves like a fluid whose *expansion* is *driven by pressure gradients*

$$(\epsilon + P)\frac{dv^i}{dt} \underset{v \ll c}{=} -\frac{\partial P}{\partial x^i} \quad \text{(Euler equation)}$$

- Spatial anisotropy is converted into momentum anisotropy;
- At freeze-out *particles* are *mostly emitted along the reaction-plane*.
- It provides information on the EOS of the produced matter (Hadron Gas vs QGP) through the *speed of sound*: $\vec{\nabla} P = c_s^2 \vec{\nabla} \epsilon$

Hydrodynamics: the general setup

- Hydrodynamics is applicable in a situation in which $\lambda_{\rm mfp} \ll L$
- In this limit the behavior of the system is entirely governed by the *conservation laws* (5 eqs. for 6 unknowns: P, ϵ, n_B, v^i)

$$\underbrace{\partial_\mu T^{\mu\nu} = 0}_{\rm four-momentum}\ , \qquad \underbrace{\partial_\mu j_B^\mu = 0}_{\rm baryon\ number}\ ,$$

where

$$T^{\mu\nu} = (\epsilon + P)u^\mu u^\nu - P g^{\mu\nu}, \quad j_B^\mu = n_B u^\mu \quad \text{and} \quad u^\mu = \gamma(1, \vec{v})$$

NB: at rest $u^\mu = (1, \vec{0})$ and $T^{\mu\nu} = {\rm diag}(\epsilon, P, P, P)$.

- Information on the medium is *entirely encoded* into the EOS

$$P = P(\epsilon, n_B) \quad (6^{\rm th}\ eq.)$$

- The transition from fluid to particles occurs at the *freeze-out hypersuface* $\Sigma^{\rm fo}$ (e.g. at $T = T_{\rm fo}$)

$$E(dN/d\vec{p}) = \int_{\Sigma^{\rm fo}} p^\mu d\Sigma_\mu \exp[-(p \cdot u)/T]$$

Relativistic hydrodynamics: the ideal case

In the absence of non-vanishing conserved charges ($n_B = 0$), the evolution of an *ideal fluid* is completely described by the *conservation of the ideal energy-momentum tensor*:

$$\partial_\mu T^{\mu\nu} = 0, \quad \text{where} \quad T^{\mu\nu} = T^{\mu\nu}_{\rm eq} = (\epsilon + P)u^\mu u^\nu - P g^{\mu\nu}$$

It is convenient to project the above equations

- along the fluid velocity ($u_\nu \partial_\mu T^{\mu\nu} = 0$)

$$D\epsilon = -(\epsilon + P)\Theta, \quad (\text{with} \quad \underbrace{D \equiv u^\mu \partial_\mu}_{\text{comov. derivative}} \quad \text{and} \quad \underbrace{\Theta \equiv \partial_\mu u^\mu}_{\text{expansion rate}})$$

- and perpendicularly to it ($\Delta_{\alpha\nu}\partial_\mu T^{\mu\nu} = 0$, with $\underbrace{\Delta_{\alpha\nu} \equiv g_{\alpha\nu} - u_\alpha u_\nu}_{\text{transv. project.}}$)

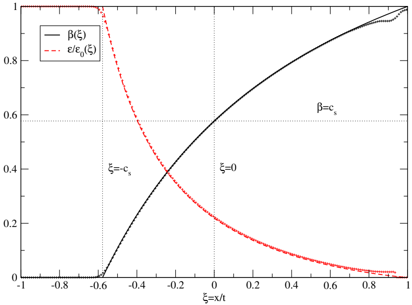
$$(\epsilon + P) D u^\alpha = \nabla^\alpha P \quad (\text{with } \nabla^\alpha \equiv \Delta^{\alpha\mu}\partial_\mu),$$

which is the *relativistic* version of the *Euler equation* (fluid acceleration driven by pressure gradients)

$$\text{non relativistic limit :} \quad \underbrace{(\epsilon + P)}_{\approx \rho} \underbrace{(\partial_t + v^k \partial_k)}_{\equiv d/dt} \vec{v} = -\vec{\nabla} P$$

The Riemann problem: rarefaction wave

As an example of plasma expanding in the vacuum we show the Riemann problem, i.e. a flow which starts from an initial condition of the kind ϵ_L, P_L, u_L^μ for $x < 0$ and ϵ_R, P_R, u_R^μ for $x > 0$. It is also very important for numerical implementations of hydrodynamic equations, which should be able to capture shocks. As an initial condition we take $\epsilon(0, \vec{x}) = \epsilon_0 \theta(-x)$ and $\vec{v}=0$. The solution is a function of the *self-similar* variable $\xi \equiv x/t$



- Curves: analytic solution
- Crosses: numerical results, with the ECHO-QGP code (L. Del Zanna et al., EPJC 73 (2013) 2524)

The head of the rarefaction wave propagates backwards with velocity $\xi_{\rm rw} = -c_s$. In the region $\xi < -c_s$ the fluid is still unperturbed ($\epsilon = \epsilon_0$ and $\vec{v}=0$), while at the origin $\beta(0) = c_s$ ($c_s = 1/\sqrt{3}$ for an ideal EoS!)

Viscous hydrodynamics

Better flow measurements required the introduction of *viscous corrections* to the energy-momentum tensor and charge current in order to reproduce the data. In the Landau frame, in which there is no heat-flow, one has

$$T^{\mu\nu} = T^{\mu\nu}_{\rm eq} + \pi^{\mu\nu} - \Pi\Delta^{\mu\nu} \quad \text{and} \quad j^\mu_B = j^\mu_{B({\rm eq})} + V^\mu_B,$$

From energy-momentum conservation $\partial_\mu T^{\mu\nu} = 0$ one gets

- Projecting along u_ν:

$$\boxed{D\epsilon + (\epsilon + P + \Pi)\Theta - \pi^{\mu\nu}\nabla_{<\mu}u_{\nu>} = 0},$$

  after replacing $\nabla_\mu u_\nu \longrightarrow \nabla_{<\mu}u_{\nu>} \equiv \frac{1}{2}(\nabla_\mu u_\nu + \nabla_\nu u_\mu) - \frac{1}{3}\Delta_{\mu\nu}\Theta$

- Projecting along $\Delta_{\alpha\nu}$:

$$\boxed{(\epsilon + P + \Pi)Du^\alpha = \nabla^\alpha(P + \Pi) - \Delta^\alpha_\nu \partial_\mu \pi^{\mu\nu}}$$

From baryon-number conservation $\partial_\mu j^\mu_B = 0$ one has

$$\boxed{Dn_B + n_B\Theta + \partial_\mu V^\mu_B = 0}$$

Fixing the viscous tensor: first order formalism ($n_B = 0$)

A way to fix the viscous tensor is through the $2^{\rm nd}$ law of thermodynamics, imposing $\partial_\mu s^\mu \geq 0$. Using the *ideal result* for the entropy current $s^\mu = s u^\mu$ and employing the thermodynamic relations

$$T s = \epsilon + P \quad \text{and} \quad T\, ds = d\epsilon$$

one gets

$$\partial_\mu s^\mu = u^\mu \partial_\mu s + s\, \partial_\mu u^\mu = \frac{1}{T}[D\epsilon + (\epsilon + P)\Theta] \geq 0$$

Employing

$$D\epsilon = -(\epsilon + P + \Pi)\Theta + \pi^{\mu\nu} \nabla_{<\mu} u_{\nu>},$$

one gets

$$\partial_\mu s^\mu = \frac{1}{T}\left[-\Pi\Theta + \pi^{\mu\nu} \nabla_{<\mu} u_{\nu>}\right] \geq 0$$

which is identically satisfied if (relativistic Navier Stokes result)

$$\Pi = -\zeta\Theta \quad \text{and} \quad \pi^{\mu\nu} = 2\eta \nabla^{<\mu} u^{\nu>},$$

where ζ and η are the bulk and shear *viscosity coefficients*.

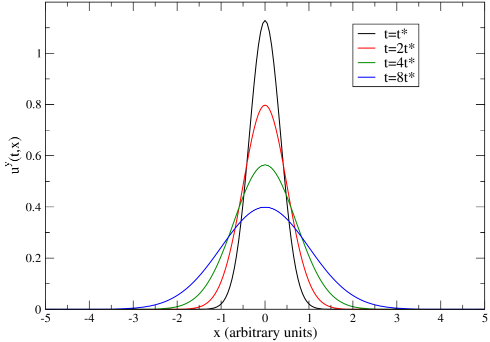
Diffusion of shear perturbations: causality problems

The propagation of shear perturbations

$$\delta u^y(t,x) \quad \text{with} \quad \delta u^y(t=0,x) \equiv \delta u_0^y(x) = \delta u_0\,\delta(x)$$

leads to the parabolic *diffusion equation*

$$(\epsilon_0 + P_0)\partial_t \delta u^y - \eta_0 \partial^2_{xx} \delta u^y = 0$$



In response to the initial perturbation one gets a *non-vanishing* fluid velocity even in *causally disconnected* regions (i.e. $x > ct$)!

Relativistic causal theory: second order formalism

The naive relativistic generalization of the Navier Stokes equations violates causality! This pathology can be cured *including viscous corrections into the entropy current*, of second order in the gradients (Israel-Stewart theory):

$$s^\mu = s^\mu_{\rm eq} + Q^\mu = su^\mu - \left(\beta_0\Pi^2 + \beta_2\pi_{\alpha\beta}\pi^{\alpha\beta}\right)\frac{u^\mu}{2T}$$

One gets then ($Df \equiv \dot{f}$):

$$T\partial_\mu s^\mu = \Pi\left[-\Theta - \beta_0\dot{\Pi} - T\Pi\,\partial_\mu(\beta_0 u^\mu/2T)\right] + \pi^{\alpha\beta}\left[\nabla_{<\alpha}u_{\beta>} - \beta_2\dot{\pi}_{\alpha\beta} - T\pi_{\alpha\beta}\,\partial_\mu(\beta_2 u^\mu/2T)\right] \geq 0,$$

which is satisfied if $\Pi \approx \zeta[-\Theta - \beta_0\dot{\Pi}]$ and $\pi_{\alpha\beta} \approx 2\eta[\nabla_{<\alpha}u_{\beta>} - \beta_2\dot{\pi}_{\alpha\beta}]$. One has then to evolve also the components of the viscous tensor (6 independent equations, due to $u_\mu\pi^{\mu\nu} = 0$ and $\pi^\mu_\mu = 0$)

$$\dot{\Pi} \approx -\frac{1}{\zeta\beta_0}[\Pi + \zeta\Theta] \quad \text{and} \quad \dot{\pi}_{\alpha\beta} \approx -\frac{1}{2\eta\beta_2}[\pi_{\alpha\beta} - 2\eta\nabla_{<\alpha}u_{\beta>}],$$

whera $\tau_\Pi \equiv \zeta\beta_0$ and $\tau_\pi \equiv 2\eta\beta_2$ play the role of *relaxation times*.

Linear perturbations: causality restoration

The equations to solve are now

$$(\epsilon_0 + P_0)\partial_t \delta u^y + \partial_x \pi^{xy} = 0 \quad \text{and} \quad \tau_\pi \partial_t \pi^{xy} + \pi^{xy} = -\eta_0 \partial_x \delta u^y$$

which in Fourier space read

$$(\epsilon_0 + P_0)(-i\omega)\delta u^y_{\omega,k} + (ik)\delta\pi^{xy}_{\omega,k} = 0$$
$$(1 - i\omega\tau_\pi)\delta\pi^{xy}_{\omega,k} + (ik)\eta_0 \delta u^y_{\omega,k} = 0$$

$$\text{Solving for } \delta u^y_{\omega,k} \text{ one gets} \quad \omega = -i\frac{\eta_0}{\epsilon_0 + P_0}\frac{k^2}{1 - i\omega\tau_\pi}$$

The *dispersion relation* of the shear mode is then

$$\omega = \frac{-i \pm \sqrt{-1 + 4[\eta_0/(\epsilon_0 + P_0)]k^2\tau_\pi}}{2\tau_\pi}$$

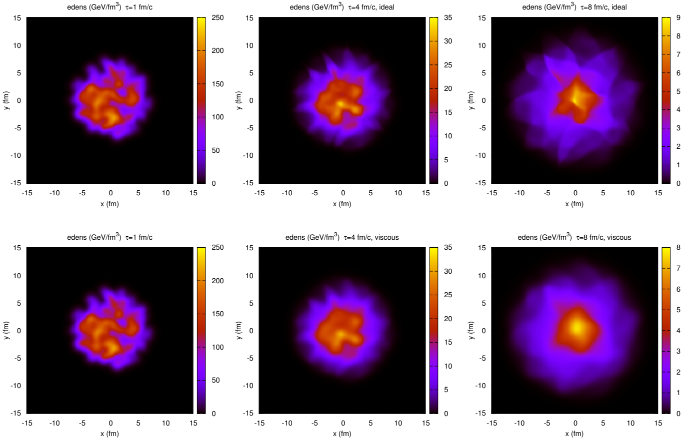
Its short-wavelength limit is given by:

$$\omega_k \underset{k\to\infty}{\sim} \sqrt{\frac{\eta_0}{\epsilon_0 + P_0}\frac{1}{\tau_\pi}}\,k \quad \Longrightarrow \quad v^T \equiv \frac{d\omega_k}{dk} \underset{k\to\infty}{\sim} \sqrt{\frac{\eta_0}{\epsilon_0 + P_0}\frac{1}{\tau_\pi}}$$

For a conformal fluid of massless particles the relaxation time is $\tau_\pi = 5\left(\frac{\eta_0}{s_0}\right)\frac{1}{T_0} = 5\frac{\eta_0}{\epsilon_0 + P_0}$, so that $v^T < c$!

Ideal vs viscous evolution

As an example, starting from the *same initial condition* (an ultra-central Au-Au collisions at $\sqrt{s_{\rm NN}} = 200 GeV$), we display the different evolution of the energy density in the ideal (upper panels) and viscous (lower panels) case



Viscosity damps short-wavelength modes!

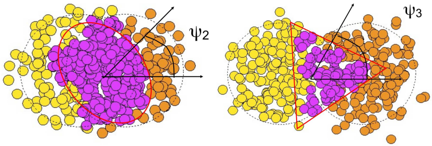
The QGP viscosity

From the comparison with the data one gets values for the *shear viscosity* close to the *universal lower bound* $\eta/s \approx 1/4\pi$ predicted by the AdS/CFT correspondence.
One can compare this with the values found for all the other known fluids:

fluid	P [Pa]	T [K]	η [Pa·s]	η/n [$\hbar$]	η/s [$\hbar/k_B$]
H_2O	$0.1\cdot10^6$	370	$2.9\cdot10^{-4}$	85	8.2
^{4}He	$0.1\cdot10^6$	2.0	$1.2\cdot10^{-6}$	0.5	1.9
H_2O	$22.6\cdot10^6$	650	$6.0\cdot10^{-5}$	32	2.0
^{4}He	$0.22\cdot10^6$	5.1	$1.7\cdot10^{-6}$	1.7	0.7
^{6}Li ($a=\infty$)	$12\cdot10^{-9}$	$23\cdot10^{-6}$	$\leq 1.7\cdot10^{-15}$	≤ 1	≤ 0.5
QGP	$88\cdot10^{33}$	$2\cdot10^{12}$	$\leq 5\cdot10^{11}$		≤ 0.4

leading to the conclusion that the QGP looks like the most *ideal fluid* ever observed

Event by event fluctuations



- Due to event-by-event fluctuations (e.g. of the nucleon positions) the initial density distribution is not smooth and can display *higher deformations*, each one with a *different azimuthal orientation*.
- Higher harmonics ($m > 2$) contribute to the angular distribution

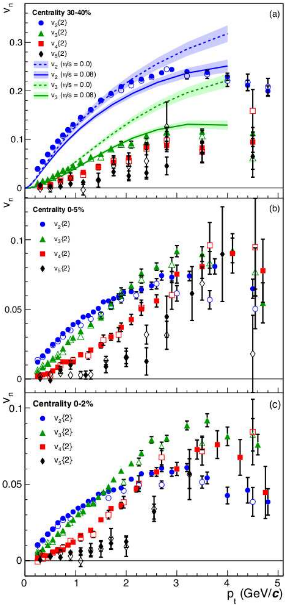
$$\frac{dN}{d\phi} = \frac{N}{2\pi}\left(1 + 2\sum_m v_m \cos[m(\phi - \psi_m)]\right)$$

of the final hadrons, where *for each event*,

$$v_m = \langle \cos[m(\phi - \psi_m)] \rangle \quad \text{and} \quad \psi_m = \frac{1}{m} \arctan \frac{\sum_i w_i \sin(m\phi_i)}{\sum_i w_i \cos(m\phi_i)}$$

The choice $w_i = p_T^i$ for the weights increase the resolution on ψ_m (one deals with a *finite number* of hadrons!)

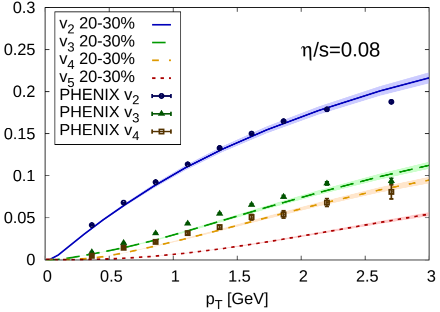
Event-by-event fluctuations: experimental consequences



Fluctuating initial conditions give rise to[a]:

- Non-vanishing v_2 in central collisions;
- Odd harmonics (v_3 and v_5)

Hydro can reproduce also higher harmonics[b]



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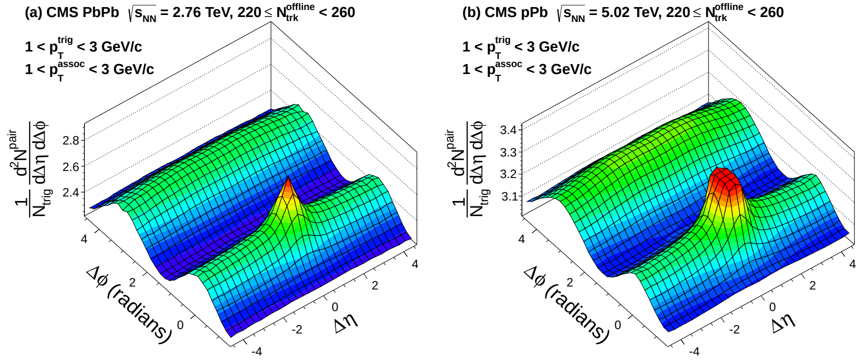
[a]ALICE, Phys.Rev.Lett. 107 (2011) 032301

[b]B: Schenke *et al.*, PRC 85, 024901 (2012)

Hydrodynamic behavior in small systems?



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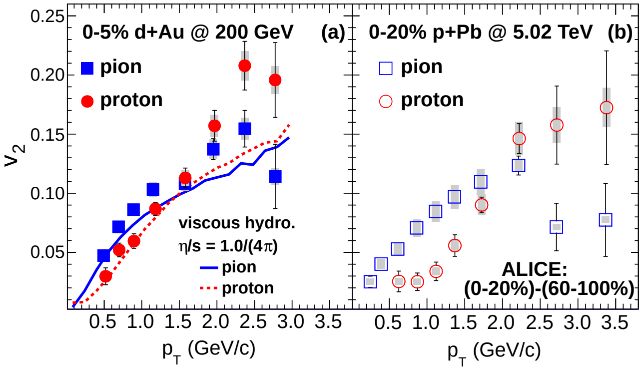

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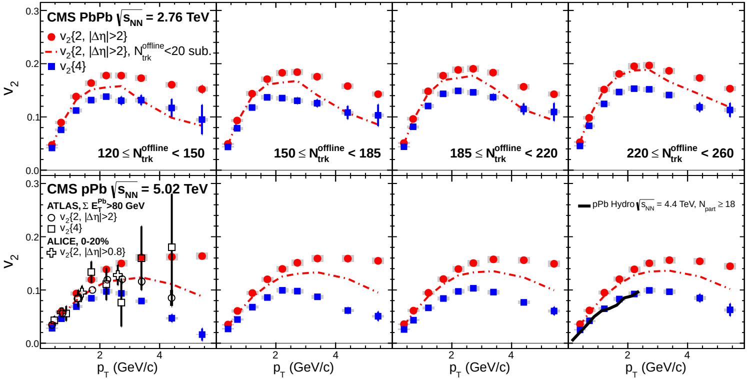

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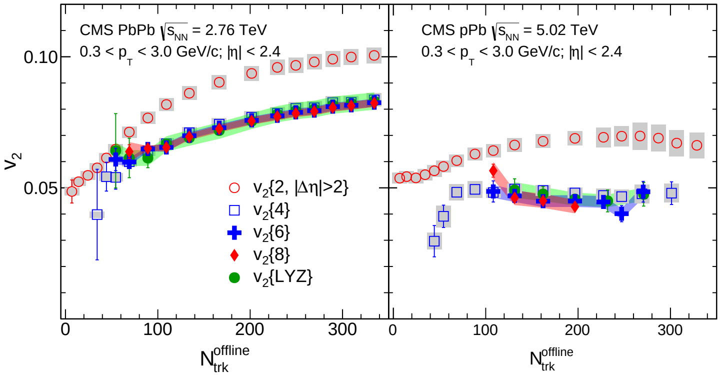

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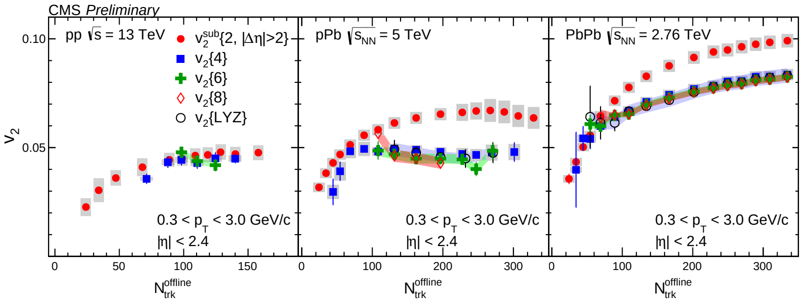

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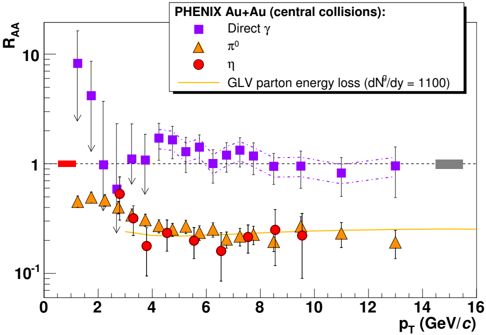
The medium is opaque: Jet-quenching in HIC's

Inclusive hadron spectra: the nuclear modification factor



$$R_{AA} \equiv \frac{\left(dN^h/dp_T\right)^{AA}}{\langle N_{\rm coll}\rangle \left(dN^h/dp_T\right)^{pp}}$$

Hard-photon $R_{AA} \approx 1$ and high-p_T hadron R_{pA}

- supports the Glauber picture (binary-collision scaling);
- entails that quenching of inclusive hadron spectra is a *final state effect* due to in-medium energy loss.

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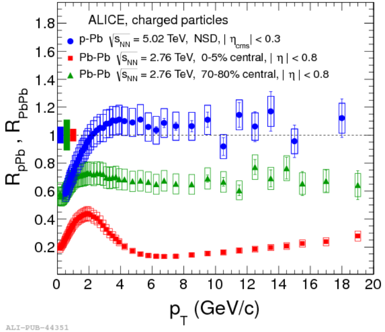

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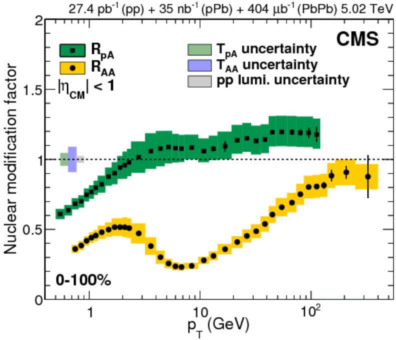

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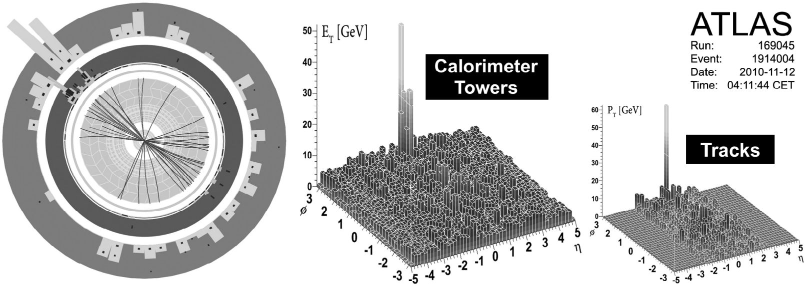
Di-jet imbalance at LHC: looking at the event display

An important fraction of events display a *huge mismatch* in E_T between the leading jet and its away-side partner



Possible to observe event-by-event, without any analysis!

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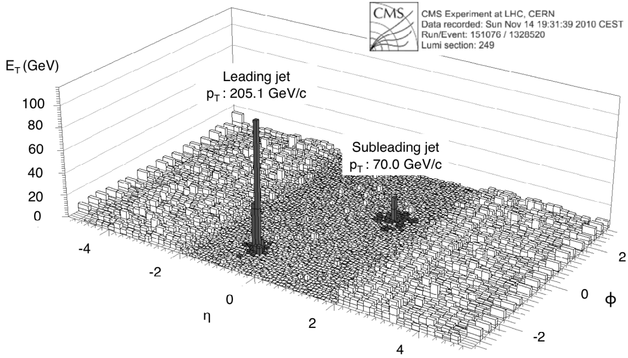
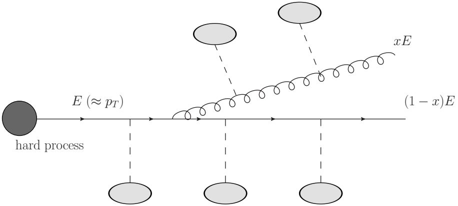


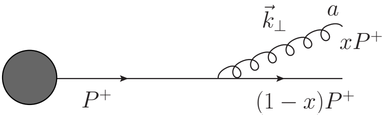
Possible to observe event-by-event, without any analysis!

Physical interpretation of the data: energy-loss *at the parton level*!



- Interaction of the high-p_T parton with the *color field of the medium* induces the radiation of (mostly) *soft* ($\omega \ll E$) and *collinear* ($k_\perp \ll \omega$) gluons;
- Radiated gluon can further re-scatter in the medium (cumulated $\mathbf{q}_\perp$ favor *decoherence* from the projectile).

QCD radiation in the vacuum



An accelerated color-charge produced in a hard event gets rid of its virtuality radiating gluons:

$$d\sigma_{\rm vac}^{\rm rad} \underset{x\to 0}{\sim} d\sigma^{\rm hard}\frac{\alpha_s}{\pi^2}C_R\frac{dk^+}{k^+}\frac{d\mathbf{k}}{\mathbf{k}^2}$$

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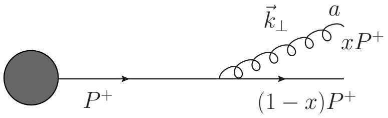

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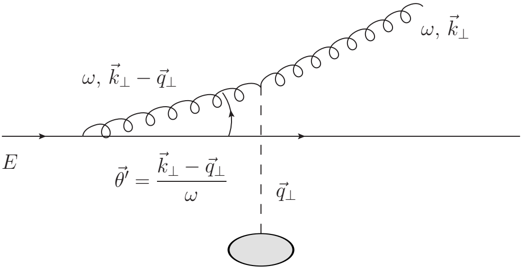
- Radiation spectrum (our benchmark): IR and collinear divergent!
- $k_\perp$ *vs* virtuality: $\mathbf{k}^2 = x\,(1-x)\,Q^2$;
- Time-scale (*formation time*) for gluon radiation:

$$\Delta t_{\rm rad} \sim Q^{-1}(E/Q) \sim \frac{\omega}{\mathbf{k}^2} \sim \frac{1}{\omega\theta^2} \quad (x \approx \omega/E)$$

Radiation of soft/collinear gluons takes a long time! Formation times will become important in the presence of a medium, whose thickness L will provide a scale to compare with!

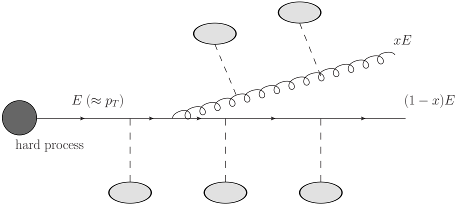
Medium-induced gluon radiation



High-energy partons can exchange momentum and color with the medium and this can induce further gluon radiation. In particular, radiated gluons can rescatter in the medium and this modifies their radiation amplitude and formation time:

$$\frac{\omega}{\mathbf{k}^2} \quad \longrightarrow \quad \frac{\omega}{(\mathbf{k}-\mathbf{q})^2}$$

Some heuristic estimates



In general the projectile system (high-E parton + rad. gluon) can interact several times with the medium. One can then estimate the *gluon formation-length* as

$$l_{\rm f} \sim \frac{\omega}{(\mathbf{k}-\mathbf{q})^2} \;\longrightarrow\; l_{\rm f} \sim \frac{\omega}{(\mathbf{k}-\sum_n \mathbf{q}_n)^2} \approx \frac{\omega}{N_{\rm scatt}\langle \mathbf{q}_n^2\rangle} = \frac{\omega}{l_{\rm f}\,\langle \mathbf{q}_n^2\rangle/\lambda_{\rm mfp}}.$$

Hence, one can identify $l_{\rm f} \equiv \sqrt{\omega/\hat{q}}$: soft gluon are formed earlier!

From $1 = \hbar c = 0.1973\,\mathrm{GeV}\cdot\mathrm{fm} \longrightarrow 1\,\mathrm{GeV}\cdot\mathrm{fm} \approx 5$...

- Gluon radiation is suppressed if $l_{\mathrm{form}}(\omega) > L$, which occurs above the *critical frequency* ω_c. Medium induces radiation of gluons with

$$l_{\mathrm{form}}(\omega) = \sqrt{\omega/\hat{q}} < L \quad \longrightarrow \quad \omega < \omega_c \equiv \hat{q}L^2$$

  For $\hat{q} \approx 1$ GeV^2/fm and $L \approx 5$ fm one gets $\omega_c \approx 125$ GeV.

- One can estimate the *typical angle at which gluons are radiated*:

$$\langle \mathbf{k}^2 \rangle \approx \hat{q}\, l_{\mathrm{form}}(\omega) = \sqrt{\hat{q}\omega} \longrightarrow \langle \theta^2 \rangle = \frac{\langle \mathbf{k}^2 \rangle}{\omega^2} = \sqrt{\frac{\hat{q}}{\omega^3}} \longrightarrow \overline{\theta} = \left(\frac{\hat{q}}{\omega^3}\right)^{1/4}$$

  For a typical $\hat{q} \approx 1$ GeV^2/fm one has:

$$\omega = 2\,\mathrm{GeV} \longrightarrow \overline{\theta} \approx 0.4 \qquad \omega = 5\,\mathrm{GeV} \longrightarrow \overline{\theta} \approx 0.2$$

  Soft gluons radiated at larger angles!

- Below the *Bethe-Heitler* frequency ω_{BH} one has $l_{\mathrm{form}}(\omega) < \lambda_{\mathrm{mfp}}$ and coherence effects are no longer important:

$$l_{\mathrm{form}}(\omega_{\mathrm{BH}}) = \sqrt{\omega_{\mathrm{BH}}/\hat{q}} = \lambda_{\mathrm{mfp}} \quad \longrightarrow \quad \omega_{\mathrm{BH}} \equiv \hat{q}\lambda^2_{\mathrm{mfp}}$$

Energy-loss: heuristic derivation

Let us estimate the spectrum of radiated gluons *in the coherent regime* $\omega_{\rm BH} < \omega < \omega_c$. One has to express the medium thickness L *in units of the gluon formation length* $l_{\rm form} = \sqrt{\omega/\hat{q}}$, getting the *effective numbers of radiators*:

$$\omega \frac{dN_g}{d\omega} \sim \alpha_s C_R \frac{L}{l_{\rm form}(\omega)} = \alpha_s C_R \sqrt{\frac{\omega_c}{\omega}}$$

Hence, for the *average energy-loss* one gets:

$$\langle \Delta E \rangle \sim \alpha_s C_R \sqrt{\omega_c} \int_{\omega_{\rm BH}}^{\omega_c} \frac{d\omega}{\sqrt{\omega}} \underset{\omega_{\rm BH} \ll \omega_c}{\sim} \alpha_s C_R \omega_c = \alpha_s C_R \hat{q} L^2$$

- The result does not depend on E
- The dependence on L^2 reflect the coherent nature of the radiation (distructive interference)

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- The result does not depend on E
- The dependence on L^2 reflect the coherent nature of the radiation (distructive interference)

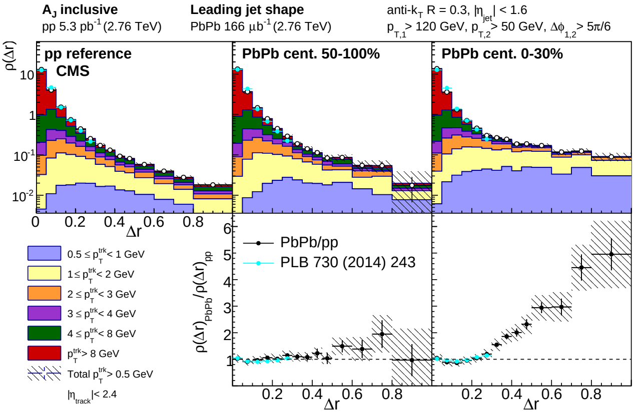
One can show that the contribution from the *incoherent regime* $\omega < \omega_{\rm BH}$ in which

$$\omega \frac{dN_g}{d\omega} \sim \alpha_s C_R \frac{L}{\lambda_{\rm mfp}}$$

is subleading by a factor $\lambda_{\rm mfp}/L$.

Modification of jet-shapes

One can study the fraction of jet-energy carried by the different tracks in rings at distance $\Delta r \equiv \sqrt{\Delta\phi^2 + \Delta\eta 2}$ from the jet-axis



Going from pp to central AA a sizable fraction of energy is carried away by soft tracks with a broad angular distribution

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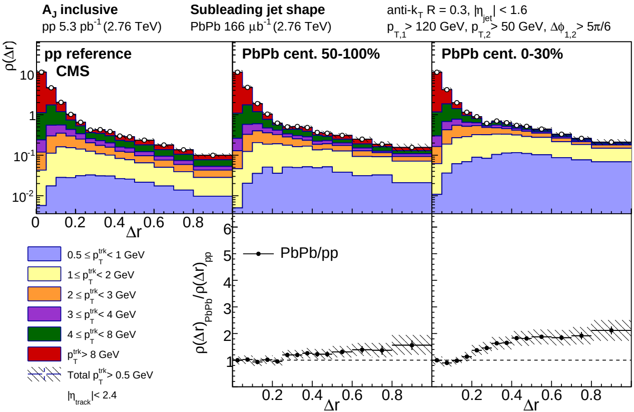
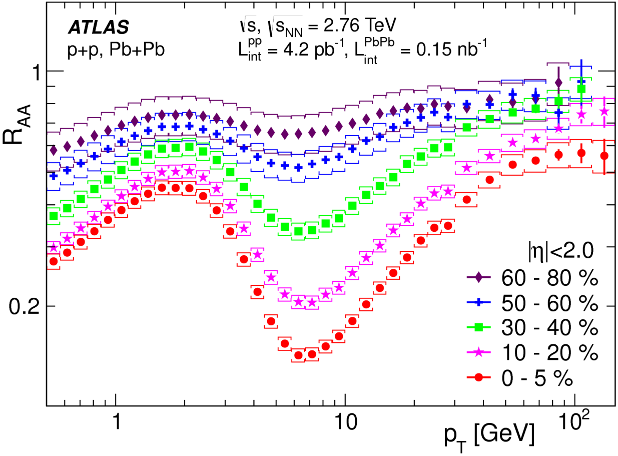


Going from pp to central AA a sizable fraction of energy is carried away by soft tracks with a broad angular distribution

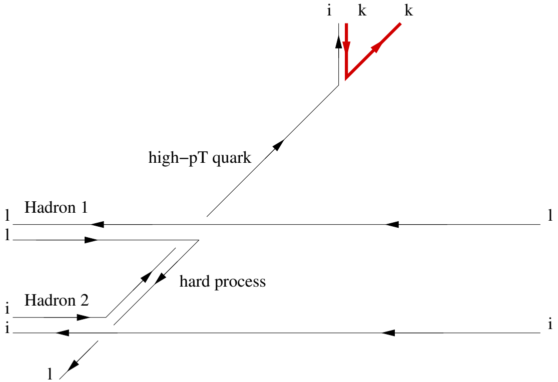
High-p_T particle suppression

Due to coherence effects, at high energy $\langle \Delta E \rangle$ becomes independent from E: this looks in agreement with the rise of the hadron R_{AA} with p_T



Medium-modification of color-flow

Hadronization convert “pre-confined” color-singlet objects into hadrons



In a vacuum parton-shower gluons remain color connected with the leading high-energy parton

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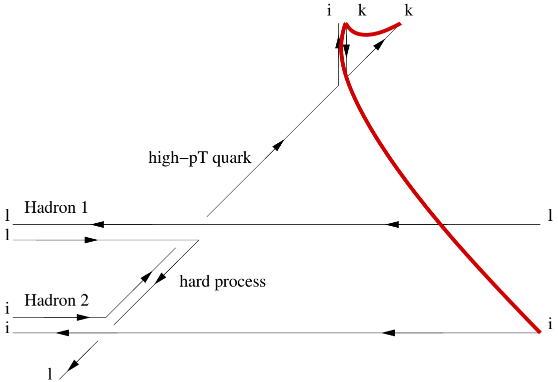

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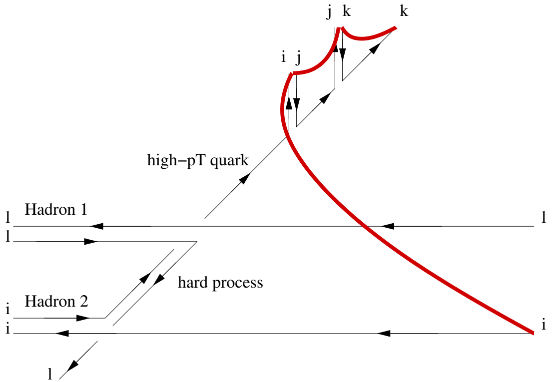
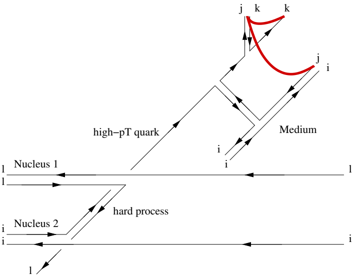


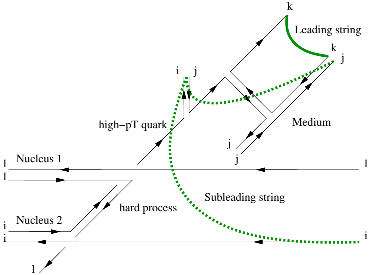
In a vacuum parton-shower gluons remain color connected with the leading high-energy parton

Medium-modification of color-flow

The interaction with the medium modifies the color-flow and hence can affect hadronization[3]



“Final State Radiation”
(gluon ∈ leading string)
Gluon contributes to leading hadron



“Initial State Radiation”
(gluon decohered: lost!)
Gluon contributes to *enhanced soft multiplicity* from subleading string

[3] A.B. et al., PRC 85 (2012) 031901 and JHEP 1207 (2012) 144

Heavy-flavor in HIC's

Heavy Flavour in the QGP: the conceptual setup

- Description of soft observables based on hydrodynamics, assuming to deal with a system close to local thermal equilibrium (no matter why);
- Description of jet-quenching based on energy-degradation of external probes (high-p_T partons);
- Description of heavy-flavour observables requires to employ/develop a setup (transport theory) allowing to deal with more general situations and in particular to describe *how particles would (asymptotically) approach equilibrium*.

NB At high-p_T the interest in heavy flavor is no longer related to thermalization, but to the study of the mass and color charge dependence of jet-quenching (not addressed in this talk)

Why are charm and beauty considered *heavy*?

- $M \gg \Lambda_{\rm QCD}$: their initial production is well described by pQCD
- $M \gg T$: thermal abundance in the plasma would be negligible; final multiplicity in the experiments (expanding fireball with lifetime $\sim$10 fm/c) set by the initial hard production
- $M \gg gT$, with gT being the *typical momentum exchange* in the collisions with the plasma particles: many soft scatterings necessary to change significantly the momentum/trajectory of the quark.

NB for realistic temperatures $g \sim 2$, so that one can wonder *whether a charm is really "heavy"*, at least in the initial stage of the evolution.

Transport theory: the Boltzmann equation

Time evolution of HQ phase-space distribution $f_Q(t,\mathbf{x},\mathbf{p})$[4]:

$$\frac{d}{dt} f_Q(t,\mathbf{x},\mathbf{p}) = C[f_Q]$$

- Total derivative along particle trajectory

$$\frac{d}{dt} \equiv \frac{\partial}{\partial t} + \mathbf{v}\frac{\partial}{\partial \mathbf{x}} + \mathbf{F}\frac{\partial}{\partial \mathbf{p}}$$

  Neglecting $\mathbf{x}$-dependence and mean fields: $\partial_t f_Q(t,\mathbf{p}) = C[f_Q]$

- Collision integral:

$$C[f_Q] = \int d\mathbf{k}[\underbrace{w(\mathbf{p}+\mathbf{k},\mathbf{k})f_Q(\mathbf{p}+\mathbf{k})}_{\text{gain term}} - \underbrace{w(\mathbf{p},\mathbf{k})f_Q(\mathbf{p})}_{\text{loss term}}]$$

  $w(\mathbf{p},\mathbf{k})$: HQ transition rate $\mathbf{p} \to \mathbf{p}-\mathbf{k}$

[4] For results based on BE see e.g. Catania-group studies

From Boltzmann to Fokker-Planck

Expanding the collision integral for *small momentum exchange*[5] (Landau)

$$C[f_Q] \approx \int d\mathbf{k} \left[k^i \frac{\partial}{\partial p^i} + \frac{1}{2} k^i k^j \frac{\partial^2}{\partial p^i \partial p^j} \right] [w(\mathbf{p}, \mathbf{k}) f_Q(t, \mathbf{p})]$$

[5] B. Svetitsky, PRD 37, 2484 (1988)

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The Boltzmann equation reduces to the *Fokker-Planck equation*

$$\frac{\partial}{\partial t} f_Q(t, \mathbf{p}) = \frac{\partial}{\partial p^i} \left\{ A^i(\mathbf{p}) f_Q(t, \mathbf{p}) + \frac{\partial}{\partial p^j} [B^{ij}(\mathbf{p}) f_Q(t, \mathbf{p})] \right\}$$

where

$$A^i(\mathbf{p}) = \int d\mathbf{k}\, k^i w(\mathbf{p}, \mathbf{k}) \longrightarrow \underbrace{A^i(\mathbf{p}) = A(p)\, p^i}_{\text{friction}}$$

$$B^{ij}(\mathbf{p}) = \frac{1}{2} \int d\mathbf{k}\, k^i k^j w(\mathbf{p}, \mathbf{k}) \longrightarrow \underbrace{B^{ij}(\mathbf{p}) = (\delta^{ij} - \hat{p}^i \hat{p}^j) B_0(p) + \hat{p}^i \hat{p}^j B_1(p)}_{\text{momentum broadening}}$$

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Problem reduced to the *evaluation of three transport coefficients*, directly derived from the scattering matrix

[5] B. Svetitsky, PRD 37, 2484 (1988)

Approach to equilibrium in the FP equation

The FP equation can be viewed as a continuity equation for the phase-space distribution of the kind $\partial_t \rho(t,\vec{p}) + \vec{\nabla}_p \cdot \vec{J}(t,\vec{p}) = 0$

$$\frac{\partial}{\partial t} \underbrace{f_Q(t,\mathbf{p})}_{\equiv \rho(t,\vec{p})} = \frac{\partial}{\partial p^i} \underbrace{\left\{ A^i(\mathbf{p}) f_Q(t,\mathbf{p}) + \frac{\partial}{\partial p^j} [B^{ij}(\mathbf{p}) f_Q(t,\mathbf{p})] \right\}}_{\equiv - J^i(t,\vec{p})}$$

admitting a steady solution $f_{\rm eq}(p) \equiv e^{-E_p/T}$ when the current vanishes:

$$A^i(\vec{p}) f_{\rm eq}(p) = -\frac{\partial B^{ij}(\vec{p})}{\partial p^j} f_{\rm eq}(p) - B^{ij}(\mathbf{p}) \frac{\partial f_{\rm eq}(p)}{\partial p^j}.$$

One gets

$$A(p) p^i = \frac{B_1(p)}{T E_p} - \frac{\partial}{\partial p^j} \left[\delta^{ij} B_0(p) + \hat{p}^i \hat{p}^j (B_1(p) - B_0(p)) \right],$$

leading to the Einstein *fluctuation-dissipation* relation

$$A(p) = \frac{B_1(p)}{T E_p} - \left[\frac{1}{p} \frac{\partial B_1(p)}{\partial p} + \frac{d-1}{p^2} (B_1(p) - B_0(p)) \right]$$

The relativistic Langevin equation

The Fokker-Planck equation can be recast into a form suitable to follow the dynamics of each individual quark: the Langevin equation

$$\frac{\Delta p^i}{\Delta t} = -\underbrace{\eta_D(p)p^i}_{\text{determ.}} + \underbrace{\xi^i(t)}_{\text{stochastic}} ,$$

with the properties of the noise encoded in

$$\langle \xi^i(\mathbf{p}_t)\xi^j(\mathbf{p}_{t'})\rangle = b^{ij}(\mathbf{p}_t)\frac{\delta_{tt'}}{\Delta t} \qquad b^{ij}(\mathbf{p}) \equiv \kappa_\parallel(p)\hat{p}^i\hat{p}^j + \kappa_\perp(p)(\delta^{ij} - \hat{p}^i\hat{p}^j)$$

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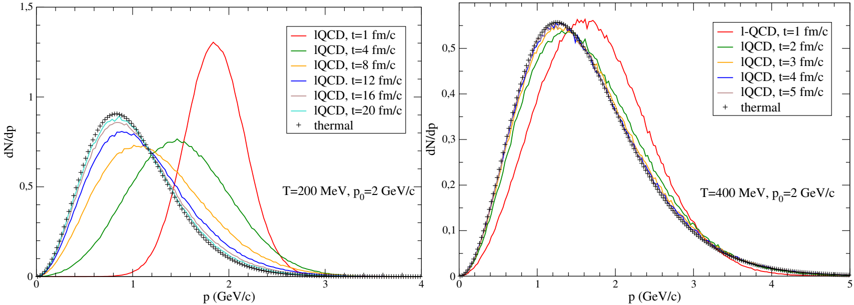
Transport coefficients to calculate:

- *Momentum diffusion* $\kappa_\perp \equiv \frac{1}{2}\frac{\langle \Delta p_\perp^2 \rangle}{\Delta t}$ and $\kappa_\parallel \equiv \frac{\langle \Delta p_\parallel^2 \rangle}{\Delta t}$;
- *Friction* term (dependent on the discretization scheme!)

$$\eta_D{}^{\text{Ito}}(p) = \frac{\kappa_\parallel(p)}{2TE_p} - \frac{1}{E_p^2}\left[(1-v^2)\frac{\partial \kappa_\parallel(p)}{\partial v^2} + \frac{d-1}{2}\frac{\kappa_\parallel(p) - \kappa_\perp(p)}{v^2}\right]$$

fixed in order to assure approach to equilibrium (Einstein relation):

A first check: thermalization in a static medium



(Test with a sample of c quarks with $p_0 = 2$ GeV/c).
For $t \gg 1/\eta_D$ one approaches a relativistic Maxwell-Jüttner distribution

$$f_{\rm MJ}(p) \equiv \frac{e^{-E_p/T}}{4\pi M^2 T\, K_2(M/T)}, \qquad \text{with} \int d^3p\, f_{\rm MJ}(p) = 1$$

The larger κ ($\kappa \sim T^3$), the faster the approach to thermalization.

From quarks to hadrons

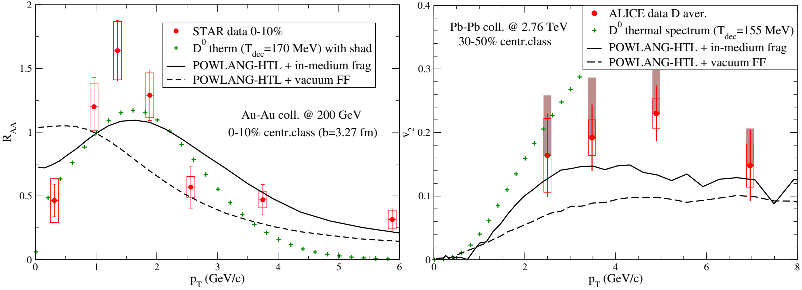
In the presence of a medium, rather then fragmenting like in the vacuum (e.g. $c \to cg \to c\overline{q}q$), HQ's can hadronize by recombining with light thermal partons from the medium. This has been implemented in several ways in the literature:

- $2 \to 1$ coalescence of partons close in phase-space: $Q + \overline{q} \to M$
- String formation: $Q + \overline{q} \to \mathrm{string} \to \mathrm{hadrons}$
- Resonance formation/decay $Q + \overline{q} \to M^{\star} \to Q + \overline{q}$

In-medium hadronization may affect the R_{AA} and v_2 of final D-mesons due to the *collective (radial and elliptic) flow* of light quarks. Furthermore, it can change the HF hadrochemistry, leading for instance to and enhanced productions of strange particles (D_s) and baryons (Λ_c): no need to excite heavy $s\overline{s}$ or diquark-antidiquark pairs from the vacuum as in elementary collisions, a lot of thermal partons available nearby! Selected results will be shown in the following.

From quarks to hadrons: effect on R_{AA} and v_2

Experimental data display a peak in the R_{AA} and a sizable v_2 one would like to interpret as a signal of *charm radial flow and thermalization* (green crosses: full thermal equilibrium, decoupling from FO hypersurface)



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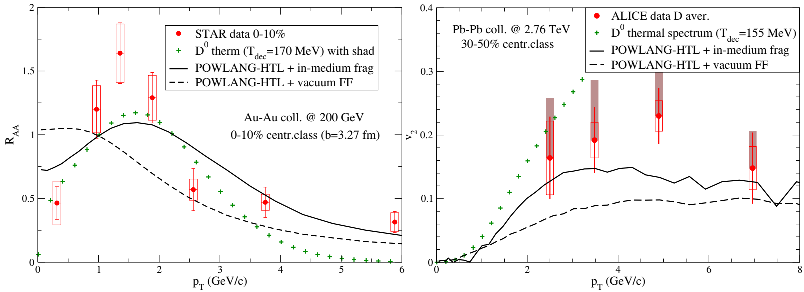
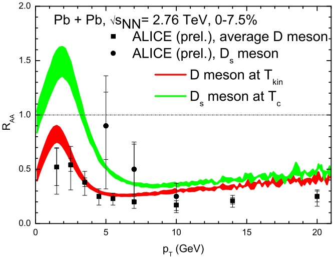


However, *comparing transport results with/without the boost* due to $u^{\mu}_{\rm fluid}$, at least part of the effect might be due to the radial and elliptic flow of the light partons from the medium picked-up at hadronization (POWLANG results A.B. et al., in EPJC 75 (2015) 3, 121).

In-medium hadronization and change in HF hadrochemistry

The abundance of strange quarks in the plasma can lead e.g. to an enhanced production of D_s mesons wrt p-p collisions via $c + \bar{s} \to D_s$



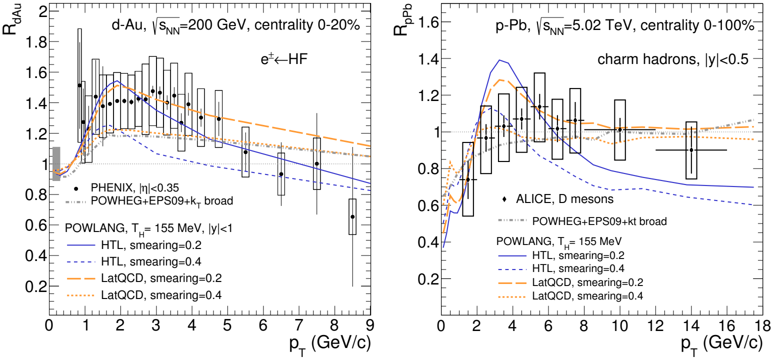
ALICE data for D and D_s mesons (A. Barbano for the ALICE Collaboration, J.Phys.Conf.Ser. 668 (2016) no.1, 012040) compared with TAMU-model predictions (M- He et al., PLB 735 (2014) 445)

Langevin transport simulation in the QGP + hadronization modeled via

$$\left(\partial_t + \vec{v} \cdot \vec{\nabla}\right) F_M(t, \vec{x}, \vec{p}) = -\underbrace{(\Gamma/\gamma_p) F_M(t, \vec{x}, \vec{p})}_{M \to Q + \bar{q}} + \underbrace{\beta(t, \vec{x}, \vec{p})}_{Q + \bar{q} \to M}$$

$$\text{with} \quad \sigma(s) = \frac{4\pi}{k^2} \frac{(\Gamma m)^2}{(s - m^2)^2 + (\Gamma m)^2}$$

Heavy-flavor in small systems



We display our predictions[6], with different initializations (source smearing) and transport coefficients (HTL vs lQCD), compared to

- HF-electron R_{dAu} by PHENIX at RHIC (left panel)
- D-mesons R_{pPb} by ALICE at the LHC (right panel)

[6] A.B. et al., JHEP 1603 (2016) 123

A window on topological aspects of QFT: the Chiral Magnetic Effect

In non-central high-energy nuclear collision huge magnetic fields $B \sim 10^{15}$ T are present during the first instants

- CME: conceptual setup[7]
- CME in condensed matter[8]
- CME in heavy-ion collisions: how to detect it?
- The necessity of a reliable decription of B+QGP evolution: RMHD

[7] D.E. Kharzeev et al. Prog.Part.Nucl.Phys. 88 (2016) 1

[8] Q. Li et al., Nature Phys. 12 (2016) 550

CME: $U_A(1)$ symmetry and quantum anomaly

La *massless* QCD Lagrangian is invariant under the $U_A(1)$ transformation

$$q \longrightarrow e^{-i\alpha\gamma^5} q, \quad \overline{q} \longrightarrow \overline{q} e^{-i\alpha\gamma^5} \quad (\text{since } \{\gamma^\mu, \gamma^5\} = 0)$$

rotating *by opposite angles* R and L components of the quark fields ($\gamma^5 q_{R/L} = \pm q_{R/L}$).
The symmetry *would be* associated to the conservation of the axial charge

$$Q_A = \int d^3x\, q^\dagger(x)\gamma^5 q(x) = \int d^3x\, [q_R^\dagger(x) q_R(x) - q_L^\dagger(x) q_L(x)] = N_R - N_L,$$

i.e. to the number of right-handed minus left-handed quarks.

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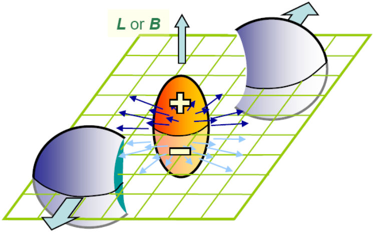
i.e. to the number of right-handed minus left-handed quarks.
However, although being a symmetry of the *classical* QCD action, $U_A(1)$ is not a symmetry of the theory, being broken by quantum fluctuations:

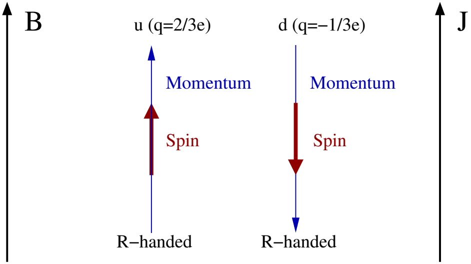
$$\frac{d}{dt}(N_R - N_L) = -N_f \frac{g^2}{16\pi^2} \int d^3x\, \frac{1}{2}\epsilon^{\alpha\beta\mu\nu} F^a_{\mu\nu} F^a_{\alpha\beta}$$

$$\equiv -N_f \frac{g^2}{16\pi^2} \int d^3x\, \widetilde{F}^{\alpha\beta,a} F^a_{\alpha\beta} \neq 0$$

Non-trivial topological configurations of the colour field can lead, event by event, to an excess of quarks of a given chirality (QCD anomaly)

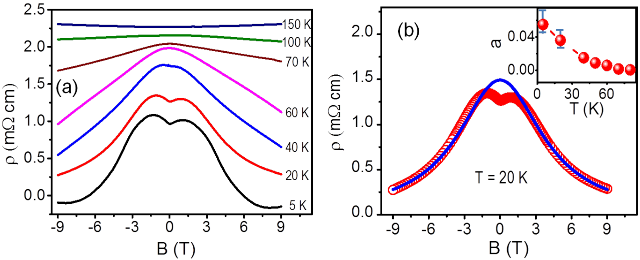
CME: the role of the magnetic field





- Huge magnetic field in the direction orthogonal to the reaction plane
- Spin of u/d quarks aligned/anti-aligned with $\vec{B}$
- Event-by-event, $U_A(1)$ anomaly leads to an excess of right or left-handed quarks
- For massless quarks chirality≡helicity $\longrightarrow$ if $N_R > N_L$ one has an excess of u-quarks moving upwards and d-quarks moving downwards: an electric current $\vec{J} \equiv \sigma_5 \vec{B}$ develops

CME in condensed matter



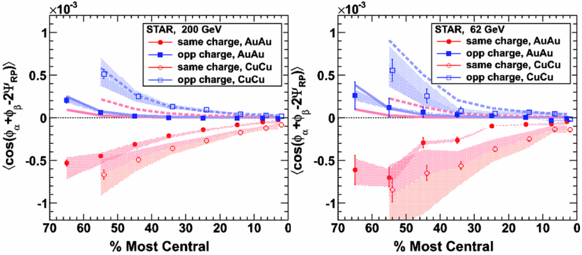
The discovery of Dirac semimetals opened the possibility of studying chiral fermions in condensed matter. Chiral imbalance induced by $\vec{E}||\vec{B}$, representing a non-trivial topological configuration ($\vec{E}\cdot\vec{B} \sim \widetilde{F}_{\mu\nu}F^{\mu\nu}$). Evolution of chiral charge-density (τ_V relaxation time for chirality-flip):

$$\frac{d\rho_5}{dt} = \frac{e^2}{4\pi^2\hbar^2 c}\vec{E}\cdot\vec{B} - \frac{\rho_5}{\tau_V} \quad \longrightarrow \quad \rho_5 \underset{t\gg\tau_V}{\sim} \frac{e^2\tau_V}{4\pi^2\hbar^2 c}\vec{E}\cdot\vec{B}$$

$$\text{From} \quad \rho_5 \sim \mu_5\left(T^2 + \frac{\mu^2}{\pi^2}\right) \quad \text{and} \quad \vec{J}_{\rm CME} = \frac{e^2}{2\pi^2}\mu_5\vec{B} \quad \text{one gets}$$

$$J^i_{\rm CME} \equiv \sigma^{ij}_{\rm CME}E^j \quad \longrightarrow \quad \sigma^{zz}_{\rm CME} \sim B^2 \quad \text{(see figure)}$$

CME in heavy-ion collisions: experimental evidence



Local $\mathcal{P}$-violation introduce odd terms in the single-particle distributions

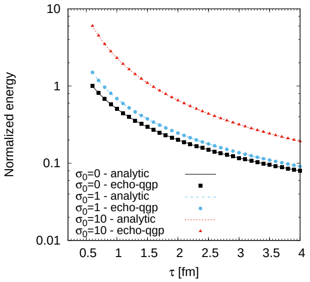
$$\frac{dN}{d\phi} \propto 1 + \sum_n 2v_n \cos[n(\phi - \psi_{RP})] + \sum_n 2a_n \sin[n(\phi - \psi_{RP})]$$

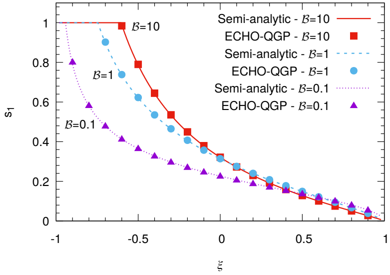
Experimentally one measures ($\alpha, \beta = \pm$ label the electric charge)

$$\langle\cos(\Delta\phi_\alpha + \Delta\phi_\beta)\rangle = \underbrace{\langle\cos(\Delta\phi_\alpha)\cos(\Delta\phi_\beta)\rangle}_{v_1^{(\alpha)}v_1^{(\beta)}+B_{in}} - \underbrace{\langle\sin(\Delta\phi_\alpha)\sin(\Delta\phi_\beta)\rangle}_{a_1^{(\alpha)}a_1^{(\beta)}+B_{out}}$$

where $v_1 \approx 0$ and most of the background cancels. In the presence of CME $a_+ = -a_-$ and one expects $\langle\cos(\Delta\phi_\alpha + \Delta\phi_\beta)\rangle$ negative/positive for same/opposite charge pairs. NB: other explanations are possible!

RMHD description of the magnetized QGP





Current studies of CME treat B as an external field, evolving as in the vacuum, with a rather fast decay. We have recently developed (G. Inghirami et al., EPJC 76 (2016) no.12, 659) a RMHD code, self-consistently solving the hydrodynamic and Maxwell equations:

$$d_\mu (T^{\mu\nu}_{\rm matt} + T^{\mu\nu}_{\rm field}) = 0, \quad d_\mu F^{\mu\nu} = -J^\nu \quad \text{and} \quad d_\mu F^{\star\mu\nu} = 0$$

The system is closed by the Ohm's law $j^\mu = \sigma^{\mu\nu} e_\nu$. We performed several tests in the ideal-conductor limit $e^\mu = 0$, i.e. vanishing electric field in the comoving frame. In the Figs. Bjorken-flow and self-similar expansion are