Two-photon form factors of the peseudoscalar mesons in Phokhara and Ekhara Monte Carlo generators

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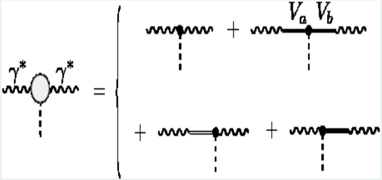
June 30, 2017

Outline

Motivation

- Monte Carlo generators base on reliable models are needed for data analysis and feasibility study.
- The knowledge of the transition form factors is important for calculation of hadronic light-by-light scattering part of anomalous magnetic moment of the muon.

$\gamma^*\gamma^*\mathcal{P}$ transition



$$\mathcal{L}_{\gamma V} = -e \sum_{i=1}^{3} f_{V_i} \partial_\mu B_\nu \left(\rho_i^{\tilde{\mu}\nu} + \frac{1}{3} \mathbf{F}_{\omega_i} \tilde{\omega}_i^{\mu\nu} - \frac{\sqrt{2}}{3} \mathbf{F}_{\phi_i} \tilde{\phi}_i^{\mu\nu} \right), \; \tilde{V}_{\mu\nu} \equiv \partial_\mu V_\nu - \partial_\nu V_\mu$$

$$\mathcal{L}_{V\gamma\mathcal{P}} = -\sum_{i=1}^{3} \frac{4\sqrt{2} e h_{V_i}}{3 f_\pi} \epsilon_{\mu\nu\alpha\beta} \partial^\alpha B^\beta \left(\rho_i^\mu + 3 \mathbf{H}_{\omega_i} \omega_i^\mu - \frac{3}{\sqrt{2}} \mathbf{A}_i^{\pi^0} \phi_i^\mu \right) \partial^\nu \pi^0 + \ldots$$

$$\mathcal{L}_{VV\mathcal{P}} = -\sum_{i=1}^{3} \frac{4\sigma_{V_i}}{f_\pi} \epsilon_{\mu\nu\alpha\beta} \left[\pi^0 \partial^\mu \omega_i^\nu \partial^\alpha \rho_i^\beta + \frac{3}{2} (\mathbf{H}_{\omega_i} - 1) \pi^0 \partial^\mu \omega_i^\nu \partial^\alpha \omega_i^\beta + \frac{3\mathbf{A}_i^{\pi^0}}{4\mathbf{F}_{\phi_i}} \pi^0 \partial^\mu \phi_i^\nu \partial^\alpha \phi_i^\beta \right] + \ldots$$

J. Prades, Z. Phys. C **63** (1994) 491 Erratum: [Z. Phys. C **11** (1999) 571] doi:10.1007/BF01580330 [hep-ph/9302246].
H. Czyz, S. Ivashyn, A. Korchin and O. Shekhovtsova, Phys. Rev. D **85** (2012) 094010 doi:10.1103/PhysRevD.85.094010 [arXiv:1202.1171 [hep-ph]].

Two-photon form factor for π^0

$$
\begin{aligned}
F_{\gamma^*\gamma^*\pi^0}(t_1,t_2) \quad = \quad & -\frac{N_c}{12\pi^2 f_\pi} + \sum_{i=1}^{3} \frac{4\sqrt{2}\mathbf{h_{V_i}}\mathbf{f_{V_i}}}{3f_\pi} t_1 \Big(D_{\rho_i}(t_1) + \mathbf{F}_{\omega_\mathbf{i}}\mathbf{H}_{\omega_\mathbf{i}} D_{\omega_i}(t_1) + \mathbf{A}_\mathbf{i}^{\pi^0}\mathbf{F}_{\phi_\mathbf{i}} D_{\phi_i}(t_1) \Big) \\
+ \quad & \sum_{i=1}^{3} \frac{4\sqrt{2}\mathbf{h_{V_i}}\mathbf{f_{V_i}}}{3f_\pi} t_2 \Big(D_{\rho_i}(t_2) + \mathbf{F}_{\omega_\mathbf{i}}\mathbf{H}_{\omega_\mathbf{i}} D_{\omega_i}(t_2) + \mathbf{A}_\mathbf{i}^{\pi^0}\mathbf{F}_{\phi_\mathbf{i}} D_{\phi_i}(t_2) \Big) \\
- \quad & \sum_{i=1}^{3} \frac{4\sigma_{\mathbf{V_i}}\mathbf{f_{V_i}^2}}{3f_\pi} t_1 t_2 \Big(D_{\rho_i}(t_2) D_{\omega_i}(t_1) + D_{\rho_i}(t_1) D_{\omega_i}(t_2) + \mathbf{A}_\mathbf{i}^{\pi^0}\mathbf{F}_{\phi_\mathbf{i}} D_{\phi_i}(t_1) D_{\phi_i}(t_2) \\
+ \quad & (\mathbf{F}_{\omega_\mathbf{i}}\mathbf{H}_{\omega_\mathbf{i}} - 1) D_{\omega_i}(t_1) D_{\omega_i}(t_2) \Big)
\end{aligned}
$$

$\mathbf{H}_{\omega_\mathbf{i}}, \mathbf{F}_{\phi_\mathbf{i}} = \mathbf{1}$ for $\mathbf{i} = \mathbf{2}, \mathbf{3}$, $\mathbf{A_3^{\pi^0}} = \mathbf{0}$. The $\mathbf{D_{V_i}}$ - vector meson propagators.

H. Czyz, S. Ivashyn, A. Korchin and O. Shekhovtsova, Phys. Rev. D **85** (2012) 094010 doi:10.1103/PhysRevD.85.094010 [arXiv:1202.1171 [hep-ph]].

Two-photon form factors for η (η')

$$
\begin{aligned}
F_{\gamma^*\gamma^*\eta}(t_1,t_2) \quad &= \quad -\frac{N_c}{12\pi^2 f_\pi}\left(\frac{5}{3}C_q - \frac{\sqrt{2}}{3}C_s\right) \\
&+ \sum_{i=1}^{3}\frac{4\sqrt{2}\mathbf{h_{V_i}}\mathbf{f_{V_i}}}{3f_\pi}t_1\left(\left(3C_qD_{\rho_i}(t_1)+\frac{1}{3}\mathbf{F}_{\omega_\mathbf{i}}C_qD_{\omega_i}(t_1)-\frac{2\sqrt{2}}{3}C_s\mathbf{F}_{\phi_\mathbf{i}}D_{\phi_i}(t_1)\right)+\left(\frac{5}{3}C_q-\frac{\sqrt{2}}{3}C_s\right)\mathbf{A_i^\eta}\mathbf{F}_{\phi_\mathbf{i}}D_{\phi_i}(t_1)\right) \\
&+ \sum_{i=1}^{3}\frac{4\sqrt{2}\mathbf{h_{V_i}}\mathbf{f_{V_i}}}{3f_\pi}t_2\left(\left(3C_qD_{\rho_i}(t_2)+\frac{1}{3}C_q\mathbf{F}_{\omega_\mathbf{i}}D_{\omega_i}(t_2)-\frac{2\sqrt{2}}{3}C_s\mathbf{F}_{\phi_\mathbf{i}}D_{\phi_i}(t_2)\right)+\left(\frac{5}{3}C_q-\frac{\sqrt{2}}{3}C_s\right)\mathbf{A_i^\eta}\mathbf{F}_{\phi_\mathbf{i}}D_{\phi_i}(t_2)\right) \\
&- \sum_{i=1}^{3}\frac{8\sigma_{\mathbf{V_i}}\mathbf{f^2_{V_i}}}{f_\pi}t_1t_2\left[\left(\frac{1}{2}C_qD_{\rho_i}(t_1)D_{\rho_i}(t_2)+\frac{1}{18}\mathbf{F}_{\omega_\mathbf{i}}C_qD_{\omega_i}(t_1)D_{\omega_i}(t_2)-\frac{\sqrt{2}}{9}C_s\mathbf{F}_{\phi_\mathbf{i}}D_{\phi_i}(t_1)D_{\phi_i}(t_2)\right)\right. \\
&\quad + \quad \left.\frac{\mathbf{A_i^\eta}\mathbf{F}_{\phi_\mathbf{i}}}{6}\left(\frac{5}{3}C_q-\frac{\sqrt{2}}{3}C_s\right)D_{\phi_i}(t_1)D_{\phi_i}(t_2)\right]
\end{aligned}
$$

$$
F_{\gamma^*\gamma^*\eta}(t_1,t_2) = F_{\gamma^*\gamma^*\eta'}(t_1,t_2)\Big|_{C_q\to C_q',\ C_s\to -C_s'}
$$

$\mathbf{H}_{\omega_\mathbf{i}}$, $\mathbf{F}_{\phi_\mathbf{i}} = \mathbf{1}$ for $\mathbf{i} = \mathbf{2}, \mathbf{3}$, $\mathbf{A_3^\eta} = \mathbf{0}$, $\mathbf{A_3^{\eta'}} = \mathbf{0}$, $\mathbf{A_1^{\eta'}} = \mathbf{A_1^\eta}$. The $\mathbf{D_{V_i}}$ - vector meson propagators.

H. Czyz, S. Ivashyn, A. Korchin and O. Shekhovtsova, Phys. Rev. D **85** (2012) 094010 doi:10.1103/PhysRevD.85.094010 [arXiv:1202.1171 [hep-ph]].

η-$\eta^{'}$ mixing parameters

$$C_q = \frac{f_\pi}{\sqrt{3}\cos(\theta_8 - \theta_0)}\left(\frac{1}{f_8}\cos\theta_0 - \frac{1}{f_0}\sqrt{2}\sin\theta_8\right)$$

$$C_s = \frac{f_\pi}{\sqrt{3}\cos(\theta_8 - \theta_0)}\left(\frac{1}{f_8}\sqrt{2}\cos\theta_0 + \frac{1}{f_0}\sin\theta_8\right)$$

$$C_q^{'} = \frac{f_\pi}{\sqrt{3}\cos(\theta_8 - \theta_0)}\left(\frac{1}{f_0}\sqrt{2}\cos\theta_8 + \frac{1}{f_8}\sin\theta_0\right)$$

$$C_s^{'} = \frac{f_\pi}{\sqrt{3}\cos(\theta_8 - \theta_0)}\left(\frac{1}{f_0}\cos\theta_8 - \frac{1}{f_8}\sqrt{2}\sin\theta_0\right)$$

$\theta_8 = -21.2^\circ \pm 1.6^\circ$, $\theta_0 = -9.2^\circ \pm 1.7^\circ$ and constants $f_8 = (1.26 \pm 0.04)f_\pi$, $f_0 = (1.17 \pm 0.03)f_\pi$, where $f_\pi = 92.4$ MeV.

$C_q \approx 0.720$, $C_s \approx 0.471$, $C_q^{'} \approx 0.590$, $C_s^{'} \approx 0.576$.

T. Feldmann, P. Kroll and B. Stech, Phys. Rev. D **58** (1998) 114006 doi:10.1103/PhysRevD.58.114006 [hep-ph/9802409].

T. Feldmann, Int. J. Mod. Phys. A **15** (2000) 159 doi:10.1142/S0217751X00000082 [hep-ph/9907491].

Asymptotic behavior

$$\lim_{\mathbf{t_1}\to\pm\infty} \mathbf{F}_{\gamma^*\gamma^*\mathcal{P}}(\mathbf{t_1},\mathbf{t_2})\Big|_{t_2=const} = 0$$

$$-\frac{N_c}{4\pi^2} + 4\sqrt{2}\sum_{i=1}^{3} h_{V_i} f_{V_i} (1 + F_{\omega_i} H_{\omega_i} + A_i^{\pi^0} F_{\phi_i}) = 0,$$

$$\sqrt{2} h_{V_i} f_{V_i} - \sigma_{V_i} f_{V_i}^2 = 0, \qquad i = 1, 2, 3$$

$$-\frac{N_c}{4\pi^2}(\frac{5}{3}C_q - \frac{\sqrt{2}}{3}C_s) + 4\sqrt{2}\sum_{i=1}^{3} h_{V_i} f_{V_i} \Big[(3C_q + \frac{1}{3}F_{\omega_i} C_q - \frac{2\sqrt{2}}{3}C_s F_{\phi_i}) + (\frac{5}{3}C_q - \frac{\sqrt{2}}{3}C_s) A_i^{\eta} F_{\phi_i}\Big] = 0,$$

$$-\frac{N_c}{4\pi^2}(\frac{5}{3}C_q^{'} + \frac{\sqrt{2}}{3}C_s^{'}) + 4\sqrt{2}\sum_{i=1}^{3} h_{V_i} f_{V_i} \Big[(3C_q^{'} + \frac{1}{3}F_{\omega_i} C_q^{'} + \frac{2\sqrt{2}}{3}C_s^{'} F_{\phi_i}) + (\frac{5}{3}C_q^{'} + \frac{\sqrt{2}}{3}C_s^{'}) A_i^{\eta^{'}} F_{\phi_i}\Big] = 0.$$

We have chosen $\sigma_{V_1} f_{V_1}^2$, $\sigma_{V_2} f_{V_2}^2$, $\sigma_{V_3} f_{V_3}^2$, $h_{V_3} f_{V_3}$, A_2^{η}, $A_2^{\eta^{'}}$ to be determined by using asymptotic relations
G. P. Lepage and S. J. Brodsky, Phys. Rev. D **22** (1980) 2157. doi:10.1103/PhysRevD.22.2157

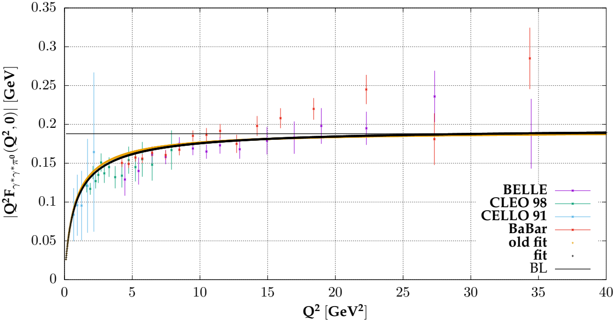
Fit the model parameters

Data in space-like region: BELLE, CLEO, CELLO, BaBar
Data in time-like region: SND, CMD2
Decay widths: $V \to e^+e^-$, $V \to \mathcal{P}\gamma$ and $\mathcal{P} \to V\gamma$
~~BaBar $F_{\gamma^*\gamma^*\pi^0}$~~
~~CMD2 $\sigma(e^+e^- \to \eta\gamma)$~~
9 parameters: $\mathbf{h_{V_1}}$, $\mathbf{A_1^{\pi^0}}$, $\mathbf{A_2^{\pi^0}}$, $\mathbf{h_{V_2} f_{V_2}}$, $\mathbf{A_1^{\eta}}$, $\mathbf{f_{V_1}}$, $\mathbf{H_{\omega_1}}$, $\mathbf{F_{\omega_1}}$, $\mathbf{F_{\phi_1}}$

$$\chi^2 = 301.89$$
$$\text{d.o.f} = 303$$

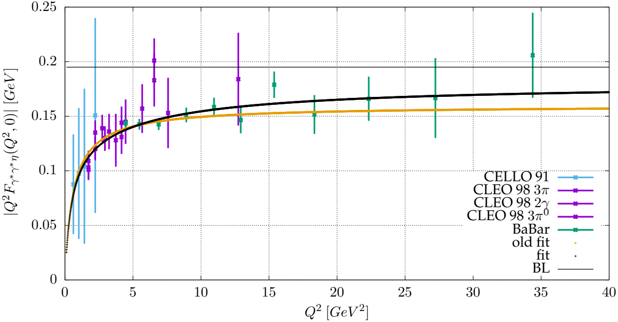
Width	exp [GeV]	th [GeV]
$\Gamma(\rho \to e^+e^-)$	$(7.04 \pm 0.06) \cdot 10^{-6}$	$(7.04 \pm 0.06) \cdot 10^{-6}$
$\Gamma(\rho \to \pi^0\gamma)$	$(8.9 \pm 1.2) \cdot 10^{-5}$	$(6.03 \pm 0.07) \cdot 10^{-5}$
$\Gamma(\omega \to e^+e^-)$	$(6.0 \pm 0.2) \cdot 10^{-7}$	$(6.06 \pm 0.14) \cdot 10^{-7}$
$\Gamma(\omega \to \pi^0\gamma)$	$(7.0 \pm 0.2) \cdot 10^{-4}$	$(7.66 \pm 0.17) \cdot 10^{-4}$
$\Gamma(\phi \to e^+e^-)$	$(1.26 \pm 0.02) \cdot 10^{-6}$	$(1.273 \pm 0.016) \cdot 10^{-6}$
$\Gamma(\phi \to \pi^0\gamma)$	$(5.4 \pm 0.3) \cdot 10^{-6}$	$(5.65 \pm 0.12) \cdot 10^{-6}$
$\Gamma(\rho \to \eta\gamma)$	$(4.5 \pm 0.3) \cdot 10^{-5}$	$(3.87 \pm 0.04) \cdot 10^{-5}$
$\Gamma(\omega \to \eta\gamma)$	$(3.9 \pm 0.3) \cdot 10^{-6}$	$(4.68 \pm 0.05) \cdot 10^{-6}$
$\Gamma(\phi \to \eta\gamma)$	$(5.6 \pm 0.1) \cdot 10^{-5}$	$(5.70 \pm 0.08) \cdot 10^{-5}$
$\Gamma(\phi \to \eta'\gamma)$	$(2.67 \pm 0.09) \cdot 10^{-7}$	$(2.65 \pm 0.05) \cdot 10^{-7}$
$\Gamma(\eta' \to \rho\gamma)$	$(5.8 \pm 0.3) \cdot 10^{-5}$	$(4.81 \pm 0.06) \cdot 10^{-5}$
$\Gamma(\eta' \to \omega\gamma)$	$(5.4 \pm 0.5) \cdot 10^{-6}$	$(4.78 \pm 0.05) \cdot 10^{-6}$

Transition form factor $\gamma^*\gamma^*\pi^0$ compared to the data



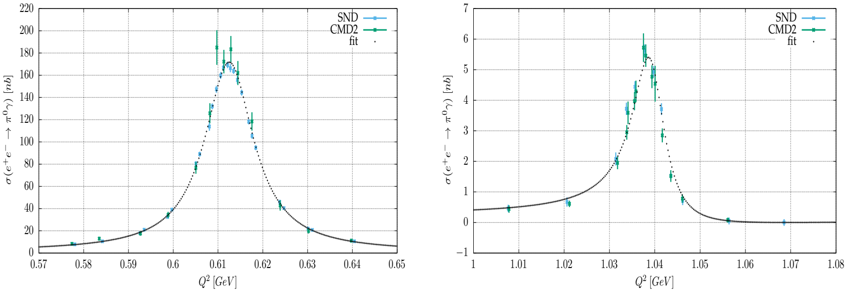
Brodsky-Lepage high-Q^2 limit at 0.1848 GeV

Transition form factor $\gamma^*\gamma^*\eta$ compared to the data



Brodsky-Lepage high-Q^2 limit at 0.1950 GeV

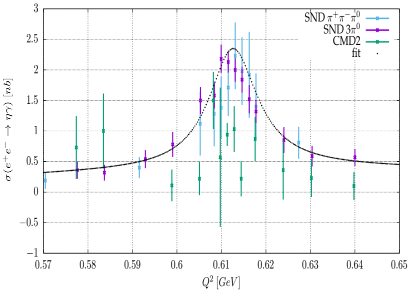
Experimental data for $\sigma(e^+e^- \to \pi^0\gamma)$ compared to the model predictions

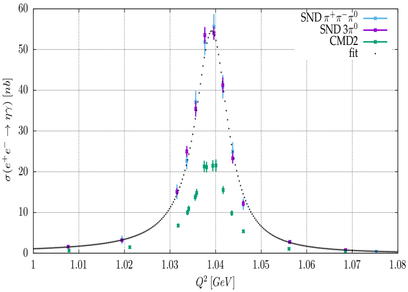


R. R. Akhmetshin *et al.* [CMD-2 Collaboration], Phys. Lett. B **605** (2005) 26 doi:10.1016/j.physletb.2004.11.020 [hep-ex/0409030].

M. N. Achasov *et al.* [SND Collaboration], Phys. Rev. D **93** (2016) no.9, 092001 doi:10.1103/PhysRevD.93.092001 [arXiv:1601.08061 [hep-ex]].

Experimental data for $\sigma(e^+e^- \to \eta\gamma)$ compared to the model predictions





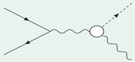
R. R. Akhmetshin *et al.* [CMD-2 Collaboration], Phys. Lett. B **605** (2005) 26 doi:10.1016/j.physletb.2004.11.020 [hep-ex/0409030].

M. N. Achasov *et al.*, Phys. Rev. D **74** (2006) 014016 doi:10.1103/PhysRevD.74.014016 [hep-ex/0605109].

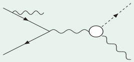
Implementation in PHOKHARA

- Scan mode

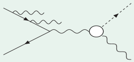
LO



NLO

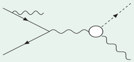


NNLO

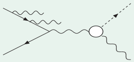


- Radiative return mode

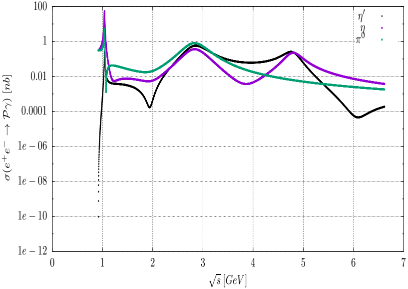
LO

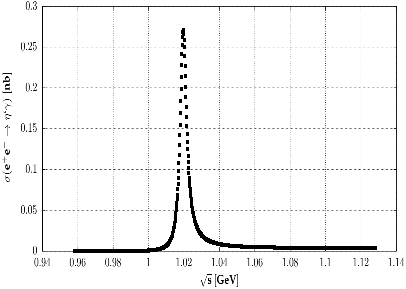


NLO

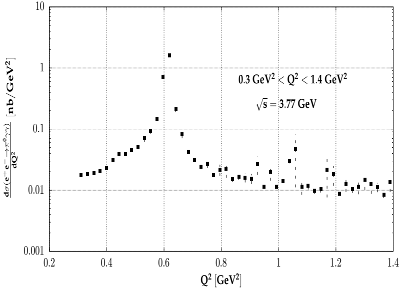


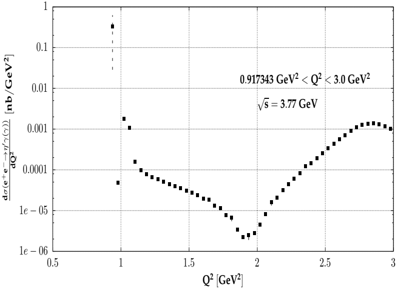
The model predictions for $e^+e^- \to \eta^{'}\gamma$ cross section.



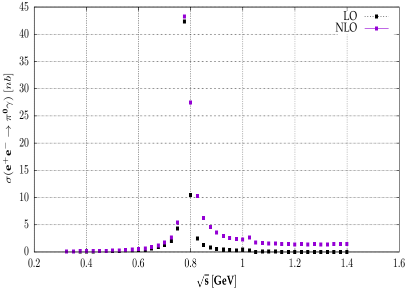


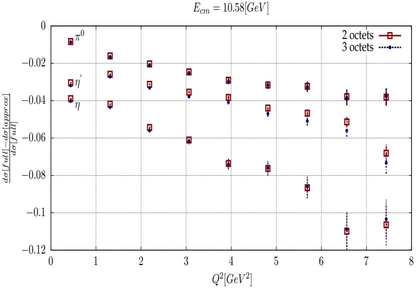
The cross section $d\sigma/dQ^2$ for the process $e^+e^- \to \pi^0\gamma\gamma$ and $e^+e^- \to \eta'\gamma\gamma$

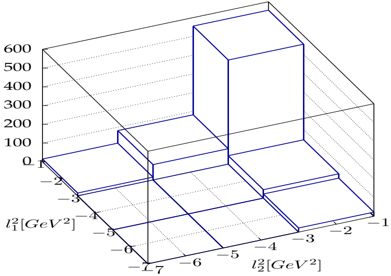




Comparison of the LO and NLO cross section for the process $e^+e^- \to \pi^0\gamma$







The relative difference of the cross sections $(d\sigma[full] - d\sigma[approx])/d\sigma[full]$ for the process $e^+e^- \to e^+e^-\pi^0$ and $e^+e^- \to e^+e^-\eta(\eta')$. The $d\sigma[approx]$ ignores the form factor dependence on q_2^2. The $d\sigma[full]$ accounts for the virtuality of both photons with $q_2^2 < 0.18$ GeV for π^0 and $q_2^2 < 0.38$ GeV for $\eta(\eta')$.

The distribution of expected number of events (N_{ev}) for π^0 production when both electron and positron are tagged. $E_{cm} = 10.58$ GeV and $\mathcal{L}_{int} = 50ab^{-1}$

Final remarks

- The transition form factors of the pseudoscalar mesons have been constructed with parameters fitted to the experimental data in the time-like and space-like region.
- The model has been implemented in MC generator Phokhara, which allows for making predictions of $e^+e^- \to \eta^{'}\gamma$ cross section.
- The impact of the radiative corrections has been studied in case of $e^+e^- \to \pi^0\gamma$ cross section.
- Implementation of higher order radiative corrections is in progress.