Radio MontecarLow WG meeting
KPH Mainz

June 30th 2017

$\bar{p}p \to e^+e^-$ @ NLO with PANDA at FAIR cross section and event generator

Manuel Zambrana[1,2], Alaa Dbeyssi[1], Frank Maas[1,2,3], Egle Tomasi-Gustafsson[4], Yury M. Bystritskiy[5], and Vladimir A. Zykunov[5].

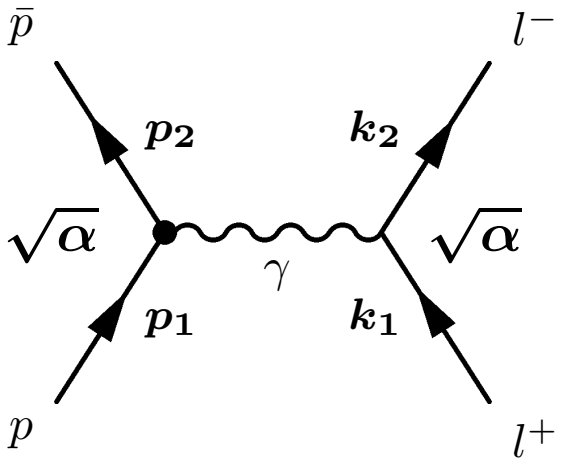
[1]Helmholtz-Institut Mainz, Germany
[2]Institut für Kernphysik, Johannes Gutenberg Universität, Mainz, Germany
[3]Prisma Cluster of Excellence, Mainz, Germany
[4]CEA, IRFU, SPhN, Saclay, France
[5]Joint Institute for Nuclear Research, Dubna, Russia

JOHANNES GUTENBERG UNIVERSITÄT MAINZ

Helmholtz-Institut Mainz

The $\bar{p}p \to e^+e^-$ cross section : leading order

- **at leading order (BORN approximation), only one diagram contributes to the amplitude:**



$$s = (p_1 + p_2)^2 \qquad t = (p_2 - k_2)^2 \qquad u = (p_1 - k_2)^2$$

$$\Rightarrow \mathcal{M}_B \sim \alpha \quad \Rightarrow \quad d\sigma_B \sim |\mathcal{M}_B|^2 \text{ is } \mathcal{O}(\alpha^2)$$

A.Zichichi et al., Nuovo Cimento XXIV, 170 (1962):

$$\frac{d\sigma_B}{d\cos\theta^*} = \frac{\pi\alpha^2}{2s\beta}\left\{|G_M|^2(1+\cos^2\theta^*) + |G_E|^2(1-\beta^2)(1-\cos^2\theta^*)\right\}$$

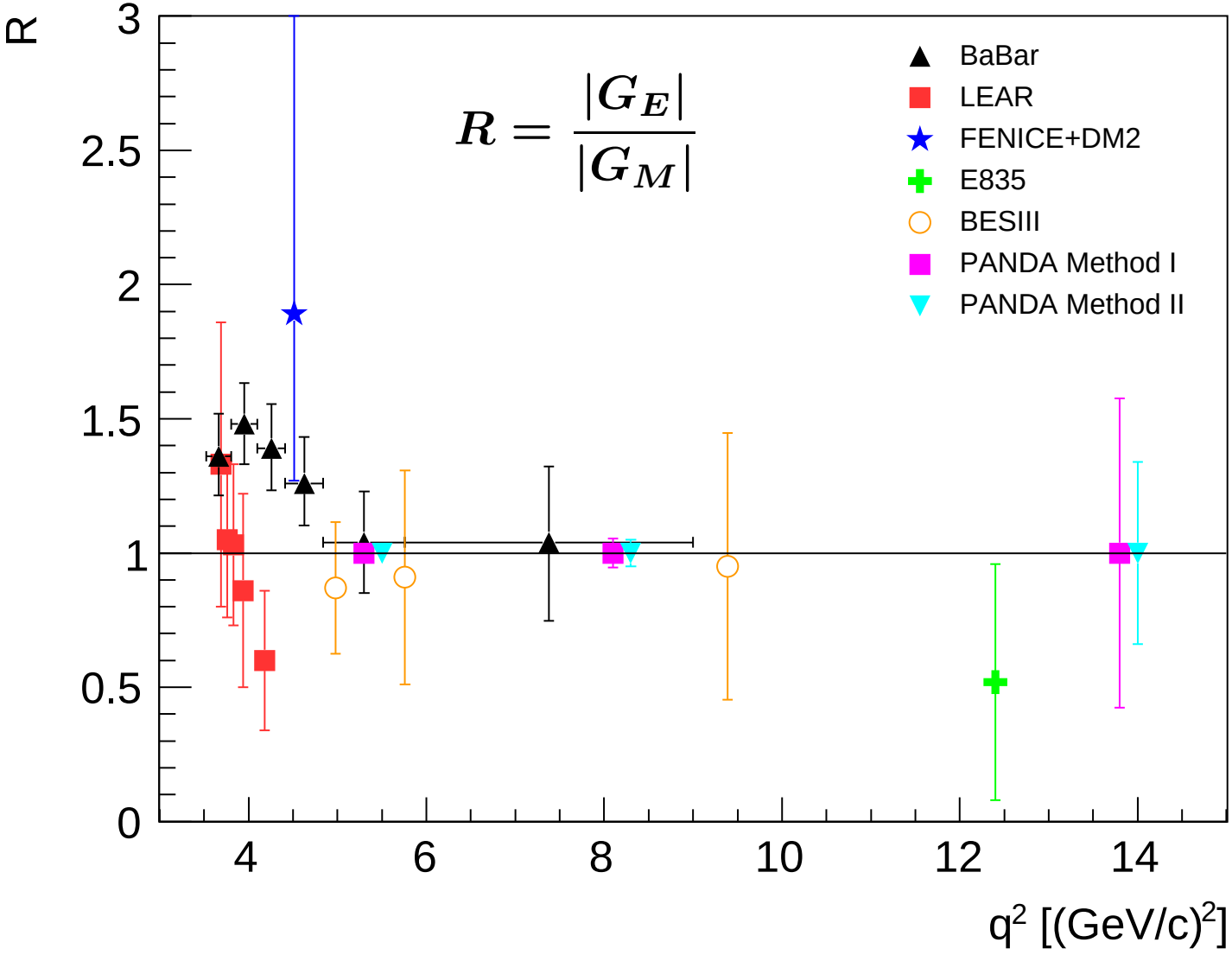
θ^* = **angle (electron, antiproton) in $\bar{p}p$ CM frame,** $\qquad 1-\beta^2 = 4M_p^2/s$

$\Rightarrow$ **fit function to extract $|G_E|$ and $|G_M|$ at leading order**

- **having sufficiently precise data, we would like to attemp a form factor extraction at next to leading order:**

$\Rightarrow$ **we need an $\mathcal{O}(\alpha^3)$ differential cross section**

Future precision measurements of proton FFs with PANDA



at low q^2,

$\frac{\Delta R}{R} \sim$ few percent

$\Rightarrow$ radiative corrections will be needed

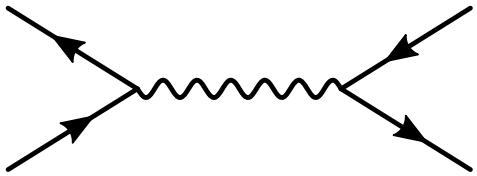
existing packages (i.e. PHOTOS) not entirely adequate:

we are developing a complete calculation / package for PANDA

"Feasibility studies of time-like proton electromagnetic form factors at PANDA at FAIR" PANDA Collaboration; Eur. Phys. J. A **52**, no. 10, 325 (2016); [arXiv:1606.01118 [hep-ex]]

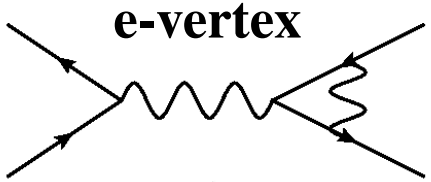
The $\bar{p}p \to e^+e^-$ cross section : next to leading order

at next to leading order (NLO), many more diagrams contribute to the amplitude:

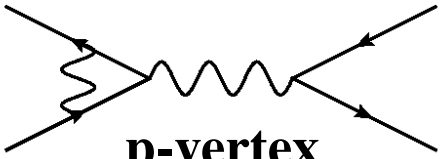


born $\mathcal{M} \sim \alpha$

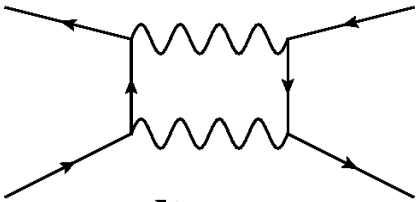
vacuum polarization



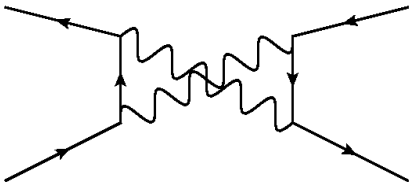
e-vertex



p-vertex

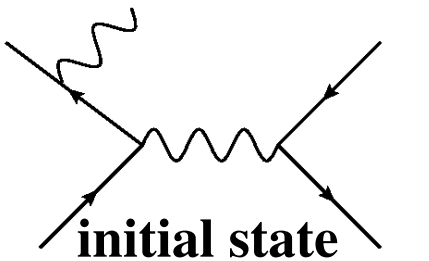


box

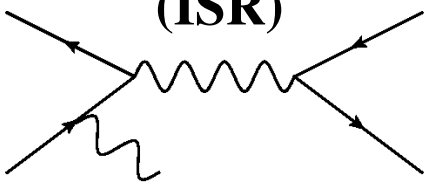


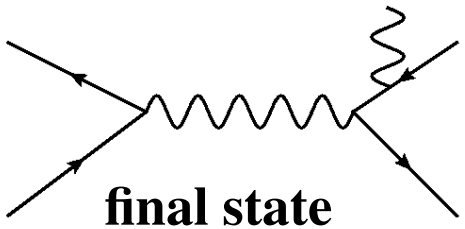
virtual corrections

$\mathcal{M} \sim \alpha^2$

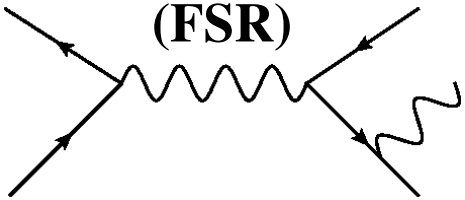


initial state radiation (ISR)





final state radiation (FSR)



real corrections

$\mathcal{M} \sim \alpha^{3/2}$

The $\bar{p}p \to e^+e^-$ cross section : next to leading order

at NLO, we have <u>two different</u> (i.e. non-interfering) final states:

- e^+e^-: **born, virtual corrections (vacuum polarization, ...)**
- $e^+e^-\gamma$: **real corrections (initial state radiation, final state radiation)**

$$\mathcal{O}(\alpha) \quad \mathcal{O}(\alpha^2) \qquad \mathcal{O}(\alpha^{3/2})$$

$$\Rightarrow d\sigma \sim |\mathcal{M}_B + \textstyle\sum_i \mathcal{M}_i|^2 + |\mathcal{M}_{ISR} + \mathcal{M}_{FSR}|^2 \qquad \text{truncated at } \mathcal{O}(\alpha^3)$$

$$\sim |\mathcal{M}_B|^2 + \textstyle\sum_i 2\mathrm{Re}(\mathcal{M}_B\mathcal{M}_i^*) + |\mathcal{M}_{ISR}|^2 + |\mathcal{M}_{FSR}|^2 + 2\mathrm{Re}(\mathcal{M}_{ISR}\mathcal{M}_{FSR}^*)$$

$$\sim |\mathcal{M}_B|^2 \left\{ 1 + \sum_i \frac{2\mathrm{Re}(\mathcal{M}_B\mathcal{M}_i^*)}{|\mathcal{M}_B|^2} + \frac{|\mathcal{M}_{ISR}|^2}{|\mathcal{M}_B|^2} + \frac{|\mathcal{M}_{FSR}|^2}{|\mathcal{M}_B|^2} + \frac{2\mathrm{Re}(\mathcal{M}_{ISR}\mathcal{M}_{FSR}^*)}{|\mathcal{M}_B|^2} \right\}$$

$$\sim |\mathcal{M}_B|^2 \left\{ 1 + \delta_{\text{vacuum}} + \delta_{\text{e-vertex}} + \delta_{\text{p-vertex}} + \delta_{\text{box}} + \underbrace{\delta_{\text{ISR}} + \delta_{\text{FSR}} + \delta_{\text{interference ISR/FSR}}}_{\equiv\, \delta_\gamma} \right\}$$

therefore, at $\mathcal{O}(\alpha^3)$, we write

$$\boxed{d\sigma = d\sigma_B \left\{ 1 + \delta_{\text{vacuum}} + \delta_{\text{e-vertex}} + \delta_{\text{p-vertex}} + \delta_{\text{box}} + \delta_\gamma \right\}}$$

interference born, virtual diagrams

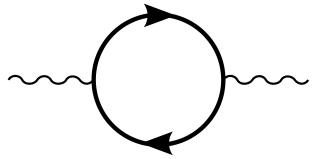
ISR, FSR, interference

Calculation at NLO : virtual corrections, real corrections

- **our calculation of virtual corrections is not novel, but it relies heavily on**

  A.I. Ahmadov et al. Phys. Rev. D **82**, 094016 (2010)
  G. Bonneu et al. Nucl. Phys. B **27**, 381 (1971)

  - **proton treated as a point-like particle**
  - **all leptons (i.e. e, μ, τ), and point-like pion are included as contributions to vacuum polarization**



- **formalism for real corrections (ISR, FSR) depends on the photon kinematics: soft photon and hard photon regime**

  - **soft photon:** i) $E_\gamma < \omega$, **no experimental detection of the photon threshold ω determined by theory/experiment**
    ii) **cross section integrated over the photon degrees of freedom**
  - **hard photon:** $E_\gamma > \omega$, **photon is detected in the experiment / experiment is sensitive to the energy loss**

currently, our calculation includes full virtual corrections and soft real corrections (hard photon emission is work in progress)

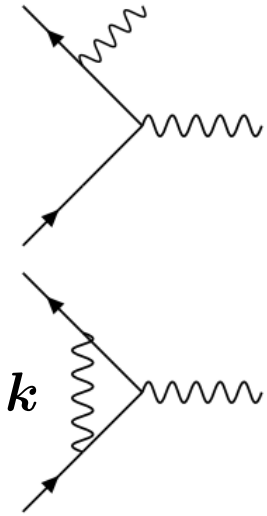
Calculation at NLO : real corrections (soft photon)

- **real corrections (ISR, FSR, interference) have been completely re-calculated**
  [Ahmadov et al. does not separate explicitly ISR/FSR terms;
  [kinematic regime $s, t, u \gg M^2, m^2$ assumed in interference term not adequate for PANDA]

  following F. A. Berends et al. Nucl. Phys. B **57** 381 (1973)
  F. A. Berends et al. Nucl. Phys. B **63** 381 (1973)
  on $e^+e^- \to \mu^+\mu^-$

  in the soft photon limit: $E_\gamma < \omega = b\sqrt{s}/2 \qquad b < 10^{-2}$

- **infrared divergences regularized by giving the photon a small mass λ (infrared cutoff)**
  - **real diagrams:** $E_\gamma > \lambda = a\sqrt{s} \qquad 10^{-6} < a < 10^{-3}$
  - **virtual diagrams:** $\int_0 dk^2 \to \int_{\lambda^2} dk^2$



- **cancellation of divergences:**

$$\delta_{\text{e-vertex}} + \delta_{\text{FSR}} = \text{finite}$$
$$\delta_{\text{p-vertex}} + \delta_{\text{ISR}} = \text{finite}$$
$$\delta_{\text{box}} + \delta_{\text{interference ISR/FSR}} = \text{finite}$$

- **full cross section is λ-independent (more accurately, λ-stable)**

Calculation at NLO : codes

- **previous cross section is used as a basis for two codes: σ-calculator and event generator, including born amplitude, virtual and real (soft photon only) corrections**
- **user can freely switch on/off correction terms (as long as cancellation of divergences is not violated), define parameters (center of mass energy $\sqrt{s}$, form factors G_E and G_M, photon cutoffs λ and ω), and kinematic region (angular range)**

```
1000000              ! #events
1                    ! RANLUX seed
1.5                  ! pbar momentum LAB-frame (GeV)
-0.999               ! cos(theta*) min
0.999                ! cos(theta*) max
1.279                ! |G_E|/|G_M|
0.00001              ! soft photon MIN energy controller
0.01                 ! soft photon MAX energy controller
1                    ! leptons (vacuum polarization)
1                    ! pion    (vacuum polarization)
1                    ! box
1                    ! electron vertex
1                    ! proton vertex
1                    ! final state radiation
1                    ! initial state radiation
1                    ! interference ISR/FSR
../output            ! pathout
output               ! outfile
```

Initialisation stage : checking parameters

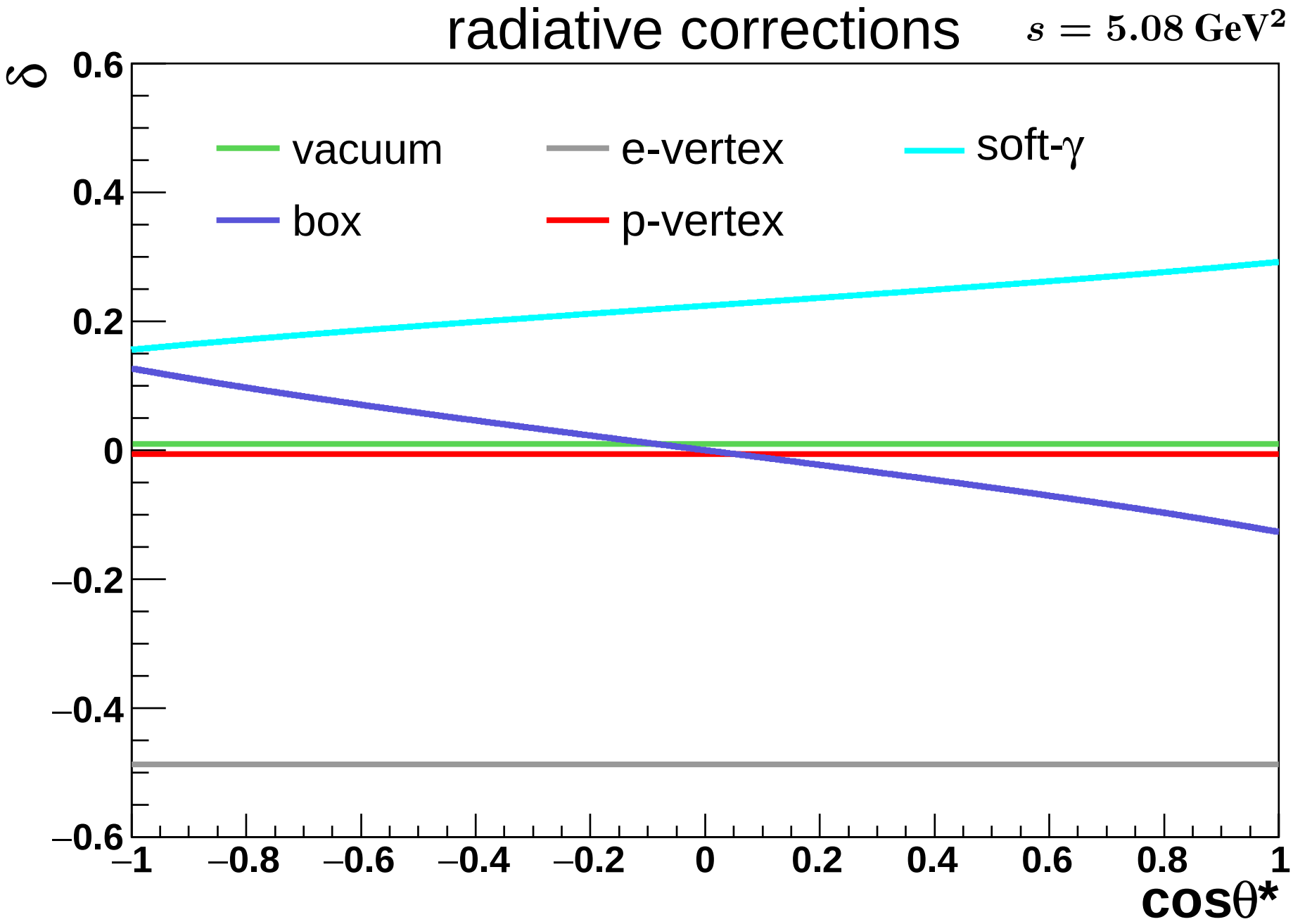
- **both programs check that all parameters are correct, otherwise printing an error message and killing the job; examples:**

```
WARNING: anti-proton momentum larger than PANDA upper limit 15.00 GeV
         [P = 20.00 GeV]

ERROR: cos(theta*) range NOT allowed! [-0.80 < cos(theta*) < -0.80]
    => cos(theta*) range [cos(theta*) min, cos(theta*) max]
       should be contained within the interval [-1.0,1.0]
    => job killed!!!

ERROR: b-parameter out of range! [b-parameter = 0.020000]
    => b-parameter should be contained within the interval
       (2.0 * a-parameter, 0.01]
       (with user's choice a-parameter = 0.000010, this interval is
       (0.000020,0.010000] )
    => job killed!!!

ERROR: vacuum polarization flag out of range! [flag = 2]
    => vacuum polarization flag should take the value 0 or 1
    => job killed!!!
```

$\Rightarrow$ no way to run a job with wrong parameters!

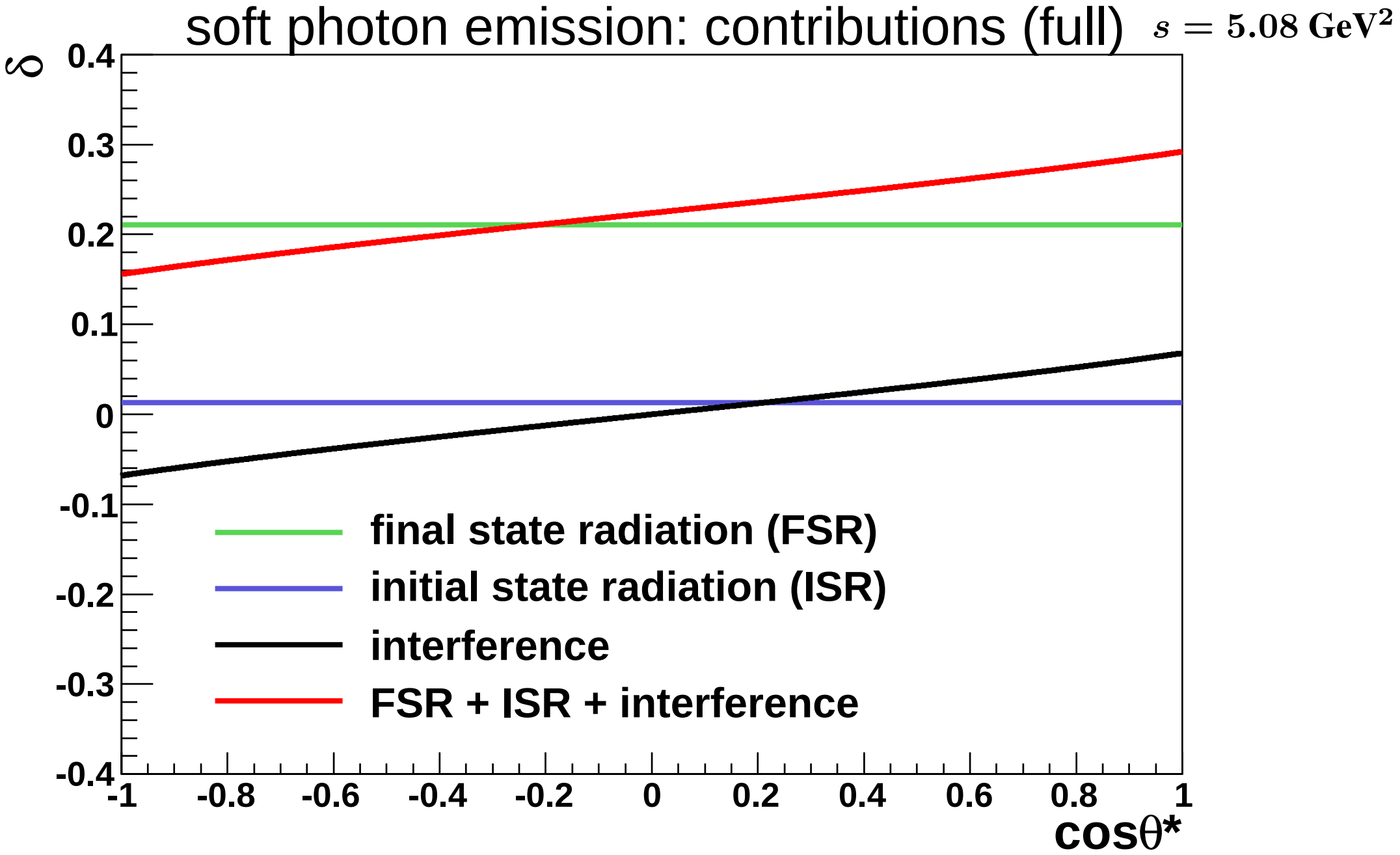
- **kinematic information printed to the log file, RANLUX initialised**

Cross section calculator : radiative corrections



box, proton vertex and ISR/FSR interference are t, u-dependent

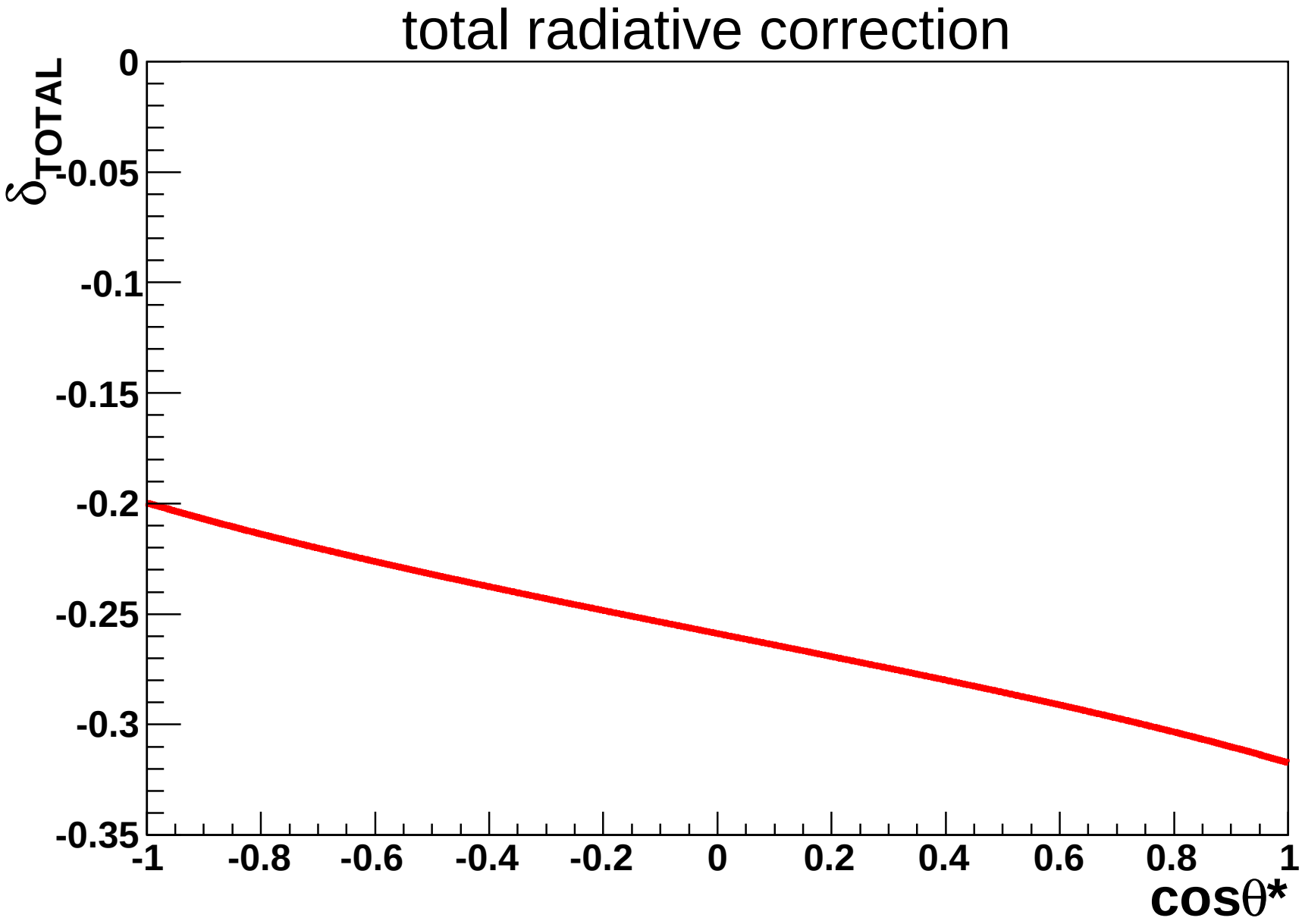
Cross section calculator : contributions to soft photon emission



Cross section calculator : total radiative correction

$$d\sigma/d\cos\theta^* = (d\sigma_B/d\cos\theta^*)(1+\delta_T)$$

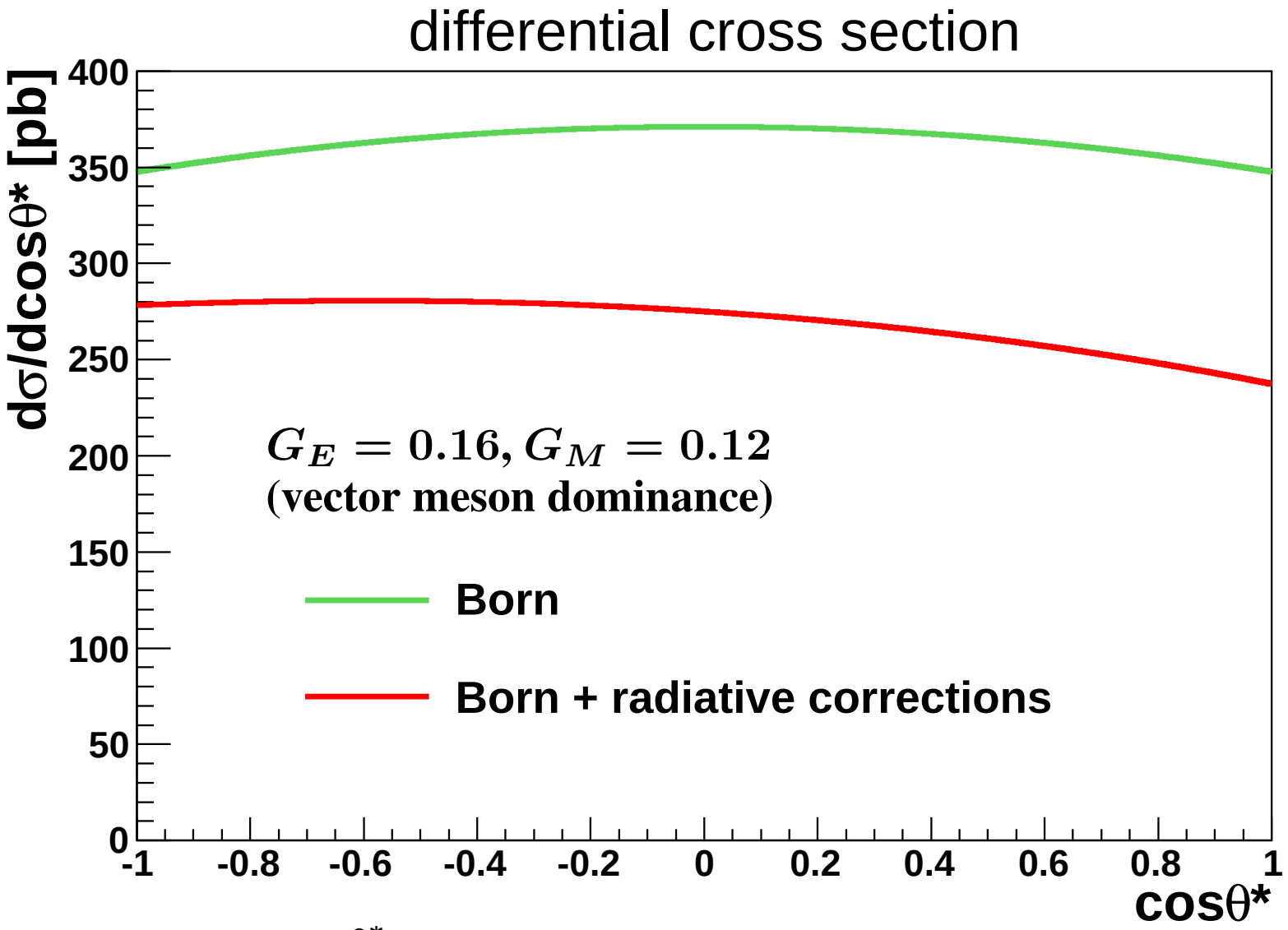
$$s = 5.08\ \mathrm{GeV}^2$$



Cross section calculator : differential and integrated cross section

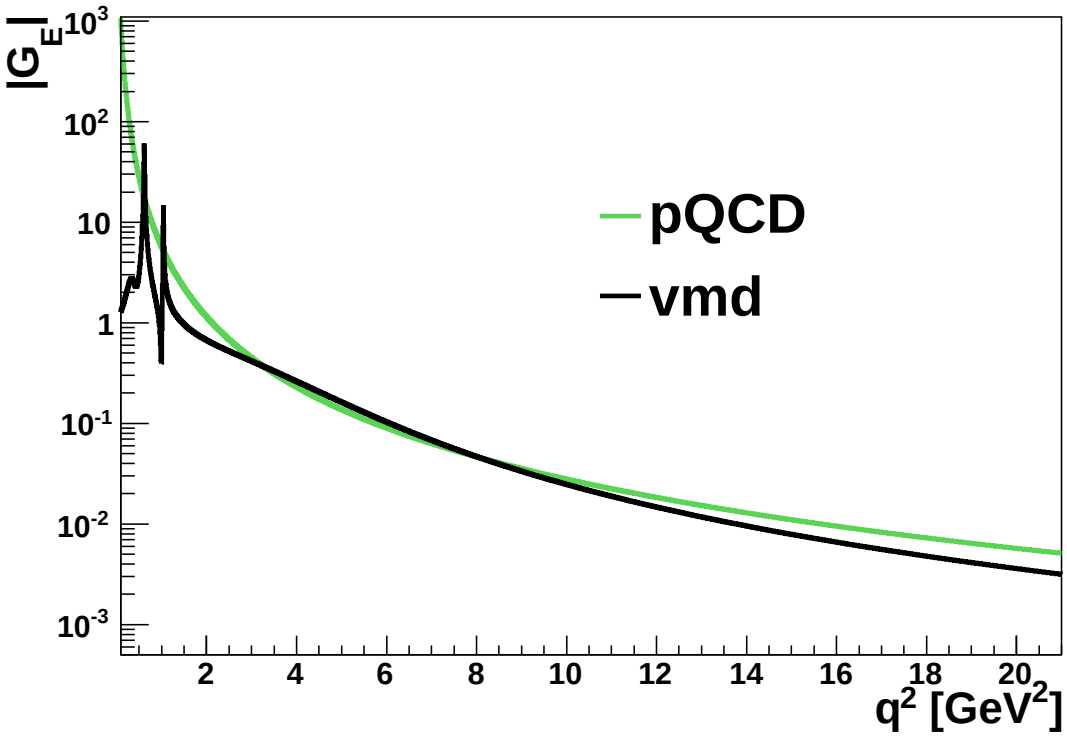
$$d\sigma/d\cos\theta^* = (d\sigma_B/d\cos\theta^*)(1+\delta_T)$$

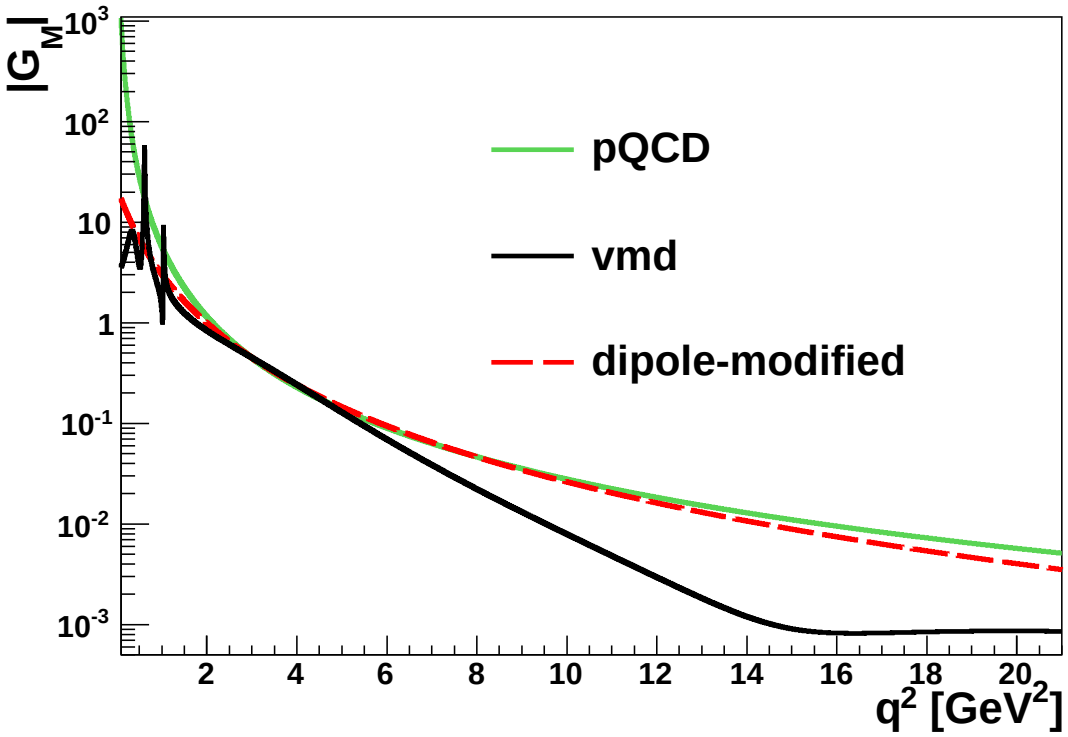
$s = 5.08\ \mathrm{GeV}^2$



$$\sigma = \int_{\cos\theta^*_{\min}}^{\cos\theta^*_{\max}} \mathrm{d}\cos\theta^* \, \frac{d\sigma}{d\cos\theta^*} = 434\ \mathrm{pb} \qquad -0.8 < \cos\theta^* < 0.8$$

Models for the form factors





- **"perturbative QCD - inspired": [1], and references therein**
- **"vector meson dominance": [2]**
- **"dipole-modified": [1], and references therein**

[1] M. Sudol et al., Eur. Phys. J. A **44**, 373 (2010)

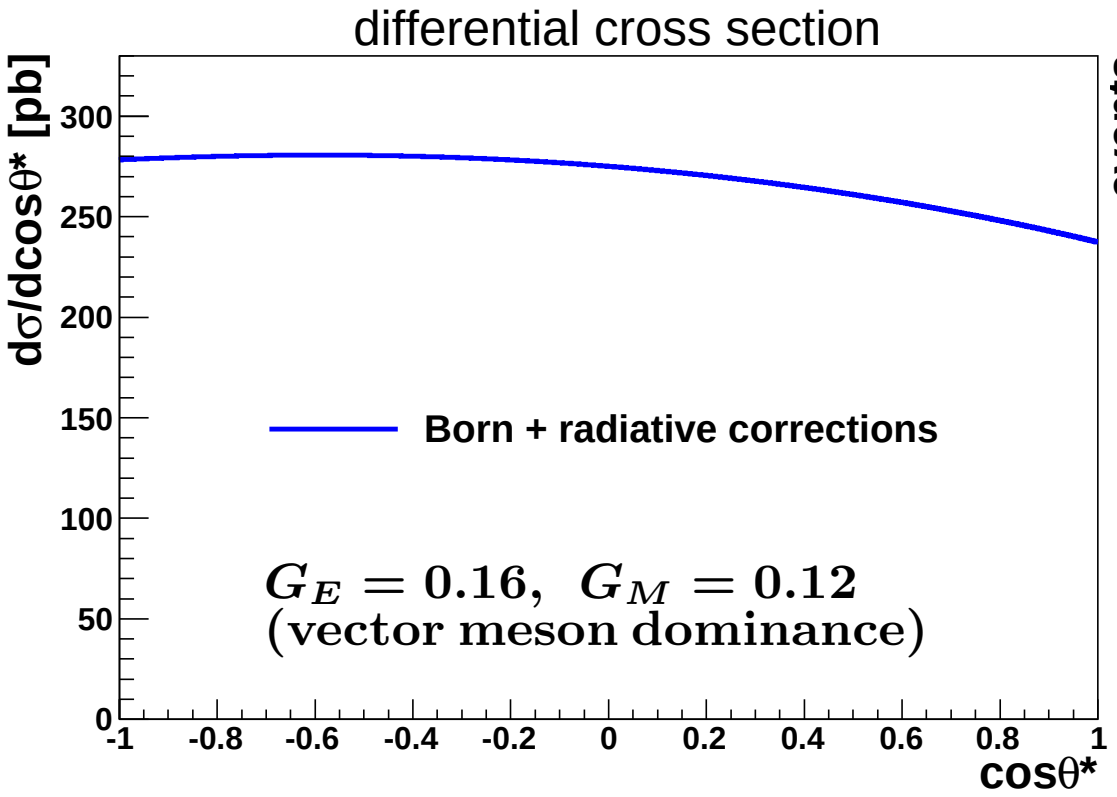
[2] F. Iachello, and Q. Wan, Phys. Rev. C **69**, 055204 (2004)

Event generator @ NLO

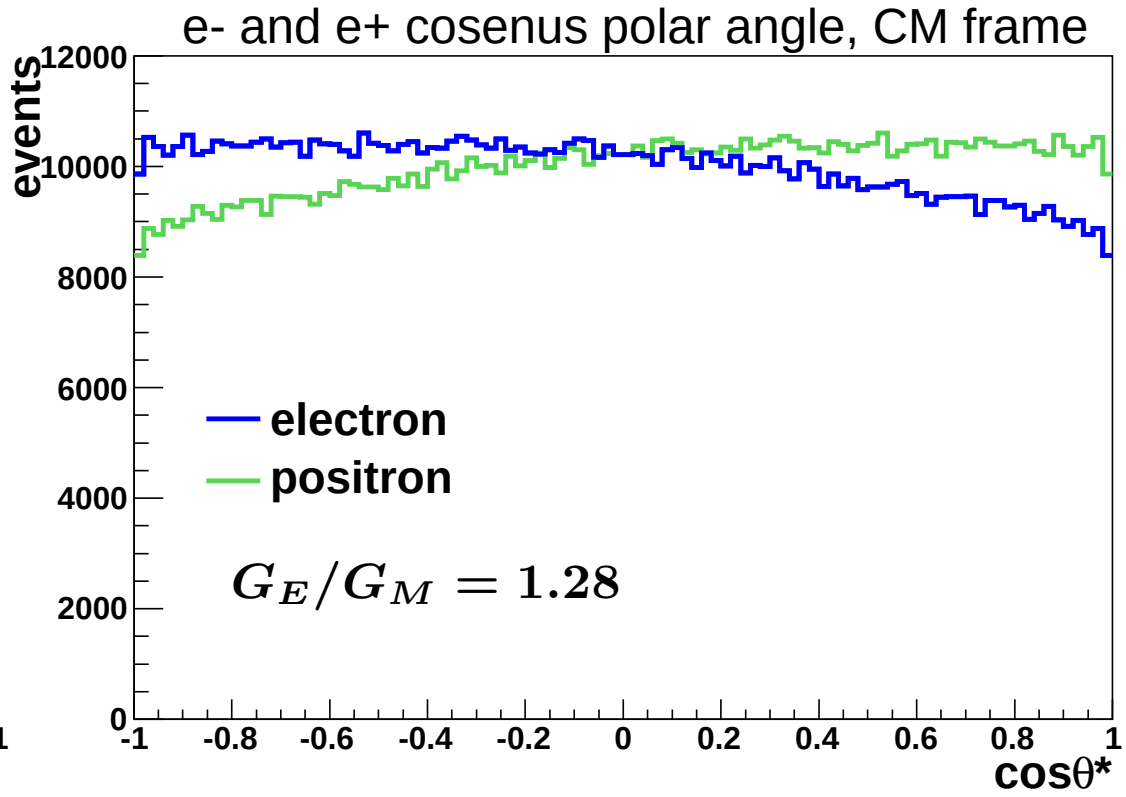
The NLO differential cross section is the basis of a Monte Carlo event generator to be used in PANDA physics analyses for the extraction of the proton form factors (currently being interfaced to PandaRoot, the full PANDA simulation software)

$s = 5.08\ \mathrm{GeV}^2$ $N = 10^6$ generated events

radiative corrected cross section

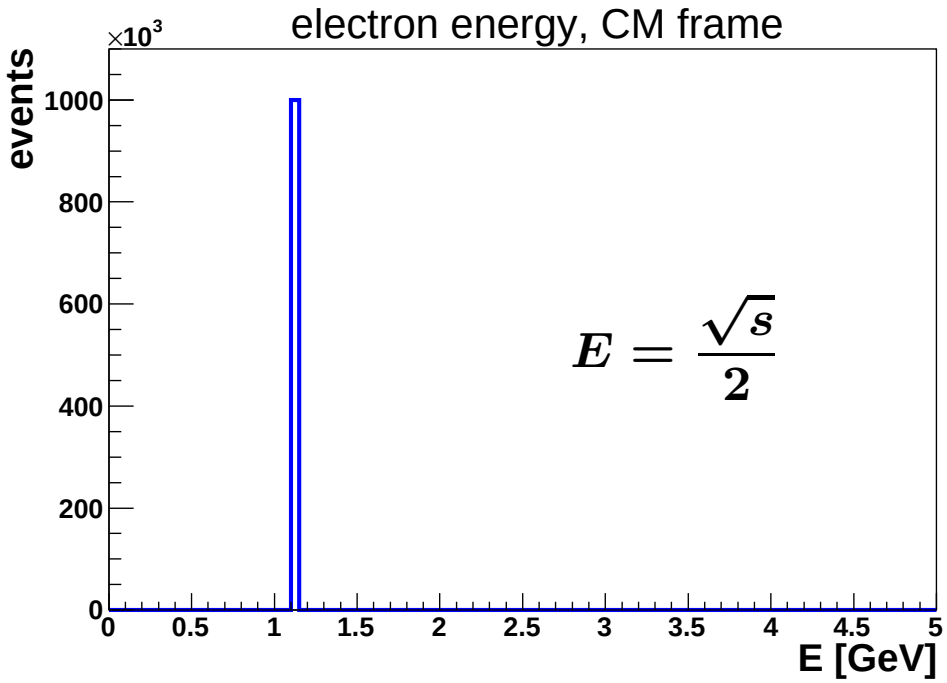


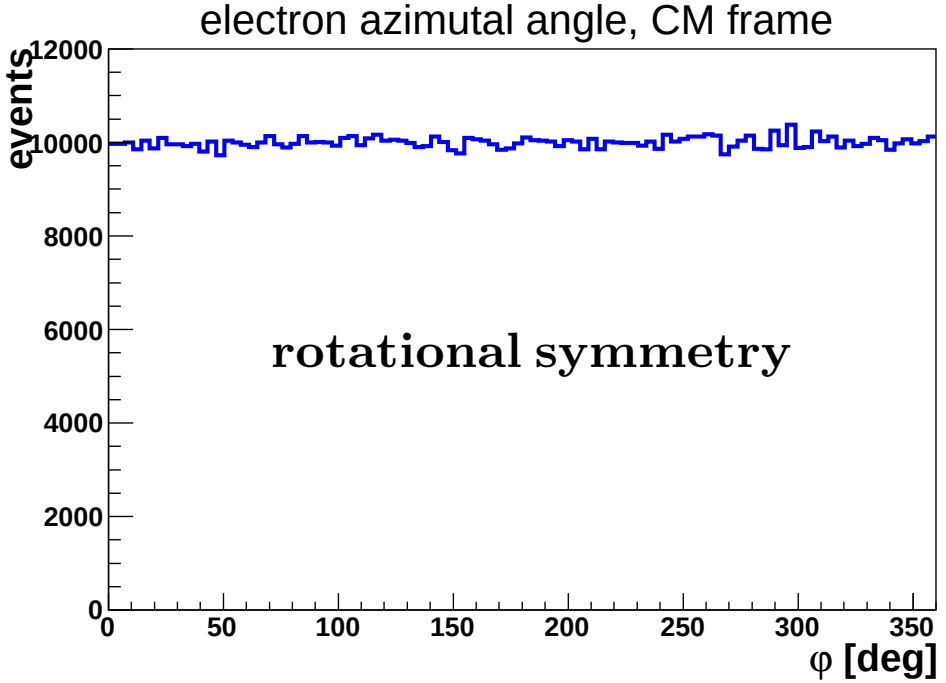
generated events

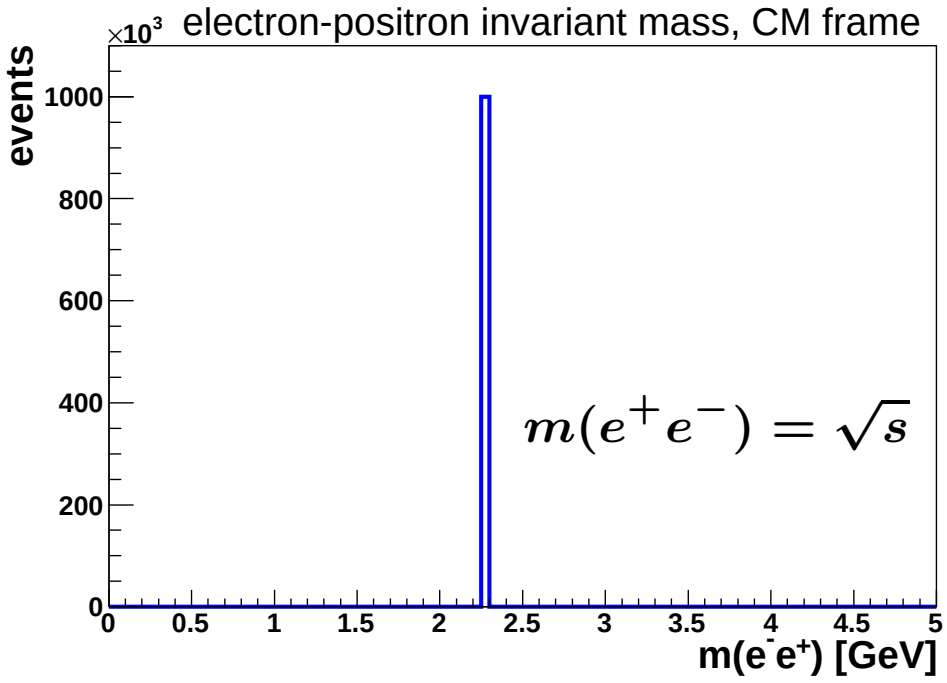


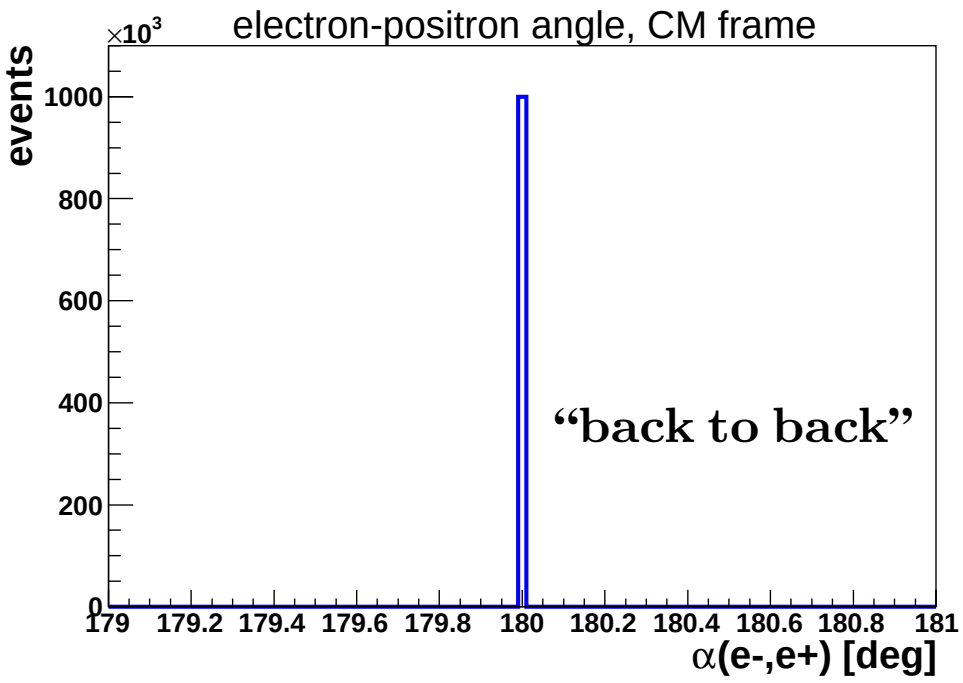
Event generator : example distributions (CM frame)

$\sqrt{s} = 2.25$ GeV



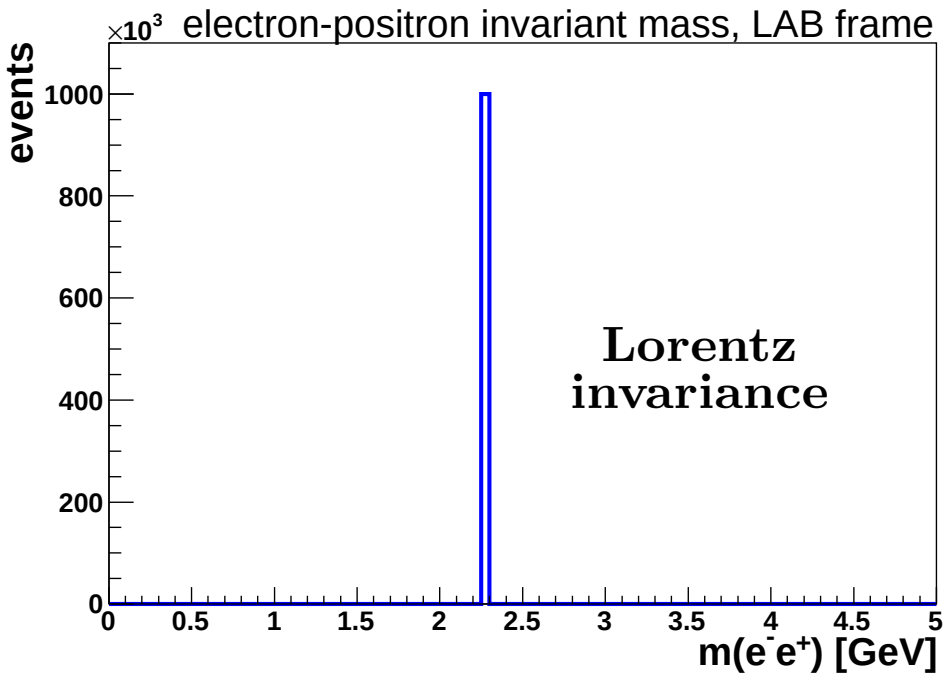


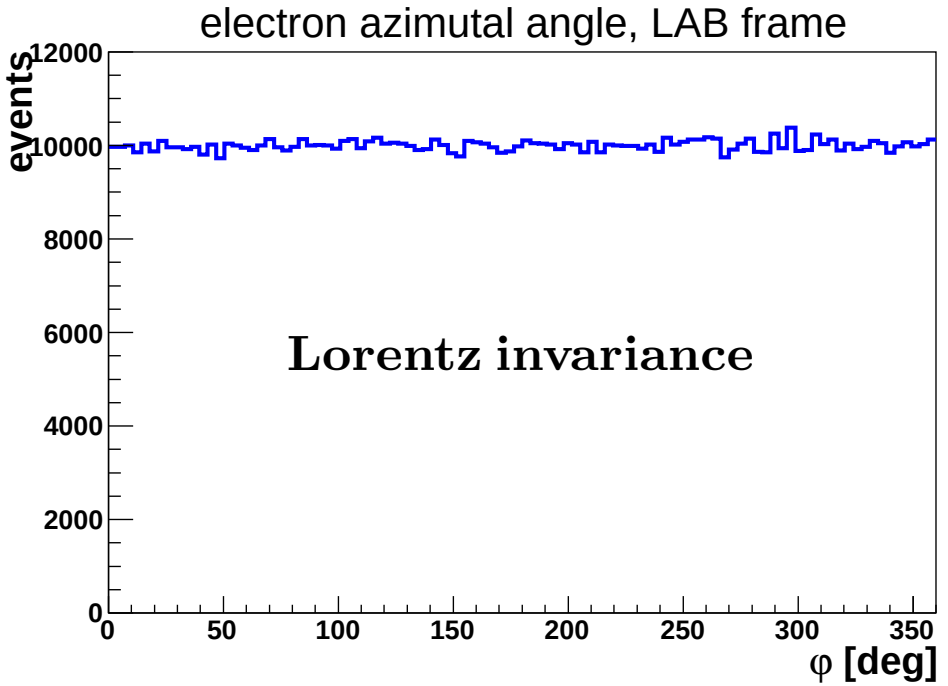


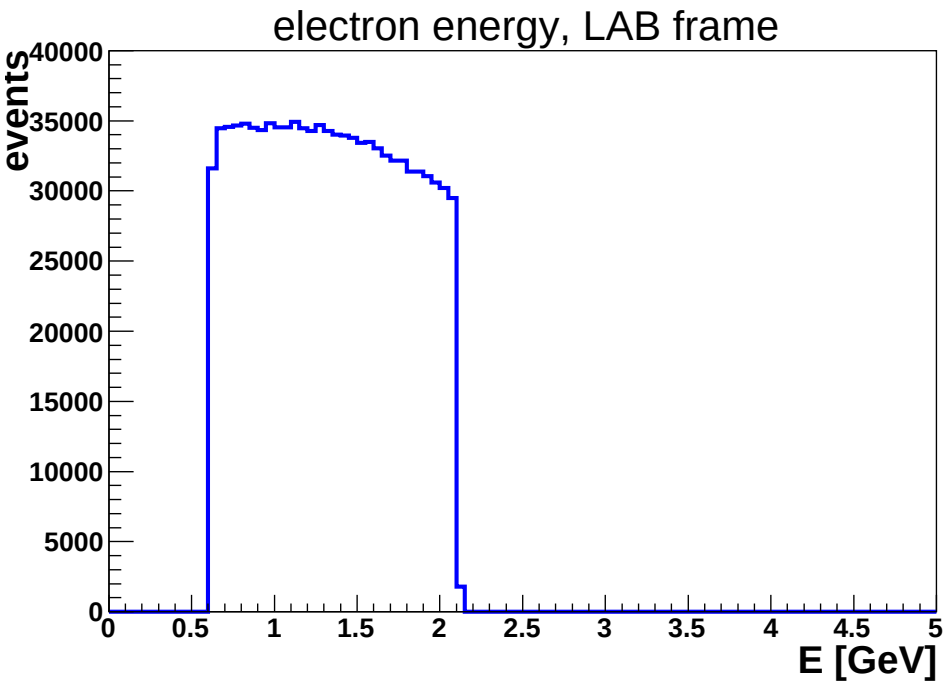


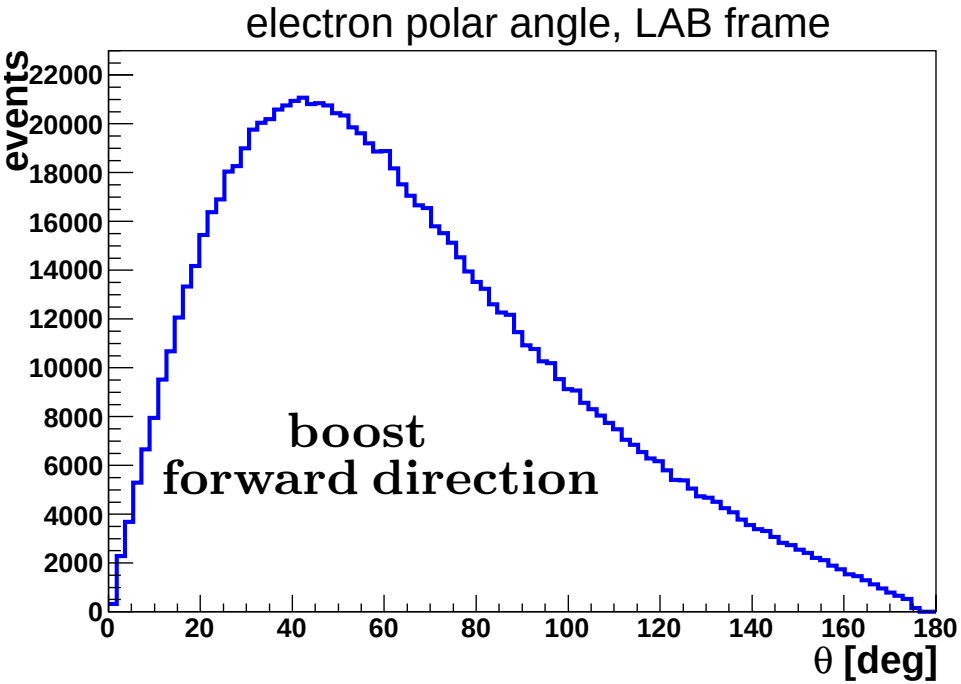
Event generator : example distributions (LAB frame)

$\sqrt{s} = 2.25$ GeV









Summary, conclusions, and perspectives

- **the NLO $\bar{p}p \to e^+e^-$ differential cross section, (including Born amplitude, full virtual corrections, and soft real corrections), has been calculated**
- **two codes fully finished and available for PANDA physics analyses: cross section calculator and event generator**

work in progress :

i) completion with the inclusion of hard photon emission

ii) interface to PANDA full simulation software (PandaRoot)