Rare and radiative decays χ_{cJ} within the NRQCD approach

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$\chi_{cJ} \to e^+e^-$ $\chi_{cJ} \to V\gamma$

Radio MonteCarLOW workshop, June 30, Mainz, Germany

NRQCD factorisation framework

There are 3 well separated scales $mv^2 \ll mv \ll m$ $v^2 \ll 1$

integrate out hard modes

Bodwin, Braaten,Lepage 1994

QCD → NRQCD

QED → NRQED

eff. Lagrangian + power counting

= systematic approach

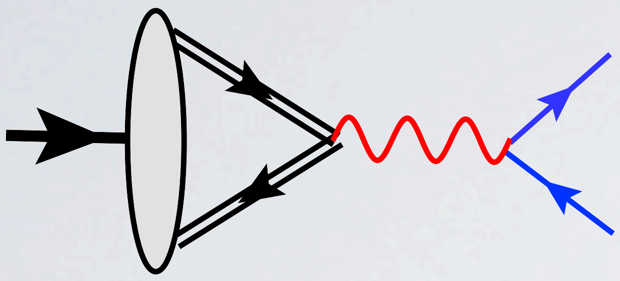
essential regions of loop momentum

Beneke, Smirnov 1997

hard	$k_0 \sim m$	$\vec{k} \sim m$	pQCD
soft	$k_0 \sim mv$	$\vec{k} \sim mv$	
potential	$k_0 \sim mv^2$	$\vec{k} \sim mv$	nonperturbative
ultrasoft	$k_0 \sim mv^2$	$\vec{k} \sim mv^2$	

NRQCD factorisation framework

$J/\Psi(^3S_1) \to e^+e^-$



$$\Gamma[J/\Psi \to ee] = \frac{4\alpha^2 e_c^2}{M^2}|R_{10}(0)|^2 \left(1 - 5.33\frac{\alpha_s}{\pi}\right)$$

$$|R_{10}(0)|^2 = 0.81 \text{ GeV}^3$$

Buchmuller-Tye potential
Eichten, Quigg 1995

theory
LO+NLO estimate

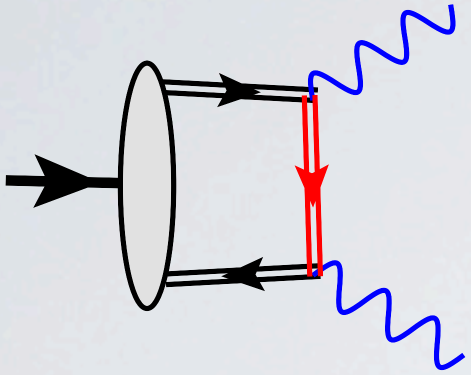
$$Br[J/\Psi \to ee] = 5.4\%$$

experiment

$$Br[J/\Psi \to ee] = 5.971 \pm 0.032\%$$

NRQCD factorisation framework

$\chi_{cJ} \to \gamma\gamma$



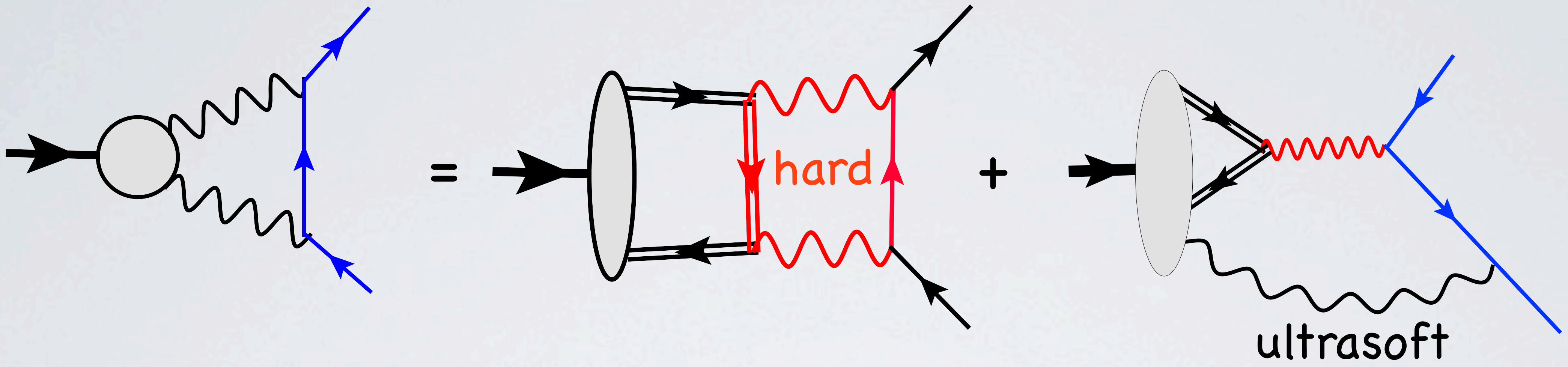
$$\Gamma[\chi_{c0} \to \gamma\gamma] = 216\alpha^2 e_c^4 \frac{|R'_{21}(0)|^2}{M^2 m_c^2}\left(1 + 0.179\frac{\alpha_s}{\pi}\right)$$

$$\Gamma[\chi_{c2} \to \gamma\gamma] = \frac{288}{5}\alpha^2 e_c^4 \frac{|R'_{21}(0)|^2}{M^2 m_c^2}\left(1 - 5.33\frac{\alpha_s}{\pi}\right)$$

Eichten, Quigg 1995 B.-T. potential $|R'_{21}(0)|^2 = 0.075\ \text{GeV}^5$ $m_c = 1.5\ \text{GeV}^2$

	theory LO+NLO estimate	experiment
$Br[\chi_{c0} \to \gamma\gamma]$	3.1×10^{-4}	$(2.23 \pm 0.13) \times 10^{-4}$
$Br[\chi_{c2} \to \gamma\gamma]$	2.1×10^{-4}	$(2.74 \pm 0.14) \times 10^{-4}$
Ratio	**1.48**	**0.81**

The amplitude for $\chi_{cJ} \to e^+e^-$ decay



The hard and ultrasoft contributions overlap and can be described consistently within NRQCD & pNRQED framework

$$\mathcal{A}_{\rm hard} \simeq R'_{21}(0) T_J [\, \bar{c}c(0) \to e^+ e^-]$$

$$T_{J=1} \sim \frac{\alpha^2}{m_c^3} \ln \frac{m_c^2}{\mu_F^2} \qquad T_{J=2} \sim \frac{\alpha^2}{m_c^3} \left\{ \ln \frac{m_c^2}{\mu_F^2} + \frac{1}{3}(\ln 2 - 1 + i\pi) \right\}$$

Kühn, Kaplan, Safiani 1979 $\qquad \mu_F \sim M - 2m_c \qquad T_{J=0} \sim m_l$

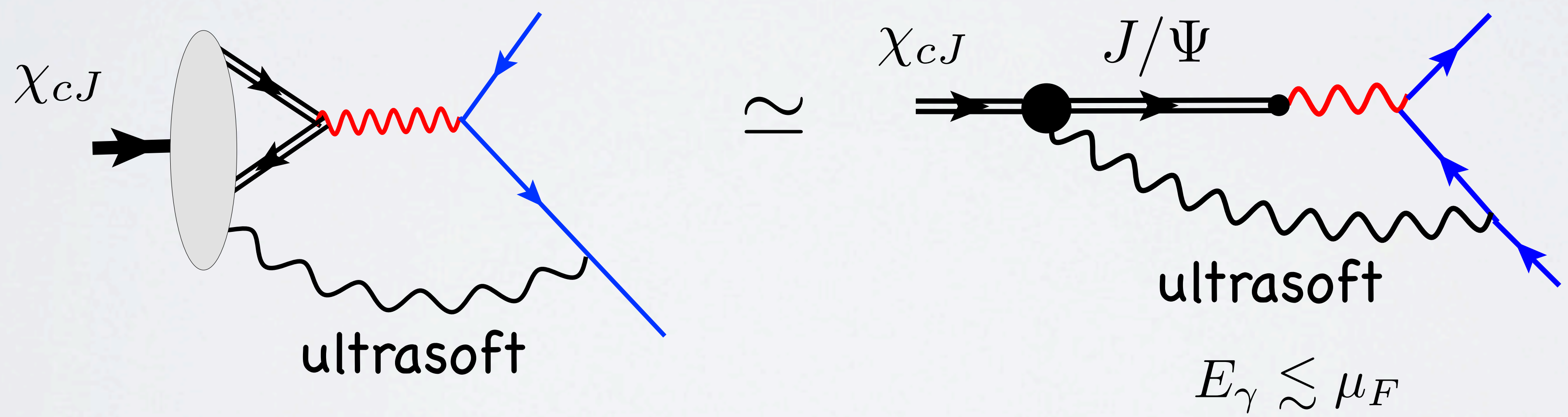
The ultrasoft contribution in $\chi_{cJ} \to e^+e^-$ decay

ultrasoft photon cannot resolve quark degrees of freedom

NRQCD → low energy effective theory

degrees of freedom: soft photons & mesons:

$\mathcal{L}_{eff}$ includes exact and approximates symmetries of NRQCD in a systematic way using 1/m expansion: HHChPT



Heavy Quark Spin Symmetry : ultrasoft photon does not resolve spin of heavy particles

Effective Lagrangian

Casalbuoni et al, 1993

$$\mathcal{L}_{int}^{eff} = ee_Q\ f_\gamma \left\{ \chi_2^{ij}\ \vec{\psi}_j\ \vec{E}_i + \sqrt{2}\,\vec{\chi}_1 \cdot (\vec{\psi} \times \vec{E}) + \frac{1}{\sqrt{3}}\chi_0\ \vec{\psi} \cdot \vec{E} \right\} + (\psi \to \psi', f_\gamma \to f'_\gamma) + \ldots$$

$$\chi_{cJ} \to J/\psi + \gamma$$

$$\psi' \to \chi_{cJ} + \gamma$$

$$|f_\gamma| = \left\{ \begin{array}{l} (\chi_{c0})\ 5.87 \\ (\chi_{c1})\ 6.05 \\ (\chi_{c2})\ 6.03 \end{array} \right\} \simeq 6.0$$

$$|f'_\gamma| = \left\{ \begin{array}{l} 6.5\ (\chi_{c0}\gamma) \\ 7.0\ (\chi_{c1}\gamma) \\ 8.1\ (\chi_{c2}\gamma) \end{array} \right\} \simeq 7.2$$

$$f_\gamma = \sqrt{2M_\chi}\sqrt{2M_\psi}\frac{1}{\sqrt{3}}\int_0^\infty dr r^3 R_{21}(r) R_{10}(r)$$

Eichten et al, 1978, 2004

$$f_\gamma > 0 \qquad f'_\gamma < 0$$

The results for widths

$$\Gamma[\chi_{c1} \to e^+e^-] = \frac{1}{12\pi} M_\chi \left| C_{\gamma\gamma}^{(1)}(\mu_0) \langle \mathcal{O}(^3P_0) \rangle + \frac{\alpha^2 e_c^2}{m_c^2} \frac{1}{\sqrt{2}} \mathcal{S}(\mu_0) \right|^2$$

$$\Gamma[\chi_{c2} \to e^+e^-] = \frac{1}{40\pi} M_\chi \left| C_{\gamma\gamma}^{(2)}(\mu_0) \langle \mathcal{O}(^3P_0) \rangle + \frac{\alpha^2 e_c^2}{m_c^2} \mathcal{S}(\mu_0) \right|^2$$

$$\mathcal{S} = f_\gamma \langle \mathcal{O}(^3S_1) \rangle \frac{\Delta}{M_\chi} \left(\ln 2 - 1 - \ln \frac{\mu_0}{\Delta} - i\pi \right) + f'_\gamma \langle \mathcal{O}'(^3S_1) \rangle \frac{\Delta'}{M_\chi} \left(\ln 2 - 1 - \ln \frac{\mu_\chi}{|\Delta'|} \right)$$

$$\Delta = (M_\chi^2 - M_{J/\Psi}^2)/2M_\chi$$

Eichten, Quigg 1995

Buchmüller-Tye potential

$$\langle \mathcal{O}(^3P_0) \rangle \sim R'_{21}(0) \qquad |R'_{21}(0)|^2 \simeq 0.075\mathrm{GeV}^5$$

$$\langle \mathcal{O}(^3S_1) \rangle \sim R_{10}(0) \qquad |R_{10}(0)|^2 \simeq 0.81\mathrm{GeV}^3$$

$$\langle \mathcal{O}'(^3S_1) \rangle \sim R_{20}(0) \qquad |R_{20}(0)|^2 \simeq 0.53\mathrm{GeV}^3$$

Numerical estimates

m_c=1.5GeV

NK, Vanderhaeghen 2016

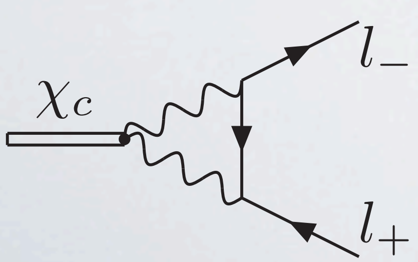
μ_0, MeV	$\Gamma[\chi_{c1} \to e^- e^+]$, eV	$\Gamma[\chi_{c2} \to e^- e^+]$, eV
300	$0.060_s + 0.009_{hs} + 0.023_h = 0.091$	$0.036_s + 0.020_{hs} + 0.016_h = 0.072$
400	$0.063_s + 0.013_{hs} + 0.011_h = 0.087$	$0.038_s + 0.017_{hs} + 0.013_h = 0.068$
500	$0.066_s + 0.011_{hs} + 0.004_h = 0.082$	$0.040_s + 0.015_{hs} + 0.010_h = 0.065$

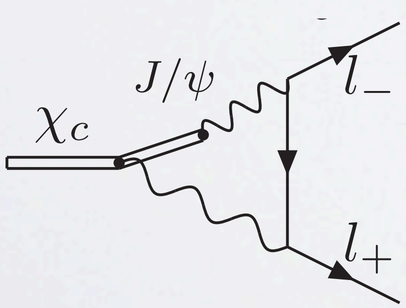
Comparison with other estimates:

Kühn, Kaplan, Safiani 1979 generalized VDM model
$$\Gamma[\chi_{c1} \to e^+ e^-] \simeq 0.46 \text{ eV}$$
$$\Gamma[\chi_{c2} \to e^+ e^-] \simeq 0.014 \text{ eV}$$

Denig, Guo, Hahnhart, Nefediev 2014 VDM model
$$\Gamma[\chi_{c1} \to e^+ e^-] \simeq 0.1 \text{ eV}$$

Czyz, Kühn, Tracz , 2016 phenom. model (VDM?)
$$\Gamma[\chi_{c1} \to e^+ e^-] \simeq 0.078 \text{ eV}$$
$$\Gamma[\chi_{c2} \to e^+ e^-] \simeq 1.35 \text{ eV}$$





	$\gamma\gamma + J/\psi\gamma$	$\gamma\gamma$	$J/\psi\gamma$	QED+Z^0
$\Gamma(\chi_{c_1} \to e^+e^-)$ [eV]	0.078	0.073	0.003	0.071
$\Gamma(\chi_{c_2} \to e^+e^-)$ [eV]	1.35	0.032	0.975	-

Radiative decays $\chi_{cJ} \to V\gamma$

Data: CLEO & BESIII

Branching fractions in units 10^{-4}

	$\chi_{c1} \to V\gamma$	$\chi_{c1} \to V_{\parallel}\gamma$	$\chi_{c1} \to V_{\perp}\gamma$	$\chi_{c0} \to V\gamma$	$\chi_{c2} \to V\gamma$
$\gamma\rho$	220 ± 18	184.8 ± 15.7	35.2 ± 7.4	< 9	< 20
$\gamma\omega$	69 ± 8	51.8 ± 8.9	17.3 ± 6.5	< 8	< 6
$\gamma\phi$	25 ± 5	17.7 ± 4.9	7.3 ± 3.6	< 6	< 8

Theory ?

Gao, Zhang, Chao 2006, '07

Chen, Dong, Liu Eur. Phys. J. (2010)

These decays must be simpler then pure hadronic decays ...

Radiative decays $\chi_{cJ} \to V\gamma$

$$A^{\perp}_{0V} : \chi_{c0} \to V(\lambda_V = \pm 1)\gamma(\lambda_\gamma = \pm 1)$$

$$A^{\perp}_{1V} \quad : \quad \chi_{c1}(\lambda_\chi = 0) \to V(\lambda_V = \pm 1)\gamma(\lambda_\gamma = \pm 1)$$

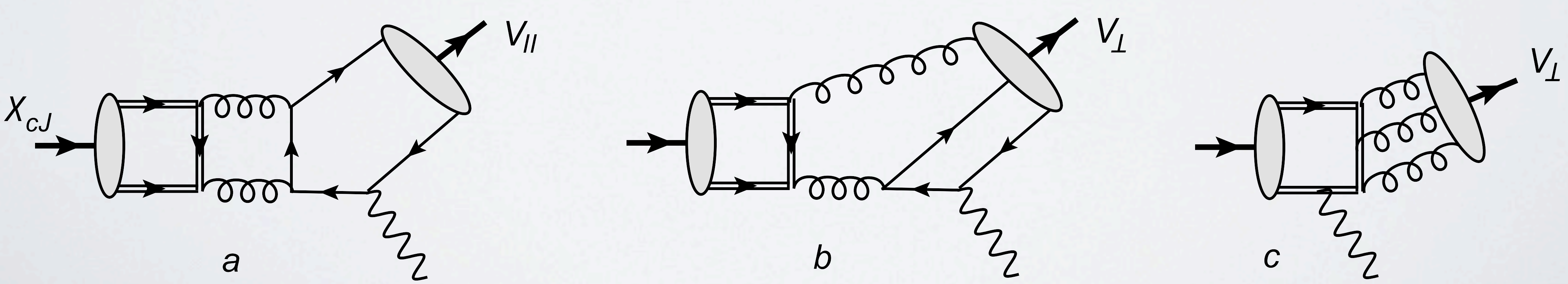
$$A^{\parallel}_{1V} \quad : \quad \underline{\chi_{c1}(\lambda_\chi = \pm 1) \to V(\lambda_V = 0)\gamma(\lambda_\gamma = \pm 1)}$$

$$A^{\perp}_{2V} \quad : \quad \chi_{c2}(\lambda_\chi = 0) \to V(\lambda_V = \pm 1)\gamma(\lambda_\gamma = \pm 1),$$

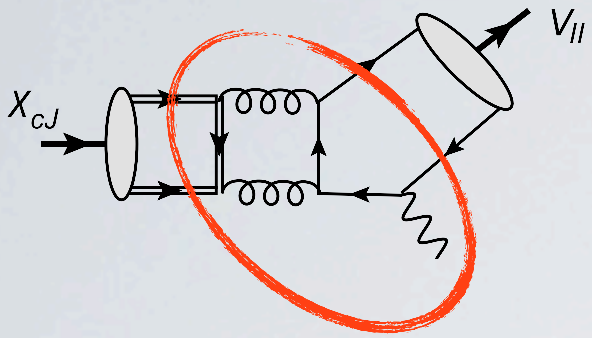
$$A^{\parallel}_{2V} \quad : \quad \underline{\chi_{c2}(\lambda_\chi = \pm 1) \to V(\lambda_V = 0)\gamma(\lambda_\gamma = \pm 1)}$$

$$T^{\perp}_{2V} \quad : \quad \chi_{c2}(\lambda_\chi = \pm 2) \to V(\lambda_V = \mp 1)\gamma(\lambda_\gamma = \pm 1)$$

Color-singlet mechanism: decays with longitudinal $V_{||}$ are dominant



Radiative decays $\chi_{cJ} \to V_{||}\, \gamma$

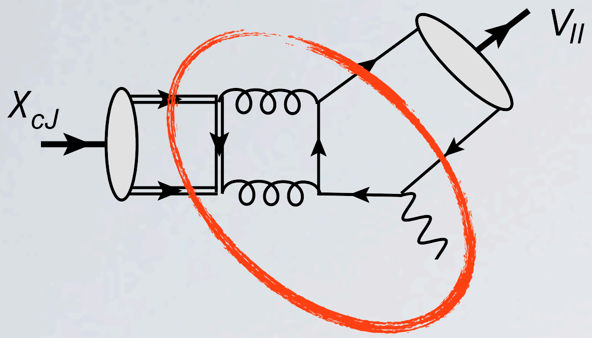


$$A[\chi_{cJ} \to V_{\|}\gamma] \sim \frac{R'_{21}(0)}{m_c^{5/2}} \sqrt{4\pi\alpha} \int_0^1 dx\, \frac{f_V}{m_c} \phi_V^{\|}(x)\, \alpha_s^2(\mu_h) T_J(x)$$

$$\begin{aligned}
\mathrm{Re}\, T_1(x) \;=\; & -\frac{\pi^2}{12}\frac{1}{\bar{x}^3} - \frac{x}{4\bar{x}^3}\ln^2 2 + \left(-\frac{1}{\bar{x}^2} - \frac{1}{4\bar{x}} - \frac{3}{4x}\right)\ln 2 + \left(-\frac{3}{4\bar{x}} + \frac{1}{4x} - \frac{1}{2x-1}\right)\ln \bar{x} \\
& + \frac{x}{\bar{x}^3}\ln x \ln \bar{x} + \left(-\frac{1}{2\bar{x}^2} + \frac{3}{4\bar{x}} - \frac{1}{4x} + \frac{1}{2x-1}\right)\ln x - \frac{3x}{4\bar{x}^3}\ln^2 x - \frac{x}{2\bar{x}^3}\ln x \ln 2 \\
& -\frac{1}{2}\frac{x}{\bar{x}^3}\left(\mathrm{Li}\left[1-\frac{1}{2x}\right] + \mathrm{Li}\,[1-2x] + \mathrm{Li}\,[1-x] + \mathrm{Li}\,[-\bar{x}/x] - \mathrm{Li}\,[2x-1]\right) + (x \to \bar{x}),
\end{aligned}$$

$$\mathrm{Im}\, T_1(x) = \frac{\pi}{4x\bar{x}^3}\left(\bar{x}(1 + x(2x-1)) + 2x^2 \ln[x]\right) + (x \to \bar{x})$$

Radiative decays $\chi_{cJ} \to V_{||}\, \gamma$



$$A[\chi_{cJ} \to V_{\|}\gamma] \sim \frac{R'_{21}(0)}{m_c^{5/2}} \sqrt{4\pi\alpha} \int_0^1 dx\, \frac{f_V}{m_c} \phi_V^{\|}(x)\, \alpha_s^2(\mu_h) T_J(x)$$

decay const. $f_\rho = 221\,\mathrm{MeV}, \quad f_\omega = 198\,\mathrm{MeV}, \quad f_\phi = 161\,\mathrm{MeV},$

$\phi_V^{\|}(x)$ Light-cone distribution amplitude

describes the momentum-fraction distribution of partons at zero transverse separation in a 2-particle Fock state

This function is well known in literature!

Model $$\phi_V(x,\mu) = 6x\bar{x}\left\{1 + a_2^V(\mu) C_2^{3/2}(2x-1)\right\}$$

$\mu = 1\,\mathrm{GeV}$ $$a_2^\rho = a_2^\omega = 0.15 \pm 0.07,\ a_2^\phi = 0.18 \pm 0.08$$

QCD SR

Ball, Braun 1996, 1999

Ball, Braun, Lenz 2006

Radiative decays $\chi_{cJ} \to V_{||}\gamma$

Theory vs. experiment : only the color singlet contribution

branching fractions in units 10^{-4} $m_c < \mu_h < 2m_c$

	$\gamma\rho$	$\gamma\omega$	$\gamma\phi$
$\chi_{c1} \to V_{\parallel}\gamma$ exp.	$153.1^{+18.2+103.7}_{-16.7-70.5}$ 184.8 ± 15.7	$13.6^{+1.6+9.2}_{-1.5-6.3}$ 51.8 ± 8.9	$31.3^{+4.2+21.4}_{-3.8-14.5}$ 17.7 ± 4.9
$\chi_{c2} \to V_{\parallel}\gamma$ exp.	$4.8^{+0.2+3.1}_{-0.2-2.1}$ < 20	$0.43^{+0.02+0.27}_{-0.02-0.19}$ < 6	$0.9^{+0.05+0.59}_{-0.04-0.41}$ < 8

Theory $$\frac{Br\left[\chi_{c1} \to \omega_{\parallel}\gamma\right]}{Br\left[\chi_{c1} \to \rho_{\parallel}\gamma\right]} \simeq \frac{1}{9}$$ (0.28)

clear indication about significance of the color octet contribution

$$\langle V(p)|J^{\mu}_{em}|\chi_{cJ}\rangle = \langle V(p)|\sum_{u,d,s} e_q\ \bar{q}\gamma^{\mu}q|\chi_{cJ}\rangle + \langle V(p)|e_c\ \bar{c}\gamma^{\mu}c|\chi_{cJ}\rangle$$

SU(2) breaking
color octet mechanism

Radiative decays $\chi_{cJ} \to V_{||}\,\gamma$

Theory vs. experiment : only the color singlet contribution

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$\chi_{c2} \to V_{\parallel}\gamma$ exp.	$2.11^{+0.09+1.3}_{-0.08-0.9}$ < 20	$0.19^{+0.008+0.12}_{-0.007-0.08}$ < 6	$0.41^{+0.02+0.26}_{-0.02-0.18}$ < 8

color octet contributions c-quark $v^2 \simeq 0.3$ $\alpha_s(2m_c^2) = 0.29$

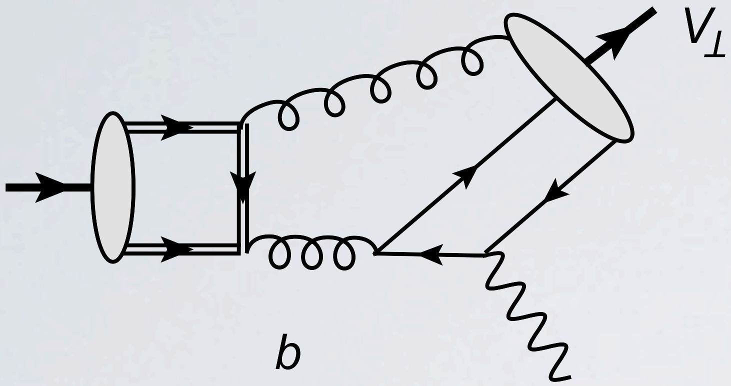
$$\left[A^{\parallel}_{1V}\right]_{oct} / \left[A^{\parallel}_{1V}\right]_{sing} \sim v^2/\alpha_s(\mu_h) \sim 1$$

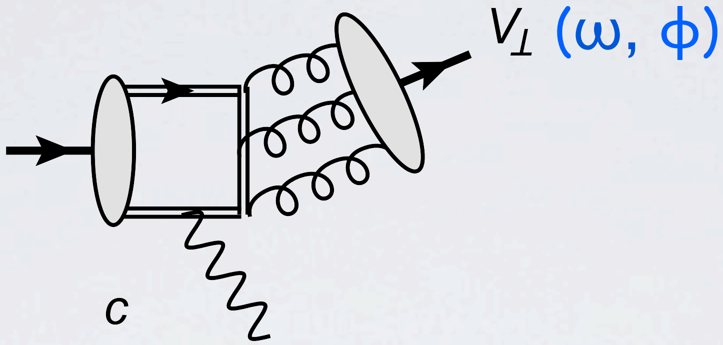
SU(2) breaking
(only ω and ϕ)

$$\left[A^{\parallel}_{1V}\right]_{oct} / \left[A^{\parallel}_{1V}\right]_{sing} \sim v^3/\alpha_s(\mu_h) \sim v \sim 0.5$$

Radiative decays $\chi_{cJ} \to V_T \gamma$

color singlet





color octet

$$[A_{1V}^{\perp}]_{oct} \,/\, [A_{1V}^{\perp}]_{sing} \sim 1$$

large ambiguity

Twist-3 DAs

$$A(\alpha_i) = 360\zeta_3 \alpha_1\alpha_2\alpha_3^2 \left(1 + \omega_3^A \frac{1}{2}(7\alpha_3 - 3)\right) \qquad G(\alpha_i) = 5040\,\zeta_3\omega_3^G \alpha_1^2\alpha_2^2\alpha_3^2$$

$$V(\alpha_i) = 540\zeta_3\omega_3^V \alpha_1\alpha_2\alpha_3^2(\alpha_2 - \alpha_1).$$

$$\mu = 1\,\text{GeV}$$

$$\begin{aligned} \rho \text{ and } \omega\text{-mesons} \quad &: \quad \zeta_3 = 0.030 \pm 0.010, \quad \omega_3^A = -3.0 \pm 1.4, \quad \omega_3^V = 5.0 \pm 2.4 \\ \phi\text{-meson} \quad &: \quad \zeta_3 = 0.024 \pm 0.008, \quad \omega_3^A = -2.6 \pm 1.3, \quad \omega_3^V = 5.3 \pm 3.0 \end{aligned}$$

$$|\omega_3^G(\mu = 1\text{GeV})| \ll 1$$

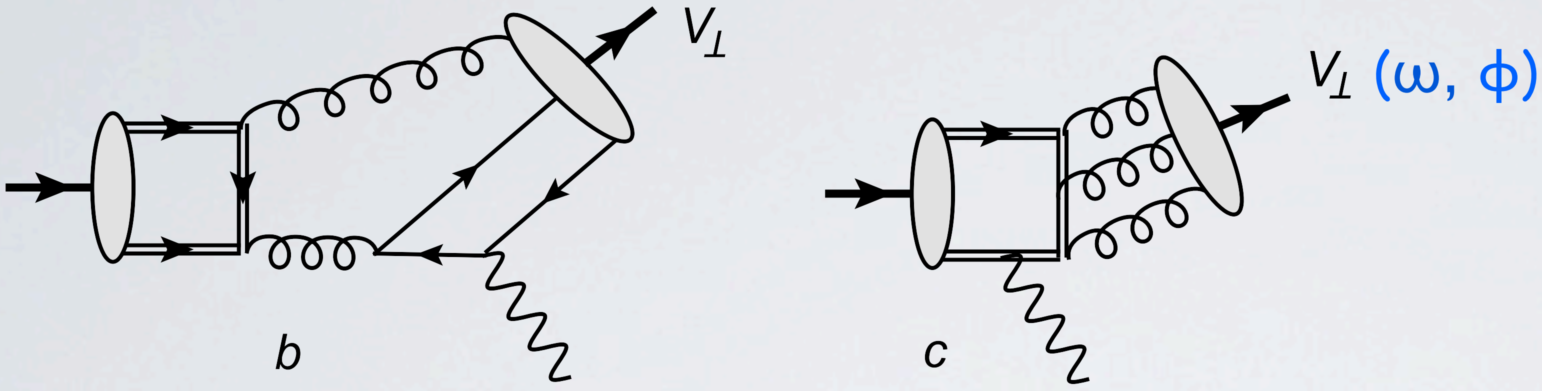
Ball, Braun 1996, 1999

Ball, Braun, Koike, Tanaka 1998

Ball, Braun, Lenz 2006

Radiative decays $\chi_{cJ} \to V_T \gamma$

color singlet



color octet

$$[A^{\perp}_{1V}]_{oct} \,/\, [A^{\perp}_{1V}]_{sing} \sim 1$$

large ambiguity

only the color singlet contribution

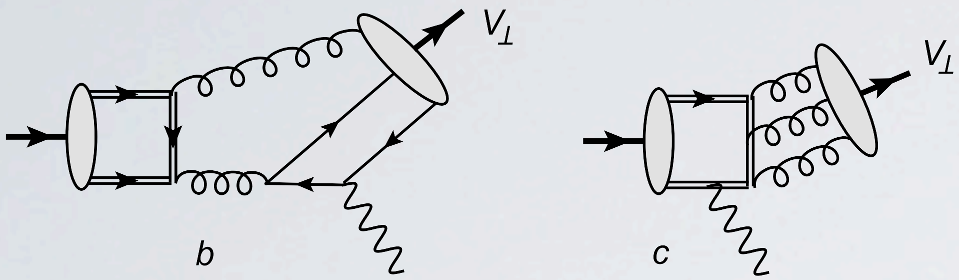
$$\Gamma^{\perp}_{\rho} = 222.4\ \zeta_3^2(-9.82 + 4.78\omega_3^A + 3.31\omega_3^V)^2$$

$$\Gamma^{\perp}_{\phi} = 168.7\ \zeta_3^2(6.5 - 3.3\ \omega_3^A + 2.9\ \omega_3^V + 733.6\ \omega_3^G)^2$$

$$\Gamma^{\perp}_{\omega} = 181.2\ \zeta_3^2(-3.3 + 1.4\ \omega_3^A + 8.3\ \omega_3^V + 735.6\ \omega_3^G)^2$$

Radiative decays $\chi_{cJ} \to V_T\,\gamma$

color singlet



(ω, ϕ)

only the color singlet

branching fractions in units 10^{-4}

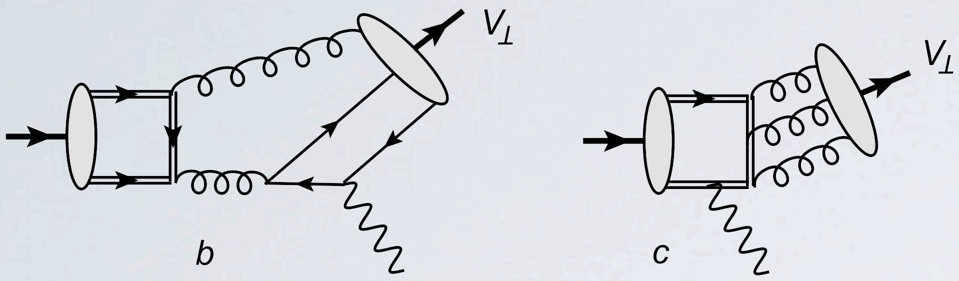
				$\chi_{c1} \to V_\perp \gamma$		
ζ_3	ω_3^A	ω_3^V	ω_3^G	ρ	ω	ϕ
				(35.2 ± 7.4)	(17.3 ± 6.5)	(7.3 ± 3.6)
0.03	-2.2	2.8	-0.037	29.6	20.8	4.8
0.03	-4.4	5.9	-0.043	30.0	13.9	8.5
0.04	-2.5	3.7	-0.041	39.2	13.5	6.5
0.04	-3.4	5.1	-0.038	33.2	14.2	6.2

Data can be described including small 3g contribution

$\chi_{c2} \to V\gamma$			$\chi_{c0} \to V\gamma$		
ρ	ω	ϕ	ρ	ω	ϕ
< 20	< 6	< 8	< 9	< 8	< 6
3.4	0.18	2.6	2.0	0.17	0.66
17.2	3.7	6.5	0.40	0.05	0.15
7.1	0.40	5.6	1.9	0.16	0.54
16.3	3.6	6.50	0.62	0.09	0.23

Radiative decays $\chi_{cJ} \to V_T \gamma$

color singlet



(ω, ϕ)

only the color singlet contribution

color octet

$$[A^{\perp}_{1V}]_{oct} \,/\, [A^{\perp}_{1V}]_{sing} \sim 1$$

large ambiguity

Data can be described including small 3g contributions

$$Br[\chi_{c1} \to \gamma\rho] > Br[\chi_{c2} \to \gamma\rho] > Br[\chi_{c0} \to \gamma\rho]$$

$$Br[\chi_{c2} \to \gamma\rho] > Br[\chi_{c2} \to \gamma\phi] > Br[\chi_{c2} \to \gamma\omega]$$

$$Br[\chi_{c2} \to \gamma\phi] \geq Br[\chi_{c1} \to \gamma\phi]$$

Precise measurements
of $Br[\chi_{c2} \to \gamma\rho]$
can help to reduce theoretical ambiguity

Thank you!