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Collinear and TMD Quark and Gluon Densities from Parton Branching Solution of QCD Evolution Equations

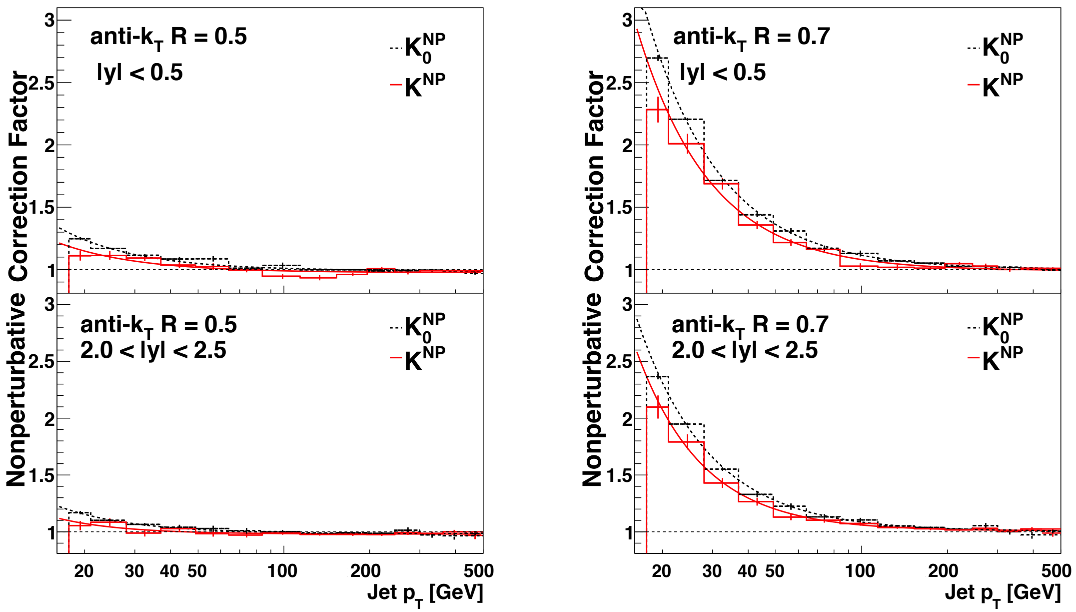
F. Hautmann

- collaboration with R. Zlebcik, V. Radescu, A Lelek and H. Jung

Phys. Lett. B 772 (2017) 446 [arXiv:1704.01757 [hep-ph]] + work in progress

MOTIVATION (I):
NONPERTURBATIVE CORRECTION FACTOR K^{NP} TO JET TRANSVERSE MOMENTUM DISTRIBUTIONS

[Dooling et al., PRD87 (2013) 094009]

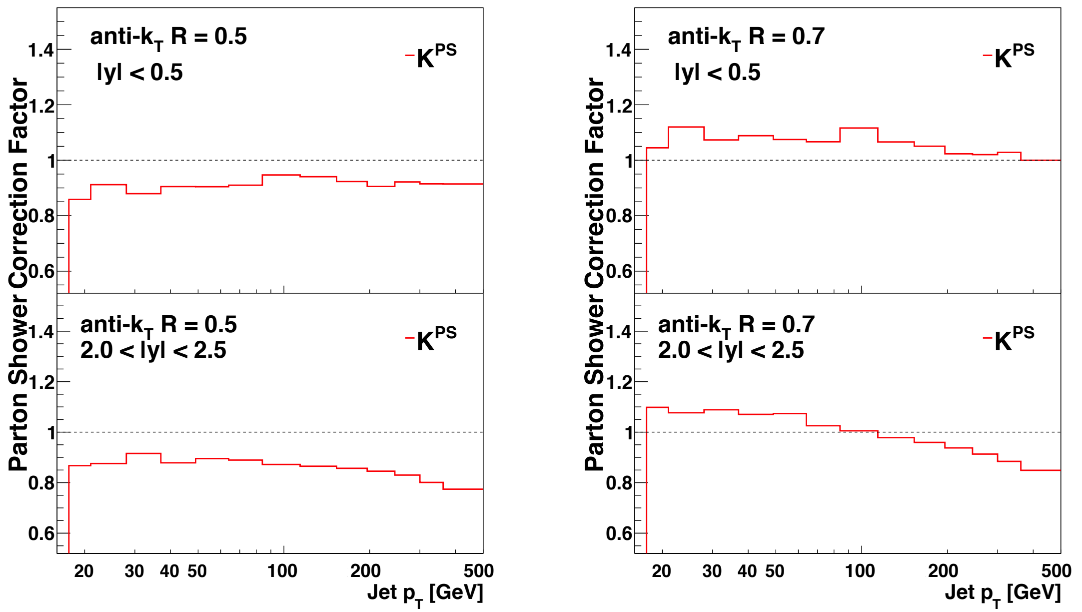


▷ non-negligible differences from definition of the hard process

▷ MPI p_T cut-off scale different in the LO and NLO cases

MOTIVATION (II):
PARTON SHOWERING CORRECTION FACTOR K^{PS} TO JET TRANSVERSE MOMENTUM DISTRIBUTIONS

[Dooling et al., PRD87 (2013) 094009]

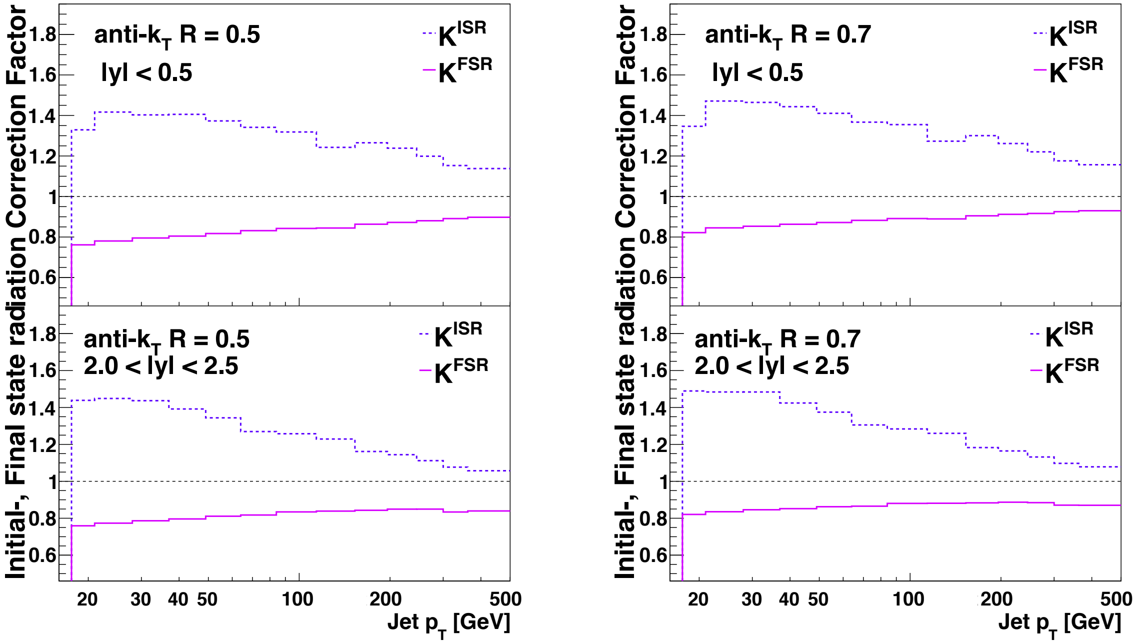


▷ not just a "K-factor" — y and p_T dependent, especially when rapidity is non-central

▷ unlike the NP correction, finite effects also at large p_T

Initial-state shower and final-state shower contributions to K^{PS}

[Dooling et al., PRD87 (2013) 094009]



▷ ISR and FSR inter-related ⇒ combined effect not additive — may dip below FSR only

▷ ISR largest at low p_T, FSR significant for all p_T

- What is the appropriate use of parton distribution functions in parton showers?

  - How can one treat the QCD shower's transverse momentun kinematics?

    - What is the role of TMDs?

▷ solve RG evolution equations by parton branching

▷ determine collinear and TMD densities

NB: soft-gluon resolution scale in evolution equations

Evolution equation and parton branching method

- use momentum-weighted PDFs: $xf(x,t)$

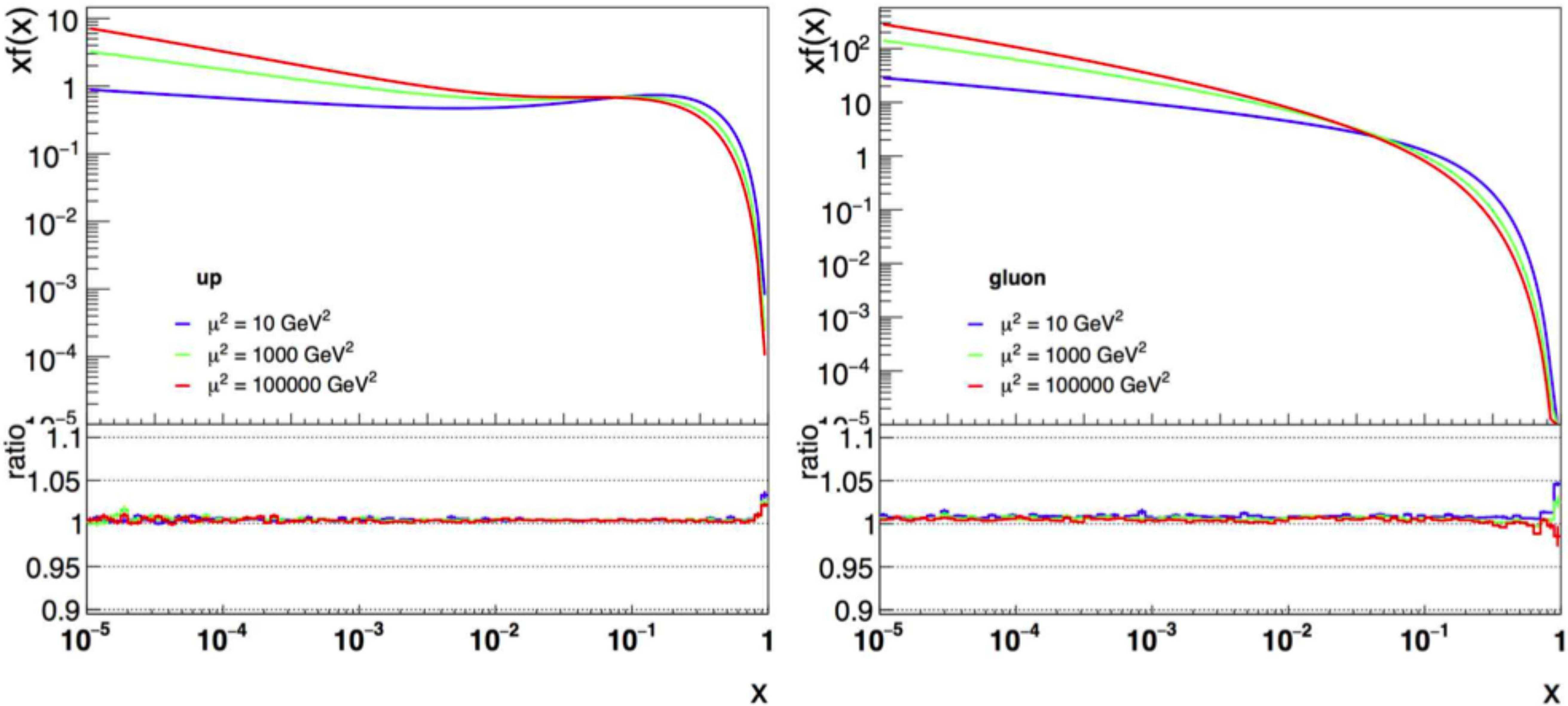
$$
\begin{aligned}
xf_a(x,\mu^2) &= \Delta_a(\mu^2)\, xf_a(x,\mu_0^2) \\
&+ \sum_b \int_{\mu_0}^{\mu^2} \frac{d\mu'^2}{\mu'^2} \frac{\Delta_a(\mu^2)}{\Delta_a(\mu'^2)} \int_x^{z_M} dz P_{ab}^{(R)}(\alpha_s, z)\, \frac{x}{z} f_b\left(\frac{x}{z}, \mu'^2\right)
\end{aligned}
$$

  - with $P_{ab}{}^{(R)}(\alpha_s(t'),z)$ real emission probability (without virtual terms)
    - z_M introduced to separate real from virtual and non-emission probability

- make use of momentum sum rule to treat virtual corrections
  - use Sudakov form factor to treat non-resolvable and virtual corrections

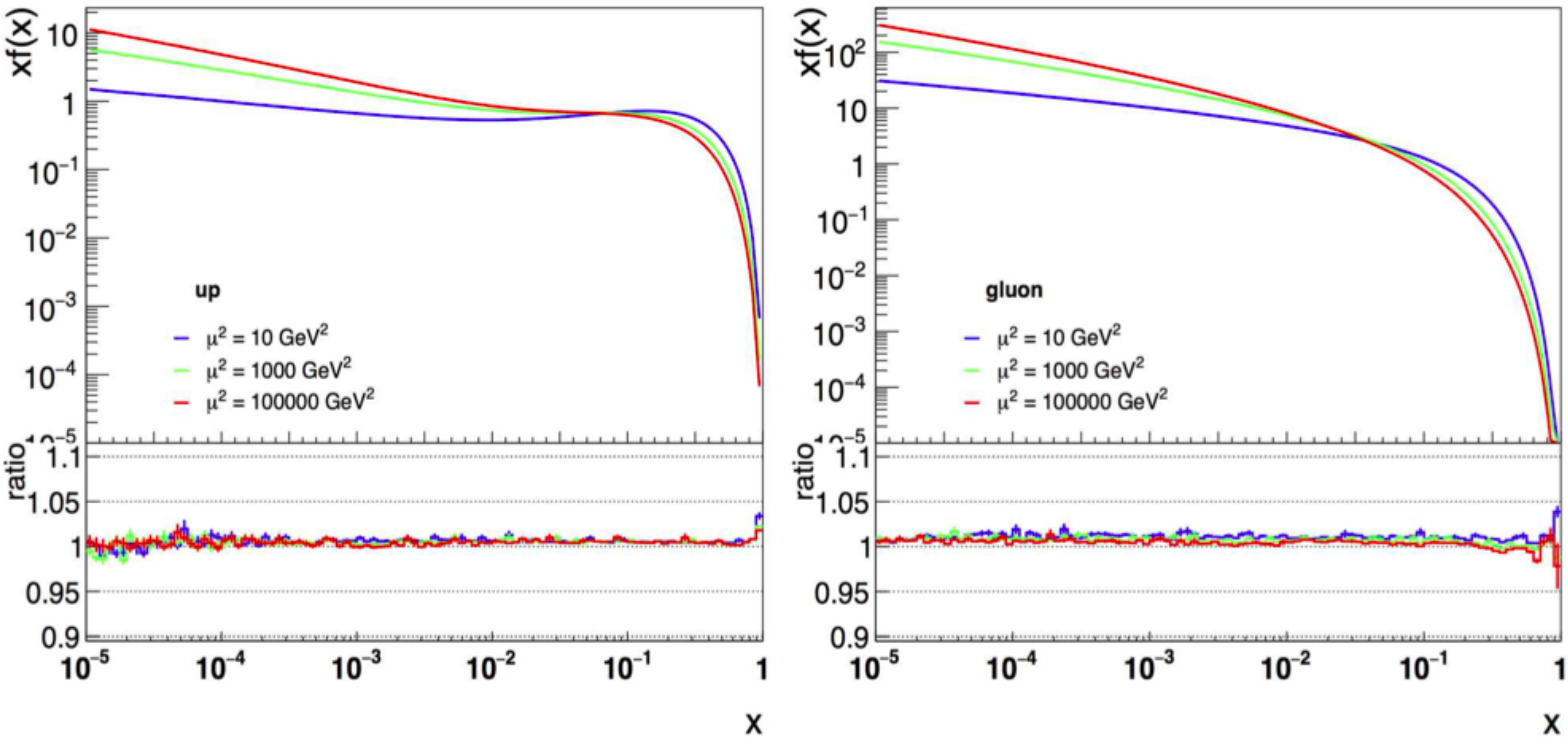
$$
\Delta_a(z_M, \mu^2, \mu_0^2) = \exp\left(- \sum_b \int_{\mu_0^2}^{\mu^2} \frac{d\mu'^2}{\mu'^2} \int_0^{z_M} dz\; z\; P_{ba}^{(R)}(\alpha_s), z) \right)
$$

Validation of method with semi-analytic result at LO



- Very good agreement with LO – semi-analytic method (QCDnum) over all x and μ^2

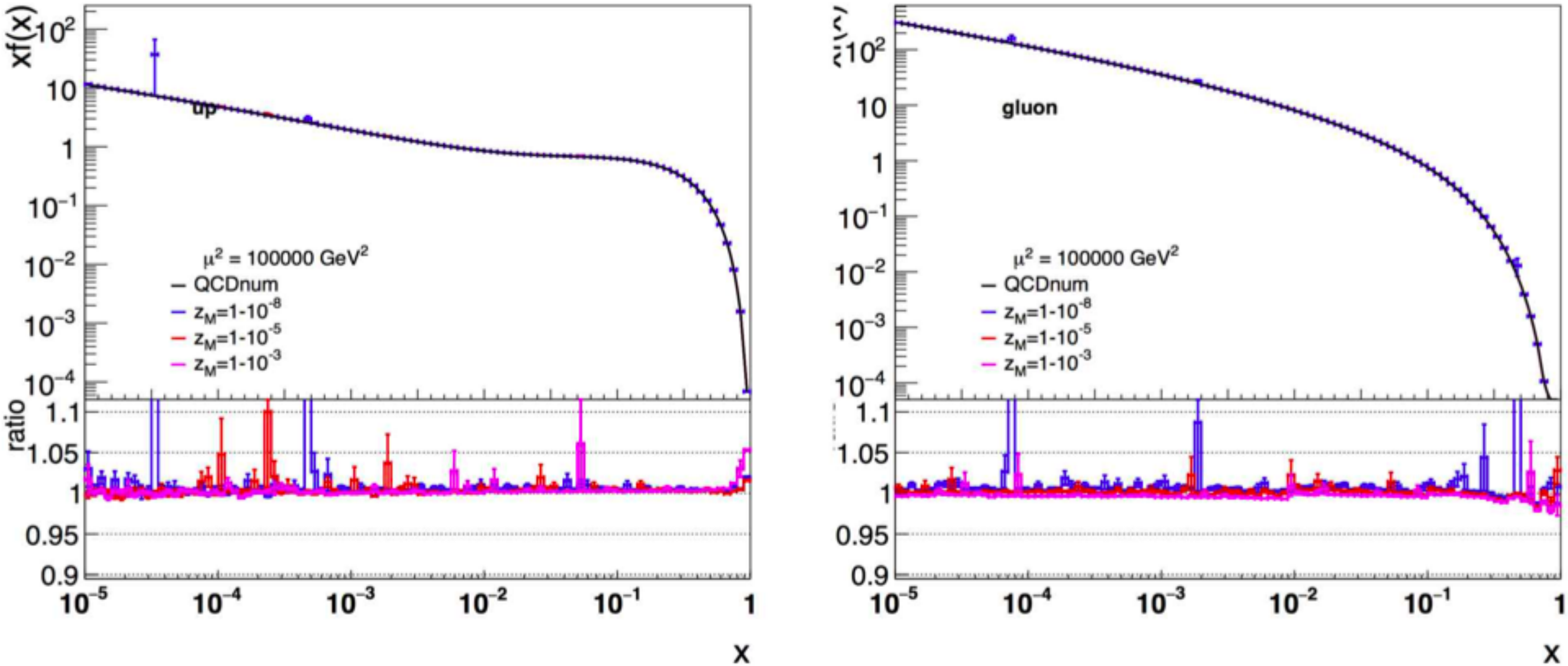
Validation of method with semi-analytic result at NLO



- Very good agreement with NLO - QCDnum over all x and μ^2
  - the same approach work also at NNLO !

Resolvable branching – at LO and NLO

- Investigate dependence on z_M : separate resolvable from virtual and non-resolvable branchings



- for large enough z_M : results are stable, both at LO and NLO (shown)
  - Sudakov treats non-resolvable and virtual branchings to all orders !

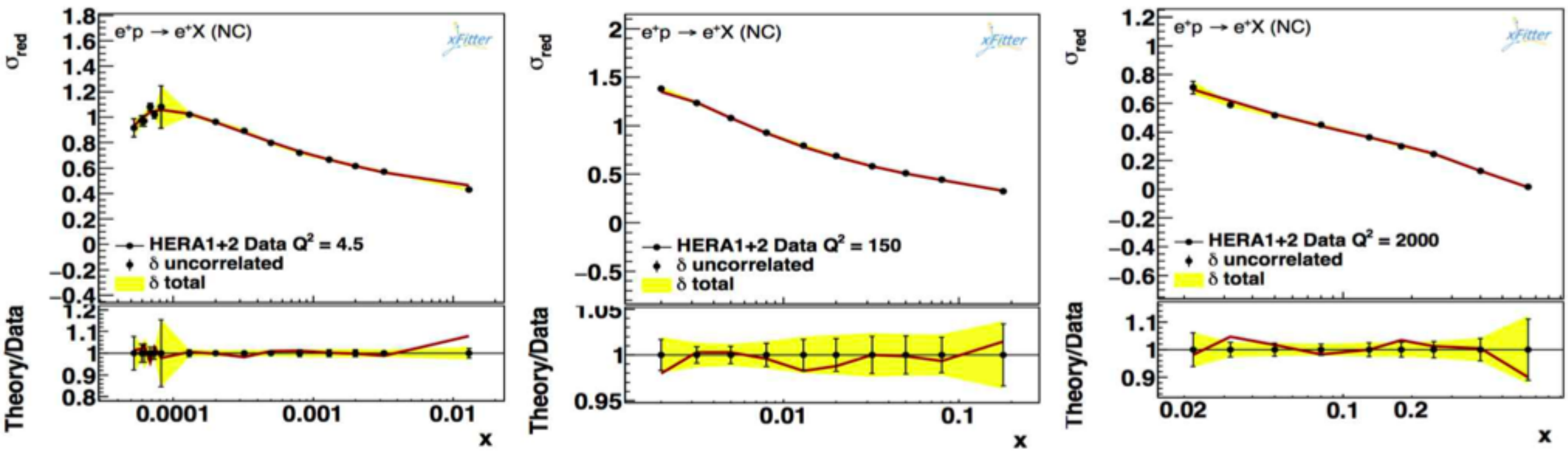
Parton branching method in xFitter

- Determine starting distribution

A. Lelek et al REF 2016

$$xf_a(x,\mu^2) = x\int dx'\int dx''\mathcal{A}_{0,b}(x')\tilde{\mathcal{A}}_a^b\left(x'',\mu^2\right)\delta(x'x''-x)$$
$$= \int dx'\mathcal{A}_{0,b}(x')\cdot\frac{x}{x'}\,\tilde{\mathcal{A}}_a^b\left(\frac{x}{x'},\mu^2\right)$$

- fit to HERA data (using xFitter) with $Q^2 \geq 3.5$ GeV^2 gives $\chi^2/ndf \sim 1.2$



- procedure to fit initial distribution is working and producing results as expected

Advantages of parton branching method

- Consistency checks with QCDnum show agreement for inclusive distributions at the 0.1% level

- Advantages of parton branching method for collinear PDFs:
  - studies of different ordering conditions possible for the first time
    - resolvable branchings as defined by:
      - angular ordering with varying $z_M = 1 - q_0/\mu'$
      - Q^2 ordering with varying $z_M = 1 - q_0/\mu'^2$
  - different choices of scales in $\alpha_s(\mu)$ possible
  - any investigation which involves details of parton branching kinematics

- further advantages – determination of TMD parton densities
  - since parton branching kinematics are known, transverse momenta of propagating partons can be calculated – determine TMD

Determination of TMD distribution

$$f(x,\mu^2) = f(x,\mu_0^2)\Delta_s(\mu^2) + \int \frac{dz}{z}\int \frac{d\mu'^2}{\mu'^2}\cdot\frac{\Delta_s(\mu^2)}{\Delta_s(\mu'^2)}P^{(R)}(z) f\left(\frac{x}{z},\mu'^2\right)$$

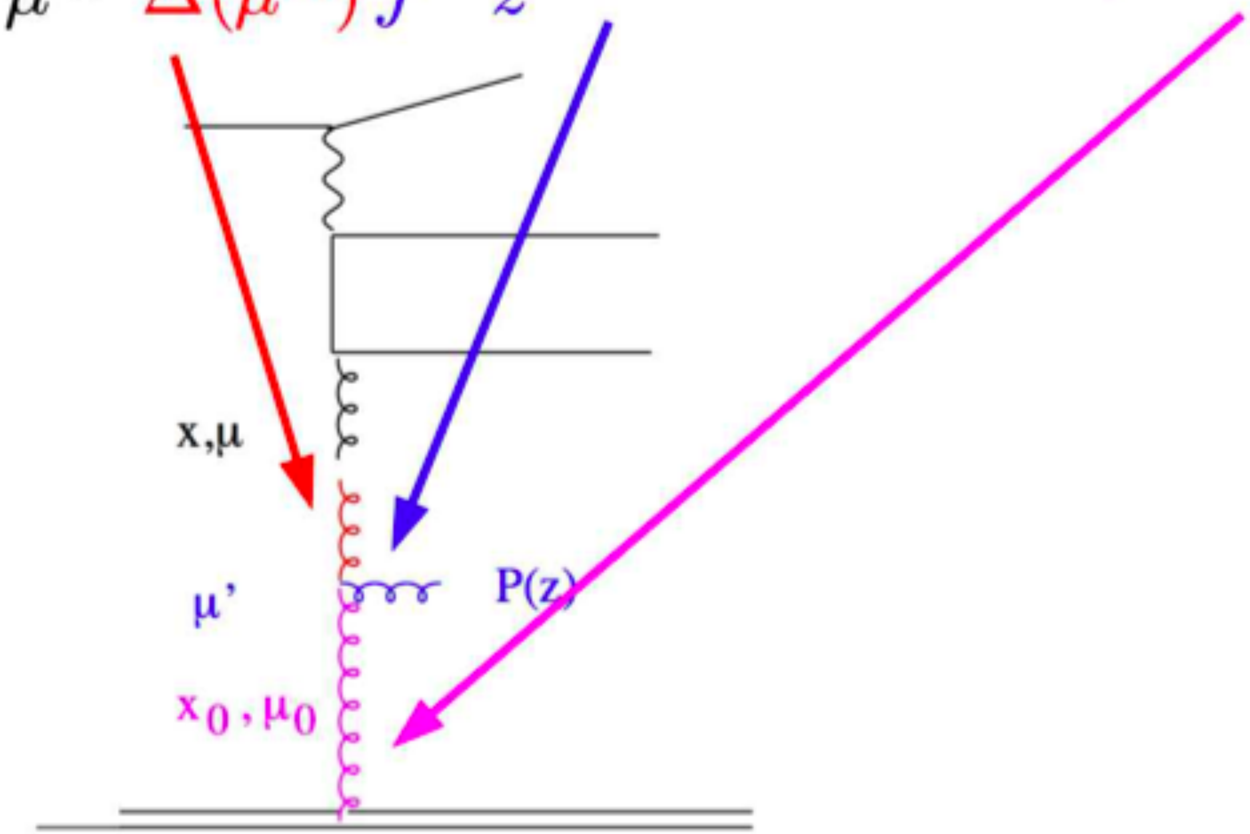
- solve integral equation via iteration:

from t' to t w/o branching | branching at t' | from t_0 to t' w/o branching

$$f_0(x,\mu^2) = f(x,\mu_0^2)\Delta(\mu^2)$$

$$f_1(x,\mu^2) = f(x,\mu_0^2)\Delta(\mu^2) + \int_{\mu_0^2}^{\mu^2}\frac{d\mu'^2}{\mu'^2}\frac{\Delta(\mu^2)}{\Delta(\mu'^2)}\int\frac{dz}{z}P^{(R)}(z) f(x/z,\mu_0^2)\Delta(\mu'^2)$$

- in every step, kinematics are known:
  - calculate k_t of propagator



Determination of TMD distribution

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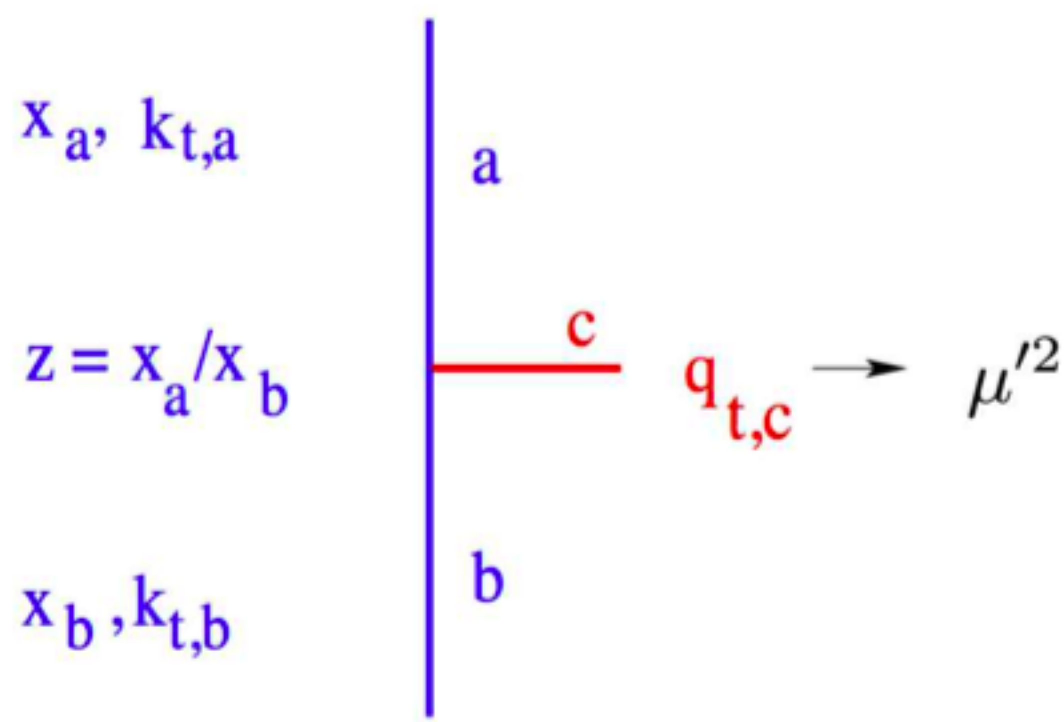
- in every step, kinematics are known:
  - calculate k_t of propagator
  - need correspondence:
    - $q_t{}^2 = \mu'^2$ with q_t emitted parton

      OR
    - $q_t{}^2 = (1-z)\,\mu'^2$, q^2 - ordering

      OR
    - $q_t{}^2 = (1-z)\,\mu'$ angular - ordering



$$\int x\,\mathcal{A}_a(x,\mathbf{k},\mu^2)\,\frac{d^2\mathbf{k}}{\pi} = \widetilde{f}_a(x,\mu^2)\,. \tag{13}$$

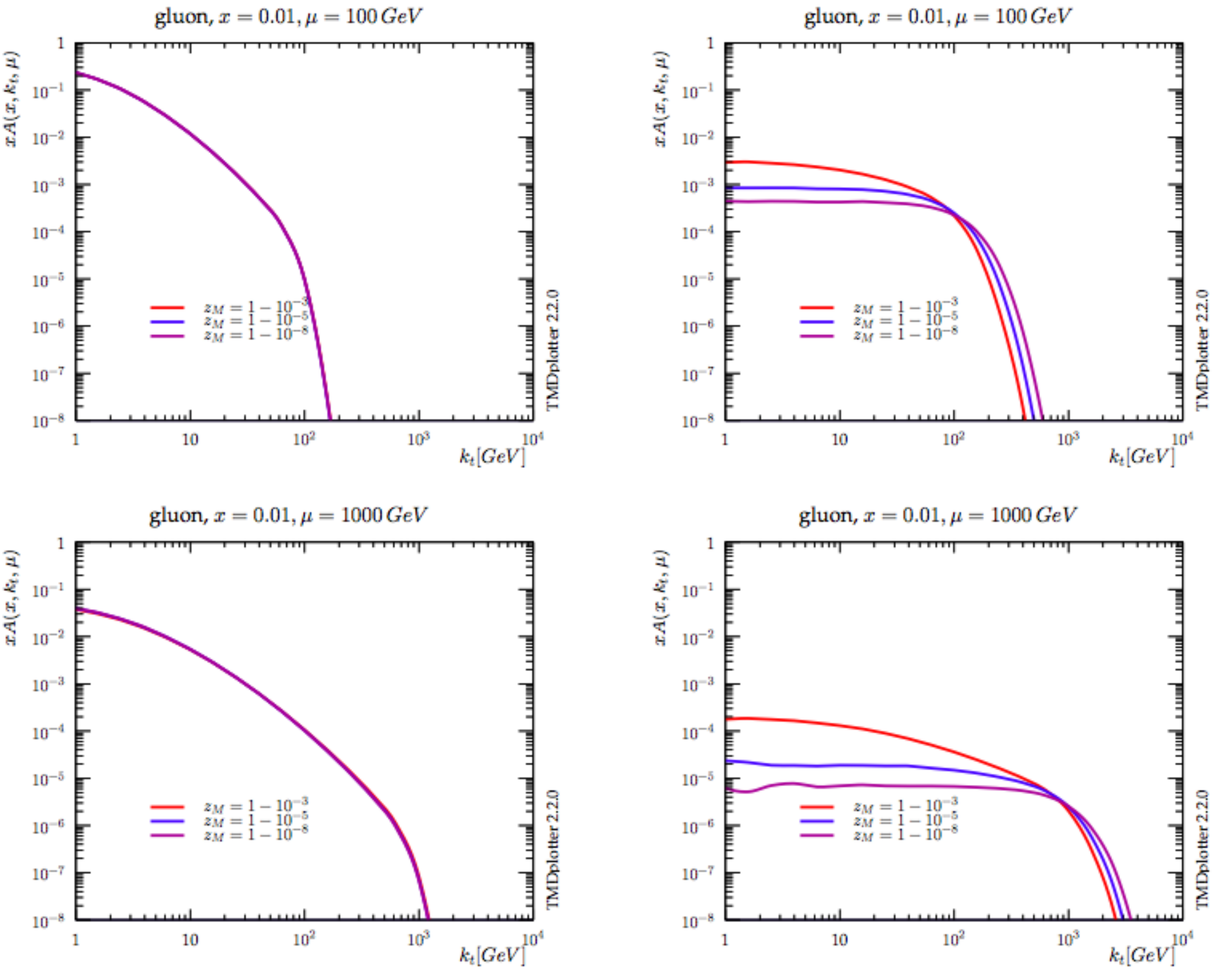
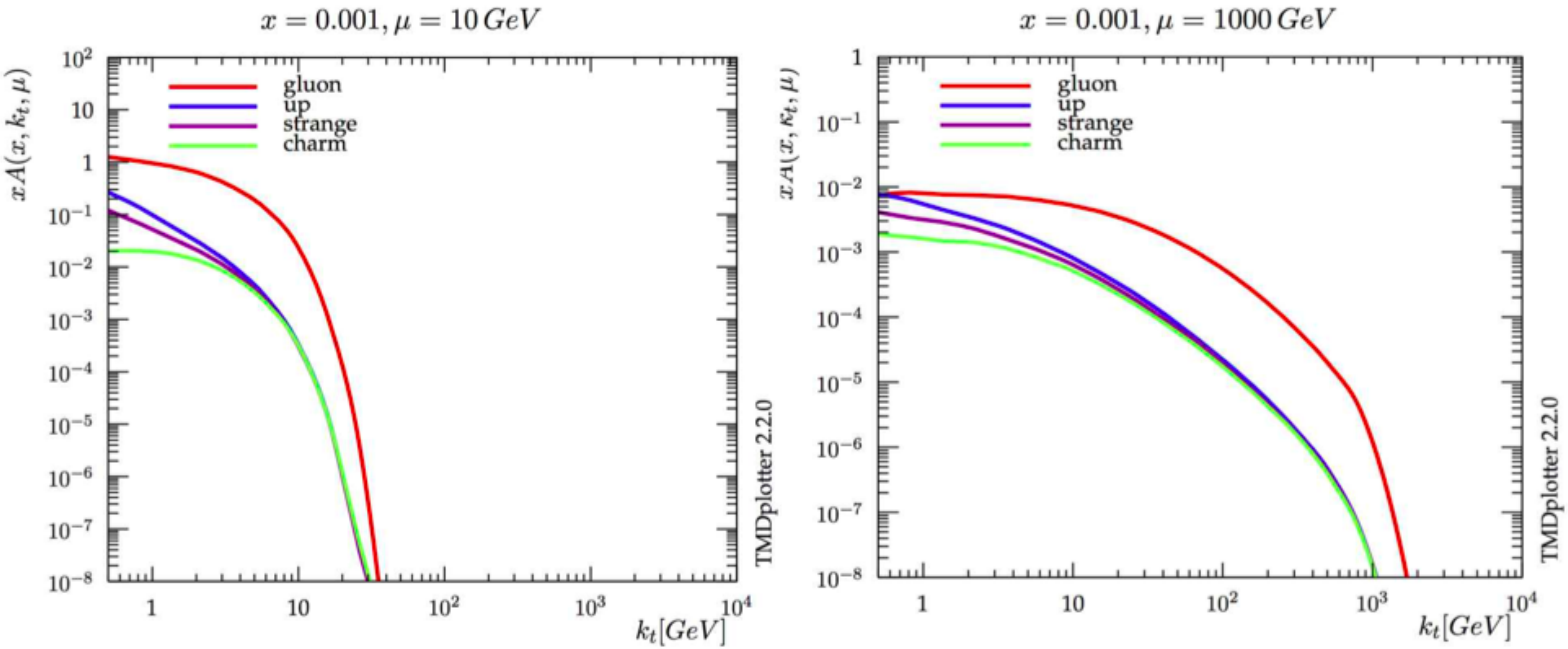


Figure 2: *Transverse momentum gluon distribution at $x = 10^{-2}$ and $\mu = 100$ GeV (upper row), $\mu = 1000$ GeV (lower row) for different values of the resolution scale parameter $1 - z_M = 10^{-3}, 10^{-5}, 10^{-8}$: (left) angular ordering; (right) transverse momentum ordering.*

TMD distributions for different flavors

- with parton-branching method, TMD distribution for all flavors can be determined.



- at small k_t intrinsic (gauss) distribution is used $\rightarrow$ subject to fit at small k_t
- at $k_t \geq Q_0$, k_t – distribution comes entirely from evolution,
  - no free parameters, except association of evolution scale with q_t

TMD distributions for different flavors

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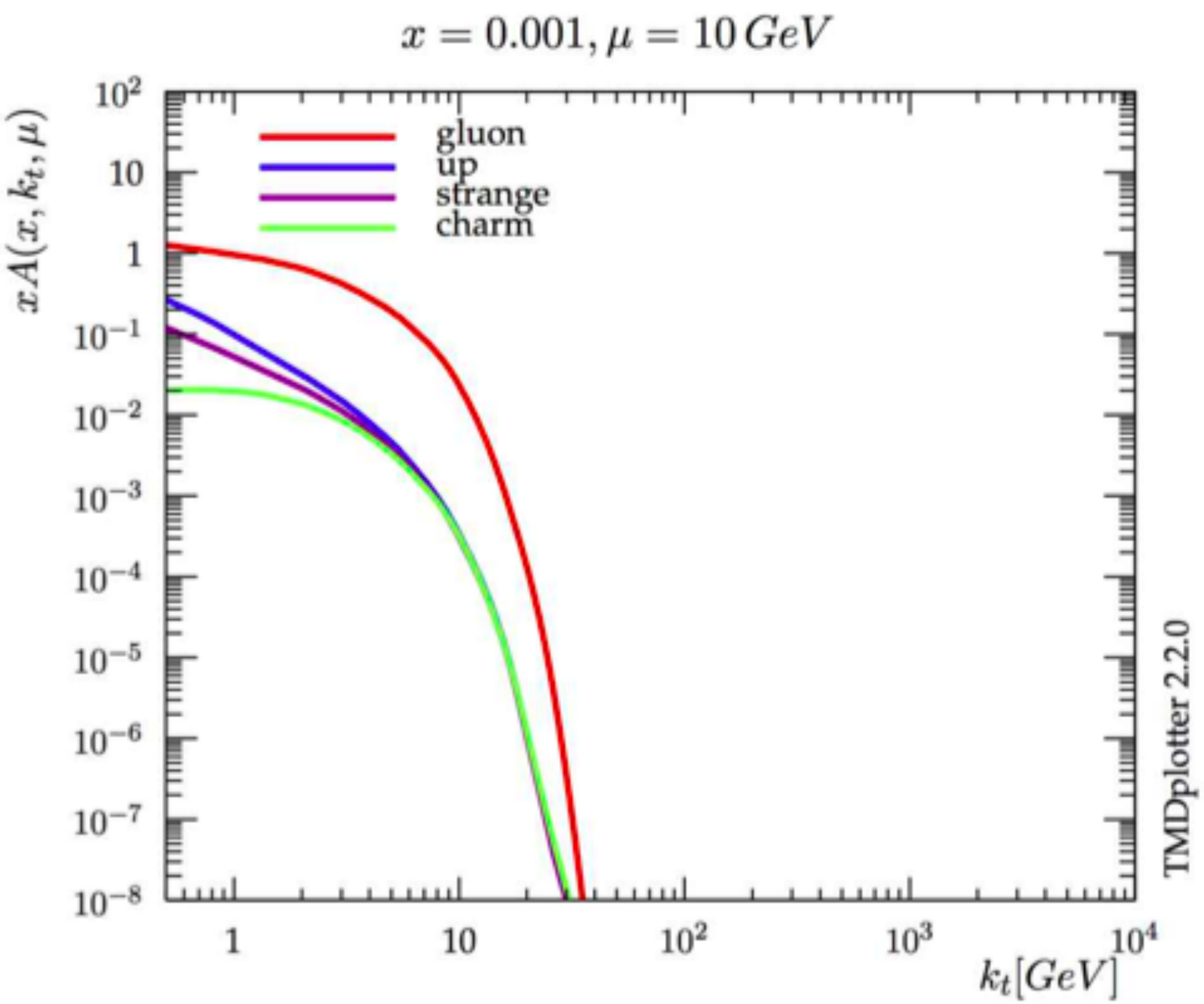


- small k_t: distributions are different

$$x\mathcal{A}(x, k_\perp^2, \mu^2) = \Delta_a(\mu^2)\; x\mathcal{A}(x, k_\perp^2, \mu_0^2) + \cdots$$

➔ depends on distribution at starting scale

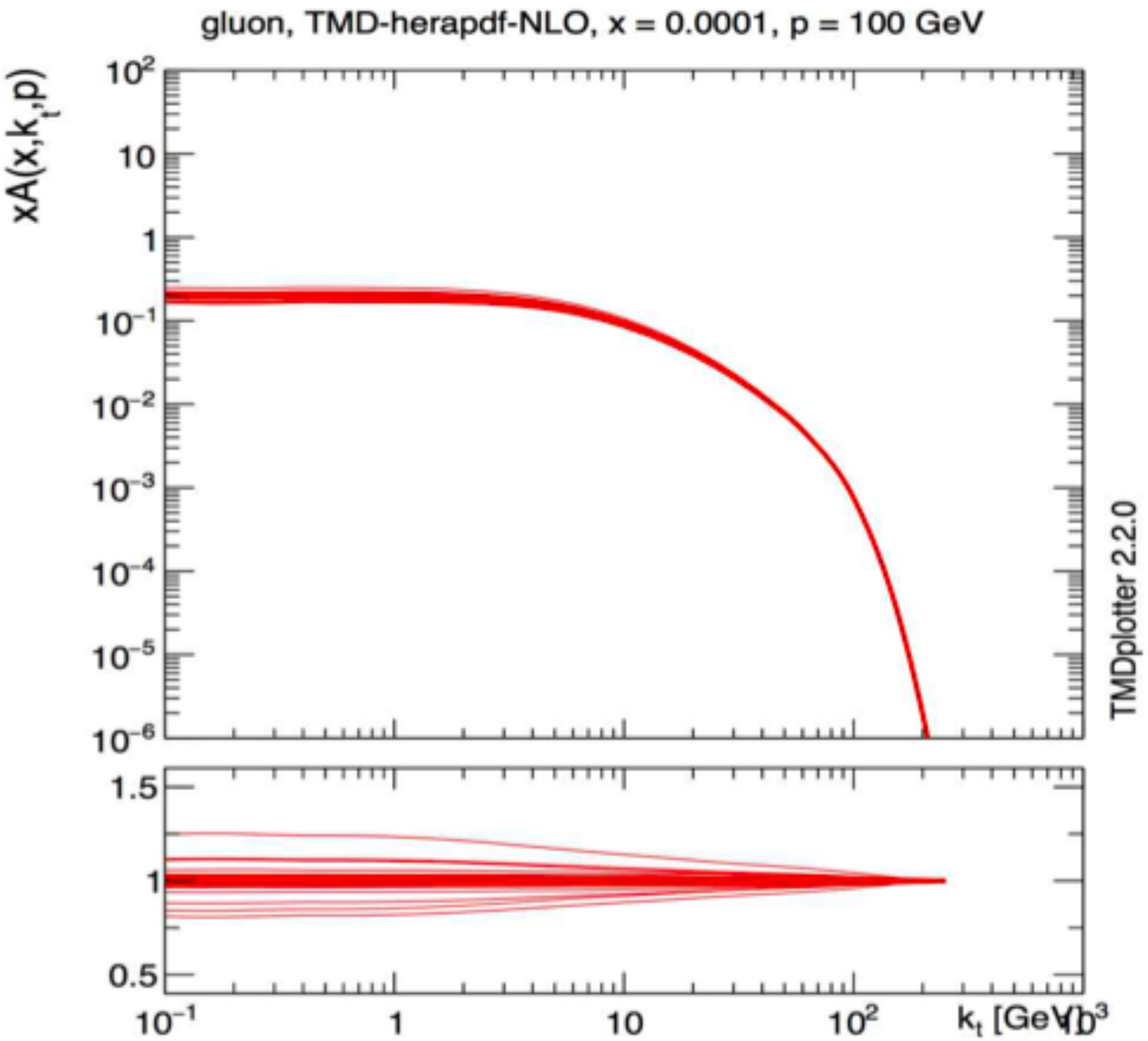
- large k_t : quarks have similar shape

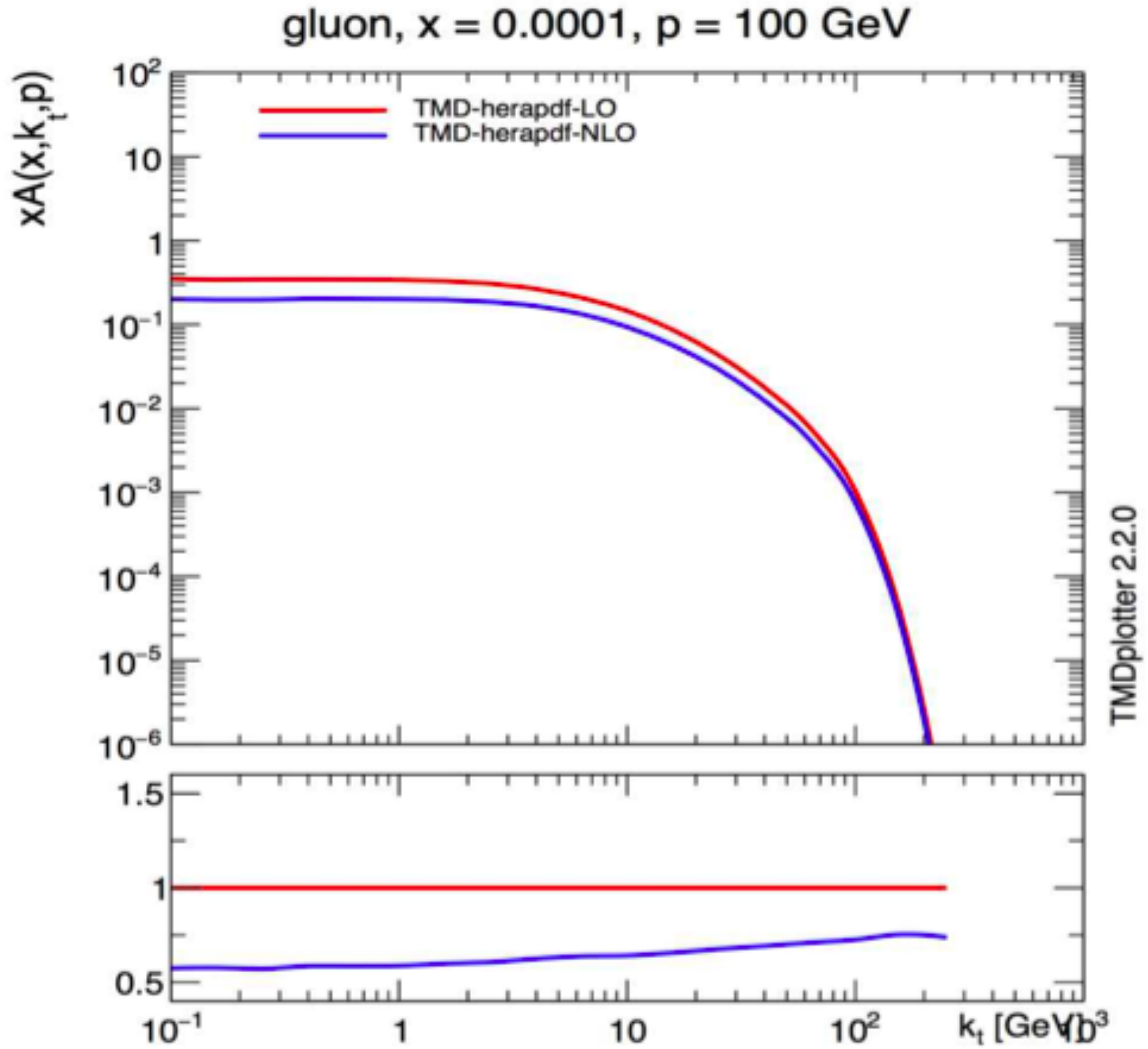
$$x\mathcal{A}(x, k_\perp^2, \mu^2) = \cdots + \sum_b \int_{\mu_0}^{\mu^2} \frac{d\mu'^2}{\mu'^2} \int_x^{z_M} dz P_{ab}^{(R)} \frac{x}{z} \mathcal{A}\left(\frac{x}{z}, k_\perp'^2, \mu'^2\right)$$

➔ distributions similar due to parton evolution

- k_t -distributions at large scales depend on initial and evolved distributions

TMD distributions from xFitter: herapdf-type





- Transverse momentum distributions including uncertainties from herapdf fit
  - only experimental uncertainties
- TMD distribution is different for NLO and LO
  - LO and NLO have different number of branchings !

Conclusion

- transverse momenta of interaction partons can be important for precision physics
  ➔ need for TMDs

- Parton Branching method developed for solving DGLAP equation at LO, NLO and NNLO
  ➔ consistence for collinear (integrated) PDFs shown

- method directly applicable to determine k_t distribution (as would be done in PS)
  ➔ TMD distributions for all flavors determined at LO and NLO, without free parameters
  ➔ TMD evolution implemented in xFitter – applicable for DIS processes