



*Ignazio Scimemi (UCM)*

# Jet TMDs and Jet Axes

Most recent results in collaboration with  
**Duff Neill,**  
**Wouter Waalewijn**  
**JHEP 1704 (2017) 020**



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# Outline & Issues

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- ❖ **Transverse Momentum Distributions (TMD) have a clear experimental meaning in DY**
- ❖ **How should we measure transverse momentum of hadrons in final states?**
- ❖ **In most cases the measure of hadrons is related to axes definitions and/or jets**
- ❖ **Does evolution of TMD depend on the these choices?**



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# ....TMD factorization ....

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.. for DY and heavy boson production we have (Collins 2011, Echevarria, Idilbi, Scimemi (EIS) 2012 )

$$\frac{d\sigma}{dQ^2 dq_T dy} = \sum_q \sigma_q^\gamma H(Q^2, \mu^2) \int \frac{d^2\mathbf{b}}{4\pi} e^{-i\mathbf{q}_T \cdot \mathbf{b}} \Phi_{q/A}(x_A, \mathbf{b}, \zeta_A, \mu) \Phi_{q/B}(x_B, \mathbf{b}, \zeta_B, \mu)$$
$$\sqrt{\zeta_A \zeta_B} = Q^2$$

...and similar formulas are valid for SIDIS (EIC) and hadron production in ee colliders

The pathological behavior is associated to a particular kind of divergences: rapidity divergences

The renormalization of the rapidity divergences is responsible for the new resummation scale

We have new nonperturbative effects which cannot be included in PDFs  
(renormalon analysis in Scimemi, Vladimirov 2016)

The latest and more elegant formulation of this in Echevarria, Scimemi, Vladimirov 2015-16:

[The NNLO era is just started! \(see the talk of A. Vladimirov and D. Gutiérrez Reyes\)](#)



# Factorization formulas for Jet TMDs

Starting point:

$$\frac{d\sigma_h}{dp_T d\eta d^2\mathbf{k} dz_h} = \sum_i \int \frac{dx}{x} \hat{\sigma}_i\left(\frac{p_T}{x}, \eta, \mu\right) \mathcal{G}_{i \rightarrow h}(x, p_T R, \mathbf{k}, z_h) [1 + \mathcal{O}(R^2)]$$

Kang, Ringer, Vitev; Dai, Kim, Leibovich

Sum over partons

Hard factor

Fragmenting Jet Function

$$z_h = \frac{p_h^-}{p_J^-}$$

$x$  = fraction of parton momentum that goes in the jet  
 $z_h \mathbf{k}$  = transverse momentum of the hadron in the jet  
 $k_A$  = component of  $\vec{k}$  along the jet axis  $\vec{A}_J$

Operator matrix element definition:

The phase space of the hadron is not included in  $J$

$$\mathcal{G}_{q \rightarrow h}(x, p_T R, \mathbf{k}, z_h) = \frac{1}{4xN_c} \sum_X \sum_{J/h} \int \frac{d\xi^+}{4\pi} e^{-ip_J^- \xi^+ / (2x)} \delta\left(z_h - \frac{p_h^-}{p_J^-}\right) \int dk_A \delta^{(3)}\left(\vec{k} - \frac{\vec{p}_h}{z_h}\right)$$

$$\times \langle 0 | T \left[ \tilde{W}_{Tn}^\dagger q_j \right]_a \left( \frac{\xi^+}{2} \right) | X, h \in J \rangle \gamma_{ij}^- \langle X, h \in J | \bar{T} \left[ \bar{q}_i \tilde{W}_{Tn} \right]_a \left( -\frac{\xi^+}{2} \right) | 0 \rangle$$

This depends on several physical scales  $p_T R$ ,  $k_T$ ,  $\Lambda_{QCD}$  Can we recover some TMD-like functions?



# Factorization formulas for Jet TMDs

A very interesting limit!!

$$p_T R \gg |\mathbf{k}| \sim \Lambda_{QCD}$$

$$\mathcal{G}_{i \rightarrow h}(x, p_T R, \mathbf{k}, z_h, \mu) = \sum_k \int \frac{dy}{y} B_{ik}(x, p_T R, y, \mu) D_{k \rightarrow h}\left(\mathbf{k}, \frac{z_h}{y}, \mu\right) \left[1 + \mathcal{O}\left(\frac{\mathbf{k}^2}{p_T^2 R^2}\right)\right]$$

**JET TMD!!!**

**AXIS DEPENDENCE**

$$D_{q \rightarrow h}(\mathbf{k}, z_h) = \frac{1}{4z_h N_c} \sum_X \int \frac{d\xi^+}{4\pi} e^{-ip_h^- \xi^+ / (2z_h)} \int dk_A \delta^{(3)}\left(\vec{k} - \frac{\vec{p}_h}{z_h}\right) \\ \times \langle 0 | T \left[ \tilde{W}_{Tn}^\dagger q_j \right]_a \left( \frac{\xi^+}{2} \right) | X, h \rangle \gamma_{ij}^- \langle X, h | \bar{T} \left[ \bar{q}_i \tilde{W}_{Tn} \right]_a \left( -\frac{\xi^+}{2} \right) | 0 \rangle$$



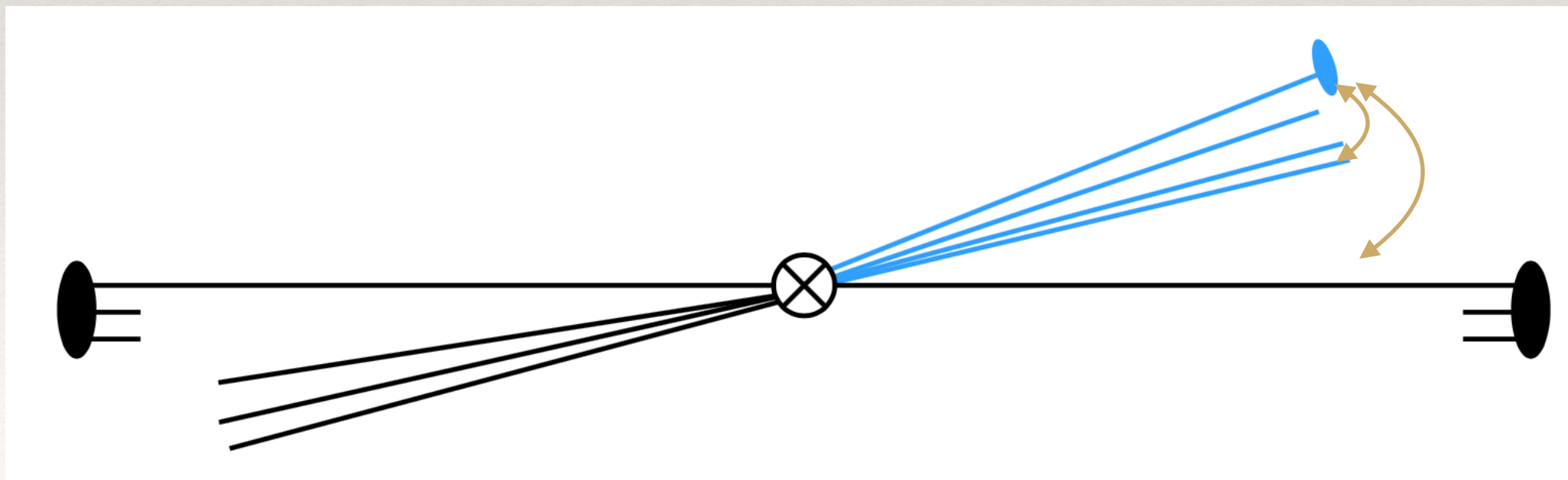
# TMDs in fragmentation...jets

The main difference with respect to the standard definition is the axis with respect to which we measure the **TMD FRAGMENTATION**

There are several possibility to define a transverse momentum depending on the reference axes:

- beam axis
- jet axis
- ...

So we have a multiplicity of information that we can use!!  
We want to study transverse momentum of hadrons inside a jet





# Recoil-free axis: Winner-Take-All axis

The definition and evolution of TMDs depend on the choice of jet axis

We have explored the possibility of recoil free axis to avoid:

- Non-global logs
- rapidity divergences

The price to pay is: the axis is not aligned with the standard jet momentum (standard jet axis)

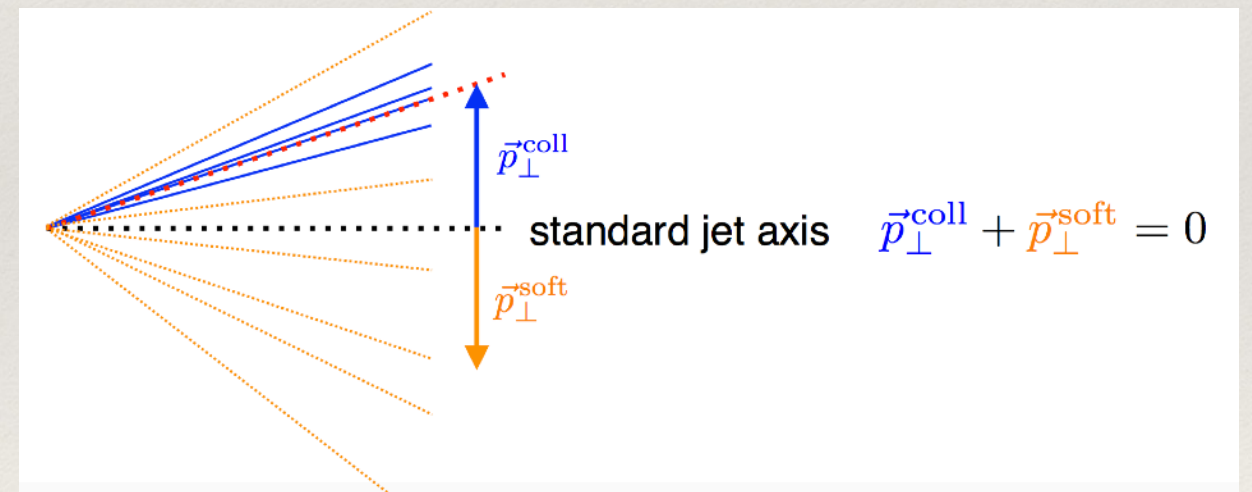
The soft radiation recoil shifts the whole collinear sector coherently in

Transverse Momentum:

We want an axis that shifts of the same

amount: WTA axis (Larkoski, Neill, Thaler)

Standard jet axis in 1705.08443 (Kang, Liu, Ringer, Xing)



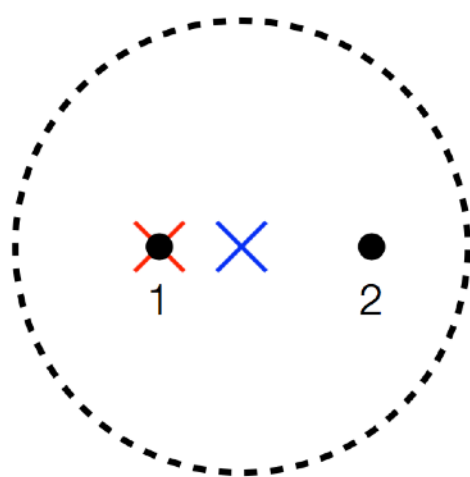


## Recoil-free axis: Winner-Take-All (WTA) axis

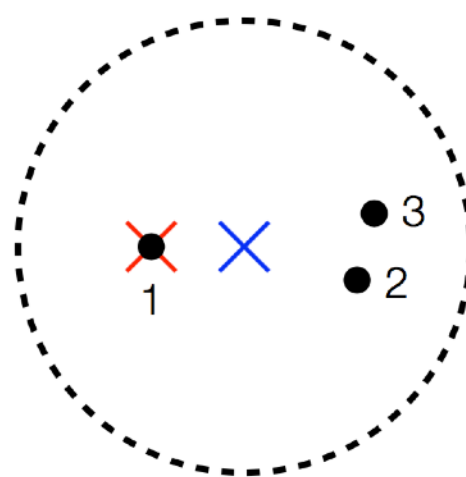
Run clustering algorithm with following recombination scheme

$$E_r = E_1 + E_2$$

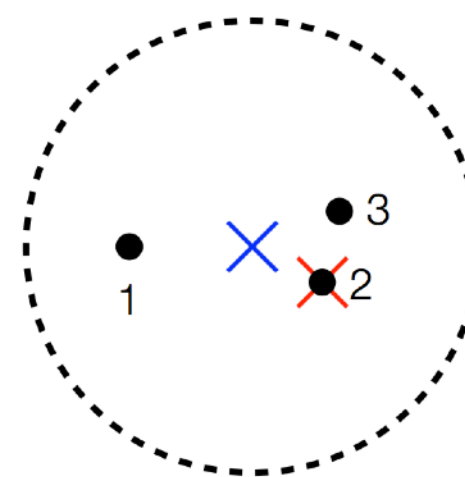
$$\hat{n}_r = \begin{cases} \hat{n}_1 & \text{if } E_1 > E_2 \\ \hat{n}_2 & \text{if } E_2 > E_1 \end{cases} \quad [\text{Salam; Bertolini, Chan, Thaler}]$$



$$E_1 > E_2$$



$$E_1 > E_2 + E_3$$



$$E_2 + E_3 > E_1$$

$$E_1 > E_2 > E_3$$



# Re-factorization limits

We can find several predictable asymptotic behaviors..

$$p_T R \sim |\mathbf{k}| \gg \Lambda_{QCD}$$

$$\mathcal{G}_{i \rightarrow h}(x, p_T R, \mathbf{k}, z_h, \mu) = \sum_j \int \frac{dz}{z} \mathcal{J}_{ij}\left(x, p_T R, \mathbf{k}, \frac{z_h}{z}, \mu\right) d_{j \rightarrow h}(z, \mu) \left[1 + \mathcal{O}\left(\frac{\Lambda_{QCD}^2}{\mathbf{k}^2}\right)\right]$$

..in this limit we can recover a classical separation in coefficients and fragmentation functions..

$\mathcal{J}_{ij}$  describes the formation of a jet with momentum fraction  $x$  of the initial parton  $i$ , containing a parton  $j$  with momentum fraction  $z_h/z$  and transverse momentum  $\mathbf{k}$ .



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$$\mathcal{J}_{ij}(x, p_T R, \mathbf{k}, z, \mu) = \sum_k \int \frac{dy}{y} B_{ik}(x, p_T R, y, \mu) C_{kj}\left(\mathbf{k}, \frac{z}{y}, \mu\right) \left[1 + \mathcal{O}\left(\frac{\mathbf{k}^2}{p_T^2 R^2}\right)\right]$$

..and all non-perturbative effects should be power suppressed..

•  $\mathcal{J}_{ij}$  describes the formation of a jet with momentum fraction  $x$  of the initial parton  $i$ , containing a  
 • parton  $j$  with momentum fraction  $z_h/z$  and transverse momentum  $\mathbf{k}$ .



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# JetTMDs with WTA axis

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$$p_T R \gg k_T \sim \Lambda_{QCD}$$

For small hadron transverse momentum the border effects can be perturbatively extracted and we can obtain Jet TMDs

$$\mathcal{G}_{i \rightarrow h}(x, p_T R, \mathbf{k}, z_h, \mu) = \sum_k \int \frac{dy}{y} B_{ik}(x, p_T R, y, \mu) D_{k \rightarrow h}\left(\mathbf{k}, \frac{z_h}{y}, \mu\right) \left[1 + \mathcal{O}\left(\frac{\mathbf{k}^2}{p_T^2 R^2}\right)\right]$$

Consistency with previous limits requires that for  $k_T \gg \Lambda_{QCD}$

$$D_{k \rightarrow h}(\mathbf{k}, z_h, \mu) = \sum_j \int \frac{dz}{z} C_{kj}\left(\mathbf{k}, \frac{z_h}{z}, \mu\right) d_{j \rightarrow h}(z, \mu) \left[1 + \mathcal{O}\left(\frac{\Lambda_{QCD}^2}{\mathbf{k}^2}\right)\right]$$



# JTMD evolution with WTA axis

## Standard fragmentation evolution...

$$d_{i \rightarrow h}^{\text{bare}}(z_h) = \sum_j \int \frac{dz}{z} Z_{ij} \left( \frac{z_h}{z}, \mu \right) d_{j \rightarrow h}(z, \mu)$$

$$\mu \frac{d}{d\mu} d_{i \rightarrow h}(z_h, \mu) = \sum_j \int \frac{dz}{z} \gamma_{ij} \left( \frac{z_h}{z}, \mu \right) d_{j \rightarrow h}(z, \mu),$$

$$\gamma_{ij}(z_h, \mu) = - \int \frac{dz}{z} Z_{ik}^{-1} \left( \frac{z_h}{z}, \mu \right) \mu \frac{d}{d\mu} Z_{kj}(z, \mu)$$

...is the same as for fragmenting jet function...

$$\mathcal{G}_{i \rightarrow h}^{\text{bare}}(x, p_T R, \mathbf{k}, z_h, \mu) = \sum_j \int \frac{dx'}{x'} Z_{ij} \left( \frac{x}{x'}, \mu \right) \mathcal{G}_{j \rightarrow h}(x', p_T R, \mathbf{k}, z_h, \mu)$$

..and is similar (but not the same!!) for fragmenting jet function...

$$D_{i \rightarrow h}^{\text{bare}}(\mathbf{k}, z) = \sum_j \int \frac{dz'}{z'} Z'_{ij} \left( \frac{z}{z'}, \mu \right) D_{j \rightarrow h}(\mathbf{k}, z', \mu)$$

$$\gamma_{ij}(z, \mu) = P_{ji}(z, \mu) \, ,$$

$$\gamma'_{ij}(z, \mu) = \theta\left(z \geq \frac{1}{2}\right) P_{ji}(z, \mu)$$

All this checked at NLO



# Summary: scales and factorization

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- Factorization depends on relevant hierarchy:

$$\frac{d\sigma_h}{dp_T d\eta d^2\mathbf{k} dz_h} = \overbrace{\hat{\sigma}(p_T, \eta)}^{\text{partonic xsec}} \otimes \underbrace{B(p_T R)}_{\text{jet boundary}} \otimes \underbrace{C(\mathbf{k}) \otimes D(\Lambda_{\text{QCD}})}_{\text{TMD fragmentation } D(\mathbf{k})} \otimes \underbrace{\mathcal{J}(p_T R, \mathbf{k})}_{\text{fragmenting jet function } \mathcal{G}(p_T R, \mathbf{k})}$$

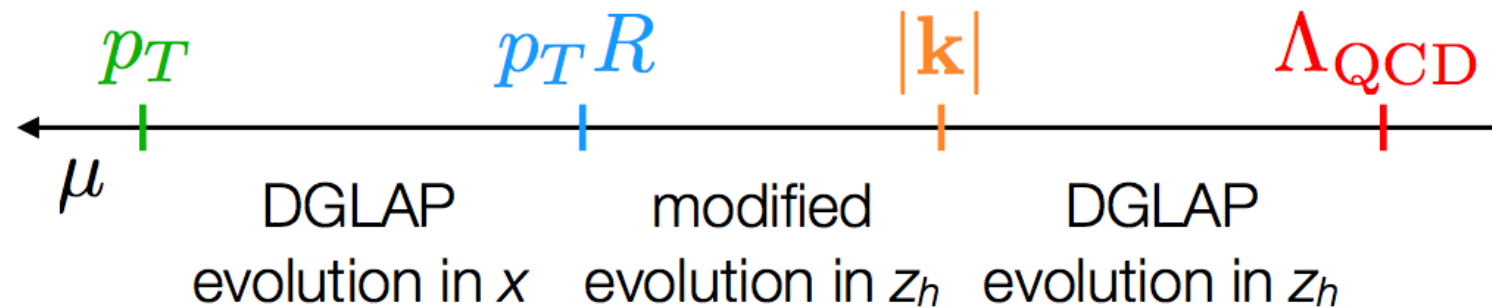
- Fragmentation scales: transverse momentum  $|\mathbf{k}|$  and  $\Lambda_{\text{QCD}}$
- Jet scales: transverse momentum  $p_T$  and radius  $R$



# Evolution and resummation

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- Single logarithms resummed by renormalization group evolution



- $\mathcal{G}_{i \rightarrow h}(x, p_T R, \mathbf{k}, z_h, \mu)$  has standard DGLAP evolution in  $x$
- $D_{i \rightarrow h}(z_h, \mu)$  satisfies DGLAP evolution in  $z_h$
- $D_{i \rightarrow h}(\mathbf{k}, z_h, \mu)$  has modified **all-orders** evolution equation:

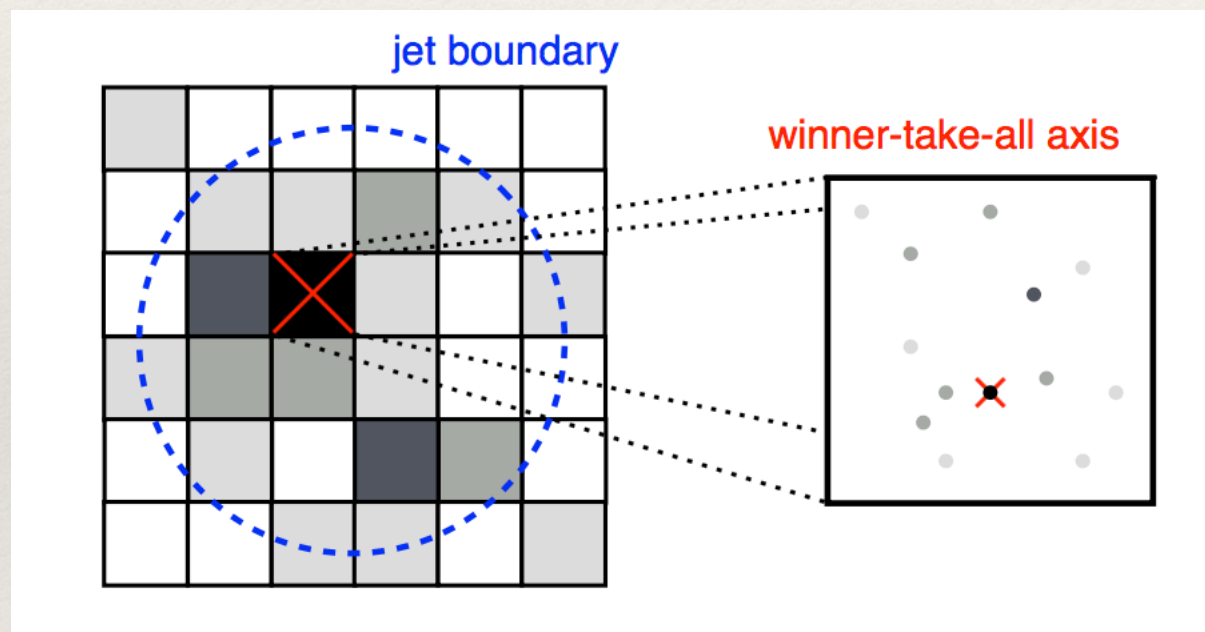
$$\mu \frac{d}{d\mu} D_{i \rightarrow h}(\mathbf{k}, z_h, \mu) = \sum_j \int \frac{dz}{z} \theta\left(z - \frac{1}{2}\right) P_{ji}(z, \mu) D_{j \rightarrow h}\left(\mathbf{k}, \frac{z_h}{z}, \mu\right)$$



# Jet Algorithms and JTMDs with WTA axis

The WTA axis is always aligned with the most energetic parton in the jet.

If  $r$  is the distance of the hadron from jet axis we want  $r \ll R$



We can pixelate the jet and look at the energy released in each pixel of radius  $r \ll R$ .

The most energetic pixel fixes the axis.  
Boundary effects are power suppressed by  $r/R$

Jet Algorithm requirements:

- ❖ radiation within the pixel that contains the jet axis will be preferentially clustered together first
- ❖ the configuration of the radiation outside of this pixel does not interfere with the constituents of the pixel
- ❖ anti-kT and Cambridge/Aachen do the job



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# Conclusions

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The possibility to define jets allows a connection between Tevatron/LHC and future experiments.

Jets allow a definition of new types of TMD which are relevant for our understanding of confinement

WTA axis analysis offer an example of these possibilities:

Jet TMDs with respect to WTA axis have a simpler evolution equation...  
they can be an opportunity for new studies



Back up



# REF 2017

## November 13-16 Madrid, Spain

For any question ask Daniel or me!

You can see the web page  
<http://jacobi.fis.ucm.es/REF2017/>

Also you can send an email to  
[dangut01@ucm.es](mailto:dangut01@ucm.es)  
[ignazios@fis.ucm.es](mailto:ignazios@fis.ucm.es)

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REF 2017

REF 2017 is the 4th workshop in the series of workshops on Resummation, Evolution, Factorization. The workshop wishes to bring together experts of different communities specialized in nuclear structure; transverse momentum dependent distributions; small-x physics; effective field theories. The topics of this workshop include: transverse momentum spectra in vector/scalar-boson production and semi-inclusive DIS; jet and heavy-quark production near the back-to-back region; TMD parton density functions and fragmentation functions; new approaches to Monte Carlo parton showering; non-perturbative effects and power corrections; TMDs and multi-parton interactions; spin, color and azimuthal asymmetries; applications to LHC, JLab, RHIC and EIC.

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# The TMD program

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- ❖ Recover / reformulate / understand QCD in the limit of high  $q_T$  (NNLO, some 3-loop results):  
M.G. Echevarría, I.S., A. Vladimirov, arXiv:1509.06392, arXiv:1511.05590, arXiv:1604.07869  
T. Lübbert, J. Oredsson, M. Stahlhofen, arXiv:1602.01829, Y. Li, H.X. Zhu, arXiv:1604.01404 A. Vladimirov, arXiv:1610.05791
- ❖ Achieve NEW results in the limit of high  $q_T$ :
  - unpolarized fragmentation at NNLO M.G. Echevarría, I.S., A. Vladimirov, arXiv:arXiv:1604.07869
  - double parton scattering factorization A. Vladimirov arXiv:1608.04920
  - Twist -2 TMD matching (D. Gutierrez-Reyes, I.S., A. Vladimirov, arXiv:1702.06558)
  -
- ❖ New inputs for the non-perturbative TMD structure:
  - renormalons (I.S., A. Vladimirov, arXiv:arXiv:1609.06047)
  - lattice (X. Ji, M. Engelhardt,..)
  - fits and data analysis...
  - jets (Kang, Ringer, Vitev, Leibovich, Mehen, Neill, I.S., Waalewijn,...  
RESUMING...
- ❖ The confinement frontier (experiment, theory, phenomenology):
  - spin physics (EIC, AFTER, COMPASS, BABAR, BELLE,..)
  - precision at LHC (Vector and Higgs Boson production, jets,...)