Finite t and target mass corrections in DVCS

A. N. Manashov

Hamburg University

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In this talk:

- Finite t and target mass corrections, t/Q^2 and m^2/Q^2

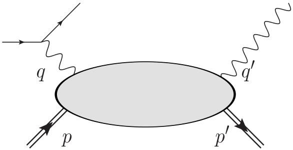
  V. Braun, A. M., PRL 107 (2011)

  V. Braun, A. M., B. Pirnay PRL 109 (2012)

  V. Braun, A. M., D. Müller, B. Pirnay PRD 89 (2014)

- Ambiguity of the leading-twist approximations

DVCS: $\gamma^* N(p) \to \gamma N(p')$ **D. Müller, X. Ji, A. Radyushkin**



Operator Product Expansion

$$\mathcal{A}_{\mu\nu} \sim \langle p'|T\Big\{j^{em}_\mu(x) j^{em}_\nu(0)\Big\}|p\rangle \quad \simeq \quad \sum_N C^N_{\mu\nu}(x^2,\mu^2)\,\langle p'|\mathcal{O}_N(\mu^2)|p\rangle$$

Kinematic variables: **hadron mass** m^2 **momentum transfer** $t = (P - P')^2$

How to calculate m^2/Q^2 **and** t/Q^2 **corrections?**

DVCS Amplitude

$$\mathcal{A}_{\mu\nu}(p,q,q') = i\int d^4x\, e^{-i(z_1 q - z_2 q', x)}\langle p'|T\{j_\mu(z_1 x) j_\nu(z_2 x)\}|p\rangle$$

$z_1 - z_2 = 1.$

translation invariance: $\mathcal{A}$ depends only on the difference $z_1 - z_2$

current conservation:

$$q^\mu \mathcal{A}_{\mu\nu}(p,q,q') = (q')^\nu \mathcal{A}_{\mu\nu}(p,q,q') = 0$$

only valid in the sum of all twists but not for each twist separately

$$T\{j_\mu(z_1 x) j_\nu(z_2 x)\} = T^{t=2}_{\mu\nu}(z_1,z_2) + T^{t=3}_{\mu\nu}(z_1,z_2) + T^{t=4}_{\mu\nu}(z_1,z_2) + \ldots$$

twist-3 : Anikin, Teryaev; Belitsky, Müller; Kivel, Polyakov, Schäfer, Teryaev, 2001
twist-4 : Braun, A.M. 2011

$$T\{j(x)j(0)\} = \sum_N a_N \mathcal{O}_N + \sum_N \Big(b_N \partial^2 \mathcal{O}_N + d_N (\partial\mathcal{O})_N \Big) + \text{all others}$$

$$= \sum_N C_N(x,\partial)\, \mathcal{O}_N + \dots$$

$\mathcal{O}_N$ are twist-two operators $\partial\mathcal{O}_N = \partial^\mu \mathcal{O}_{\mu\mu_2\dots\mu_N} = 0$ (free theory)

Conformal OPE: S. Ferrara, A. F. Grillo, G. Parisi and R. Gatto

$d = 4 - 2\epsilon \mapsto$ QCD at the critical point: $\beta(\alpha_*) = 0$, $\alpha_* \sim \epsilon$

$$C_N(x,\partial) = C_N(\alpha_s, \epsilon, x, \partial) = C_N^{LO}(x,\partial) + O(\alpha_s, \epsilon)$$

Braun, A.M., Pirnay: PRD **86** (2012) 014003

$$\begin{aligned}\mathcal{A}_{\mu\nu}(q,q',p) &= i\int d^4x\, e^{-i(z_1 q - z_2 q')x}\langle p',s'|\,T\{J_\mu(z_1x)J_\nu(z_2x)\}|p,s\rangle \\ &= \varepsilon^+_\mu\varepsilon^-_\nu\mathcal{A}^{++} + \varepsilon^-_\mu\varepsilon^+_\nu\mathcal{A}^{--} + \varepsilon^0_\mu\varepsilon^-_\nu\mathcal{A}^{0+} \\ &+ \varepsilon^0_\mu\varepsilon^+_\nu\mathcal{A}^{0-} + \varepsilon^+_\mu\varepsilon^+_\nu\mathcal{A}^{+-} + \varepsilon^-_\mu\varepsilon^-_\nu\mathcal{A}^{-+} + q'_\nu\mathcal{A}^{(3)}_\mu\end{aligned}$$

for the calculation to the twist-4 accuracy one needs

- $\mathcal{A}^{++}, \mathcal{A}^{--}$: $1 + \frac{1}{Q^2}$
- $\mathcal{A}^{0+}, \mathcal{A}^{0-}$: $\frac{1}{Q}$ $\longleftarrow$ agree with existing results
- $\mathcal{A}^{-+}, \mathcal{A}^{+-}$: $\frac{1}{Q^2}$ $\longleftarrow$ straightforward

BMP Compton form factors (CFFs)

- Photon helicity amplitudes can be expanded in a given set of spinor bilinears

$$\mathcal{A}_q^{a\pm} = \mathbb{H}^q_{a\pm} h + \mathbb{E}^q_{a\pm} e \mp \widetilde{\mathbb{H}}^q_{a\pm} \tilde{h} \mp \widetilde{\mathbb{E}}^q_{a\pm} \tilde{e}$$

with, e.g. Belitsky, Müller, Ji: NPB **878** (2014) 214

$$h = \frac{\bar{u}(p')\,(\not{q} + \not{q}')u(p)}{P\cdot(\not{q} + \not{q}')} \qquad \dots$$

- The results read Braun, A.M., Pirnay: PRL109 (2012) 242001

$$\mathbb{H}_{++} = T_0 \circledast H + \frac{t}{Q^2}\Big[-\frac{1}{2}T_0 + T_1 + 2\xi\mathbf{D}_\xi\, T_2\Big] \circledast H + \frac{2t}{Q^2}\xi^2\partial_\xi\xi T_2 \circledast (H+E)$$

$$\mathbb{H}_{0+} = -\frac{4|\xi P_\perp|}{\sqrt{2}Q}\Big[\xi\partial_\xi T_1 \circledast H + \frac{t}{Q^2}\partial_\xi\xi\, T_1 \circledast (H+E)\Big] - \frac{t}{\sqrt{2}Q|\xi P_\perp|}\xi T_1 \circledast \Big[\xi\,(H+E) - \widetilde{H}\Big]$$

$$\mathbb{H}_{-+} = \frac{4|\xi P_\perp|^2}{Q^2}\Big[\xi\partial_\xi^2\xi\, T_1^{(+)} \circledast H + \frac{t}{Q^2}\partial_\xi^2\xi^2\, T_1^{(+)} \circledast (H+E)\Big] + \frac{2t}{Q^2}\xi\Big[\xi\partial_\xi\xi\, T_1^{(+)} \circledast (H+E) + \partial_\xi\,\xi\, T_1 \circledast \widetilde{H}\Big]$$

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- The results read Braun, A.M., Pirnay: PRL109 (2012) 242001

$$\mathbb{E}_{++} = T_0 \circledast E + \frac{t}{Q^2}\Big[-\frac{1}{2} T_0 + T_1 + 2\xi \mathbf{D}_\xi\, T_2\Big] \circledast E - \frac{8m^2}{Q^2}\xi^2 \partial_\xi \xi\, T_2 \circledast (H+E)$$

$$\mathbb{E}_{0+} = -\frac{4|\xi P_\perp|}{\sqrt{2}Q}\Big[\xi\partial_\xi T_1 \circledast E\Big] + \frac{4m^2}{\sqrt{2}Q|\xi P_\perp|}\xi T_1 \circledast \Big[\xi\,(H+E) - \widetilde{H}\Big]$$

$$\mathbb{E}_{-+} = \frac{4|\xi P_\perp|^2}{Q^2}\Big[\xi\partial_\xi^2 \xi\, T_1^{(+)} \circledast E\Big] - \frac{8m^2}{Q^2}\xi\Big[\xi\partial_\xi\xi\, T_1^{(+)} \circledast (H+E) + \partial_\xi\, \xi\, T_1 \circledast \widetilde{H}\Big]$$

etc.

where $F = H, E, \widetilde{H}, \widetilde{E}$ are C-even GPDs

$$T \circledast F = \sum_q e_q^2 \int_{-1}^{1} \frac{dx}{2\xi}\, T\left(\frac{\xi + x - i\epsilon}{2(\xi - i\epsilon)}\right) F(x, \xi, t)$$

the coefficient functions $T_k^{\pm}$ are given by the following expressions:

$$T_0(u) = \frac{1}{1-u}$$

$$T_1(u) \equiv T_1^{(-)}(u) = -\frac{\ln(1-u)}{u}$$

$$T_1^{(+)}(u) = \frac{(1-2u)\ln(1-u)}{u}$$

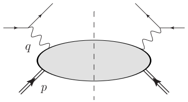
$$T_2(u) = \frac{\mathrm{Li}_2(1) - \mathrm{Li}_2(u)}{1-u} + \frac{\ln(1-u)}{2u}$$

and

$$\mathbf{D}_\xi = \partial_\xi + 2\frac{|\xi P_\perp|^2}{t}\partial_\xi^2 \xi \;=\; \partial_\xi - \frac{t - t_{\min}}{2t}(1-\xi^2)\partial_\xi^2 \xi$$

- **Ambiguity of leading order approximation**

DIS

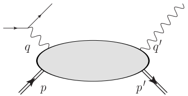


Define (p, q) as longitudinal plane:

$$p = (p_0, \vec{0}_\perp, p_z)$$
$$q = (q_0, \vec{0}_\perp, q_z)$$

$\Rightarrow$ parton fraction = Bjorken x

DVCS



Many choices possible:

$$p = (p_0, \vec{0}_\perp, p_z), \quad q = (q_0, \vec{0}_\perp, q_z)$$

or

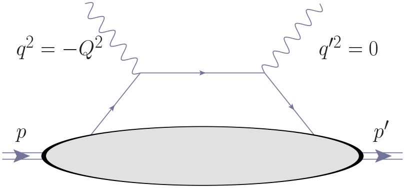
$$p + p' = (P_0, \vec{0}_\perp, P_z), \quad q = (q_0, \vec{0}_\perp, q_z)$$

etc.

$\Rightarrow$ parton fraction $2\xi = x_B[1+\mathcal{O}\left(\frac{t}{Q^2}\right)]$, redefinition of helicity amplitudes

- **Ambiguity is resolved by adding "kinematic" power corrections t/Q^2, m^2/Q^2**

Braun, A.M., Pirnay: PRD **86** (2012) 014003



longitudinal plane (q, q')

$$n = q', \qquad \tilde{n} = -q + \frac{Q^2}{Q^2 + t}\, q'$$

with this choice $\Delta = q - q'$ is longitudinal and

$$|P_\perp|^2 = -m^2 - \frac{t}{4}\frac{1-\xi^2}{\xi^2} \;\sim\; t_{\min} - t$$

where

$$P = \frac{1}{2}(p + p')\,, \qquad \xi_{\rm BMP} = -\frac{(\Delta\cdot q')}{2(P\cdot q')} \;=\; \frac{x_B(1 + t/Q^2)}{2 - x_B(1 - t/Q^2)}$$

photon polarization vectors

$$\varepsilon^0_\mu = -\left(q_\mu - q'_\mu q^2/(qq')\right)/\sqrt{-q^2}\,,$$
$$\varepsilon^\pm_\mu = (P^\perp_\mu \pm i\bar{P}^\perp_\mu)/(\sqrt{2}|P_\perp|)\,, \qquad \bar{P}^\perp_\mu = \epsilon^\perp_{\mu\nu}P^\nu$$

$$\mathcal{F}^{\text{lab}}_{++} = \mathcal{F}^{\text{phot}}_{++} + \frac{\varkappa}{2}\left[\mathcal{F}^{\text{phot}}_{++} + \mathcal{F}^{\text{phot}}_{-+}\right] - \varkappa_0\,\mathcal{F}^{\text{phot}}_{0+},$$
$$\mathcal{F}^{\text{lab}}_{0+} = -\,(1+\varkappa)\,\mathcal{F}^{\text{phot}}_{0+} + \varkappa_0\left[\mathcal{F}^{\text{phot}}_{++} + \mathcal{F}^{\text{phot}}_{-+}\right]$$

$$\mathcal{F} \in \{\mathcal{H}, \mathcal{E}, \widetilde{\mathcal{H}}, \widetilde{\mathcal{E}}\}$$

where

$$\varkappa_0 \sim \sqrt{(t_{\min} - t)/Q^2}\,, \qquad \varkappa \sim (t_{\min} - t)/Q^2$$

and different skewedness parameter

$$\xi^{\text{lab}} \simeq \frac{x_B}{2 - x_B} \quad \text{vs.} \quad \xi^{\text{phot}} = \frac{x_B(1 + t/Q^2)}{2 - x_B(1 - t/Q^2)}$$

Defining the Leading Twist approximation

Kumerički-Müller convention (KM)	Braun-Manashov-Pirnay convention (BMP)

$$\mathsf{LT}_{\rm KM}: \begin{cases} \mathcal{F}^{\sf lab}_{++} = T_0 \otimes F, & \mathcal{F}^{\sf lab}_{0+} = 0, \\ \mathcal{F}^{\sf lab}_{-+} = 0, & \xi_{\rm KM} = \xi^{\sf lab} \end{cases}$$

$$\mathsf{LT}_{\rm BMP}: \begin{cases} \mathcal{F}^{\sf phot}_{++} = T_0 \otimes F, & \mathcal{F}^{\sf phot}_{0+} = 0, \\ \mathcal{F}^{\sf phot}_{-+} = 0, & \xi_{\rm BMP} = \xi^{\sf phot} \end{cases}$$

$$\Downarrow$$

$$\mathsf{LT}_{\rm BMP}: \begin{cases} \mathcal{F}^{\sf lab}_{++} = \left(1 + \frac{\varkappa}{2}\right) T_0 \otimes F, & \mathcal{F}_{0+} = \varkappa_0\, T_0 \otimes F \\ \mathcal{F}^{\sf lab}_{-+} = \frac{\varkappa}{2} T_0 \otimes F, & \xi = \xi_{\rm BMP}, \end{cases}$$

- **Changing frame of reference results in**
  - **Different skewedness parameter for a given x_B**
  - **Numerically significant excitation of helicity-flip CFFs**
- **Different results for experimental observables**

Goloskokov, Kroll model (GK12)

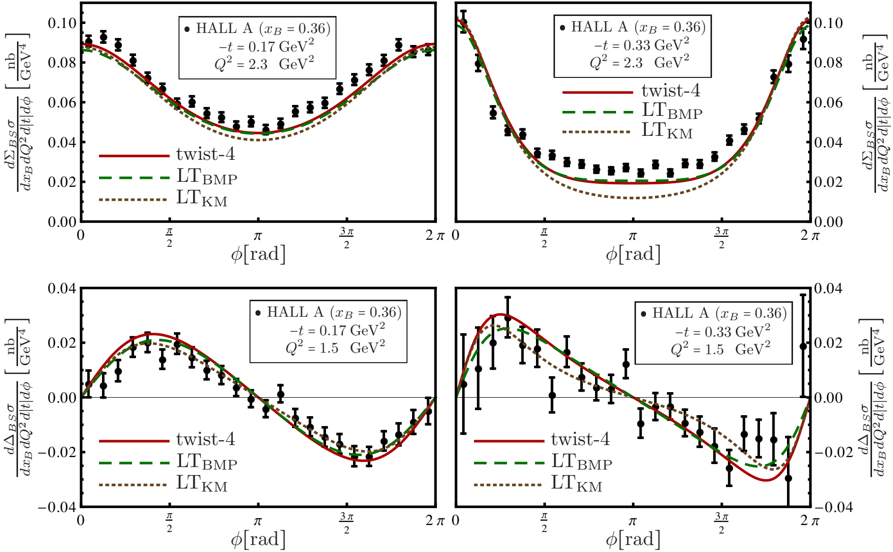
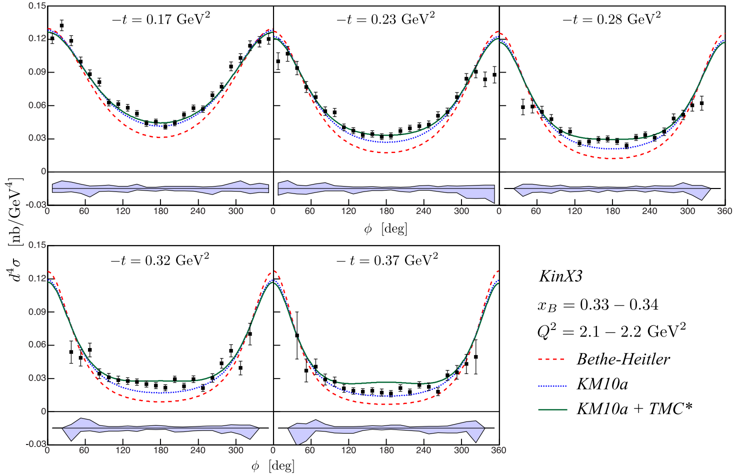


Figure: The unpolarized cross section for $x_B = 0.36$ and $Q^2 = 2.3\,\text{GeV}^2$ [upper panels] and

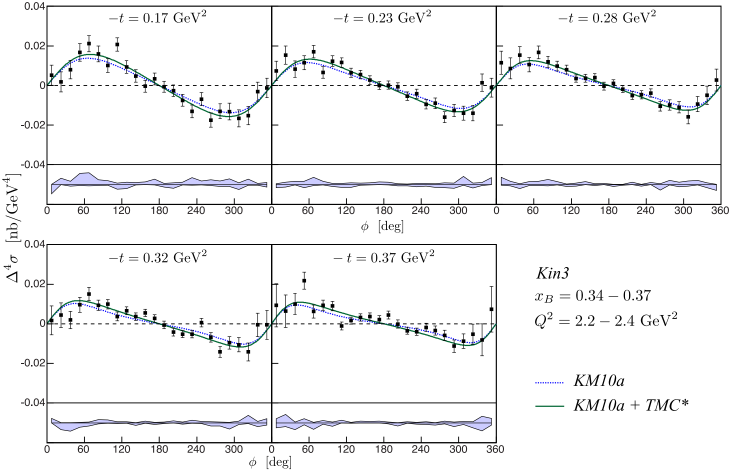
M. Defurne *et al.* [Hall A Collaboration] arXiv:1504.05453



- **TMC* curves very close to BMP LT**

GPD model: KM10a (Kumericki, Mueller, Nucl. Phys. B **841** (2010) 1)

M. Defurne *et al.* [Hall A Collaboration] arXiv:1504.05453



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Summary :

- **noncomplanarity makes separation of collinear directions ambiguous**

  — hence "leading twist approximation" ambiguous

  — related to violation of translation invariance and EM Ward identities

- **have to be repaired by adding power corrections of special type, "kinematic" PC**

Main features:

- **Complete results available to t/Q^2, m^2/Q^2 accuracy**

  — translation and gauge invariance restored

  — factorization valid

  — correct threshold behavior $t \to t_{\min}$, $\xi \to 1$

  — for many observables, complete results close to LT in "photon frame"

- **Two expansion parameters**

$$\frac{t}{Q^2}; \qquad \frac{t - t_{\min}}{Q^2} \sim \frac{|\xi P_\perp|^2}{Q^2}$$

- **Most of mass corrections absorbed in $t_{\min} = -4m^2\xi^2/(1-\xi^2)$;**
  always overcompensated by finite-t corrections in the physical region

- **Some extra m^2/Q^2 corrections for nucleon due to spinor algebra;**
  disappear in certain CFF combinations and for scalar targets

What can/should be done?

short/medium term

- Bulk of the twist-four corrections captured in “photon” frame for generic H, E?
- Direct calculation of DVCS observables starting from “photon” frame
- “Standard” code combining twist-4 + NLO

long(er) term

- resummation of $(t/Q^2)^k$ and $(m^2/Q^2)^k$ corrections to all powers
- NLO corrections to $(t/Q^2)^k$ and $(m^2/Q^2)^k$, gluon constributions