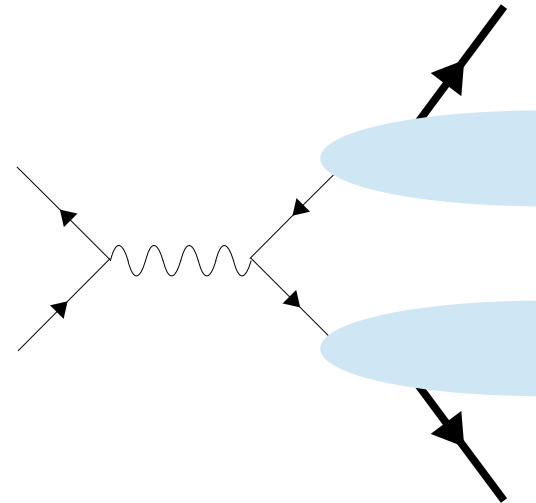
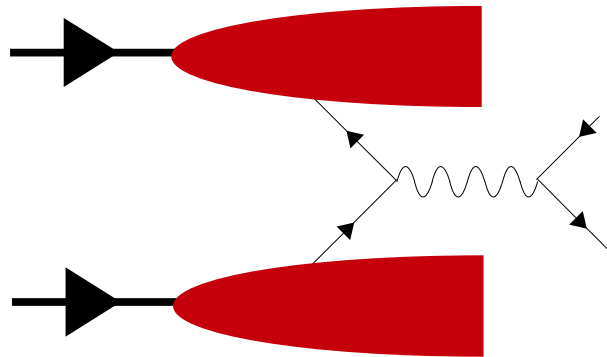


Opportunities for the Extraction Of TMD Fragmentation Functions

Electron Ion Collider User Group Meeting 2017

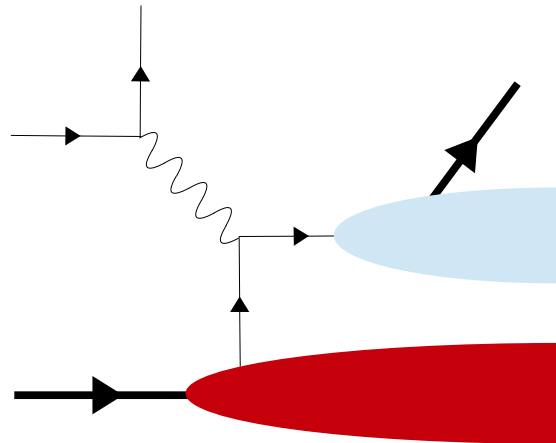
**J. Osvaldo Gonzalez-Hernandez
University of Turin
&
INFN**

Drell Yan



e^+e^-

PDFs



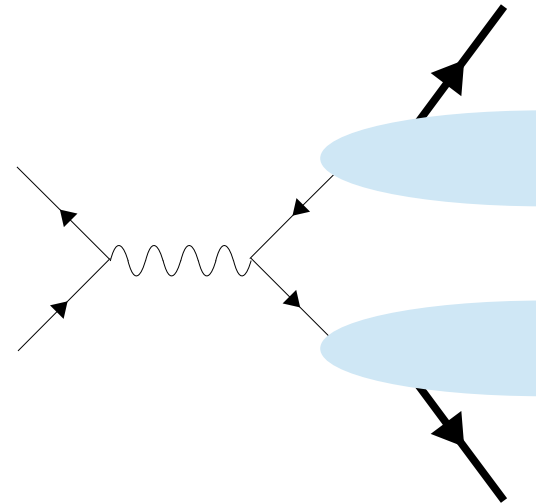
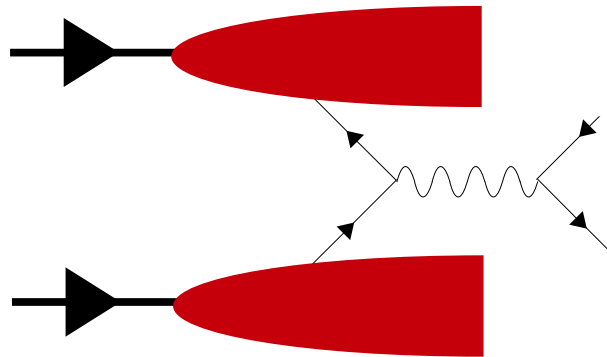
Fragmentation
Functions

SIDIS

Global Fits?

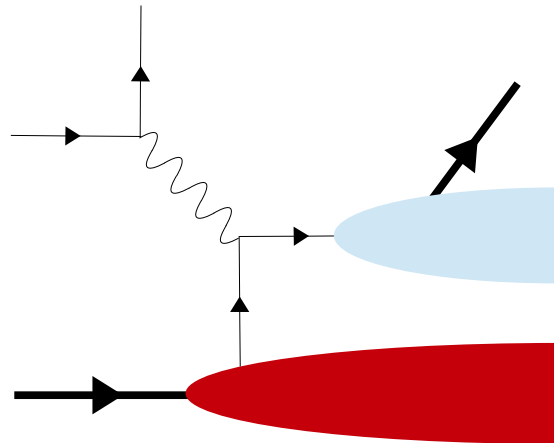
A. Bacchetta, F. Delcarro, C. Pisano,
M. Radici, A. Signori
arXiv:1703.10157

Drell Yan



e^+e^-

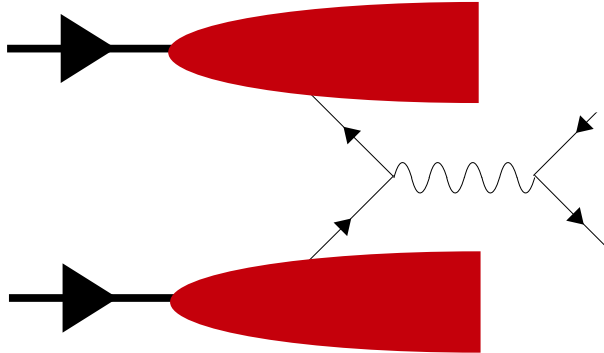
PDFs



Fragmentation
Functions

SIDIS

Drell Yan



**Under control, high
precision phenomenology:**

See for example:

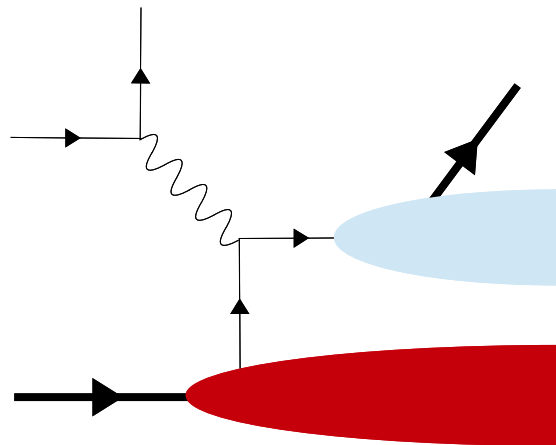
arXiv:1706.01473

Ignazio Scimemi, Alexey Vladimirov

Must still address some issues.

Delicate kinematics of available
multidimensional data

The matching between low and large
transverse momentum

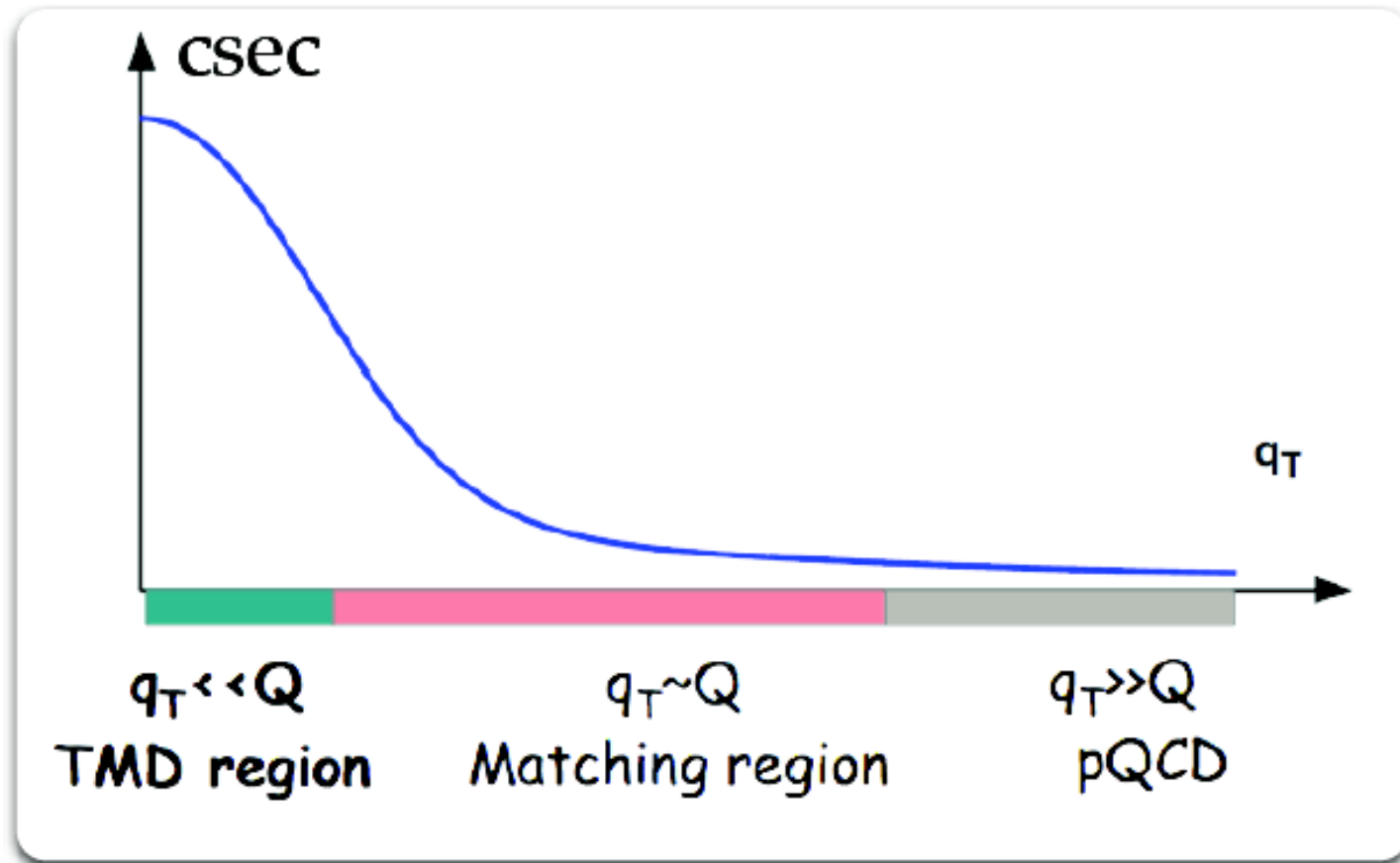


SIDIS

The Matching Problem in SIDIS

$$\{Q^2, x_B, P_{hT}, z_h\}$$

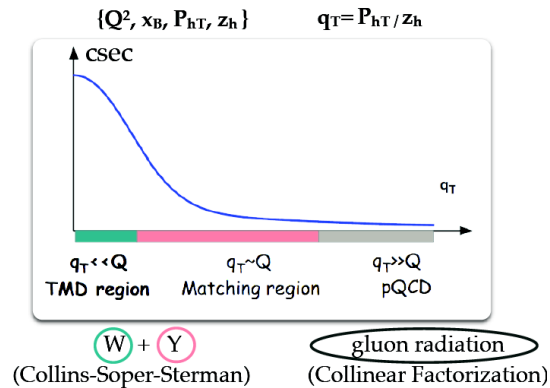
$$q_T = P_{hT}/z_h$$



$\textcircled{W} + \textcircled{Y}$
(Collins-Soper-Sterman)

gluon radiation
(Collinear Factorization)

The Matching Problem in SIDIS



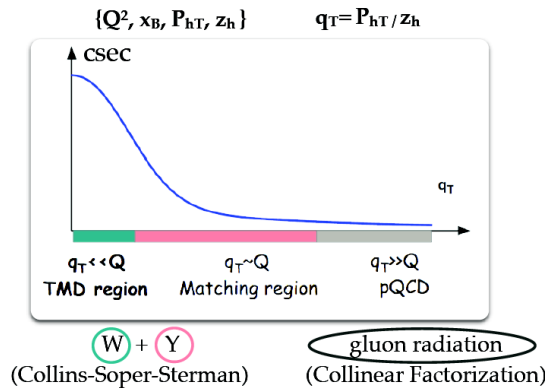
Works for SIDIS at high enough, $Q^2 > 10 \text{ GeV}^2$,
energy flow (**integration over z_h**)

Nadolsky, Stump, Yuan

DOI: [10.1103/PhysRevD.64.059903](https://doi.org/10.1103/PhysRevD.64.059903)

However, information about z -dependence gets washed out. Also, integration over z mixes TMD and collinear factorization effects.

The Matching Problem in SIDIS



Works for SIDIS at high enough, $Q^2 > 10 \text{ GeV}^2$,
energy flow (**integration over z_h**)

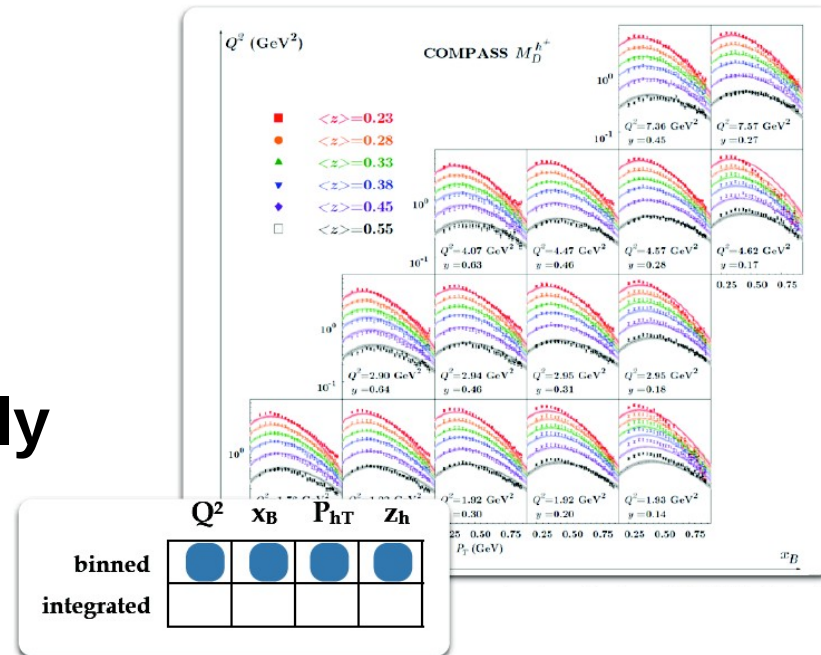
Nadolsky, Stump, Yuan

DOI: [10.1103/PhysRevD.64.059903](https://doi.org/10.1103/PhysRevD.64.059903)

However, information about z -dependence gets washed out. Also, integration over z mixes TMD and collinear factorization effects.

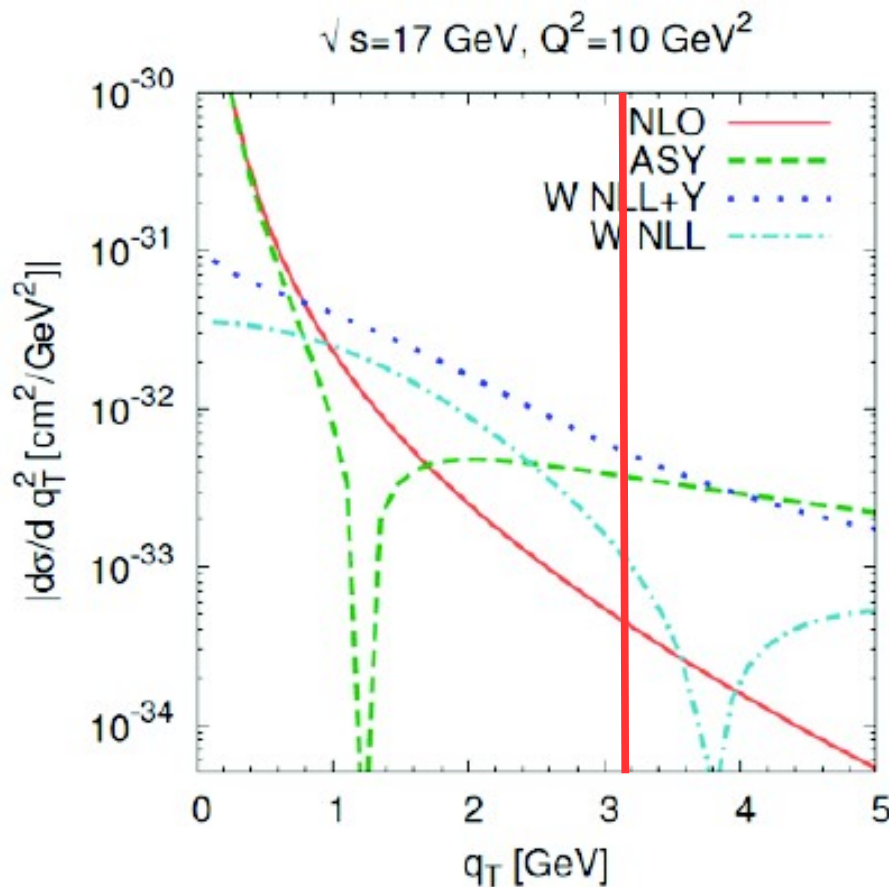
Multidimensional data
are ideal.

Can CSS be successfully
Implemented?



M. Anselmino, M. Boglione, J.O.G.H., S. Melis, A. Prokudin: Published in JHEP 1404 (2014) 005

Large q_T corrections are hard to implement.



- Large Y-term at small q_T
 - Small cross section at large q_T
 - No smooth matching
 - Delicate kinematics
- Delicate kinematics**

Source of Errors?

Unpolarized SIDIS cross section (current region)

$$\frac{d\sigma^{\ell+p \rightarrow \ell' h X}}{dx_B dQ^2 dz_h dP_T^2} = \frac{2\pi^2\alpha^2}{(x_B s)^2} \frac{[1 + (1-y)^2]}{y^2} F_{UU}$$

$$F_{UU} = \sum_q \mathcal{H}_q \text{ F.T. } \left\{ \tilde{D}_{h/q}(z, z \mathbf{b}_\perp; Q) \tilde{f}_{q/P}(x, \mathbf{b}_\perp; Q) \right\}$$

+ large q_T corrections + power suppressed terms

Perturbation Theory

Factorization

(Re)Calculation of large q_T SIDIS cross section

Work in progress:

J.O.G.H., T. Rogers, N. Sato, A. Signori, B. Wang

$$F_{UU} = \sum_q \mathcal{H}_q \text{ F.T. } \left\{ \tilde{D}_{h/q}(z, z \mathbf{b}_\perp; Q) \tilde{f}_{q/P}(x, \mathbf{b}_\perp; Q) \right\} \\ + \text{large } q_T \text{ corrections} + \text{power suppressed terms}$$

Perturbation Theory

Source of Errors?

Unpolarized SIDIS cross section (current region)

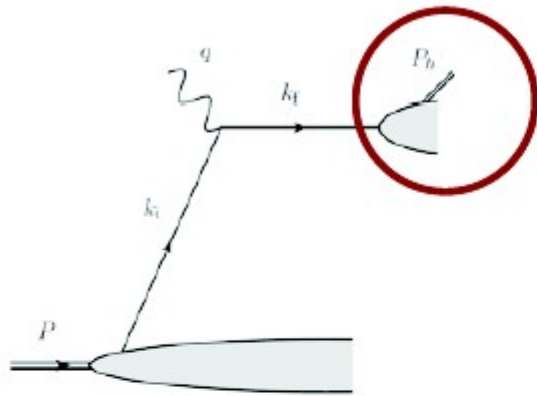
$$\frac{d\sigma^{\ell+p\rightarrow\ell' hX}}{dx_B dQ^2 dz_h dP_T^2} = \frac{2\pi^2\alpha^2}{(x_B s)^2} \frac{[1 + (1-y)^2]}{y^2} F_{UU}$$

$$F_{UU} = \sum_q \mathcal{H}_q \text{ F.T. } \left\{ \tilde{D}_{h/q}(z, z \mathbf{b}_\perp; Q) \tilde{f}_{q/P}(x, \mathbf{b}_\perp; Q) \right\}$$

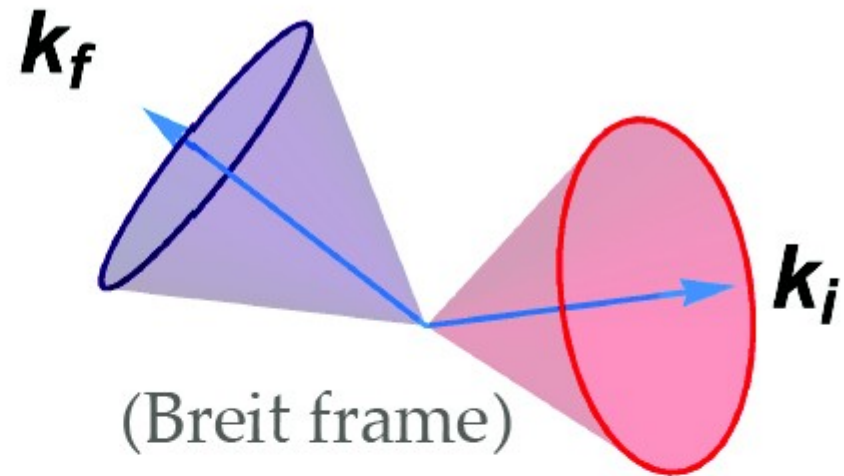
+ large q_T corrections + power suppressed terms

Factorization

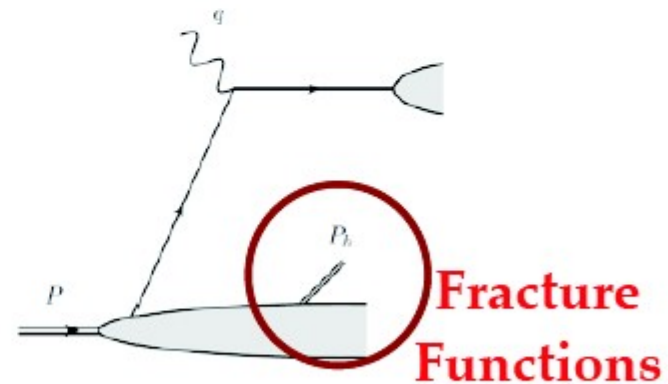
Which Region?



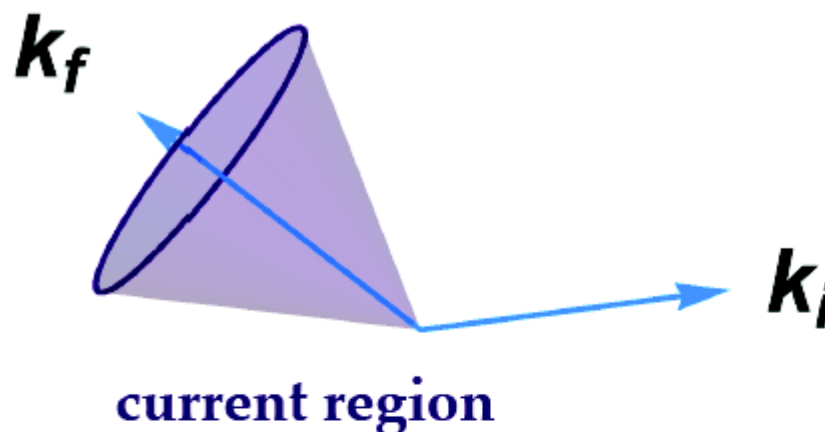
TMDs



**factorization theorems for
different leading regions**



Power counting and kinematics of the current region



small masses

$$P_h \cdot k_f = O(m^2)$$

$$P_h \cdot k_i = O(Q^2)$$

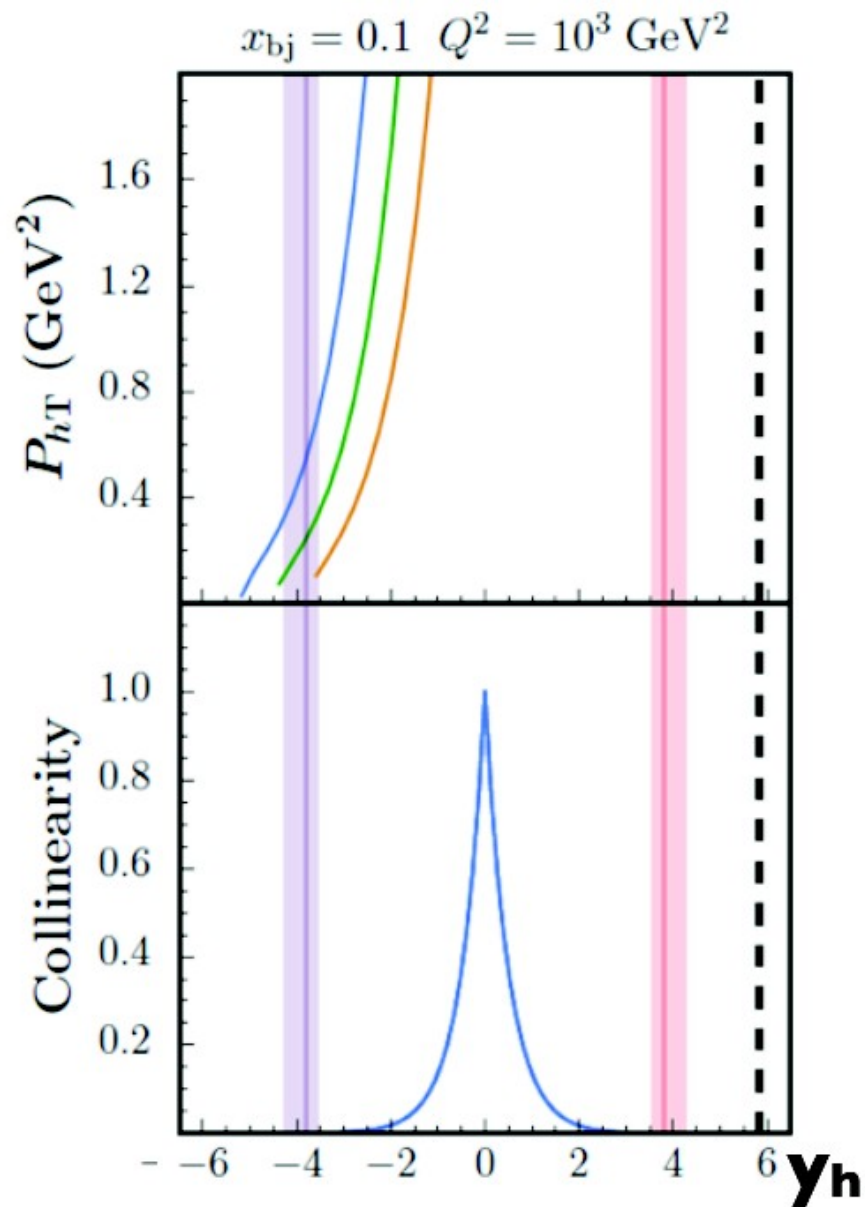
hard scale

require small values
for

$$R \equiv \frac{P_h \cdot k_f}{P_h \cdot k_i}$$

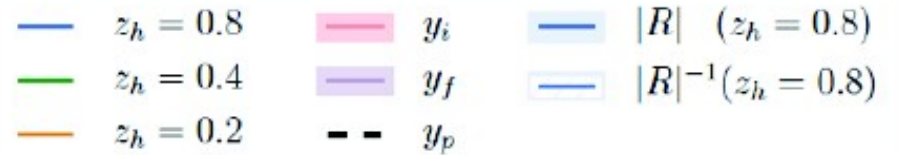
notice quark momenta
have to be estimated

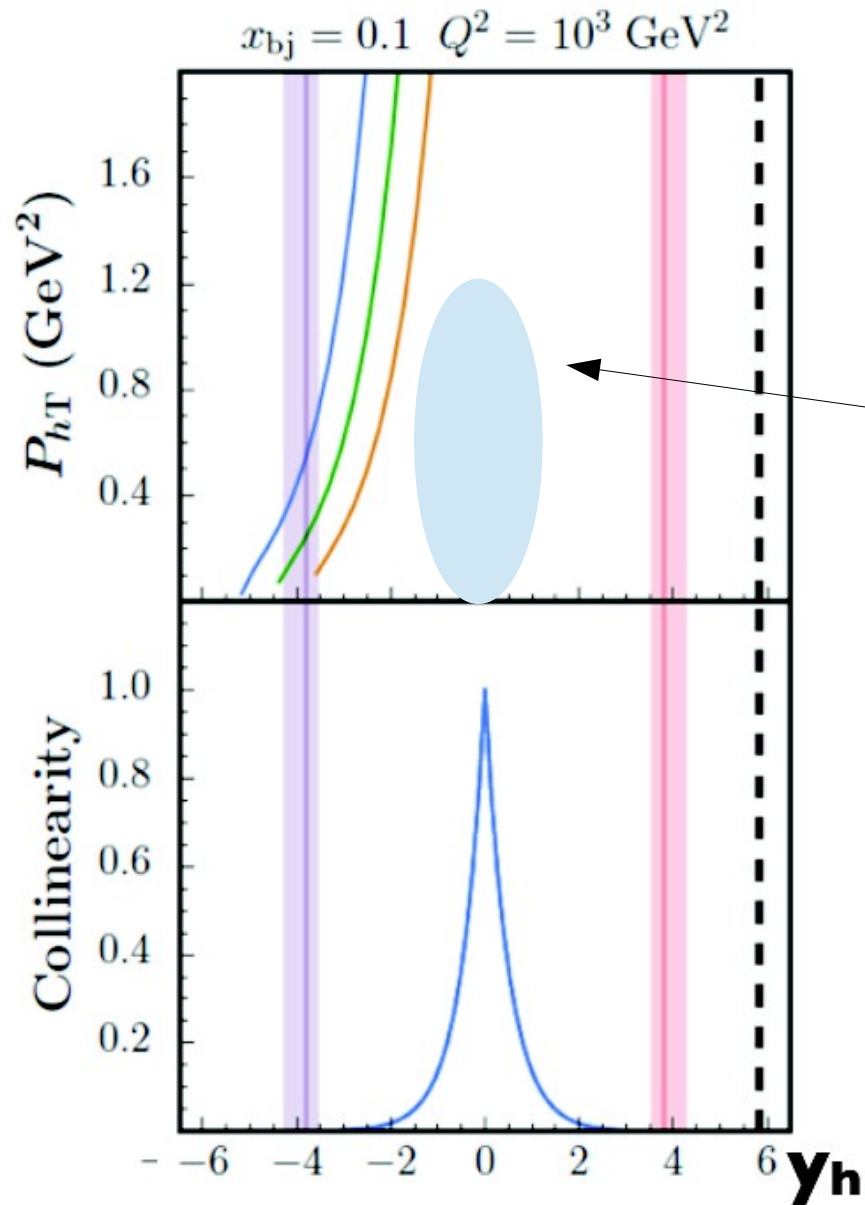
M. Boglione, J. Collins, L. Gamberg, JOGH, T. C. Rogers,
and N. Sato, Phys. Lett. B 766, 245 (2017), 1611.10329.



$$y_h \equiv \frac{1}{2} \log \frac{P_h^+}{P_h^-}$$

$$R \equiv \frac{P_h \cdot k_f}{P_h \cdot k_i}$$

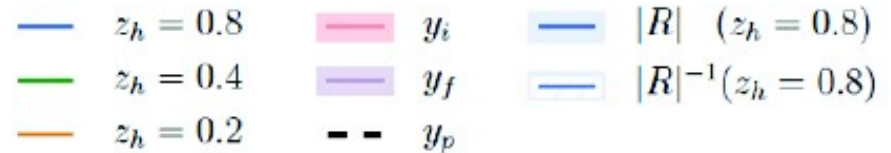


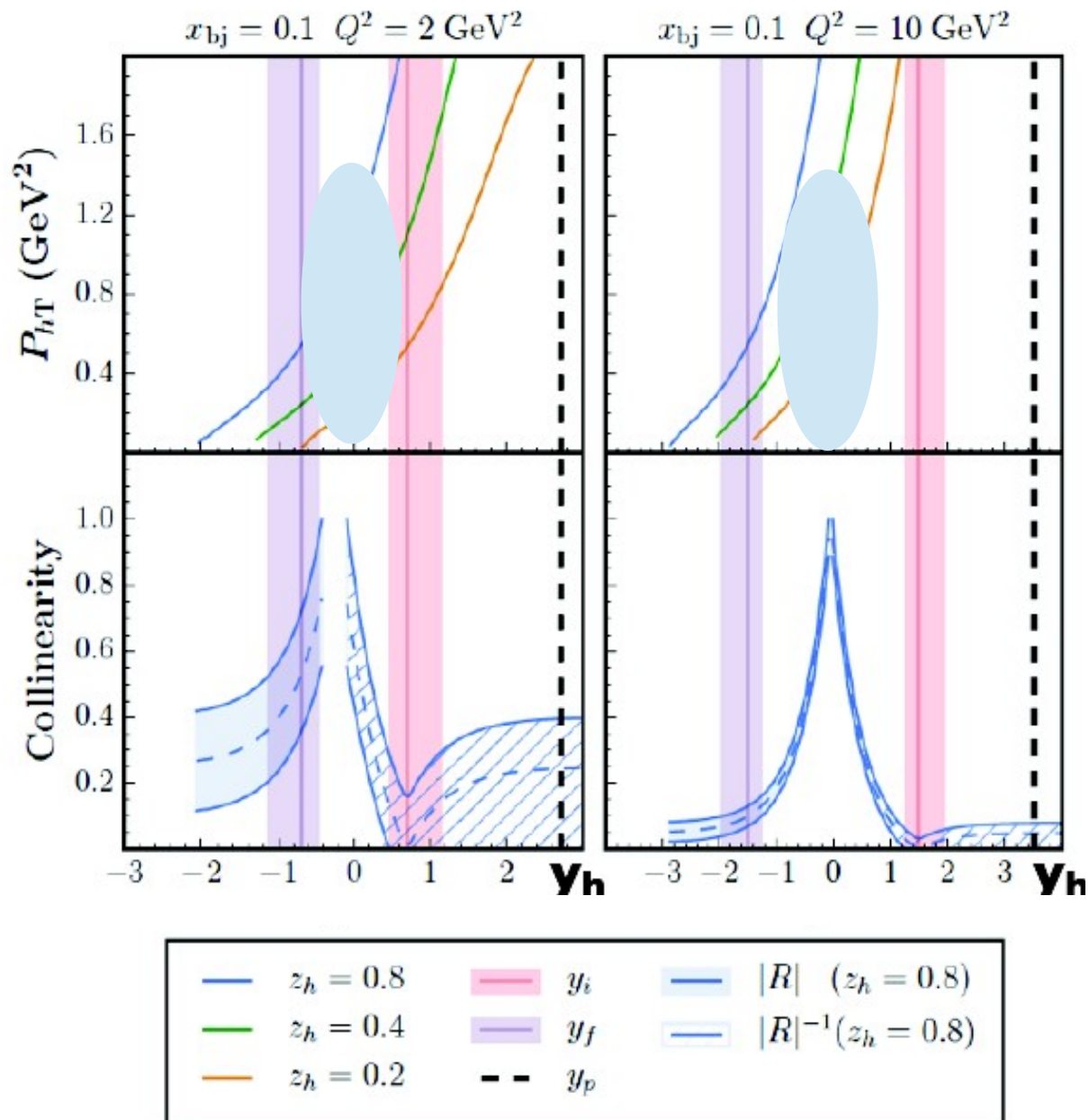


EIC projected energies.

Avoid region of central rapidity, soft non-TMD effects

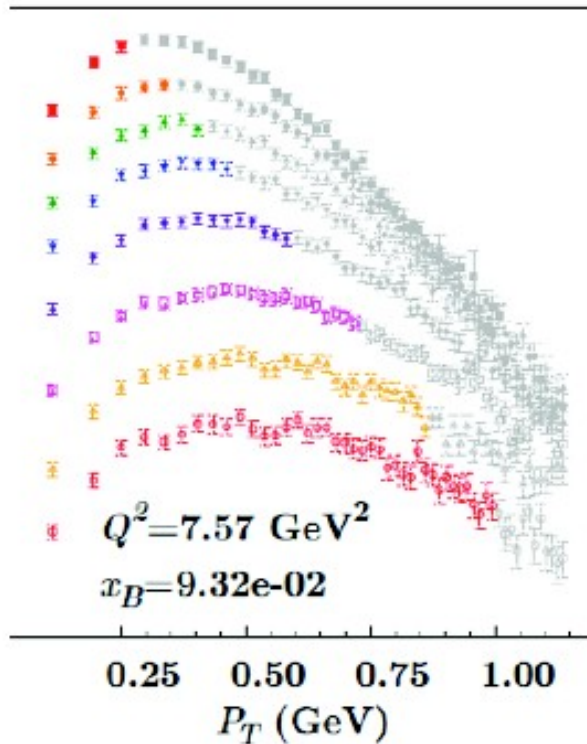
$$R \equiv \frac{P_h \cdot k_f}{P_h \cdot k_i}$$





**Available data is likely
to receive contributions
from non-TMD physics.**

$$R \equiv \frac{P_h \cdot k_f}{P_h \cdot k_i}$$



precise implementation of
the R criterion on data is
work in progress

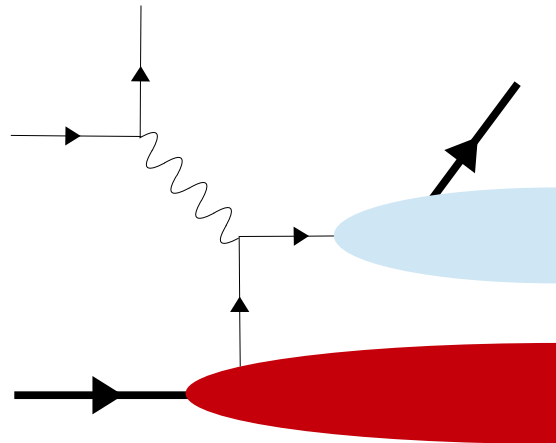
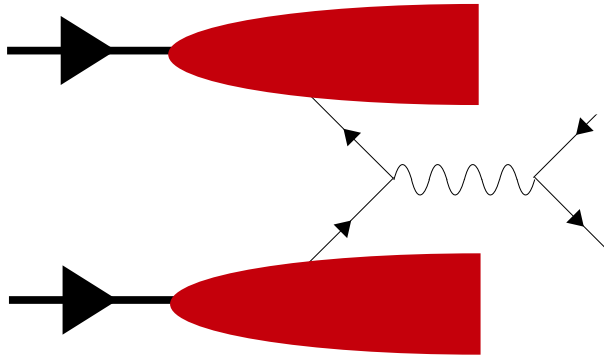
a better set of variables?

$$\{Q^2, x_B, P_{hT}, z_h\}$$

$$q_T = P_{hT} / z_h \quad y_h$$

***ONLY AN
EXAMPLE**

Drell Yan

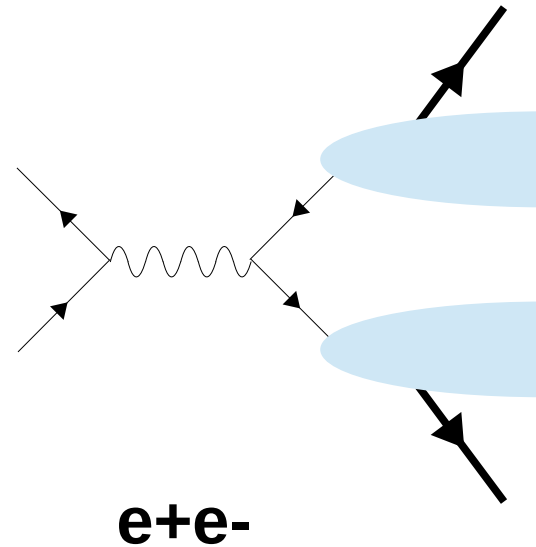


?

SIDIS

Recently, BELLE, BaBar, BES III
Collins asymmetries.

No modern unpolarized
measurements are available.

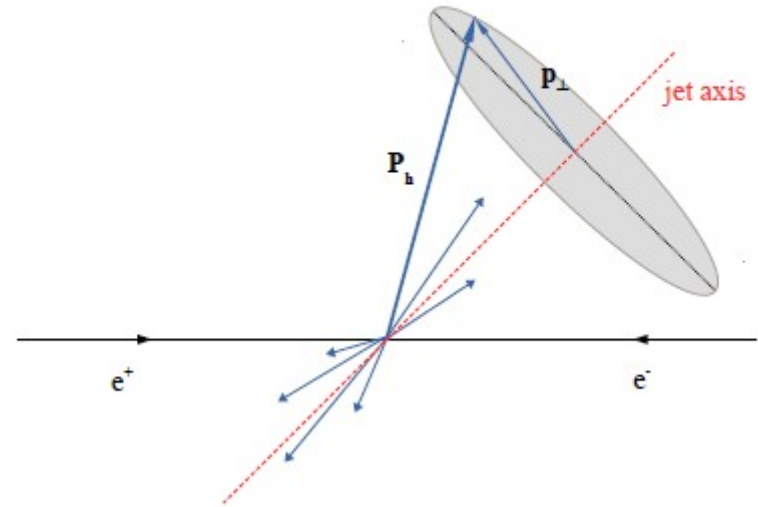


Recently, BELLE, BaBar, BES III
Collins asymmetries.

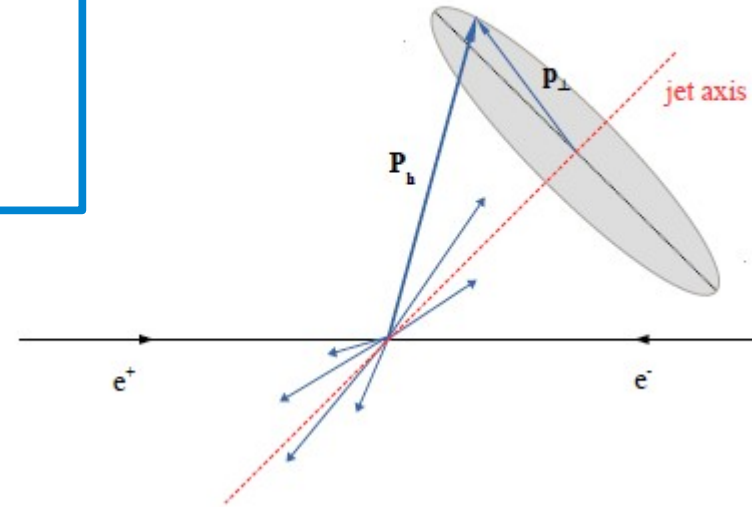
No modern unpolarized
measurements are available.

TASSO, MARK II available for
 $e^+e^- \rightarrow X h$

- **p_T** distributions
- different energies
- integrated over **z**



Boglione, JOGH, R. Taghavi
Phys.Lett. B772 (2017) 78
arXiv:1704.08882



TASSO, MARK II available for
 $e^+e^- \rightarrow X h$

- p_T distributions
- different energies
- integrated over z

Big Limitation

New analysis:

how much information about the **unpolarized TMD FF**
can we get from these data sets?

Use this...



$$D_{h/q}(z, p_{\perp}) = d_{h/q}(z) h_d(p_{\perp})$$

Assuming factorization

To get information
about this

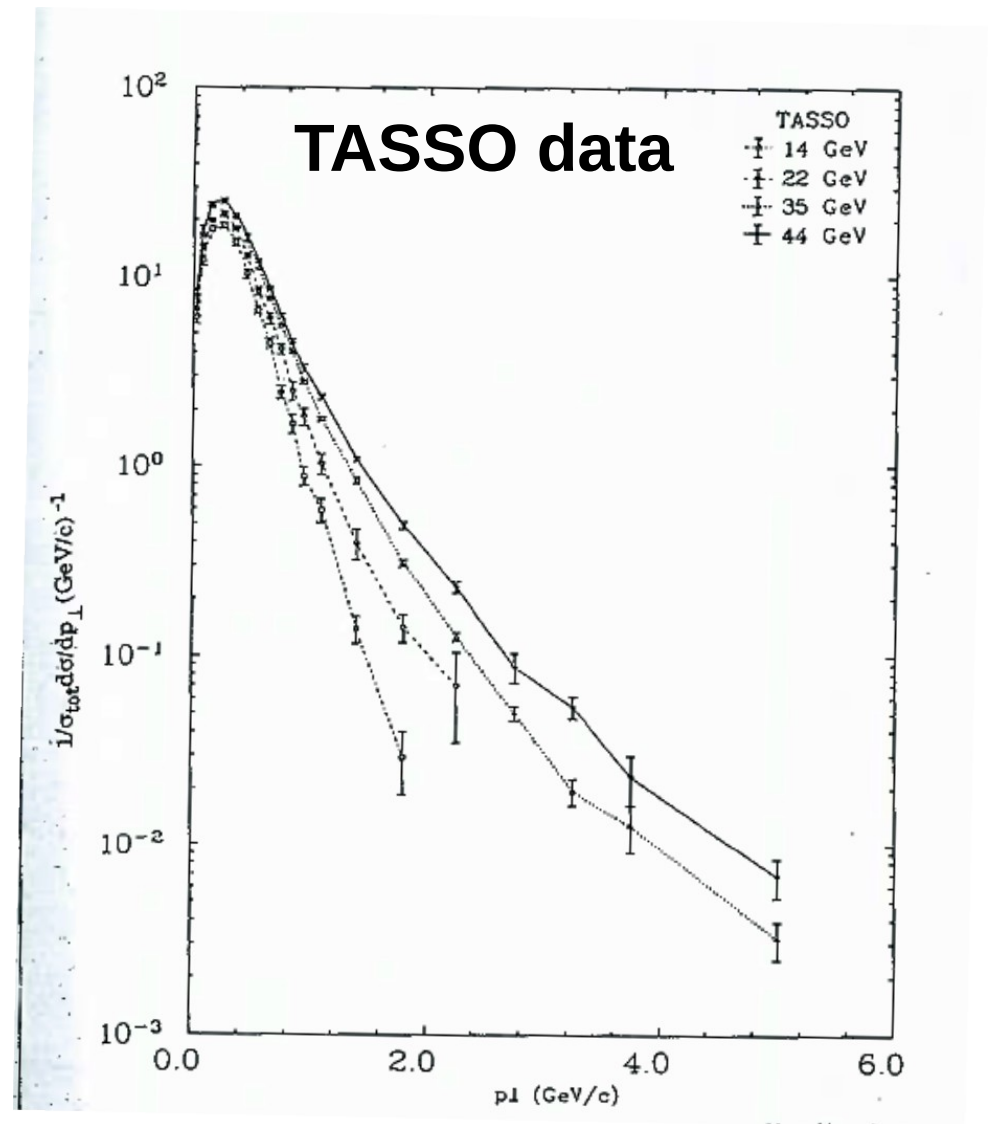


QCD picture

$$\tilde{D}_{h/q}(z, \mathbf{b}_{\perp}; Q) = \sum_j \left[\left(\tilde{C}_{j/q} \otimes \frac{d_{h/j}}{z^2} \right) e^{\Gamma_D(Q)} \right] \exp \left\{ g_{j/P}(x, b_{\perp}) + g_K(b_{\perp}) \log \left(\frac{Q}{Q_0} \right) \right\}$$

Things to investigate:

- appropriate functional form for $\mathbf{g}_{j/P}$
- scale evolution regulated by \mathbf{g}_K



$$\tilde{D}_{h/q}(z, \mathbf{b}_{\perp}; Q) = \sum_j \left[\left(\tilde{C}_{j/q} \otimes \frac{d_{h/j}}{z^2} \right) e^{\Gamma_D(Q)} \right] \exp \left\{ g_{j/P}(x, b_{\perp}) + g_K(b_{\perp}) \log \left(\frac{Q}{Q_0} \right) \right\}$$

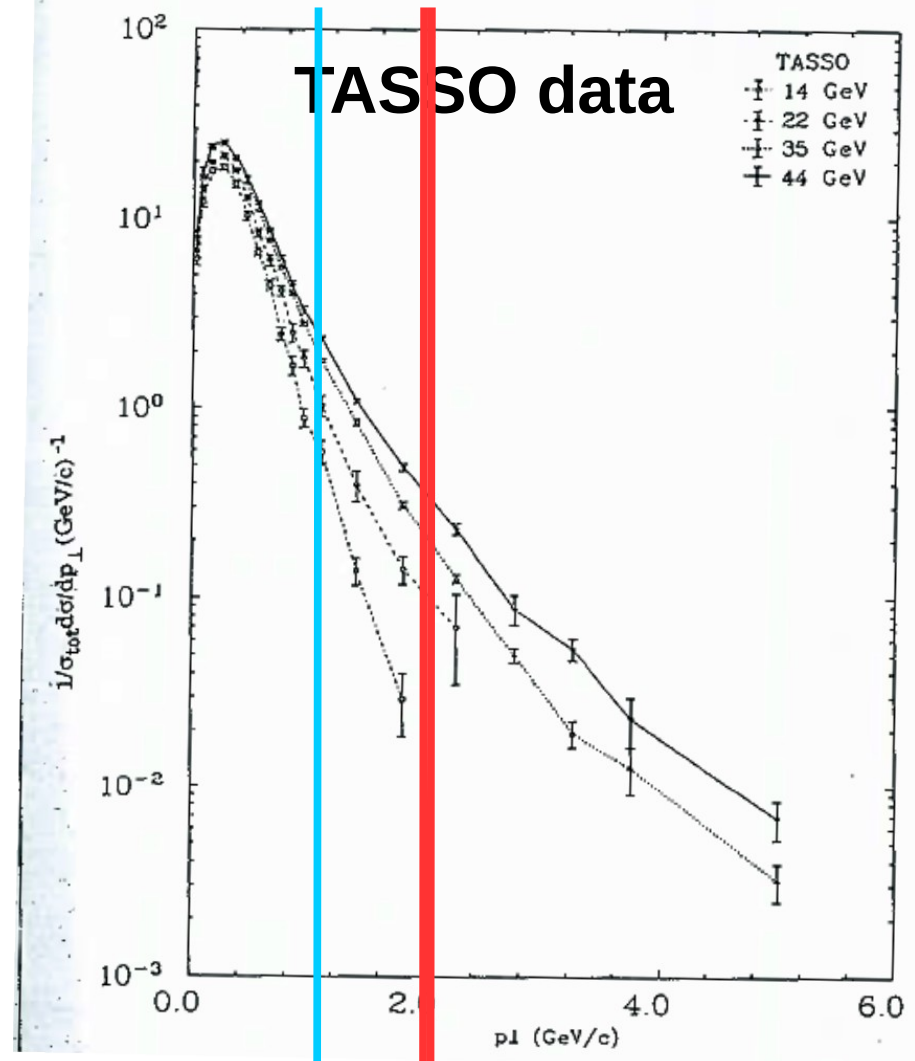
Identify region where TMD Effects dominate:

For fully differential cross sections, matching region is Expected to be at

$$p_{\perp} \sim zQ$$

Use experimental $\langle z \rangle$ to make an estimate

$$p_{\perp} \sim 2 \text{ GeV}$$



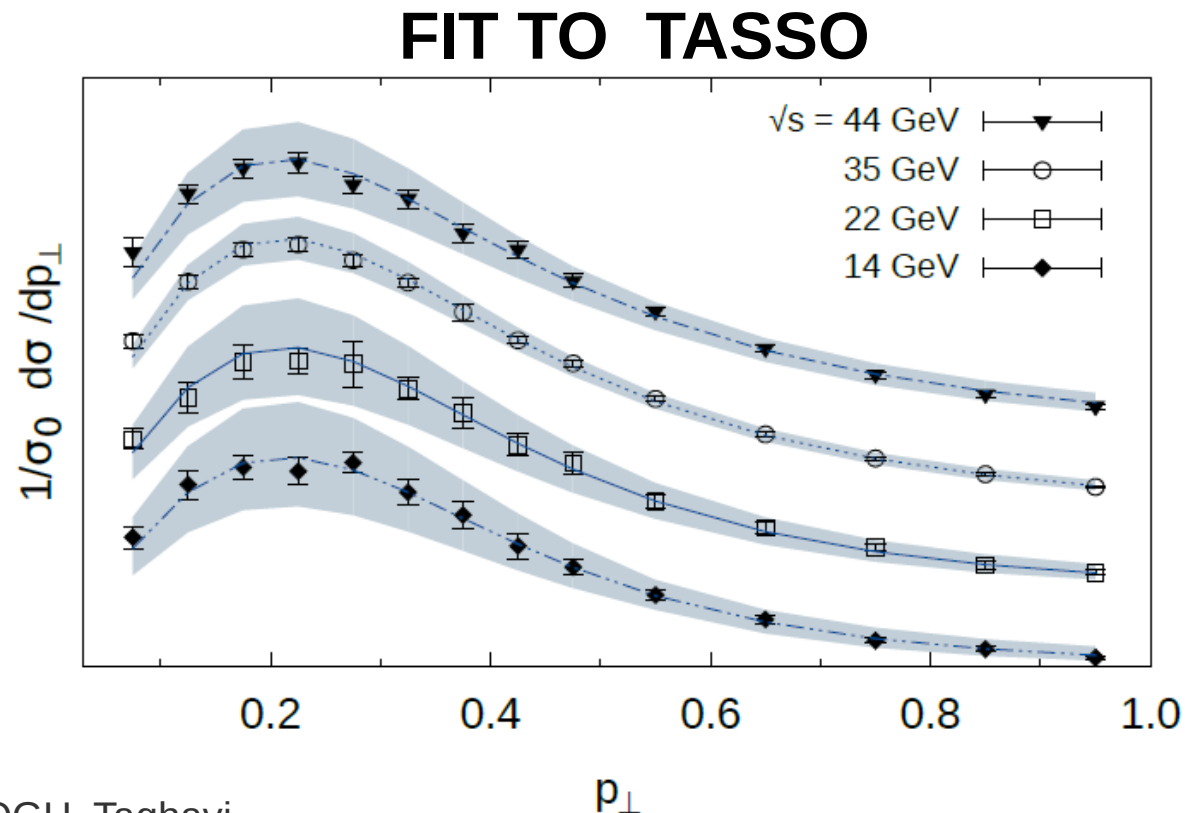
We looked at a restricted range:

$$p_{\perp} < 1 \text{ GeV}$$

Power law to model transverse momentum dependence

$$D_{h/q}(z, p_{\perp}) = d_{h/q}(z) h_d(p_{\perp})$$

$$h(p_{\perp}) = 2(\alpha - 1)M^{2(\alpha-1)} \frac{1}{(p_{\perp}^2 + M^2)^{\alpha}}$$

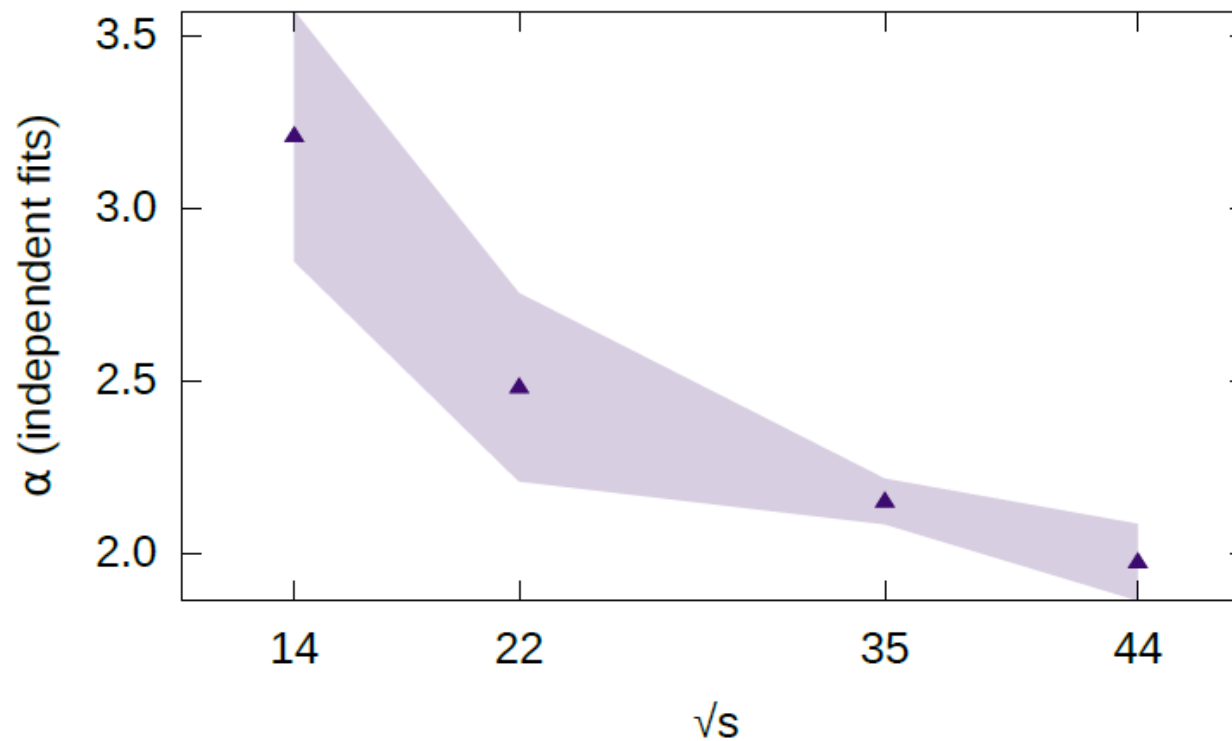


Boglione, JOGH, Taghavi

Phys.Lett. B772 (2017) 78-86

Power law parameters follow a logarithmic trend

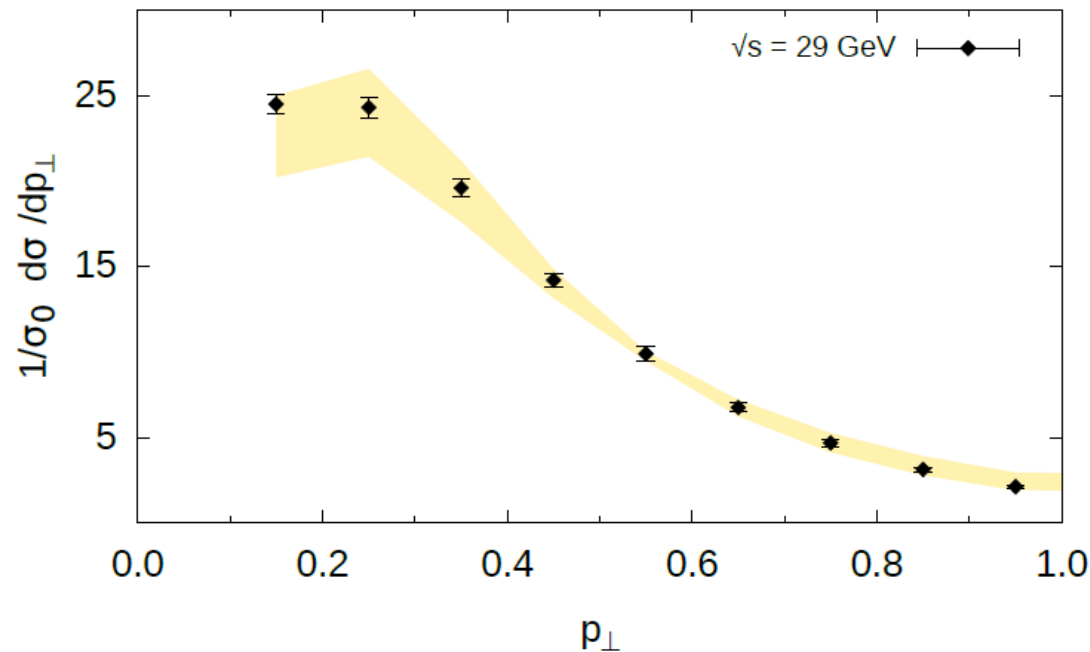
$$h(p_{\perp}) = 2(\alpha - 1)M^{2(\alpha-1)} \frac{1}{(p_{\perp}^2 + M^2)^{\alpha}}$$



Power law parameters follow a logarithmic trend
Consistent with MARK II data.

$$h(p_{\perp}) = 2(\alpha - 1)M^{2(\alpha-1)} \frac{1}{(p_{\perp}^2 + M^2)^{\alpha}}$$

COMPARISON TO MARK II



TMD

$$\mathcal{F}^{-1} \left\{ \frac{d\sigma^h}{dz d^2 \mathbf{p}_\perp} \right\} \propto \exp \left\{ \left(\lambda_\Gamma(b_*) + g_K(b_\perp) \right) \log \left(\frac{Q}{Q_0} \right) \right\} \Big|_{b_\perp \rightarrow z b_\perp}$$

$$\lambda_\Gamma(b_*) \equiv \frac{32}{27} \log \left(\log \frac{2e^{-\gamma_E}}{\Lambda_{QCD} b_*} \right)$$

MODEL $h(p_\perp) = 2(\alpha - 1)M^{2(\alpha-1)} \frac{1}{(p_\perp^2 + M^2)^\alpha}$

$$\mathcal{F}^{-1} \left\{ \frac{1}{(p_\perp^2 + M^2)^\alpha} \right\} \xrightarrow{\text{large } b_\perp} \frac{1}{2^\alpha \pi \Gamma(\alpha)} \left(\frac{b_\perp}{M} \right)^{\alpha-1} \sqrt{\frac{\pi}{2}} \frac{e^{-b_\perp M}}{\sqrt{b_\perp M}} \left[1 + \mathcal{O} \left(\frac{1}{b_\perp M} \right) \right]$$

TMD

$$\mathcal{F}^{-1} \left\{ \frac{d\sigma^h}{dz d^2 \mathbf{p}_\perp} \right\} \propto \exp \left\{ \left(\lambda_\Gamma(b_*) + g_K(b_\perp) \right) \log \left(\frac{Q}{Q_0} \right) \right\} \Big|_{b_\perp \rightarrow z b_\perp}$$

$$\lambda_\Gamma(b_*) \equiv \frac{32}{27} \log \left(\log \frac{2e^{-\gamma_E}}{\Lambda_{QCD} b_*} \right)$$

Logarithmic behavior of alpha may be interpreted as a consequence of the **Log** in the definition of the **TMD FF**.

$$\alpha = \alpha_0 + \tilde{\alpha} \log \left(\frac{Q}{Q_0} \right)$$

$$g_K(b_\perp) \xrightarrow{\text{large } b_\perp} \tilde{\alpha} \log(\nu b_\perp)$$

TMD

There are caveats on this interpretation, while consistent with theoretical expectations, it's not the only possibility.

(loss of information through z-integration)

Logarithmic behavior of alpha may be interpreted as a consequence of the **Log** in the definition of the **TMD FF**.

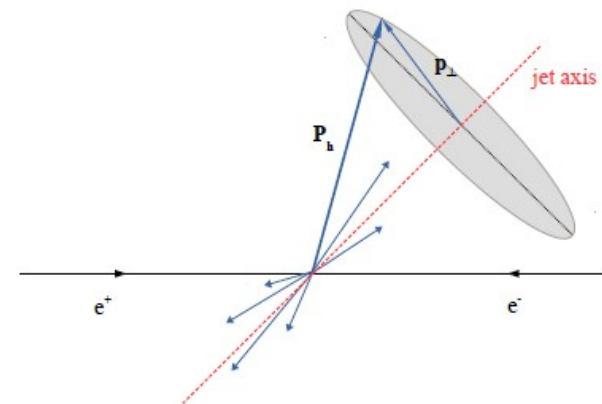
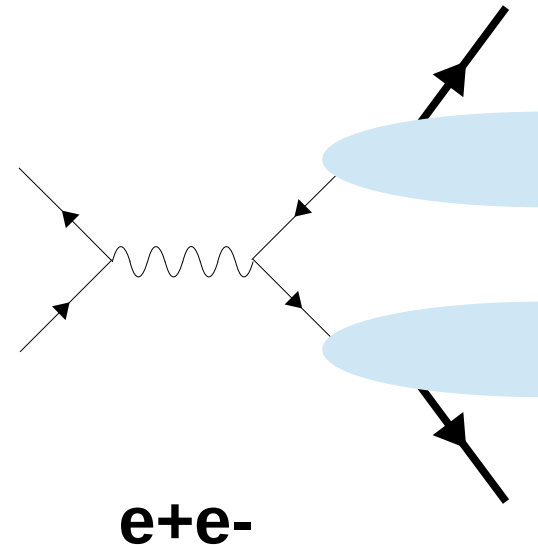
$$\alpha = \alpha_0 + \tilde{\alpha} \log \left(\frac{Q}{Q_0} \right)$$

$$g_K(b_\perp) \xrightarrow{\text{large } b_\perp} \tilde{\alpha} \log(v b_\perp)$$

The lack of information about \mathbf{z} hinders a full TMD extraction of the FF.

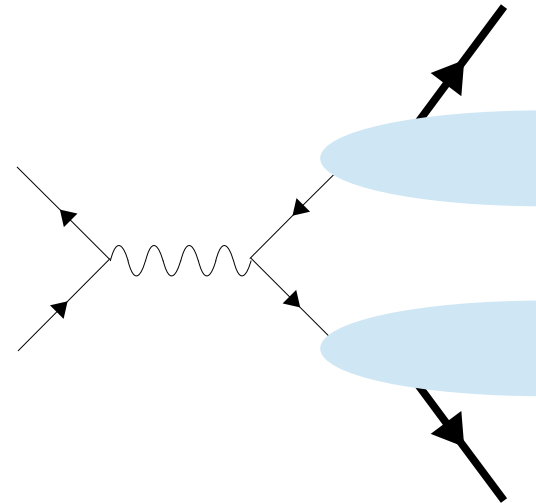
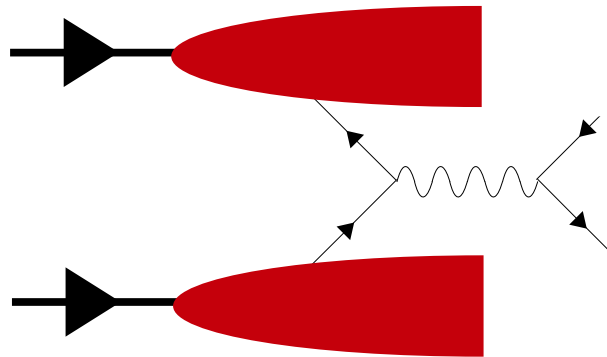
Future upcoming data by BELLE on unpolarized one-hadron production may allow for a combined analysis with TASSO and MARK II data.

Phenomenological Test for factorization

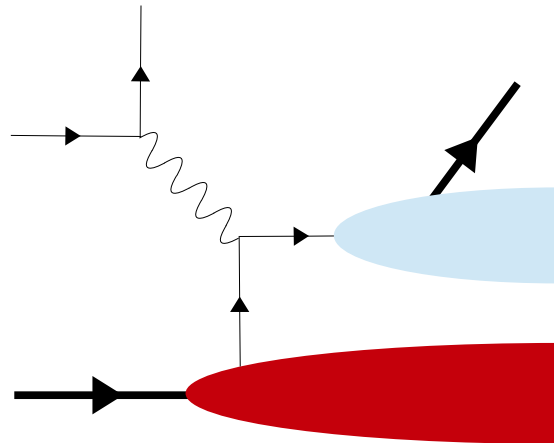


Extraction from data?

Drell Yan



e^+e^-



Let's keep working,
but let us
be careful with
interpretations

SIDIS

**Fragmentation
Functions**

Final Remarks

Currently, we are attempting to do phenomenology within **full QCD picture**.

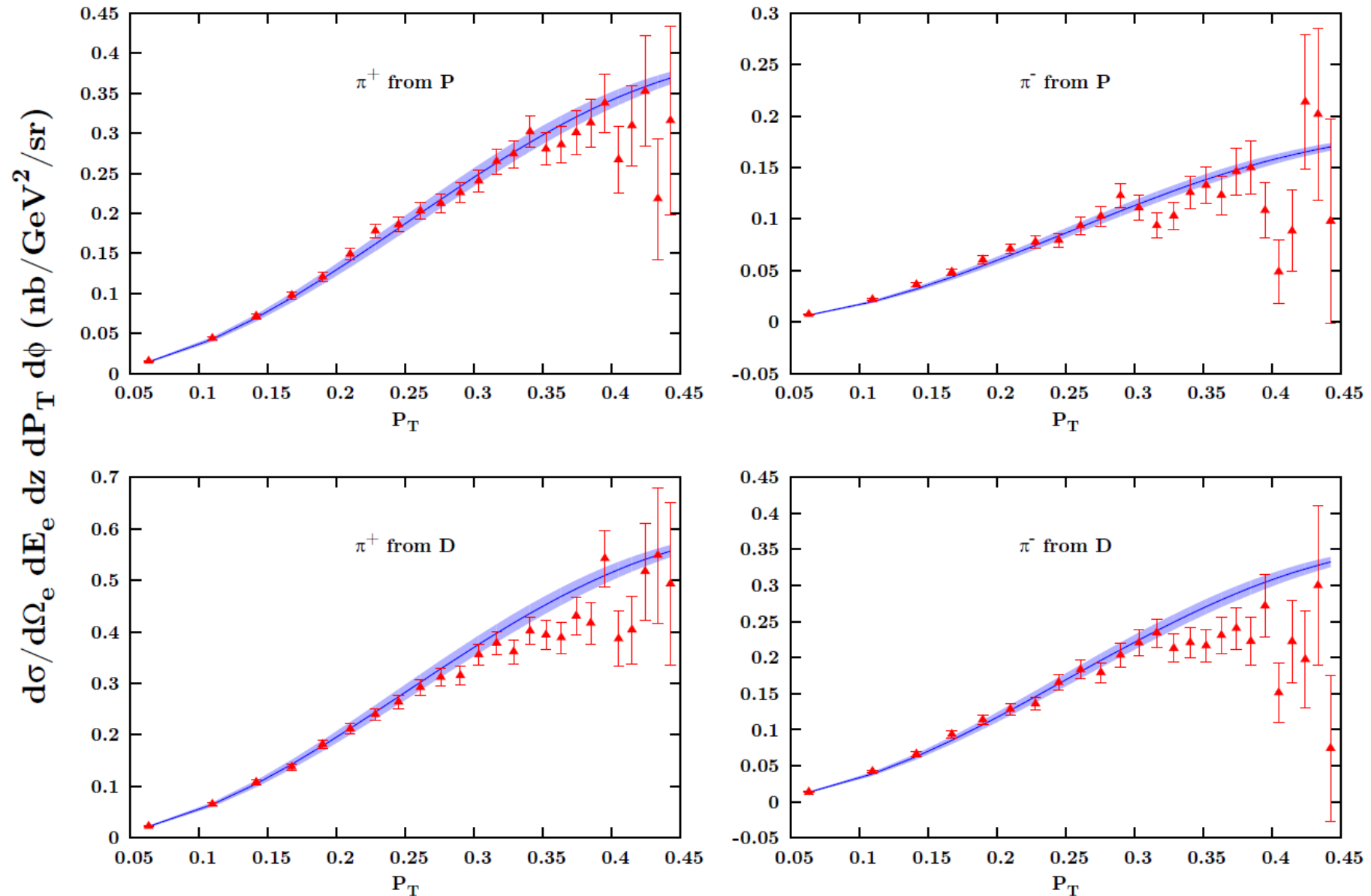
Recent SIDIS multidimensional data is so far the most suitable way to access information about the unpolarized TMD FF. Must solve some *theoretical issues*.

On the side of e^+e^- one hadron production, in the near future unpolarized cross sections by BELLE may allow for an analysis of the older sets, TASSO MARKII within a full TMD picture.

TMD Factorization for e^+e^- one hadron production?

Thank you.

Jlab SIDIS data (2012) (Parameters from HERMES extraction).



Ingredients for extraction of Collins function.

$e^+e^- \rightarrow \pi\pi X$

SIDIS

Unpolarized TMDFF

Collins TMDFF

$$\frac{d\sigma^{e^+e^- \rightarrow h_1 h_2 X}}{dz_1 dz_2 d^2\mathbf{P}_{1T} d\cos\theta_2} = \frac{3\pi\alpha^2}{2s} \left\{ \boxed{D_{h_1 h_2}} + \boxed{N_{h_1 h_2}} \cos 2\phi_1 \right\}$$

$$P_0^{U,L,C} = \frac{N^{U,L,C}}{D^{U,L,C}}$$

Ratio

$$\begin{aligned} D^U &= D_{\pi^+\pi^-} + D_{\pi^-\pi^+} & N^U &= N_{\pi^+\pi^-} + N_{\pi^-\pi^+} \\ D^L &= D_{\pi^+\pi^+} + D_{\pi^-\pi^-} & N^L &= N_{\pi^+\pi^+} + N_{\pi^-\pi^-} \\ D^C &= D^U + D^L & N^C &= N^U + N^L, \end{aligned}$$

$$\frac{A_0^U}{A_0^{L(C)}} \equiv 1 + \cos(2\phi_1) \boxed{A_0^{UL(C)}} \quad \text{Double Ratio}$$

$$\begin{aligned} \frac{d\sigma^{\ell(S_\ell)+p(S)\rightarrow\ell'hX}}{dx_B dQ^2 dz_h d^2\mathbf{P}_T d\phi_S} = \\ \frac{2\alpha^2}{Q^4} \left\{ \frac{1+(1-y)^2}{2} F_{UU} + \dots \right. \\ \left. + S_T(1-y)(\sin(\phi_h + \phi_S) F_{UT}^{\sin(\phi_h + \phi_S)}) \right\}. \end{aligned}$$

$$\boxed{A_{UT}^{\sin(\phi_h + \phi_S)}} \sim \frac{\boxed{F_{UT}^{\sin(\phi_h + \phi_S)}}}{\boxed{F_{UU}}}$$

Unpolarized
TMDFF
& TMDPDF

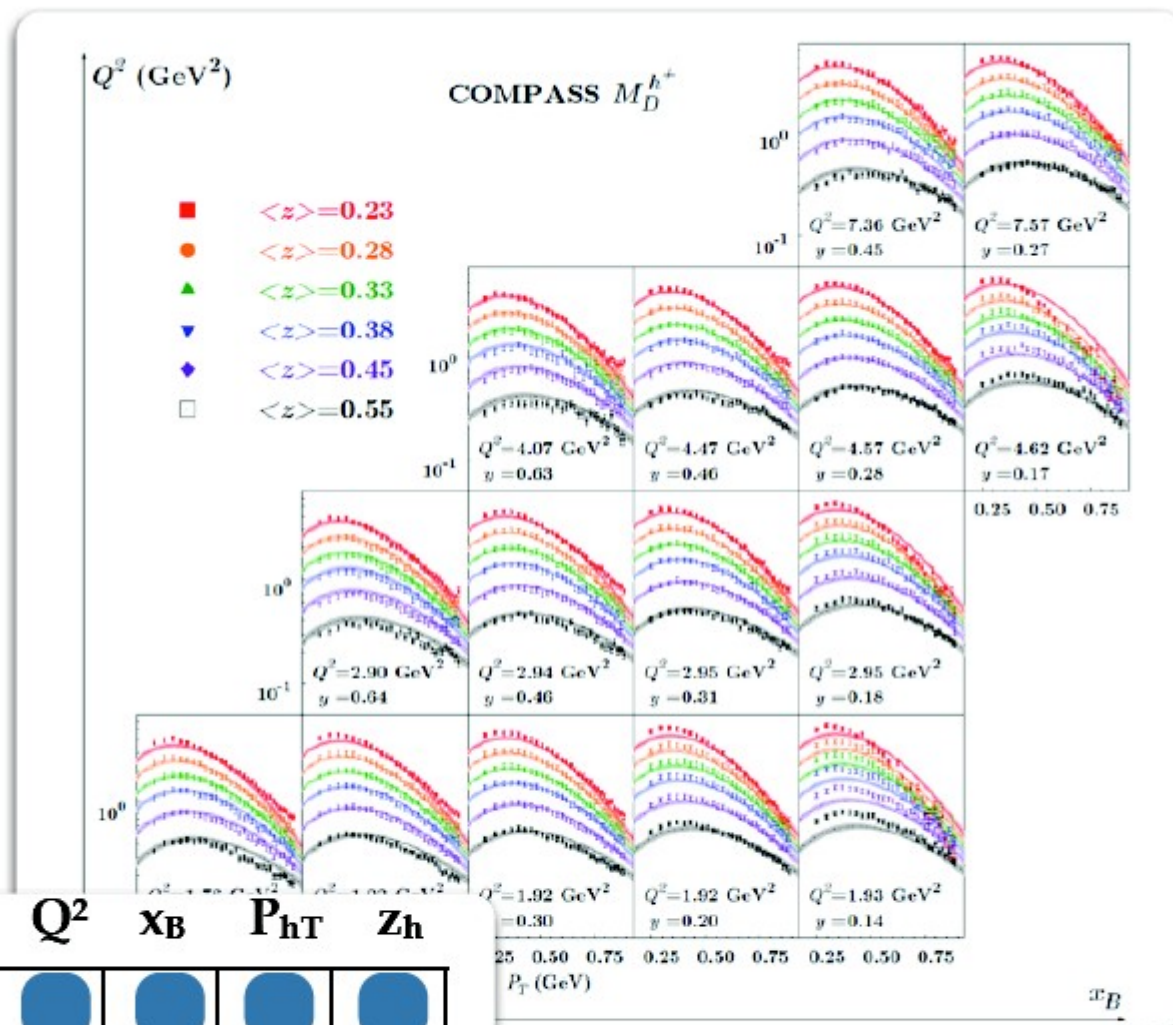
TMD Transversity
& Collins function

Unpolarized SIDIS cross section (current region)

$$\frac{d\sigma^{\ell+p\rightarrow\ell' hX}}{dx_B dQ^2 dz_h dP_T^2} = \frac{2\pi^2\alpha^2}{(x_B s)^2} \frac{[1 + (1-y)^2]}{y^2} F_{UU}$$

$$F_{UU} = \sum_q \mathcal{H}_q \text{ F.T. } \left\{ \tilde{D}_{h/q}(z, z \mathbf{b}_\perp; Q) \tilde{f}_{q/P}(x, \mathbf{b}_\perp; Q) \right\}$$

+ large q_T corrections + power suppressed terms



binned
integrated

<input checked="" type="checkbox"/>	<input checked="" type="checkbox"/>	<input checked="" type="checkbox"/>	<input checked="" type="checkbox"/>
<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>