EICUG, Trieste, July 2017

Bases and first results of a recursive quark fragmentation model with spin

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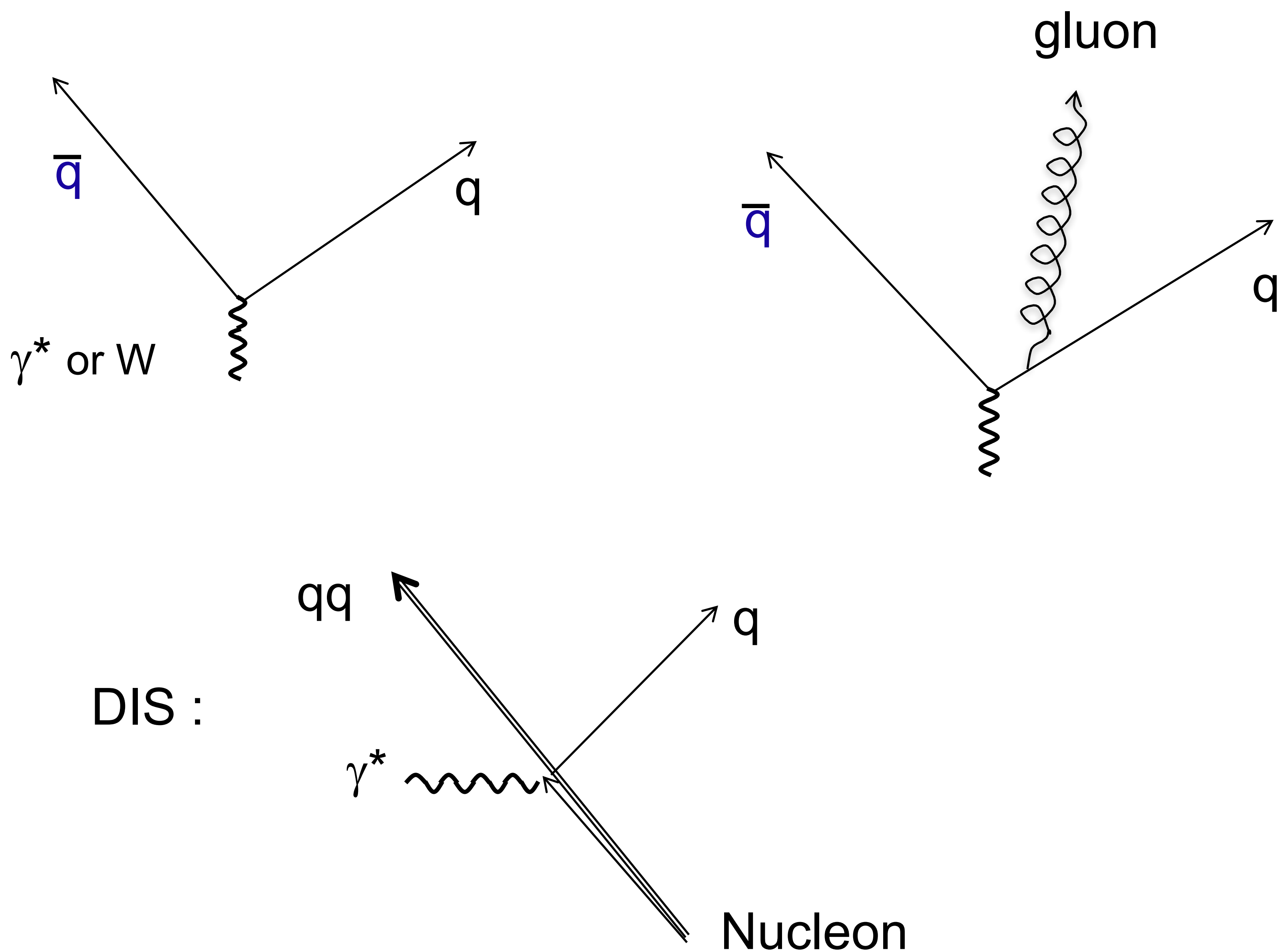
Outlines

- The string, multiperipheral and recursive points of view
- *Quark* ***L****ine* ***R****eversal* or "**L**eft-**R**ight" symmetry
- General method of introducting the quark spin :
    - the splitting *matrix*
    - the Monte-Carlo algorithm
- Input from the string fragmentation model:

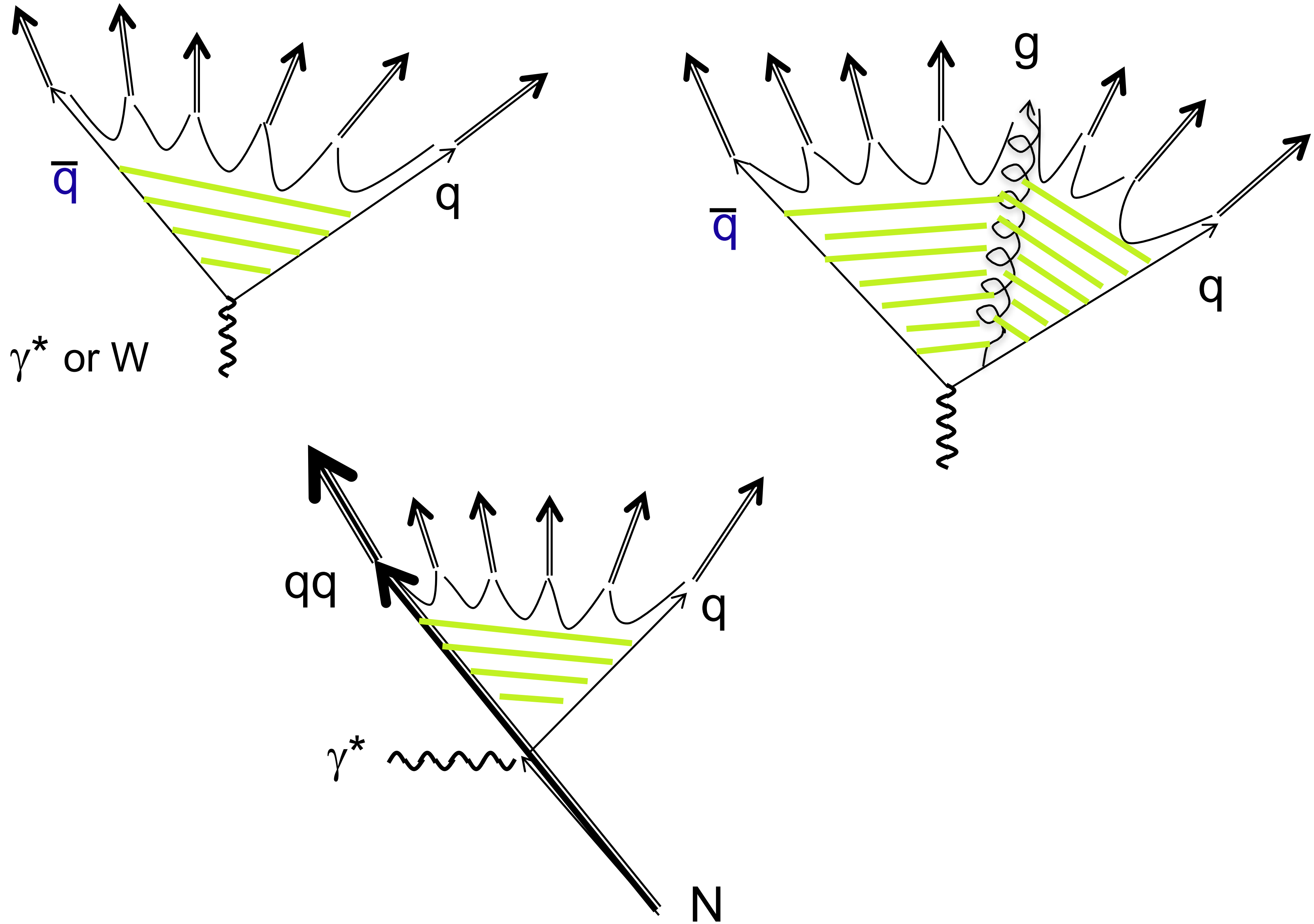
  transformation of the spinless Lund symmetric splitting function

  into a splitting matrix
- Input from the $^3\mathbf{P}_0$ mechanism of quark pair creation
- Comparison of simulation results and COMPASS experiments

Naked final partons

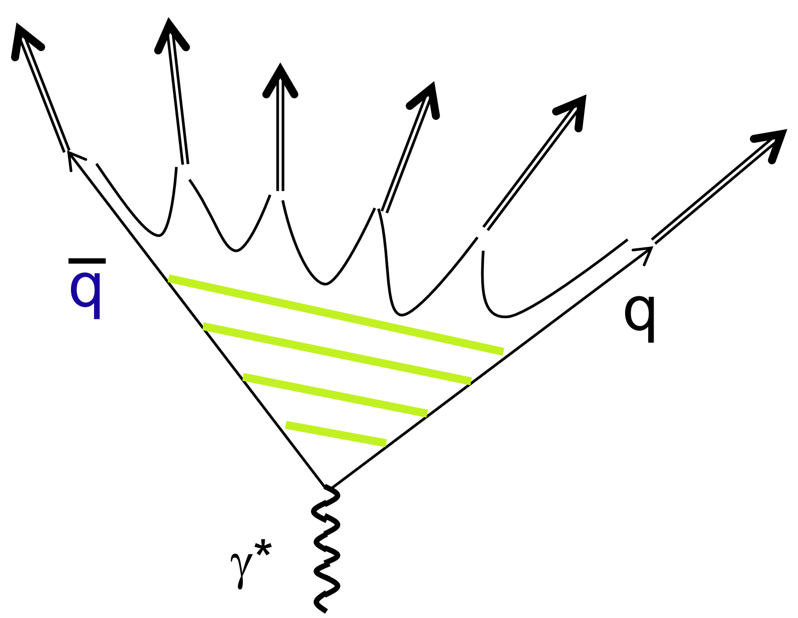


dressing



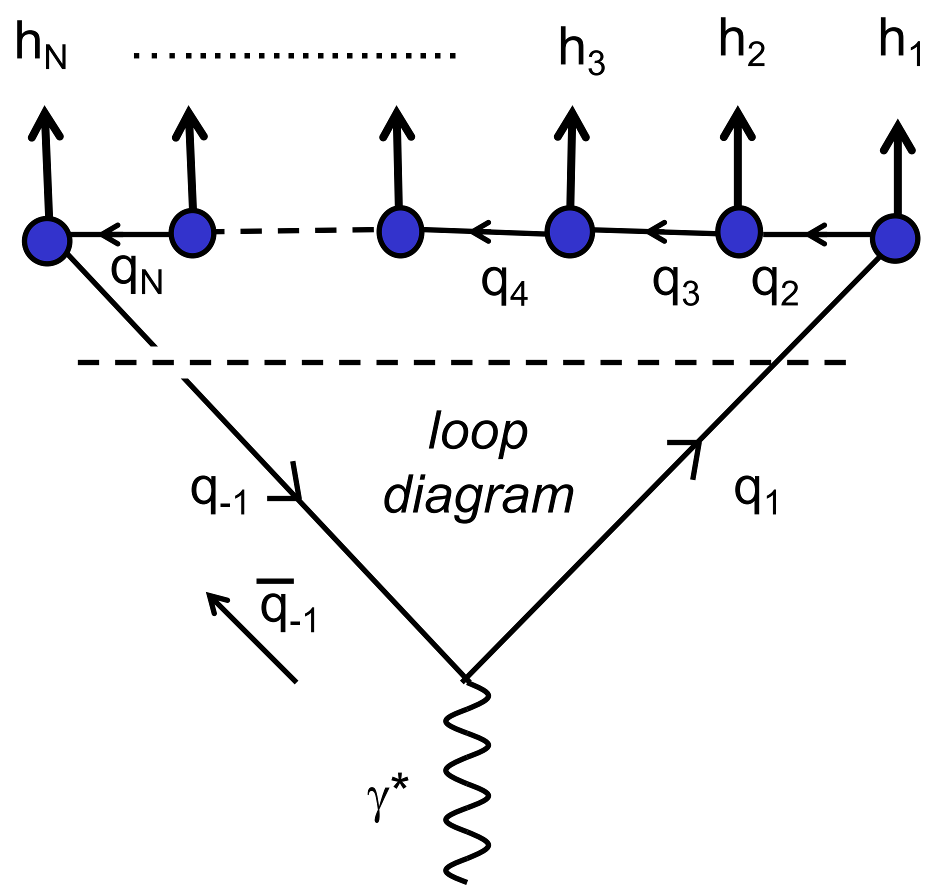
Theory : two complementary models

String Fragmentation (SFM)

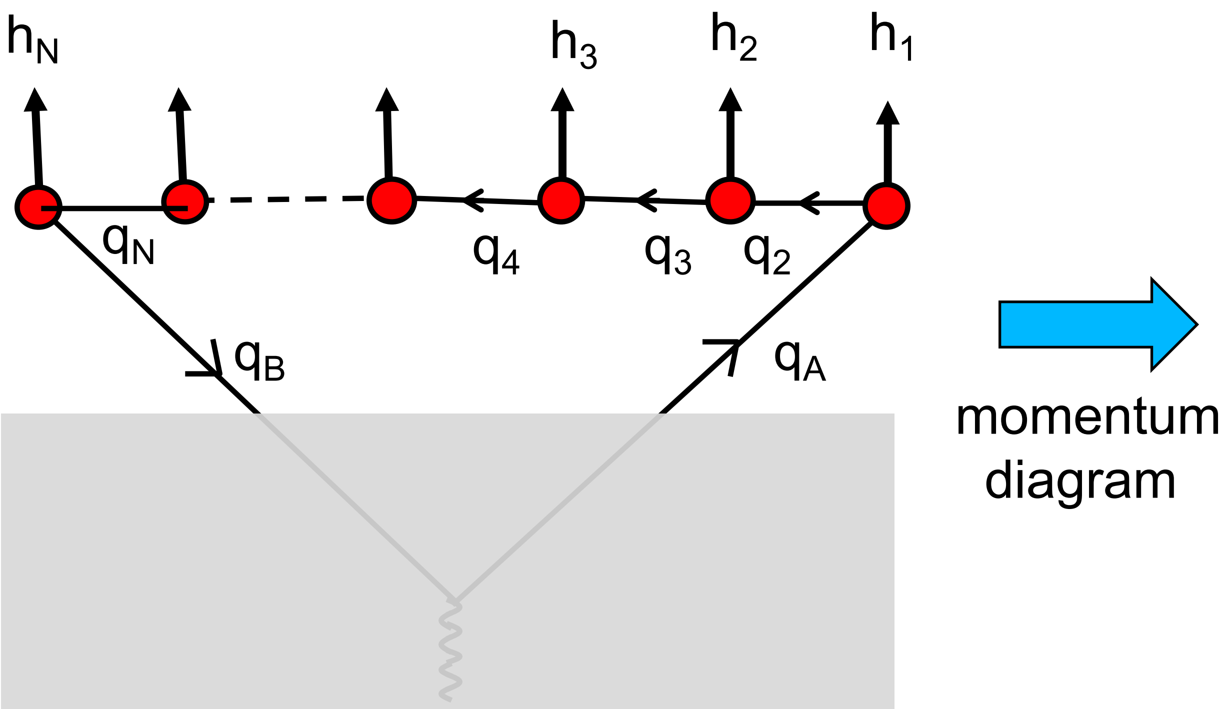


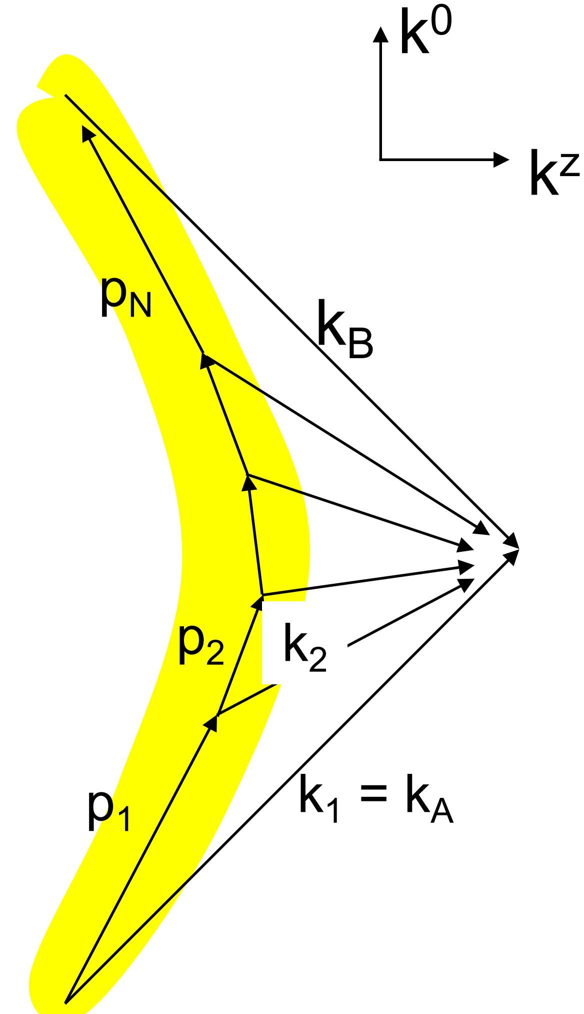
confinement
built-in

Quark Multiperipheral (QMPM)



QMPM diagram





quark virtuality : $k^2 = -\mathbf{k}^2_T - |k^+k^-|$ = spacelike

Cutoff in $\mathbf{k}_T$ ➔ Cutoff in $\mathbf{p}_T$

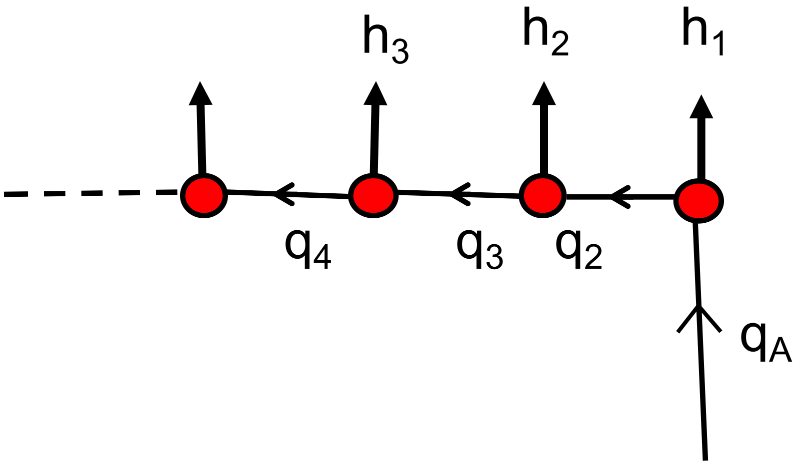
Cutoff in $|k^+k^-|$ ➔ $h_1, h_2, h_3, \ldots$ ~ ordered in rapidity

Recursive model

$q_A\ (k_A) \rightarrow h_1(p_1) + q_2(k_2)$,

$q_2\ (k_2) \rightarrow h_2(p_2) + q_3(k_3)$,

… etc.



= iteration of the *splitting* process $q(k) \rightarrow h(p) + q'(k')$

momentum conservation: $k' = k - p$

In a Monte Carlo method, **p** is drawn with the *splitting distribution*

$$F(p,k)\, dZ/Z\, d^2\mathbf{p}_T \qquad (dZ/Z\, d^2\mathbf{p}_T = d^3\mathbf{p}/p^0)$$

$Z = p^+/k^+$, $\quad p^\pm = p^0 \pm p^z$

Z different from $z = p^+/k_A^+$ $\qquad z_r = Z_r\,(1- Z_{r-1})(1- Z_{r-2}) \dots (1- Z_1)$

How many arguments in the splitting function ?

$F(Z)\, dZ/Z$ [Niedermayer1974]

$F(Z, \mathbf{k'}_T)\, dZ/Z\, d^2\mathbf{p}_T$ [Feynman-Field 1978]

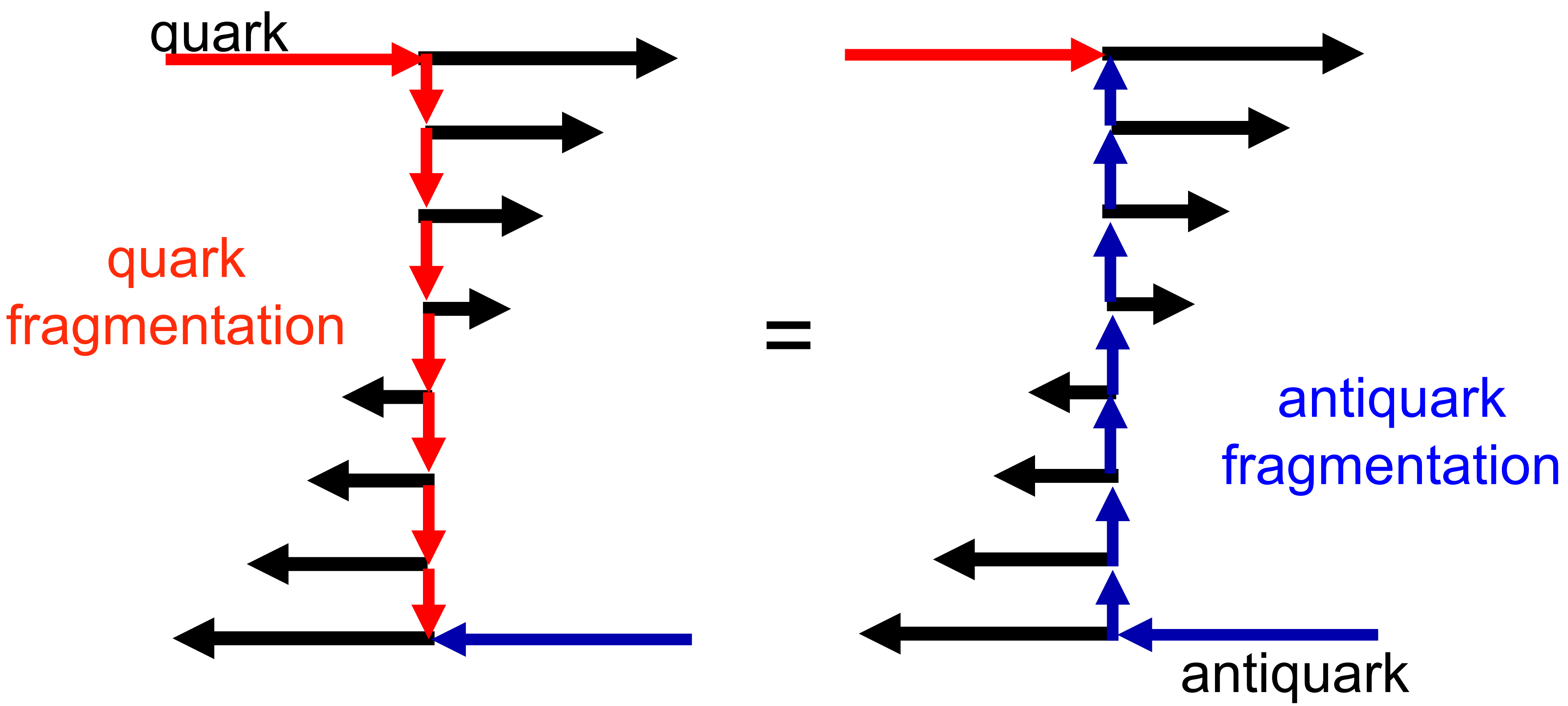
$F(Z, \mathbf{p}_T, \mathbf{k'}_T)\, dZ/Z\, d^2\mathbf{p}_T$ [Lund group]

More generally,

$F(Z, \mathbf{p}_T, \mathbf{k'}_T, k^2)\, dZ/Z\, d^2\mathbf{p}_T$?

The **LR symmetry**

Symmetry under *quark **L**ine **R**eversal* (or *"Left-Right"* symmetry)



→ Lund-***symmetric*** model

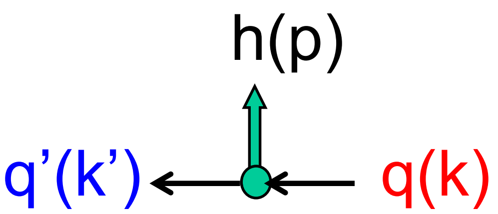
How to build a model satisfying LR symmetry

The recursive momenta k_2, k_3, etc. build up a *single-quark* distribution in momentum space,

$$U_q(k)\, d^4k$$

and a *double distribution* for pairs of consecutive quarks (vertices):

$$W_{q'hq}(k',k)\, \delta(p^2 - m_h^2)\, d^4k\, d^4k'$$



We choose a function $W_{q'hq}(k',k)$ as *input* of the model,

with $W_{q'hq}(k',k) = W_{q\underline{h}q'}(k,k')$ for LR symmetry.

Then, we calculate $U_q(k) = \int d^4k\; W_{q'hq}(k',k)\, \delta(p^2 - m_h^2)$

The splitting function is $F_{q'hq}(p,k) = W_{q'hq}(k',k) / U_q(k)$

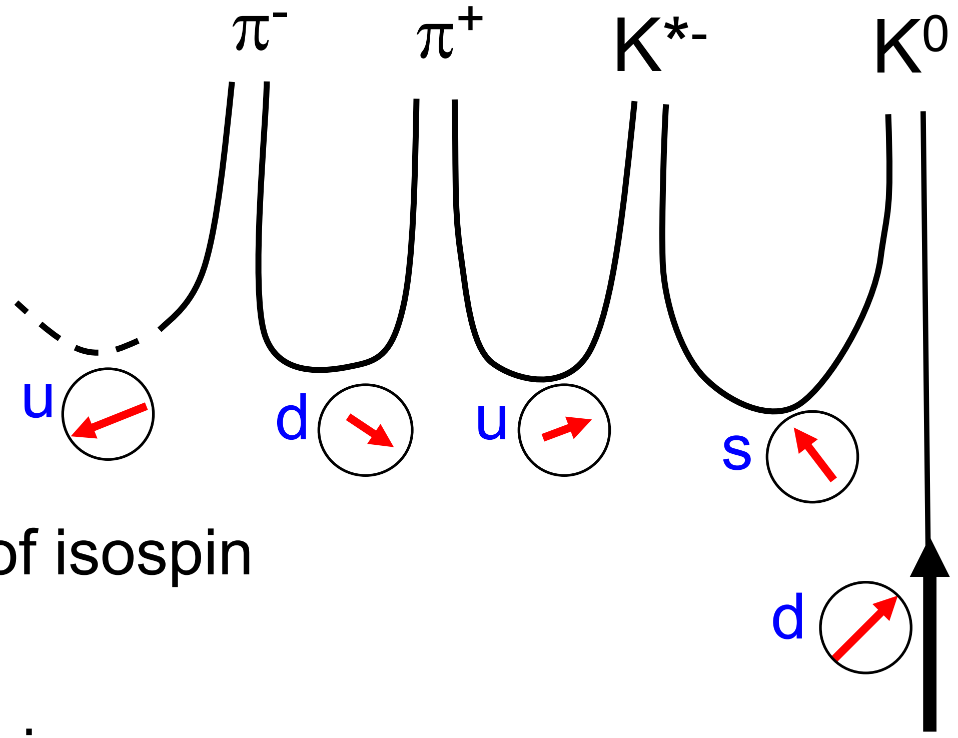
Inclusion of quark spin

Spin needs a **quantum-mechanical** model.

We encode the quark spin degree of fredom
in a 2×2 density matrix : $\rho(q) = ½ (1 + \sigma.\mathbf{S}_q)$

Why not 4×4, since quarks are relativistic
and off-mass-shell ?
Partial answer : for economy !...

Difference between **flavor** and **spin** degrees of freedom



Flavor : only 3rd component of isospin

Spin : *three* components of spin

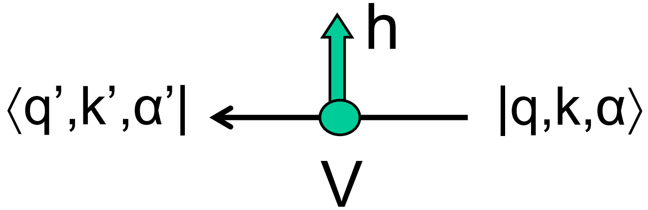
General method :

1) we replace the vertex density $W_{q'hq}(k',k)$

by a vertex ***amplitude*** $V_{q'hq}(k',k)$,

which is a 2×2 spin matrix

$$\langle \alpha' | V_{q'hq}(k',k) | \alpha \rangle$$

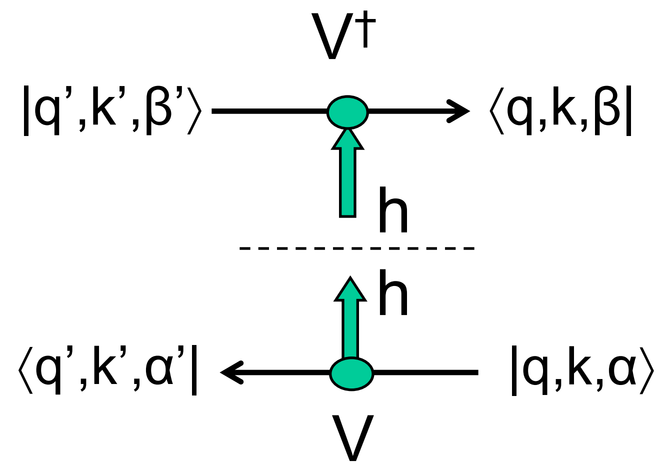


(α = helicity, for instance)

2) we build the ***double-density matrix***

$$\langle \alpha' | \langle \beta | W_{q'hq}(k',k) | \alpha \rangle | \beta' \rangle$$

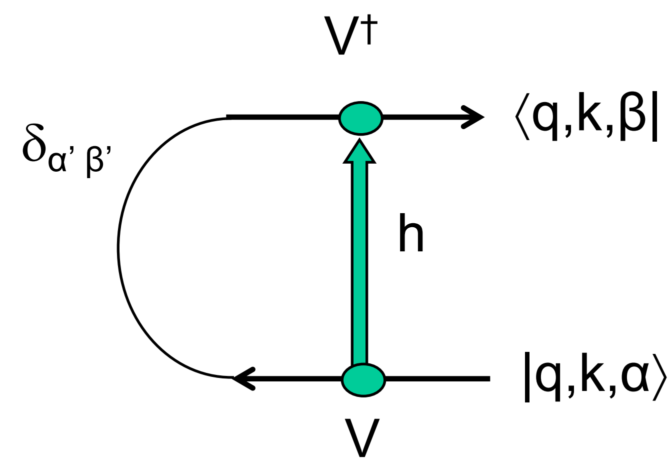
$$= \langle \alpha' | V | \alpha \rangle \quad \langle \beta | V^\dagger | \beta' \rangle$$



3) integrating over k’ and summing over the polarization of q’ gives the ***single-quark density matrix***

$$\langle \beta | U_q(k) | \alpha \rangle =$$

$$\Sigma_h \int d^4k' \, \delta(p^2 - m_h^2) \;\; \Sigma_{s'} \langle \alpha' | \langle \beta | V^\dagger V | \alpha \rangle | \alpha' \rangle$$



4) the spinless splitting function F = W / U is replaced by the ***splitting matrix***

$$T_{q'hq}(k',k) = V_{q'hq}(k',k) \;\; U_q^{-1/2}(k)$$

5) use of the ***splitting matrix*** $T_{q'hq}(k',k)$ in a recursive Monte-Carlo simulation

- At the begining, we know the density matrix $\rho_1 = ½\,(1+\mathbf{S}_1.\boldsymbol{\sigma})$ of the initial quark $q_1 = q_A$.
- We perform the 1rst splitting $q_1 \to h_1 + q_2$ in two steps:

Step 1 : draw the momentum $\mathbf{p}_1$ and the hadron species h_1

following the distribution

$$dZ_1/Z_1\; d^2\mathbf{p}_{1T}\;\; \mathrm{Trace}\{T\, \rho_1\, T^\dagger\} \qquad\qquad T = T_{q2,\, h1,\, q1}(k_2,k_1)$$

Step 2 : calculate the polarization of the new quark

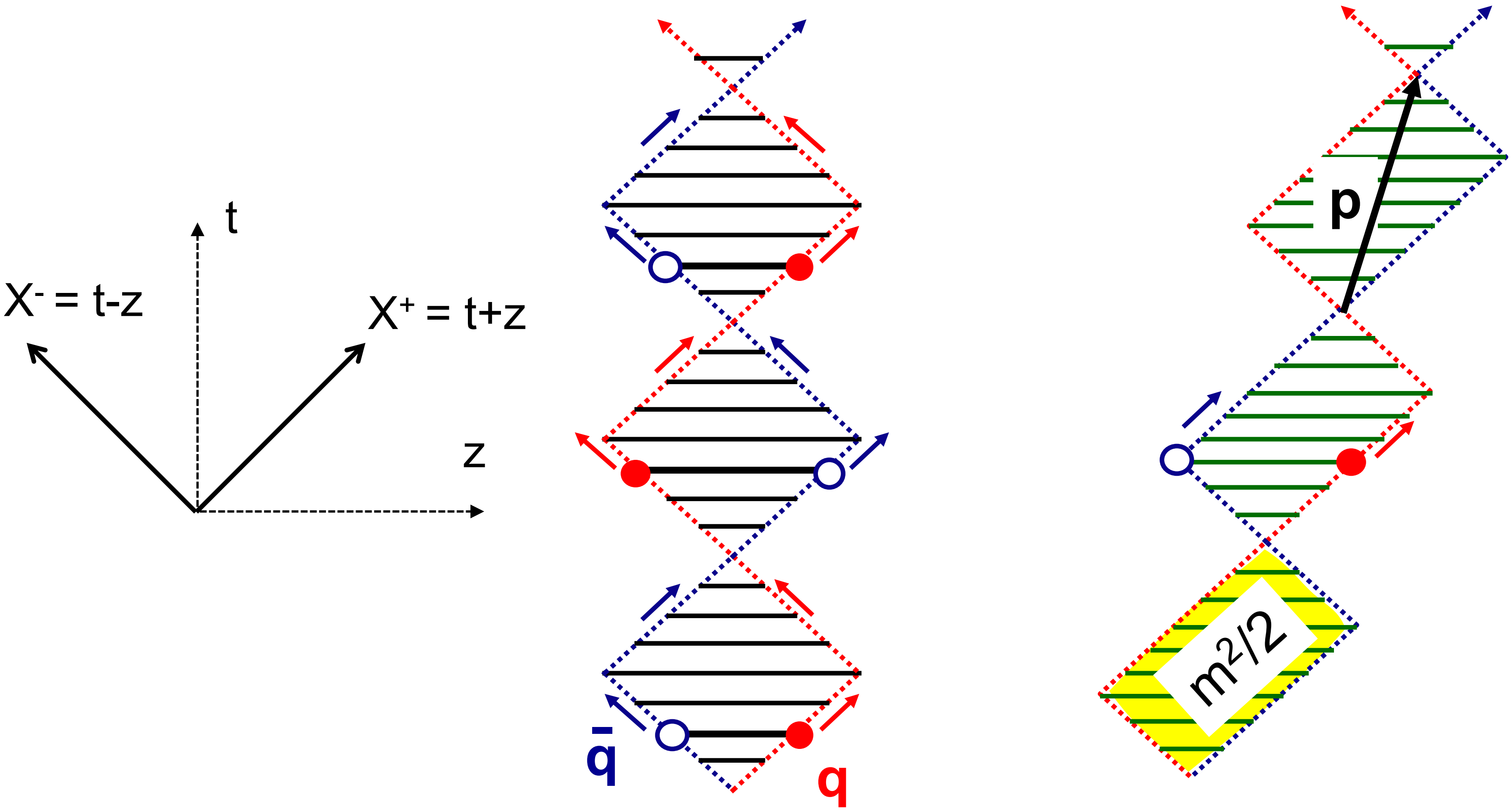
$$\rho_2 = T\, \rho_1\, T^\dagger \,/\, \mathrm{Trace}\{T\, \rho_1\, T^\dagger\}$$

- We iterate steps 1 and 2

Input from the
String **F**ragmentation **M**odel

1) Recall on the SFM without spin

Simplest string motion : the relativistic yo-yo

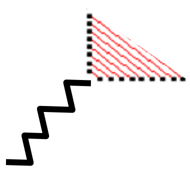


yo-yo "at rest"

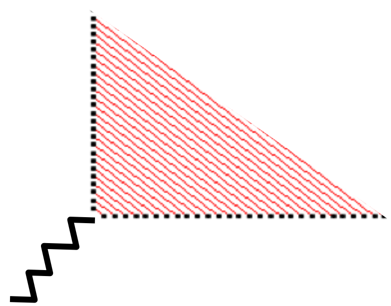
yo-yo in motion

(string tension $\kappa \approx 1$ GeV/fermi taken as unity)

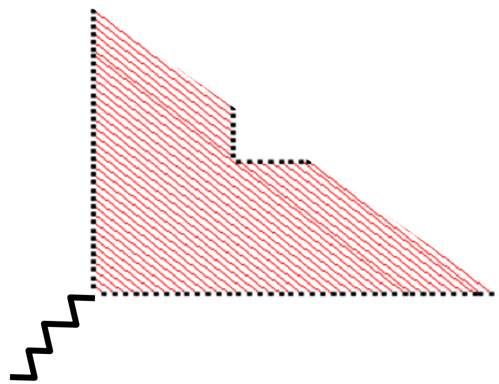
Decay of a massive yoyo



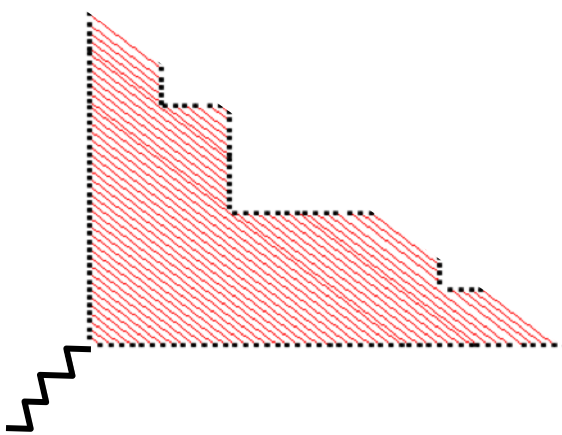
Decay of a massive yoyo



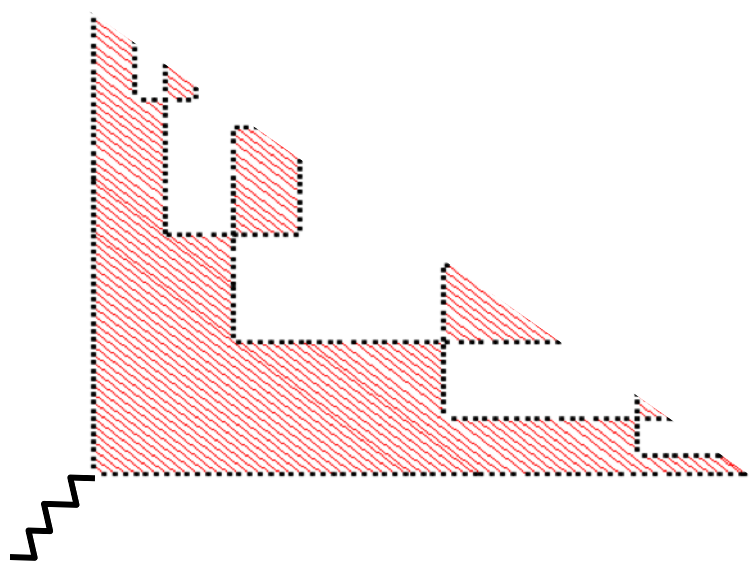
Decay of a massive yoyo



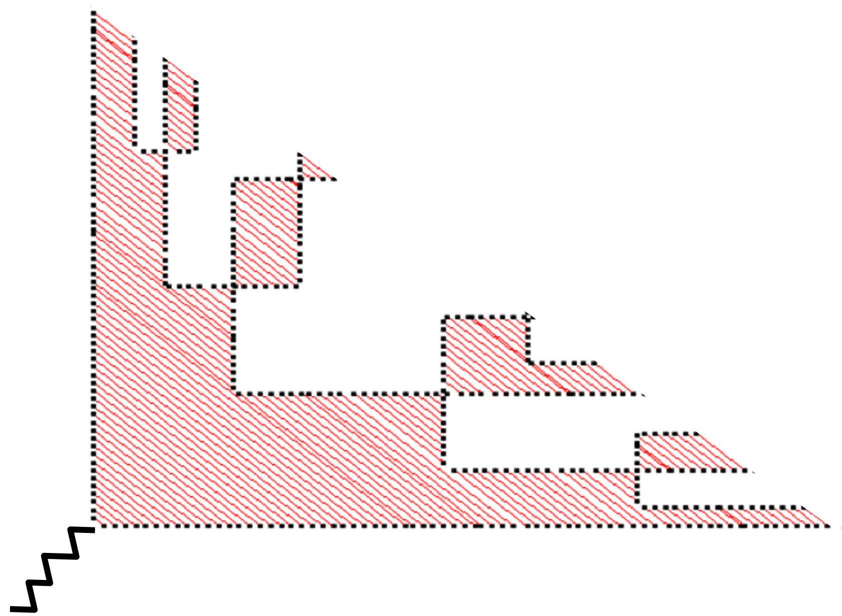
Decay of a massive yoyo



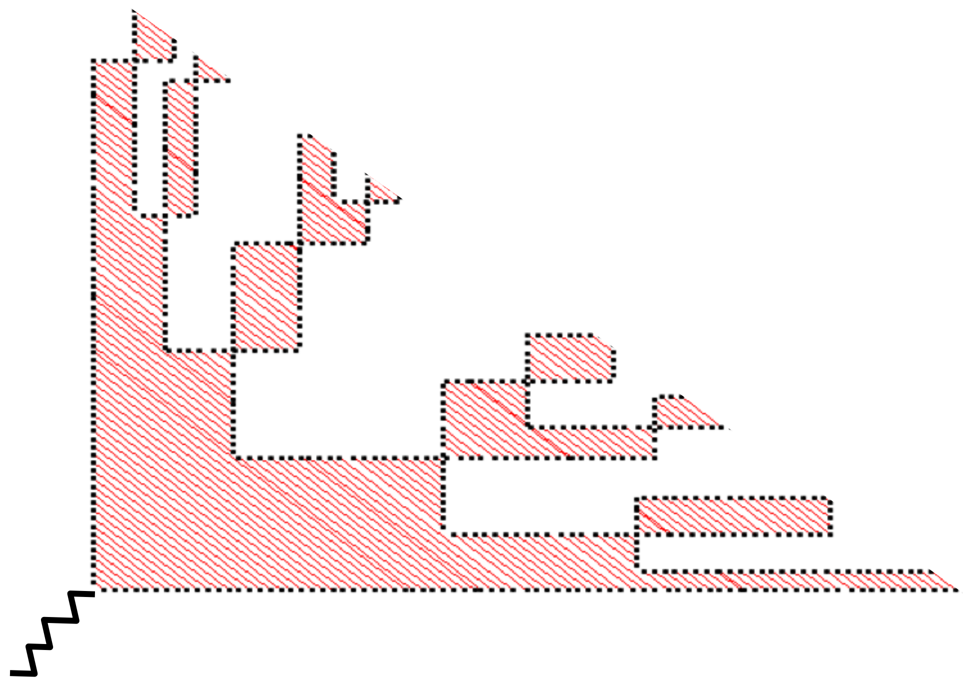
Decay of a massive yoyo



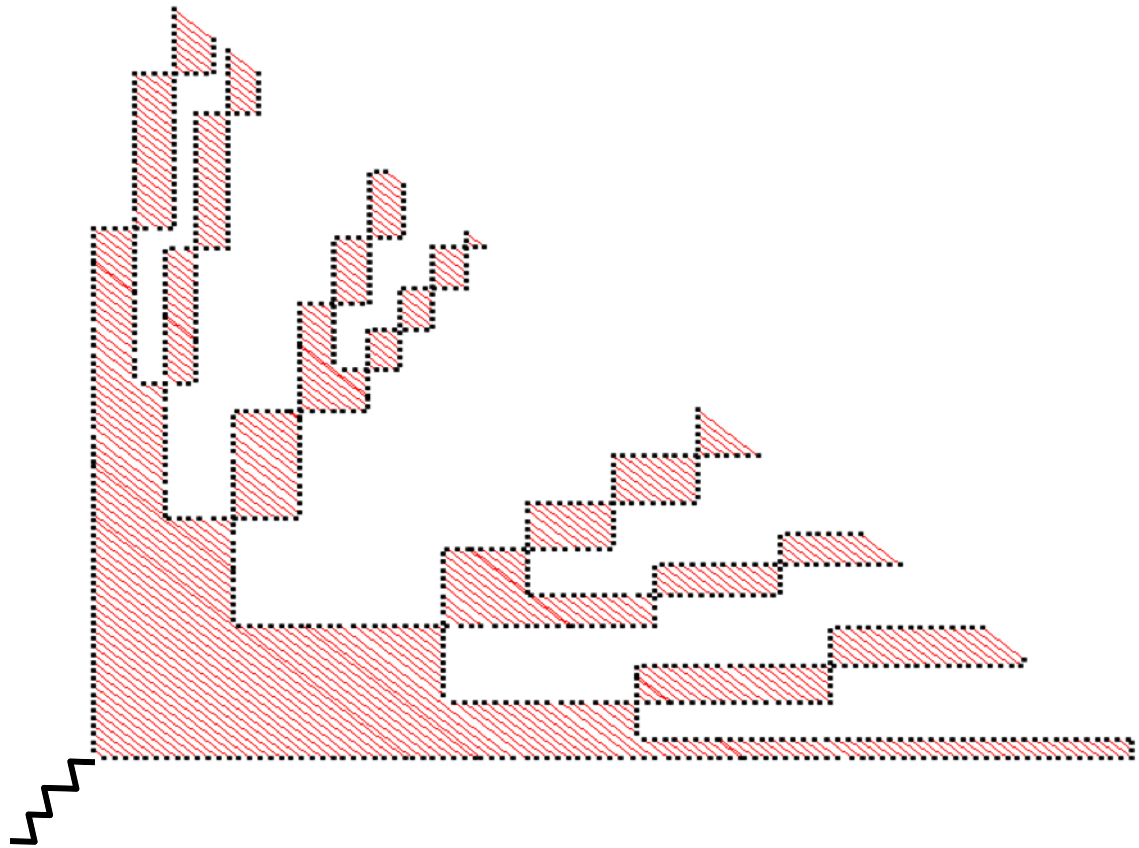
Decay of a massive yoyo



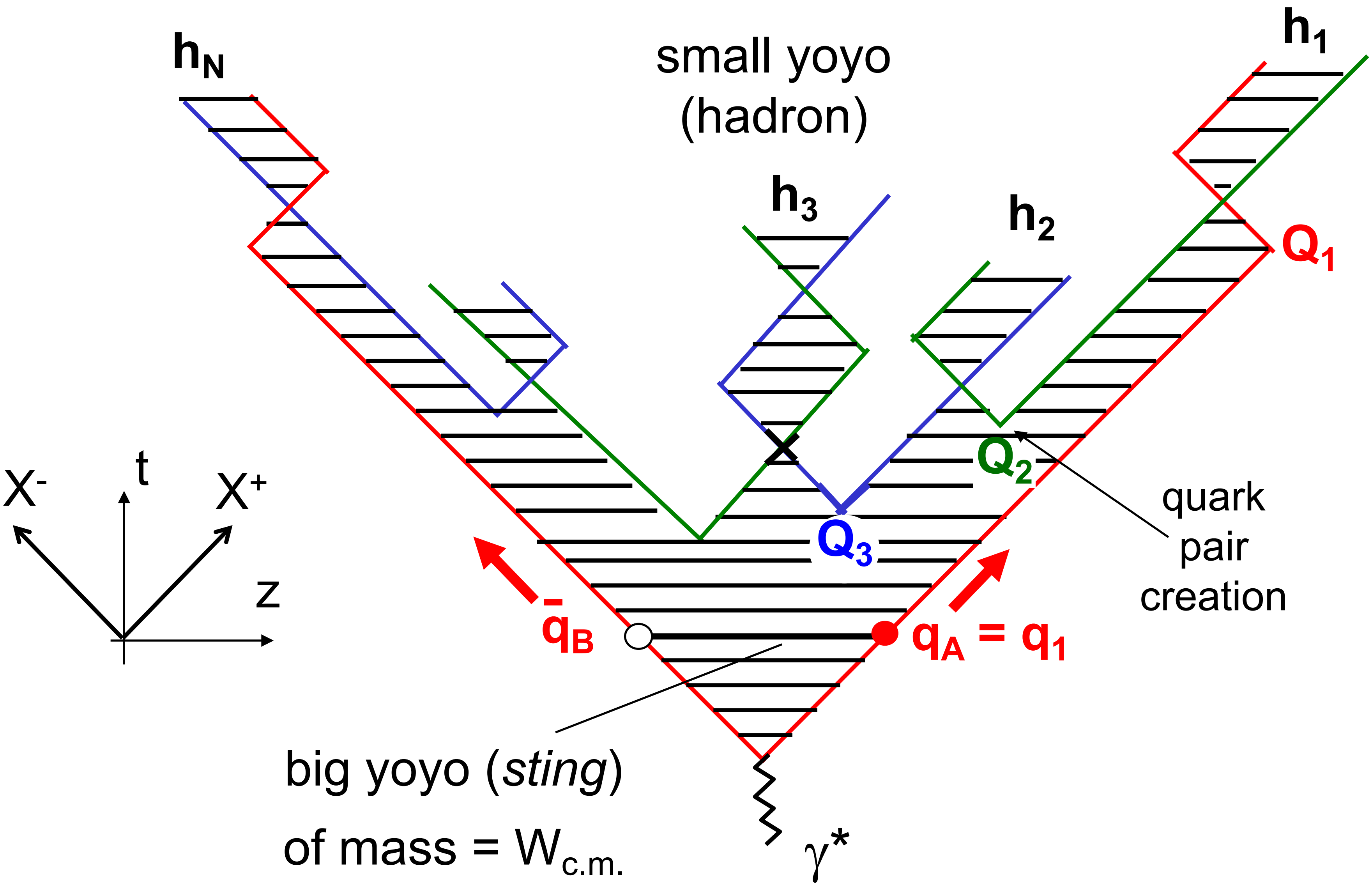
Decay of a massive yoyo



Decay of a massive yoyo



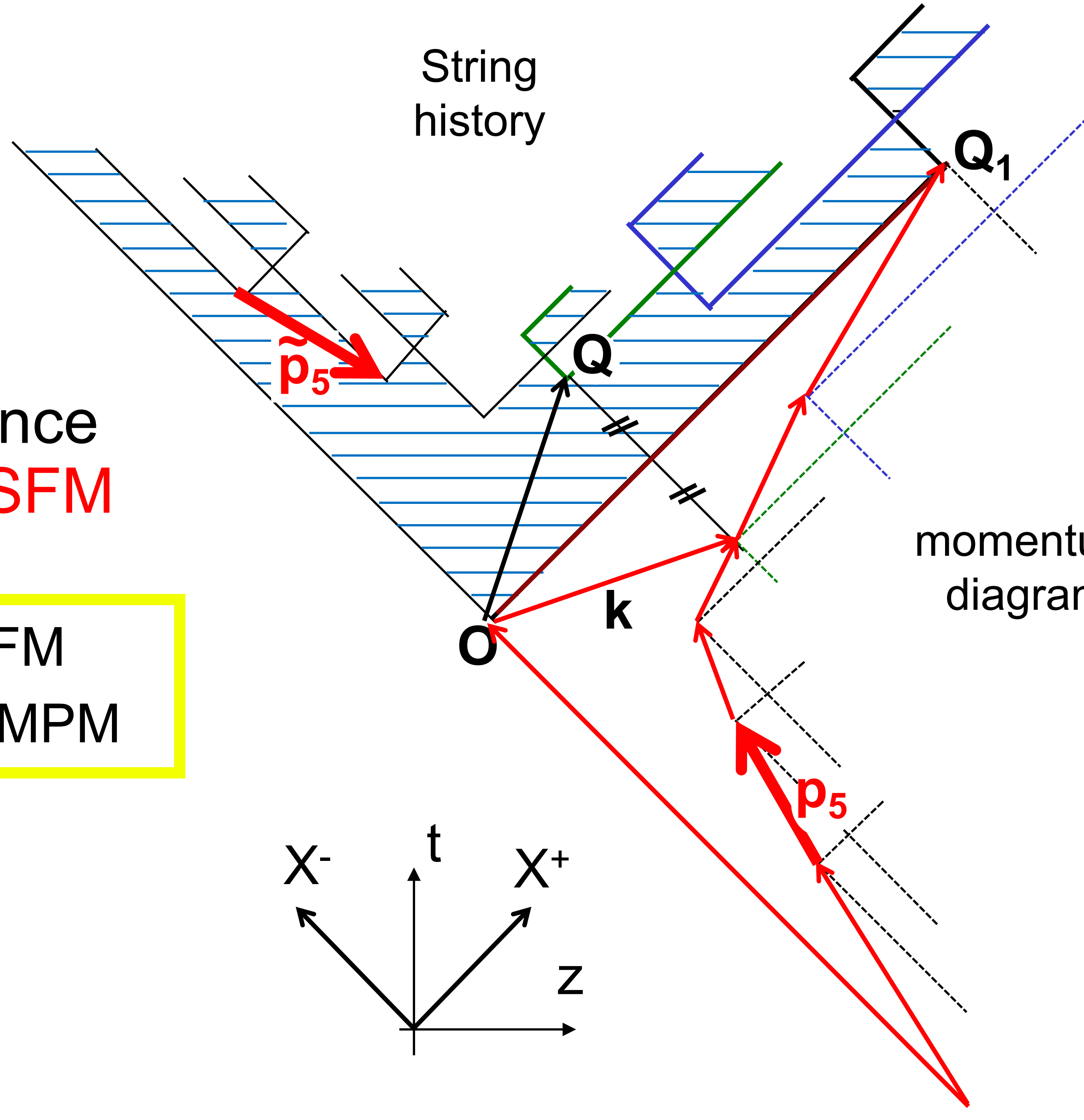
Space-time history of string fragmentation



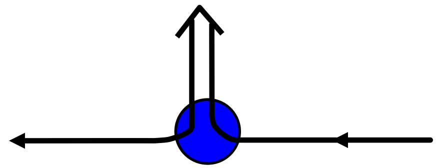
Kinematical correspondance
QMPM ⟷ SFM

$(\mathbf{OQ})^2$ in SFM

$= |k^+k^-|$ in QMPM



The Lund Symmetric Splitting Function



$$F(q \to h + q') \propto \exp(-b_T \mathbf{k}'^2_T)$$
$$\times \exp(-b_L \varepsilon^2/Z) \ [(1-Z)/\varepsilon^2]^a$$
$$\times g^2(\varepsilon^2)$$

$\varepsilon^2 = m_h^2 + \boldsymbol{p}_T^2$: tranverse energy square

b_L : string fragility

In PYTHIA: $g^2(\varepsilon^2) = 1 / N(\varepsilon^2)$, with $N(\varepsilon^2)$ = integral of the 2nd line

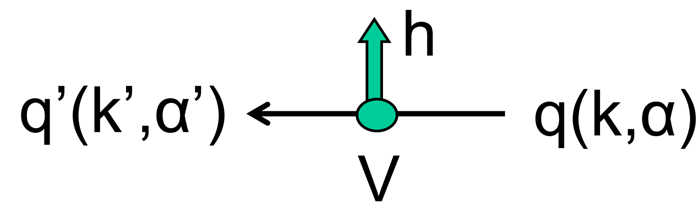
$\rightarrow$ no ($\mathbf{k}_T$, $\mathbf{k}'_T$) correlation

We choose another one [*] : $g(\varepsilon^2) = \varepsilon^a$ $\rightarrow$ ($\mathbf{k}_T$, $\mathbf{k}'_T$) correlation

[*] X.A., Z.Belghobsi, E. Redouane-Salah, Phys. Rev. 94 (2016)]

2) SFM with quark spin

For the vertex ***amplitude*** we take



$$V_{q'hq}(k',k) = [W_{q'hq}(k',k)_{spinless}]^{1/2} \times \gamma_{q'hq}(\mathbf{k}'_T,\mathbf{k}_T)$$

$\gamma = 2 \times 2$ spin matrix. For instance [*] $\gamma = \mu\sigma_z + \boldsymbol{\sigma}.\mathbf{p}_T$

Complex $\mu \rightarrow$ Collins effect

The double quark density giving the Lund Symmetric Splitting Function is

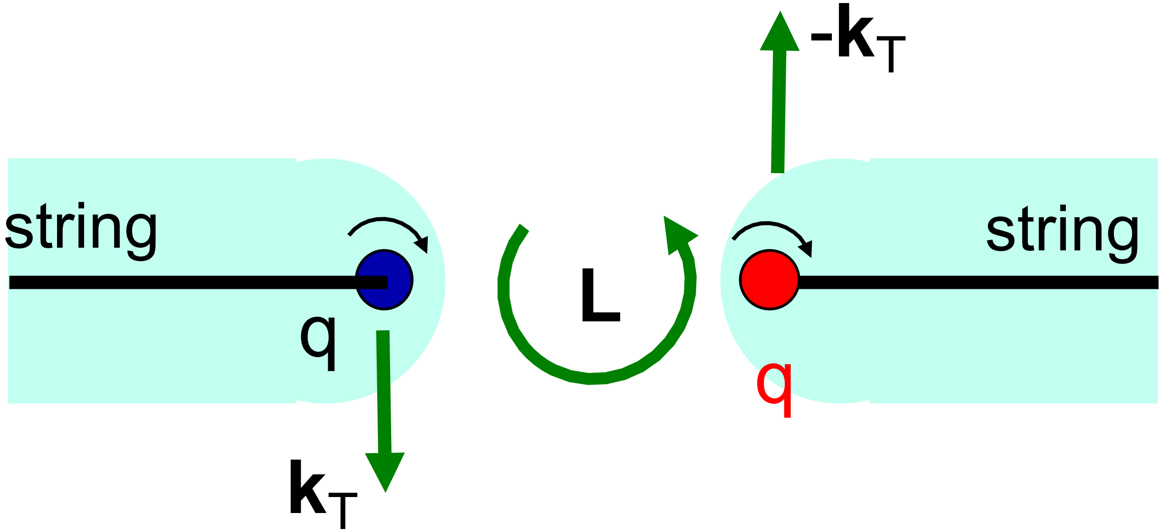
$$W_{q'hq}(k',k)_{spinless} \propto g^2(\varepsilon^2)\ \exp(-b_T \mathbf{k}'^2_T)\ \exp(-b_T \mathbf{k}_T^2)$$
$$\times\ (k'^+/p^+)^a\ |k^-/p^-|^a\ \ \exp(-b_L\,|k'^- k^+|)$$

[*] A. Kerbizi et al, proceedings of SPIN-16

3) input from the $^3P_0$ mechanism [*]

[*] [Orsay group for hadron decay, Lund group for hyperon polarization]

<u>Semi-classical model</u> : the (q-qbar) pair is created with the **vacuum quantum numbers**, therefore in the $\mathbf{0^{++}} = \mathbf{^3P_0}$ state

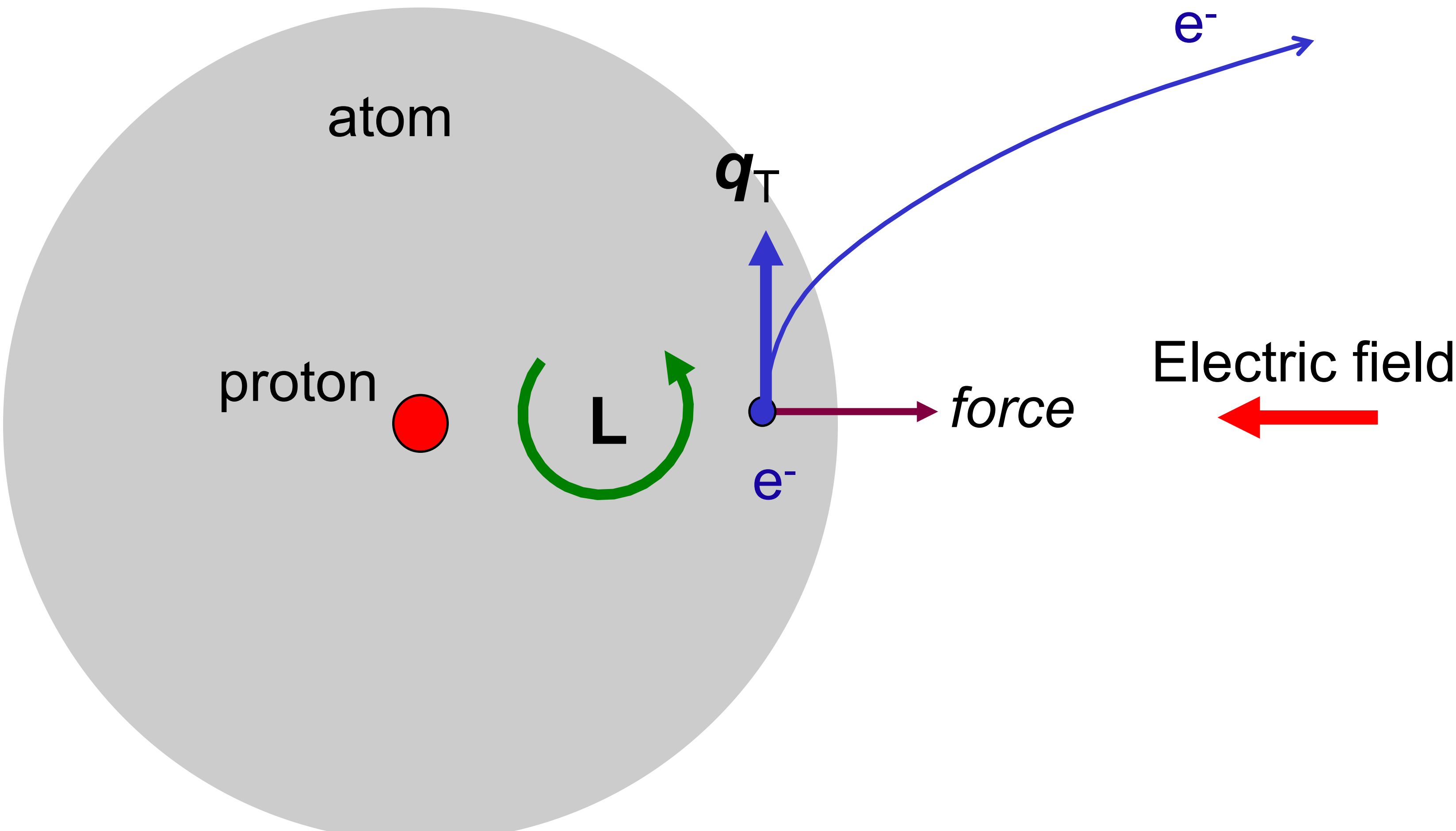


→ azimutal correlation between $\mathbf{k}_T$ and $\mathbf{S}_q$

Analogue in atomic physics :

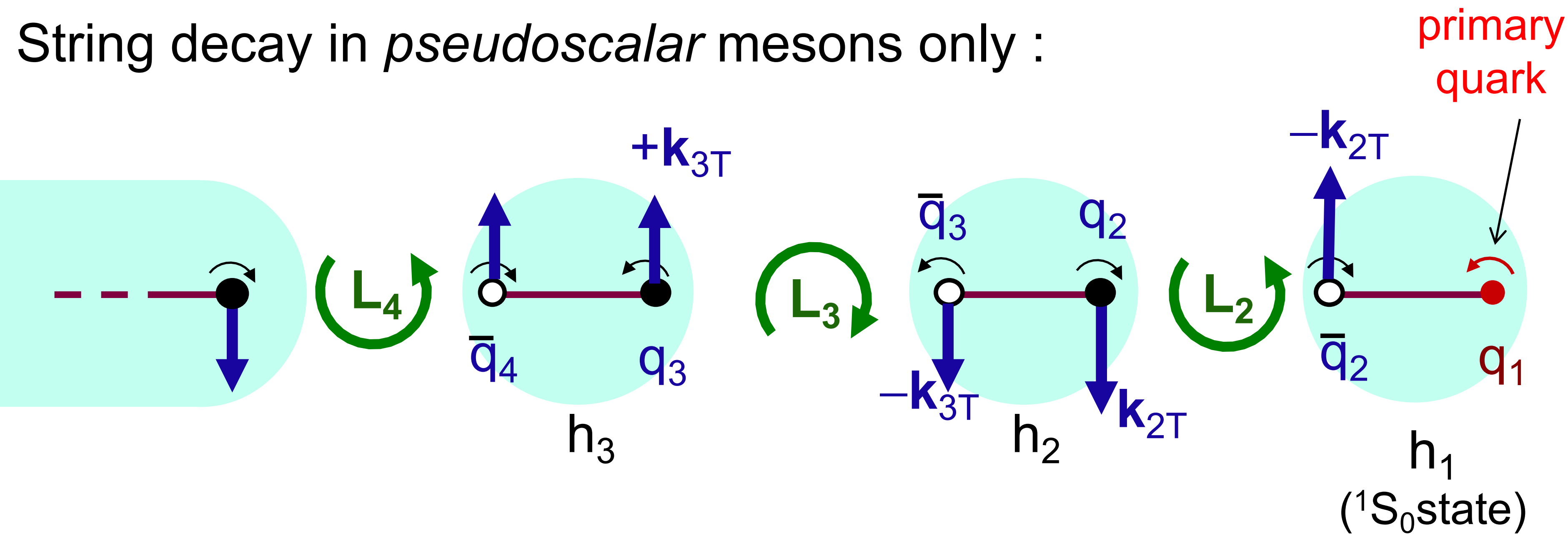
Strong field ionisation of a hydrogen (by tunnel effect)

[X.A. and E. Redouane-Salah, Phys. Rev. A **93** (2016)]



Iteration of the *string* + $^3P_0$ mechanism

String decay in *pseudoscalar* mesons only :



→ Collins asymmetry of alternate signs

→ large Collins asymmetry for the “unfavored” fragmentation

→ "hidden spin" effect: $\langle \mathbf{p}_T^2 \rangle_{\rho\ \mathbf{meson}} < 2 \langle \mathbf{k}_T^2 \rangle_{\mathbf{quark}} < \langle \mathbf{p}_T^2 \rangle_{\mathbf{pion}}$

Quantum model of the ${}^3P_0$ mechanism

For the spin-dependent part γ of $V_{q'hq}(k',k)$, take

$$\gamma_{q'hq}(\mathbf{k'}_T,\mathbf{k}_T) = (\mu + \sigma_z\, \boldsymbol{\sigma}.\mathbf{k'}_T) \times \Gamma_h \times (\mu + \sigma_z\, \boldsymbol{\sigma}.\mathbf{k}_T)$$

$\Gamma_h = \sigma_z$ for a pseudoscalar meson h (analogue of γ_5)

The factor $(\mu + \sigma_z\, \boldsymbol{\sigma}.\mathbf{k}_T)$, with Im(μ) >0, reproduces the semi-classical ${}^3P_0$ azimutal correlation between $\mathbf{k}_T$ and $\mathbf{S}_q$ [*]

Nonrelativistic argument : $\sigma_z\, (\mu + \sigma_z\, \boldsymbol{\sigma}.\mathbf{k}_T) = \boldsymbol{\sigma}.\mathbf{k}$ if k_z 🎭 μ

↑

${}^3P_0$ operator

[*] X.A., DSPIN-09

Synthesis of string and 3P0 inputs (pseudoscalar mesons only)

$T_{q'hq}(k',k) = C_{q'hq}$ ← Flavor

$\times \exp(-\tfrac{1}{2} b_T \mathbf{k}'^2_T)$ ← transverse momentum cutoff

$\times (1-Z)^{a/2} \exp(-b_L \varepsilon^2/2Z)$ ← string model

$\times (\mu + \sigma_z \boldsymbol{\sigma}.\mathbf{k}'_T)$ ← $^3P_0$

$\times \sigma_z$ ← Pseudoscalar

$\times \hat{u}_q^{-1/2}(\mathbf{k}_T)$ ← reduced single quark density matrix

$$\hat{u}_q(\mathbf{k}_T) = \Sigma_h \int dZ/Z \; d^2\mathbf{p}_T \; \mathbf{t}^\dagger \; \mathbf{t}$$

$$= [\hat{u}_0(\mathbf{k}_T^2) + \hat{u}_1(\mathbf{k}_T^2)\, \boldsymbol{\sigma} \cdot (\mathbf{z} \times \mathbf{k}_T) / |\mathbf{k}_T| \,]$$

($\mathbf{t}$ = T without the last line)

Simulations with (k_T, k'_T) correlations and comparison with Compass proton data

- Fragmentation (only) of transversely polarized u quarks into pseudoscalar mesons
- (k_T, k'_T) correlations (contrary to Pythia)
- No primordial transverse momentum
- We have used a sample of x_B, Q^2 values from Compass real events
- We require $z > 0.1$ and $p_T > 0.1\ GeV/c$
- Parameters used in these simulations (fixed by comparison with data)

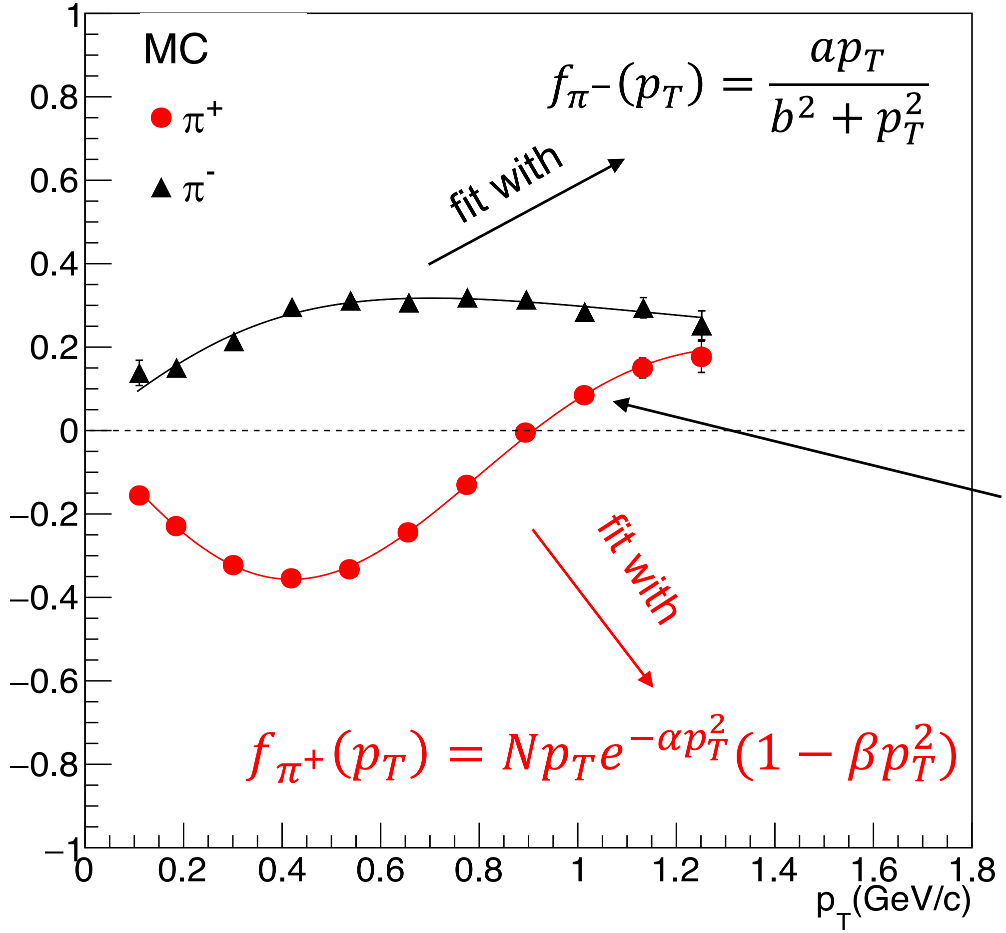
$$b_L = 0.5 GeV^{-2}, \qquad b_T = 5.17\ GeV^{-2}$$
$$a = 0.9, \qquad \mu = (0.42, 0.76)\ GeV$$

Preliminary version: [A. Kerbizi et al. Proceedings of Spin-16, **arXiv:1701.08543**]

Collins analysing power as function of p_T from Monte Carlo

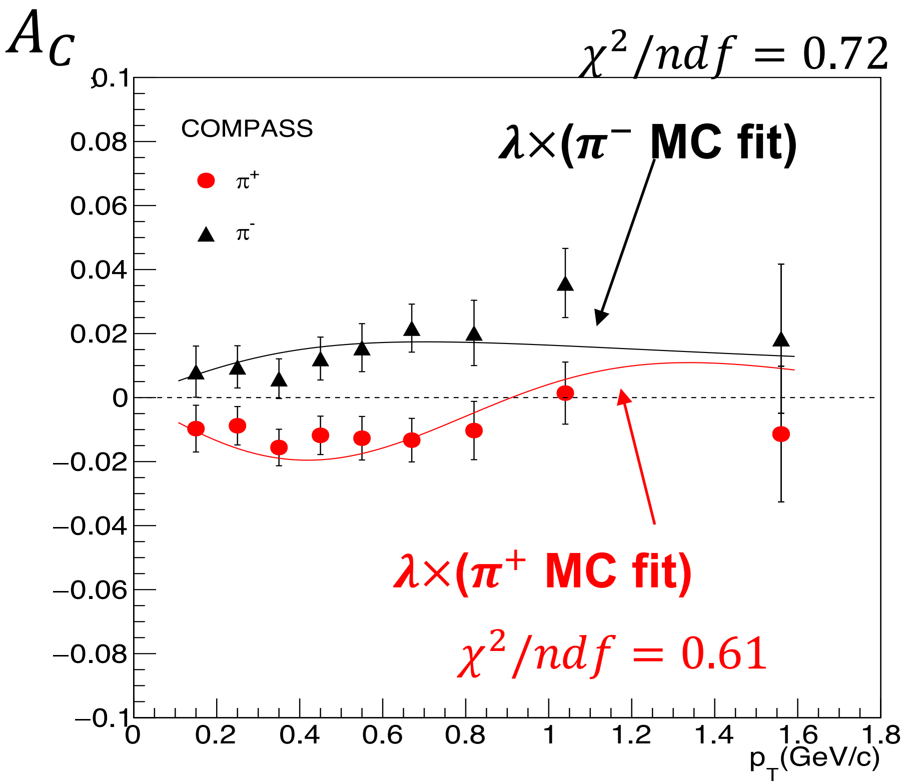
$2\langle\sin\phi_C\rangle$

$\phi_C = \phi_h - \phi_{s_q}$



Change of sign due to a second rank π^+ produced after a first rank π^0

Comparison with COMPASS Collins asymmetry
1) as function of p_T



Points -> Compass

$$\boldsymbol{\lambda = 0.055 \pm 0.010}$$

- λ is a scale parameter estimated minimizing

$$\chi^2 = \sum_i \frac{\left(A_{C_i} - \lambda f_{\pi^-}(p_{T,i})\right)^2}{\sigma_i^2}$$

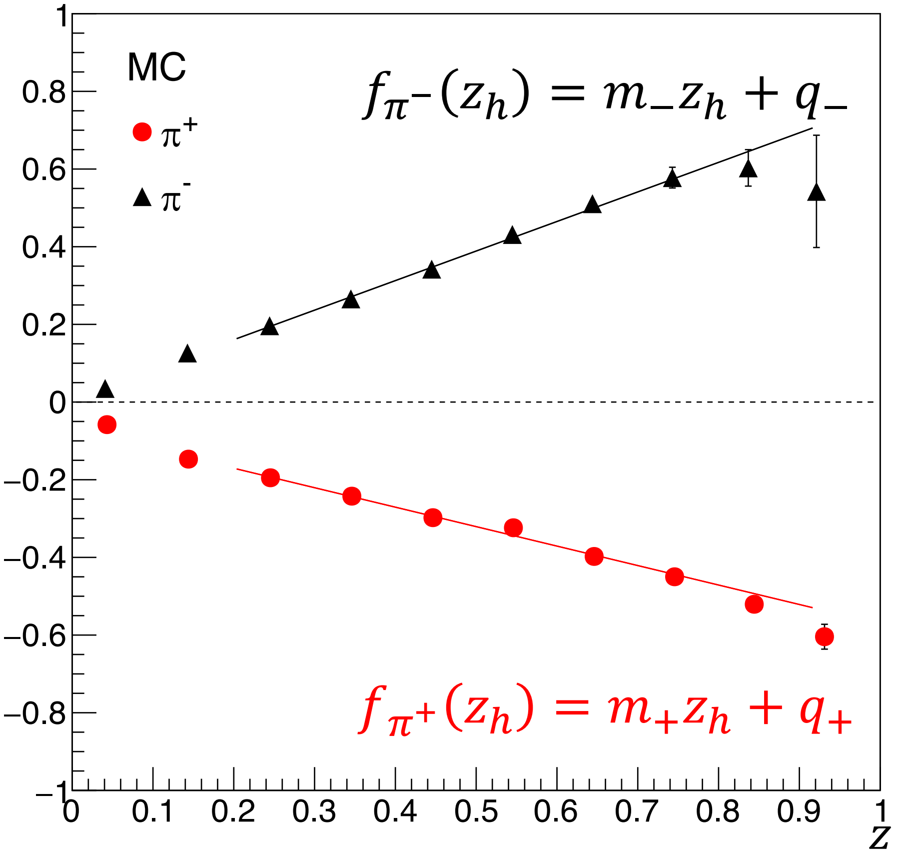
- $\lambda \sim \langle h_1^u / f_1^u \rangle$
- $f_{\pi^-}(p_T)$: preceding MC fit for π^-
- A_C: experimental value of the π^- asymmetry
- σ: statistical error

Collins analysing power as function of z from Monte Carlo

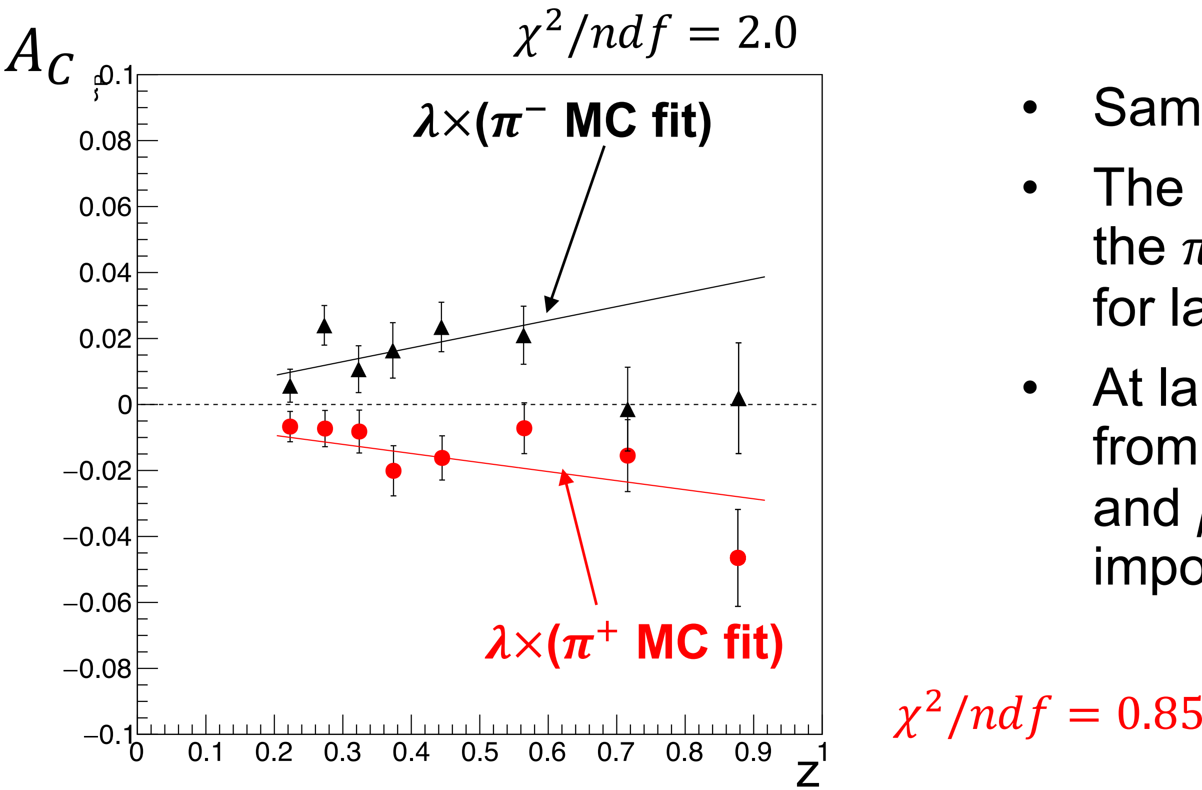
$2\langle \sin\phi_C \rangle$

$\phi_C = \phi_h - \phi_{s_q}$

$z = E_h/\nu$



Comparison with COMPASS Collins asymmetry
2) as function of z



Points -> Compass

$\boldsymbol{\lambda = 0.055}$

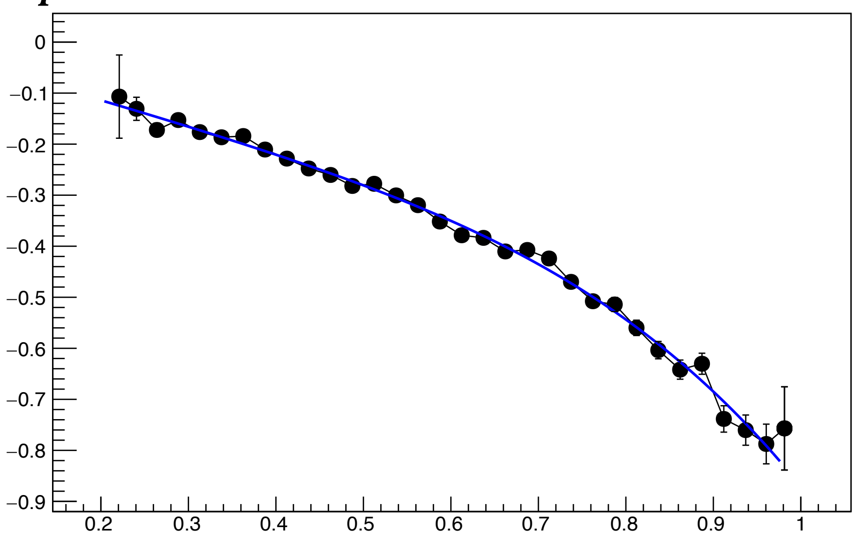
- Same λ
- The MC overestimates the π^- analysing power for large z
- At large z contributions from primary d quarks and ρ^0 decay should be important

Di-hadron asymmetry: simulation result for h^+h^- as function M_{inv} and $z = z_1 + z_2$

$$2\langle \sin(\phi_{R_T} - \phi_{s_q}) \rangle$$

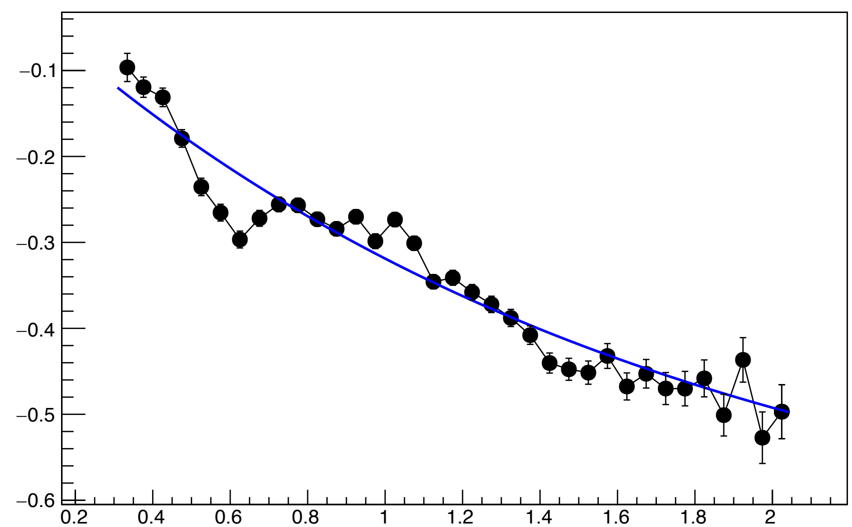
$$\boldsymbol{R}_T = \frac{z_2 \boldsymbol{p}_{1T} - z_1 \boldsymbol{p}_{2T}}{z_1 + z_2}$$

$a_P^{u \to h^+ h^-}$



z

$a_P^{u \to h^+ h^-}$

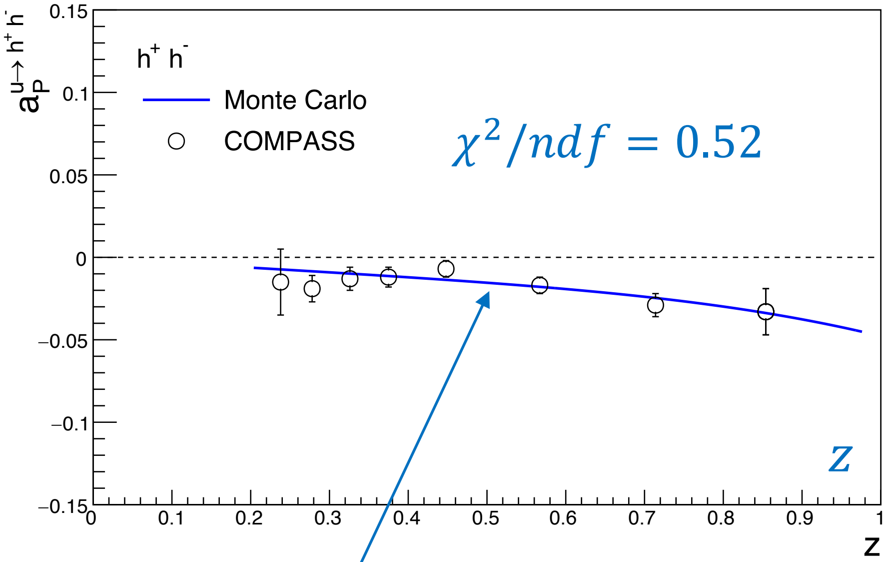


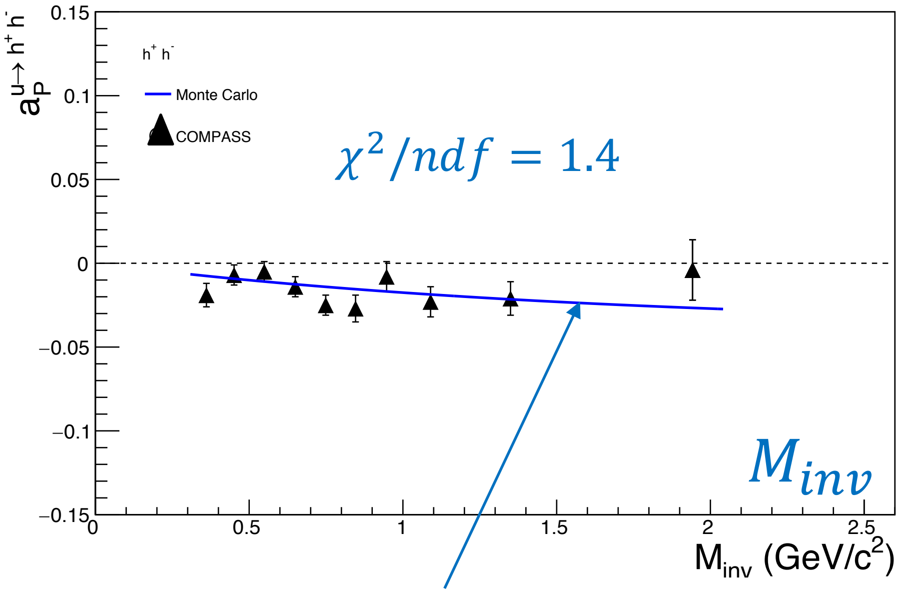
M_{inv}

$$f(z) = N_z(1 + a_1 z + a_2 z^2 + a_3 z^3 + a_4 z^4)$$

$$f(M_{inv}) = N_{M_{inv}} \left[\frac{1 - e^{-p_1 M_{inv}}}{1 + e^{-p_1 (M_{inv} - p_2)}} \right]$$

Di-hadron asymmetry: comparison with COMPASS





$\lambda\times$MC fit

$\lambda\times$MC fit

- Same λ also for 2h asymmetries

$\boldsymbol{\lambda = 0.055}$

Conclusions

- We have presented a model of jet including quark spin, satisfying LR (quark - antiquark) symmetry, consistent with confinement (by strings) and the rules of quantum mechanics.

- Simulations show that the model reproduces the main observed features of the Collins asymmetries:
    - Opposite signs for h^+ and h^-
    - Stronger asymmetry for unfavoured fragmentation
    - Rough quantitative agreement with data for single Collins and di-hadron asymmetries

- Further improvements: include spin1 resonances and baryons.

- This model can be incorporated in existing package like PYTHIA.

- We started a collaboration with Diefenthaler et al. on the JLab LDRD proposal *Phenomenological study of hadronization in nuclear and high-energy physics experiments.*

Thank you for attention !