TMDs and DPDs on the lattice

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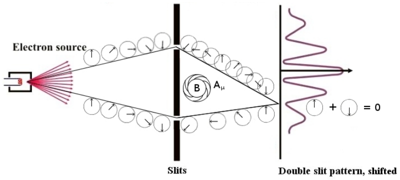
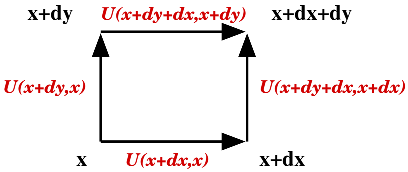
- 1706.03406: Moments of TMDs from the lattice
- DPDs, a link between EIC and RHIC+LHC physics
- Lattice results for DPDs
- Conclusion

Like General Relativity local gauge theories have non-trivial parallel transport. TMDs and DPDs are sensitive to these.

Aharanov Bohm effect

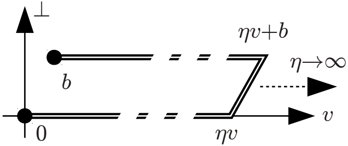


generic gauge links



TMDs are related to correlators of the type

$$\widetilde{\Phi}^{[\Gamma]}_{\text{unsubtr.}}(b, P, S, \ldots) \equiv \frac{1}{2}\langle P, S|\, \bar{q}(0)\, \Gamma\, \mathcal{U}[0, \eta v, \eta v + b, b]\, q(b)\, |P, S\rangle$$



We simulate for spatial, not light-like separations, but the limit $\hat{\zeta} \to \infty$ of

$$\hat{\zeta} := \frac{v \cdot P}{\sqrt{v^2}\sqrt{P^2}}$$

reproduces the light-cone behavior. For fixed target experiments $\zeta \sim O(1)$

We used RBC/UKQCD (domain wall) and W&M (Clover) ensembles, $N_f = 2 + 1$

ID	Clover	DWF
Fermion Type	Clover	Domain-wall
Geometry	$32^3 \times 96$	$32^3 \times 64$
$a(\mathrm{fm})$	0.11403(77)	0.0840(14)
$m_\pi(\mathrm{MeV})$	317(2)(2)	297(5)
# confs.	967	533
# meas.	23208	4264

only connected diagrams, i.e. $u - d$

$$
\begin{aligned}
\widetilde{\Phi}^{[\Gamma]}_{\text{subtr.}}(b,P,S,\ldots) &= \widetilde{\Phi}^{[\Gamma]}_{\text{unsubtr.}}(b,P,S,\ldots)\cdot\mathcal{S}\cdot Z_{\mathrm{TMD}}\cdot Z_2 \\
\Phi^{[\Gamma]}(x,\boldsymbol{k}_T,P,S,\ldots) &= \int\frac{d^2\boldsymbol{b}_\mathrm{T}}{(2\pi)^2}\int\frac{d(b\cdot P)}{2\pi P^+}e^{ix(b\cdot P)-i\boldsymbol{b}_\mathrm{T}\cdot\boldsymbol{k}_\mathrm{T}}\,\widetilde{\Phi}^{[\Gamma]}_{\text{subtr.}}\Big|_{b^+=0} \\
\Phi^{[\gamma^+]} &= f_1-\frac{\epsilon_{ij}\boldsymbol{k}_i\boldsymbol{S}_j}{m_N}f_{1T}^{\perp} \\
\Phi^{[\gamma^+\gamma^5]} &= \Lambda g_1+\frac{\boldsymbol{k}_\mathrm{T}\cdot\boldsymbol{S}_\mathrm{T}}{m_N}g_{1T} \\
\Phi^{[i\sigma^{i+}\gamma^5]} &= \boldsymbol{S}_i h_1+\frac{(2\boldsymbol{k}_i\boldsymbol{k}_j-\boldsymbol{k}_\mathrm{T}^2\delta_{ij})\boldsymbol{S}_j}{2m_N^2}h_{1T}^{\perp}+\frac{\Lambda\boldsymbol{k}_i}{m_N}h_{1L}^{\perp}+\frac{\epsilon_{ij}\boldsymbol{k}_j}{m_N}h_1^{\perp} \\
\tilde{f}^{[m](n)}(\boldsymbol{b}_\mathrm{T}^2,\ldots) &= n!\left(-\frac{2}{m_N^2}\partial_{\boldsymbol{b}_\mathrm{T}^2}\right)^n\int_{-1}^{1}dx\,x^{m-1}\int d^2\boldsymbol{k}_\mathrm{T}\,e^{i\boldsymbol{b}_\mathrm{T}\cdot\boldsymbol{k}_\mathrm{T}}f(x,\boldsymbol{k}_\mathrm{T}^2) \\
\langle\vec{k}_y\rangle_{TU}(\boldsymbol{b}_\mathrm{T}^2;\ldots) &= m_N\frac{\tilde{f}_{1T}^{\perp[1](1)}(\boldsymbol{b}_\mathrm{T}^2;\ldots)}{\tilde{f}_1^{[1](0)}(\boldsymbol{b}_\mathrm{T}^2;\ldots)}
\end{aligned}
$$

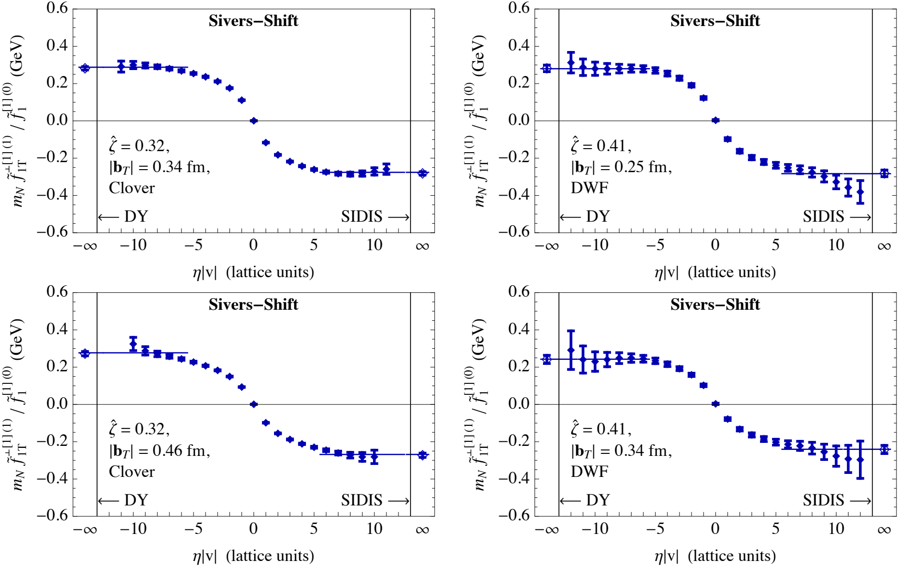
the generalized tensor charge

$$\tilde{h}_1^{[1](0)}(\boldsymbol{b}_\mathrm{T}^2;\ldots)/\tilde{f}_1^{[1](0)}(\boldsymbol{b}_\mathrm{T}^2;\ldots)$$

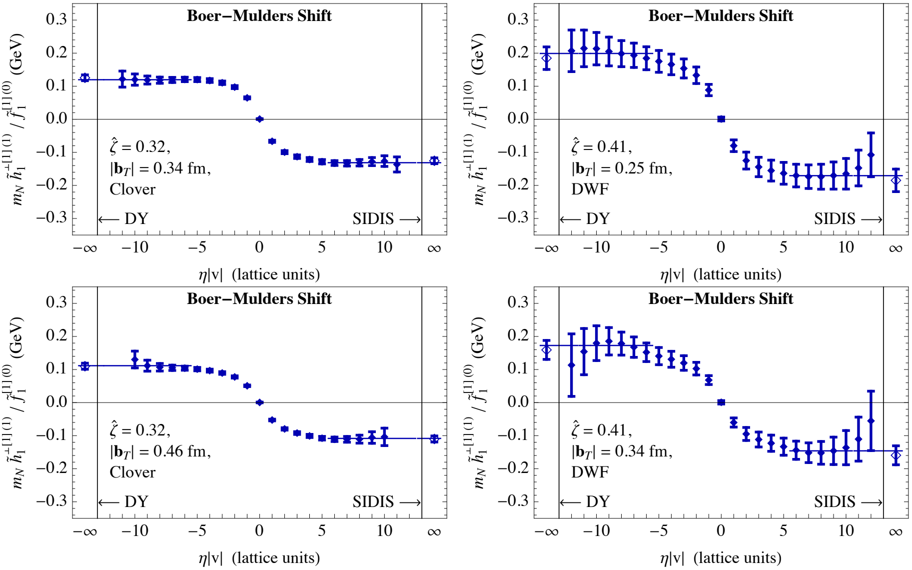
$$g_T^{u-d} = \int dx\, d^2\boldsymbol{k}_\mathrm{T}\, h_1(x,\boldsymbol{k}_\mathrm{T}^2) = \tilde{h}_1^{[1](0)}(\boldsymbol{b}_\mathrm{T}^2 = 0)$$

limits:

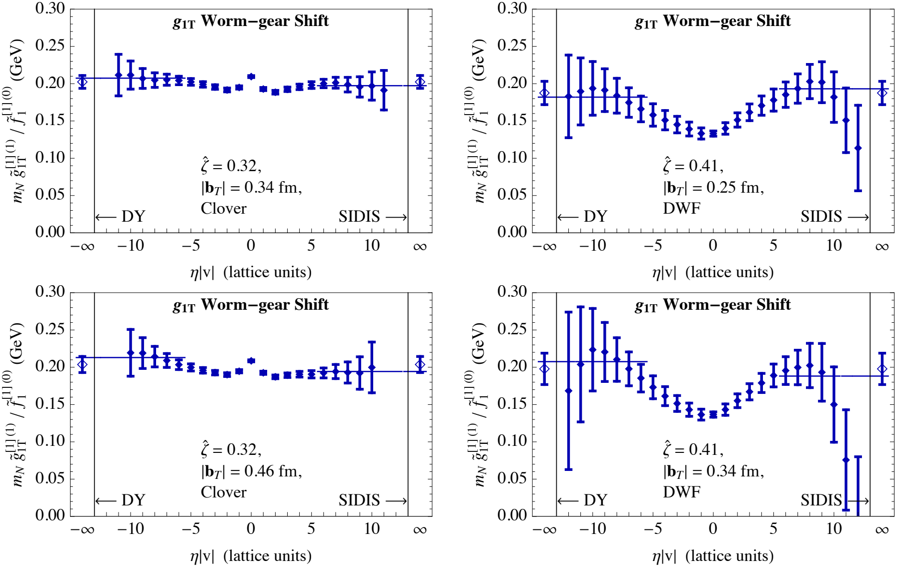
$$\begin{aligned} \eta|\boldsymbol{v}| &\to \infty \\ b_T &\gg a \\ \hat{\zeta} &\to \infty \end{aligned}$$



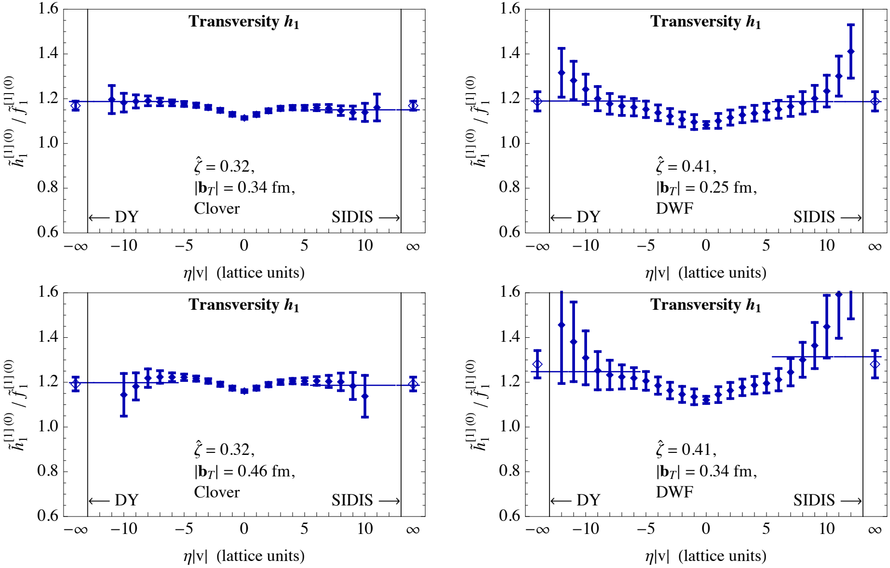
Sivers shift



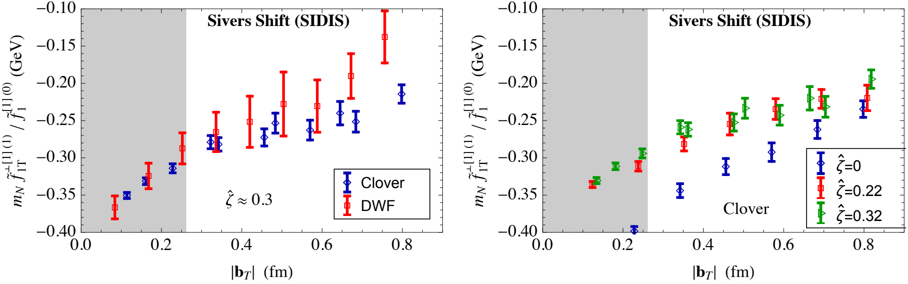
Boer-Mulders shift



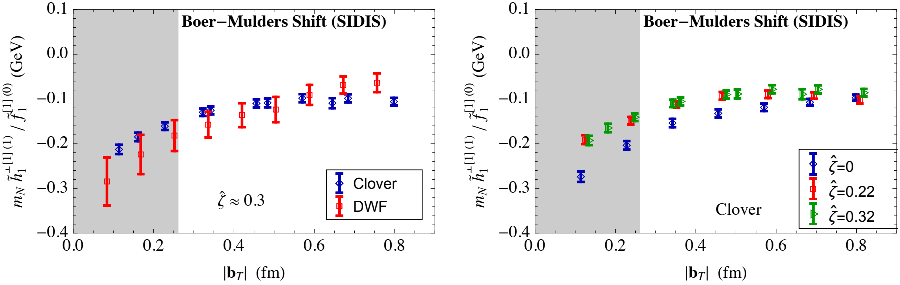
Worm-gear shift, for g_{1T}



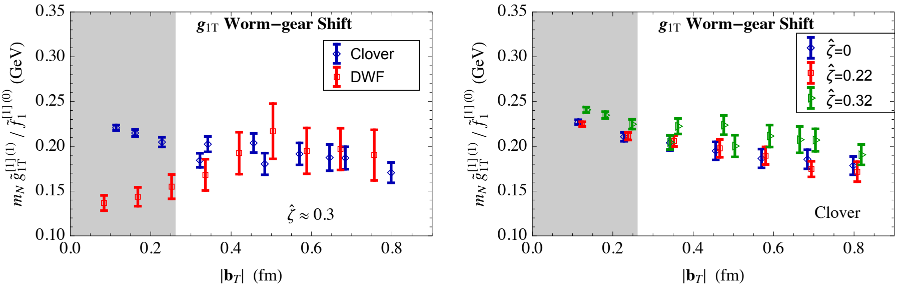
transversity ratio



Sivers shift

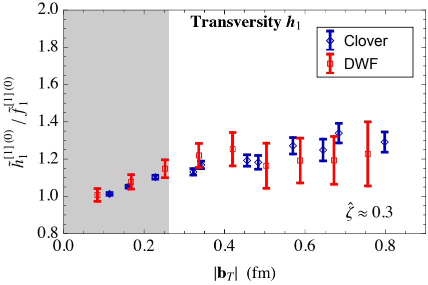


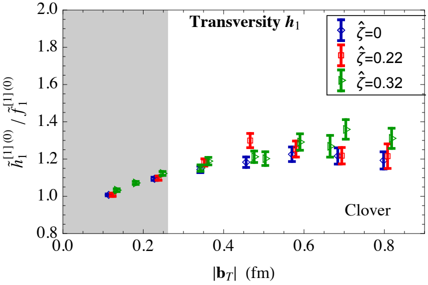
Boer-Mulders shift



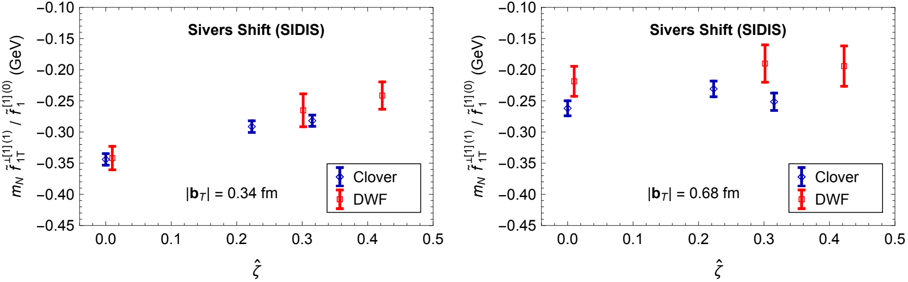
Worm-gear shift, for g_{1T}

Constantinou and Panagopoulos, arXiv:1705.11193, lattice perturbation theory $\Rightarrow$ operator mixing for Clover fermions

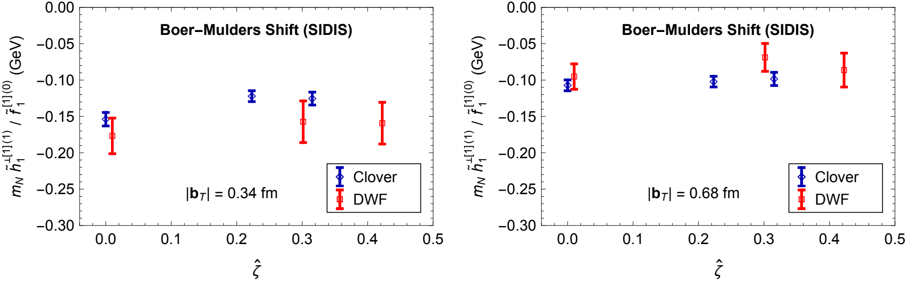




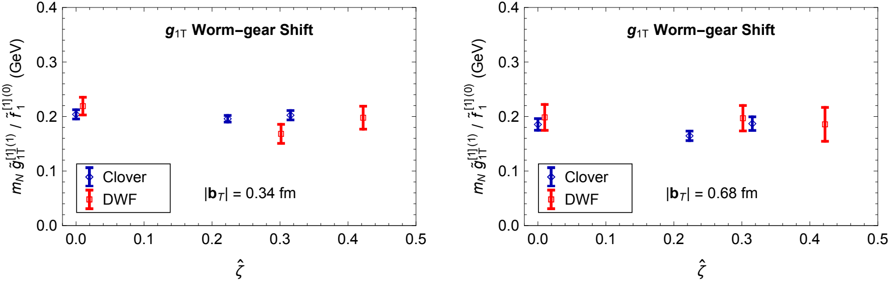
transversity ratio



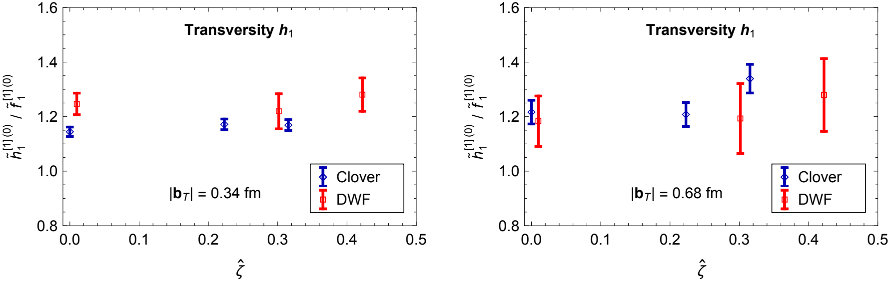
Sivers shift



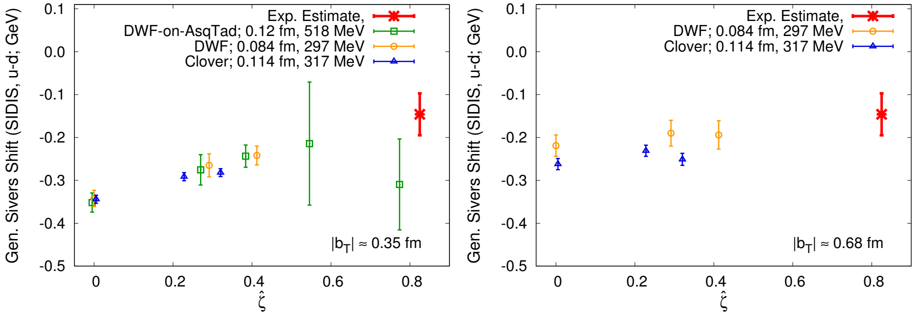
Boer-Mulders shift



Worm-gear shift, for g_{1T}



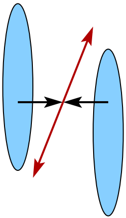
transversity ratio



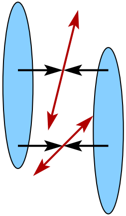
Comparison with experiment

Double Parton Distribbutions and MPIs at LHC

Full use of discovery potential requires a better of the "underlying event", example: Double hard interactions



Single parton interaction

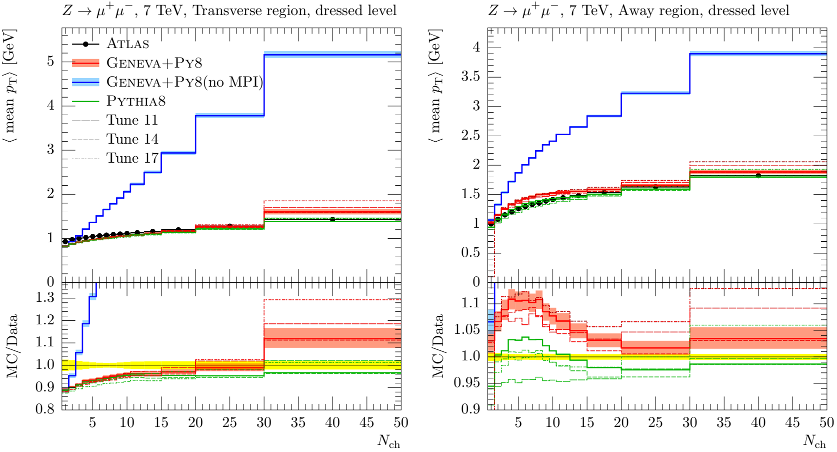


Double parton interaction

Naive (?) assumption

$$d\sigma_{DPS} = \frac{d\sigma_{SPS} d\sigma_{SPS}}{2\sigma_{eff}}$$

Two plots from Alioli, Bauer, Guns, Tackmann 1605.07192
mean charged particle p_T as function of N_{ch}.
Events with large N_{ch} have a higher MPI contribution. MPIs produce many particles with p_T of O(1 GeV).



Can the pocket formular be correct?

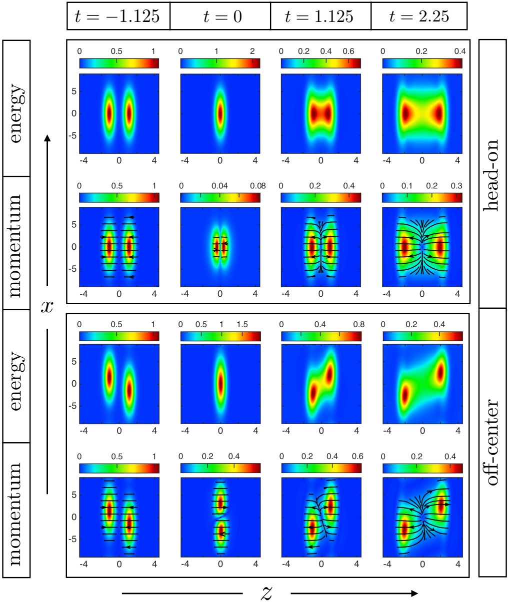
Experiment as well as AdS/CFT studies suggest that p+p is similar to A+A, e.g.

P. M. Chesler, “How big are the smallest drops of quark-gluon plasma?” JHEP **1603** (2016) 146 [arXiv:1601.01583]

Rapid entropy production ($\tau \sim 0.1\mathrm{fm/c}$) also in p+p

Non trivial MPI contributions or dynamical suppresion of quark correlations in p+p ???

TMDs and DPDs are intimately connected. E.g. A. Vladimirov arXiv:1608.04920 soft factor TMDs $\Rightarrow$ soft factor DPDs.


$t = -1.125$
$t = 0$
$t = 1.125$
$t = 2.25$
energy
momentum
energy
momentum
x
z
head-on
off-center

Quark-quark correlations in the pion; RQCD; $N_f = 2$, Clover-Wilson fermions, down to nearly physical mass ($m_\pi = 150$ MeV).

Many direct Tests of naive factorization, e.g., Integrals based on

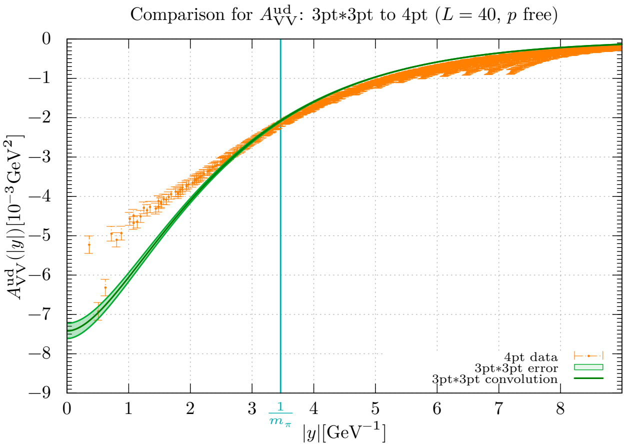
$$\int \; 4pt - correlator \;\stackrel{?}{=}\; \int \; (formfactor)^2$$

$$\int_{-\infty}^{\infty} d(\vec{p}_\perp \cdot \vec{y}_\perp)\, A_{VV}(y^2, py) \;=\; \frac{1}{2p^+} \int_{-\infty}^{\infty} dy^- \; \langle \pi(p)| V^+(0) V^+(y) |\pi(p)\rangle \Big|_{y^+=0}$$

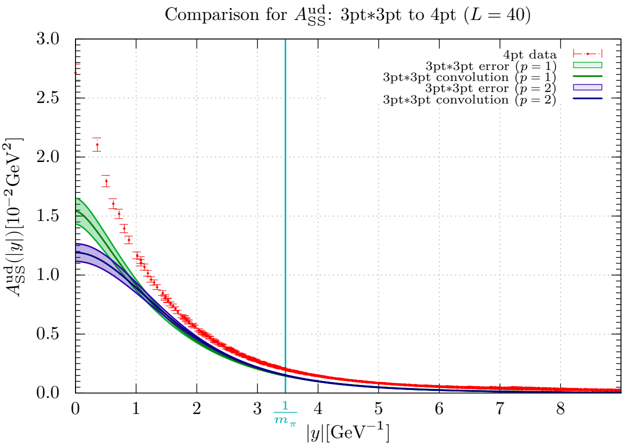
$$\int_{-\infty}^{\infty} d(py) A^{ud}_{VV}(y^2, yp) \;\stackrel{?}{\approx}\; -\int \frac{d^2 r_\perp}{(2\pi)^2}\, e^{-\mathrm{i}\vec{y}_\perp \cdot \vec{r}_\perp} \, |F_V(r^2)|^2$$

Quark-quark correlations in the pion are sizeable. On theoretical grounds they should be stronger in nucleons.

Direct tests of naive factorization: Test 1 VV case

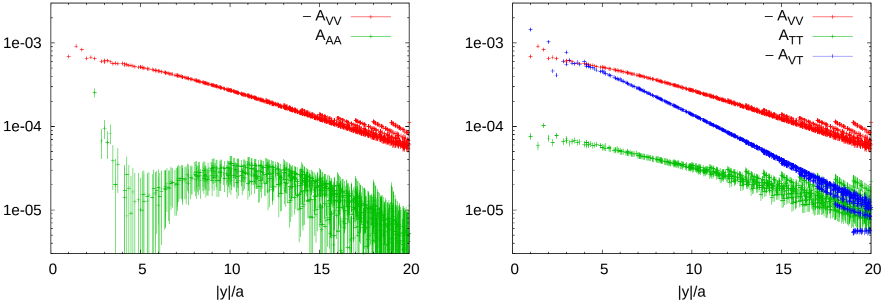


Direct tests of naive factorization: Test 1 SS case



Spin correlations

AA: longitudinal spin correlation $u^{\uparrow}\bar{d}^{\uparrow} + u^{\downarrow}\bar{d}^{\downarrow} - u^{\uparrow}\bar{d}^{\downarrow} - u^{\downarrow}\bar{d}^{\uparrow}$
TT: transverse spin correlation $\vec{s}_u \cdot \vec{s}_d$
VT: $\vec{y} \cdot \vec{s}_{\bar{d}}$



Conclusions

- TMDs and DPDs depend on local gauge links, the most characteristic feature of QCD
- Calculations on the lattice are feasible, but are still exploratory
- Results for TMDs agree with expectations (operator mixing for Wilson fermions and g_{1T})
- Results for DPDs in the pion show substantial quark correlations
- Comparing TMDs and DPDs in p+p, p+A and A+A promises to be interesting