TMDs of a Spin-One Target

Impacts of orbital angular momentum and DCSB

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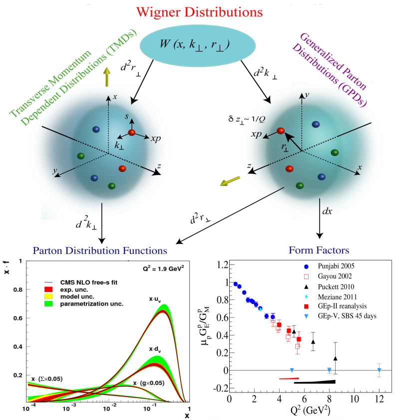
Momentum Tomography

leading twist		quark operator γ^+	$\gamma^+\gamma_5$	$\gamma^+\gamma^i\gamma_5$
target polarization	U	$f_1 =$ unpolarized		$h_1^\perp =$ — Boer-Mulders
	L		$g_1 =$ — helicity	$h_{1L}^\perp =$ — worm gear 1
	T	$f_{1T}^\perp =$ — Sivers	$g_{1T} =$ — worm gear 2	$h_1 =$ — transversity $h_{1T}^\perp =$ — pretzelosity
	TENSOR	$\theta_{LL}(x, \boldsymbol{k}_T^2)$ $\theta_{TT}(x, \boldsymbol{k}_T^2)$ $\theta_{LT}(x, \boldsymbol{k}_T^2)$	$g_{1TT}(x, \boldsymbol{k}_T^2)$ $g_{1LT}(x, \boldsymbol{k}_T^2)$	$h_{1LL}^\perp(x, \boldsymbol{k}_T^2)$ $h_{1TT}, \quad h_{1TT}^\perp$ $h_{1LT}, \quad h_{1LT}^\perp$



- A spin-1 target can have tensor polarization [associated with $\lambda = 0$]
  - 3 additional T-even and 7 additional T-odd quark TMDs compared to nucleon

    [A. Bacchetta and P. J. Mulders, Phys. Rev. D **62**, 114004 (2000)]
- Analogous situation for gluon TMDs
  - to fully expose role of quarks and gluons in nuclei need polarized nuclear targets, both transverse and longitudinal with all spin projections, e.g., for $J = 1$: ^{2}H, ^{6}Li

TMDs of Spin-One Targets

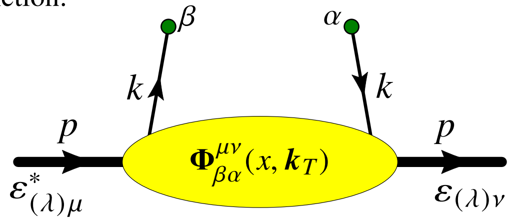
- Spin 4-vector of a spin-one particle moving in z-direction – with spin quantization axis $\boldsymbol{S} = (\boldsymbol{S}_T, S_L)$ reads:

$$S^\mu(p) = \left(\frac{p_z}{m_h} S_L, \boldsymbol{S}_T, \frac{p_0}{m_h} S_L \right)$$

  - for given direction $\boldsymbol{S}$ the particle has the three possible spin projections $\lambda = \pm 1, 0$
  - longitudinal polarization $\Longrightarrow$ $\boldsymbol{S}_T = 0, S_L = 1$; transverse $\Longrightarrow$ $|\boldsymbol{S}_T| = 1, S_L = 0$

- Define quark TMDs of a spin-one target with respect to the $\boldsymbol{k}_T$ dependent quark correlation function:

$$\boldsymbol{\Phi}^{(\lambda)s}_{\beta\alpha}(x, \boldsymbol{k}_T) =$$



- At leading-twist

$$\langle \gamma^+, \ \gamma^+\gamma_5, \ \gamma^+\gamma^i\gamma_5 \rangle^{(\lambda)}_{\boldsymbol{S}}(x, \boldsymbol{k}_T) = \frac{1}{2}\, \mathrm{Tr}_D \left[\left\{ \gamma^+, \ \gamma^+\gamma_5, \ \gamma^+\gamma^i\gamma_5 \right\} \Phi^{(\lambda)s}(x, \boldsymbol{k}_T) \right]$$

Spin-One TMD Decomposition

- Leading-twist decomposition which is independent of constraints on spin quantization axis $\boldsymbol{S}$:

$$\left\langle \gamma^+ \right\rangle_{\boldsymbol{S}}^{(\lambda)}(x, \boldsymbol{k}_T) \equiv f(x, \boldsymbol{k}_T^2) - \frac{3\lambda^2 - 2}{2}\left[\left(S_L^2 - \frac{1}{3}\right)\theta_{LL}(x, \boldsymbol{k}_T^2) + \frac{(\boldsymbol{k}_T \cdot \boldsymbol{S}_T)^2 - \frac{1}{3}\boldsymbol{k}_T^2}{m_h^2}\,\theta_{TT}(x, \boldsymbol{k}_T^2) + S_L\,\frac{\boldsymbol{k}_T \cdot \boldsymbol{S}_T}{m_h}\,\theta_{LT}(x, \boldsymbol{k}_T^2)\right]$$

$$\left\langle \gamma^+ \gamma_5 \right\rangle_{\boldsymbol{S}}^{(\lambda)}(x, \boldsymbol{k}_T) \equiv \lambda\left[S_L\, g_L(x, \boldsymbol{k}_T^2) + \frac{\boldsymbol{k}_T \cdot \boldsymbol{S}_T}{m_h}\, g_T(x, \boldsymbol{k}_T^2)\right]$$

$$\left\langle \gamma^+ \gamma^i \gamma_5 \right\rangle_{\boldsymbol{S}}^{(\lambda)}(x, \boldsymbol{k}_T) \equiv \lambda\left[S_T^i\, h_T(x, \boldsymbol{k}_T^2) + k_T^i\,\frac{\boldsymbol{k}_T \cdot \boldsymbol{S}_T}{m_h^2}\, h_T^{\perp}(x, \boldsymbol{k}_T^2) + S_L\,\frac{k_T^i}{m_h}\, h_L^{\perp}(x, \boldsymbol{k}_T^2)\right]$$

- The TMDs $\theta_{LL},\ \theta_{TT},\ \theta_{LT}$ are associated with tensor polarization

PDFs of Spin-One Targets

- Integrating over $\boldsymbol{k}_T^2$ gives 4 leading-twist quark PDFs for a spin-1 target

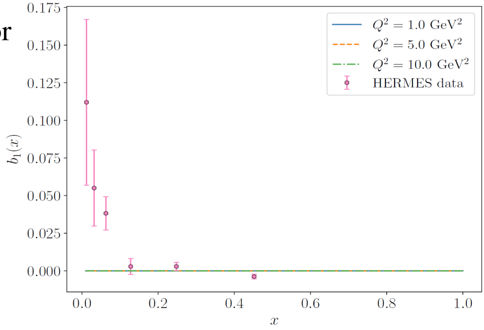
$$f(x) = \int d\boldsymbol{k}_T \; f(x, \boldsymbol{k}_T^2), \quad \theta(x) = \int d\boldsymbol{k}_T \left[\theta_{LL}(x, \boldsymbol{k}_T^2) - \frac{\boldsymbol{k}_T^2}{2\, m_h^2}\, \theta_{TT}(x, \boldsymbol{k}_T^2)\right], \ldots$$

- For DIS on spin-1 target 4 additional structure functions $b_{1\ldots4}(x)$ appear; in Bjorken limit just one $b_1(x)$ [Hoodbhoy, Jaffe and Manohar, Nucl. Phys. B **312**, 571 (1989)]

$$b_1(x) = \sum\nolimits_q e_q^2 \left[b_1^q(x) + b_1^{\bar{q}}(x)\right], \quad b_1^q = \tfrac{1}{2}\,\theta_q = \tfrac{1}{4}\left[2\, q^{(\lambda=0)} - q^{(\lambda=1)} - q^{(\lambda=-1)}\right]$$

- To measure $b_1(x)$ in DIS need tensor polarized target; HERMES has ^{2}H data, experiment planned at JLab

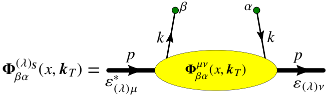
- Seems impossible to explain HERMES data with only bound nucleon degrees of freedom
  - need exotic QCD states: $6q$ bags, etc
  - JLab experiment is needed



TMD Positivity Constraints

- Positivity conditions must be imposed on [Bourrely, Soffer and Leader, Phys. Rept. **59**, 95 (1980)]

$$M^{(\lambda)s}(x, \boldsymbol{k}_T) = \left[\Phi^{(\lambda)s}(x, \boldsymbol{k}_T)\gamma^+\right]^T$$



  - the matrix M is the antiquark–hadron forward scattering matrix
  - in hadron rest-frame M is a 6×6 matrix in quark and hadron spin space

- Positivity implies that eigenvalues of M must be non-negative for all x & $\boldsymbol{k}_T$
  - imposes 6 *sufficient conditions* on the 9 spin-1 quark TMDs (very complicated)
  - also sub-minors of M must be semi-positive – imposes 63 *necessarily conditions*

- For quark PDFs of a spin-one target this gives 3 sufficient conditions:

$$f(x) \geqslant 0, \qquad |g(x)| \leqslant f(x) - \frac{1}{3}\theta(x)$$

$$2\,h(x)^2 \leqslant \left(f(x) + \frac{2}{3}\theta(x)\right)\left(f(x) + g(x) - \frac{1}{3}\theta(x)\right)$$

spin-1 Soffer bound

[A. Bacchetta and P. J. Mulders, Phys. Lett. B **518**, 85 (2001)]

- *Positivity conditions place tight constraints on experiment and calculations*

Measuring TMDs of Spin-1 Targets

- Need longitudinal and tensor polarized spin-1 targets, e.g., deuteron and ^{6}Li

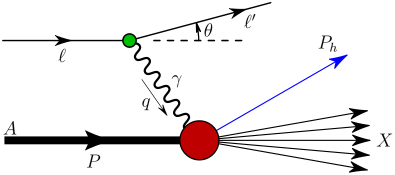
- For SIDIS there are 41 structure functions; 18 for U+L which also appear for spin-half and 23 associated with tensor polarization

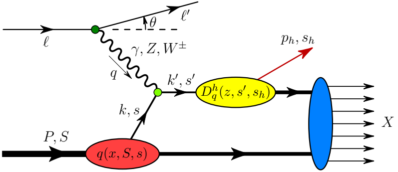
  [W. Cosyn, M. Sargsian and C. Weiss, PoS DIS **2016**, 210 (2016)]

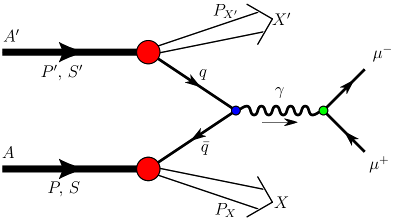
- For proton + deuteron Drell-Yan there are 108 structure functions; 60 associated with tensor structure of deuteron

  [S. Kumano, J. Phys. Conf. Ser. **543**, no. 1, 012001 (2014)]

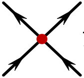
- Very challenging experimentally
  - need solid physics motivation and likely an EIC





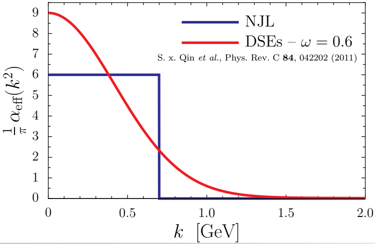


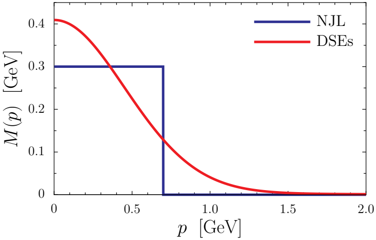
DSE Contact Interaction

Continuum QCD *"integrate out gluons"* →  $\frac{1}{m_G^2}\,\Theta(\Lambda^2 - k^2)$

- this is just a modern interpretation of the Nambu–Jona-Lasinio (NJL) model
- model is a Lagrangian based covariant QFT, exhibits dynamical chiral symmetry breaking & quark confinement; elements can be QCD motivated via the DSEs

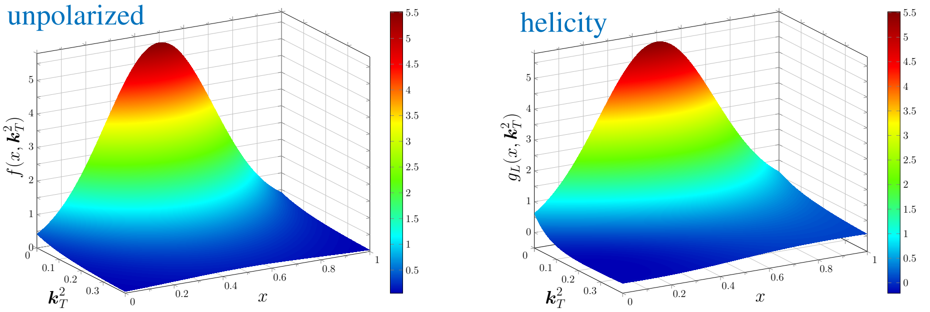
- Quark confinement is implemented via proper-time regularization
  - quark propagator: $[\not{p} - m + i\varepsilon]^{-1} \;\rightarrow\; Z(p^2)[\not{p} - M + i\varepsilon]^{-1}$
  - wave function renormalization vanishes at quark mass-shell: $Z(p^2 = M^2) = 0$
  - *confinement critical for our description of hadrons e.g.* $2\,M \simeq m_\rho,\;\; 3\,M \simeq m_\Delta$





TMDs for a Rho Meson

[Yu Ninomiya, ICC and Wolfgang Bentz, arXiv:1707.03787 [nucl-th]]



- Are spin-one TMDs interesting – do they contain new information?
- For each of the six T-even spin-one TMDs that have a nucleon analogy find:
  - each TMD is comparable in magnitude and shape
  - however arguably contain few surprises; peak near $x \sim 1/2$, essentially Gaussian in $\boldsymbol{k}_T$
  - as $\boldsymbol{k}_T^2$ becomes large each TMDs develops a weaker dependence on x – therefore x-dependent correlations are getting suppressed

TMDs for a Rho Meson

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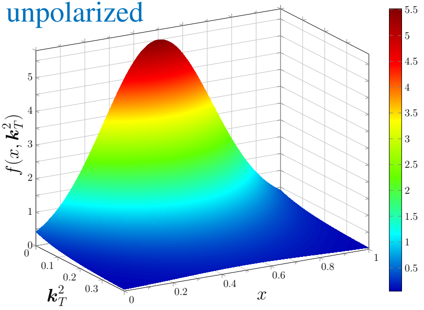
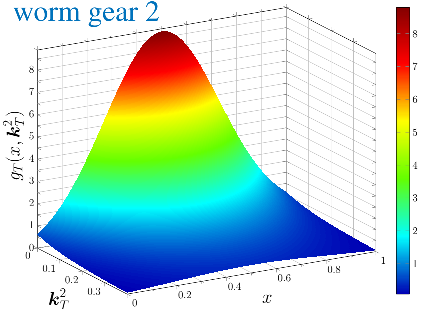
unpolarized



worm gear 2



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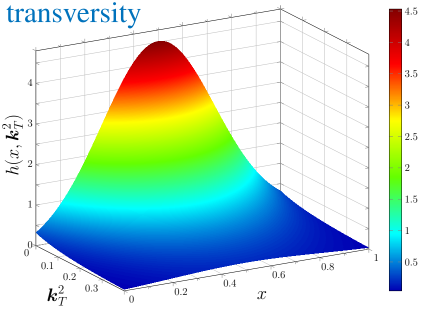
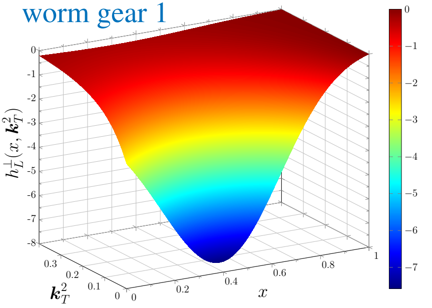
TMDs for a Rho Meson – $\gamma^+\gamma_T\gamma_5$





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- For the chiral-odd TMDs find that they are only non-zero because of DCSB:

$$h_i(x, \boldsymbol{k}_T^2) \;\equiv\; M\, \ell_i(x, \boldsymbol{k}_T^2) \;\overset{M\to 0}{=}\; 0$$

  - using the Drell–Yan–Levy relation expect that chiral-odd $q \to \rho$ TMD fragmentation functions are also directly sensitive to DCSB

- With only 2.2 MeV binding energy the deuteron helicity and transversity TMDs are likely much smaller ... but maybe there are surprises c.f. $b_1(x)$

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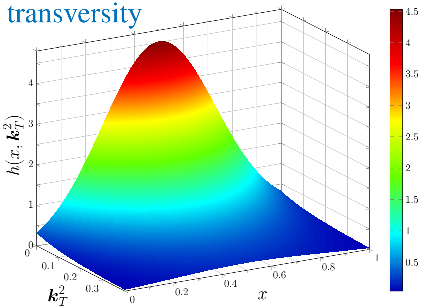

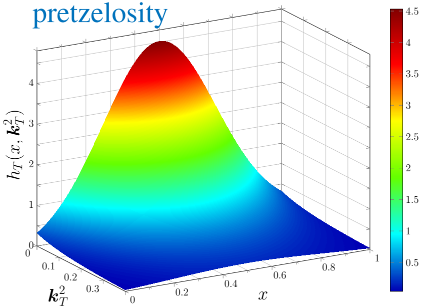

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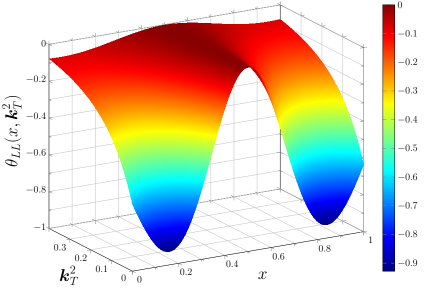
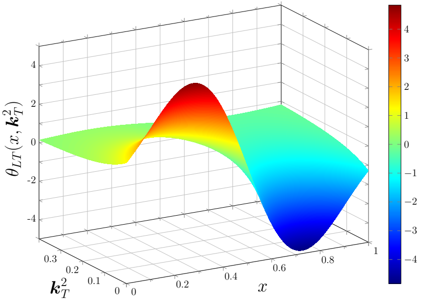
Rho Meson TMDs – Tensor Polarization





- Tensor polarized TMDs have a number of surprising features

$$\theta(x, \boldsymbol{k}_T^2) = \theta_{LL}(x\,\boldsymbol{k}_T^2) - \frac{\boldsymbol{k}_T^2}{2\,m_h^2}\,\theta_{TT}(x\,\boldsymbol{k}_T^2)$$

- TMDs $\theta_{LL}(x\,\boldsymbol{k}_T^2)$ & $\theta_{LT}(x\,\boldsymbol{k}_T^2)$ identically vanishes at $x = 1/2$ for all $\boldsymbol{k}_T^2$
  - $x = 1/2$ corresponds to zero relative momentum between (the two) constituents, that is, s-wave contributions
  - therefore θ_{LL} & θ_{LT} only receive contributions from $L \geqslant 1$ components of the wave function – *sensitive measure of orbital angular momentum*
- Features hard to determine from a few moments – difficult for lattice QCD

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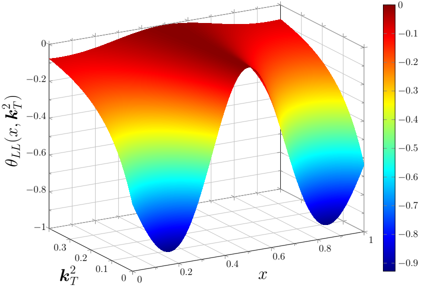

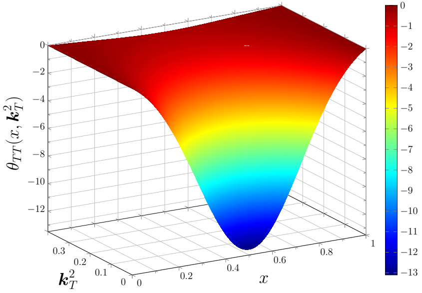

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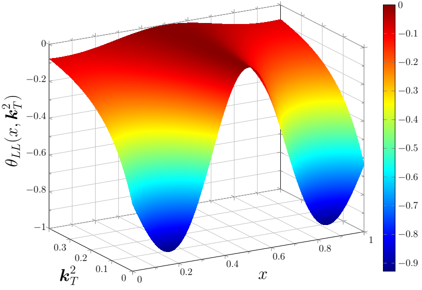

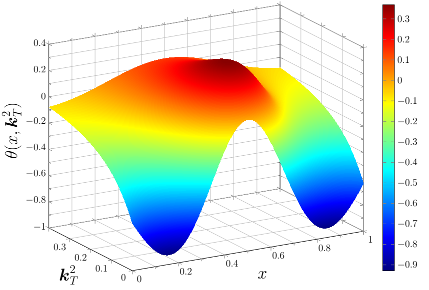

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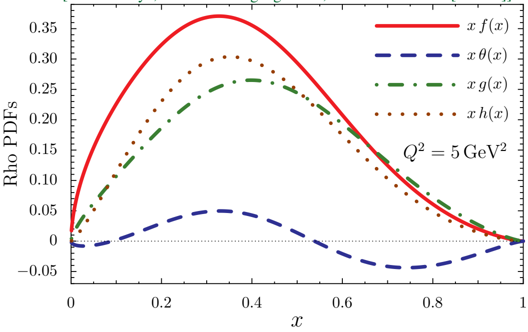
Rho Meson PDFs

- Results satisfy positivity relations e.g.

$$|g(x)| \leqslant f(x) - \tfrac{1}{3}\,\theta(x)$$

- Find that 44% of the spin of the ρ is carried by orbital angular momentum

[Yu Ninomiya, ICC and Wolfgang Bentz, arXiv:1707.03787 [nucl-th]]



- There is a *fundamental* sum rule for the $b_1^q(x)$ $[\propto \theta(x)]$ PDF

$$\int_0^1 dx\,\left[b_1^q(x) - b_1^{\bar{q}}(x)\right] = 0$$

[F. E. Close and S. Kumano, Phys. Rev. D42, 2377 (1990)]

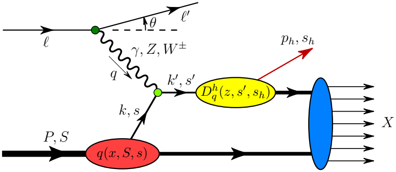
[A. V. Efremov and O. V. Teryaev, Sov. J. Nucl. Phys. 36, 557 (1982)]

  - interpretation: valence quark number does not depend on the hadron's spin state

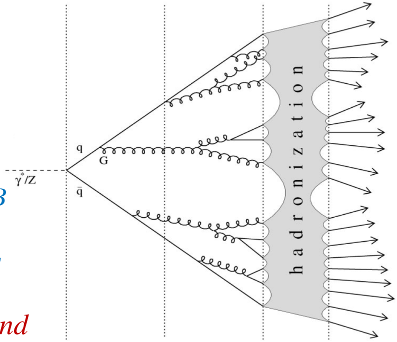
- Comparison with deuteron data and calculations, find that $b_1(x)$ for the ρ has similar behavior but with opposite sign
  - analogous situation found for the ρ and deuteron quadruple moments

Spin-1 Fragmentation Functions: $q \to \rho + X$

- Measuring the ρ TMDs is clearly not possible for the forseeable future
  - for spin-one need nuclear target



- However, measuring the $q \to \rho$ TMD fragmentation functions is forseeable



- Fragmentation functions are particularly important
  - *potentially fragmentation functions can shed the most light on confinement and DCSB – because they describe how a fast moving (massless) quark becomes a tower of hadrons*

- *Understanding the nature of confinement and its relation to DCSB is one of the most important challenges in hadron physics – origin of ∼98% of mass in visible universe*

Conclusion

- Spin-1 targets present a rich quark and gluon structure that can help expose novel aspects of QCD
  - e.g. gluon chiral-odd PDFs/TMDs only possible in targets with $J \geqslant 1$ $\stackrel{?}{\Longrightarrow}$ gluon content of NN interaction
  - find that TMDs associated with tensor polarization are sensitive to quark orbital angular momentum
  - ρ meson results a stepping stone to deuteron calculations
- Jefferson Lab EIC design is better suited to studing 3D tomography of $J \geqslant 1$ targets
  - critical to explore physics content of these observables
- Deuteron is arguably best neutron target