Nucleon structure from lattice QCD

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Outline

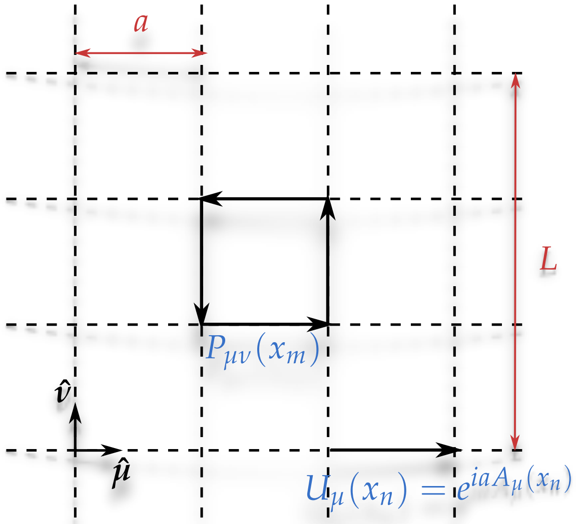
- **Lattice QCD short overview**
  - Landscape of simulations
  - Overview of nucleon structure methods
- **Selected nucleon observables from lattice QCD**
  - Nucleon axial charge, spin and momentum decomposition
  - Direct calculation of Parton Distribution Functions
  - Nucleon σ-terms, scalar and tensor charges
  - Nucleon Electromagnetic Form Factors (time permitting)

Lattice QCD — ab initio simulation of QCD

- Freedom in choice of:
  - quark masses (heavier is cheaper)
  - lattice spacing a (larger is cheaper)
  - lattice volume $L^3 \times T$ (smaller is cheaper)
- Choice of discretisation scheme

  e.g. Clover, Twisted Mass, Staggered, Overlap, Domain Wall

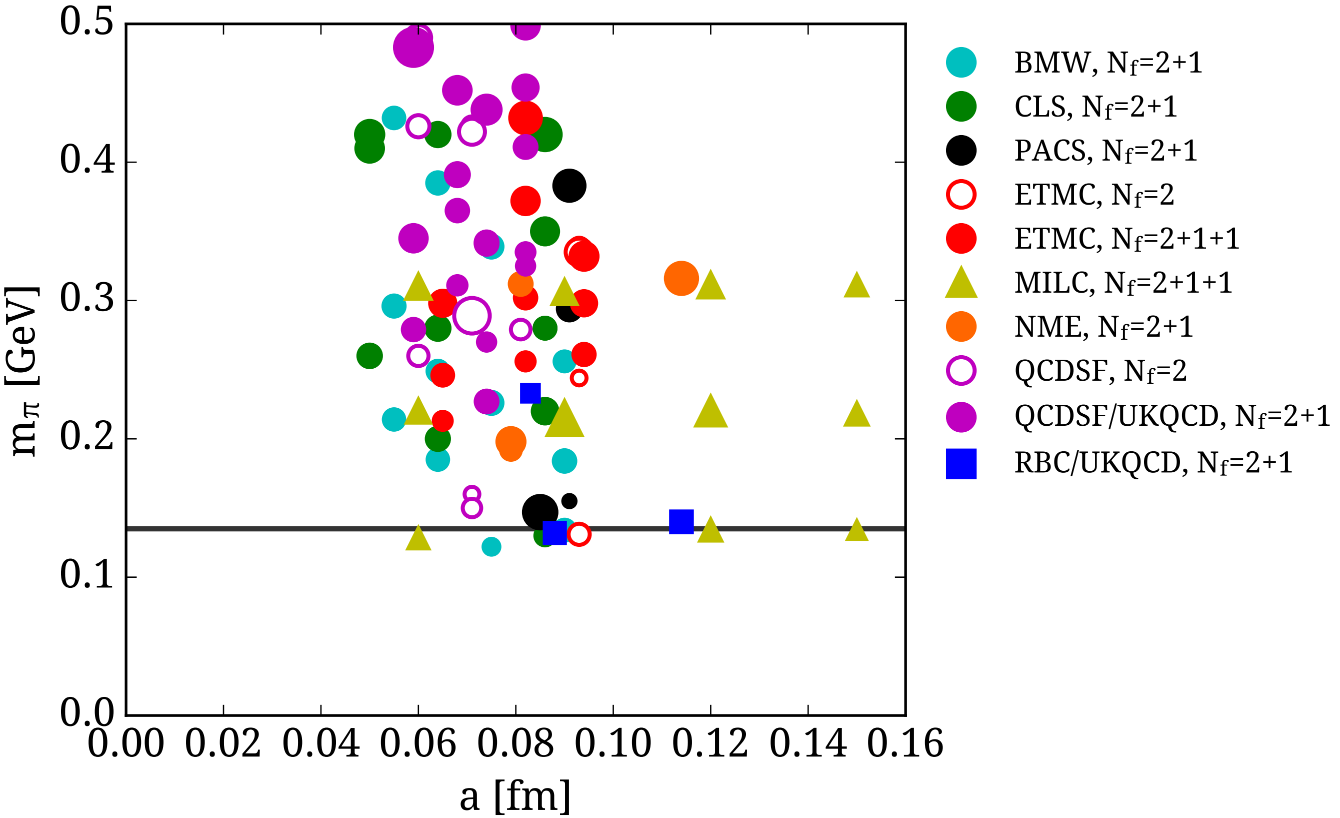
  Trade — offs and advantages for each differ



Eventually, all schemes must agree:

- At the continuum limit: $a \to 0$
- At infinite volume limit $L \to \infty$
- At physical quark mass

Simulations landscape



Selected lattice simulation points used for hadron structure

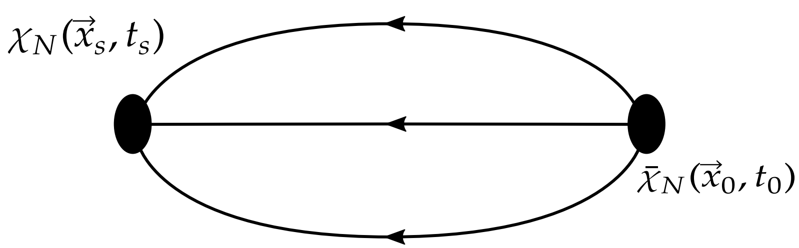
- Multiple collaborations simulating at physical pion mass
- Size of points indicates $m_\pi L$

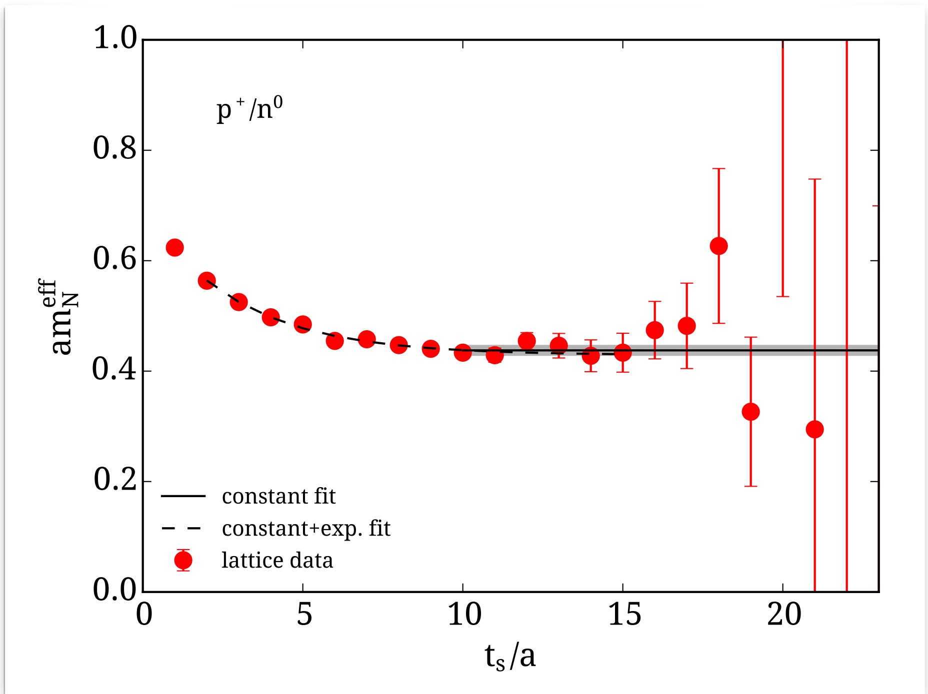
Sources of uncertainty

- Statistical error: $1/\sqrt{N}$, with MC samples
- Correlation functions: exponentially decay with time-separation

Systematic uncertainties

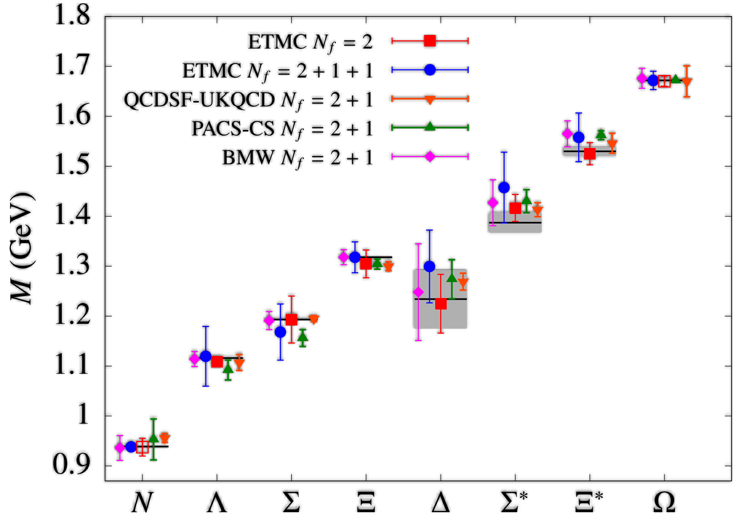
- Extrapolations: a, L, m_π
- Contamination from higher energy states

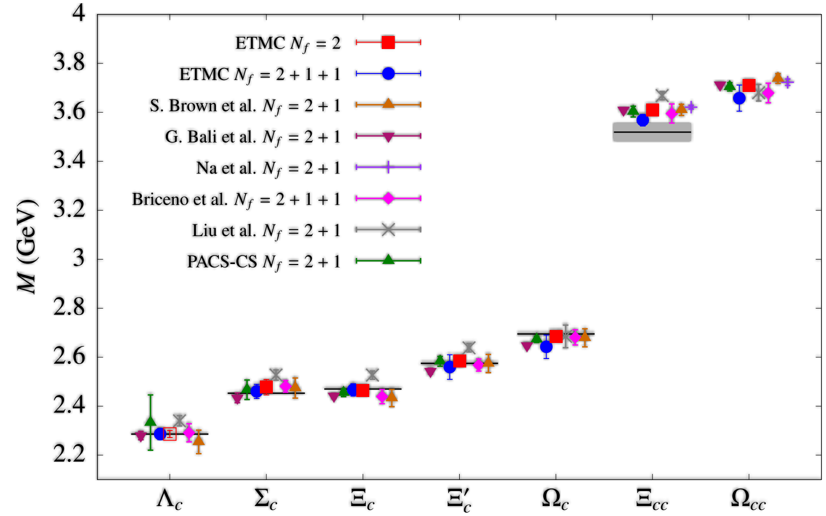




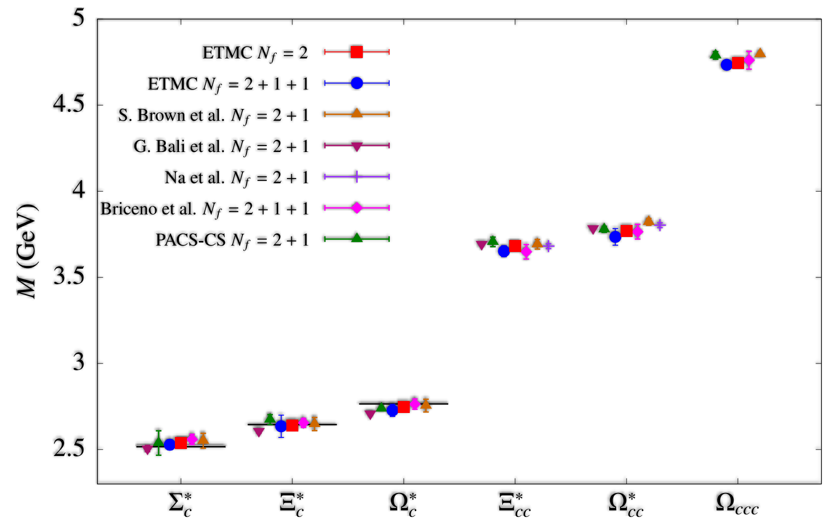
$$\sum_{\vec{x}_s} \Gamma^{\alpha\beta} \langle \bar{\chi}_N^{\beta}(x_s) | \chi_N^{\alpha}(0) \rangle = c_0 e^{-E_0 t_s} + c_1 e^{-E_1 t_s} + \ldots$$

Baryon spectrum





Summary plots from arXiv:1704.02647



Reproduction of light baryon masses

- Agreement between lattice discretisations
- Reproduction of experiment

Prediction of yet to be observed baryons

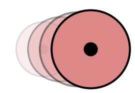
- Confidence through agreement between lattice schemes

Nucleon structure on the lattice

– Lattice: moments are readily accessible

Unpolarised

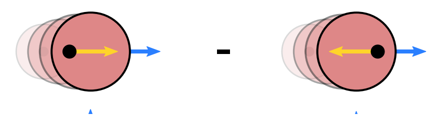
$$\mathcal{O}_{V}^{\mu\mu_1\mu_2\ldots\mu_n} = \bar{\psi}\gamma^{\{\mu}iD^{\mu_1}iD^{\mu_2}\ldots iD^{\mu_n\}}\psi$$



$$\langle 1\rangle_{u-d} = g_V\,,\quad \langle x\rangle_{u-d},\quad \cdots$$

Helicity

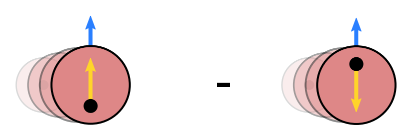
$$\mathcal{O}_{A}^{\mu\mu_1\mu_2\ldots\mu_n} = \bar{\psi}\gamma_5\gamma^{\{\mu}iD^{\mu_1}iD^{\mu_2}\ldots iD^{\mu_n\}}\psi$$



$$\langle 1\rangle_{\Delta u-\Delta d} = g_A,\quad \langle x\rangle_{\Delta u-\Delta d},\quad \cdots$$

Transverse

$$\mathcal{O}_{T}^{\nu\mu\mu_1\mu_2\ldots\mu_n} = \bar{\psi}\sigma^{\nu\{\mu}iD^{\mu_1}iD^{\mu_2}\ldots iD^{\mu_n\}}\psi$$

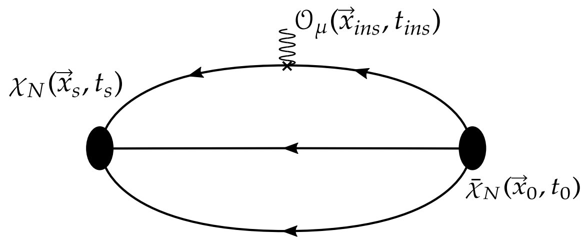


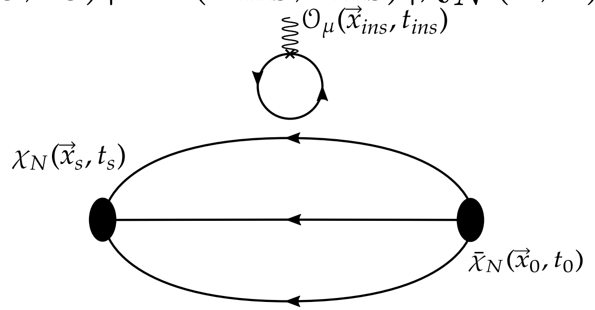
$$\langle 1\rangle_{\delta u-\delta d} = g_T,\quad \langle x\rangle_{\delta u-\delta d},\quad \cdots$$

Lattice evaluation of matrix elements

Three-point function:

$$G_\mu(\Gamma;\vec{q};t_s,t_{\rm ins}) = \sum_{\vec{x}_s\,\vec{x}_{\rm ins}} e^{-i\vec{q}.\vec{x}_{\rm ins}}\Gamma^{\alpha\beta}\langle\bar{\chi}_N^{\beta}(\vec{x}_s;t_s)|\mathcal{O}^{\mu}(\vec{x}_{\rm ins};t_{\rm ins})|\chi_N^{\alpha}(\vec{0};0)\rangle$$





Analyses for identifying excited state contributions

- Plateau:

$$R(t_s,t_{\rm ins},t_0) \xrightarrow[t_{\rm ins}-t_0\to\infty]{t_s-t_{\rm ins}\to\infty} \mathcal{M}[1+\mathcal{O}(e^{-\Delta(t_{\rm ins}-t_0)},e^{-\Delta'(t_s-t_{\rm ins})})]$$

  fit to constant w.r.t $t_{\rm ins}$ for multiple values of t_s

- Sum over $t_{\rm ins}$

$$\sum_{t_{\rm ins}} R(t_s,t_{\rm ins},t_0) \xrightarrow{t_s-t_0\to\infty} \text{Const.} + \mathcal{M}(t_s-t_0) + \mathcal{O}(t_s e^{-\Delta t_s})$$

  fit to linear form, matrix element is the slope

- Fit, including first excited states

Agreement between methods signals excited state suppression

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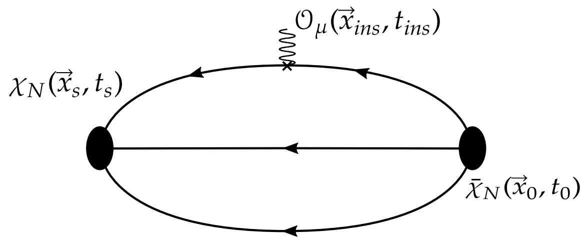
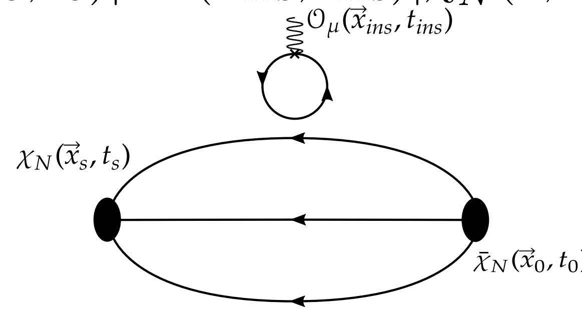




tions

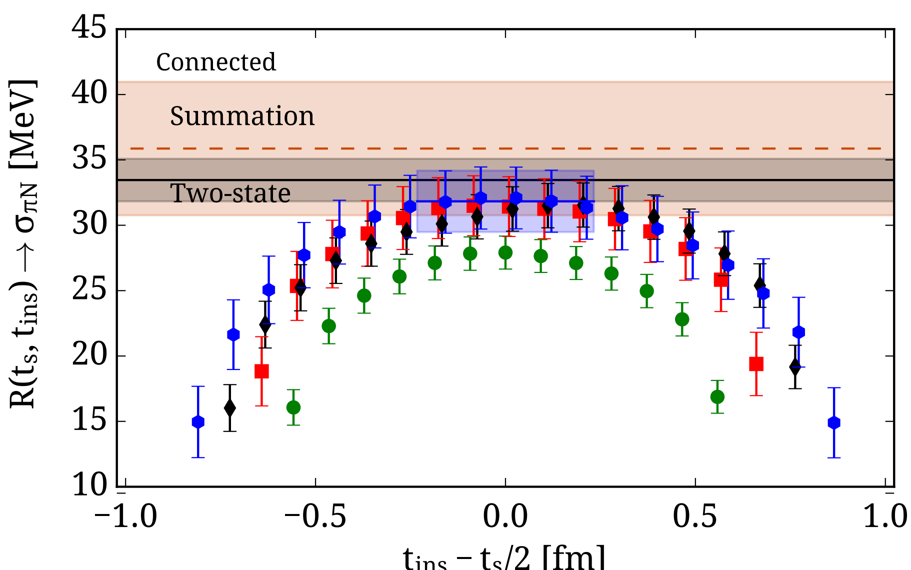
$+\,\mathcal{O}(e^{-\Delta(t_{\rm ins}-t_0)}, e^{-\Delta'(t_s-t_{\rm ins})})]$

of t_s

st. $+\,\mathcal{M}(t_s-t_0)+\mathcal{O}(t_s e^{-\Delta t_s})$

ope

state suppression



Connected, ~12,500 statistics

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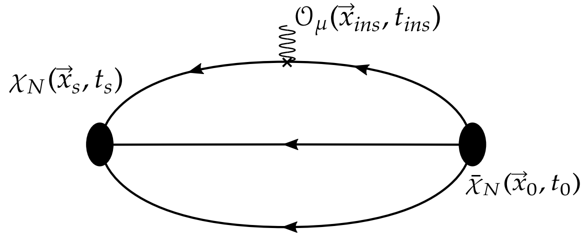
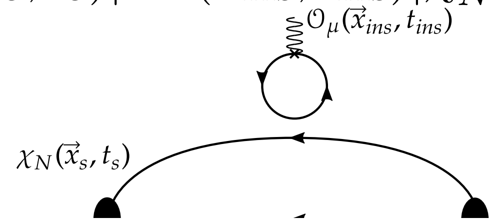
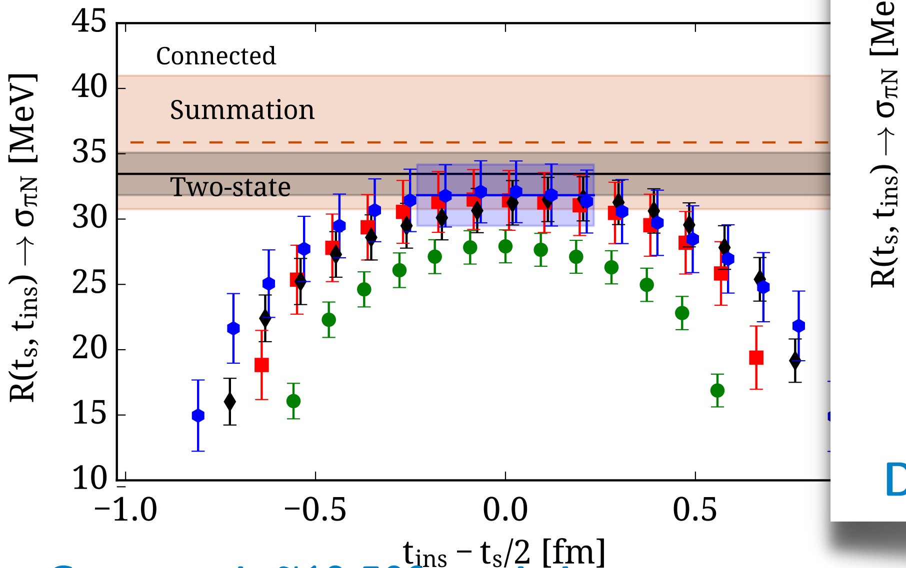
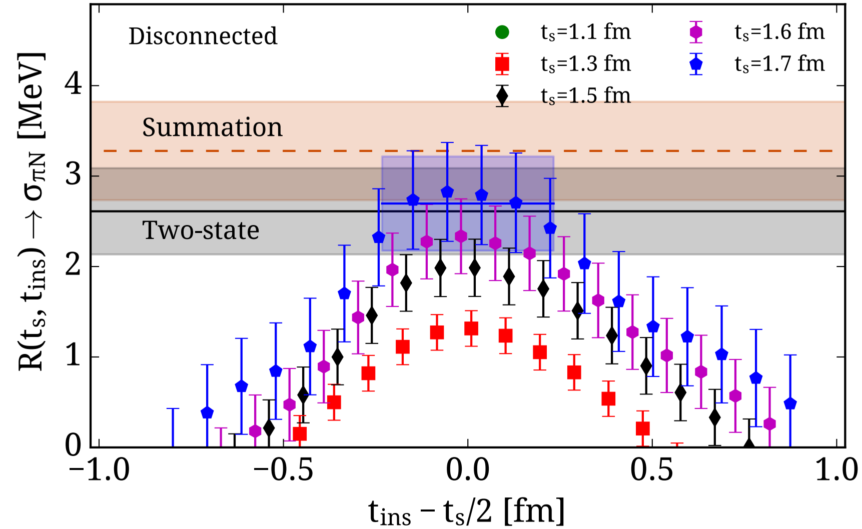






Connected, ~12,500 statistics



Disconnected, ~200,000 statistics

state suppression

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Axial matrix elements

Isovector axial charge

- Well known from β-decay
- Readily accessible on the lattice: $\mathcal{O}^A = \bar{u}\gamma_5\gamma_k u - \bar{d}\gamma_5\gamma_k d$
- Benchmark quantity in lattice QCD

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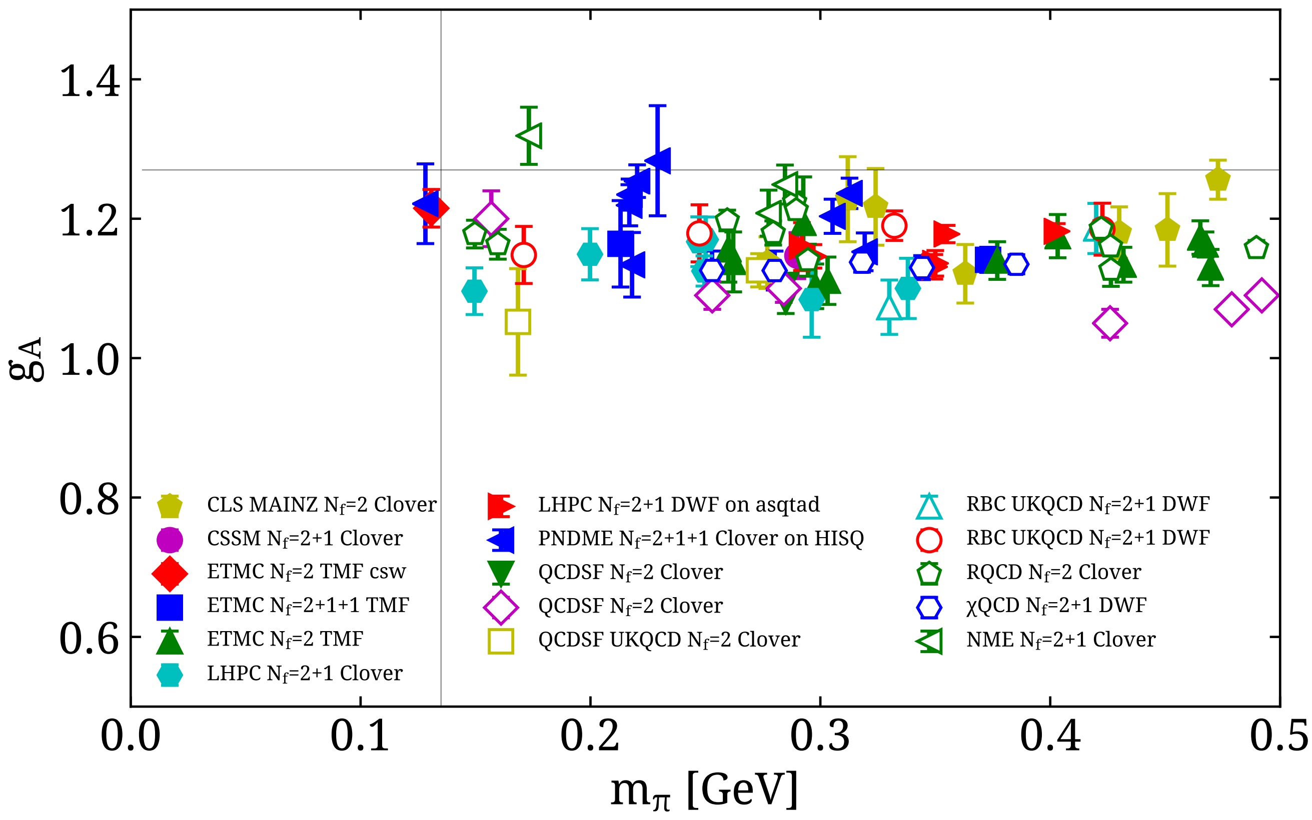


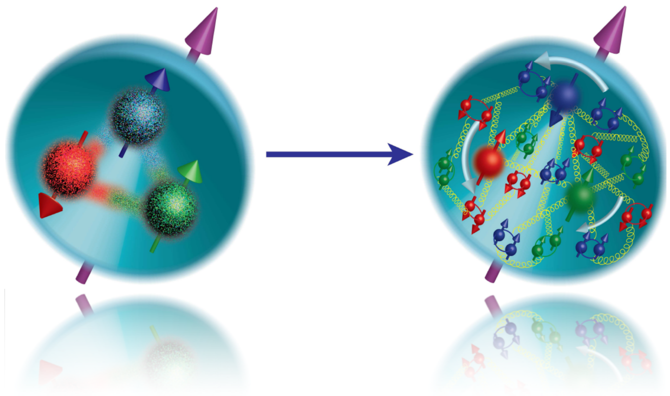
Results at near physical pion mass only available recently

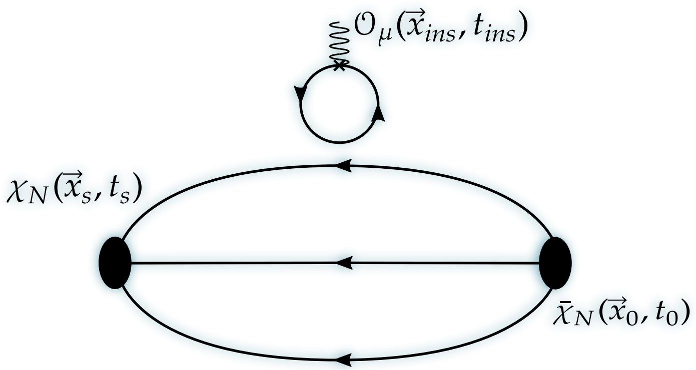
- General agreement between lattice schemes
- Still small tension with experiment at physical point

Nucleon spin

Quark intrinsic spin contributions to nucleon spin

$$\frac{1}{2}\Delta\Sigma = \frac{1}{2}\sum_{q=u,d,s,\ldots} g_A^q$$



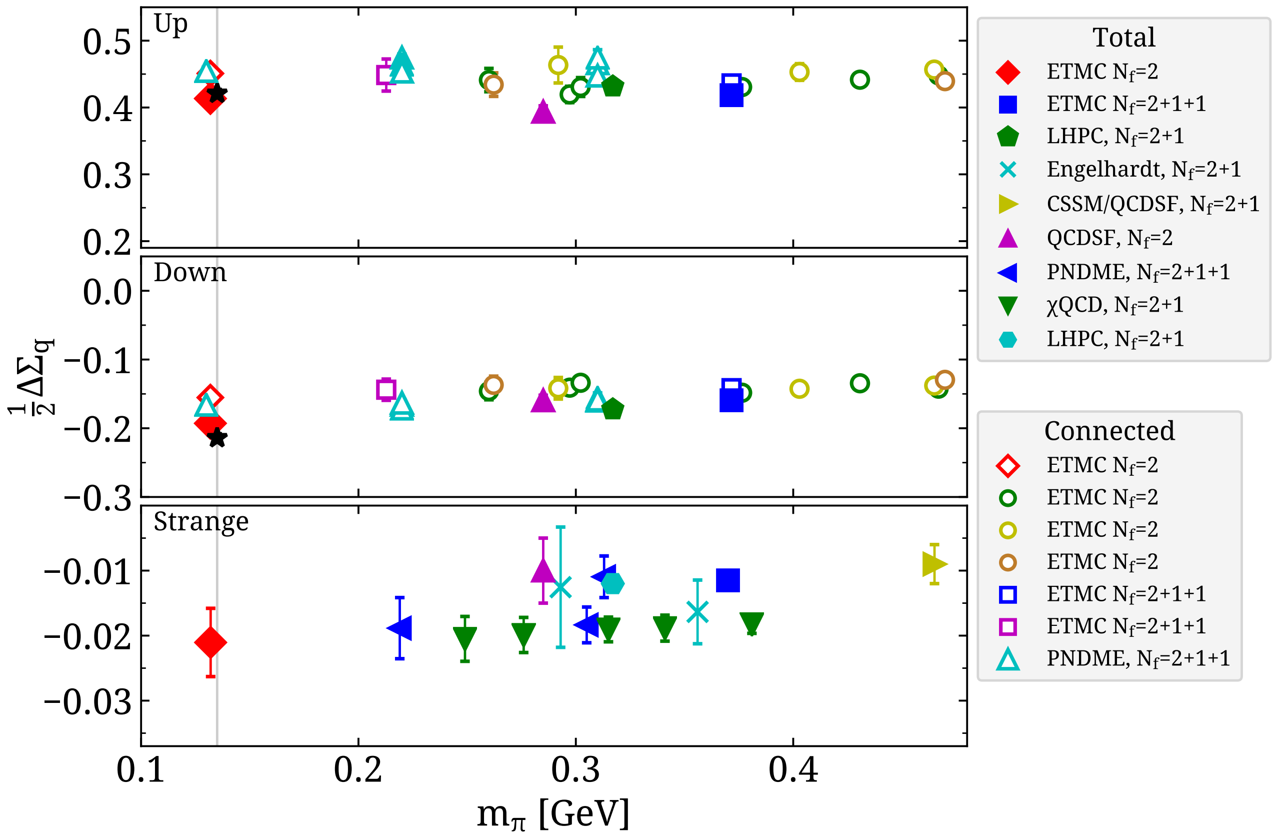


- Need linear combination of isovector and isoscalar contributions for individual up- and down-quarks
- Strange quark contribution is sea-quark contribution only (disconnected diagrams)
- Very demanding on the lattice, need O(10) - O(100) times more statistics

Nucleon spin

Quark intrinsic spin contributions to nucleon spin

– Overall agreement between formulations, and with experimental determinations
– Strange and down-quark contributions negative
– u, d, and s intrinsic spin contributions at 20(2)% of 1/2, at physical pion mass



Nucleon spin

Total parton spin contributions to nucleon spin

- Ji's spin sum rule: $J_N = \sum_{q=u,d,s,c\cdots} \left(\frac{1}{2}\Delta\Sigma_q + L_q \right) + J_g$
- Quark contribution:

$$\frac{1}{2}\Delta\Sigma_q + L_q = J_q = \frac{1}{2}[A^q_{20}(0) + B^q_{20}(0)]$$

where $A^q_{20}(0)$ and $B^q_{20}(0)$ are obtained from the matrix element of the first derivative operator: $\mathcal{O}_V^{\mu\mu_1} = \bar{\psi}\gamma^{\{\mu}iD^{\mu_1\}}\psi$, i.e. $A^q_{20}(0) = \langle x \rangle_q$

- Similarly for gluon contribution need disconnected diagram:

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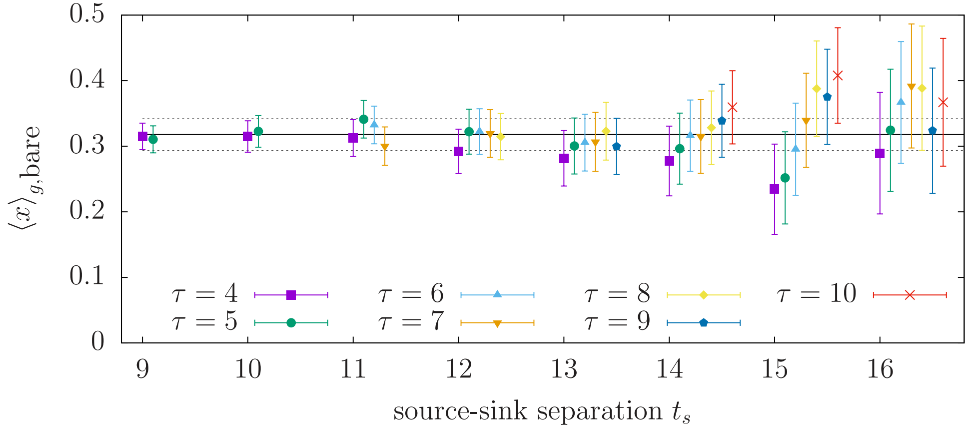
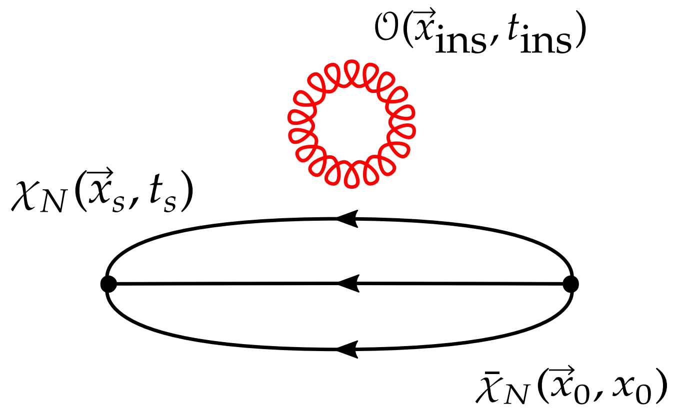
– Similarly for gluon contribution need disconnected diagram of gluonic operator: $2Tr[G_{\mu\sigma}G_{\nu\sigma}]$





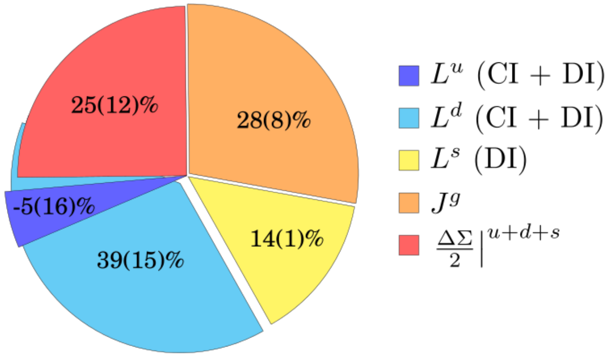
C. Alexandrou et al. (ETM collaboration), arXiv:1611.06901

Nucleon spin

Parton spin and momentum contributions to nucleon spin

- Calculations including u, d, s, and gluons
- Spin and momentum sums satisfied within errors

$$J_N = \sum_{q=u,d,s,c\cdots} \left(\frac{1}{2}\Delta\Sigma_q + L_q\right) + J_g$$



← m_π=330 MeV

- L_q obtained via J_q-½$\Delta\Sigma_q$
- Update at physical m_π, S_G=0.251(47)(16)

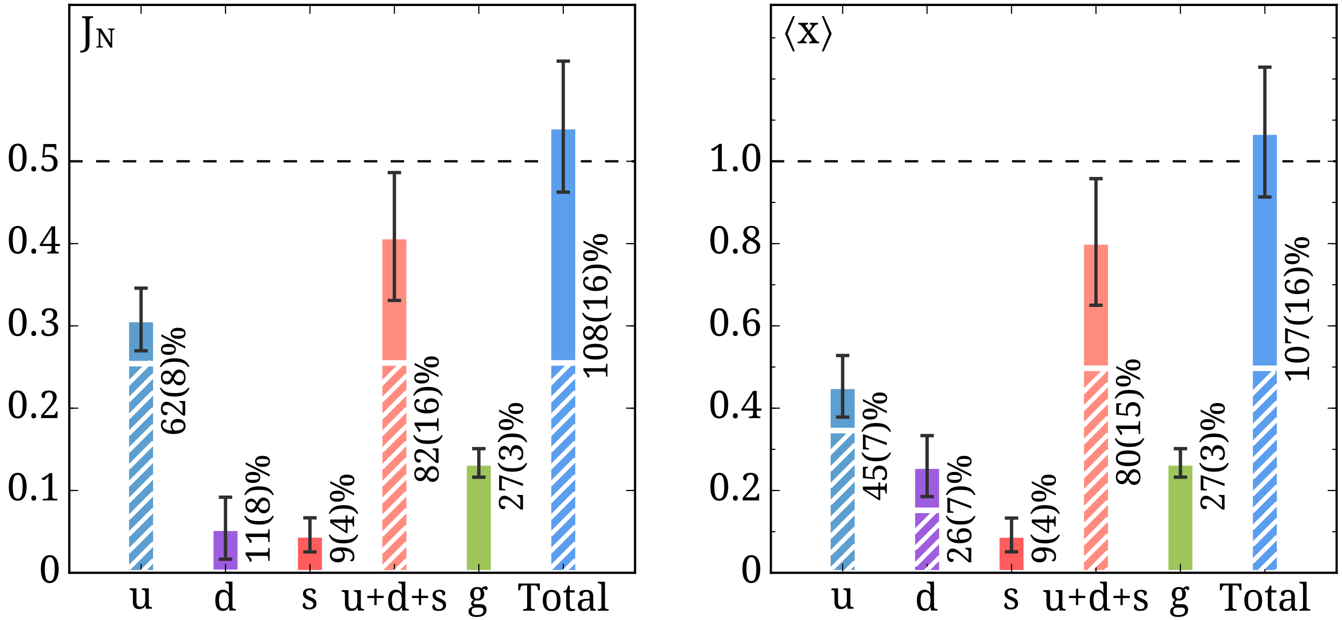
Phys. Rev. Lett. 118 (2017) no.10, 102001

K.-F. Liu, Int. J. Mod. Phys. Conf. Ser. 40 (2016) 1660005

Nucleon spin

Parton spin and momentum contributions to nucleon spin

- Includes u, d, s, and gluons simulated at physical pion mass
- Spin and momentum sums satisfied within errors



$J^N_{u+d+s+g} = 0.541(62)(49)$ $\langle x \rangle_{u+d+s+g} = 1.07(12)(10)$

C. Alexandrou et al., arXiv:1706.02973 (under review)

Direct calculation of PDFs on the lattice

Higher order Mellin moments on the lattice are hard:

$$\langle x^n \rangle_q = \int_{-1}^{1} x^n q(x) dx$$

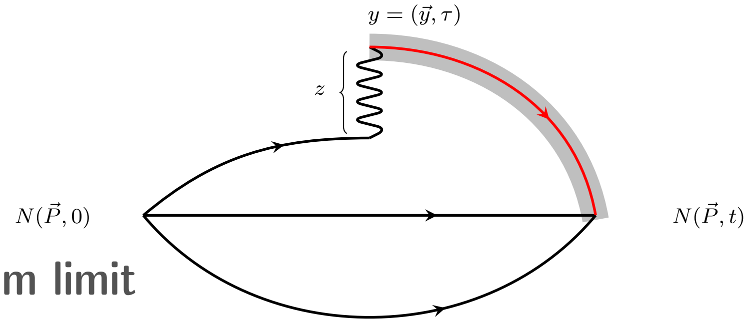
For n large:

- Noisier correlation functions
- Complicated renormalisation
- Mixing at high derivatives

Proposal by X. Ji [Phys. Rev. Lett. 110 (2013) 26] for the calculation of quasi-PDFs on a Euclidean lattice:

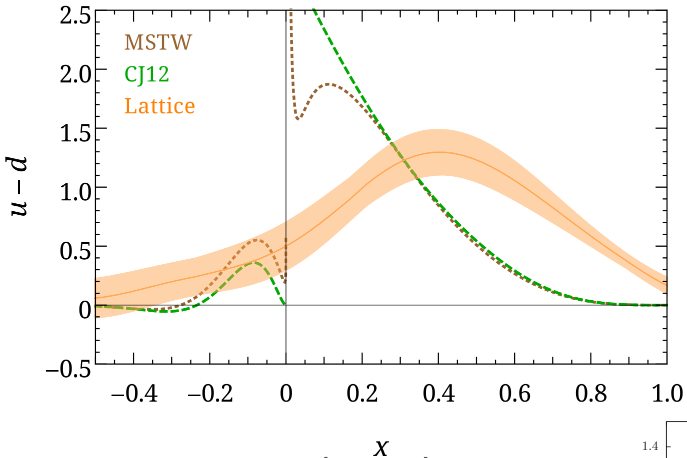
$$\tilde{q}(x, P_3) = \int \frac{dz}{4\pi} e^{-ixP_3 z} \langle N(P_3) | \bar{\psi}(z) \gamma_z A_z(z, 0) \psi(0) | N(P_3) \rangle$$

- Three-point correlation function
- Boosted nucleon
- Line of glue connecting quark lines
- Match quasi-PDF to PDF at infinite momentum limit
- Non-trivial renormalisation



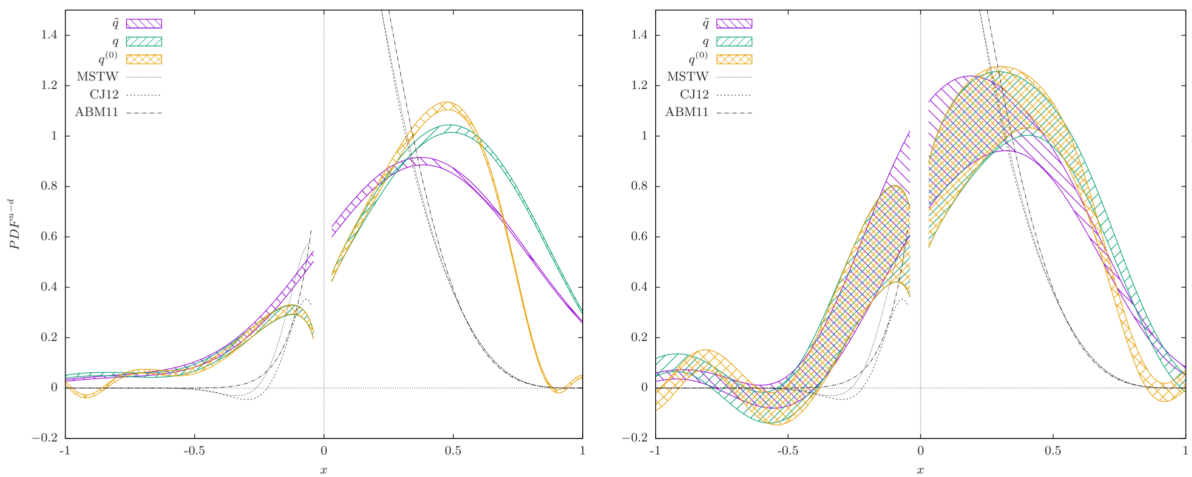
Direct calculation of PDFs on the lattice

Two first calculations in lattice QCD



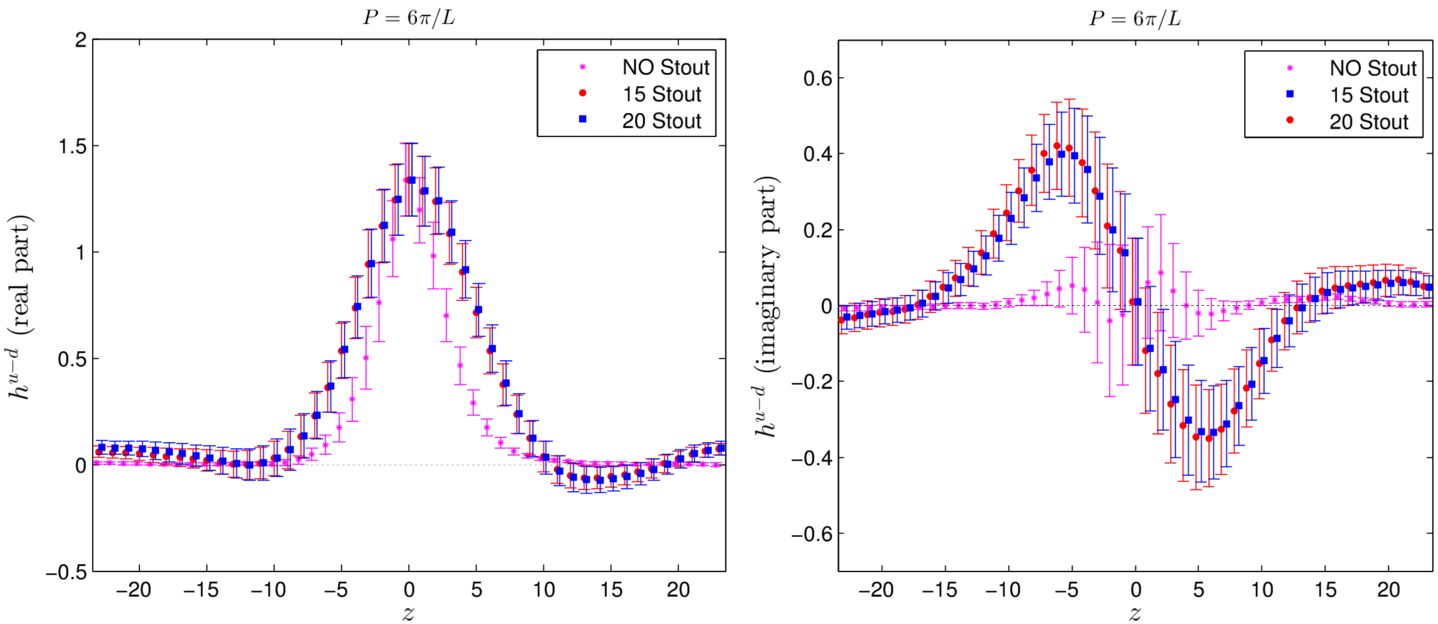
Phys. Rev. D91 (2015) 054510
2+1+1 Clover on HISQ
m_π=310 MeV

Phys.Rev. D92 (2015) 014502
2+1+1 twisted mass,
m_π=370 MeV



Direct calculation of PDFs on the lattice

Progress towards physical point:



Preliminary - ETM collaboration: K. Cichy, talk at Lattice 2017, m_π=135 MeV

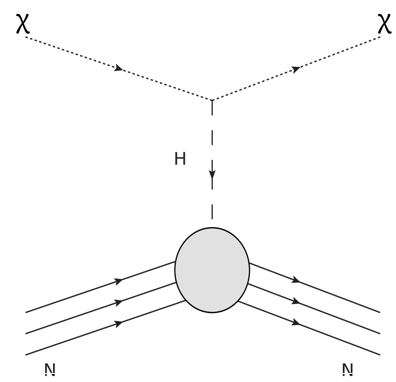
Progress in renormalisation:
- ETMC arXiv:1706.00265
- J-W Chen et al. arXiv:1706.01295

TMDs:

- Fri. 12:25, A. Schafer, “TMDs and DPDs on the lattice” (Parallel 4)

Nucleon σ-terms

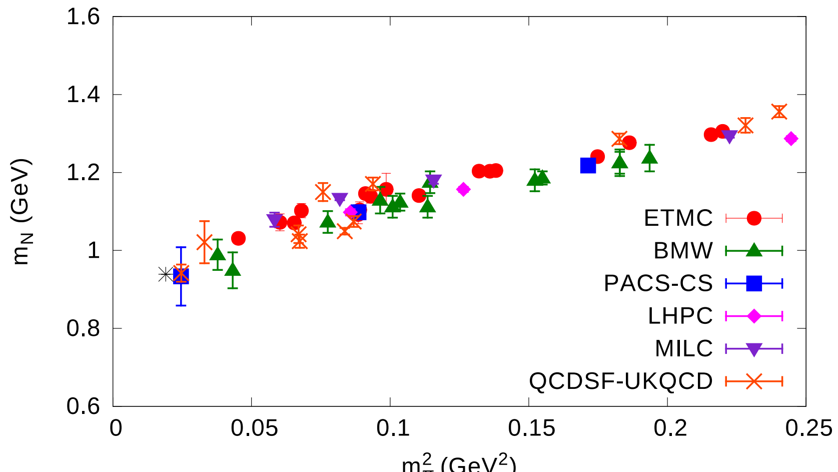
- Pion nucleon σ-term: $\sigma_{\pi N} = m_{ud}\langle N|\bar{u}u + \bar{d}d|N\rangle$
- Strange σ-term: $\sigma_s = m_s\langle N|\bar{s}s|N\rangle$
- Enter super-symmetric candidate particle scattering cross sections with nucleon (e.g. neutralino through Higgs)



1. Direct calculation of matrix elements

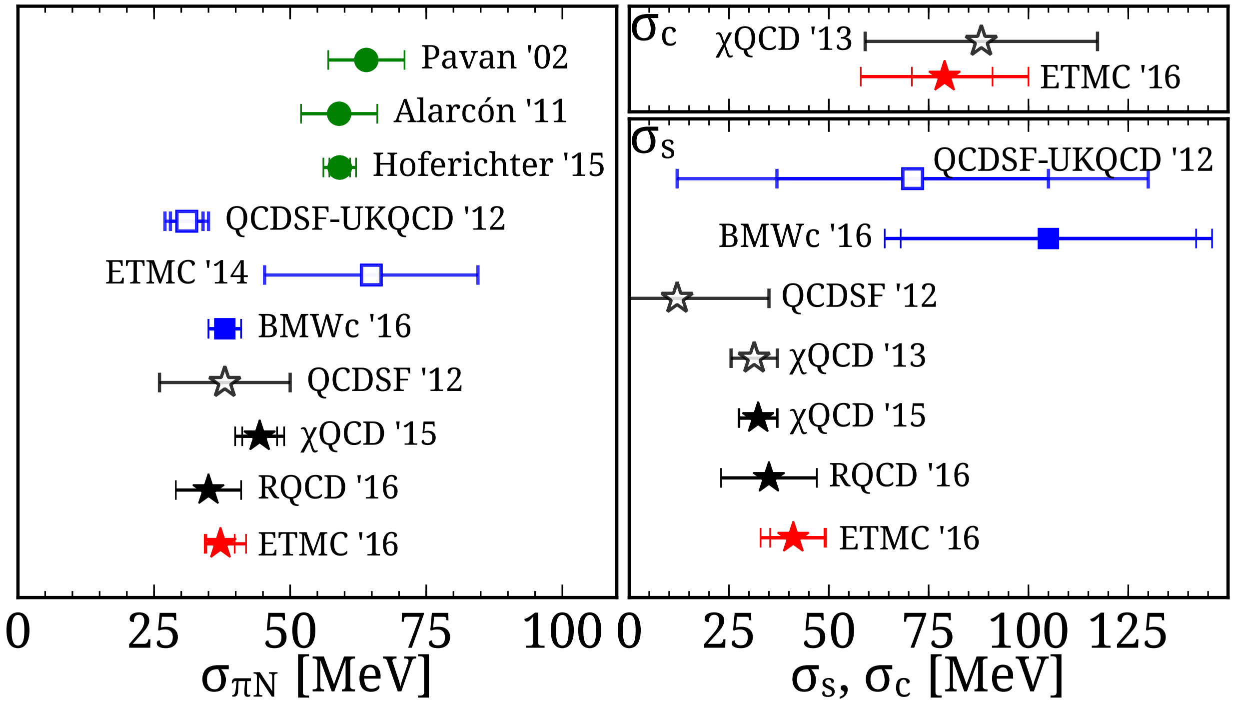
Involves disconnected contributions

2. Through Feynman - Hellmann theorem: $\sigma_{\pi N} = m_{ud}\frac{\partial m_N}{\partial m_{ud}}$ $\sigma_s = m_s\frac{\partial m_N}{\partial m_s}$



- Reliance on effective theories for dependence on m_π
- Weak dependence on m_s

Nucleon σ-terms



Summary from Phys. Rev. Lett. **116** (2016) 252001

- Recent results using direct matrix element evaluation [★]
- Large errors in FH-method [■](especially strange) due to sensitivity to quark mass
- Compare to phenomenology [●]

Tensor and scalar charges

Matrix elements of scalar and tensor operators

$$\mathcal{O}_{S^a} = \bar{\psi}\frac{\tau^a}{2}\psi$$

$$\mathcal{O}^{\mu\nu}_{T^a} = \bar{\psi}\sigma^{\mu\nu}\frac{\tau^a}{2}\psi$$

Scalar: related to σ-terms, cross-sections with WIMPs

Tensor: novel CP-violating interactions, non-zero nEDM

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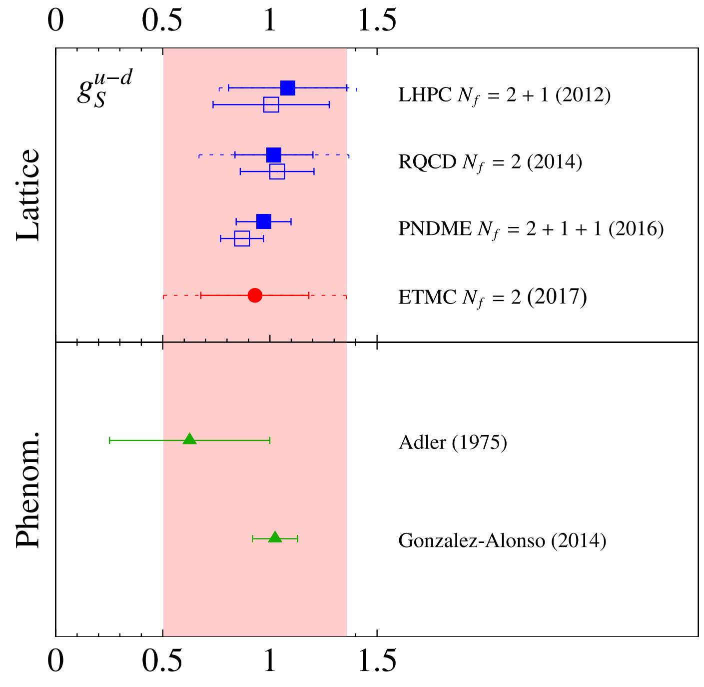

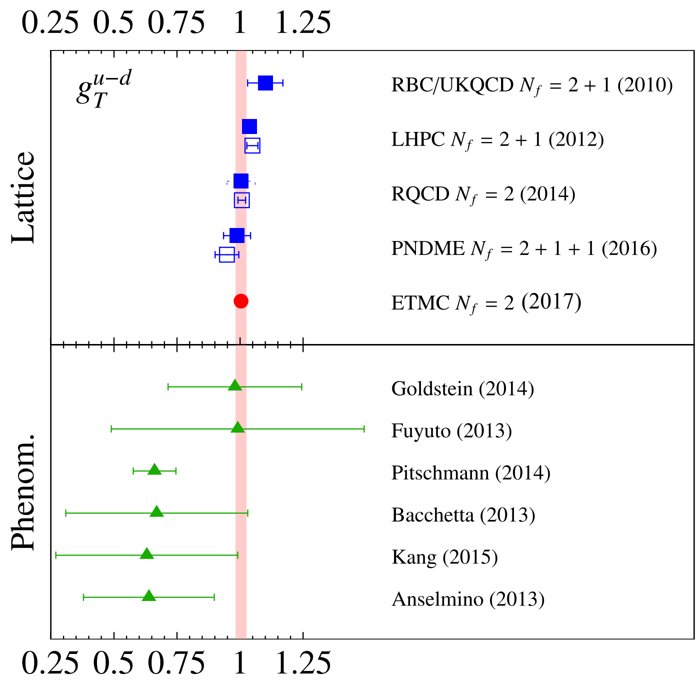

Review plot from Phys.Rev. D95 (2017) no.11, 114514

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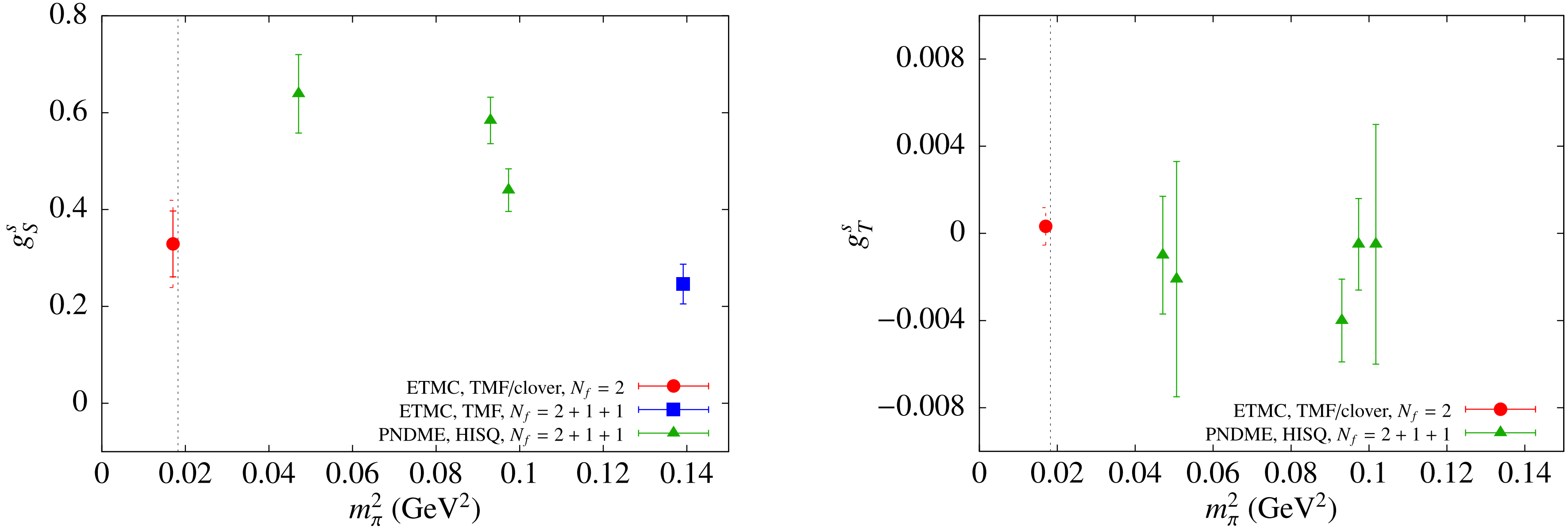
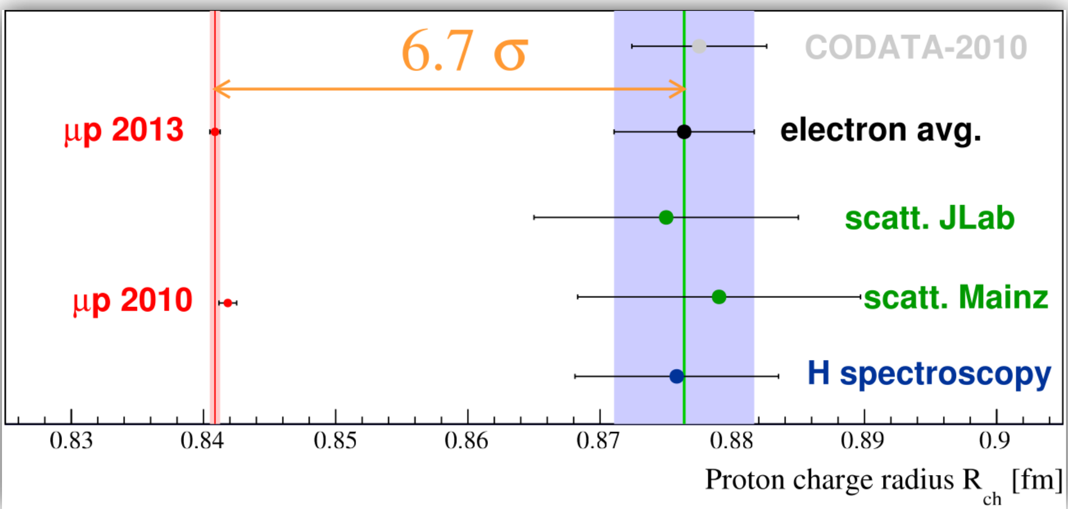


Review plot from Phys.Rev. D95 (2017) no.11, 114514

Nucleon form factors

Electromagnetic form factors

- Distribution of charges in proton
- Magnetic moment, electric and magnetic radii



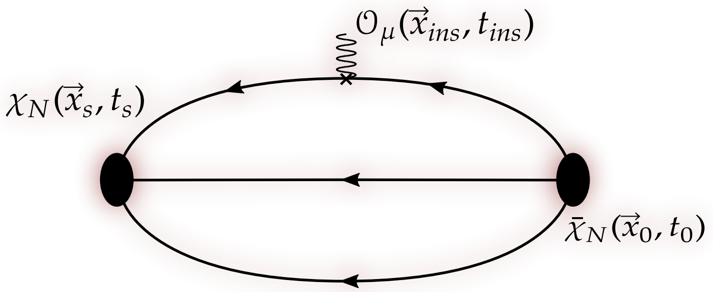
- Discrepancy between proton radius measured using μH Lamb shifts vs e-p scattering
- R. Pohl et al., Nature 466 (2010) 213, R. Pohl et al., 353 (2016) 669-673 (muonic deuterium)
- vs. CODATA, Rev. Mod. Phys. 88 (2016) 035009

Nucleon form factors

Electromagnetic form factors

$$\langle N(p',s')|j^\mu|N(p,s)\rangle = \sqrt{\frac{M_N^2}{E_N(\mathbf{p}')E_N(\mathbf{p})}}\,\bar{u}(p',s')\mathcal{O}^\mu u(p,s)$$

$$\mathcal{O}^\mu = \gamma_\mu F_1(q^2) + \frac{i\sigma_{\mu\nu}q^\nu}{2M_N}F_2(q^2), \quad q = p' - p$$



Sachs form factors:

$$G_M(q^2) = F_1(q^2) + F_2(q^2)$$

$$G_E(q^2) = F_1(q^2) + \frac{q^2}{(2m_N)^2}F_2(q^2)$$

$$G_E(Q^2) = 1 - \frac{1}{6}\langle r_E^2\rangle Q^2 + \mathcal{O}(Q^4)$$

Isovector and isoscalar combinations

$$j_\mu^v = \bar{u}\gamma_\mu u - \bar{d}\gamma_\mu d, \quad j_\mu^s = \bar{u}\gamma_\mu u + \bar{d}\gamma_\mu d$$

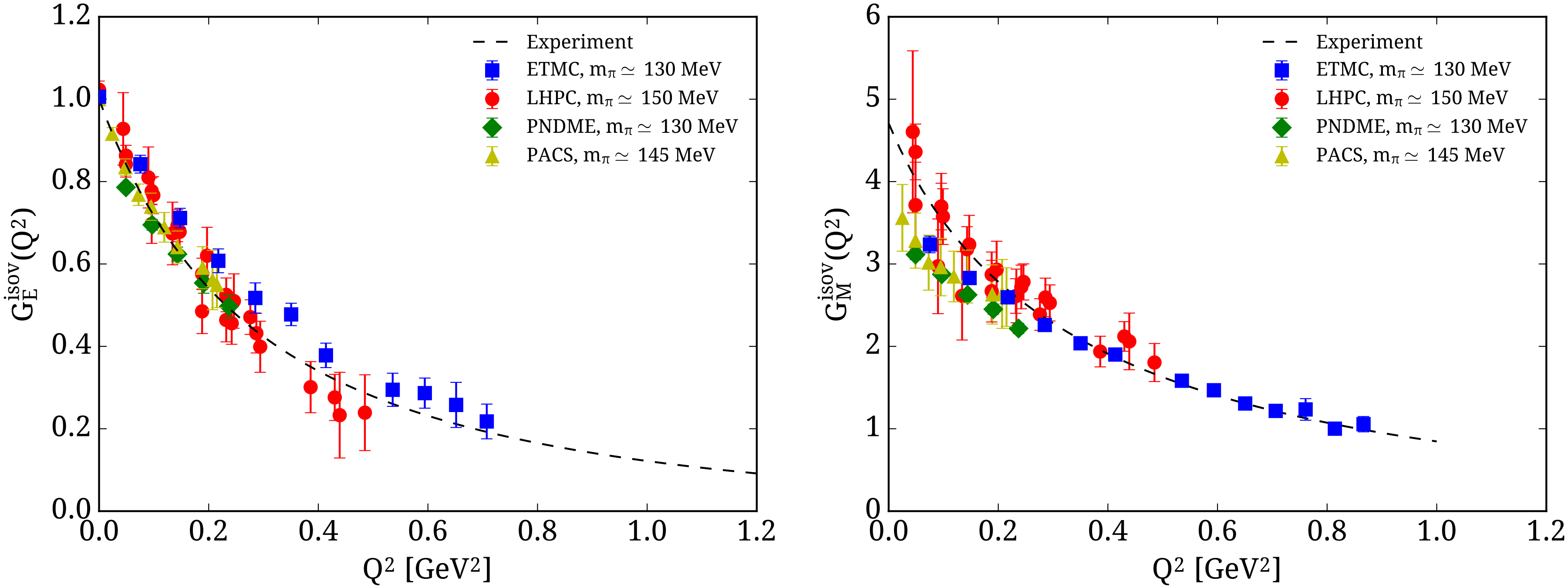
$$F^p - F^n = F^u - F^d$$

$$F^p + F^n = \frac{1}{3}(F^u + F^d)$$

Assuming flavour isospin symmetry

Nucleon form factors

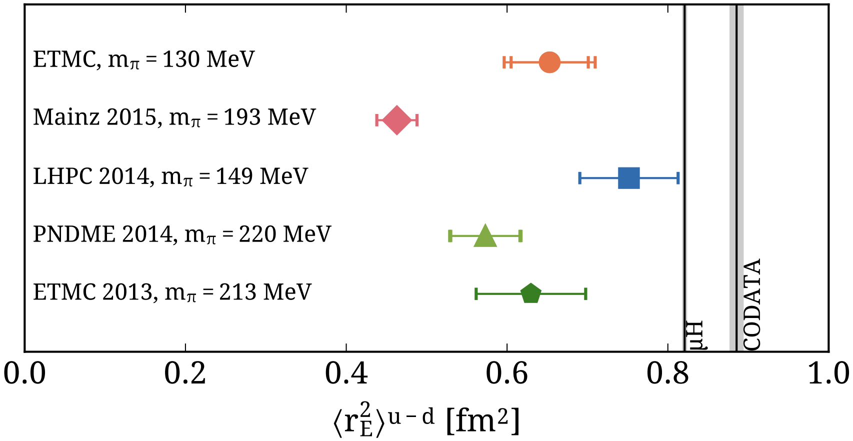
Electromagnetic form factors

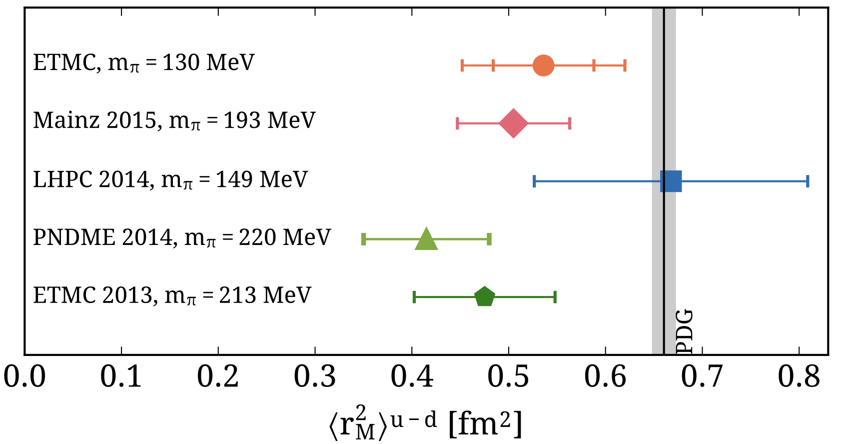


- Multiple collaborations at near-physical pion mass
- Overall consistency between formulations
- Some discrepancy with experiment (e.g. G_M at low Q^2)

Nucleon form factors

Electromagnetic form factors





- Isovector case, no disconnected contributions
- Need slope at Q2⟶0:

$$\frac{\partial}{\partial Q^2} G_E(Q^2)|_{Q^2=0} = -\frac{1}{6} G_E(0)\langle r_E^2 \rangle,$$

- Model Q2 dependence:

a. Dipole

$$G_E(Q^2) = \frac{1}{(1 + \frac{Q^2}{M_E^2})}$$

b. z-expansion

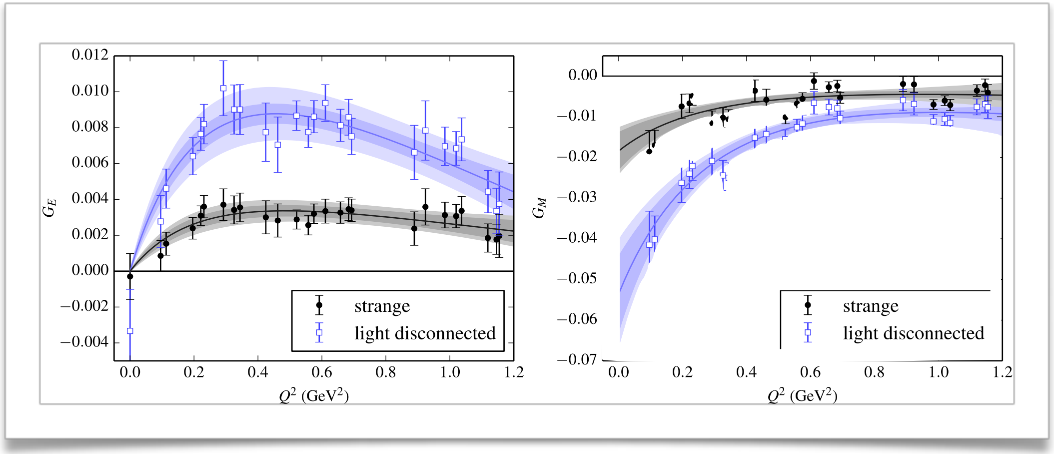
$$G_E(Q^2) = \sum_{k=0}^{k_{\max}} a_k z^k$$

$$z = \frac{\sqrt{t_{\text{cut}} + Q^2} - \sqrt{t_{\text{cut}}}}{\sqrt{t_{\text{cut}} + Q^2} + \sqrt{t_{\text{cut}}}}$$

- Smallest momentum: 2π/L

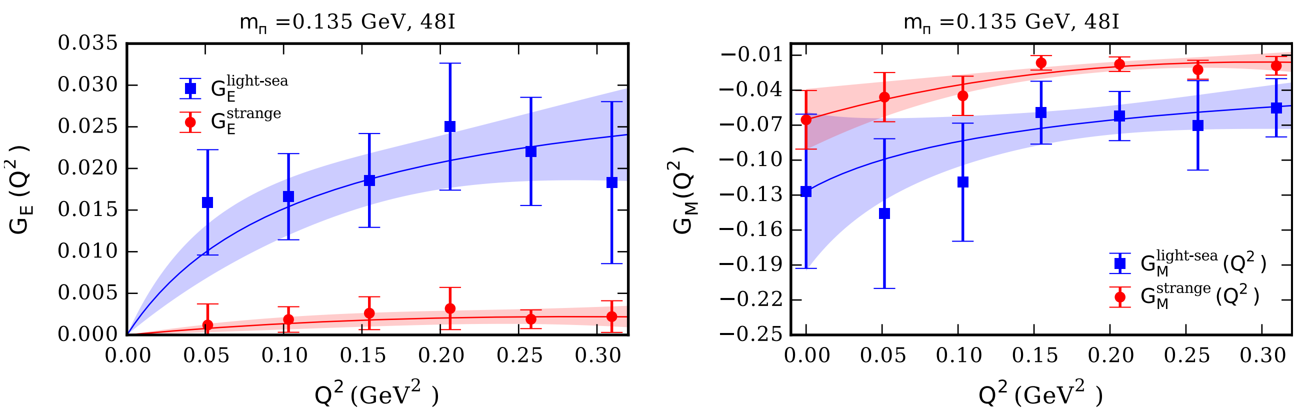
Comparison plot from: C. Alexandrou et al. arXiv:1706.00469 PRD (to appear)

Nucleon form factors

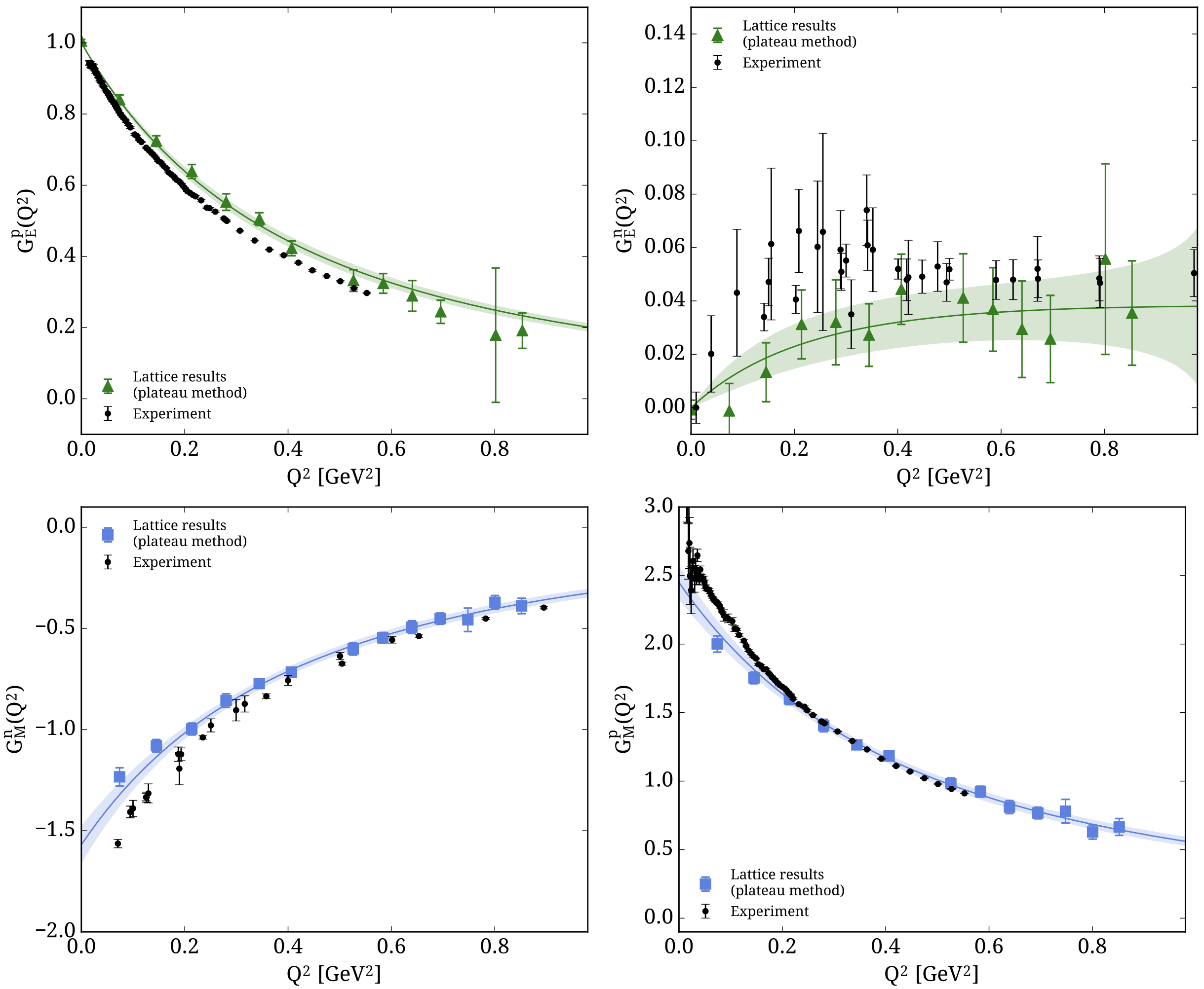


Disconnected contributions are at the % level

- Disconnected (sea-quark) contributions needed for isoscalar ⟶ proton, neutron
- J. Green et al., Phys. Rev. D92 (2015) 3, 031501, at ~317 MeV pion mass (above)
- χQCD collab., arXiv:1705.05849



Nucleon form factors



First complete result (connected+disconnected) at physical point

C. Alexandrou et al. arXiv:1706.00469 PRD (to appear)

Summary and outlook

★Lattice QCD in new era

- Physical pion mass simulations from a number of collaborations
- Other systematic uncertainties coming under control

★Nucleon spin

- Spin decomposition of proton possible. Results coming out at physical point
- First results including gluon contribution
- Errors can be reduced further in coming years

Summary and outlook

★ Nucleon charges

- Large effort for evaluating disconnected (sea) contributions
- Simulations yielding observables important for BSM searches at high accuracy
- Results for σ-terms, g_S, g_T, with statistical errors allowing direct comparison with experiment or prediction when experimental result not available

★ Some noteworthy new directions not reviewed here

- Fri. 12:25, A. Schafer, "TMDs and DPDs on the lattice" (Parallel 4)
- Isospin breaking effects, separation of breaking into QED contribution and from up- and down-quark mass difference
  - BMWc, Phys.Rev.Lett. 111 (2013) no.25, 252001
- CP-odd matrix elements (e.g. neutron EDM)
  - C. Alexandrou et al., Phys.Rev. D93 (2016) no.7, 074503
  - M. Abramczyk et al., Phys.Rev. D96 (2017) no.1, 014501

★ETM Collaboration

Cyprus (Univ. of Cyprus, Cyprus Inst.), **France** (Orsay, Grenoble), **Germany** (Berlin/Zeuthen, Bonn, Frankfurt, Hamburg, Münster), **Italy** (Rome I, II, III, Trento), **Netherlands** (Groningen), **Poland** (Poznan), **Spain** (Valencia), **Switzerland** (Bern), **UK** (Liverpool), **US** (Temple, PA)

★Collaborators:

- C. Alexandrou, S. Bacchio, K. Cichy, M. Constantinou, J. Finkenrath, K. Hadjiyiannakou, K. Jansen, Ch. Kallidonis, K. Ottnad, M. Petschlies, A. Scapellato, F. Steffens, A. Vaquero, C. Wiese