Based on the work arXiv:1702.06558

# Features of spin dependent TMDs

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# Outline

- Introduction to TMDs
- Factorization theorems with TMDs
- Small-$b$ operator product expansion
- Helicity distribution
- Transversity and pretzelosity distributions
- Conclusions

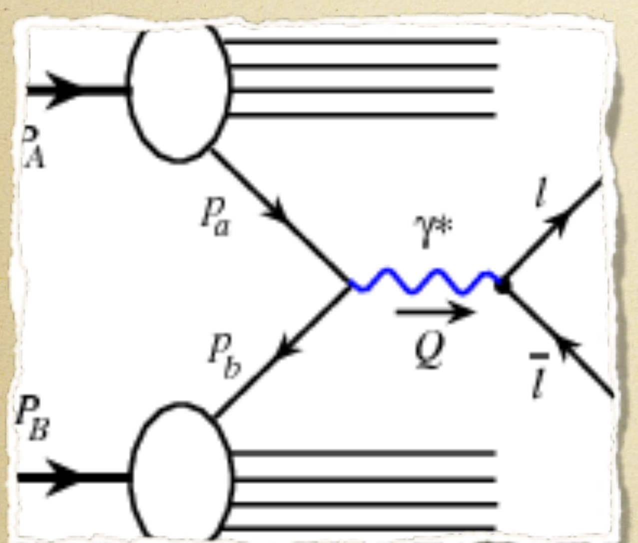
# Introduction to TMDs

TMDs offer an unified description of different experiments ⇨ Process independent

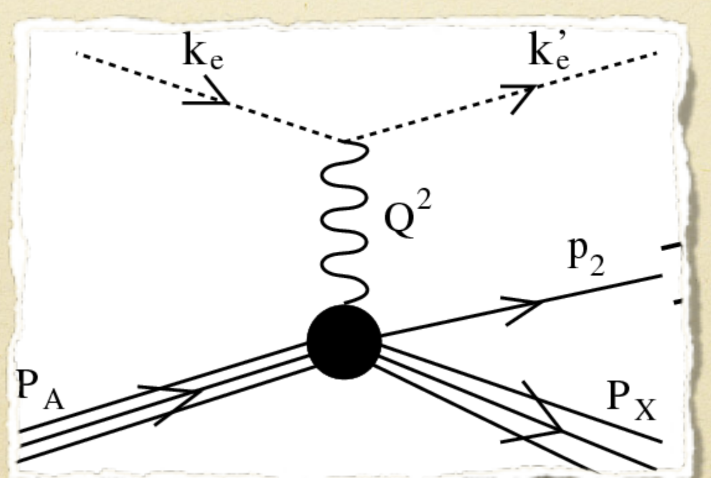
DY type experiments ⟶ TMDPDFs

SIDIS ⟶ TMDPDF and TMDFF

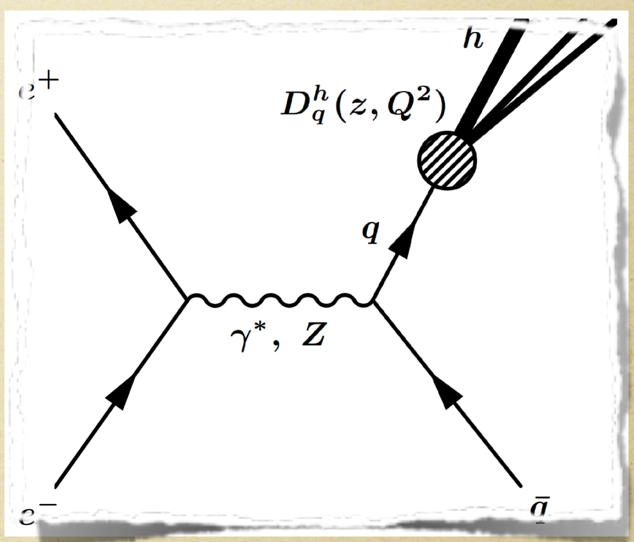
$e^+e^-$ to 2 hadrons ⟶ TMDFFs




DY




SIDIS




e+e- -> 2h

# Factorization theorems with TMDs
# Definition of Operators

TMD factorization theorems for SIDIS and DY type processes ⇨ Consistent treatment of rapidity divergences in Spin (in)dependent TMDs

⇨

Self contained definition of TMD operators ⇨ Considered individually in QFT ⇨ Without referring to a scattering process

- Quark and gluon components of the generic TMDs

$$\Phi_{ij}(x,\boldsymbol{b}) = \int \frac{d\lambda}{2\pi} e^{-ixp^+\lambda} \bar{q}_i\left(\lambda n + \boldsymbol{b}\right) \mathcal{W}(\lambda,\boldsymbol{b}) q_j\left(0\right)$$

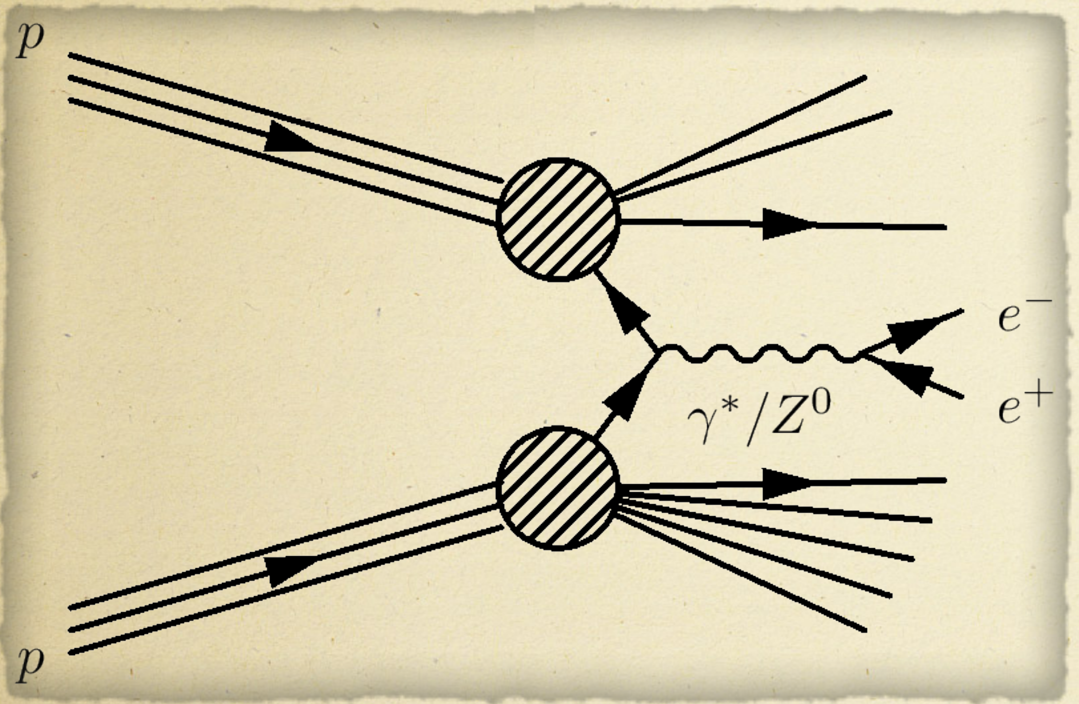
$$\Phi_{\mu\nu}(x,\boldsymbol{b}) = \frac{1}{xp^+} \int \frac{d\lambda}{2\pi} e^{-ixp^+\lambda} F_{+\mu}\left(\lambda n + \boldsymbol{b}\right) \mathcal{W}(\lambda,\boldsymbol{b}) F_{+\nu}\left(0\right)$$

- The soft function renormalizes the rapidity divergences

R-factor

$$S(\boldsymbol{b}) = \frac{\mathrm{Tr}_{\mathrm{color}}}{N_c} \langle 0| \left[ S_n^{T\dagger} \tilde{S}_{\bar{n}}^T \right] (\boldsymbol{b}) \left[ \tilde{S}_{\bar{n}}^{T\dagger} S_n^T \right] (0) |0\rangle \Longrightarrow \boxed{R_{\delta\text{-reg.}} = \frac{1}{\sqrt{S(\boldsymbol{b})}}}$$

# Factorization theorems with TMDs
# Drell-Yan cross section




We write the cross section in terms of a product of TMDPDFs!

↓

**DIFFERENT POLARIZATIONS!**

Factorization theorems allow us to write cross sections as

$$\frac{d\sigma}{dQ^2 dy d(q_T^2)} = \frac{4\pi}{3N_c}\frac{\mathcal{P}}{sQ^2}\sum_{GG'} z_{ll'}^{GG'}(q)\sum_{ff'} z_{FF'}^{GG'}|C_V(q,\mu)|^2$$

$$\int \frac{d^2\boldsymbol{b}}{4\pi} e^{i(\boldsymbol{b}\boldsymbol{q})} F_{f\leftarrow h_1}(x_1,\boldsymbol{b};\mu,\zeta) F_{f'\leftarrow h_2}(x_2,\boldsymbol{b};\mu,\zeta) + Y$$

# Spin dependent TMD decomposition

Hadron matrix elements of TMD operators with open vector and spinor indices are to be decomposed over all posible Lorentz variants ⇒ TMDPDFs

Naturally defined

Momentum space

$b$-space (IPS)

A.Vladimirov 1610.05791,
K.Goeke et al. 0504130,
A.Bachetta et al. 0803.0227

D.Boer et al. 1107.5294
M.G.Echevarria et al. 1502.05354

## Decomposition over Lorentz variants

Unpolarized quarks

Helicity quarks

$$\Phi_{q\leftarrow h,ij}(x,\boldsymbol{b})=\langle h|\Phi_{ij}(x,\boldsymbol{b})|h\rangle=\frac{1}{2}\Big(f_1\gamma^-_{ij}+g_{1L}S_L(\gamma_5\gamma^-)_{ij}$$

$$(S_T^\mu i\gamma_5\sigma^{+\mu})_{ij}h_1+(i\gamma_5\sigma^{+\mu})_{ij}\left(\frac{g_T^{\mu\nu}}{2}+\frac{b^\mu b^\nu}{\boldsymbol{b}^2}\right)\frac{S_T^\nu}{2}h_{1T}^\perp+\ldots\Big)$$

Transversity

Pretzelosity

$$\Phi_{g\leftarrow h,\mu\nu}(x,\boldsymbol{b})=\langle h|\Phi_{\mu\nu}(x,\boldsymbol{b})|h\rangle=\frac{1}{2}\Big(-g_T^{\mu\nu}f_1^g-i\epsilon_T^{\mu\nu}S_Lg_{1L}^g+2h_1^{\perp g}\left(\frac{g_T^{\mu\nu}}{2}+\frac{b^\mu b^\nu}{\boldsymbol{b}^2}\right)+\ldots\Big)$$

Unpolarized gluons

Helicity gluons

Linearly polarized gluons

# Small-$b$ operator product expansion

**Small-$b$ OPE** ⇨ Relation between TMD operators and lightcone operators

$$\Phi_{ij}(x,\boldsymbol{b}) = \Big[(C_{q\leftarrow q}(\boldsymbol{b}))^{ab}_{ij}\otimes\phi_{ab}\Big](x)+\Big[(C_{q\leftarrow g}(\boldsymbol{b}))^{\alpha\beta}_{ij}\otimes\phi_{\alpha\beta}\Big](x)+...,$$

$$\Phi_{\mu\nu}(x,\boldsymbol{b}) = \Big[(C_{g\leftarrow q}(\boldsymbol{b}))^{ab}_{\mu\nu}\otimes\phi_{ab}\Big](x)+\Big[(C_{g\leftarrow g}(\boldsymbol{b}))^{\alpha\beta}_{\mu\nu}\otimes\phi_{\alpha\beta}\Big](x)+...$$

**Notation**

$$\Phi_q^{[\Gamma]} = \frac{\mathrm{Tr}(\Gamma\Phi)}{2} \qquad \Phi_g^{[\Gamma]} = \Gamma^{\mu\nu}\Phi_{\mu\nu}$$

**Renormalization** of TMD operators

$$\Phi^{\mathrm{ren}}(x,\boldsymbol{b};\mu,\zeta) = Z(\mu,\zeta|\epsilon)R(\boldsymbol{b},\mu,\zeta|\epsilon,\delta)\Phi(x,\boldsymbol{b}|\epsilon,\delta)$$

# Small-$b$ OPE: Cancellation of rapidity divergences

- Small-$b$ OPE for a generic TMD quark operator

$$\Phi_q^{[\Gamma]} = \Gamma^{ab}\phi_{ab} + a_s C_F \boldsymbol{B}^\epsilon \Gamma(-\epsilon)\Big[\dots$$

$$+\left(\frac{1}{(1-x)_+} - \ln\left(\frac{\delta}{p^+}\right)\right)\left(\gamma^+\gamma^-\Gamma + \Gamma\gamma^-\gamma^+ + \frac{i\epsilon\gamma^+\not{b}\Gamma}{2\boldsymbol{B}} + \frac{i\epsilon\Gamma\not{b}\gamma^+}{2\boldsymbol{B}}\right)^{ab} + \dots\Big] \otimes \phi_{ab} + \mathcal{O}(a_s^2)$$

- General R-factor

$$R = 1 + 2a_s C_F \boldsymbol{B}^\epsilon \Gamma(-\epsilon)\left(\mathbf{L}_{\sqrt{\zeta}} + 2\ln\left(\frac{\delta}{p^+}\right) - \psi(-\epsilon) - \gamma_E\right) + \mathcal{O}(a_s^2)$$

**Cancellation of rapidity divergences in $R\Phi$** $\longrightarrow$

$$\boxed{\begin{array}{l}\gamma^+\Gamma = \Gamma\gamma^+ = 0 \\ \Gamma^{+\mu} = \Gamma^{-\mu} = \Gamma^{\mu+} = \Gamma^{\mu-} = 0\end{array}}$$

$$\boxed{\Gamma^q = \{\gamma^+, \gamma^+\gamma^5, \sigma^{+\mu}\}} \quad \boxed{\Gamma^g = \{g_T^{\mu\nu}, \epsilon_T^{\mu\nu}, b^\mu b^\nu/\boldsymbol{b}^2\}}$$

**Lorentz structures of "leading dynamical twist" TMDs**

| | LO | NLO | NNLO |
|---|---|---|---|
| Unpolarized | ✓ | ✓ | ✗ |
| Helicity | ✓ | ✓ | ✗ |
| Transvesity | ✓ | ✓ | ✓ |
| Pretzelosity | ✓ | ✓ | ✓ In process |
| Linearly polarized | ✓ | ✓ | ✗ |

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# Helicity distribution

# Schemes for $\gamma^5$ in DR

Lorentz structures

$\Gamma = \gamma^+\gamma^5 \quad \Gamma^{\mu\nu} = i\epsilon_T^{\mu\nu}$

$\gamma^5$ needs a definition in DR!

Projectors

$\overline{\Gamma} = \mathcal{N}_{\text{sch.}} \frac{\gamma^-\gamma^5}{2} \quad \overline{\Gamma}^{\mu\nu} = i\mathcal{N}_{\text{sch.}} \frac{\epsilon_T^{\mu\nu}}{2}$

**HVBM**

**Larin**

$$\gamma^+\gamma^5 = \frac{i}{3!}\epsilon^{+\nu\alpha\beta}\gamma_\nu\gamma_\alpha\gamma_\beta$$

$\epsilon^{\mu\nu\alpha\beta}$ is 4-dimensional

$\epsilon^{\mu\nu\alpha\beta}$ is $d$-dimensional

# $Larin^{+}$ scheme

Larin scheme is more convenient than HVBM because it does not violate Lorentz invariance, but it violates the definition of the leading dynamical twist

$$\gamma^{+}\Gamma = \gamma^{+}\left(\gamma^{+}\gamma^{5}\right)_{\text{Larin}} = \frac{i}{3!}\epsilon^{+\nu\alpha\beta}\gamma^{+}\gamma_{\nu}\gamma_{\alpha}\gamma_{\beta} \neq 0$$

Light modification of Larin scheme ⇨ $Larin^{+}$

$$(\gamma^{+}\gamma^{5})_{\text{Larin}^{+}} = \frac{i\epsilon^{+-\alpha\beta}}{2!}\gamma^{+}\gamma_{\alpha}\gamma_{\beta} = \frac{i\epsilon_{T}^{\alpha\beta}}{2!}\gamma^{+}\gamma_{\alpha}\gamma_{\beta}$$

# Small-$b$ OPE

The NLO calculation is now striaghtforward to the unpolarized calculation in

M.G.Echevarria, I.Scimemi, A.Vladimirov: 1604.07869

Matching onto integrated functions

$$R\Phi_q^{[\gamma^+\gamma_5]} = \Delta C_{q\leftarrow q} \otimes \phi_q^{[\gamma^+\gamma_5]} + \Delta C_{q\leftarrow g} \otimes \phi_g^{[\epsilon_T]}$$

$$R\Phi_g^{[\epsilon_T]} = \Delta C_{g\leftarrow q} \otimes \phi_q^{[\gamma^+\gamma_5]} + \Delta C_{g\leftarrow g} \otimes \phi_g^{[\epsilon_T]}$$

Helicity TMD distribution in the regime of small-$b$

$$g_{1L}(x,\boldsymbol{b}) = [\Delta C_{q\leftarrow q}(\boldsymbol{b}) \otimes \Delta f_q](x) + [\Delta C_{q\leftarrow g}(\boldsymbol{b}) \otimes \Delta f_g](x) + \mathcal{O}(\boldsymbol{b}^2)$$

$$g_{1L}^g(x,\boldsymbol{b}) = [\Delta C_{g\leftarrow q}(\boldsymbol{b}) \otimes \Delta f_q](x) + [\Delta C_{g\leftarrow g}(\boldsymbol{b}) \otimes \Delta f_g](x) + \mathcal{O}(\boldsymbol{b}^2)$$

# Matching coefficients: scheme dependence

$$\Delta C_{q\leftarrow q} = \delta(\bar{x}) + a_s C_F \left\{ 2\boldsymbol{B}^{\epsilon}\Gamma(-\epsilon)\Big[\frac{2}{(1-x)_+} - 2 + \bar{x}(1+\epsilon)\mathcal{H}_{\text{sch.}} + \delta(\bar{x})\left(\mathbf{L}_{\sqrt{\zeta}} - \psi(-\epsilon) - \gamma_E\right)\Big] \right\}_{\epsilon\text{-finite}}$$

$$\Delta C_{q\leftarrow g} = a_s C_F \left\{ 2\boldsymbol{B}^{\epsilon}\Gamma(-\epsilon)\Big[x - \bar{x}\mathcal{H}_{\text{sch.}}\Big] \right\}_{\epsilon\text{-finite}}$$

$$\Delta C_{g\leftarrow q} = a_s C_F \left\{ 2\boldsymbol{B}^{\epsilon}\Gamma(-\epsilon)\Big[1 + \bar{x}\mathcal{H}_{\text{sch.}}\Big] \right\}_{\epsilon\text{-finite}}$$

$$\Delta C_{g\leftarrow g} = \delta(\bar{x}) + a_s C_A \left\{ 2\boldsymbol{B}^{\epsilon}\Gamma(-\epsilon)\frac{1}{x}\Big[\frac{2}{(1-x)_+} - 2 - 2x^2 + 2x\bar{x}\mathcal{H}_{\text{sch.}} + \delta(\bar{x})\left(\mathbf{L}_{\sqrt{\zeta}} - \psi(-\epsilon) - \gamma_E\right)\Big] \right\}_{\epsilon\text{-finite}}$$

$$\mathcal{H}_{\text{sch.}} = \begin{cases} 1+2\epsilon & \text{HVBM} \\ \dfrac{1+\epsilon}{1-\epsilon} & Larin^{+} \end{cases}$$

$\longrightarrow$ **At NLO there is not scheme dependence!**

# Transversity and pretzelosity distributions

# Lorentz structure and matching

Usual spinor structure

$$\Gamma = i\gamma_5\sigma^{+\mu}$$

Scheme dependent

Not mixture with gluons at leading twist

Common spinor structure

$$\Gamma = \sigma^{+\mu}$$

Scheme independent!

Calculating $R\Phi$ and comparing with the general parameterization

$$R\Phi_q^{[\sigma^{+\mu}]} = g_T^{\mu\nu}\,\delta C_{q\leftarrow q}\otimes\phi_q^{[\sigma^{+\nu}]} + \left(\frac{b^\mu b^\nu}{\boldsymbol{b}^2} + \frac{g_T^{\mu\nu}}{2(1-\epsilon)}\right)\delta^\perp C_{q\leftarrow q}\otimes\phi_q^{[\sigma^{+\nu}]}$$

Transversity - Transversity matching

Pretzelosity - Transversity matching

# Matching coefficients

**Transversity - Transversity** small-$b$ expression

$$h_1(x,\boldsymbol{b}) = \Big[\delta C_{q\leftarrow q}(\boldsymbol{b}) \otimes \delta f_q\Big](x) + \mathcal{O}(\boldsymbol{b}^2)$$

Agrees with A.Bachetta, A.Prokudin 1303.2129!

NLO matching coefficient

$$\delta C_{q\leftarrow q} = \delta(\bar{x}) + a_s C_F\Big(-2\mathbf{L}_\mu \delta p_{qq} + \delta(\bar{x})\left(-\mathbf{L}_\mu^2 + 2\mathbf{L}_\mu \mathbf{l}_\zeta - \zeta_2\right)\Big) + \mathcal{O}(a_s^2)$$

Calculations at NNLO are in progress!! 

**Pretzelosity - Transversity** small-$b$ expression

Non zero!

$$h_{1T}^\perp(x,\boldsymbol{b}) = \Big[\delta^\perp C_{q\leftarrow q}(\boldsymbol{b}) \otimes \delta f_q\Big](x) + \mathcal{O}(\boldsymbol{b}^2) = \Big[\left(0 + \mathcal{O}(a_s^2)\right) \otimes \delta f_q\Big](x) + \mathcal{O}(\boldsymbol{b}^2)$$

NLO matching coefficient

At NLO the coefficient is $\sim \epsilon$

$$\delta^\perp C_{q\leftarrow q} = -4 a_s C_F \boldsymbol{B}^\epsilon \Gamma(-\epsilon)\bar{x}\epsilon^2$$

This observation is supported by the measurement of $\sin(3\phi_h - \phi_S)$ asymmetries by HERMES and COMPASS!
C.Lefky, A.Prokudin 1411.0580

# Conclusions

- We have provided a complete discussion on the matching of TMDs to the twist-2 ⇨ Small-$b$ OPE ⇨ complete set of NLO TMD matching coefficients

- The evaluation of the OPE for a general operator restricts the Lorentz structures ⇨ Leading dynamical twist

- Different schemes for $\gamma^5$ in DR ⇨ Larin scheme does not support the condition of leading dynamical twist ⇨ $Larin^+$ ⇨ At NLO the difference between schemes arises only in the $\epsilon$ -suppressed terms

- Pretzelosity has $\epsilon$-suppressed matching coefficient ⇨ non zero at NNLO. Natural explanation of its smallness in phenomenological analyses

- Complete $\epsilon$-dependent expressions ⇨ Open the path to the NNLO evaluation (work in progress!) and phenomenological studies (fits planned!, for unpolarized fits see A. Vladimirov talk)


# REF 2017

November 13-16
Madrid, Spain

For any question ask me!

You can see the web page
http://jacobi.fis.ucm.es/REF2017/

Also you can send an email to
dangut01@ucm.es
ignazios@fis.ucm.es

Thanks!!!

# Back up

# Drawback of schemes. $Z^5_{qq}$ renormalization constant

**Drawback** of both schemes $\Rightarrow$ Violation of Adler-Bardeen theorem $\Rightarrow$ Non renormalization of the axial anomaly

Fixed by an extra renormalization constant, $Z^5_{qq}$ $\Rightarrow$ Derived from a external condition

S.A. Larin 9302240, Y.Matiouine et al 076002, V.Ravindran et al. 0311304

Only affect to the quark-to-quark part

- At large $q_T$ TMD factorization reproduces collinear factorization $\Rightarrow$ It is natural to normalize Helicity TMDPDF $\Rightarrow$ It reproduces polarized DY which is normalized to unpolarized DY
- Equivalent in TMDs $\Rightarrow$ Equality in polarized and unpolarized coefficients

$$\Big[Z^5_{qq}(\boldsymbol{b}) \otimes \Delta C_{q\leftarrow q}(\boldsymbol{b})\Big](x) = C_{q\leftarrow q}(x,\boldsymbol{b})$$

$$\downarrow$$

$$\boxed{Z^5_{qq} = \delta(\bar{x}) + 2a_s C_F \boldsymbol{B}^{\epsilon}\Gamma(-\epsilon)\left(1-\epsilon-(1+\epsilon)\mathcal{H}_{\text{sch.}}\right)\bar{x}}$$

# Transversity and pretzelosity matchings at NNLO

We have some preliminar results for the transversity matching at NNLO for the $q \leftarrow q$ and $q \leftarrow \bar{q}$ cases.
We show part of the the $q \leftarrow q$ result for the $T_r N_f$ part written in the form

$$\delta C^{[2]}_{T_r N_f} = \sum_k \delta C^{(2;k)}_{T_r N_f} L^k_\mu$$

$$\delta C^{(2;0)}_{T_r N_f} = \left(-\frac{4}{3} + \frac{296x}{27} - \frac{4x^2}{3} + \frac{40}{9} x \ln x + \frac{4}{3} x \ln^2 x\right) \frac{1}{(1-x)_+} + \delta(\bar{x}) \left(\frac{2}{3} + \frac{10\pi^2}{9}\right)$$

For the pretzelosity matching at NNLO we have only a preliminar result for the $q \leftarrow \bar{q}$ case.

It is different for zero!

# Helicity matching coefficients: NLO results

At $\epsilon \to 0$ we have the NLO coefficients

$$\Delta C_{q\leftarrow q} \equiv C_{q\leftarrow q} = \delta(\bar{x}) + a_s C_F \Big( -2\mathbf{L}_\mu \Delta p_{qq} + 2\bar{x} + \delta(\bar{x}) \left(-\mathbf{L}_\mu^2 + 2\mathbf{L}_\mu \mathbf{l}_\zeta - \zeta_2\right) \Big) + \mathcal{O}(a_s^2)$$

$$\Delta C_{q\leftarrow g} = a_s T_F \left(-2\mathbf{L}_\mu \Delta p_{qg} + 4\bar{x}\right) + \mathcal{O}(a_s^2)$$

$$\Delta C_{g\leftarrow q} = a_s C_F \left(-2\mathbf{L}_\mu \Delta p_{gq} - 4\bar{x}\right) + \mathcal{O}(a_s^2)$$

$$\Delta C_{g\leftarrow g} = \delta(\bar{x}) + a_s C_A \Big( -2\mathbf{L}_\mu \Delta p_{gg} - 8\bar{x} + \delta(\bar{x}) \left(-\mathbf{L}_\mu^2 + 2\mathbf{L}_\mu \mathbf{l}_\zeta - \zeta_2\right) \Big) + \mathcal{O}(a_s^2)$$

where $\Delta p_{ij}$ ⇨ Splitting kernels + anomalous dimensions

These results agree with the obtained in
M.G.Echevarría et al. 1502.05354
A.Bachetta A.Prokudin 1303.2129!!

# Linearly polarized gluons matching coefficients

Small-$b$ expression for the linearly polarized gluon TMDPDF

$$h_1^{\perp g}(x,\boldsymbol{b}) = [\delta^L C_{g\leftarrow q}(\boldsymbol{b})\otimes f_q](x) + [\delta^L C_{g\leftarrow g}(\boldsymbol{b})\otimes f_g](x) + \mathcal{O}(\boldsymbol{b}^2)$$

NLO matching coefficients

$$\delta^L C_{g\leftarrow g} = -4a_s C_A \frac{\bar{x}}{x} + \mathcal{O}(a_s^2) \qquad \delta^L C_{g\leftarrow q} = -4a_s C_F \frac{\bar{x}}{x} + \mathcal{O}(a_s^2)$$

These results agree with the obtained in
T. Becher et al. 1212.2621!!