Probing the Gluon Sivers Function in $p^{\uparrow}p \rightarrow J/\psi + X$ and $p^{\uparrow}p \rightarrow D + X$

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Summary

- Introduction and motivations
- The GPM approach and its CGI extension: Theoretical framework
- GSF: Universality and process dependence in the (CGI)-GPM
- Results for $p^{\uparrow}p \rightarrow J/\psi + X$ and $p^{\uparrow}p \rightarrow D + X$ and discrimination power
- Conclusions and outlook for EIC

Work done in collaboration with U. D'Alesio, C. Pisano, P. Taels, arXiv :1705.04169

Introduction and motivations

- Gluon Sivers function(s) [GSF] almost unknown
- (TMD) Gluon distributions crucial for EIC physics programme
- Requires study of gluon-dominated processes
- Generalized Parton Model: most simple generalization of QCD-improved parton model
- Assumes factorization and universality of TMD PDFs and FFs: strong constraint, hopefully easier to check experimentally, useful as a first step to assess the size of possible/expected factorization-breaking terms, also for single energy-scale processes
- Color Gauge Invariant (CGI) extension of GPM: first performed for quark subprocesses by L. Gamberg and Z.B. Kang [PLB 696 (2011)] here extended to gluon subprocesses for the first time; for process dependence of quark SF in $p^{\uparrow}p \to \text{jet}\,\pi + X$ see [D'Alesio et al. - PLB 704 (2011)]
- $p^{\uparrow}p \to J/\psi + X$ and $p^{\uparrow}p \to D + X$: a case study for testing universality and process dependence of the GSF(s), and the GPM versus its CGI version
- Comparison and discrimination between the CGI-GPM, the collinear twist-3 approach, and the TMD factorization scheme for processes with two distinct energy scales

Cross section and SSA for pp → J/ψ + X: GPM and Color Singlet Model

Partonic level subprocess

$$g(p_a) + g(p_b) \;\to\; Q\overline{Q}[^3S_1^{(1)}](p_Q) + g(p_g)$$

$$\mathrm{d}\sigma \equiv E_Q \frac{\mathrm{d}\sigma}{\mathrm{d}^3\boldsymbol{p}_Q} = \frac{\alpha_s^3}{s} \int \frac{\mathrm{d}x_a}{x_a} \frac{\mathrm{d}x_b}{x_b}\, \mathrm{d}^2\boldsymbol{k}_{\perp a}\, \mathrm{d}^2\boldsymbol{k}_{\perp b}\, f_{g/p}(x_a, k_{\perp a})\, f_{g/p}(x_b, k_{\perp b})\, H^U_{gg\to J/\psi g}(\hat{s}, \hat{t}, \hat{u})\, \delta(\hat{s} + \hat{t} + \hat{u} - M^2)$$

$$H^U_{gg\to J/\psi g} = \frac{5}{9}\, |R_0(0)|^2\, M\, \frac{\hat{s}^2(\hat{s}-M^2)^2 + \hat{t}^2(\hat{t}-M^2)^2 + \hat{u}^2(\hat{u}-M^2)^2}{(\hat{s}-M^2)^2(\hat{t}-M^2)^2(\hat{u}-M^2)^2}$$

$$f_{g/p}(x, k_\perp) = f_{g/p}(x)\, \frac{1}{\pi \langle k_\perp^2 \rangle}\, e^{-k_\perp^2/\langle k_\perp^2\rangle}$$

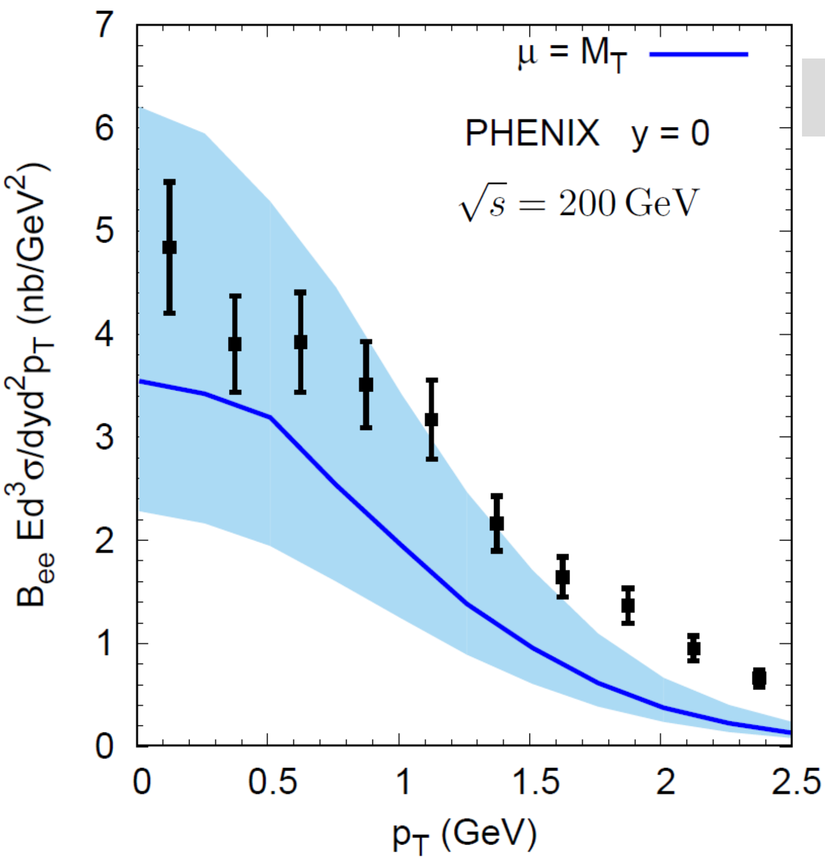
Collinear DGLAP evolution implemented for unpol. PDFs

$$\langle k_\perp^2 \rangle = 1\mathrm{GeV}^2, \qquad M_T/2 \le \mu \le 2M_T\,, \qquad M_T = \sqrt{\boldsymbol{p}_T^2 + M^2}$$

$$|R_0(0)|^2 = 1.01\,\mathrm{GeV}^3\,, \qquad \mathrm{Br}(J/\psi \to e^+e^-) = 0.0597\,, \qquad M = 3.097\,\mathrm{GeV}$$

Data include feed-down contributions; expected fraction of direct J/ψ production: 0.58

Unpolarized cross section for pp → J/ψ + X: results vs. RHIC data



PHENIX PRD82 012001 (2010)

Reasonably well reproduced at $p_T \leq 2$ GeV

In agreement with Brodsky and Lansberg results, PLB 695, 149 (2011), PRD 81, 051502 (2010)

NLO QCD corrections and intrinsic charm can further improve the theoretical description

Role of color octet states, relevant at high p_T, much less important in this kinematical regime

FIG. 1: Unpolarized cross section for the process $pp \to J/\psi\, X \to e^+e^-\, X$, at $\sqrt{s} = 200$ GeV in the central rapidity region $y = 0$, as a function of the transverse momentum p_T of the J/ψ. The theoretical curve is obtained adopting the Generalized Parton Model and the color singlet production mechanism for the quarkonium. Data are taken from Ref. [50]. The uncertainty band results from varying the factorization scale in the range $M_T/2 \leq \mu \leq 2M_T$.

Single spin asymmetry for $p^{\uparrow}p \to J/\psi + X$ in the GPM framework

$$A_N \equiv \frac{\mathrm{d}\sigma^{\uparrow} - \mathrm{d}\sigma^{\downarrow}}{\mathrm{d}\sigma^{\uparrow} + \mathrm{d}\sigma^{\downarrow}} \equiv \frac{\mathrm{d}\Delta\sigma}{2\mathrm{d}\sigma}$$

$$\mathrm{d}\Delta\sigma^{\mathrm{GPM}} \equiv \frac{E_{\mathcal{Q}}\,\mathrm{d}\sigma^{\uparrow}}{\mathrm{d}^3\boldsymbol{p}_{\mathcal{Q}}} - \frac{E_{\mathcal{Q}}\,\mathrm{d}\sigma^{\downarrow}}{\mathrm{d}^3\boldsymbol{p}_{\mathcal{Q}}} = \frac{2\alpha_s^3}{s}\int \frac{\mathrm{d}x_a}{x_a}\,\frac{\mathrm{d}x_b}{x_b}\,\mathrm{d}^2\boldsymbol{k}_{\perp a}\,\mathrm{d}^2\boldsymbol{k}_{\perp b}$$
$$\times \left(-\frac{k_{\perp a}}{M_p}\right) f_{1T}^{\perp g}(x_a, k_{\perp a}) \cos\phi_a \; f_{g/p}(x_b, k_{\perp b})\; H^{U}_{gg\to J/\psi g}(\hat{s},\hat{t},\hat{u})\,\delta(\hat{s}+\hat{t}+\hat{u}-M^2)$$

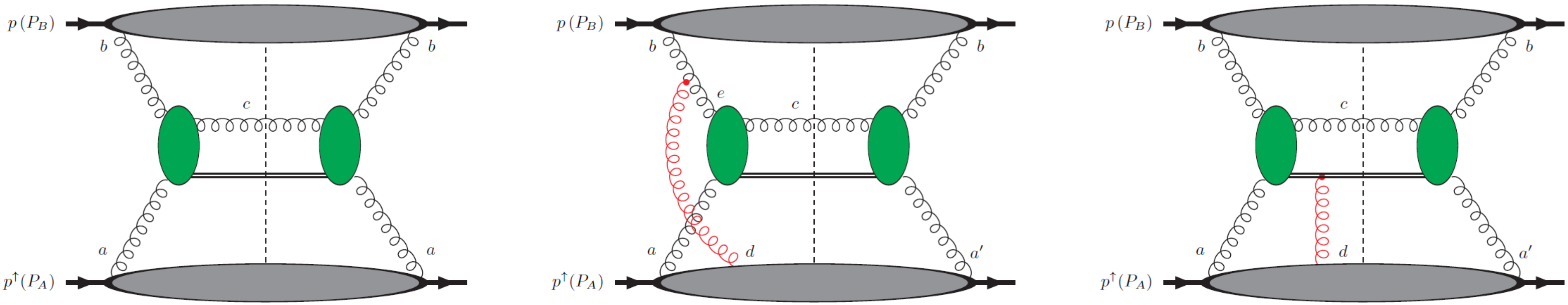
Positivity bound $$\frac{k_{\perp}}{M_p}\,|f_{1T}^{\perp a}(x_a, k_{\perp a})| \le f_{a/p}\,(x_a, k_{\perp a})$$ **Automatically fulfilled for any $(x, k_{\perp})$ values**

$$\Delta^N f_{g/p^{\uparrow}}(x,k_{\perp}) = \left(-2\frac{k_{\perp}}{M_p}\right) f_{1T}^{\perp g}(x,k_{\perp}) = 2\,\frac{\sqrt{2e}}{\pi}\,\mathcal{N}_g(x)\, f_{g/p}(x)\,\sqrt{\frac{1-\rho}{\rho}}\,k_{\perp}\,\frac{e^{-k_{\perp}^2/\rho\langle k_{\perp}^2\rangle}}{\langle k_{\perp}^2\rangle^{3/2}}$$

$$\mathcal{N}_g(x) = N_g x^{\alpha}(1-x)^{\beta}\,\frac{(\alpha+\beta)^{(\alpha+\beta)}}{\alpha^{\alpha}\beta^{\beta}}\,, \qquad |N_g| \le 1\,, \qquad 0<\rho<1$$

Sivers function also constrained by Burkardt Sum Rule; available fits from SIDIS data for q, $\bar{q}$ SFs almost saturate, within uncertainties, the BSR; consistent with large-N_c QCD arguments

SSA for $p^{\uparrow}p \rightarrow J/\psi + X$ in the Color-Gauge-Invariant GPM framework



(a) (b) (c)

- **In the CGI-GPM initial (ISI) and final (FSI) state interactions are approximated by a single eikonal gluon exchange, corresponding to the LO contribution of the Wilson line in a power expansion in terms of the strong coupling constant**
- **FSIs do not contribute for a J/ψ produced in a color singlet state**
- **Moreover, FSIs vanish for the final unobserved gluon when summing over all different cut diagrams**

SSA for $p^{\uparrow}p \to J/\psi + X$ in the Color-Gauge-Invariant GPM framework - 2

The squared amplitude for the ISI contribution (b) can be obtained from the Born level one (a) with the replacement

$$\varepsilon^{\nu}_{\lambda_b}(p_b) \longrightarrow \varepsilon^{\sigma}_{\lambda_b}(p_b)\left[g_s A^{+}(k)\,\frac{1}{k^{+}+i\epsilon}\right] T^{d}_{eb}\,, \qquad \frac{1}{k^{+}\pm i\epsilon} = \mathrm{P}\,\frac{1}{k^{+}} \mp i\pi\delta(k^{+})$$

- **The imaginary part of the eikonal propagator provides the phase needed to generate the Sivers asymmetry. The corresponding amplitude is given by the Born amplitude multiplied by a different color factor, because of the extra gluon**
- **Due to the 3-gluon vertex involved, there are two different ways of neutralize color. Therefore, two independent GSFs with different properties (charge-conjugation, evolution, contribution to BSR, small-x behaviour, etc.)**
- **Formally, all this corresponds to the following substitution, in the numerator of the asymmetry, when going from the GPM to its CGI version**

$$f_{1T}^{\perp\,g}\,H^{U}_{gg\to J/\psi g} \longrightarrow f_{1T}^{\perp\,g\,(f)}\,H^{\mathrm{Inc}\,(f)}_{gg\to J/\psi g} + f_{1T}^{\perp\,g\,(d)}\,H^{\mathrm{Inc}\,(d)}_{gg\to J/\psi g}$$

SSA for $p^{\uparrow}p \to J/\psi + X$ in the Color-Gauge-Invariant GPM framework - 3

$$H^{\mathrm{Inc}\,(f/d)}_{gg\to J/\psi g} \equiv \frac{C_I^{\mathrm{Inc}\,(f/d)}}{C_U}\, H^U_{gg\to J/\psi g} = \frac{C_I^{(f/d)} + C_{F_c}^{(f/d)}}{C_U}\, H^U_{gg\to J/\psi g}$$

$$C_U = \frac{5}{288}\,, \quad C_I^{(f)} = -\frac{1}{2}\, C_U\,, \quad C_I^{(d)} = 0\,, \quad C_{F_c}^{(f,d)} = 0 \quad \text{for} \quad J/\psi[{}^3S_1^{(1)}]$$

Therefore the numerator of the SSA for $p^{\uparrow}p \to J/\psi + X$ in the CGI-GPM/color singlet model reads

$$\begin{aligned} \mathrm{d}\Delta\sigma^{\mathrm{CGI}} \equiv\; & \frac{E_{\mathcal{Q}}\,\mathrm{d}\sigma^{\uparrow}}{d^3\boldsymbol{p}_{\mathcal{Q}}} - \frac{E_{\mathcal{Q}}\,\mathrm{d}\sigma^{\downarrow}}{\mathrm{d}^3\boldsymbol{p}_{\mathcal{Q}}} = \frac{2\alpha_s^3}{s} \int \frac{\mathrm{d}x_a}{x_a}\,\frac{\mathrm{d}x_b}{x_b}\, \mathrm{d}^2\boldsymbol{k}_{\perp a}\, \mathrm{d}^2\boldsymbol{k}_{\perp b} \\ & \times \left(-\frac{k_{\perp a}}{M_p}\right) f_{1T}^{\perp\, g\,(f)}(x_a, k_{\perp a}) \cos\phi_a\, f_{g/p}(x_b, k_{\perp b}) \left(-\frac{1}{2}\, H^U_{gg\to J/\psi g}(\hat{s}, \hat{t}, \hat{u})\right) \delta(\hat{s} + \hat{t} + \hat{u} - M^2) \end{aligned}$$

Very useful to gather separate and direct information on the f-type GSF
(in kinematical regimes where the color singlet contribution dominates)

SSA for p↑p → J/ψ + X in the (CGI) GPM approaches - Results

$$\Delta^N f_{g/p^\uparrow}(x,k_\perp) = \left(-2\frac{k_\perp}{M_p}\right) f_{1T}^{\perp g}(x,k_\perp) = 2\,\frac{\sqrt{2e}}{\pi}\,\mathcal{N}_g(x)\,f_{g/p}(x)\,\sqrt{\frac{1-\rho}{\rho}}\,k_\perp\,\frac{e^{-k_\perp^2/\rho\langle k_\perp^2\rangle}}{\langle k_\perp^2\rangle^{3/2}}$$

$$\mathcal{N}_g(x) = N_g x^\alpha (1-x)^\beta \,\frac{(\alpha+\beta)^{(\alpha+\beta)}}{\alpha^\alpha \beta^\beta}\,, \qquad |N_g| \le 1\,, \qquad 0 < \rho < 1$$

No use of previous (scarce) information on GSF
Generation of the bands of all allowed values up to the maximized ones

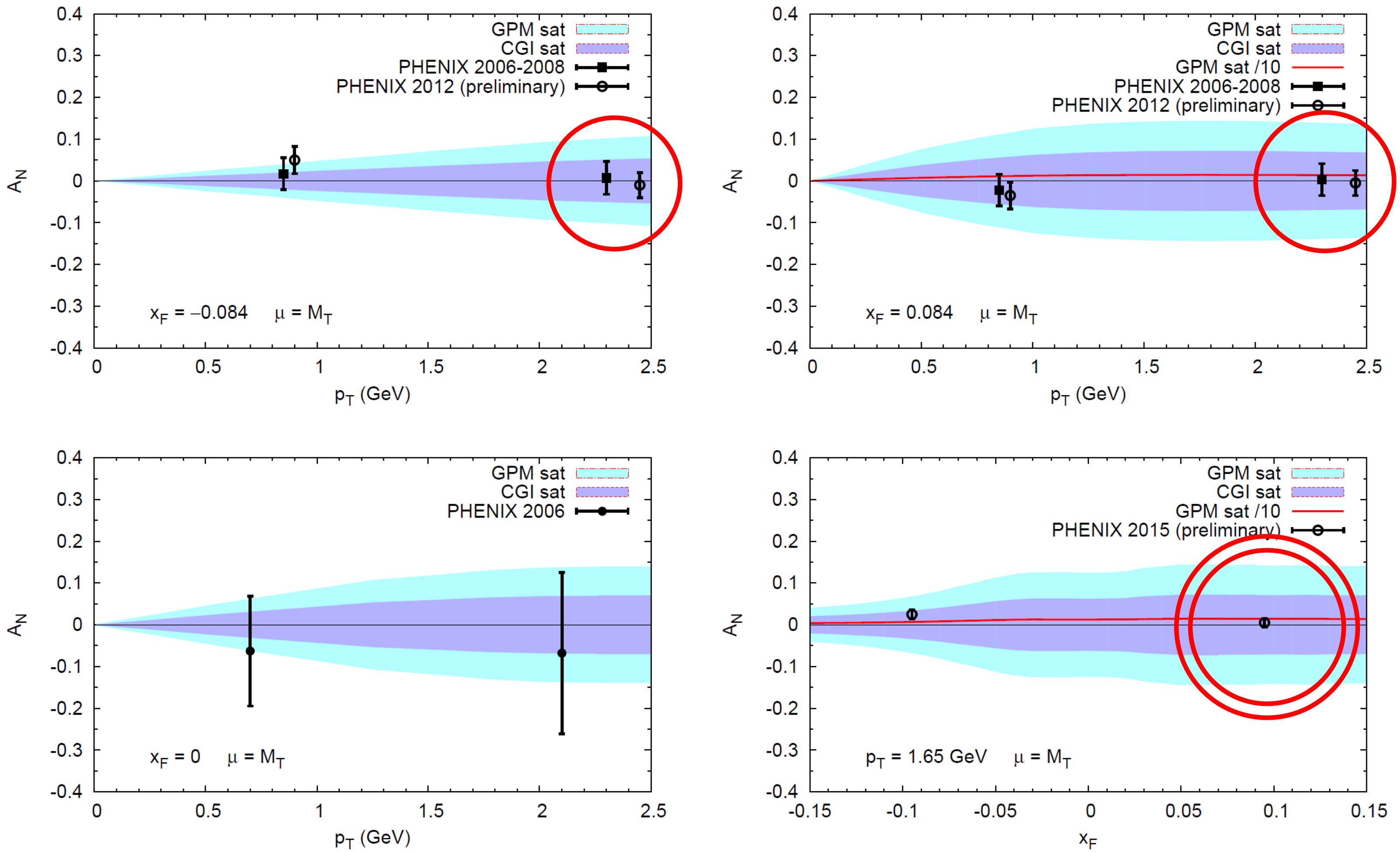
$$-1 \le \mathcal{N}_g(x) \le +1\,, \qquad \rho = 2/3$$

D'Alesio, FM, Pisano
JHEP 1509 (2015) 119

$$\langle k_\perp^2\rangle = 1\,\mathrm{GeV}^2, \qquad \mu = M_T = \sqrt{\boldsymbol{p}_T^2 + M^2}$$

$$\mathrm{CTEQ6-LO\ set\ for\ collinear\ unpolarized\ PDFs}$$

SSA for $p^{\uparrow}p \to J/\psi + X$ in the (CGI) GPM approaches – Results (2)



Red solid lines: GPM with $N_g(x) = +0.1$

SSA for $p^{\uparrow}p \to J/\psi + X$ in the (CGI) GPM approaches – Remarks

- **CGI-GPM (maximized) results are a factor 2 smaller in size w.r.t. GPM ones (and have opposite sign)**
- **SSAs much less sensitive to factorization/evolution scale than the unpol. cross sect.**
- **PHENIX 2006 data at $x_F = 0$ unable to constraint/discriminate the two approaches**
- **Combined 2006-2008 and preliminary 2012 data at $x_F = \pm\ 0.084$ vs. p_T precise enough to constrain in magnitude the GSF in the GPM approach**
- **Latest preliminary RUN 2015 data at fixed $p_T = 1.65$ vs. x_F even more precise: they could constrain the GSF in size both in the GPM and CGI_GPM schemes**
- **Overall precision and amount of present data does not allow to reject any of two approaches or discriminate between them**
- **Additional information on the size and sign of the f-type GSF (and of the d-type GSF not playing a role here) may be gathered by a combined study of the $p^{\uparrow}p \to D + X$ process**

Cross section for $p^{\uparrow}p \to D + X$ in the (CGI) GPM approaches

$$2\mathrm{d}\sigma \equiv \frac{E_D\,\mathrm{d}\sigma^{\uparrow}}{\mathrm{d}^3\boldsymbol{p}_D} + \frac{E_D\,\mathrm{d}\sigma^{\downarrow}}{\mathrm{d}^3\boldsymbol{p}_D} = \frac{2\alpha_s^2}{s}\int \frac{\mathrm{d}x_a}{x_a}\frac{\mathrm{d}x_b}{x_b}\,\mathrm{d}z\,\mathrm{d}^2\boldsymbol{k}_{\perp a}\,\mathrm{d}^2\boldsymbol{k}_{\perp b}\,\mathrm{d}^3\boldsymbol{k}_D\,\delta(\boldsymbol{k}_D\cdot\hat{\boldsymbol{p}}_Q)\,\delta(\hat{s}+\hat{t}+\hat{u}-2m_c^2)$$

$$\times\quad \mathcal{J}(z,\boldsymbol{k}_D)\Big\{\sum_q\Big[f_{q/p}(x_a,k_{\perp a})\,f_{\bar{q}/p}(x_b,k_{\perp b})\,H^U_{q\bar{q}\to Q\overline{Q}}(\hat{s},\hat{t},\hat{u})\,D_{D/Q}(z,\boldsymbol{k}_D)\Big]$$

$$+\Big[f_{g/p}(x_a,k_{\perp a})\,f_{g/p}(x_b,k_{\perp b})\,H^U_{gg\to Q\overline{Q}}(\hat{s},\hat{t},\hat{u})\,D_{D/Q}(z,\boldsymbol{k}_D)\Big]\Big\}$$

$q = u, \bar{u}, d, \bar{d}, s, \bar{s}$ $\quad Q = c$ for D^+, D^0, $\quad Q = \bar{c}$ for $D^-, \bar{D}^0$

cross section is the same for D^+, D^- and for $D^0, \bar{D}^0$

$$H^U_{q\bar{q}\to c\bar{c}} = \frac{N_c^2-1}{2N_c^2}\left(\frac{\tilde{t}^2+\tilde{u}^2+2m_c^2\tilde{s}}{\tilde{s}^2}\right),$$

$$H^U_{gg\to c\bar{c}} = \frac{N_c}{N_c^2-1}\,\frac{1}{\tilde{t}\tilde{u}}\left(\frac{N_c^2-1}{2N_c^2}-\frac{\tilde{t}\tilde{u}}{\tilde{s}^2}\right)\left(\tilde{t}^2+\tilde{u}^2+4m_c^2\tilde{s}-\frac{4m_c^4\tilde{s}^2}{\tilde{t}\tilde{u}}\right)$$

$$\tilde{s} \equiv (p_a+p_b)^2 = \hat{s}\,, \qquad \tilde{t} \equiv (p_a-p_c)^2 - m_c^2 = \hat{t} - m_c^2\,, \qquad \tilde{u} \equiv (p_b-p_c)^2 - m_c^2 = \hat{u} - m_c^2$$

Single spin asymmetry for p$^\uparrow$p → D + X in the GPM framework

$$
\begin{aligned}
d\Delta\sigma^{\rm GPM} \equiv \; & \frac{E_D\, d\sigma^\uparrow}{d^3\boldsymbol{p}_D} - \frac{E_D\, d\sigma^\downarrow}{d^3\boldsymbol{p}_D} = \frac{2\alpha_s^2}{s}\int \frac{dx_a}{x_a}\frac{dx_b}{x_b}\, dz\, d^2\boldsymbol{k}_{\perp a}\, d^2\boldsymbol{k}_{\perp b}\, d^3\boldsymbol{k}_D\, \delta(\boldsymbol{k}_D\cdot\hat{\boldsymbol{p}}_c)\,\delta(\hat{s}+\hat{t}+\hat{u}-2m_c^2) \\
\times \; & \mathcal{J}(z,\boldsymbol{k}_D)\left\{\sum_q \left[\left(-\frac{k_{\perp a}}{M_p}\right) f_{1T}^{\perp q}(x_a,k_{\perp a})\cos\phi_a\, f_{\bar{q}/p}(x_b,k_{\perp b})\, H^U_{q\bar{q}\to Q\overline{Q}}(\hat{s},\hat{t},\hat{u})\, D_{D/Q}(z,\boldsymbol{k}_D)\right]\right. \\
+ \; & \left.\left[\left(-\frac{k_{\perp a}}{M_p}\right) f_{1T}^{\perp g}(x_a,k_{\perp a})\cos\phi_a\, f_{g/p}(x_b,k_{\perp b})\, H^U_{gg\to Q\overline{Q}}(\hat{s},\hat{t},\hat{u})\, D_{D/Q}(z,\boldsymbol{k}_D)\right]\right\}
\end{aligned}
$$

- **Gluons do not carry any transverse polarization ⟶ unpolarized final heavy quarks**
- **Quark s-channel annihilation gives no spin transfer to the final heavy quarks too**
- **No Collins fragmentation contribution to the SSA**

- **All allowed contributions to the SSA other than the Sivers effect are strongly suppressed after integration over transverse momenta (because of azimuthal phase factors).**
- **They can be safely neglected**

SSA for p$^\uparrow$p → D + X in the Color-Gauge-Invariant GPM framework

Again the Sivers functions become process dependent
Both ISIs and FSIs are present and are taken into account

- $q\bar{q} \to Q\bar{Q}$ **subprocesses**

- **(anti)quark Sivers distributions extracted from fits to SIDIS data can still be used**
- **Calculable process-dependent coefficients are absorbed into the modified hard functions**
- **Results are in agreement with the corresponding ones in the twist-3 approach[Koike, Yoshida PRD84,014026 (2011)] and, in the massless limit, with the CGI-GPM calculations of Gamberg and Kang [PLB696, 109 (2011)]**

$$H^{\rm Inc}_{q\bar{q}\to c\bar{c}} = -H^{\rm Inc}_{\bar{q}q\to \bar{c}c} = \frac{N_c^2-1}{2N_c^2}\left(\frac{\tilde{t}^2+\tilde{u}^2+2m_c^2\tilde{s}}{\tilde{s}^2}\right) = H^U_{q\bar{q}\to c\bar{c}}$$

$$H^{\rm Inc}_{q\bar{q}\to \bar{c}c} = -H^{\rm Inc}_{\bar{q}q\to c\bar{c}} = \frac{3}{2}\,\frac{1}{N_c^2}\left(\frac{\tilde{t}^2+\tilde{u}^2+2m_c^2\tilde{s}}{\tilde{s}^2}\right) = \frac{3}{N_c^2-1}\,H^U_{q\bar{q}\to c\bar{c}}$$

SSA for $p^{\uparrow}p \to D + X$ in the Color-Gauge-Invariant GPM framework

- $gg \to Q\bar{Q}$ subprocesses

Effects of ISIs and FSIs have to be estimated diagram by diagram

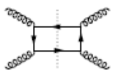
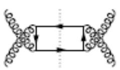
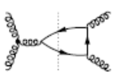
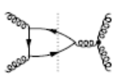
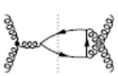
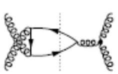
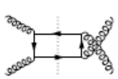
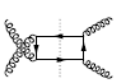
- C_U : usual unpol. CF for a specific diagram D
- $C_I^{(f,d)}$: CF for extra gluon attached to the gluon from unpol. proton in D
- $C_{F_c}^{(f,d)}$: CF for extra gluon attached to the fragmenting heavy (anti)quark in D
- $C_{F_d}^{(f,d)}$: CF for extra gluon attached to the unobserved heavy (anti)quark in D

$$C_G^{(f/d)} \equiv \frac{C_I^{(f/d)} + C_{F_c}^{(f/d)} + C_{F_d}^{(f/d)}}{C_U}$$

gluonic pole strenghts

Our results are in complete agreement with those, obtained adopting a different method, by C. J. Bomhof and P. J. Mulders, JHEP 0702 (2007) 029, for the study of the GSF in less inclusive processes, like e.g. $p^{\uparrow}p \to \pi\,\pi + X$, for which FSIs for the unobserved parton d need to be taken into account as well

CGI-GPM color factors for LO diagrams contributing to the gg →$c\bar{c}$ process

D	C_U	$C_I^{(f)}$	$C_{F_c}^{(f)}$	$C_{F_d}^{(f)}$	$C^{\mathrm{Inc}\,(f)}$	$C_I^{(d)}$	$C_{F_c}^{(d)}$	$C_{F_d}^{(d)}$	$C^{\mathrm{Inc}\,(d)}$
	$\frac{1}{4N_c}$	$-\frac{N_c}{8(N_c^2-1)}$	$\frac{1}{8N_c}$	$-\frac{1}{8N_c(N_c^2-1)}$	$-\frac{1}{8N_c(N_c^2-1)}$	$\frac{N_c}{8(N_c^2-1)}$	$\frac{1}{8N_c}$	$\frac{1}{8N_c(N_c^2-1)}$	$\frac{2N_c^2-1}{8N_c(N_c^2-1)}$
	$\frac{1}{4N_c}$	$-\frac{N_c}{8(N_c^2-1)}$	$-\frac{1}{8N_c(N_c^2-1)}$	$\frac{1}{8N_c}$	$-\frac{N_c^2+1}{8N_c(N_c^2-1)}$	$-\frac{N_c}{8(N_c^2-1)}$	$-\frac{1}{8N_c(N_c^2-1)}$	$-\frac{1}{8N_c}$	$-\frac{N_c^2+1}{8N_c(N_c^2-1)}$
	$\frac{N_c}{2(N_c^2-1)}$	$-\frac{N_c}{4(N_c^2-1)}$	$\frac{N_c}{8(N_c^2-1)}$	$\frac{N_c}{8(N_c^2-1)}$	$-\frac{N_c}{8(N_c^2-1)}$	0	$\frac{N_c}{8(N_c^2-1)}$	$-\frac{N_c}{8(N_c^2-1)}$	$\frac{N_c}{8(N_c^2-1)}$
	$\frac{N_c}{4(N_c^2-1)}$	$-\frac{N_c}{8(N_c^2-1)}$	$\frac{N_c}{8(N_c^2-1)}$	0	0	$\frac{N_c}{8(N_c^2-1)}$	$\frac{N_c}{8(N_c^2-1)}$	0	$\frac{N_c}{4(N_c^2-1)}$
	$\frac{N_c}{4(N_c^2-1)}$	$-\frac{N_c}{8(N_c^2-1)}$	$\frac{N_c}{8(N_c^2-1)}$	0	0	$\frac{N_c}{8(N_c^2-1)}$	$\frac{N_c}{8(N_c^2-1)}$	0	$\frac{N_c}{4(N_c^2-1)}$
	$-\frac{N_c}{4(N_c^2-1)}$	$\frac{N_c}{8(N_c^2-1)}$	0	$-\frac{N_c}{8(N_c^2-1)}$	$\frac{N_c}{8(N_c^2-1)}$	$\frac{N_c}{8(N_c^2-1)}$	0	$\frac{N_c}{8(N_c^2-1)}$	$\frac{N_c}{8(N_c^2-1)}$
	$-\frac{N_c}{4(N_c^2-1)}$	$\frac{N_c}{8(N_c^2-1)}$	0	$-\frac{N_c}{8(N_c^2-1)}$	$\frac{N_c}{8(N_c^2-1)}$	$\frac{N_c}{8(N_c^2-1)}$	0	$\frac{N_c}{8(N_c^2-1)}$	$\frac{N_c}{8(N_c^2-1)}$
	$-\frac{1}{4N_c(N_c^2-1)}$	0	$-\frac{1}{8N_c(N_c^2-1)}$	$-\frac{1}{8N_c(N_c^2-1)}$	$-\frac{1}{8N_c(N_c^2-1)}$	0	$-\frac{1}{8N_c(N_c^2-1)}$	$\frac{1}{8N_c(N_c^2-1)}$	$-\frac{1}{8N_c(N_c^2-1)}$
	$-\frac{1}{4N_c(N_c^2-1)}$	0	$-\frac{1}{8N_c(N_c^2-1)}$	$-\frac{1}{8N_c(N_c^2-1)}$	$-\frac{1}{8N_c(N_c^2-1)}$	0	$-\frac{1}{8N_c(N_c^2-1)}$	$\frac{1}{8N_c(N_c^2-1)}$	$-\frac{1}{8N_c(N_c^2-1)}$

SSA for $p^{\uparrow}p \to D + X$ in the Color-Gauge-Invariant GPM framework

$$
\begin{aligned}
d\Delta\sigma^{\rm CGI} \;\equiv\; & \frac{E_D\, d\sigma^{\uparrow}}{d^3\boldsymbol{p}_D} - \frac{E_D\, d\sigma^{\downarrow}}{d^3\boldsymbol{p}_D} = \frac{2\alpha_s^2}{s}\int \frac{dx_a}{x_a}\frac{dx_b}{x_b}\, dz\, d^2\boldsymbol{k}_{\perp a}\, d^2\boldsymbol{k}_{\perp b}\, d^3\boldsymbol{k}_D\, \delta(\boldsymbol{k}_D\cdot\hat{\boldsymbol{p}}_c)\,\delta(\hat{s}+\hat{t}+\hat{u}-2m_c^2) \\
& \times\; \mathcal{J}(z,\boldsymbol{k}_D)\Bigg\{\sum_q \Bigg[\left(-\frac{k_{\perp a}}{M_p}\right) f_{1T}^{\perp q}(x_a,k_{\perp a})\cos\phi_a\, f_{\bar{q}/p}(x_b,k_{\perp b})\, H^{\rm Inc}_{q\bar{q}\to Q\overline{Q}}(\hat{s},\hat{t},\hat{u})\, D_{D/Q}(z,\boldsymbol{k}_D)\Bigg] \\
& +\; \Bigg[\left(-\frac{k_{\perp a}}{M_p}\right) f_{1T}^{\perp g\,(f)}(x_a,k_{\perp a})\cos\phi_a\, f_{g/p}(x_b,k_{\perp b})\, H^{\rm Inc\,(f)}_{gg\to Q\overline{Q}}(\hat{s},\hat{t},\hat{u})\, D_{D/Q}(z,\boldsymbol{k}_D) \\
& +\; \left(-\frac{k_{\perp a}}{M_p}\right) f_{1T}^{\perp g\,(d)}(x_a,k_{\perp a})\cos\phi_a\, f_{g/p}(x_b,k_{\perp b})\, H^{\rm Inc\,(d)}_{gg\to Q\overline{Q}}(\hat{s},\hat{t},\hat{u})\, D_{D/Q}(z,\boldsymbol{k}_D)\Bigg]\Bigg\}
\end{aligned}
$$

$$
H^{\rm Inc\,(f)}_{gg\to c\bar{c}} = H^{\rm Inc\,(f)}_{gg\to \bar{c}c} = -\frac{N_c}{4(N_c^2-1)}\frac{1}{\tilde{t}\tilde{u}}\left(\frac{\tilde{t}^2}{\tilde{s}^2}+\frac{1}{N_c^2}\right)\left(\tilde{t}^2+\tilde{u}^2+4m_c^2\tilde{s}-\frac{4m_c^4\tilde{s}^2}{\tilde{t}\tilde{u}}\right),
$$

$$
H^{\rm Inc\,(d)}_{gg\to c\bar{c}} = -H^{\rm Inc\,(d)}_{gg\to \bar{c}c} = -\frac{N_c}{4(N_c^2-1)}\frac{1}{\tilde{t}\tilde{u}}\left(\frac{\tilde{t}^2-2\tilde{u}^2}{\tilde{s}^2}+\frac{1}{N_c^2}\right)\left(\tilde{t}^2+\tilde{u}^2+4m_c^2\tilde{s}-\frac{4m_c^4\tilde{s}^2}{\tilde{t}\tilde{u}}\right)
$$

In agreement with the hard partonic cross sections evaluated in the collinear twist-three approach,[Kang, Qiu, Vogelsang, Yuan, PRD78, 114013 (2008)]

SSA for p↑p → D + X in the (CGI) GPM approaches - Results

$$f_{g/p}(x, k_\perp) = f_{g/p}(x)\,\frac{1}{\pi\langle k_\perp^2\rangle}\,e^{-k_\perp^2/\langle k_\perp^2\rangle}$$

Similar functional shape (in z and $k_{\perp D}$) for the unpolarized TMD FFs for D mesons

Only one nonzero (favoured) FF for D meson production
[with such a choice the unpol. cross sections for D^0, $\bar{D}^0$ and D^+, D^- mesons are the same]

$$D_{D^0/c}(z) = D_{\bar{D}^0/\bar{c}}(z) = D_{D^+/c}(z) = D_{D^-/\bar{c}}(z)$$

$$\Delta^N f_{g/p^\uparrow}(x, k_\perp) = \left(-2\frac{k_\perp}{M_p}\right) f_{1T}^{\perp g}(x, k_\perp) = 2\,\frac{\sqrt{2e}}{\pi}\,\mathcal{N}_g(x)\,f_{g/p}(x)\,\sqrt{\frac{1-\rho}{\rho}}\,k_\perp\,\frac{e^{-k_\perp^2/\rho\langle k_\perp^2\rangle}}{\langle k_\perp^2\rangle^{3/2}}$$

$$\mathcal{N}_g(x) = N_g x^\alpha (1-x)^\beta\,\frac{(\alpha+\beta)^{(\alpha+\beta)}}{\alpha^\alpha\beta^\beta}\,, \qquad |N_g| \le 1\,, \qquad 0 < \rho < 1$$

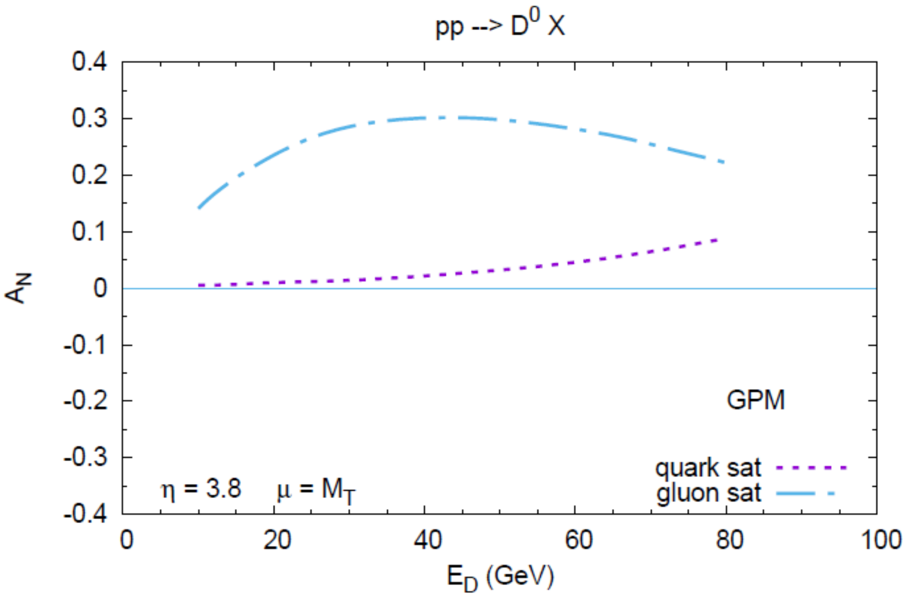
$$\mathcal{N}_{q,g}(x) = +1\,, \quad \rho = 2/3\,, \quad \mu = M_T = \sqrt{\boldsymbol{p}_T^2 + M_D^2} \quad M_D = 1.869\,\mathrm{GeV} \quad m_c = 1.3\,\mathrm{GeV}$$

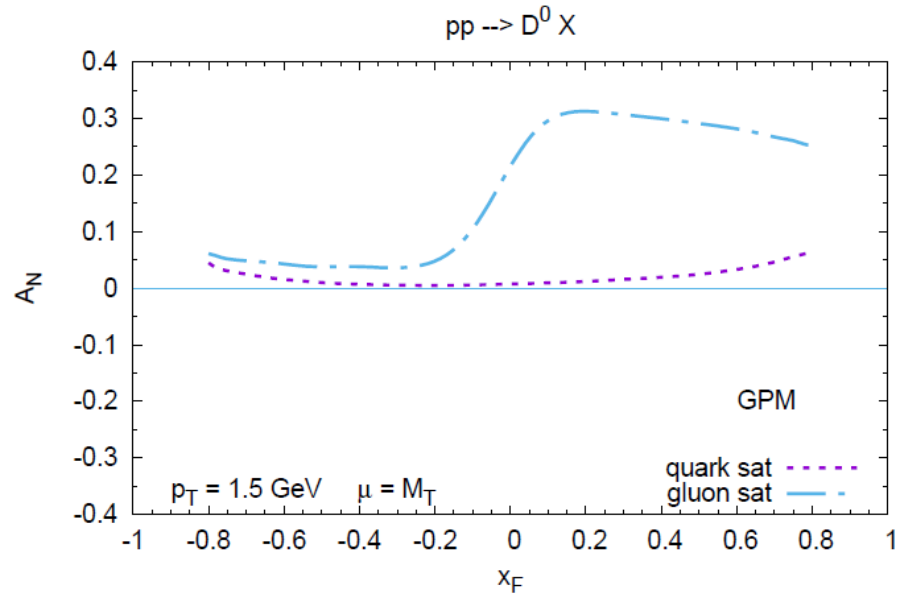
$$\langle k_{\perp q}^2\rangle = 0.25\,\mathrm{GeV}^2\,, \quad \langle k_{\perp g}^2\rangle = 1.00\,\mathrm{GeV}^2\,, \quad \langle k_{\perp D}^2\rangle = 0.20\,\mathrm{GeV}^2\,,$$

CTEQ6 – LO set for collinear unpolarized PDFs

Kniehl, Kramer FF set PRD74, 037502 (2006)

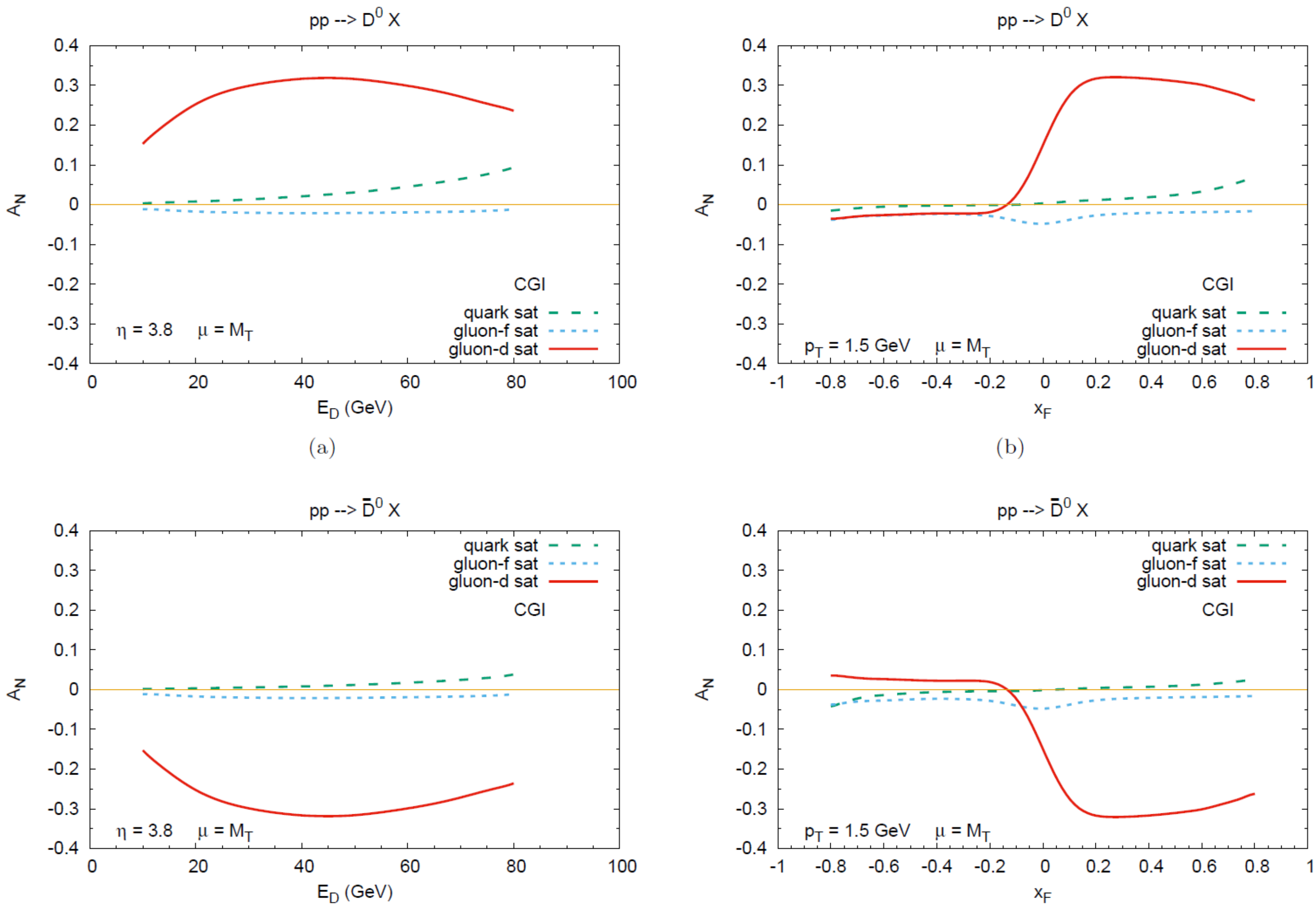
SSA for $p^{\uparrow}p \to D + X$ in the GPM approach – Results





- In the GPM, the GSF contribution can be relatively large in size for $x_F > 0$ and in the whole range of E_D
- In the GPM scheme the SSA for D^0 and $\bar{\mathrm{D}}^0$ mesons are the same; like in the twist-three formalism, this is not the case for the CGI-GPM approach
- Both in GPM and CGI-GPM (see next slide) the maximized quark contributions to the SSA are almost negligible for $E_D \leq 40$ GeV and for $x_F \leq 0.6$
- Adopting any of the GPM quark Sivers functions as extracted from SIDIS data would lead to almost negligible contributions to the SSA everywhere, leaving at work the GSF term alone

SSA for $p^{\uparrow}p \rightarrow D + X$ in the CGI-GPM approach – Results



(a) (b) (c) (d)

SSA for $p^{\uparrow}p \to D + X$ in the CGI-GPM approach – Results

- D^0 production in the CGI-GPM approach:
f-type gluon Sivers contribution always very small
d-type contribution similar to f-type one for $x_F < 0$ and to the GPM gluon contribution for $x_F > 0$

- D^0 vs. $\bar{D}^0$ production in the CGI-GPM approach:
No differences between the (very small in size) f-type contributions
Tiny differences for the (also very small in size) quark Sivers contributions
overall change of sign for the (potentially large in size) d-type gluon Sivers terms

- For $x_F > 0$, a sizable difference in the SSAs for D^0 and $\bar{D}^0$ production would validate the CGI-GPM (disprove the GPM), providing indications on the size of the unknown GSF $f_{1T}^{\perp g(d)}$

- If $f_{1T}^{\perp g(d)}$ is very small, both the GPM and the CGI-GPM would predict the same (small) asymmetry for D^0 and $\bar{D}^0$, making it impossible to distinguish between the two schemes

- In our scenario $\mathrm{d}\sigma(D^0) = \mathrm{d}\sigma(\bar{D}^0)$, so that

$$A_N(D^0 + \bar{D}^0) = \frac{1}{2}\left[A_N(D^0) + A_N(\bar{D}^0)\right]$$

In the GPM scheme, since $A_N(D^0) = A_N(\bar{D}^0)$, this SSA would be the same as for D^0, $\bar{D}^0$ production
Instead, in the CGI-GPM case, since the (potentially large) contribution to the SSA from $f_{1T}^{\perp g(d)}$ cancels in the sum, only the small contribution from $f_{1T}^{\perp g(f)}$ survives.
Therefore, a sizable value of $A_N(D^0 + \bar{D}^0)$ at forward rapidities could be expected only in the GPM approach

SSA for $p^{\uparrow}p \to D + X$ - CGI-GPM vs. twist-three approaches

- The simultaneous study of the Sivers SSA for inclusive D^0 and $\bar{D}^0$ meson production as been also suggested as a tool for disentangling the two possible trigluon correlation functions in the twist-three formalism [Kang, Qiu, Vogelsang, Yuan, PRD78, 114013 (2008); Koike, Yoshida PRD84,014026 (2011)]

- Both in the GPM and CGI-GPM approaches, the SSA in the backward ($x_F < 0$) region cannot be sizable, because of cancellations occurring when integrating over the azimuthal phases involved.
This is in contrast to what happens in the twist-three formalism, where one could get A_N values of the order of 30% for $x_F < 0$

Remarks and outlook for EIC physics

- $p^{\uparrow}\, p \to J/\psi, D + X$ processes are a very useful tool for testing TMD gluon distributions, in particular the GSFs

- A combined and unified analysis with $\ell\, p^{\uparrow} \to \ell'\, J/\psi, D + X$ and $\ell p^{\uparrow} \to J/\psi, D + X$ at EIC will be very important for testing factorization breaking effects, process dependence and evolution properties of gluon TMDs

- Work along these lines is currently in progress