Scaling behaviour in string cosmology

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1. Example of attractor solutions driven by presence of matter.
2. Applications to stabilising of dilaton and volume moduli
3. Kahler moduli inflation - basin of attraction.
4. CMB constraints on cosmic strings and cosmic superstrings.
5. PBB and axion-dilator solutions.

Post Inflationary String Cosmology

Bologna - Sept 19th 2017

Many approaches have been taken to String Cosmology. They are driven by the need to address a number of questions including the origin of inflation, the nature of primordial density fluctuations and the resolution of the initial singularity. A few types include:

1. Pre Big Bang Cosmology (Veneziano & Gasperini 91) -- low energy string action, has singular collapsing phase in low curvature regimes but require higher order curvatures to avoid the singularity and bounce -- not well controlled.

2. Warped D3-brane inflation (Kachru et al 03) -- inflation arising from type IIB flux compactifications involving Calabi-Yau spaces. Moduli fields can be stabilised and flat directions lifted by inclusion of extra D3-branes. Have interesting consequences like production of cosmic superstrings. Requires fine tunings for inflation.

3. Axion inspired cosmology (Kim et al 04). Example of large field inflation. Use PQ symmetry to protect the inflaton as it evolves from super-Planckian distances. Models based on N-flation (collection of many axions) or axion monodromy (McAllister et al 08). Linear potential and non-perturbative oscillating corrections leads to modulations in the power spectrum and to interesting non-gaussian features in CMB.

4. Eternal inflation models -- example of the string landscape in action (Susskind, Linde ...). Lots of issues over how to properly define the measure in such a landscape and to make predictions of what we expect to see in the CMB.

5. Ekpyrotic and Cyclic models (Khoury et al 01, Steinhardt and Turok 01). Returning to idea of PBB, replacing HBB singularity with prior-contracting phase. Once again have to control the higher curvature effects as enter bounce regime, and there is a lot to be done to understand the propagation of perturbations through the bounce. Not clear how it is embedded in string theory.

6. String gas cosmology (Brandenberger and Vafa 89). The intercommuting of a gas of strings on T^9 dynamically favours the emergence of three space dimensions. Still quite a bit to work through, including the true equations that describe the background of the gas of strings.

I think it is fair to say that all of the models proposed have technical issues concerning the detailed predictive powers they have and in justifying the assumptions made in formulating them in the first place.

Stabilising moduli -- big issue.

1. Light moduli in string compactification are a problem -- they can destabilise the compact dimensions, overclose the universe ...

2. They need to be stabilised, but not that straightforward.

A few methods have been proposed. Best known is probably inflation in KKLT, KKLMMT which uses a particular string compactification with stable moduli and inflates into de Sitter like regime.

But involves first finding stabilised solution in AdS_4 type vacuum before up-lifting to dS_4. (see also Conlon and Quevedo 05).

In cosmology as in many areas of physics we often deal with systems that are inherently described through a series of coupled non-linear differential equations.

Analysing them by determining the late time behaviour of some combination of the variables, we often see that they may approach some form of attractor solution.

By determining the nature of these attractor solutions (their stability for example) one can learn a great deal about the system in general.

Moreover the phase plane description of the system is often highly intuitive enabling easy analysis and understanding of the system.

Examples include the relative energy densities in scalar fields compared to the background radiation and matter densities, as well as the relative energy density in cosmic strings.

Tracker solutions

Wetterich,

Peebles and Ratra,

EJC, Liddle and Wands

Scalar field: $\phi:\ \rho_\phi = \frac{\dot{\phi}^2}{2} + V(\phi)\ ;\ p_\phi = \frac{\dot{\phi}^2}{2} - V(\phi)$

EoM:

$$\dot{H} = -\frac{\kappa^2}{2}(\dot{\phi}^2 + \gamma\rho_B)$$

$$\dot{\rho}_B = -3\gamma H\rho_B$$

$$\ddot{\phi} = -3H\dot{\phi} - \frac{dV}{d\phi}$$

+ constraint:

$$H^2 = \frac{\kappa^2}{3}(\rho_\phi + \rho_B)$$

Intro:

$x = \frac{\kappa\dot{\phi}}{\sqrt{6}H}$ $y = \frac{\kappa\sqrt{V}}{\sqrt{3}H}$ $\lambda \equiv \frac{-1}{\kappa V}\frac{dV}{d\phi}$ $\Gamma - 1 \equiv \frac{d}{d\phi}\left(\frac{1}{\kappa\lambda}\right)$

Eff eqn of state: $\gamma_\phi = \dfrac{\dot{\phi}^2}{V + \dfrac{\dot{\phi}^2}{2}} = \dfrac{2x^2}{x^2 + y^2}$ $\quad \Omega_\phi = \dfrac{\kappa^2 \rho_\phi}{3H^2} = x^2 + y^2$

Friedmann eqns and fluid eqns become:

$$x' = -3x + \lambda\sqrt{\frac{3}{2}}y^2 + \frac{3}{2}x\left[2x^2 + \gamma\left(1 - x^2 - y^2\right)\right]$$

$$y' = -\lambda\sqrt{\frac{3}{2}}xy + \frac{3}{2}y\left[2x^2 + \gamma\left(1 - x^2 - y^2\right)\right]$$

$$\lambda' = -\sqrt{6}\lambda^2(\Gamma - 1)$$

$\dfrac{\kappa^2 \rho_\gamma}{3H^2} + x^2 + y^2 = 1$ where ${}' \equiv d/d(\ln a)$

Note: $0 \le \gamma_\phi \le 2 : 0 \le \Omega_\phi \le 1$

Scaling solutions: (x`=y`=0)

No:	x_c	y_c	Existance	Stability	Ω_ϕ	γ_ϕ
1	0	0	$\forall\lambda,\gamma$	SP: $0<\gamma$ SN: $\gamma=0$	0	Undefined
2a	1	0	$\forall\lambda,\gamma$	UN: $\lambda<\sqrt{6}$ SP: $\lambda>\sqrt{6}$	1	2
2b	-1	0	$\forall\lambda,\gamma$	UN: $\lambda>-\sqrt{6}$ SP: $\lambda<-\sqrt{6}$	1	2
3	$\frac{\lambda}{\sqrt{6}}$	$\left(1-\frac{\lambda^2}{6}\right)^{1/2}$	$\lambda^2\le 6$	SP: $3\gamma<\lambda^2<6$ SN: $\lambda^2<3\gamma$	1	$\frac{\lambda^2}{3}$
4	$\left(\frac{3}{2}\right)^{1/2}\frac{\gamma}{\lambda}$	$\left[\frac{3(2-\gamma)\gamma}{2\lambda^2}\right]^{1/2}$	$\lambda^2\ge 3\gamma$	SN: $3\gamma<\lambda^2<\frac{24\gamma^2}{9\gamma-2}$ SS: $\lambda^2>\frac{24\gamma^2}{9\gamma-2}$	$\frac{3\gamma}{\lambda^2}$	γ

$$V = V_0 e^{-\lambda\kappa\phi}$$

Late time attractor is scalar field dominated

$\lambda^2 \le 6$

Field mimics background fluid.

$\lambda^2 \ge 3\gamma$

EJC, Liddle and Wands

$$V = V_0 e^{-\lambda\kappa\phi}$$

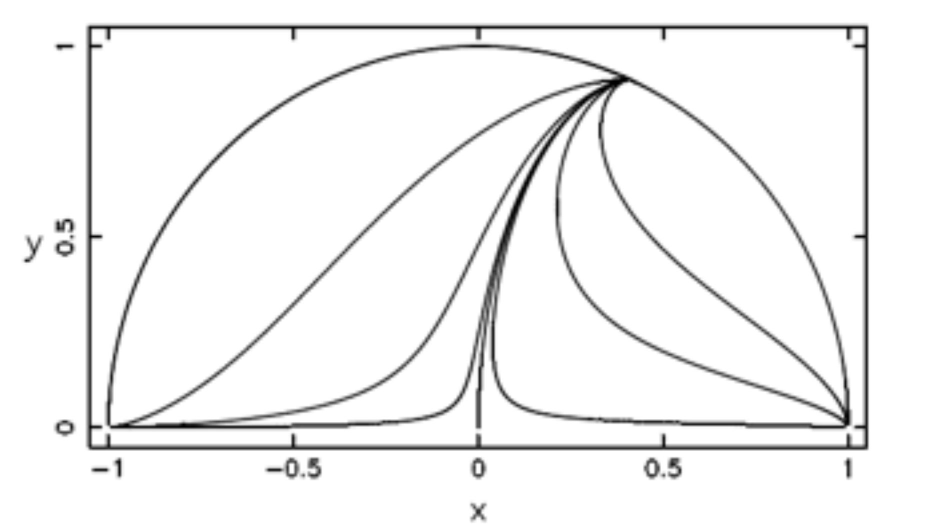


FIG. 2. The phase plane for $\gamma = 1$, $\lambda = 1$. The late-time attractor is the scalar field dominated solution with $x = \sqrt{1/6}$, $y = \sqrt{5/6}$.

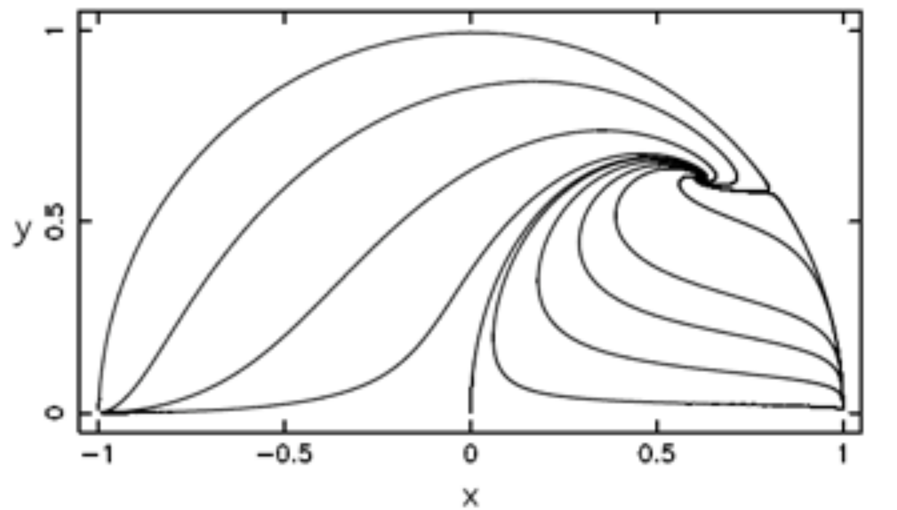


FIG. 3. The phase plane for $\gamma = 1$, $\lambda = 2$. The scalar field dominated solution is a saddle point at $x = \sqrt{2/3}$, $y = \sqrt{1/3}$, and the late-time attractor is the scaling solution with $x = y = \sqrt{3/8}$.

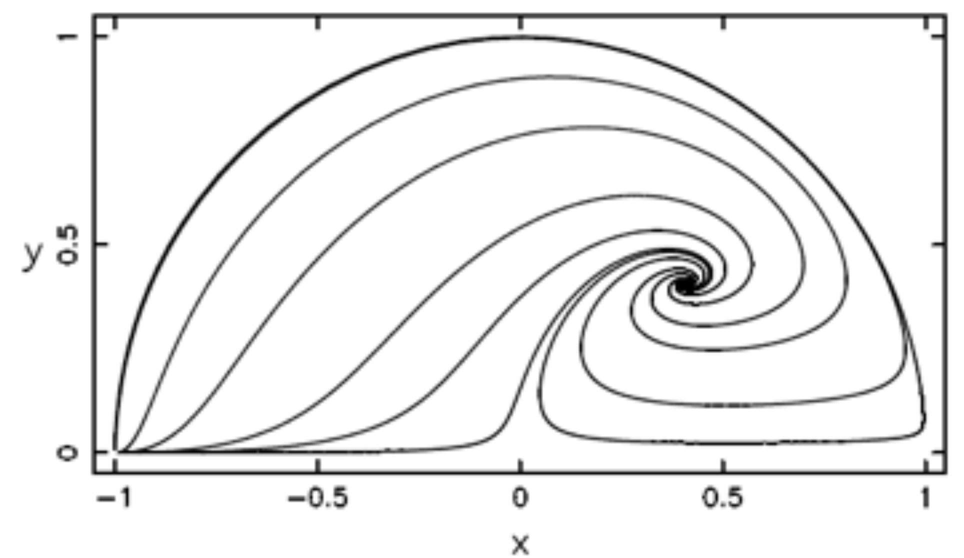
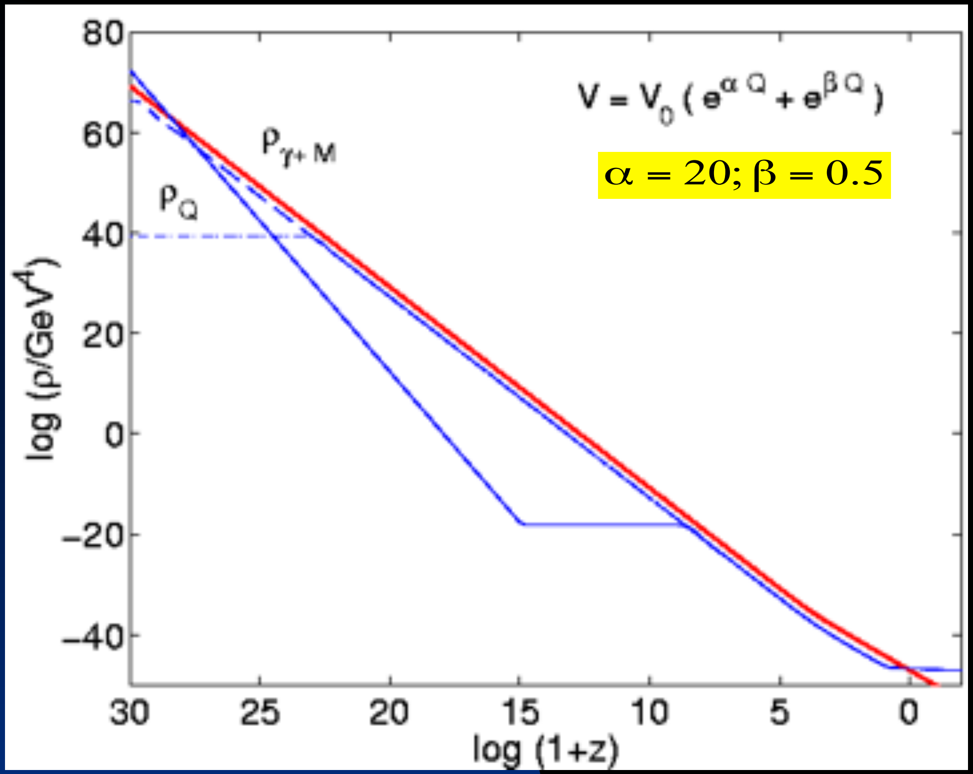
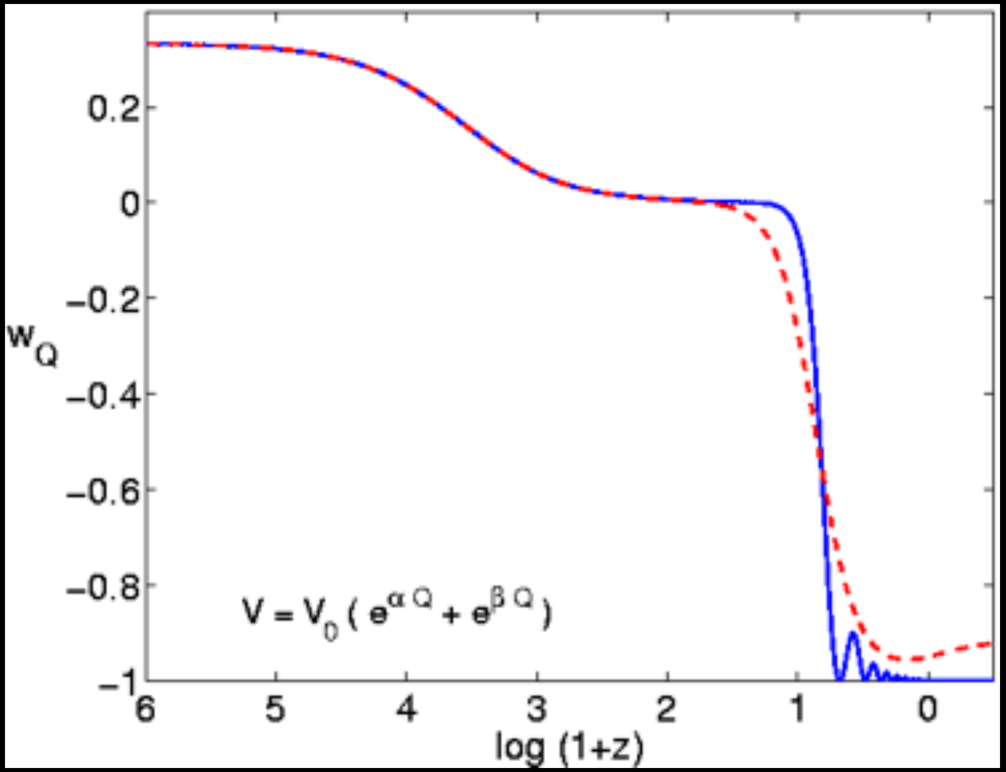


FIG. 4. The phase plane for $\gamma = 1$, $\lambda = 3$. The late-time attractor is the scaling solution with $x = y = \sqrt{1/6}$.

1.Scaling solutions in Dark Energy - Quintessence





Scaling for wide range of i.c.

Fine tuning:

$$V_0 \approx \rho_\phi \approx 10^{-47}\ \mathrm{GeV}^4 \approx (10^{-3}\ \mathrm{eV})^4$$

Mass:

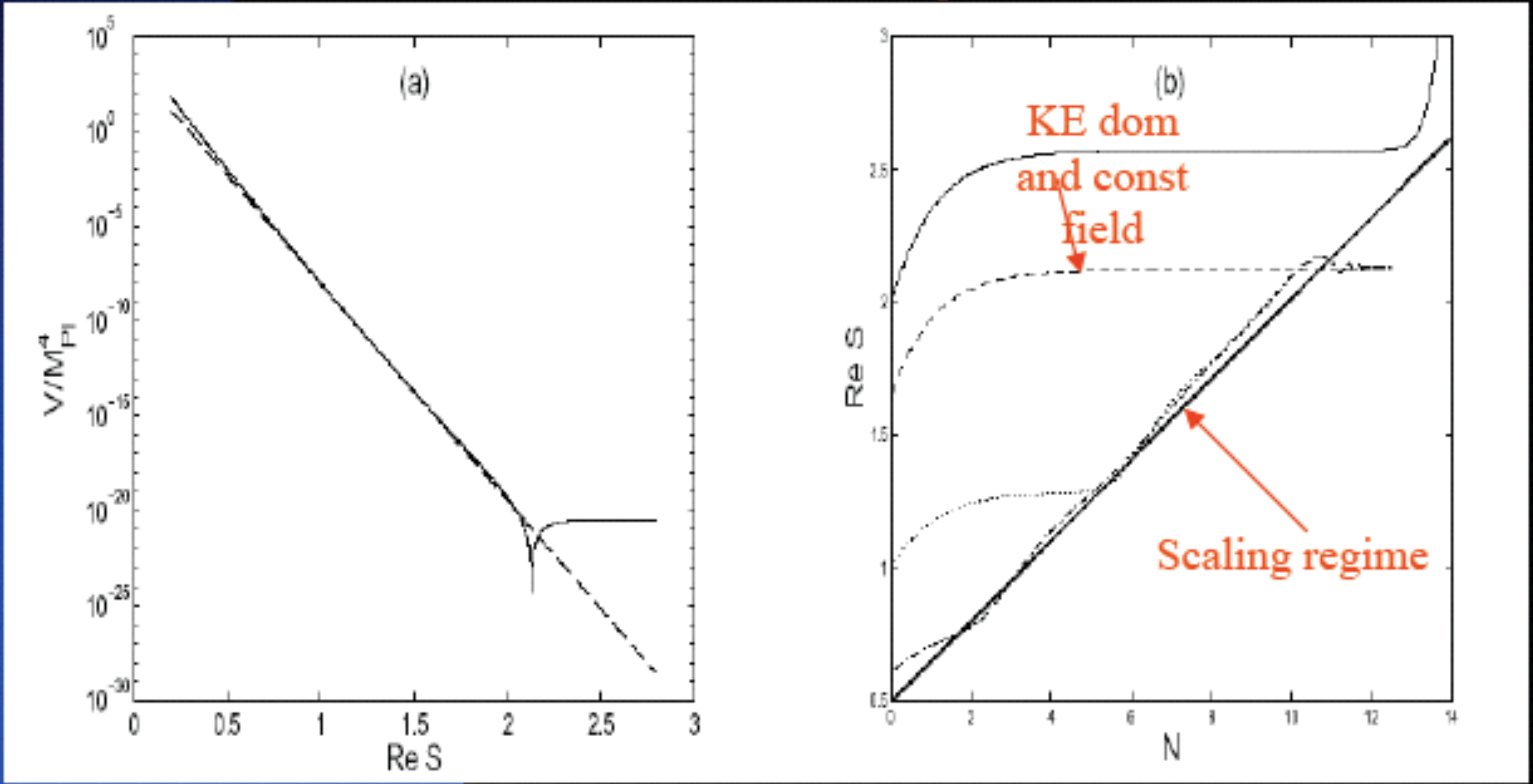
$$m \approx \sqrt{\frac{V_0}{M_{pl}^2}} \approx 10^{-33}\ \mathrm{eV}$$

Fifth force !

2. **Useful way of stabilising moduli in string cosmology. Sources provide extra friction when potentials steep.**

Barreiro, de Carlos and EC : hep/th-9805005

Brustein, Alwis and Martins : hep-th/0408160



Two condensate model with $V \sim e^{-a\mathrm{Re}S}$ as approach minima

Barreiro et al : hep-th/0506045

3. Stabilising volume moduli ($\sigma=\sigma_r+\sigma_i$) in KKLT [Kachru et al 2003]

$$
\begin{aligned}
\ddot{\sigma}_r + 3H\dot{\sigma}_r - \frac{1}{\sigma_r}(\dot{\sigma}_r{}^2 - \dot{\sigma}_i{}^2) + \frac{2\sigma_r^2}{3}\partial_{\sigma_r}V &= 0 \\
\ddot{\sigma}_i + 3H\dot{\sigma}_i - \frac{2}{\sigma_r}\dot{\sigma}_r\dot{\sigma}_i + \frac{2\sigma_r^2}{3}\partial_{\sigma_i}V &= 0 \\
\dot{\rho}_b + 3H\gamma\rho_b &= 0
\end{aligned}
$$

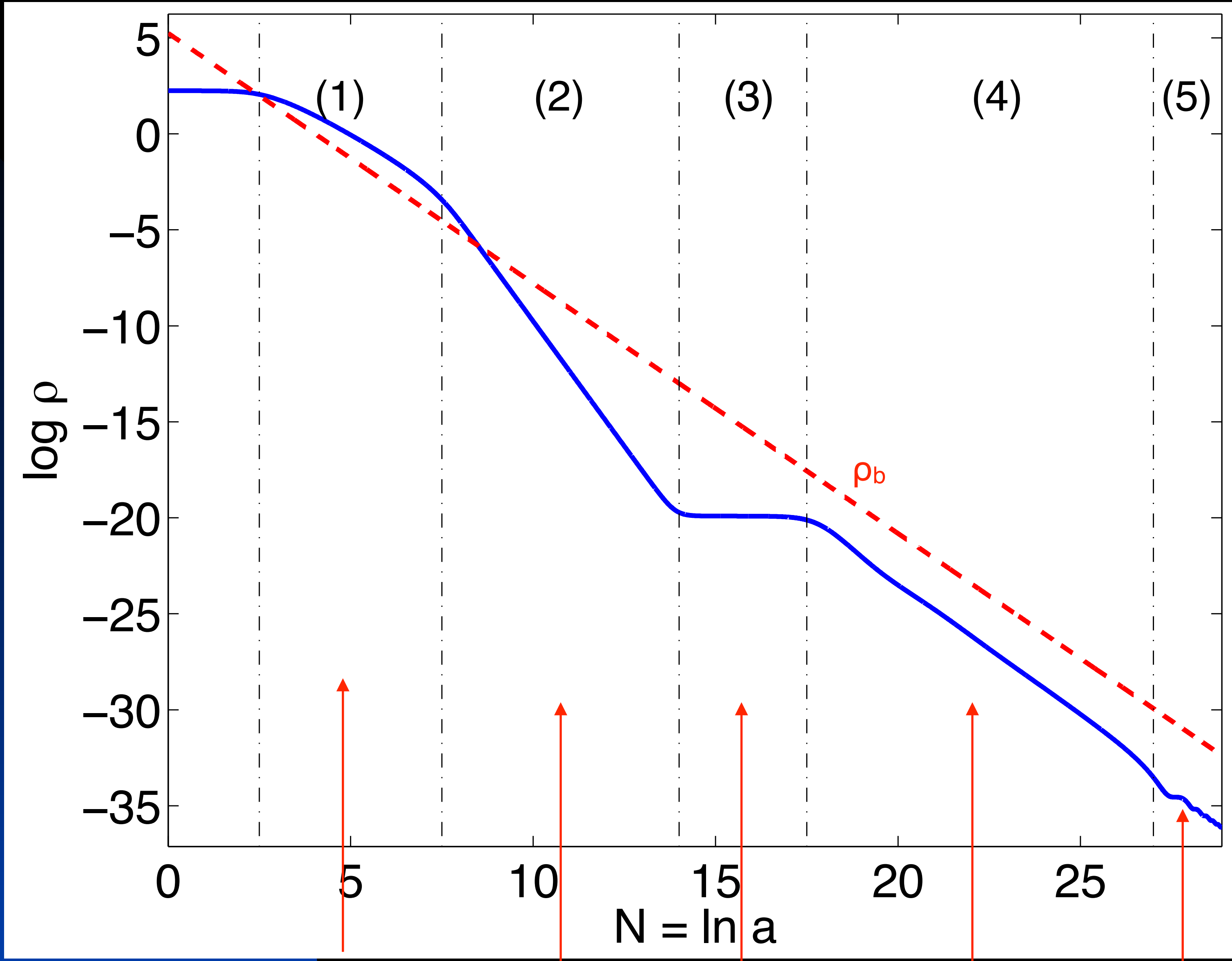
$$
3H^2 = \frac{3}{4\sigma_r^2}(\dot{\sigma}_r{}^2 + \dot{\sigma}_i{}^2) + V + \rho_b
$$

$$
\begin{aligned}
V &= \frac{\alpha A \mathrm{e}^{-\alpha\sigma_r}}{2\sigma_r^2}\left[A\left(1+\frac{\alpha\sigma_r}{3}\right)\mathrm{e}^{-\alpha\sigma_r} + W_0\cos(\alpha\sigma_i)\right] \\
&+ \frac{C}{\sigma_r^3}\ . \qquad (15
\end{aligned}
$$

[including contribution from D term to uplift the potential to de Sitter]

[for discussion on validity of D term addition see also Burgess et al 2003; Achucarro et al 2006]

Evolution of energy density of $\phi \propto \ln\sigma_r$ in KKLT and Kallosh Linde type potentials



Flat potential: Scalar field dominated

Steeper pot Kinetic field dominated

Field frozen in pot

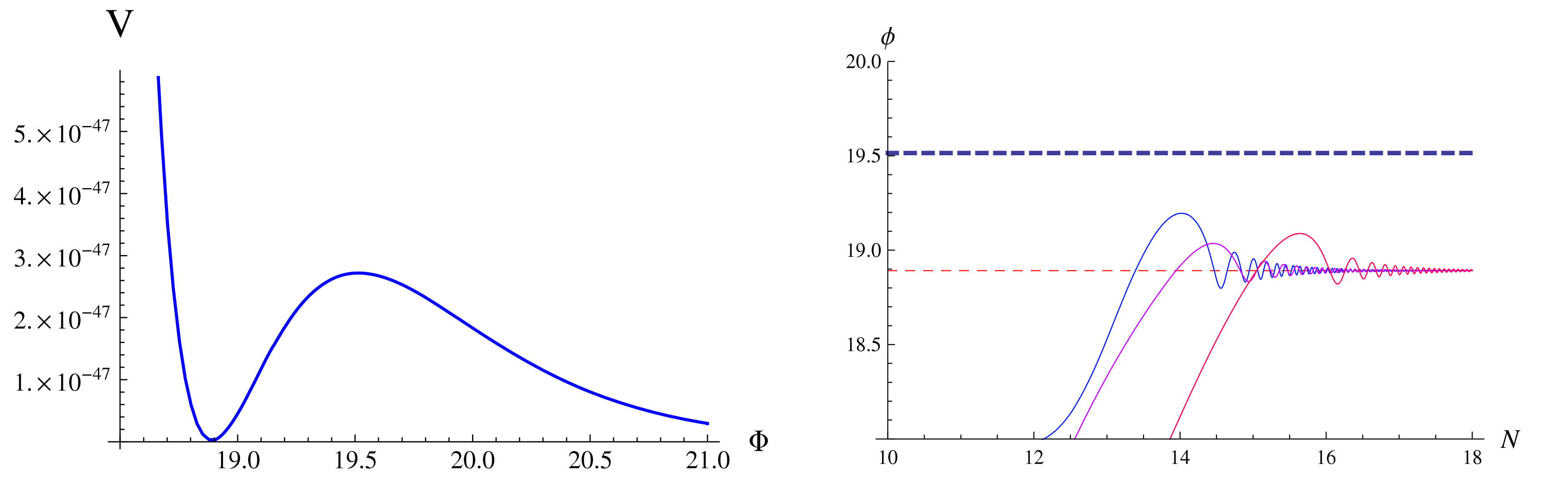
Scaling or tracking regime

Added friction from scaling regime slows field down and stabilises it in min of potential

[Brustein et al 2004; Barreiro et al 2005]

4.Large volume modulus inflation - high scale inflation & low scale SUSY co-existing [Conlon et al 2008]

$$V = V_0\left((1-\epsilon\,\Phi^{3/2})e^{-\sqrt{27/2}\Phi} + C\,e^{-10\Phi/\sqrt{6}} + De^{-11\Phi/\sqrt{6}} + \delta e^{-\sqrt{6}\Phi}\right)$$



Steep potential after inflation would normally have runaway solutions but presence of radiation leads to additional Hubble Friction which leads to attractor behaviour and field settles in its minimum.

Kahler Moduli Inflation [Conlon and Quevedo 2006]

They consider large volume scenarios within a class of Type IIB flux compactifications on a CY orientifold.

Internal volume of CY

$$\mathcal{V} = \frac{\alpha}{2\sqrt{2}} \left[(T_1 + \bar{T}_1)^{\frac{3}{2}} - \sum_{i=2}^{n} \lambda_i (T_i + \bar{T}_i)^{\frac{3}{2}} \right] = \alpha \left(\tau_1^{3/2} - \sum_{i=2}^{n} \lambda_i \tau_i^{3/2} \right)$$

Complex Kahler moduli $T_i = \tau_i + i\theta_i$

τ_i – volume of internal four cycles in CY

θ_i – axionic partners

Full scalar potential for moduli fields

$$\begin{aligned} V = & \sum_{\substack{i,j=2 \\ i<j}}^{n} \frac{A_i A_j \cos(a_i\theta_i - a_j\theta_j)}{(4\mathcal{V} - \xi)(2\mathcal{V} + \xi)^2} e^{-(a_i\tau_i + a_j\tau_j)} \left(32(2\mathcal{V} + \xi)(a_i\tau_i + a_j\tau_j + 2a_i a_j \tau_i \tau_j) + 24\xi\right) \\ & + \frac{12 W_0^2 \xi}{(4\mathcal{V} - \xi)(2\mathcal{V} + \xi)^2} + \sum_{i=2}^{n} \left[\frac{12 e^{-2a_i\tau_i} \xi A_i^2}{(4\mathcal{V} - \xi)(2\mathcal{V} + \xi)^2} + \frac{16 (a_i A_i)^2 \sqrt{\tau_i} e^{-2a_i\tau_i}}{3\alpha\lambda_i (2\mathcal{V} + \xi)} \right. \\ & \left. + \frac{32 e^{-2a_i\tau_i} a_i A_i^2 \tau_i (1 + a_i\tau_i)}{(4\mathcal{V} - \xi)(2\mathcal{V} + \xi)} + \frac{8 W_0 A_i e^{-a_i\tau_i} \cos(a_i\theta_i)}{(4\mathcal{V} - \xi)(2\mathcal{V} + \xi)} \left(\frac{3\xi}{2\mathcal{V} + \xi} + 4a_i\tau_i \right) \right] + V_{uplift} \, . \end{aligned}$$

Slow roll inflation supported when $\mathcal{V} \gg 1$ implies $\tau_1 \gg \tau_i, i = 2..n$

Idea: displace just one moduli from its minimum, keeping the others fixed and show consistent slow roll inflation can be obtained with that moduli evolving back to its minima

Displace τ_2 with parameter $\rho \ll 1$ where

$$\rho \equiv \frac{\lambda_2}{a_2^{3/2}} : \sum_{i=2}^{n} \frac{\lambda_i}{a_i^{3/2}}$$

$$V_{LARGE} = \frac{BW_0^2}{\mathcal{V}^3} - \frac{4W_0 a_2 A_2 \tau_2 e^{-a_2\tau_2}}{\mathcal{V}^2}$$

Note axions assumed fixed in their minima

Intriguing results obtained for 50-60 efolds:

$$\eta \simeq -\frac{1}{N_e}, \qquad \epsilon < 10^{-12},$$
$$0.960 < n_s < 0.967, \quad 0 < |r| < 10^{-10}$$
$$10^5 l_s^6 \leq \mathcal{V} \leq 10^7 l_s^6$$

Planck 2015 : $n_s = 0.968 \pm 0.006; \quad r < 0.09$

Relaxing the assumptions [Blanco-Pillado et al 2009]

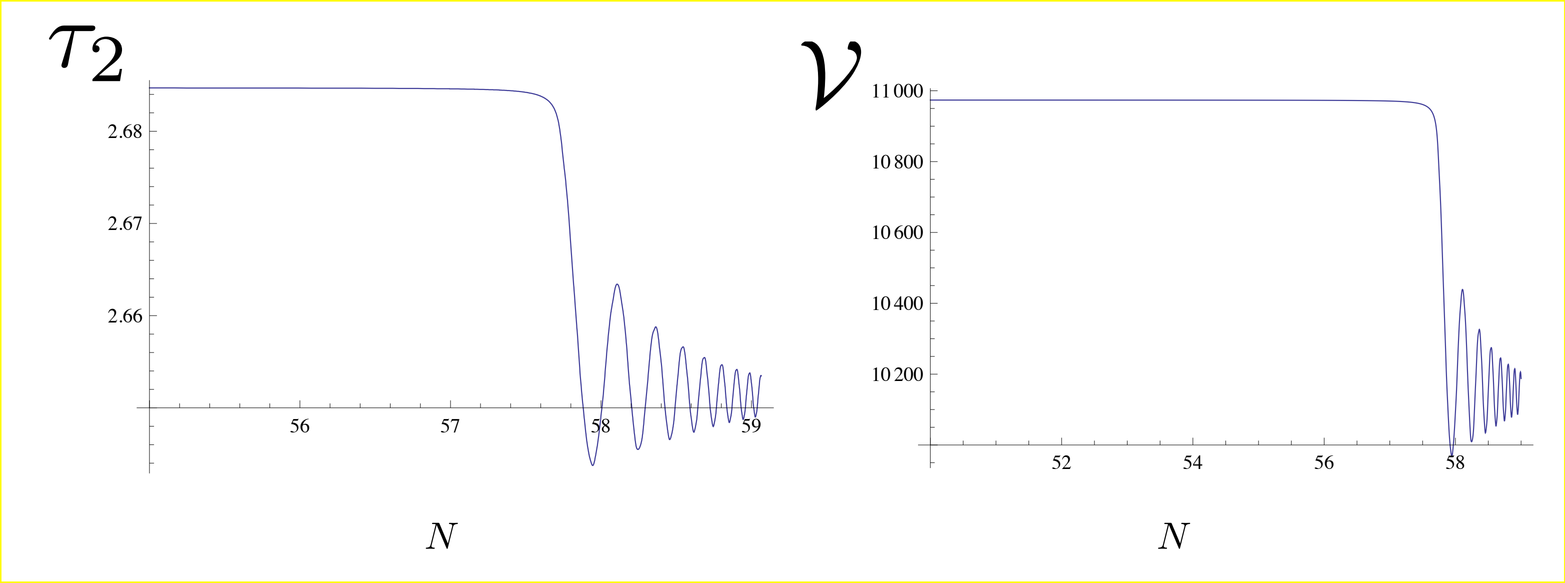
Numerically solve the full equations. The question is what happens if we allow the moduli to evolve so that they all have to find their minima. Do we find the kind of evolution that Conlon and Quevedo assumed in their analytic model ?

$$\mathrm{Ex}: \quad \rho \sim 0.99$$

Global min: $$\tau_1^f = 2555.95, \quad \tau_2^f = 4.7752, \quad \tau_3^f = 2.6512, \quad \mathcal{V}^f = 10143.94363$$

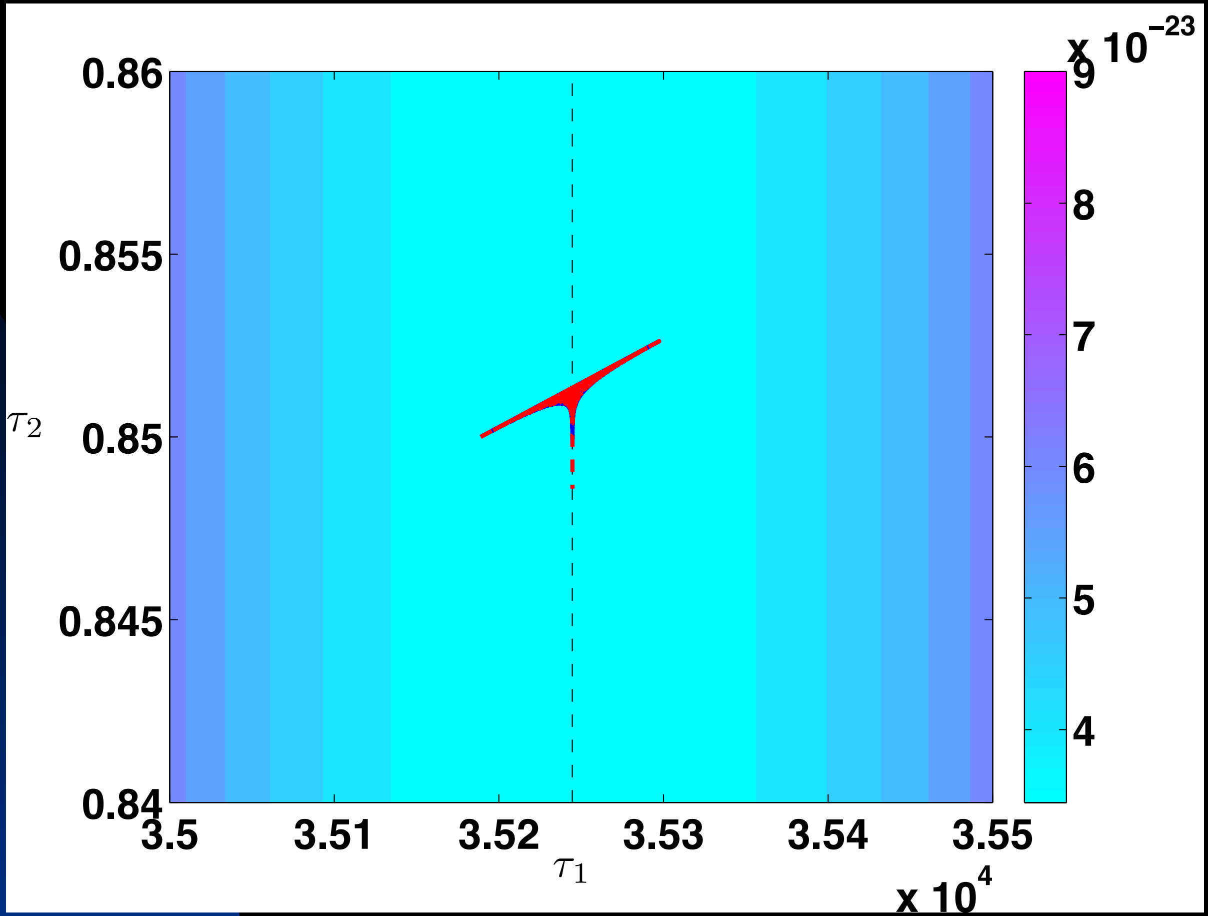
Displace: $$\tau_2^i = 78.7752067$$

New min: $$\tau_1^i = 2781.185086997, \quad \tau_3^i = 2.684717126, \quad \mathcal{V}^i = 10973.9$$



$$n_s = 0.960$$

Basin of attraction [Blanco-Pillado et al 2009]



Original model: Dashed line - trajectory which maintains const vol at fixed

$$\tau_3$$

$$\text{Fix } \tau_3 \text{ allow } \tau_1, \tau_2, (\theta_2 \text{ red line) to evolve}$$

The model works for broader range of parameters and sizes of moduli fields than might be expected. Two potential issues:

$$\text{gravitino mass generally too large}$$

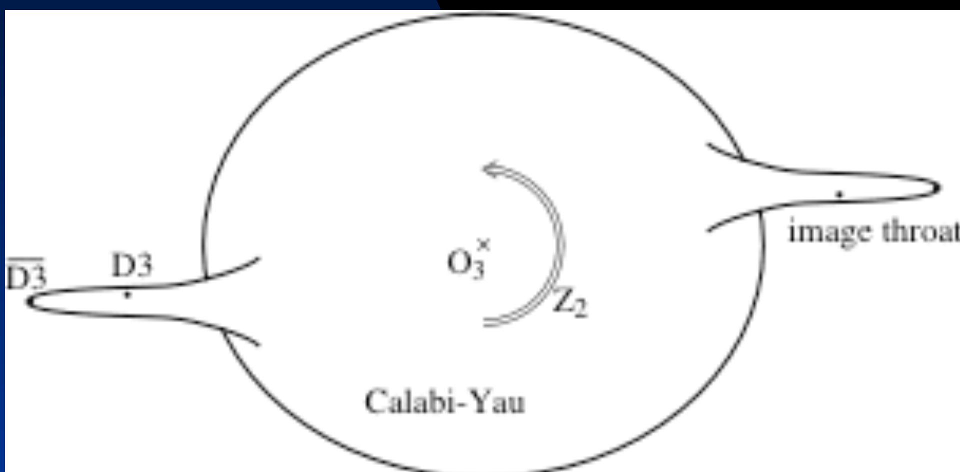
$$\delta\theta_3 \ll 1 \text{ to avoid runaway decompactifiction}$$

Strings in KLMT © model -- an example.

[Kachru, Kallosh, Linde, Maldacena, McAllister & Trivedi 03]

IIB string theory on CY manifold, orientifolded by Z_2 sym with isolated fixed points, become O3 planes. Warped metric:

$$ds^2 = e^{2A(x_\perp)}\eta_{\mu\nu}dx^\mu dx^\nu + ds_\perp^2 \; .$$



Inflaton: sep of D3 and anti D3 in throat.

Annihilation in region of large grav redshift,

$$min\{e^{A(x_\perp)}\} = e^{A_0} \ll 1$$

Redshift in throat important. Inflation scale and string tension, as measured by a 10 dim inertial observer, are set by string physics -- close to the four-dimensional Planck scale. Corresponding energy scales as measured by a 4 dim obs are suppressed by a factor of e^{A_0}

Strings surviving inflation:

D-brane-antibrane inflation leads to formation of D1 branes in non-compact space [**Dvali & Tye; Burgess et al; Majumdar & Davis; Jones, Sarangi &Tye; Stoica & Tye**]

Form strings, not domain walls or monopoles.

$$10^{-11} \leq G\mu \leq 10^{-6}$$

In general for cosmic strings to be cosmologically interesting today we require that they are not too massive (from CMB constraints), are produced after inflation (or survive inflation) and are stable enough to survive until today [**Dvali and Vilenkin (2004); EJC,Myers and Polchinski (2004)**].

What sort of strings? Expect strings in non-compact dimensions where reheating will occur: F1-brane (fundamental IIB string) and D1 brane localised in throat. [Jones,Stoica & Tye, Dvali & Vilenkin]

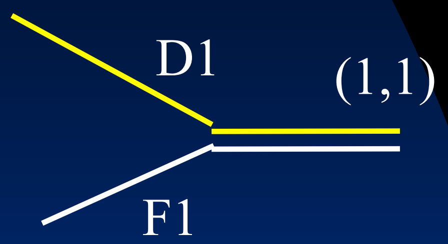
D1 branes - defects in tachyon field describing D3-anti D3 annihilation, so produced by Kibble mechanism.

Strings created at end of inflation at bottom of inflationary throat. Remain there because of deep pot well. Eff 4d tensions depend on warping and 10d tension $\bar{\mu}$

$$\mu = e^{2A(x_\perp)}\bar{\mu}$$

F1-branes and D1-branes --> also (p,q) strings for relatively prime integers p and q. [Harvey & Strominger; Schwarz]

Interpreted as bound states of p F1-branes and q D1-branes [Polchinski;Witten]



Tension in 10d theory:

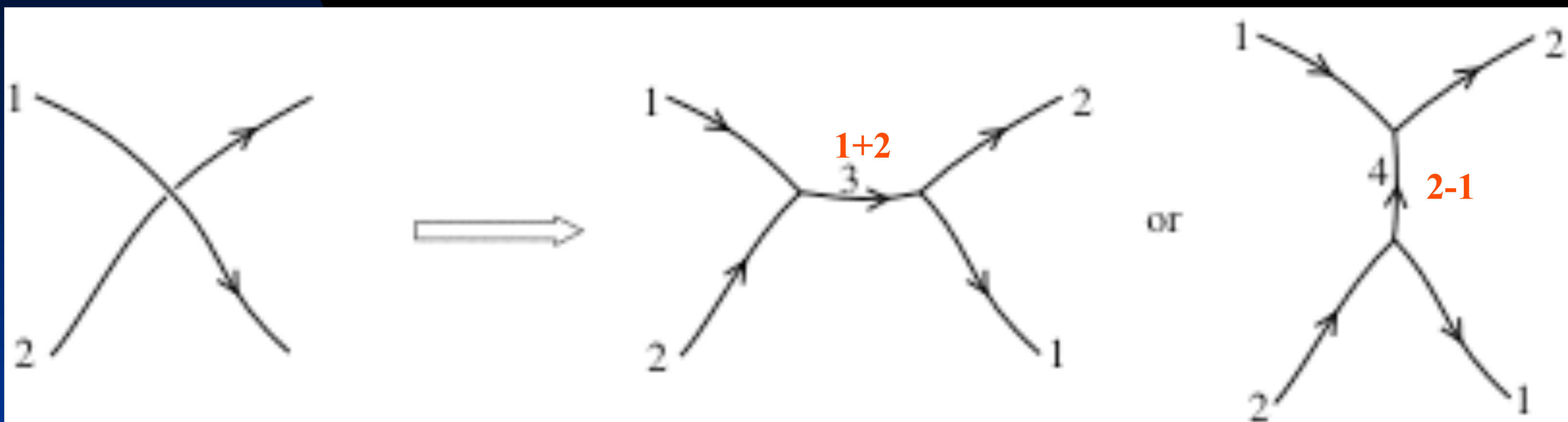
$$\mu_i \equiv \mu_{(p_i,q_i)} = \frac{\mu_F}{g_s}\sqrt{p_i^2 g_s^2 + q_i^2}$$

Distinguishing cosmic superstrings

1. Intercommuting probability for gauged strings P~1 always ! In other words when two pieces of string cross each other, they reconnect. Not the case for superstrings -- model dependent probability [Jackson et al 04].
2. Existence of new `defects’ D-strings allows for existence of new hybrid networks of F and D strings which could have different scaling properties, and distinct observational effects.

(p,q) string networks -- exciting prospect.

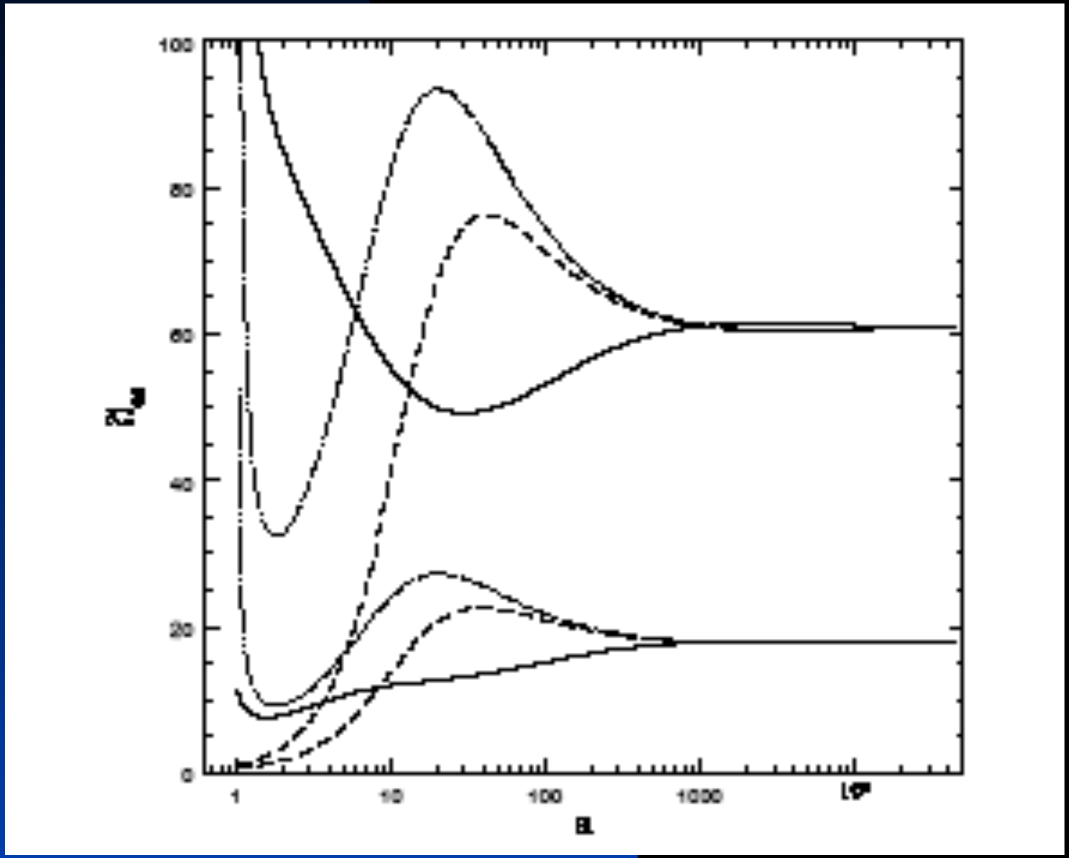
Two strings of different type cross, can not intercommute in general -- produce pair of trilinear vertices connected by segment of string.



What happens to such a network in an expanding background? Does it scale or freeze out in a local minimum of its PE [Sen]?Then it could lead to a frustrated network scaling as w=-1/3

Including multi-tension cosmic superstrings

[Tye et al 05, Avgoustidis and Shellard 07, Urrestilla and Vilenkin 07, Avgoustidis and EJC 10].



Density of (p,q) cosmic strings.

Density of D1 strings.

Scaling achieved indep of initial conditions, and indep of details of interactions.

Recap single one-scale model: (Kibble + many...)

Infinite string density $\rho = \frac{\mu}{L^2}$

$$\dot{\rho} = -2\frac{\dot{a}}{a}\rho - \frac{\rho}{L}$$

Expansion Loss to loops

Correlation length $L(t) = \xi(t)t, \quad a(t) \sim t^{\beta}$ Scale factor

$$\frac{\dot{\xi}}{\xi} = \frac{1}{2t}\left(2(\beta - 1) + \frac{1}{\xi}\right)$$

Scaling solution $\xi = [2(1-\beta)]^{-1}.$

Need this to understand the behaviour with the CMB.

Velocity dependent model: (Shellard and Martin)

$$\dot{\rho} = -2\frac{\dot{a}}{a}(1+v^2)\rho - \frac{\tilde{c}\,v\rho}{L},$$

RMS vel of segments

$$\dot{v} = (1-v^2)\left(\frac{k}{L} - 2\frac{\dot{a}}{a}v\right)$$

Curvature type term encoding small scale structure

$$k = \frac{2\sqrt{2}}{\pi}\left(\frac{1-8v^6}{1+8v^6}\right)$$

$$\xi^2 = \frac{k(k+\tilde{c})}{4\beta(1-\beta)}, \qquad v^2 = \frac{k(1-\beta)}{\beta(k+\tilde{c})}$$

Both correlation length and velocity scale

Multi tension string network: (Avgoustidis & Shellard 08, Avgoustidis & EJC 10)

$$\dot{\rho}_i = -2\frac{\dot{a}}{a}(1+v_i^2)\rho_i - \frac{c_i v_i \rho_i}{L_i} - \sum_{a,k} \frac{d_{ia}^k \bar{v}_{ia} \mu_i \ell_{ia}^k(t)}{L_a^2 L_i^2} + \sum_{b,\, a\leq b} \frac{d_{ab}^i \bar{v}_{ab} \mu_i \ell_{ab}^i(t)}{L_a^2 L_b^2}$$

Expansion

Loop of `i' string

Segment of `i' collides with `a' to form segment `k' -- removes energy

Segment of `i' forms from collision of `a' and `b' -- adds energy

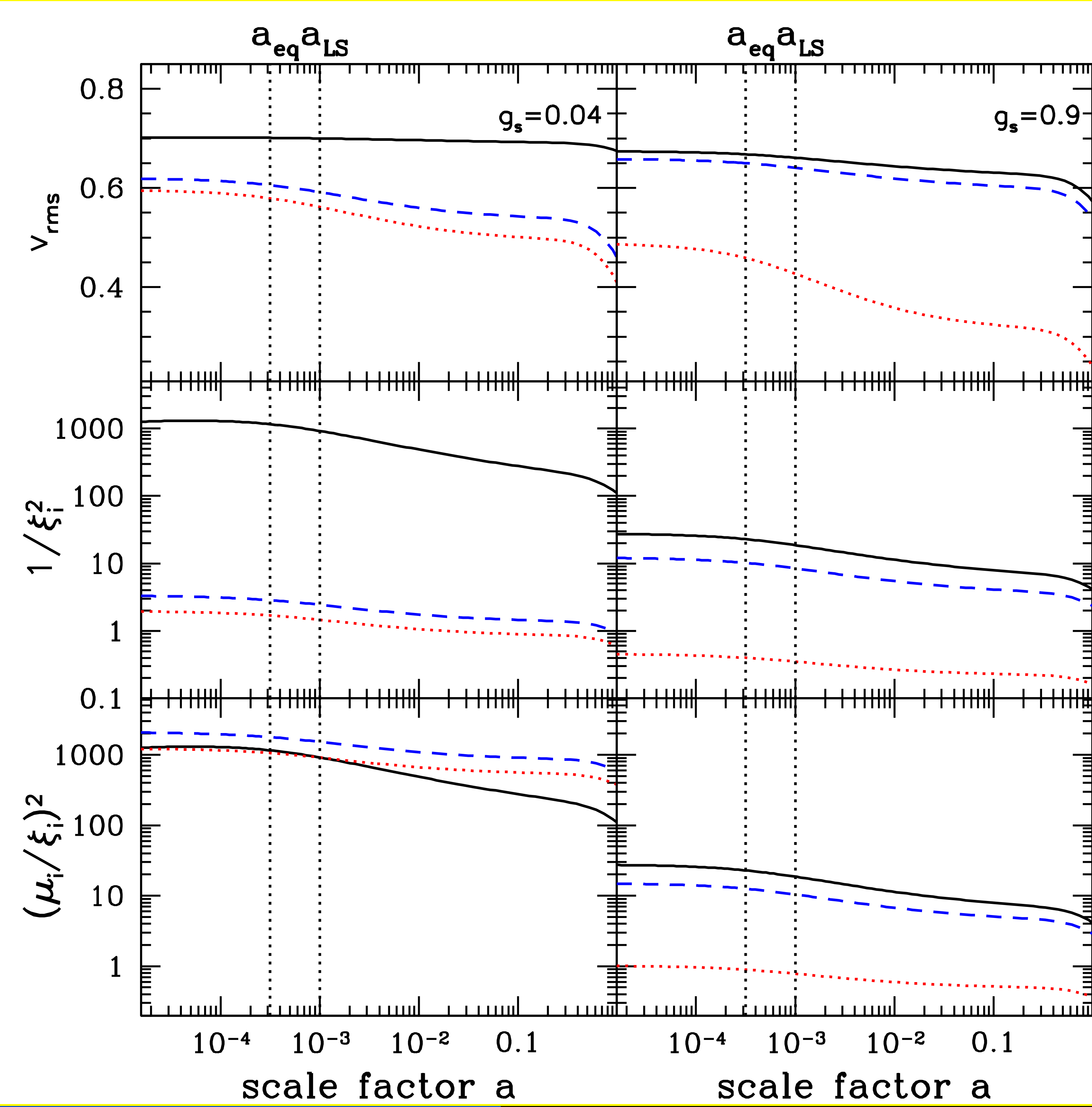
$$\dot{v}_i = (1-v_i^2)\left[\frac{k_i}{L_i} - 2\frac{\dot{a}}{a}v_i + \sum_{b,\, a\leq b} b_{ab}^i \frac{\bar{v}_{ab}}{v_i} \frac{(\mu_a + \mu_b - \mu_i)}{\mu_i} \frac{\ell_{ab}^i(t) L_i^2}{L_a^2 L_b^2}\right]$$

$$v_{ab} = \sqrt{v_a^2 + v_b^2} \qquad \mu_i \equiv \mu_{(p_i,q_i)} = \frac{\mu_F}{g_s}\sqrt{p_i^2 g_s^2 + q_i^2} \qquad \rho_i = \frac{\mu_i}{L_i^2}$$

`k' segment length $\quad \ell_{ij}^k = \frac{L_i L_j}{L_i + L_j}$

d_{ia}^k incorporate the probabilities of intercommuting and the kinetic constraints. They have a strong dependence on the string coupling g_s

$$\{(p,q)_i\} = \{(1,0),(0,1),(1,1),(1,2),(2,1),(1,3),(3,1)\}\ , \quad (i = 1, ..., 7)$$



Avgoustidis et al
(PRL 2011)

Example - 7 types of (p,q) string. Only first three lightest shown - scaling rapidly reached in rad and matter.

Densities of rest suppressed.

Black -- (1,0) -- Most populous
Blue dash -- (0,1)
Red dot dash -- (1,1)

Deviation from scaling at end as move into Λ domination.

Note lighter F strings dominate number density whilst heavier and less numerous D strings dominate power spectrum for at smaller g_s, where as they are comparable at large g_s ~1

General Network Behaviour

- Scaling for all string types

  (though we keep the first 7 lightest strings)

- Only 3 lightest components

  (F, D, FD strings)

- Hierarchy in number densities

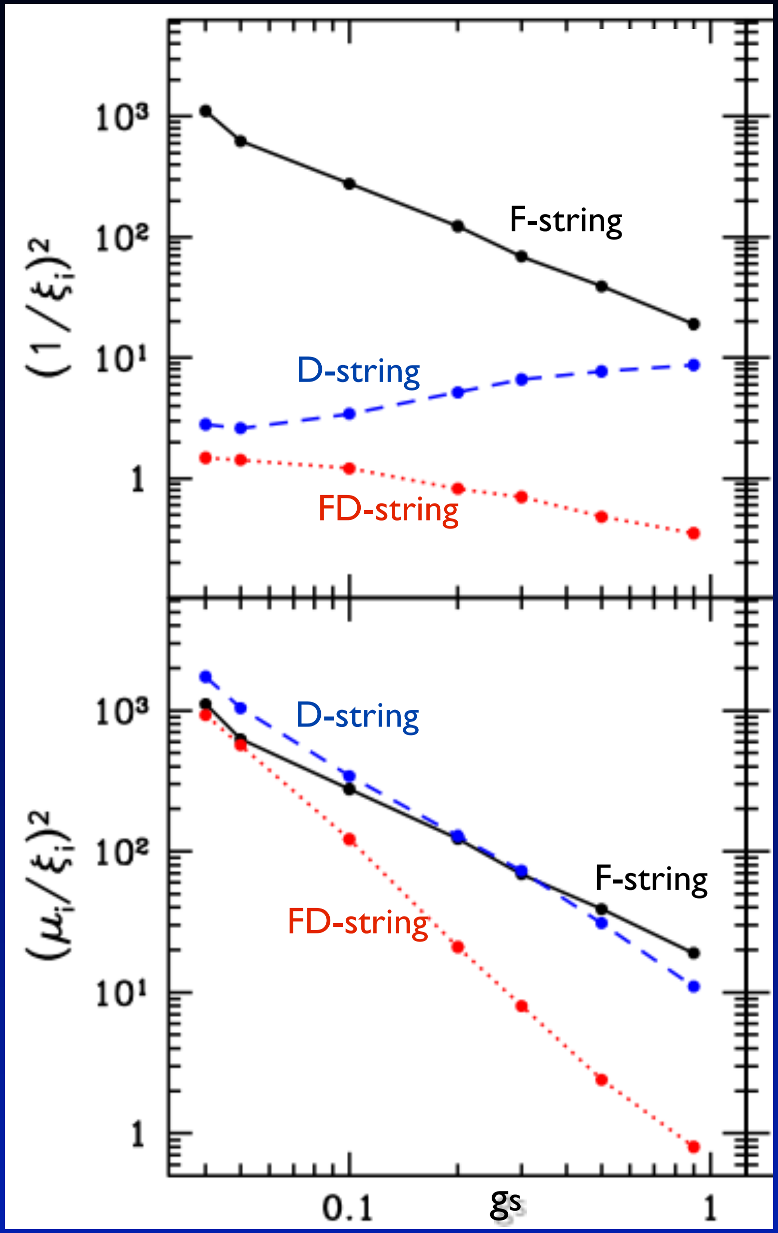
  $N_F > N_D > N_{FD}$

- Hierarchy in tensions

  $\mu_{FD} > \mu_D > \mu_F$

- Number density vs "CMB" density

  Competition depending on g_s



Strings and the CMB

Modified CMBACT (Pogosian) to allow for multi-tension strings.
Shapes of string induced CMB spectra mainly obtained form large scale properties of string such as correlation length and rms velocity given from the earlier evolution eqns.
Normalisation of spectrum depends on:

$$C_l^{strings} \propto \sum_{i=1}^{N} \left(\frac{G\mu_i}{\xi_i} \right)^2$$

i.e. on tension and correlation lengths of each string

Since strings can not source more than 10% of total CMB anisotropy, we use that to determine the fundamental F string tension which is otherwise a free parameter. So μ_F chosen to be such that:

$$f_s = C^{TT}_{strings} / C^{TT}_{total} = 0.1$$

where

$$C^{TT} \equiv \sum_{\ell=2}^{2000} (2\ell + 1) C_\ell^{TT}$$

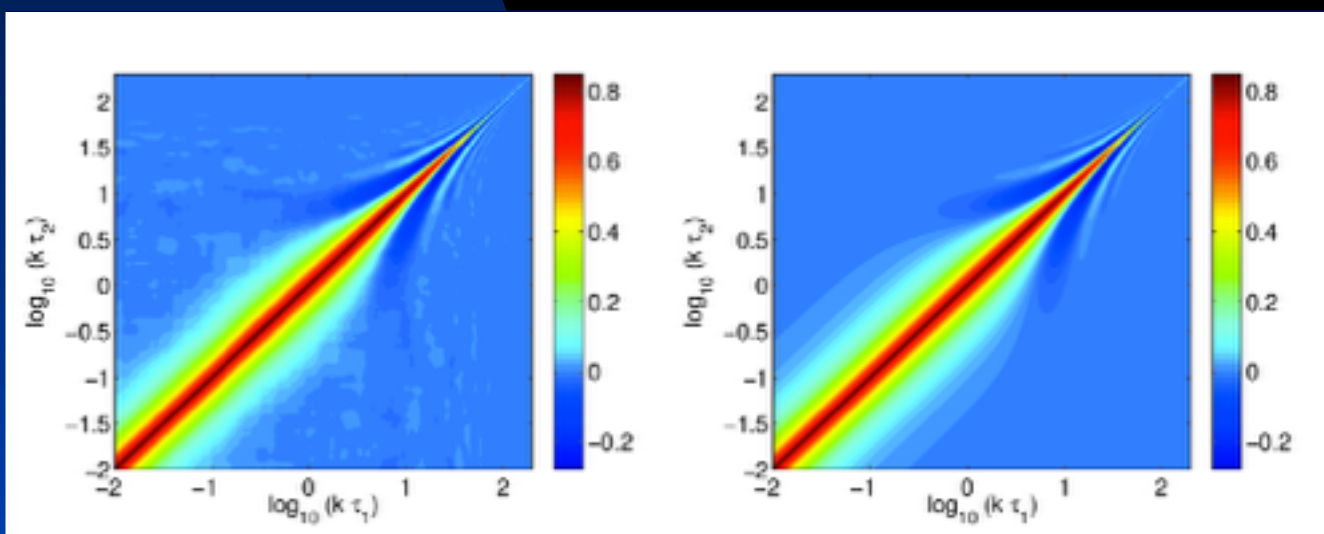
Strings and the CMB

Strings are active, incoherent sources —> require UETC:

$$\langle \Theta(k,\tau_1)\Theta(k,\tau_2)\rangle = \frac{2f(\tau_1,\tau_2,\xi,L_f)}{16\pi^3}\int_0^{2\pi} d\phi \int_0^{\pi} \sin\theta\, d\theta \int_0^{2\pi} d\psi \int_0^{2\pi} d\chi\, \Theta(k,\tau_1)\Theta(k,\tau_2)$$

Model network as made of unconnected string segments with lengths and velocities given by VOS model

Compute integrals analytically [Avgoustidis et al 2012]

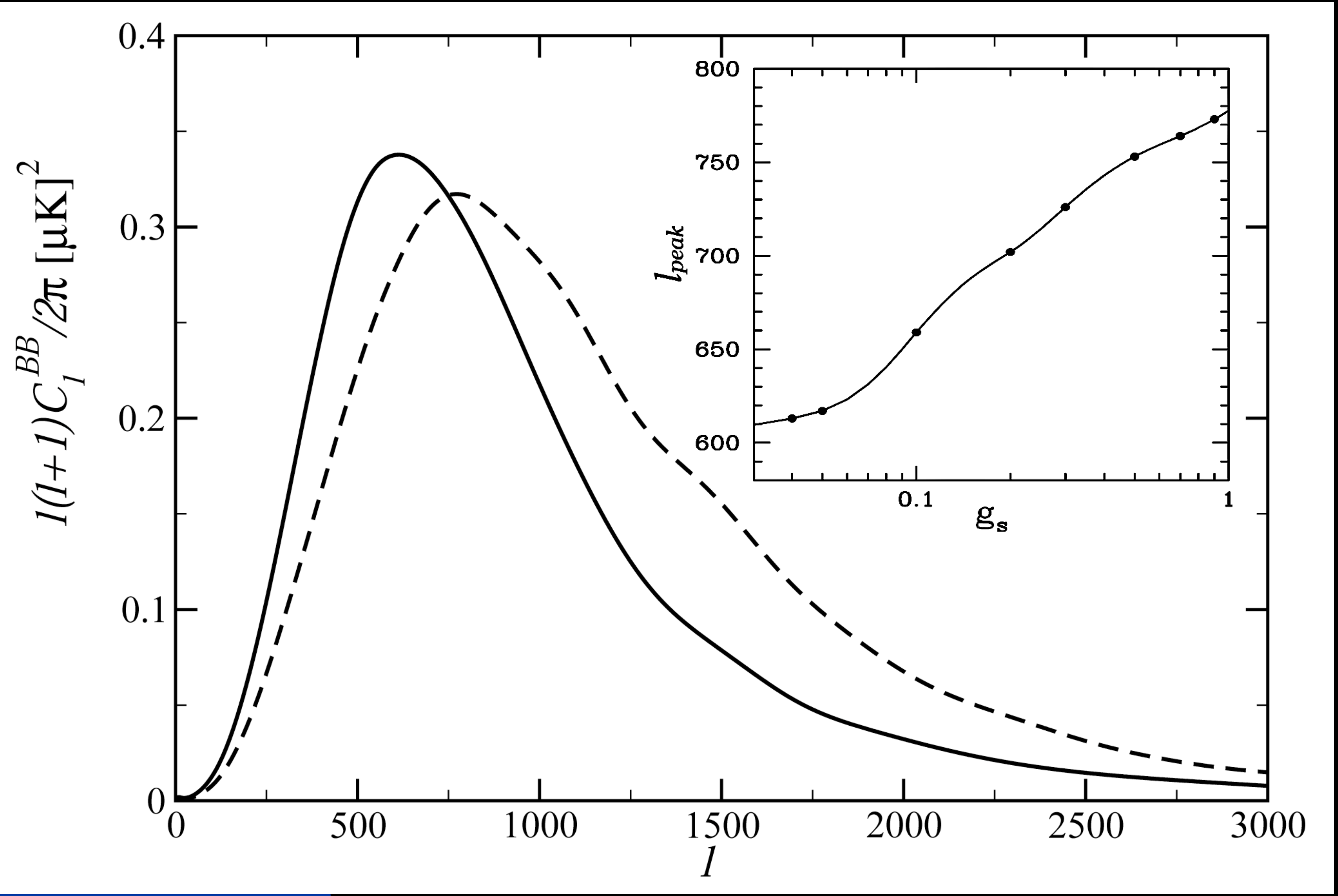


Can get Cl's in a few minutes: MCMC analysis including network parameters now possible [Charnock et al 2016]

USM - 8 hours

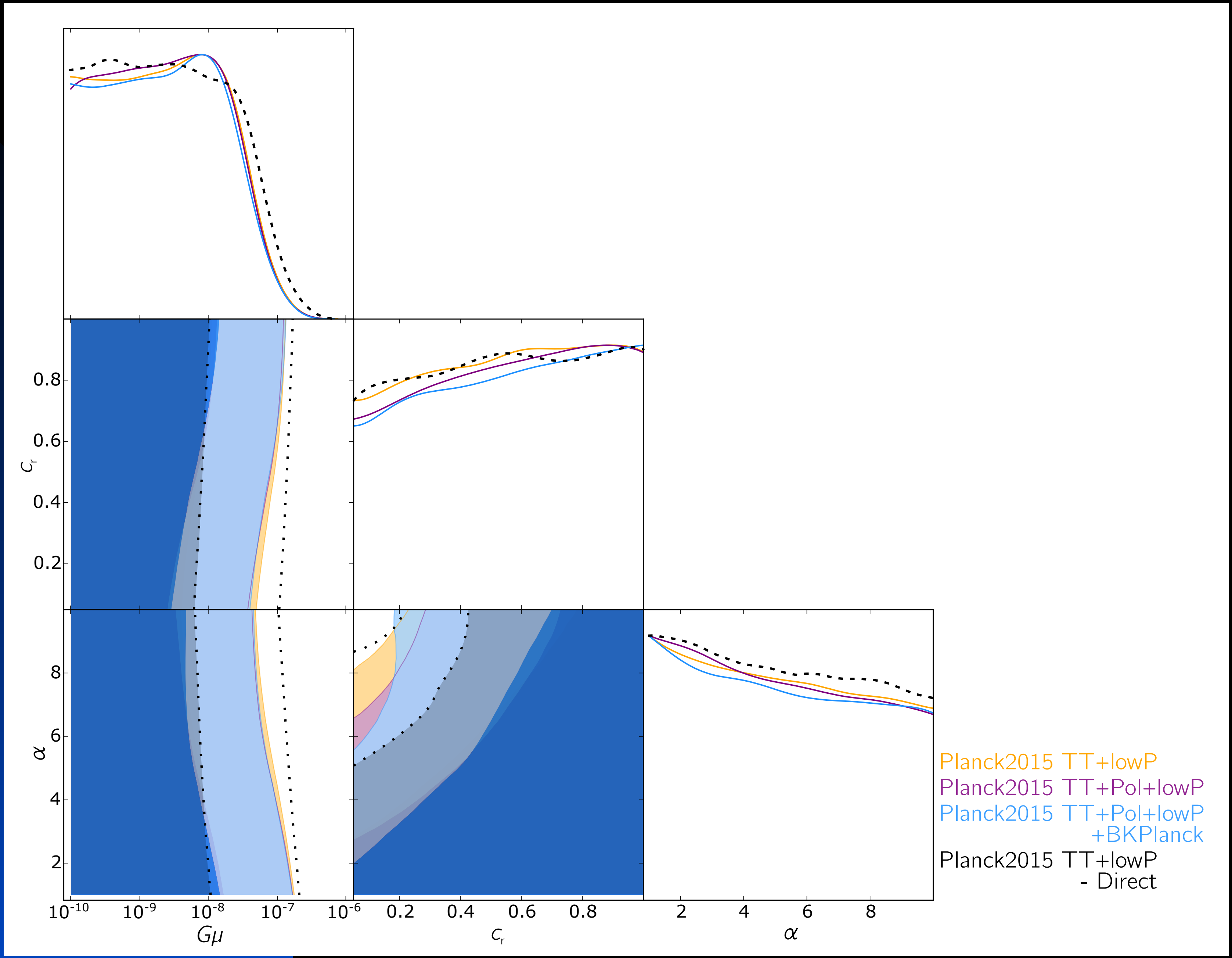
Analytic - 20 secs

B-mode Power Spectrum due to strings



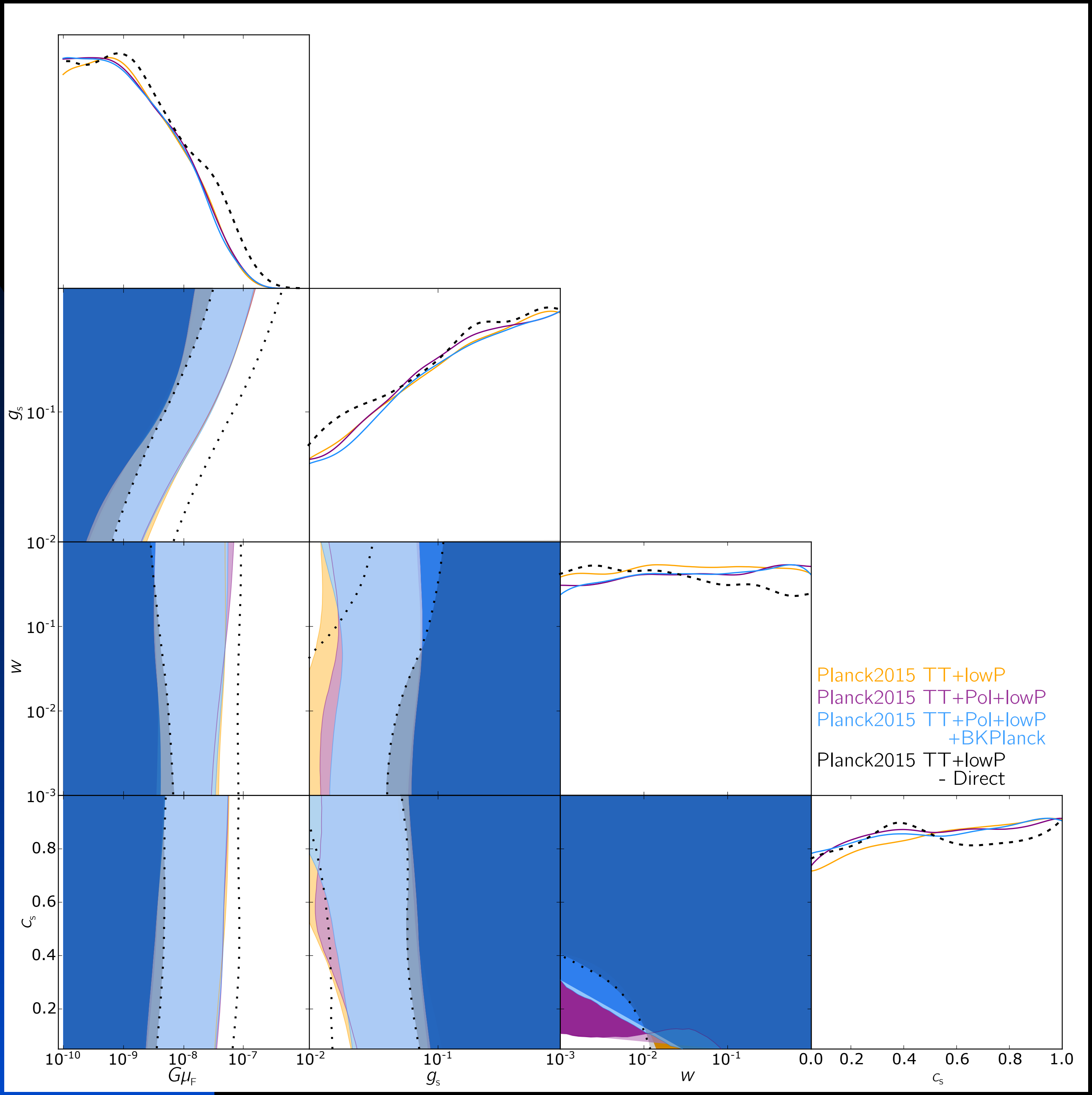
B-mode power spectra for $g_s = 0.04$ (solid) and $g_s = 0.9$ (dash) normalised so that strings contribute 10% of the total CMB anisotropy.
Inset figure -- the position of the peak as a function of string coupling. Note the shift of the peak to lower l values as the string coupling is reduced.
Possible to discriminate them in future experiments like QUIET and Polarbear.

CMB constraints on cosmic strings [Charnock et al 2016]



2σ likelihood contours for Gμ, c_r and α

CMB constraints on cosmic superstrings [Charnock et al 2016]



2σ likelihood contours for Gμ, c_s, g_s and w

Results - cosmic strings:

$$G\mu < 1.1 \times 10^{-7} - \text{ Planck2015 TT}$$
$$G\mu < 9.6 \times 10^{-8} - \text{ Planck2015 TT } + \text{ Pol } + \text{ lowP}$$
$$G\mu < 8.9 \times 10^{-8} - \text{ Planck2015 TT } + \text{ Pol } + \text{ lowP } + \text{ BKPlanck}$$

No constraints on c_r and α, slight preference for higher values of c_r

and lower values of α

Results - cosmic superstrings:

$$G\mu_F < 2.8 \times 10^{-8} - \text{ Planck2015 TT } + \text{ lowP}$$

when marginalised over c_s, g_s and w

Currently looking at three point correlation function for evidence of non-gaussianity and B mode polarisation effects - initial results show signal is extremely small and in fact analytically tensor bi-spectrum vanishes.

Pre-Big Bang

Veneziano and Gasperini

First attempt develop new cosmology based on underlying string sym.

Allows solutions for negative time.

Inflation – based on kinetic energy of massless moduli rather than pot.

Density perturbations produced by flucns in massless fields.

Will discuss origin of perturbations, solns of pre big bang, fine tuning issues and avoiding the singularity – the graceful exit problem in string cosmology.

Recall NS-NS:
$$S_* = \int d^4x\sqrt{|g|}e^{-\varphi}\left[R + (\nabla\varphi)^2 - \frac{1}{2}(\nabla\beta)^2 - \frac{1}{2}e^{2\varphi}(\nabla\sigma)^2\right]$$

Reduced action:
$$S = \int dt e^{-\bar{\phi}}\left[3\dot{\alpha}^2 - \dot{\bar{\phi}}^2 + \frac{1}{2}\dot{\beta}^2 + \frac{1}{2}e^{2\varphi}\dot{\sigma}^2\right]$$

where $\alpha \equiv \ln a.$ and shifted dilaton is defined by: $\bar{\phi} \equiv \varphi - 3\alpha\,.$

Amplitude of the pertns grow towards small scales:

Big for modes outside horizon ($|k\eta| < 1$) only near Planck era, $\widetilde{H}^2 \sim l_{pl}^{-2}$.

Spectral tilt: $n - 1 \equiv \Delta n_x = \dfrac{d \ln \mathcal{P}_{\delta x}}{d \ln k} \longrightarrow \Delta n_\varphi = \Delta n_\beta = 3$

Same blue spectra as for scalar metric perturbation.

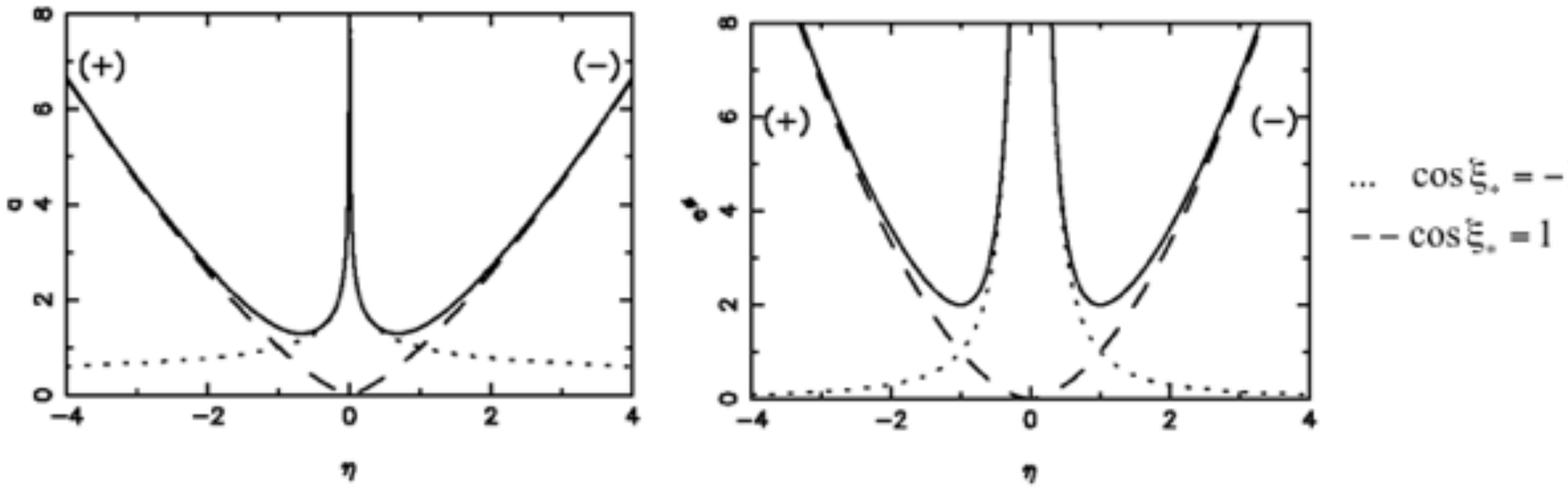
Axion perturbation spectra – hope at last!

Non-minimal coupling to dilaton vital! Canonically norm field: $v \equiv \dfrac{1}{\sqrt{16\pi} l_{\rm Pl}} e^{\varphi} \tilde{a} \delta\sigma \,,$

Spectral index for axion pertn: $\Delta n_\sigma = 3 - 2\sqrt{3} |\cos \xi_*|$ can be -0.04

Since bgd axion field is constant, resulting density pertns only second order in the axion pertn → neglect backreaction from metric to linear order.

Solutions



$$
\begin{aligned}
e^{\varphi} &= e^{\varphi_*}\left|\frac{t}{t_*}\right|^{2\cos\xi_*/(\sqrt{3}+\cos\xi_*)}, \\
a &= a_*\left|\frac{t}{t_*}\right|^{(1+\sqrt{3}\cos\xi_*)/(3+\sqrt{3}\cos\xi_*)}, \\
e^{\beta} &= e^{\beta_*}\left|\frac{t}{t_*}\right|^{2\sin\xi_*/(\sqrt{3}+\cos\xi_*)},
\end{aligned}
$$

For $\cos\xi_* < -1/\sqrt{3}$ inf lation in string frame for $\eta < 0$ and e^{φ} ® 0 as t ® $-\infty$, weak coupling regime.

Note + and - branches

Evolution in Einstein frame – dilaton minimally coupled to gravity

Scale factor:

$$\tilde{a} = \tilde{a}_* \left| \frac{\tilde{t}}{\tilde{t}_*} \right|^{1/3} .$$

As η ® 0 on the (+) branch,
universe collapsing, $\tilde{a}$ ® 0,
comoving Hubble length
$\left| {d(\ln(\tilde{a}))}/{d\eta} \right|^{-1} = 2|\eta|$ decreases.
Inflation also takes place.

Importance

Comoving scale starts in past inside Hubble radius, becomes larger than Hubble radius as $t \to 0$.

Can produce perturbations in dilaton, graviton and other matter fields on scales much larger than the present Hubble radius, from quantum fluctuations in flat spacetime at earlier times – a vital property of any inflationary scenario.

But:

- Axion interpolates between two strong coupling dilaton-moduli vac solutions.
- Curavture tends to prevent enough inflation.
- String coupling must be initially v. small
- Intial size of homog region large in string units.

Where does initial state come from?

Ensemble of grav and dilatonic grav waves? Known as Asymptotic past triviality. In Einstein frame, waves undergo collapse when certain conditions satisfied. In string frame, these gravitational unstable areas expand into homo regions on large scales. – still not clear!

Summary

1. Radiation in the early universe can be the moduli’s friend guiding it to its minima and stopping it running away and decompactifying everything - scaling solutions.

2. Large Volume Inflation appears robust to allowing many moduli fields to evolve - maybe issues over the light axions which can’t evolve too far way from their minima

3. Beginning now to get CMB constraints on the cosmic superstring parameters through the B mode Power spectrum, although much more to do through the bispectrum.

4. Future constraints will be enhanced through GW signatures and Pulsar bounds.

5. Pre Big Bang model can lead to nearly scale invariant spectral index with the presence of axions but then it is isocurvature modes and would have to be converted to curvature perturbations.