Rapid Oscillation of Gravitational Constant in the Scalar-Tensor Theory of Gravity:

the early-time constraints on its induced energy density from cosmology

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Outline

I. Introduction

II. An Expanding Universe in the Scalar-Tensor Theory

III. Higgs Field as a Source of G Oscillation

IV. Phenomenological Constraints on NMC Scalar Field

V. Summary

Introduction

Introduction

The scalar-tensor theory of gravity

- Brans-Dicke Theory (1961)

$$S_{BD} = \int d^4x\sqrt{-g}\left(\boxed{\varphi R} - \frac{\omega}{\varphi}g^{\mu\nu}\nabla_\mu\varphi\nabla_\nu\varphi + \mathcal{L}_{\mathcal{M}}\right)$$

- A **non-minimal coupling (NMC)** between φ and gravity
- In general, the theory reduces to general relativity as $\omega \to \infty$

$$S_{EH} = \int d^4x\sqrt{-g}\left(\frac{1}{16\pi G}R\right)$$

In this sense, we have an "**effective gravitational constant**" with intrinsic dynamics from φ:

$$G_{\text{eff}}(\varphi) \equiv \frac{1}{16\pi\varphi}$$

Introduction

Motivations

- Non-Minimal Coupling (NMC)

$$S_{BD} = \int d^4x\sqrt{-g}\left(\boxed{\varphi R} - \frac{\omega}{\varphi}g^{\mu\nu}\,\nabla_\mu\varphi\nabla_\nu\varphi + \mathcal{L}_{\mathcal{M}}\right)$$

- Characteristic size of the extra dimensions in the Kaluza-Klein theory
- Dilaton from the string theory
- Low-energy limit of the bosonic string theory
- ….

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- ….

Theories of unification

Introduction

Motivations

- Rapid Oscillation of Gravitational Constant

VOLUME 67, NUMBER 3 PHYSICAL REVIEW LETTERS 15 JULY 1991

Cosmological Consequences of High-Frequency Oscillations of Newton's Constant

Frank S. Accetta and Paul J. Steinhardt

Department of Physics, University of Pennsylvania, Philadelphia, Pennsylvania 19104

(Received 15 November 1990; revised manuscript received 19 February 1991)

We show that high-frequency, small-amplitude oscillations of Newton's constant G can dramatically alter cosmology even if the frequency is very high compared to the expansion rate. For example, it is possible to have a spatially flat universe in which dynamical tests of $\Omega \equiv$(matter density)/(critical density)—tests which attempt to directly measure the mass density of the universe—obtain values less than unity ($\Omega = 0.1-0.3$, say). The cosmological effects can be obtained in a frequency-amplitude range allowed by all known constraints on G and its time derivative.

PACS numbers: 98.80.Dr, 04.50.+h

Accetta and Steinhardt [PRL 67, 298-301 (1991)]

Introduction

Motivations

- Rapid Oscillation of Gravitational Constant

  - During the 1980s …

    Theoretical consequence of inflation: flat universe with $\Omega_{total,0} = 1$

    Observations suggested: $\Omega_{m,0} = 0.1 - 0.3$

    Accelerated expansion of the universe (dark energy)

  - The Friedmann equation from the scalar-tensor theory:

$$\mathbf{H} = \frac{\dot{\mathbf{G}}}{2\mathbf{G}} + \left[\frac{8\pi \mathbf{G}}{3}(\rho_{\mathrm{m}} + \rho_{\mathrm{r}} + \rho_{\phi}) + \left(\frac{\dot{\mathbf{G}}}{2\mathbf{G}}\right)^{2}\right]^{1/2}$$

$$A \sin\ (mt) \;\downarrow\; m \cdot A \cos\ (mt)$$

Accetta and Steinhardt [PRL 67, 298-301 (1991)]

$$\mathbf{H} = \frac{\dot{\mathbf{G}}}{2\mathbf{G}} + \left[\frac{8\pi\mathbf{G}}{3}(\rho_{\mathbf{m}} + \rho_{\mathbf{r}} + \rho_{\phi}) + \left(\frac{\dot{\mathbf{G}}}{2\mathbf{G}}\right)^{\mathbf{2}}\right]^{1/2}$$

- Rapid Oscillation of Gravitational Constant

$$\left(\frac{\dot{G}}{2G}\right)^2 \equiv \frac{8\pi G}{3}\rho_G$$

ρ_G : **Energy density induced by *G* oscillation**

- Phenomenology:

■ Rough constraints from local experiments: $\nu > 10^{16} Hz$ ($\sim 10^{-3} eV$)

■ Consider the ***G* Oscillation (*GO*)** with $\begin{cases} \text{High frequency} \\ \text{Small amplitude } \Delta G \ll G_0 \end{cases}$

Accetta and Steinhardt [PRL 67, 298-301 (1991)]

$$\mathbf{H} = \frac{\dot{\mathbf{G}}}{2\mathbf{G}} + \left[\frac{8\pi\mathbf{G}}{3}(\rho_{\mathbf{m}} + \rho_{\mathbf{r}} + \rho_{\phi}) + \left(\frac{\dot{\mathbf{G}}}{2\mathbf{G}}\right)^{2}\right]^{1/2}$$

- Rapid Oscillation of Gravitational Constant

$$\left(\frac{\dot{G}}{2G}\right)^2 \equiv \frac{8\pi G}{3}\rho_G$$

ρ_G : **Energy density induced by *G* oscillation**

Theoretically, the idea of inflation can lead to G oscillation if the inflaton is non-minimally coupled to gravity

- Phenomenology:

■ Rough constraints from local experiments: $\nu > 10^{16} Hz$ ($\sim 10^{-3} eV$)

■ Consider the ***G* Oscillation (*GO*)** with High frequency / Small amplitude $\Delta G \ll G_0$

Steinhardt and Will [Phys. Rev. D 52, 628 (1995)]

Accetta and Steinhardt [PRL 67, 298-301 (1991)]

Introduction

This Work

- The impact of the **non-minimal coupling (NMC)** on the scalar field
- **Dynamical evolution** behind the rapid oscillation of G
- **Phenomenological constraints** on the effective energy density of the non-minimally coupled scalar field from cosmology and the rapid oscillation of G

An Expanding Universe in the Scalar-Tensor Theory

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An Expanding Universe in the Scalar-Tensor Theory

- Begin with the Lagrangian:

$$\mathcal{L} = \mathbf{f}(\phi)\mathbf{R} - \frac{1}{2}\mathbf{g}^{\mu\nu}\nabla_\mu\phi\nabla_\nu\phi - \mathbf{V}(\phi)$$

$$\mathbf{f}(\phi) \equiv \frac{1}{16\pi\mathbf{G}(\phi)}$$

$$\mathbf{S} = \int \mathbf{d}^4\mathbf{x}\sqrt{-\mathbf{g}}\,\left(\mathcal{L} + \mathcal{L}_{\mathcal{M}} - \mathbf{M}_\mathbf{P}^2\mathbf{\Lambda}\right)$$

$$\hbar = 1 = c$$

The reduced Planck mass:

$$M_P \equiv \left(\frac{1}{8\pi G_0}\right)^{1/2} \approx 2.435 \times 10^{18} GeV$$

$f(\phi)$: coupling function

$f = \frac{M_P^2}{2} = \frac{1}{16\pi G_0}$ for General Relativity

$f(\phi) = \frac{1}{16\pi\, G_{eff}(\phi)}$ for the scalar-tensor theory

An Expanding Universe in the Scalar-Tensor Theory

- The field equations

$$\boldsymbol{f(\phi) = \frac{1}{16\pi\, G_{eff}(\phi)}}$$

  - The "modified Einstein equations" :

$$\begin{aligned} R_{\mu\nu} - \frac{1}{2}R\, g_{\mu\nu} = {} & \frac{1}{2f}\, T_{\mu\nu} \\ & + \frac{1}{2f}\left\{ \nabla_\mu\phi\nabla_\nu\phi - g_{\mu\nu}\left(\frac{1}{2}\nabla^\lambda\phi\nabla_\lambda\phi + V\right)\right\} \\ & + \frac{1}{f}\left(\nabla_\mu\nabla_\nu f - g_{\mu\nu}\Box f\right) - \frac{M_P^2}{2f}\,\Lambda g_{\mu\nu} \end{aligned}$$

  - Equation of motion of the scalar field :

$$-\Box\,\phi = f'(\phi)R - V'(\phi)$$

An Expanding Universe in the Scalar-Tensor Theory

- The flat FLRW background
  + homogeneous, isotropic scalar field $\phi(t)$ non-minimally coupled to gravity

$$\mathbf{H} = -\frac{\mathbf{f}'}{\mathbf{2f}}\dot{\phi} + \left[\frac{\mathbf{1}}{\mathbf{6f}}\left(\rho_{\mathbf{G}} + \sum_{\mathbf{i}} \rho_{\mathbf{i}}\right)\right]^{\mathbf{1/2}}$$

i = matter, radiation, Λ, ϕ

$$\boxed{\mathcal{F}(\phi) \equiv \sqrt{1 + \frac{\mathbf{3f}'^{\mathbf{2}}}{\mathbf{f}}}}$$

$$\ddot{\phi} + \mathbf{3H}\dot{\phi} = \frac{\mathbf{-1}}{\mathcal{F}^{\mathbf{2}}} \cdot \left\{\mathbf{V}' + \frac{\mathbf{f}'}{\mathbf{2f}}\left[\mathbf{6f}''\dot{\phi}^{\mathbf{2}} - \sum_{\mathbf{i}}(\rho_{\mathbf{i}} - \mathbf{3}\mathcal{P}_{\mathbf{i}})\right]\right\}$$

$$\rho_\phi = \frac{\mathbf{1}}{\mathbf{2}}\dot{\phi}^{\mathbf{2}} + \mathbf{V}(\phi)$$

- The function $\mathcal{F}$ reduces to 1 if there is no non-minimal coupling

$$\rho_{\mathbf{G}} \equiv \frac{\mathbf{3f}'^{\mathbf{2}}}{\mathbf{2f}}\dot{\phi}^{\mathbf{2}}$$

- We will show that $\mathcal{F}$ could significantly influence the evolution of ϕ in some situations

$$\mathbf{H} = -\frac{\mathbf{f}'}{2\mathbf{f}}\dot{\phi} + \left[\frac{1}{6\mathbf{f}}\left(\rho_{\mathbf{G}} + \sum_{\mathbf{i}} \rho_{\mathbf{i}}\right)\right]^{1/2}$$

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$$\ddot{\phi} + 3\mathbf{H}\dot{\phi} = \frac{-1}{\mathcal{F}^2} \cdot \left\{\mathbf{V}' + \frac{\mathbf{f}'}{2\mathbf{f}}\left[6\mathbf{f}''\dot{\phi}^2 - \sum_{\mathbf{i}}(\rho_{\mathbf{i}} - 3\mathcal{P}_{\mathbf{i}})\right]\right\}$$

$$\boxed{\mathcal{F}(\phi) \equiv \sqrt{1 + \frac{3\mathbf{f}'^2}{\mathbf{f}}}}$$

Effective energy density

j = matter, radiation, Λ

$$\boxed{\mathbf{H} = \underbrace{-\frac{\mathbf{f}'}{2\mathbf{f}}\dot{\phi}}_{\delta\mathbf{H}} + \underbrace{\left[\frac{1}{6\mathbf{f}}\left(\widetilde{\rho}_{\phi} + \sum_{\mathbf{j}} \rho_{\mathbf{j}}\right)\right]^{1/2}}_{\bar{\mathbf{H}}}}$$

Fluctuation *Average*

$$\Longrightarrow \quad \widetilde{\rho}_{\phi} \equiv \rho_{\phi} + \rho_{\mathbf{G}} = \frac{1}{2}\mathcal{F}(\phi)^2\dot{\phi}^2 + \mathbf{V}(\phi)$$

$$\boxed{\frac{\mathbf{d}}{\mathbf{dt}}\widetilde{\rho}_{\phi} = \underbrace{-3\mathcal{F}^2\bar{\mathbf{H}}\dot{\phi}^2}_{\textit{Damped}} + \frac{\mathbf{f}'}{2\mathbf{f}}\left(4\widetilde{\rho}_{\phi} + \rho_{\mathbf{m}} + 4\rho_{\Lambda}\right)\dot{\phi}}$$

$$\mathbf{H} = -\frac{\mathbf{f}'}{2\mathbf{f}}\dot{\phi} + \left[\frac{1}{6\mathbf{f}}\left(\rho_{\mathbf{G}} + \sum_{\mathbf{i}} \rho_{\mathbf{i}}\right)\right]^{1/2}$$

i = matter, radiation, Λ, ϕ

$$\ddot{\phi} + 3\mathbf{H}\dot{\phi} = \frac{-1}{\mathcal{F}^2} \cdot \left\{ \mathbf{V}' + \frac{\mathbf{f}'}{2\mathbf{f}}\left[6\mathbf{f}''\dot{\phi}^2 - \sum_{\mathbf{i}}(\rho_{\mathbf{i}} - 3\mathcal{P}_{\mathbf{i}})\right]\right\}$$

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Fluctuation *Average*

Effective energy density

$$\Longrightarrow \quad \tilde{\rho}_{\phi} \equiv \rho_{\phi} + \rho_{\mathbf{G}} = \frac{1}{2}\mathcal{F}(\phi)^2\dot{\phi}^2 + \mathbf{V}(\phi)$$

- This suggests that ρ_G and ρ_ϕ are complementary and should be considered together as $\tilde{\rho}_\phi$

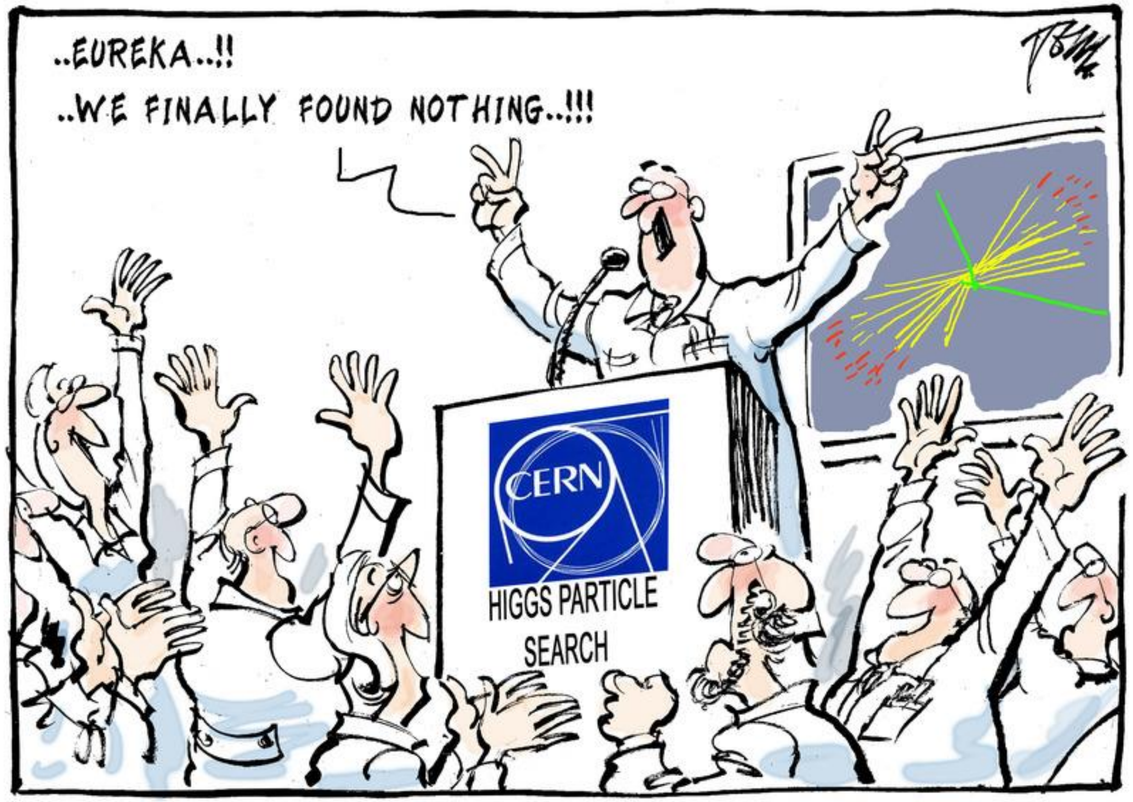
$$\boxed{\frac{\mathbf{d}}{\mathbf{dt}}\tilde{\rho}_{\phi} = \underbrace{-3\mathcal{F}^2\bar{\mathbf{H}}\dot{\phi}^2}_{Damped} + \frac{\mathbf{f}'}{2\mathbf{f}}\left(4\tilde{\rho}_{\phi} + \rho_{\mathbf{m}} + 4\rho_{\Lambda}\right)\dot{\phi}}$$

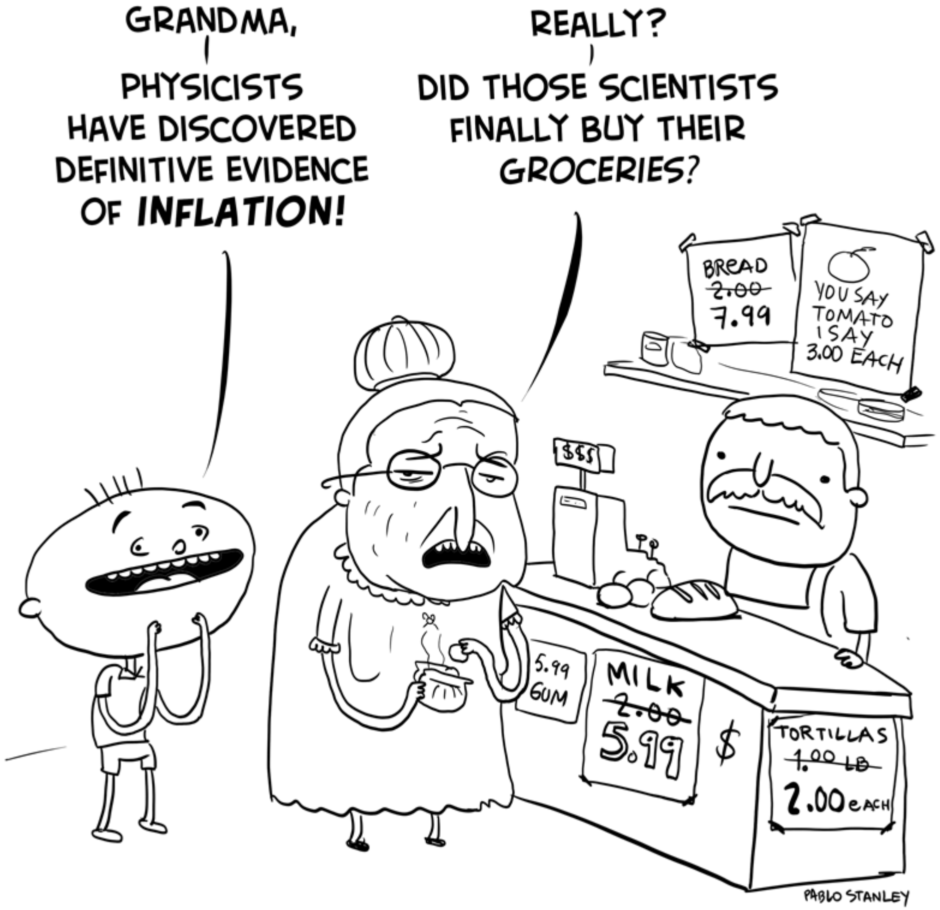
- For certain form of potential $V(\phi)$: it is possible to find the oscillatory solution of $\phi(t)$

Higgs Field as a Source of G Oscillation

Higgs Field
as a Source of G Oscillation

Higgs Inflation





Higgs Inflation

- Can we produce the cosmic inflation under the framework of the standard model?
- Is it favored by the current CMB measurements?

Higgs Field as a Source of G Oscillation

Higgs Inflation

- Can we produce the cosmic inflation under the framework of the standard model?
- Is it favored by the current CMB measurements?

■ First attempt without the non-minimal coupling between the Higgs field and gravity:

After the electroweak phase transition: $\mathbf{V(h) = \frac{1}{4}\lambda(h^2 - v^2)^2}$

Standard Model: $\lambda \sim \mathcal{O}(1)$

CMB constraint: $\lambda \sim \left(\frac{\Delta T}{T}\right)^2 \sim 10^{-10}$

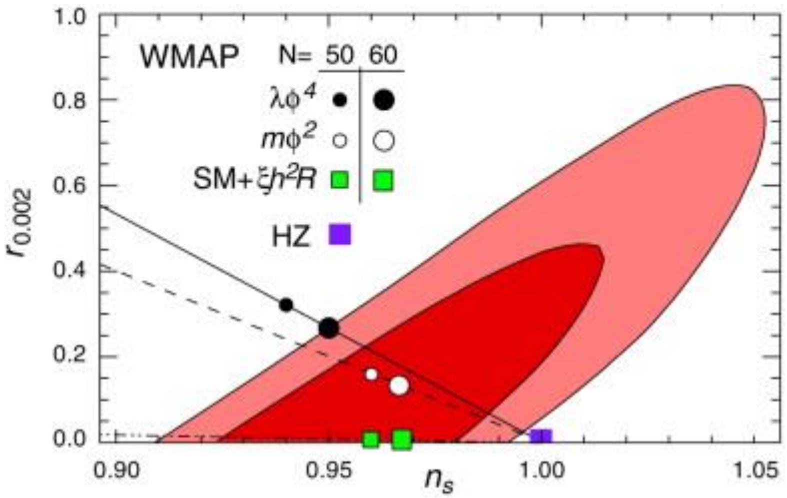
Higgs Field as a Source of G Oscillation

Higgs Inflation

- Can we produce the cosmic inflation under the framework of the standard model?
- Is it favored by the current CMB measurements?

■ What if: Strong non-minimal coupling between the Higgs field and gravity

$$\mathbf{S} = \int \mathbf{d}^4\mathbf{x}\sqrt{-\mathbf{g}}\left\{\mathbf{f}(\mathbf{h})\mathbf{R} - \frac{1}{2}\partial_\mu \mathbf{h}\,\partial^\mu \mathbf{h} - \frac{\lambda}{4}\left(\mathbf{h}^2 - \mathbf{v}^2\right)^2\right\} \qquad \mathbf{f}(\mathbf{h}) = \frac{\mathbf{M}^2}{2}\left(1 + \xi\frac{\mathbf{h}^2}{\mathbf{M}^2}\right)$$



Standard Model: $\lambda \sim \mathcal{O}(1)$

CMB constraint: $\dfrac{\lambda}{\xi^2} \sim \left(\dfrac{\Delta T}{T}\right)^2 \sim 10^{-10}$

Bezrukov and Shaposhnikov [Phys. Lett. B, 659 (2008), p. 703]

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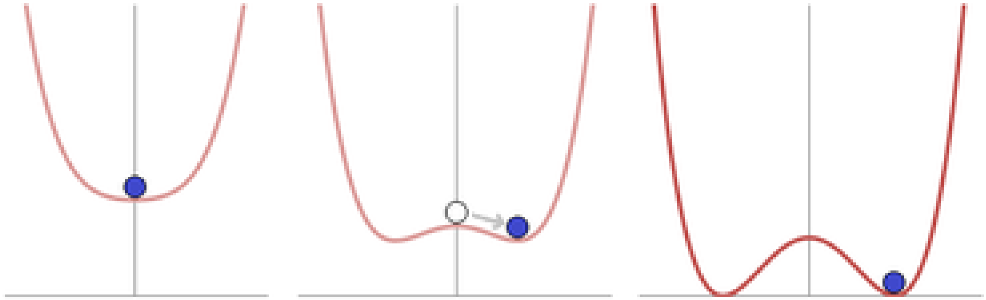

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Bezrukov and Shaposhnikov [Phys. Lett. B, 659 (2008), p. 703]

- Inspired by the Higgs Inflation…

$$\mathbf{V}(\phi) = \frac{1}{2}\mathbf{m}^2\phi^2$$

Recall that $\mathcal{F}(\phi) \equiv \sqrt{1 + \frac{3f'^2}{f}}$

- *Linear NMC:* $f_L(\phi) = \frac{M_P^2}{2}\left(1 + \alpha\frac{\phi}{M_P}\right)$ $\quad \mathcal{F}_L(\phi) = \sqrt{1 + \frac{3}{2}\alpha^2\left(1 + \alpha\frac{\phi}{M_P}\right)^{-1}}$

- *Quadratic NMC:* $f_Q(\phi) = \frac{M_P^2}{2}\left(1 + \xi\frac{\phi^2}{M_P^2}\right)$ $\quad \mathcal{F}_Q(\phi) = \sqrt{1 + 6\frac{\xi^2\phi^2}{M_P^2}\left(1 + \xi\frac{\phi^2}{M_P^2}\right)^{-1}}$

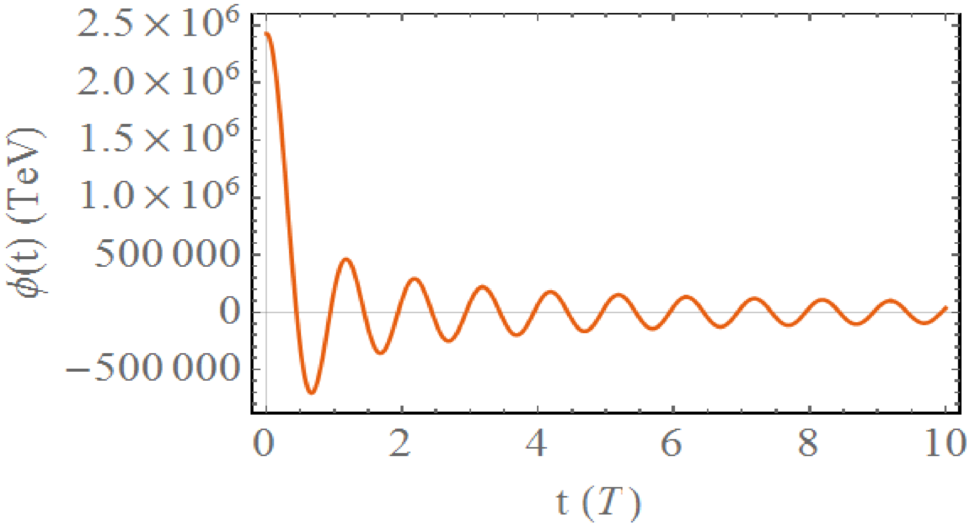
- Our Equation: $$\frac{d}{dt}\widetilde{\rho}_\phi = -3\mathcal{F}^2\bar{H}\dot{\phi}^2 + \frac{f'}{2f}\left(4\widetilde{\rho}_\phi + \rho_m + 4\rho_\Lambda\right)\dot{\phi}$$

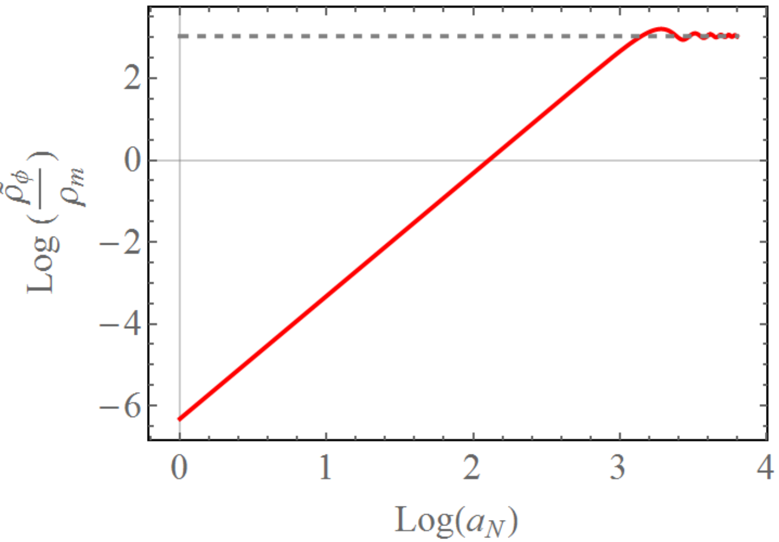
- For both ***Linear*** and ***Quadratic*** NMC, this can be the case…

Slow-roll: $\nu \lesssim H,\ \Delta G \ll G_0$

- The damping force from the cosmic expansion dominates the dynamics of ϕ

- Typically, this happens in the early universe (i.e., H is extremely large)

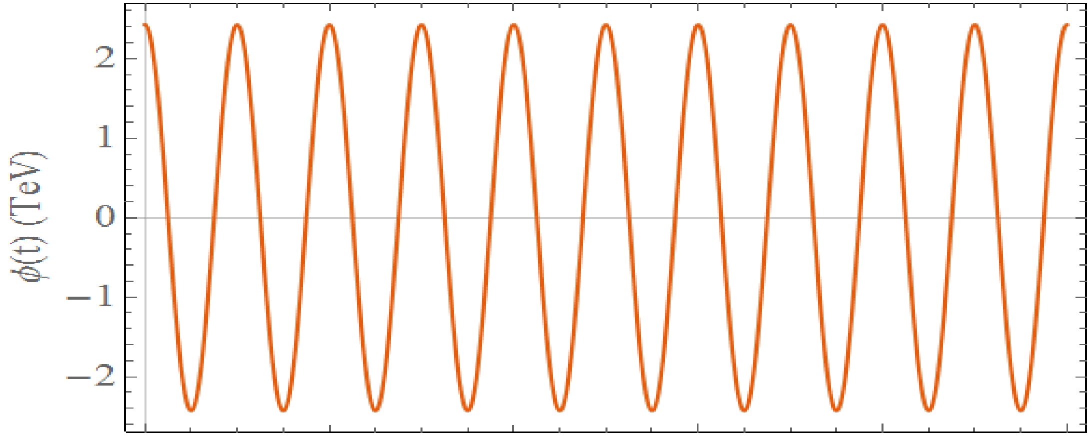


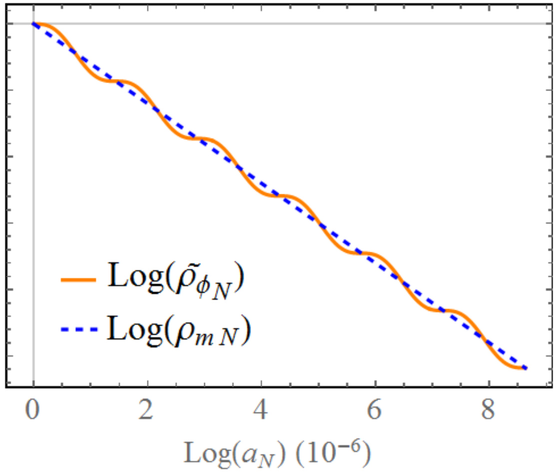


- For *Linear* NMC: $\mathbf{f_L}(\phi) = \frac{\mathbf{M_P^2}}{\mathbf{2}}\left(\mathbf{1} + \alpha\frac{\phi}{\mathbf{M_P}}\right)$ $\quad \mathbf{V}(\phi) = \frac{\mathbf{1}}{\mathbf{2}}\mathbf{m}^2\phi^2$

Type-1 dissipation: $\nu \gg H,\ \Delta G \ll G_0 \quad (\alpha\phi \ll M_P)$

- The cosmic expansion can be ignored within few periods of oscillation
- Frequency of oscillation: $\nu \sim \frac{m}{\mathcal{F}} \simeq \frac{m}{\sqrt{1+3\alpha^2/2}}$
- The dissipation of the effective energy density: $\tilde{\rho}_\phi \propto a^{-3}$





- For *Quadratic* NMC: $$f_Q(\phi) = \frac{M_P^2}{2}\left(1 + \xi\frac{\phi^2}{M_P^2}\right) \quad V(\phi) = \frac{1}{2}m^2\phi^2$$

Type-1 dissipation: $\nu \gg H,\ \Delta G \ll G_0\ \ (\xi\phi^2 \ll M_P^2),\ \mathcal{F}(\phi_{\mathbf{max}}) \simeq 1$

- The cosmic expansion can be ignored within few periods of oscillation
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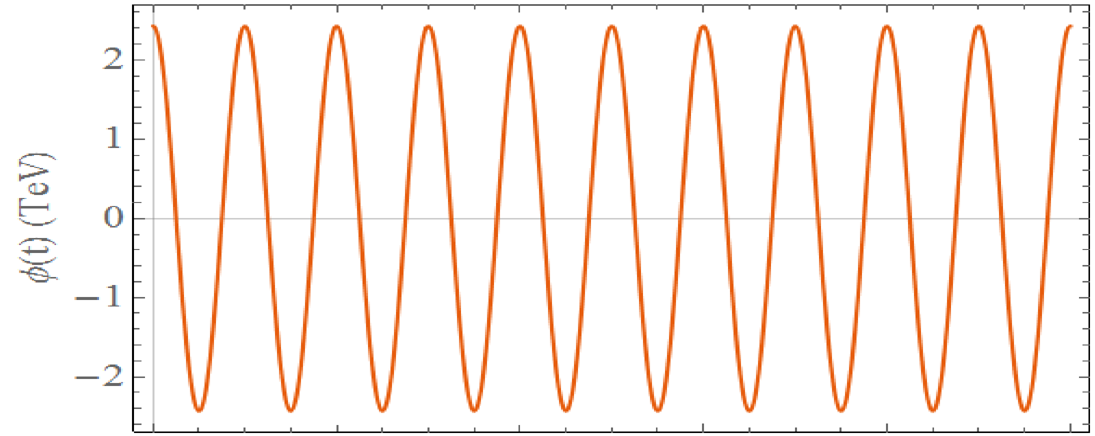
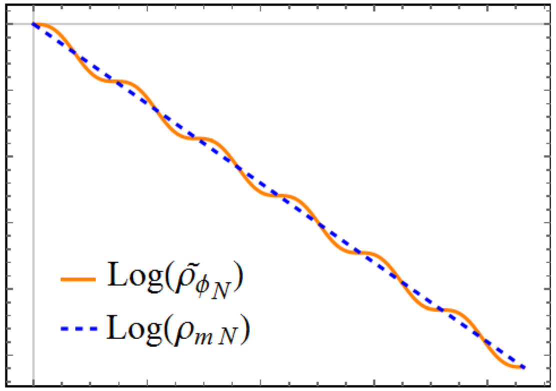




Essentially, the Type-1 dissipation is not so surprising. Since one might expect that as:

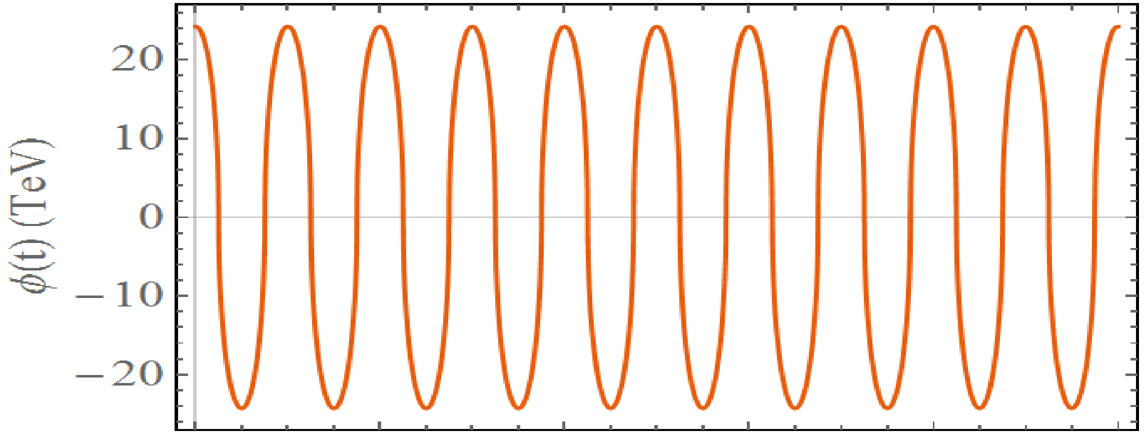
$$f \approx \frac{M_P^2}{2} (i.e., \Delta G \ll G_0),$$

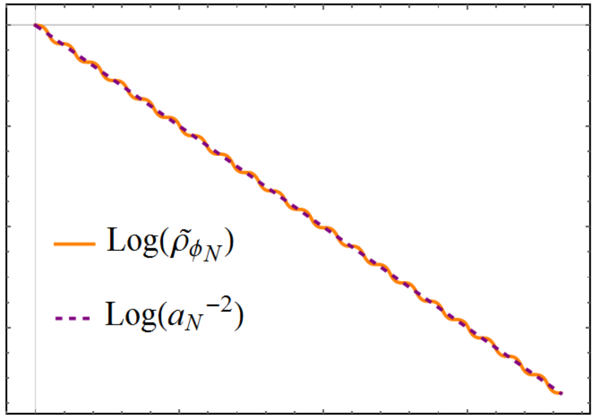
NMC seems to disappear, and the theory would simply return to GR with single scalar field…

- For *Quadratic* NMC: $f_Q(\phi) = \frac{M_P^2}{2}\left(1 + \xi \frac{\phi^2}{M_P^2}\right)$ $V(\phi) = \frac{1}{2} m^2 \phi^2$

Type-2 dissipation: $\nu \gg H,\ \Delta G \ll G_0\ (\xi\phi^2 \ll M_P^2)$, $\boxed{\mathcal{F}(\phi_{max}) \gg 1}$

- The cosmic expansion can be ignored within few periods of oscillation
- ***However, the oscillatory behavior changes a lot :*** $\nu(\phi_{max}) \approx \frac{m M_P}{4\sqrt{6}\,\xi} \cdot \frac{1}{\phi_{max}}$
- The dissipation of the effective energy density: $\tilde{\rho}_\phi \propto a^{-2}$





- For *Quadratic* NMC: $\mathbf{f_Q(\phi) = \frac{M_P^2}{2}\left(1 + \xi\frac{\phi^2}{M_P^2}\right)}$ $\mathbf{V(\phi) = \frac{1}{2}m^2\phi^2}$

Type-2 dissipation: $\nu \gg H,\ \Delta G \ll G_0\ (\xi\phi^2 \ll M_P^2)$, $\boxed{\mathcal{F}(\phi_{\mathbf{max}}) \gg 1}$

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- ***However, the oscillatory behavior changes a lot :*** $\nu(\phi_{max}) \approx \frac{mM_P}{4\sqrt{6}\,\xi} \cdot \frac{1}{\phi_{max}}$
- The dissipation of the effective energy density: $\tilde{\rho}_\phi \propto a^{-2}$

Even $f \approx \frac{M_P^2}{2}$, as the case for GR, NMC can make non-trivial effect on the scalar field!!

Phenomenological Constraints on NMC Scalar Field

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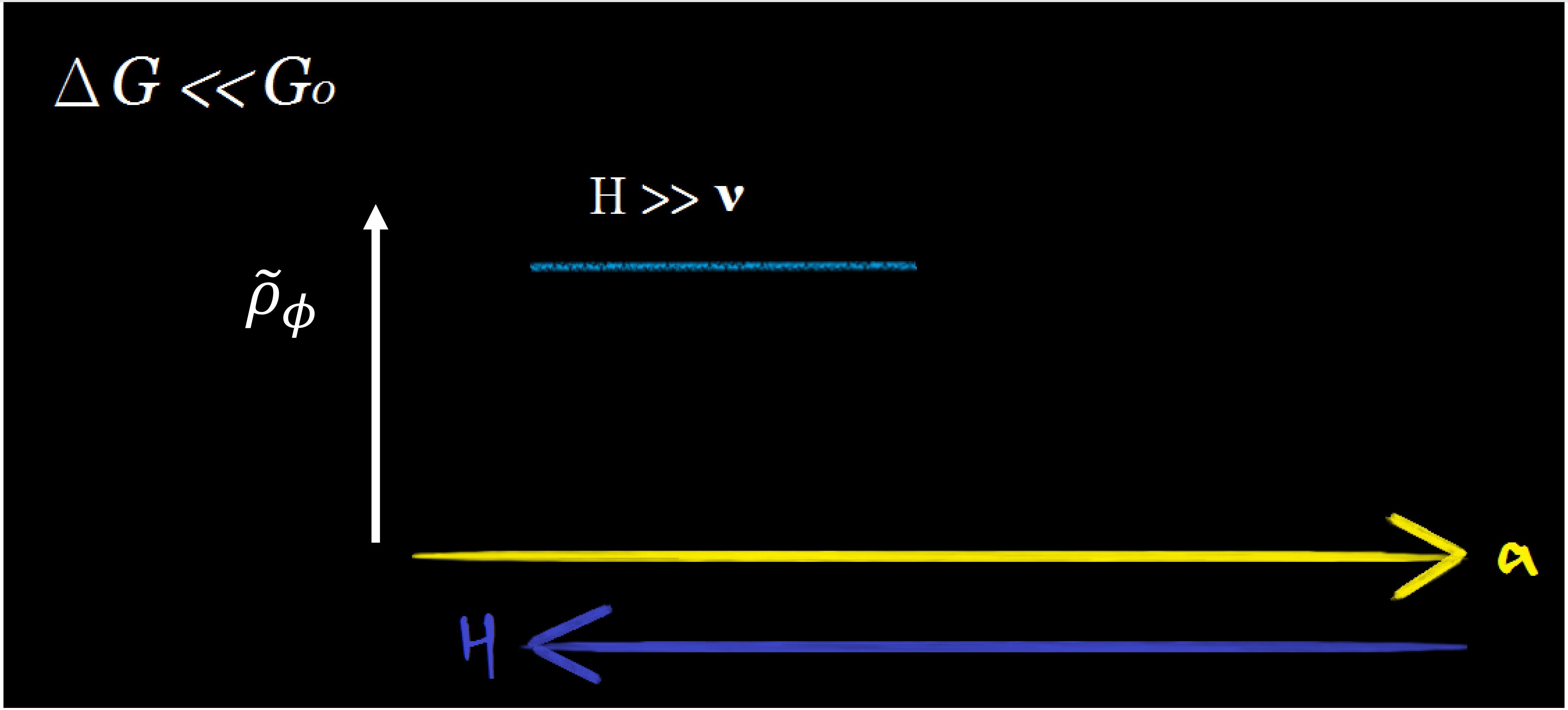
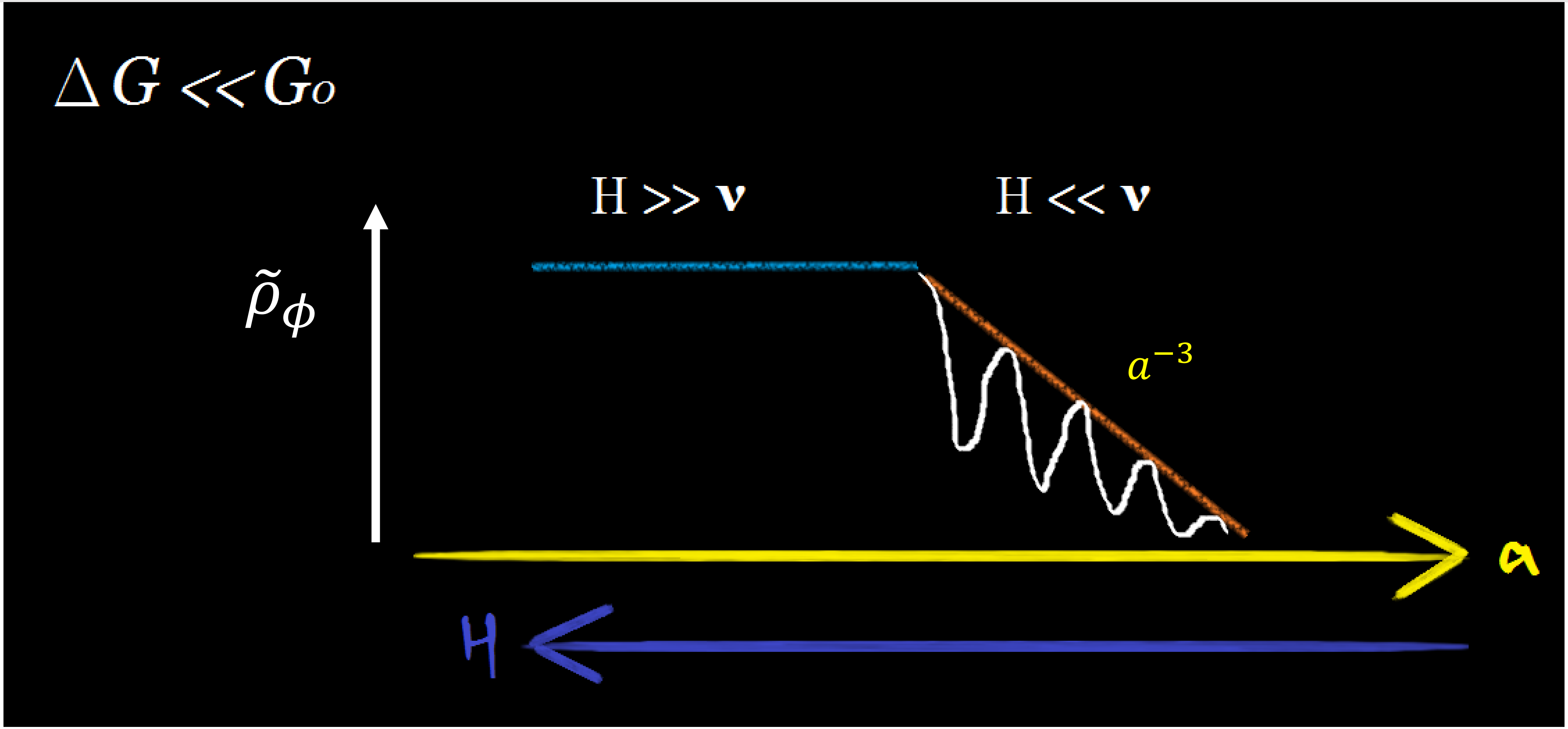
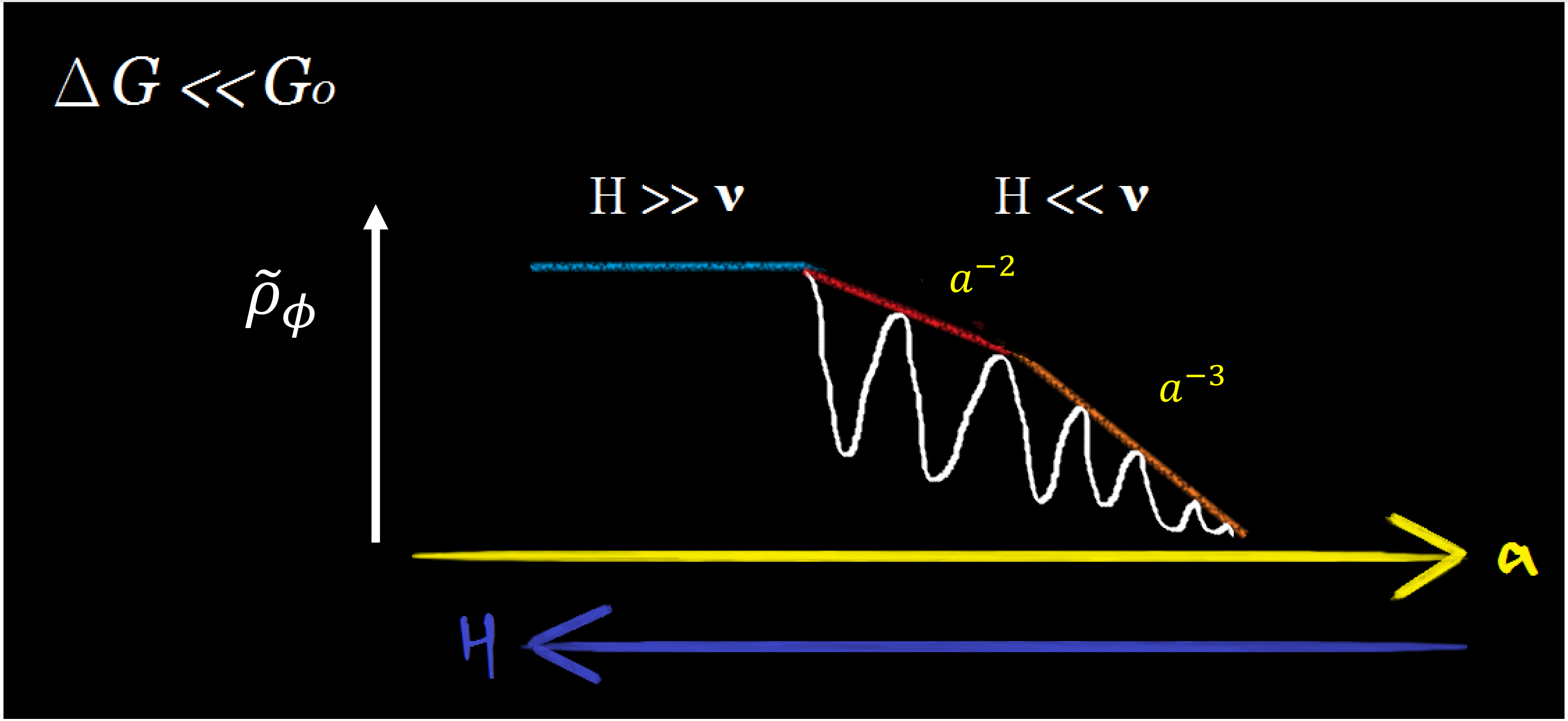
Evolution of the energy density of the NMC scalar field

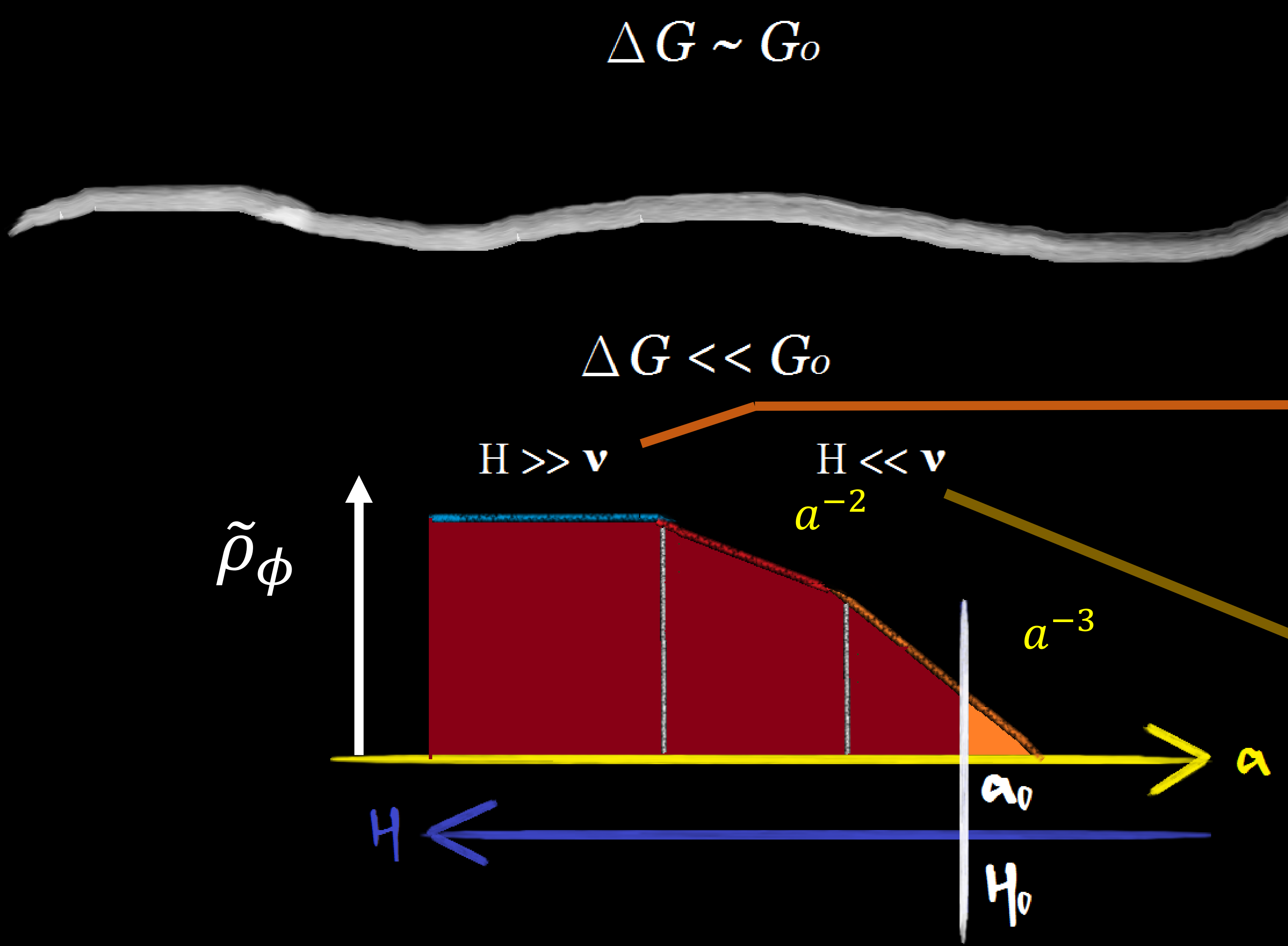


- **Evolution for the** { **Linear-NMC models** / **Quadratic-NMC models with small ξ** }



■ (Typical) Evolution for the Quadratic-NMC models with large ξ



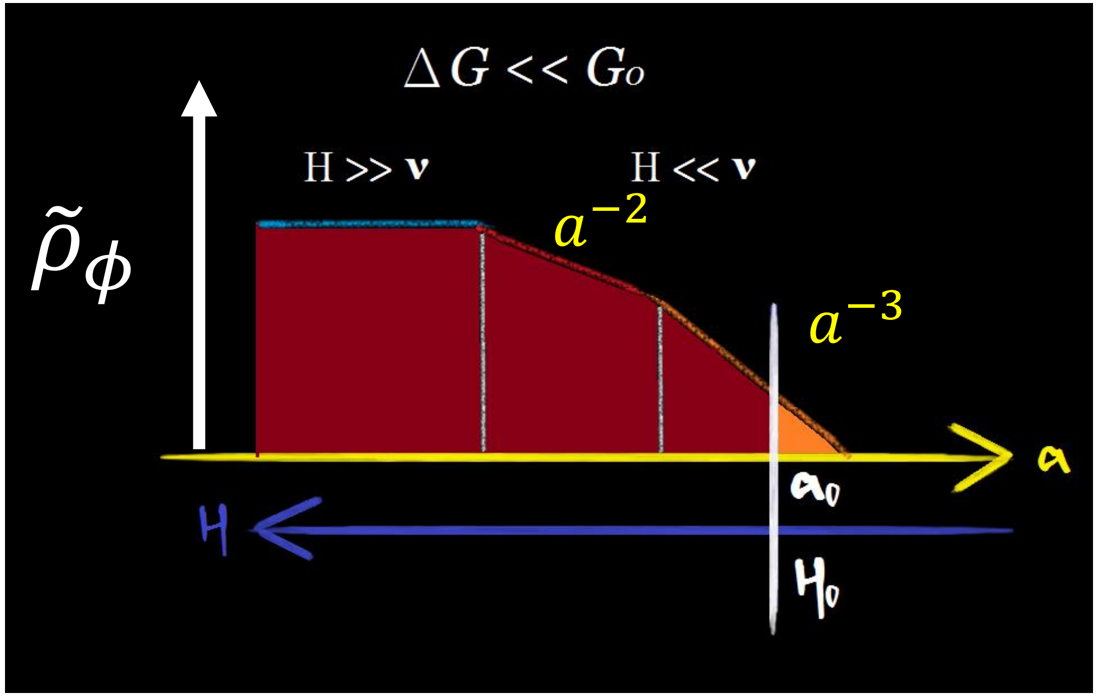


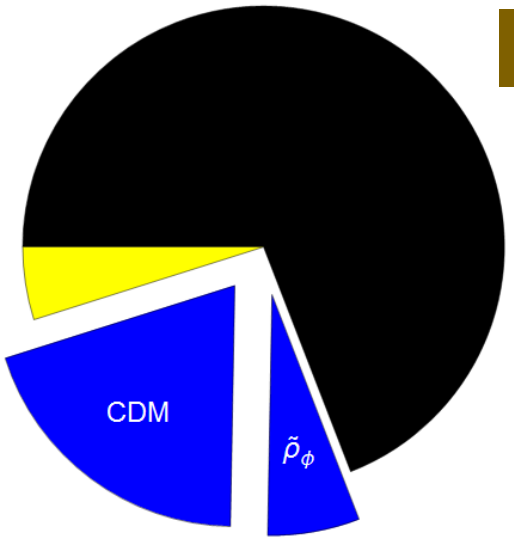
- Damping force dominates
- **Slow-roll**

For **oscillatory solution** -

- **High frequency**
- Cosmic expansion can be ignored within a period of oscillation

Phenomenological Constraints on NMC Scalar Field





Planck Collaboration. Planck 2015 results

- Dark Energy: 69.11%
- Cold Dark Matter (CDM) + $\tilde{\rho}_\phi$ (ϕ + GO): 26%
- Baryon: 4.89%

$$\Omega_{\mathcal{M}0} = \tilde{\Omega}_{\phi 0} + \underbrace{\Omega_{\mathrm{cdm},0} + \Omega_{\mathrm{b}0}}_{\Omega_{\mathrm{m}0}} \approx 0.3089$$

- G oscillation in the present-time universe:

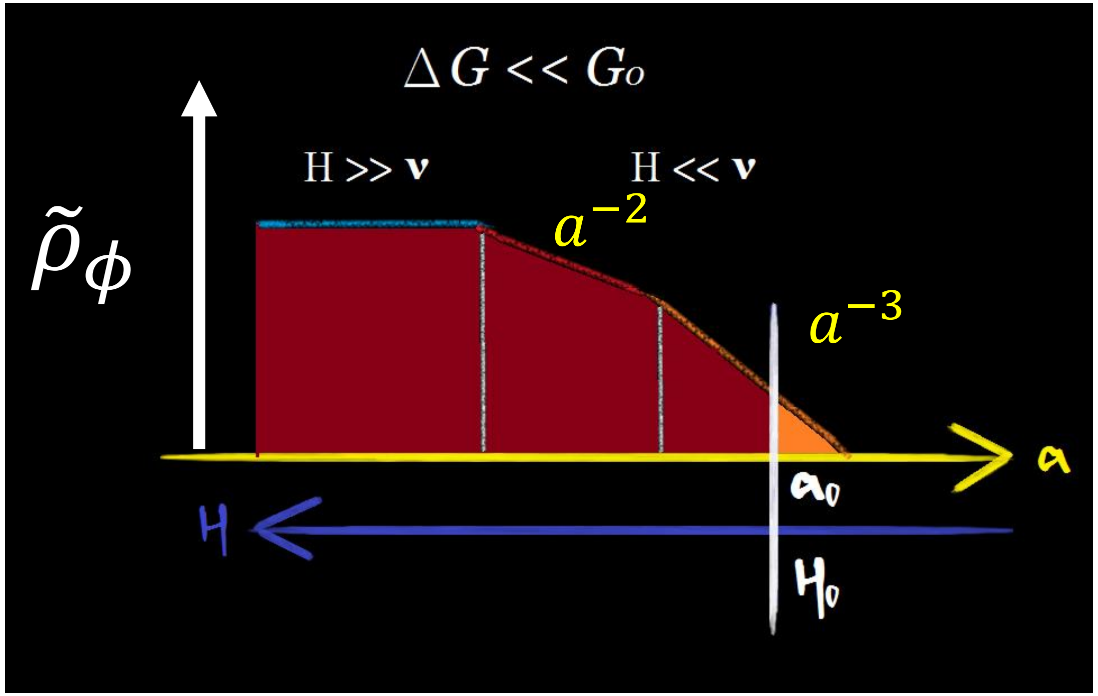
  Should be *High-frequency, Small-amplitude*

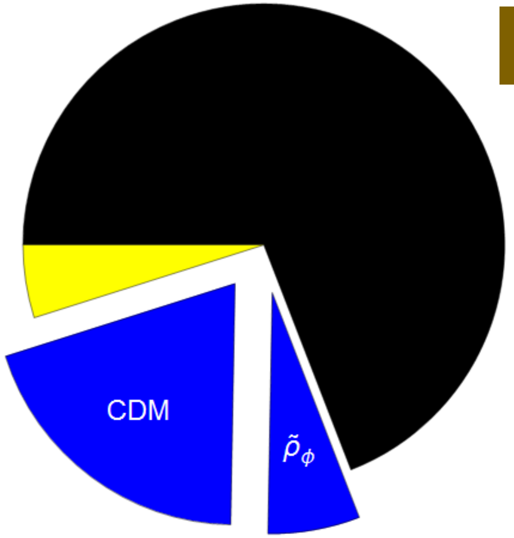
  $\Downarrow$

- **The effective energy density $\tilde{\rho}_\phi$ contributes as a matter sector in the universe at present**

$$\boxed{\tilde{\Omega}_{\phi 0} < \Omega_{\mathcal{M}0} - \Omega_{\mathrm{b}0} \approx 0.26}$$

Phenomenological Constraints on NMC Scalar Field





Planck Collaboration. Planck 2015 results

- Dark Energy: 69.11%
- Cold Dark Matter (CDM) + $\tilde{\rho}_\phi$ (ϕ + GO): 26%
- Baryon: 4.89%

$$\Omega_{\mathcal{M}0} = \tilde{\Omega}_{\phi 0} + \underbrace{\Omega_{\mathrm{cdm},0} + \Omega_{\mathrm{b}0}}_{\Omega_{\mathrm{m}0}} \approx 0.3089$$

- G oscillation in the present-time universe:

  Should be *High-frequency, Small-amplitude*

  $\Downarrow$

- **The effective energy density $\tilde{\rho}_\phi$ contributes as a matter sector in the universe at present**

A very loose upper bound!
Expected to be more stringent if taking the DM in galaxy clusters into account.

$$\boxed{\tilde{\Omega}_{\phi 0} < \Omega_{\mathcal{M}0} - \Omega_{\mathrm{b}0} \approx 0.26}$$

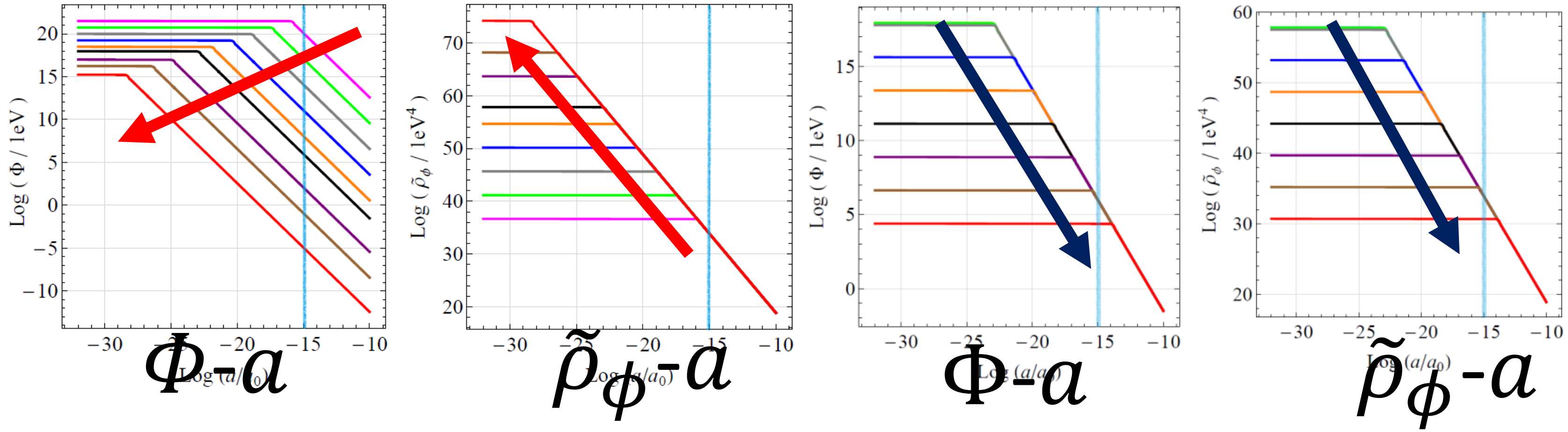
m increases

α, ξ increases

Linear

Fix: $\alpha = 10^{-12}$

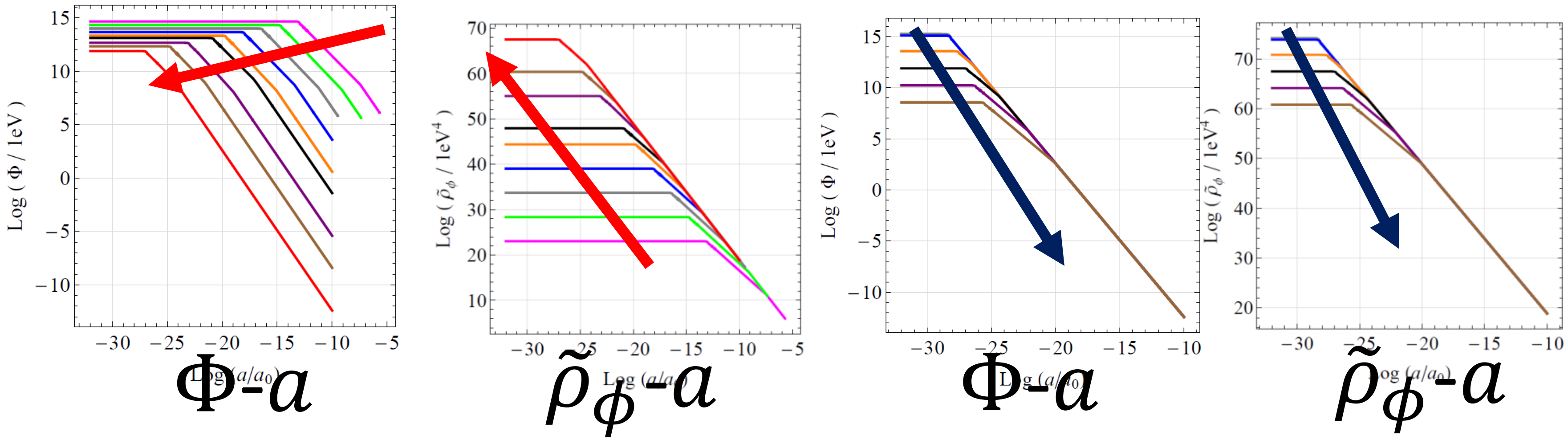
Fix: $m = 125$ GeV



Quadratic

Fix: $\xi = 10^{18}$

Fix: $m = 10^{13}$ GeV



Summary

Summary

- Theoretically, we have several motivations to consider the scalar-tensor theory of gravity and the existence of the oscillation of G (e.g., Higgs inflation).

- We have derived the field equations of the theory, and shown that the complicated equations could be recast into a fairly graceful form, which enables us to analyze the time evolution of the energy density relevant to the non-minimally coupled scalar field.

- Realizing the evolution of $\tilde{\rho}_\phi$ along the cosmic history is crucial for setting the upper bounds on ϕ in any epoch of the universe. It is straightforward to see that the evolution of $\tilde{\rho}_\phi$ is characterized by both the effects from non-minimal coupling and the cosmic expansion.

Summary

- Based on the properties of G oscillation set by local experiments (i.e., high-frequency + small-amplitude) and the standard ΛCDM cosmology, we could set a loose constraint on $\tilde{\rho}_{\phi 0}$ — the effective energy density from the non-minimally coupled scalar field at present.

- In other words, given certain kinds of the scalar-tensor models with condensate of single scalar field oscillating around the local minimum of potential, one could apply the phenomenological constraints from G oscillation and the standard ΛCDM cosmology to examine the ideas.

Grazie

Appendix-1

To analyze the motion for the quadratic-coupling models with $\mathcal{F}(\phi_{\mathbf{max}}) \gg 1$:
Resort to the method of averaging.

$$\frac{d}{dt}\tilde{\rho}_\phi = -3\mathcal{F}^2\bar{H}\dot{\phi}^2 + \frac{f'}{2f}\left(4\tilde{\rho}_\phi + \rho_m + 4\rho_\Lambda\right)\dot{\phi} \qquad \mathcal{F}^{\mathbf{2}} \approx \frac{\mathbf{6}\xi^{\mathbf{2}}}{\mathbf{M_P^2}}\phi^{\mathbf{2}} \gg \mathbf{1}$$

$$\frac{d}{dt}\left(\frac{1}{2}\cdot\frac{6\,\xi^2}{M_P^2}\phi^2\dot{\phi}^2 + \frac{1}{2}m^2\phi^2\right) \approx -3\frac{6\,\xi^2}{M_P^2}\bar{H}\phi^2\dot{\phi}^2$$

$$\Rightarrow \quad \frac{d}{dt}\left(\frac{1}{2}\dot{\chi}^2 + \frac{m^2 M_P}{\sqrt{6}\,\xi}\chi\right) \approx -3\bar{H}\dot{\chi}^2\,,$$

with $\chi \equiv \dfrac{\sqrt{6}\,\xi}{2M_P}\phi^2$ and the effective potential $\tilde{V}(\chi) \equiv \dfrac{m^2 M_P}{\sqrt{6}\,\xi}\chi$

Appendix-1

Let $y \equiv \widetilde{V}/\widetilde{V}_{\max} = \chi/\chi_{\max}$, we can get the approximated period of oscillation:

$$T \approx \oint_{\chi(\tau)} \frac{1}{\dot{\chi}}\, d\chi \approx \oint_{\chi(\tau)\geq 0} \frac{1}{\sqrt{2(\widetilde{V}_{\max} - \widetilde{V})}}\, d\chi$$

$$\approx \left(2\widetilde{V}_{\max}\right)^{-1/2} \oint_{\chi(\tau)\geq 0} \left(1 - \frac{\chi}{\chi_{\max}}\right)^{-1/2} d\chi$$

$$\approx \left(2\widetilde{V}_{\max}\right)^{-1/2} \cdot 4\chi_{\max} \underbrace{\int_0^1 (1-y)^{-1/2}\, dy}_{B(\,0\,,\,\frac{1}{2}\,)\,=\,2} = \frac{4\sqrt{6}\,\xi}{mM_P}\Phi\,.$$

Thus, $\boxed{\nu(\Phi) \approx \frac{mM_P}{4\sqrt{6}\,\xi} \cdot \frac{1}{\Phi}}$

Appendix-1

To examine the dissipating behavior of $\widetilde{\rho}_\phi$ for the quadratic-coupling models with $F(\phi_{max}) \gg 1$: Averaging over individual period τ of oscillation.

$$\left\langle \frac{d}{dt}\left(\frac{1}{2}\dot{\chi}^2 + \widetilde{V}(\chi)\right)\right\rangle_\tau \approx -3\left\langle \bar{H}\dot{\chi}^2\right\rangle_\tau$$

Assuming $\nu \gg H$, we have $\dfrac{d}{dt}\widetilde{V}_{\rm max}(\chi) \approx -3\bar{H}\left\langle\dot{\chi}^2\right\rangle_\tau$.

Since

$$\begin{aligned}\left\langle\dot{\chi}^2\right\rangle_\tau &\approx 2\widetilde{V}_{\rm max}\cdot \oint_{\chi(\tau)\geq 0}\left(1-\frac{\widetilde{V}}{\widetilde{V}_{\rm max}}\right)^{1/2} d\chi \oint_{\chi(\tau)\geq 0}\left(1-\frac{\widetilde{V}}{\widetilde{V}_{\rm max}}\right)^{-1/2} d\chi \\ &= 2\widetilde{V}_{\rm max}\cdot\int_0^1 (1-y)^{1/2}\,dy \int_0^1 (1-y)^{-1/2}\,dy = \frac{2}{3}\widetilde{V}_{\rm max}\ .\end{aligned}$$

The relation is obtained: $\dfrac{d}{dt}\widetilde{V}_{\rm max} \approx -2\widetilde{V}_{\rm max}\dfrac{\dot{a}}{a}$.

Appendix-1

Consequently,

$$\widetilde{V}_{\max} \propto a(t)^{-2}$$

$$\Rightarrow \quad \boxed{\langle\ \chi_{\max}\ \rangle \propto \langle\ \Phi^2\ \rangle \propto a(t)^{-2}}$$

$$\boxed{\langle\ \widetilde{\rho}_\phi\ \rangle \propto a(t)^{-2}}\ .$$

Appendix-2

Starting form the action of the Higgs inflation:

$$S = \int d^4x\sqrt{-g}\left\{f(h)R - \frac{1}{2}\partial_\mu h\,\partial^\mu h - \frac{\lambda}{4}\left(h^2 - v^2\right)^2\right\} ,$$

$$f(h) = \frac{M^2}{2}\left(1 + \xi\frac{h^2}{M^2}\right) ,$$

where the the parameter M is defined by the equality: $f(v) = \dfrac{M_P^2}{2}$

Appendix-2

Consider a perturbation on *h*: $\delta h \equiv h - v \ll v$

The effective potential of the Higgs field can be approximated as:

$$V \approx \frac{\lambda}{4} \cdot (2v \cdot \delta h)^2 = \lambda v^2 \ (\delta h)^2 \ .$$

Identifying the effective potential with $\frac{1}{2}m^2(\delta h)^2$,

The Higgs mass is $\boxed{m \equiv \sqrt{2\lambda}\ v}$

Appendix-2

Similarly, the quadratic non-minimal coupling function $f(h)$ can be approximated as follows:

$$
\begin{aligned}
f(h) &= \frac{M^2}{2}\left\{1+\xi\frac{(\delta h+v)^2}{M^2}\right\} \\
&\approx \frac{M^2}{2}\left(1+\xi\frac{v^2}{M^2}+\frac{2\,\xi v}{M^2}\delta h\right) \\
&= \frac{M_P^2}{2}\left(1+\frac{2\,\xi v}{M_P^2}\delta h\right) \quad .
\end{aligned}
$$

We obtain a linear non-minimal coupling function with coupling constant:

$$
\boxed{\alpha \equiv \frac{2\,\xi v}{M_P}}
$$