

CMB and Inflaton Couplings to Radiation at Reheating

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ICTP & Kim Il Sung U.

With M. Drewes, U. Mun: arXiv:1708.01197

E. Cheung, M. Drewes, J. Kim: 1504.04444 (JCAP)

M. Drewes: 1305.0267 (NPB), 1510.05646 (JHEP)

Why reheating?

- Reheating: post-inflationary transition to the initial condition for the standard hot big bang ($>10\text{MeV}$).
- affects inflationary prediction for CMB Liddle, Leach 2003
--> CMB constraints on reheating era. Martin, Ringeval 2010
- may provide a probe into microphysics at high scales: “Cosmic Collider”

Fundamentally interesting to extract information about microphysical properties at reheating, such as inflaton couplings to matter.

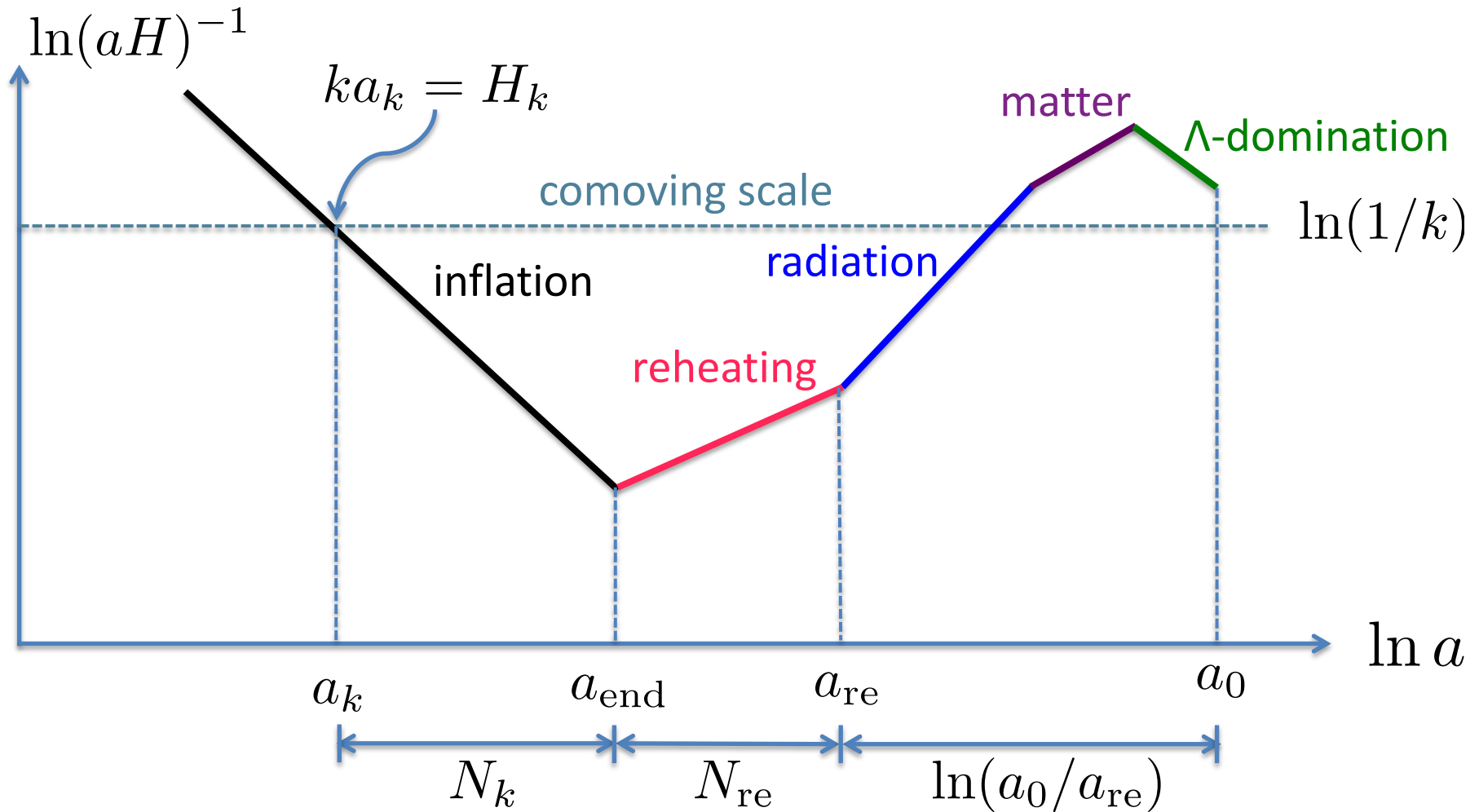
“measurement” of the inflaton couplings by CMB?

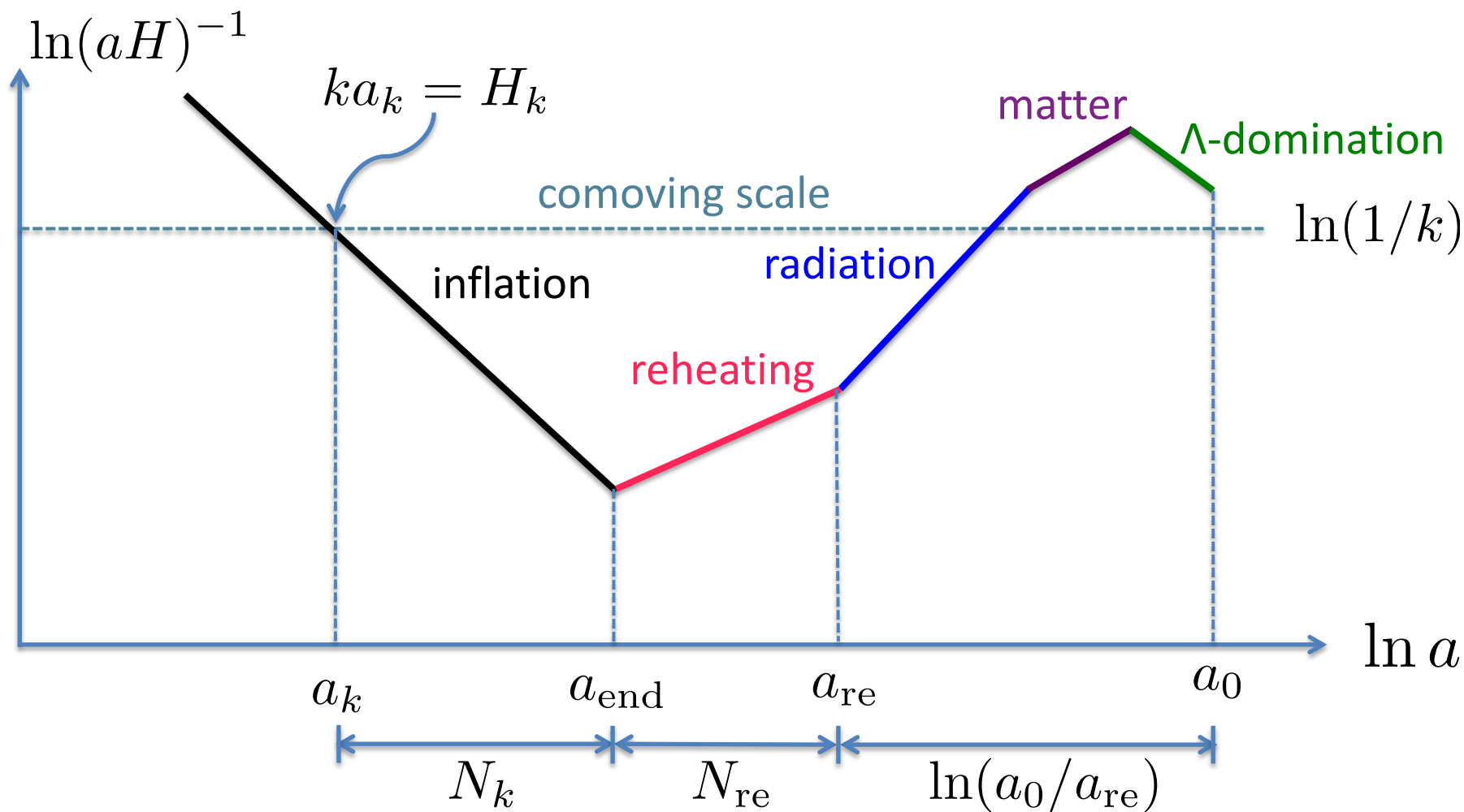
Plan

- General ideas and methodology
- Work out the analysis of the reheating in a simple class of inflationary models
 - Take the **alpha-tractor** inflation.
 - Consider explicit **interactions** between inflaton and other **scalars** or **fermions**
 - Focus on **perturbative** reheating, without triggering preheating, but take into account backreaction (**thermal**) effects.

General considerations

CMB constraint on reheating phase





$$N_k + N_{\text{re}} + \ln\left(\frac{a_0}{a_{\text{re}}}\right) = \ln\left(\frac{a_0}{a_k}\right) = \ln\left(\frac{H_k}{(k/a_0)}\right)$$

$$H_k = \frac{\pi M_{\text{pl}} \sqrt{r} A_s}{\sqrt{2}}$$

r : tensor to scalar ratio
 A_s : amplitude of the scalar perturbation

Matching equation

Liddle, Leach 2003
Martin, Ringeval 2010

Entropy
conserv. \rightarrow

$$\frac{a_0}{a_{\text{re}}} \simeq \left(\frac{11g_{s*}}{43} \right)^{1/3} \frac{T_{\text{re}}}{T_0} = \left(\frac{11g_{s*}}{43} \right)^{1/3} \left(\frac{30\rho_{\text{re}}}{\pi^2 g_* T_0^4} \right)^{1/4}$$

$$\rho_{\text{re}} = \pi^2 g_* T_{\text{re}}^4 / 30,$$

Energy

density:

$$\rho_{\text{re}} = \rho_{\text{end}} e^{-3N_{\text{re}}(1+\bar{w}_{\text{re}})}, \quad \bar{w}_{\text{re}} = \frac{1}{N_{\text{re}}} \int_0^{N_{\text{re}}} w(N) dN$$

\downarrow
equation of state

$$N_{\text{re}} = \frac{4}{3\bar{w}_{\text{re}} - 1} \left[N_k + \ln \left(\frac{k}{a_0 T_0} \right) + \frac{1}{4} \ln \left(\frac{40}{\pi^2 g_*} \right) + \frac{1}{3} \ln \left(\frac{11g_{s*}}{43} \right) - \frac{1}{2} \ln \left(\frac{\pi^2 M_{pl}^2 r A_s}{\sqrt{3}\rho_{\text{end}}} \right) \right]$$

Reheating

temperature:

$$T_{\text{re}} = \exp \left[-\frac{3}{4} (1 + \bar{w}_{\text{re}}) N_{\text{re}} \right] \left(\frac{30\rho_{\text{end}}}{g_* \pi^2} \right)^{1/4}$$

Matching equation

$$N_{\text{re}} = \frac{4}{3\bar{w}_{\text{re}} - 1} \left[N_k + \ln \left(\frac{k}{a_0 T_0} \right) + \frac{1}{4} \ln \left(\frac{40}{\pi^2 g_*} \right) + \frac{1}{3} \ln \left(\frac{11 g_{s*}}{43} \right) - \frac{1}{2} \ln \left(\frac{\pi^2 M_{pl}^2 r A_s}{\sqrt{3} \rho_{\text{end}}} \right) \right]$$

- Obtained using only effect of cosmic expansion (red-shifting), independent of model for inflation and reheating.
- For given CMB data and slow-roll potential, this equation fixes the combination $\ln R_{\text{rad}} \equiv N_{\text{re}}(3\bar{w}_{\text{re}} - 1)/4$.
This allows to constrain the reheating phase from CMB.
- $(N_{\text{re}}, T_{\text{re}}, \bar{w}_{\text{re}})$ depend on the post-inflationary properties, such as potential around minimum, and dissipation rate, which is sensitive to the microphysics deriving the reheating.

Last two statements indicates that the CMB data can tell us something about the microphysics of the reheating!

Slow-roll phase

- Background
EOMs:

$$3H\dot{\phi} \simeq -V'(\phi), \quad 3M_{pl}^2 H^2 \simeq V(\phi)$$

- Slow-roll
parameters:

$$\epsilon = \frac{1}{2} M_{pl}^2 \left(\frac{V'(\phi)}{V(\phi)} \right)^2, \quad \eta = M_{pl}^2 \frac{V''(\phi)}{V(\phi)},$$

- Inflation ends when $\epsilon(\phi_{\text{end}}) \simeq 1$, $\rho_{\text{end}} \simeq \frac{3V(\phi_{\text{end}})}{4}$,

- Horizon
crossing: $\phi_k \rightarrow N_k = \int_{\phi_k}^{\phi_{\text{end}}} \frac{H d\phi}{\dot{\phi}} \simeq -\frac{1}{M_{pl}^2} \int_{\phi_k}^{\phi_{\text{end}}} d\phi \left(\frac{V(\phi)}{V'(\phi)} \right)$

- Relation to the CMB parameters:

$$n_s = 1 - 6\epsilon(\phi_k) + 2\eta(\phi_k), \quad r = 16\epsilon(\phi_k), \quad A_s = \frac{2V(\phi_k)}{3\pi^2 M_{pl}^4 r},$$



$n_s \rightarrow \phi_k$



$n_s \rightarrow N_k(n_s)$

Reheating phase

$$\ddot{\phi} + (3H + \Gamma)\dot{\phi} + V'(\phi) = 0$$

↳ full dissipation rate

Reheating
completes :

$$\Gamma = H$$



$$\Gamma = \frac{1}{M_{pl}} \left(\frac{\rho_{\text{end}}}{3} \right)^{1/2} e^{-3(1+\bar{w}_{\text{re}})N_{\text{re}}/2}$$



$$N_{\text{re}} = \frac{1}{3(1 + \bar{w}_{\text{re}})} \ln \left(\frac{\rho_{\text{end}}}{3\Gamma^2 M_{pl}^2} \right)$$

Equation
of state :

e.g. $V(\phi) \sim \phi^{2n} \longrightarrow \bar{w}_{\text{re}} \stackrel{?}{\simeq} \frac{n-1}{n+1}$

Amin's talk

Next : compute Γ in terms of the microphysical parameter.

Dissipation rate

Use **Closed-Time-Path** (Schwinger-Keldysh or “in-in”) formalism of the Non-Equilibrium QFT:

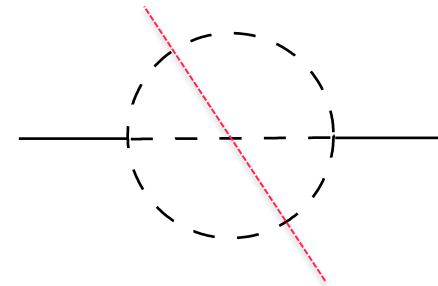
Cheung, Drewes, JK, Kim 2015;
Drewes, JK 2013 & 2015; Drewes, JK, Mun 2017



Dissipation rate: $\Gamma = \frac{\text{Im}\Pi_0^-(m_\phi)}{2m_\phi}$

Π^- : spectral self-energy

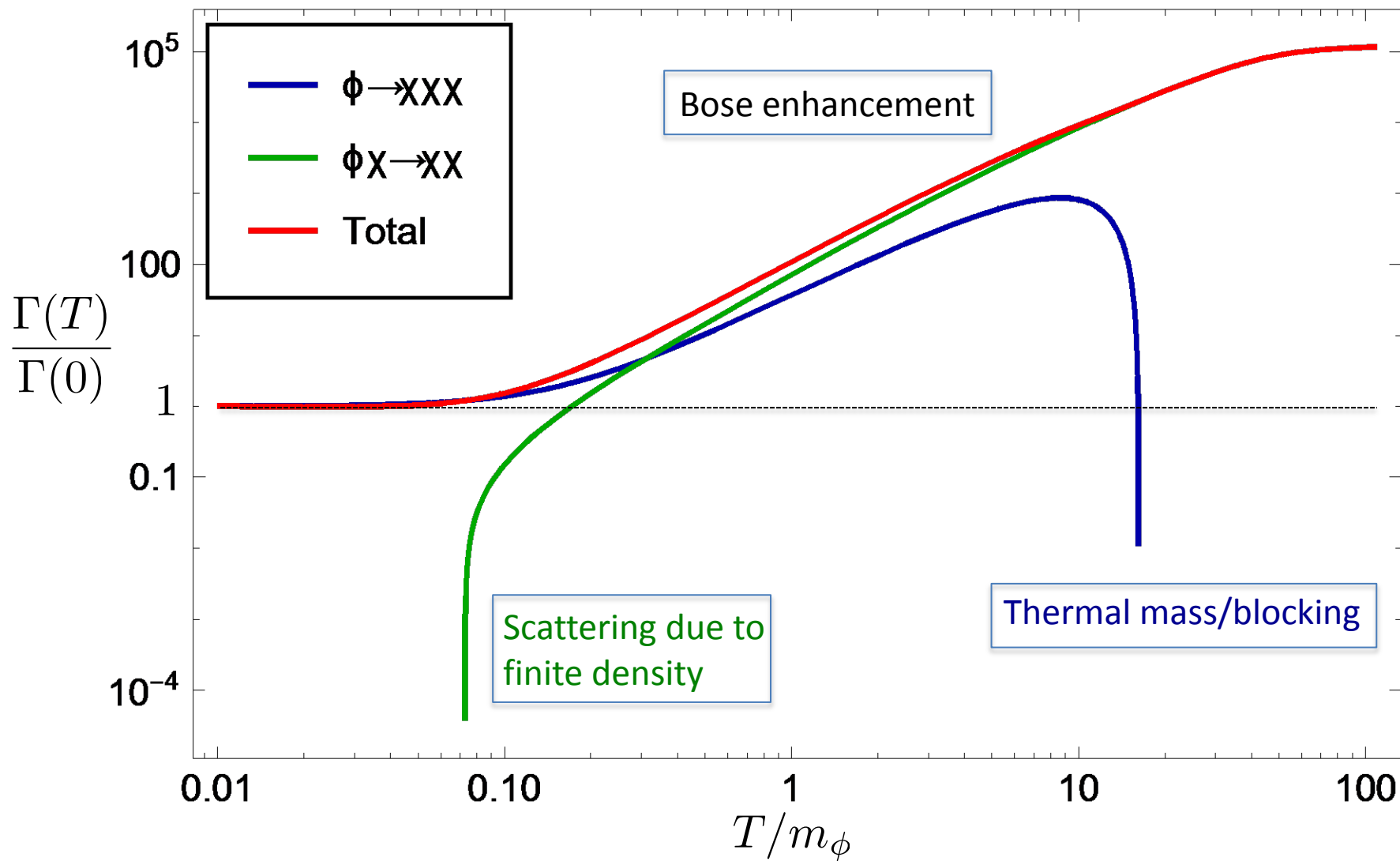
Example: $\mathcal{L}_{\text{int}} = -h\phi\chi^3/3!$



$$\Pi_{\mathbf{p}}^-(p_0) = -\frac{ih^2}{6} \int \frac{d^4q}{(2\pi)^4} \frac{d^4k}{(2\pi)^4} \frac{d^4l}{(2\pi)^4} (2\pi)^4 \delta^{(4)}(p - q - k - l) \rho_\chi(q) \rho_\chi(k) \rho_\chi(l) \\ \times \left[(1 + f_B(q_0)) (1 + f_B(k_0)) (1 + f_B(l_0)) - f_B(q_0) f_B(k_0) f_B(l_0) \right]$$

ρ_χ : spectral density

$$f_B(\omega) = 1/(e^{\omega/T} - 1)$$

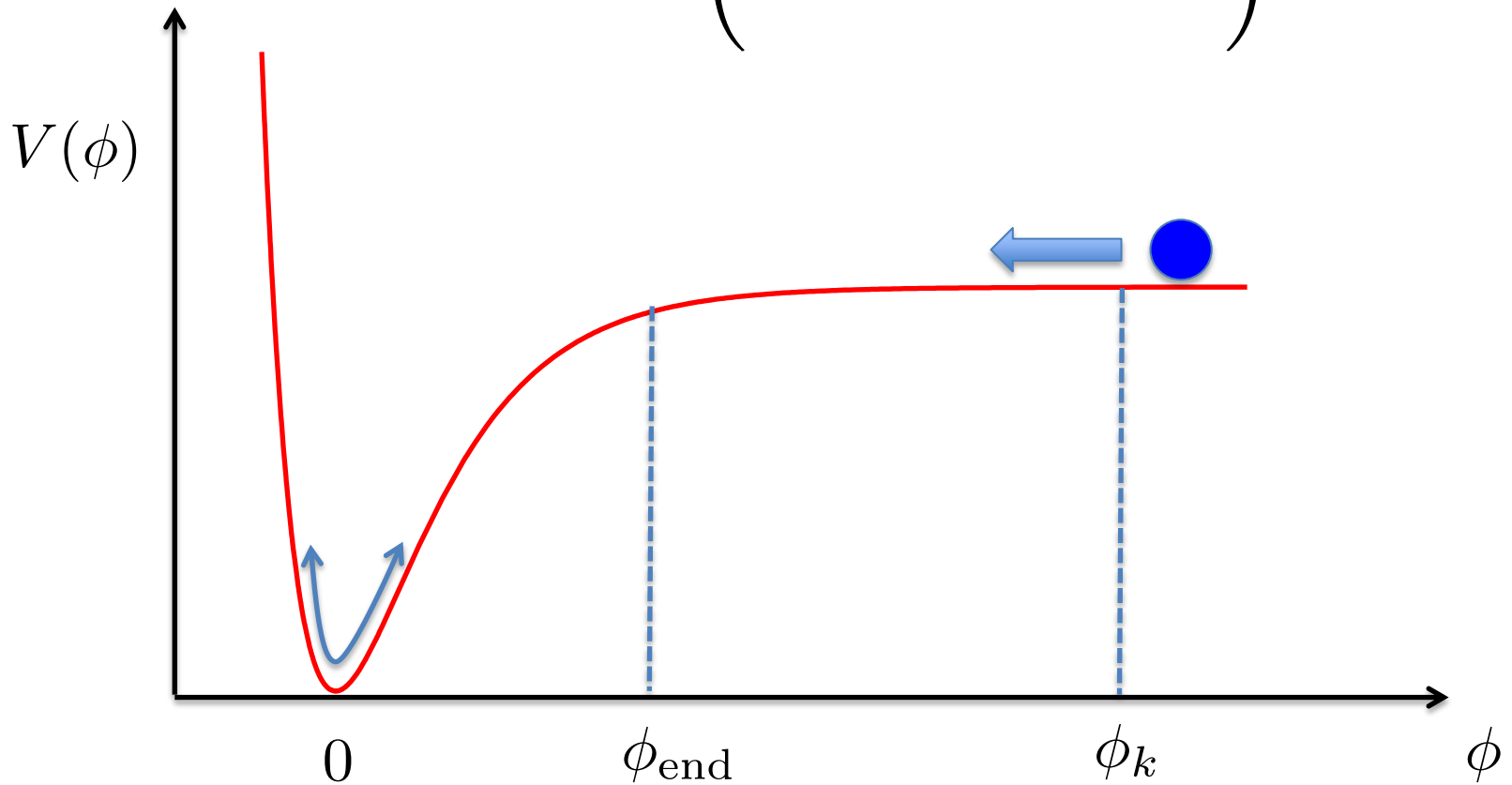


Vacuum decay rate: $\Gamma(0) = \frac{h^2 m_\phi}{3!64(2\pi)^3}$

Concrete example:
 α -attractor + interaction

α -attractor model: e.g. Kallosh, Linde 2013

$$V = \Lambda^4 \left(1 - e^{-\sqrt{\frac{2}{3\alpha}} \frac{\phi}{M_{pl}}} \right)^{2n}$$



$$V = \Lambda^4 \left(1 - e^{-\sqrt{\frac{2}{3\alpha}} \frac{\phi}{M_{pl}}} \right)^{2n}$$

$$\epsilon(\phi_{\text{end}}) \simeq 1 \quad \longrightarrow \quad \phi_{\text{end}} = \sqrt{\frac{3\alpha}{2}} M_{pl} \ln \left(\frac{2n}{\sqrt{3\alpha}} + 1 \right)$$

$$n = 1 \quad \longrightarrow \quad \left\{ \begin{array}{l} m_\phi = \frac{2\Lambda^2}{\sqrt{3\alpha} M_{pl}} \\ \bar{w}_{\text{re}} = 0 \end{array} \right.$$

Possible input parameters: $(n_s, r, A_s) \longrightarrow (N_k, \Lambda, \alpha)$

However, we trade r for α due to the current uncertainty of r :

$$(n_s, \alpha, A_s) \longrightarrow (N_k, \Lambda, r)$$

$$n_s = 1 - \frac{8n \left(e^{\sqrt{\frac{2}{3\alpha}} \frac{\phi_k}{M_{pl}}} + n \right)}{3\alpha \left(e^{\sqrt{\frac{2}{3\alpha}} \frac{\phi_k}{M_{pl}}} - 1 \right)^2}$$



$$\phi_k = \sqrt{\frac{3\alpha}{2}} M_{pl} \ln \left(1 + \frac{4n + \sqrt{16n^2 + 24\alpha n (1 - n_s)(1 + n)}}{3\alpha(1 - n_s)} \right)$$

$$N_k = \int_{\phi_k}^{\phi_{\text{end}}} \frac{H d\phi}{\dot{\phi}} \simeq -\frac{1}{M_{pl}^2} \int_{\phi_k}^{\phi_{\text{end}}} d\phi \left(\frac{V(\phi)}{V'(\phi)} \right)$$



$$N_k = \frac{3\alpha}{4n} \left[e^{\sqrt{\frac{2}{3\alpha}} \frac{\phi_k}{M_{pl}}} - e^{\sqrt{\frac{2}{3\alpha}} \frac{\phi_{\text{end}}}{M_{pl}}} - \sqrt{\frac{2}{3\alpha}} \frac{(\phi_k - \phi_{\text{end}})}{M_{pl}} \right]$$

$$H_k^2 \simeq \frac{V(\phi_k)}{3M_{pl}^2} \quad \text{and} \quad H_k = \frac{\pi M_{pl} \sqrt{r A_s}}{\sqrt{2}}$$

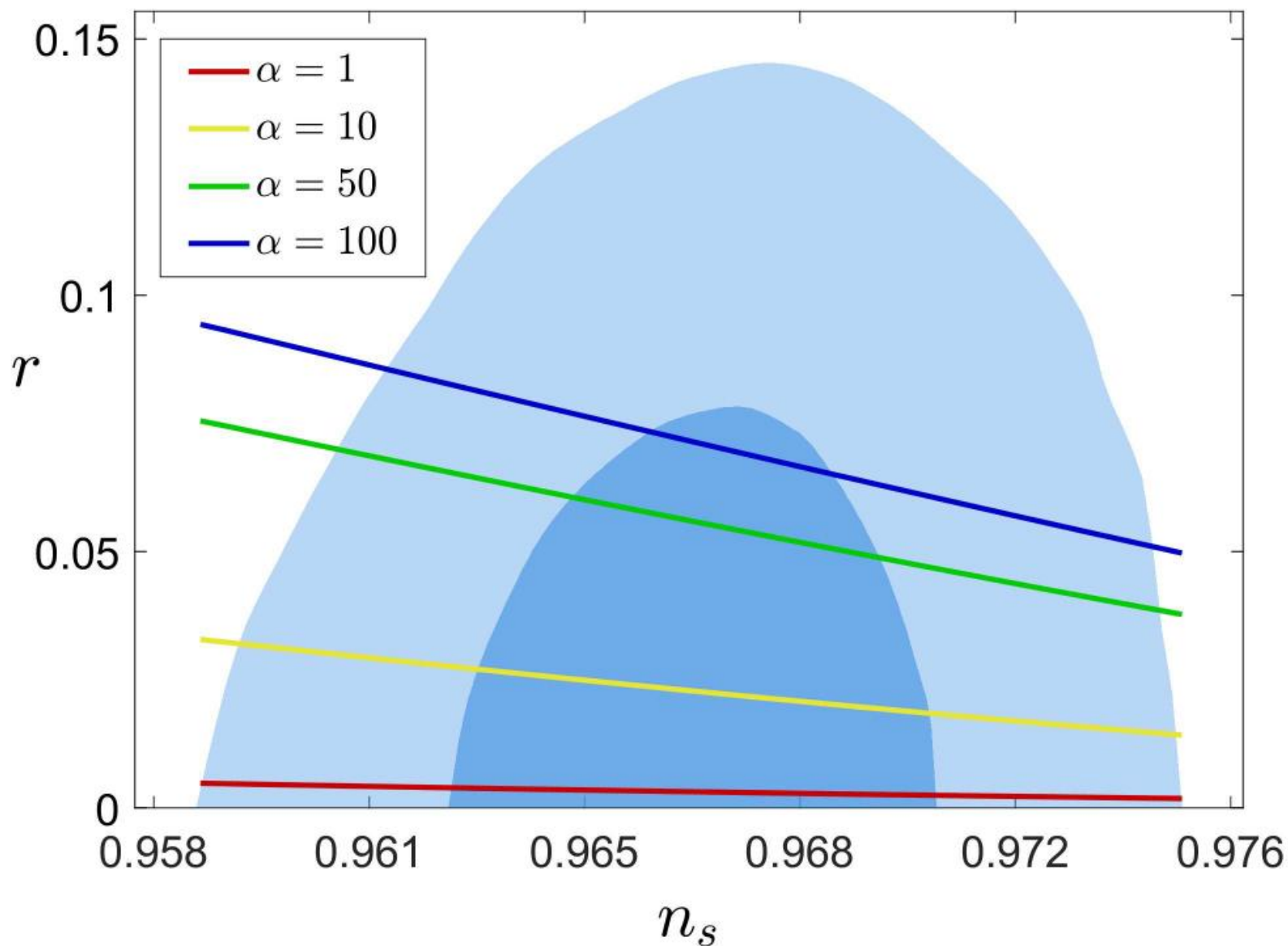


$$\Lambda = M_{pl} \left(\frac{3\pi^2 r A_s}{2} \right)^{1/4} \left[\frac{2n(1+2n) + \sqrt{4n^2 + 6\alpha(1+n)(1-n_s)}}{4n(1+n)} \right]^{n/2}$$

$$r = 16 \epsilon(\phi_k),$$



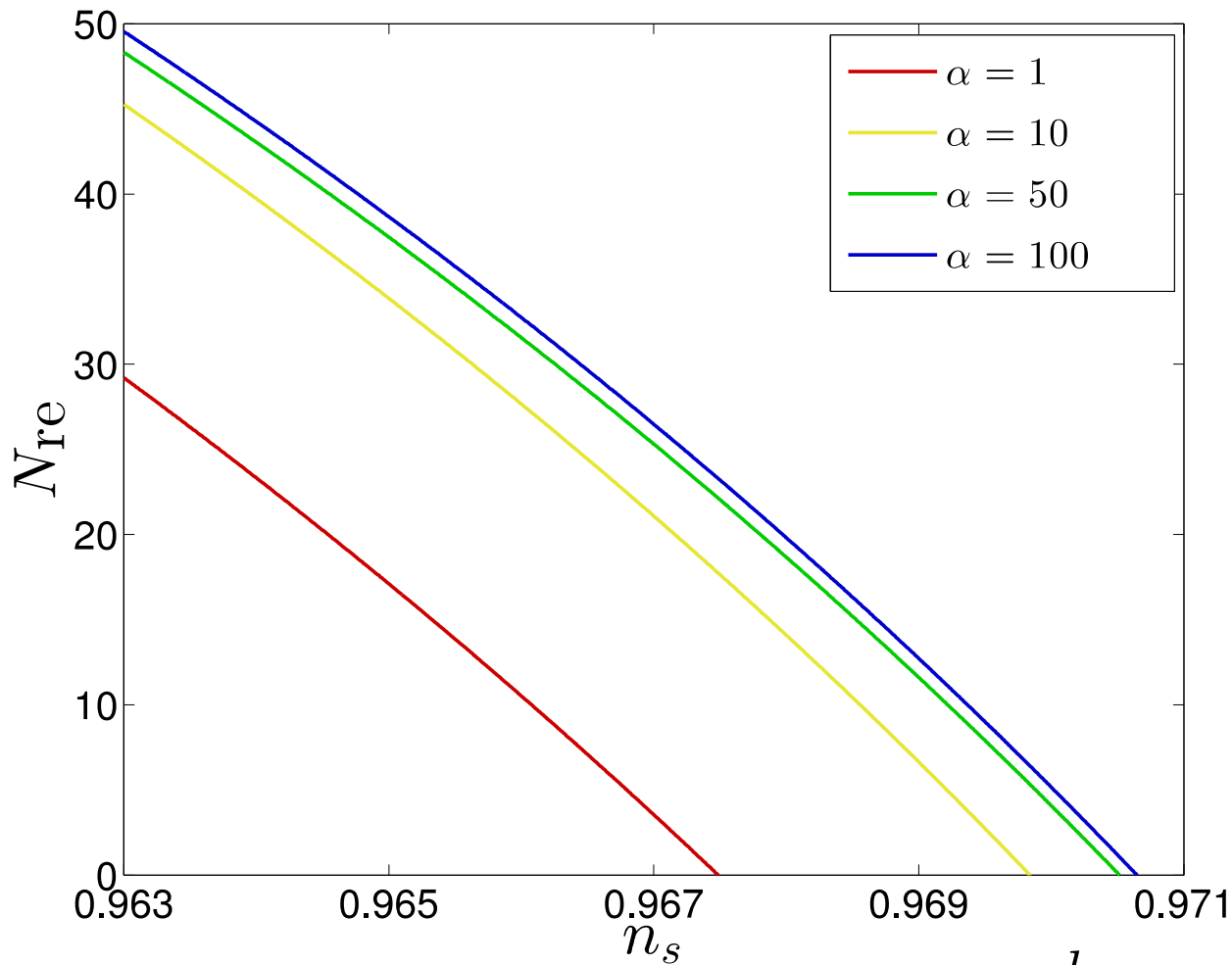
$$r = \frac{64n^2}{3\alpha \left(e^{\sqrt{\frac{2}{3\alpha}} \frac{\phi_k}{M_{pl}}} - 1 \right)^2} = \frac{192 \alpha n^2 (1-n_s)^2}{\left[4n + \sqrt{16n^2 + 24\alpha n(1-n_s)(1+n)} \right]^2}$$



$$n = 1, \quad A_s = 10^{-10} e^{3.064}, \quad \frac{k}{a_0} = 0.002 \text{ Mpc}^{-1}$$

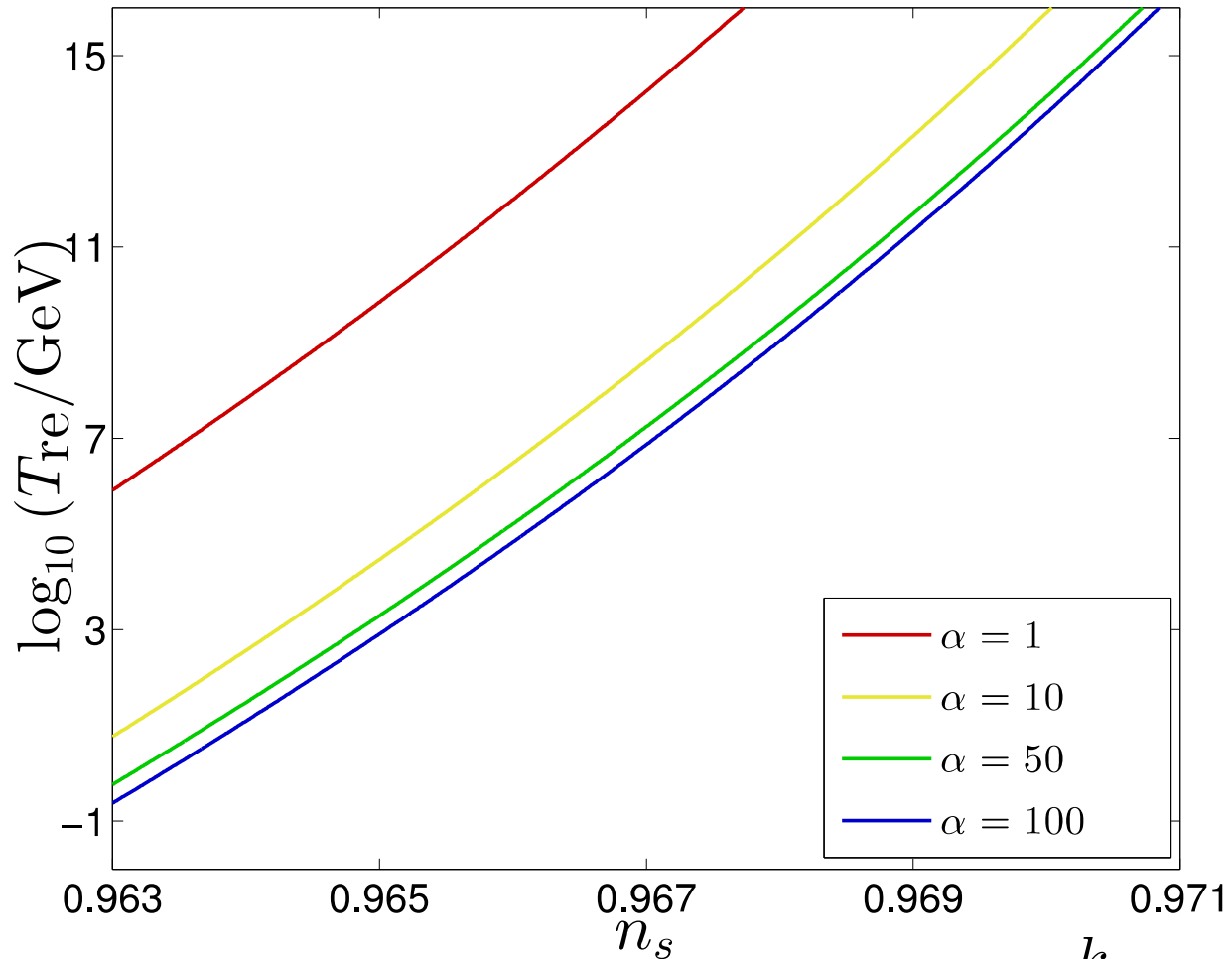
Ade et al.
Planck 2015

$$N_{\text{re}} = \frac{4}{3\bar{w}_{\text{re}} - 1} \left[N_k + \ln \left(\frac{k}{a_0 T_0} \right) + \frac{1}{4} \ln \left(\frac{40}{\pi^2 g_*} \right) + \frac{1}{3} \ln \left(\frac{11 g_{s*}}{43} \right) - \frac{1}{2} \ln \left(\frac{\pi^2 M_{pl}^2 r A_s}{\sqrt{3} \rho_{\text{end}}} \right) \right]$$



$$n = 1, \quad \bar{w}_{\text{re}} = 0, \quad A_s = 10^{-10} e^{3.064}, \quad \frac{k}{a_0} = 0.002 \text{ Mpc}^{-1}$$

$$T_{\text{re}} = \exp \left[-\frac{3}{4}(1 + \bar{w}_{\text{re}})N_{\text{re}} \right] \left(\frac{40V_{\text{end}}}{g_*\pi^2} \right)^{1/4}$$



$$n = 1, \quad \bar{w}_{\text{re}} = 0, \quad A_s = 10^{-10} e^{3.064}, \quad \frac{k}{a_0} = 0.002 \text{ Mpc}^{-1}$$

Scalar interaction: $g\phi\chi^2$

$$\mathcal{L}_\chi = \frac{1}{2}\partial^\mu\chi\partial_\mu\chi - \frac{1}{2}m_\chi^2\chi^2 - \frac{\lambda}{4!}\chi^4 - g\phi\chi^2$$

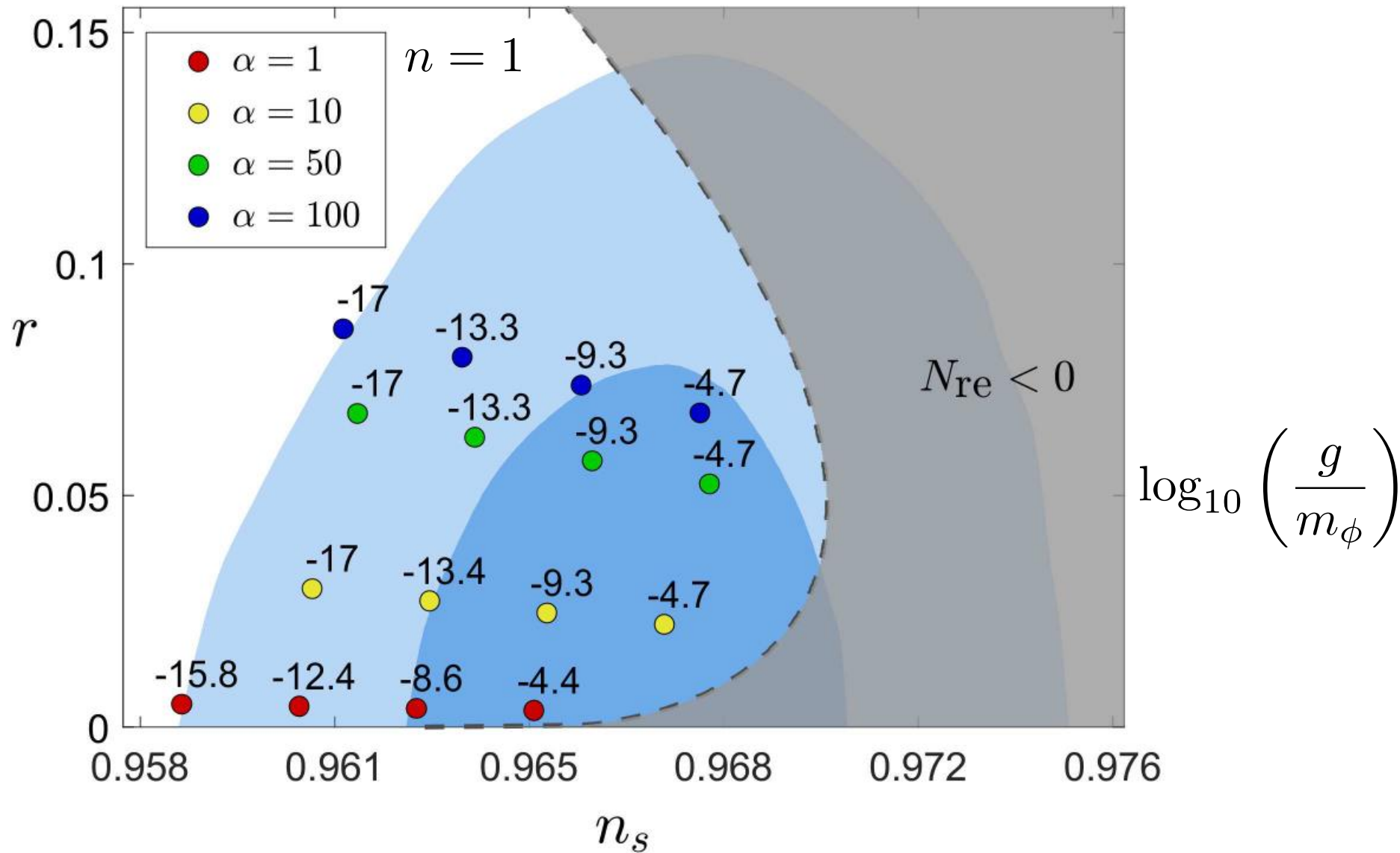
$$\Gamma_{\phi\rightarrow\chi\chi} = \frac{g^2}{8\pi m_\phi} \left[1 - \left(\frac{2M_\chi}{m_\phi} \right)^2 \right]^{1/2} [1 + 2f_B(m_\phi/2)]$$

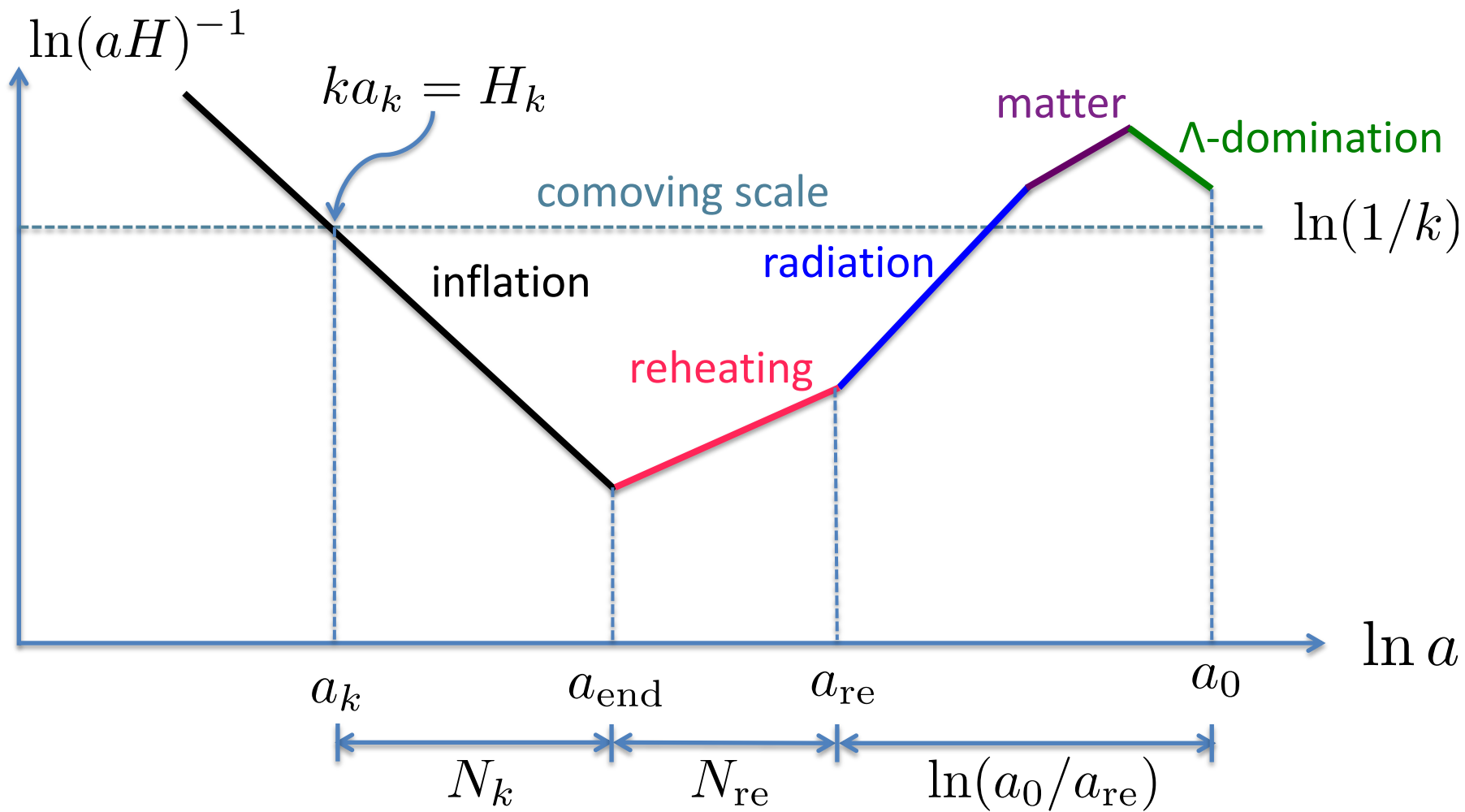
$$f_B(\omega) = 1/(e^{\omega/T} - 1), \quad M_\chi^2 = m_\chi^2 + \frac{\lambda T^2}{24}, \quad m_\chi \ll m_\phi$$

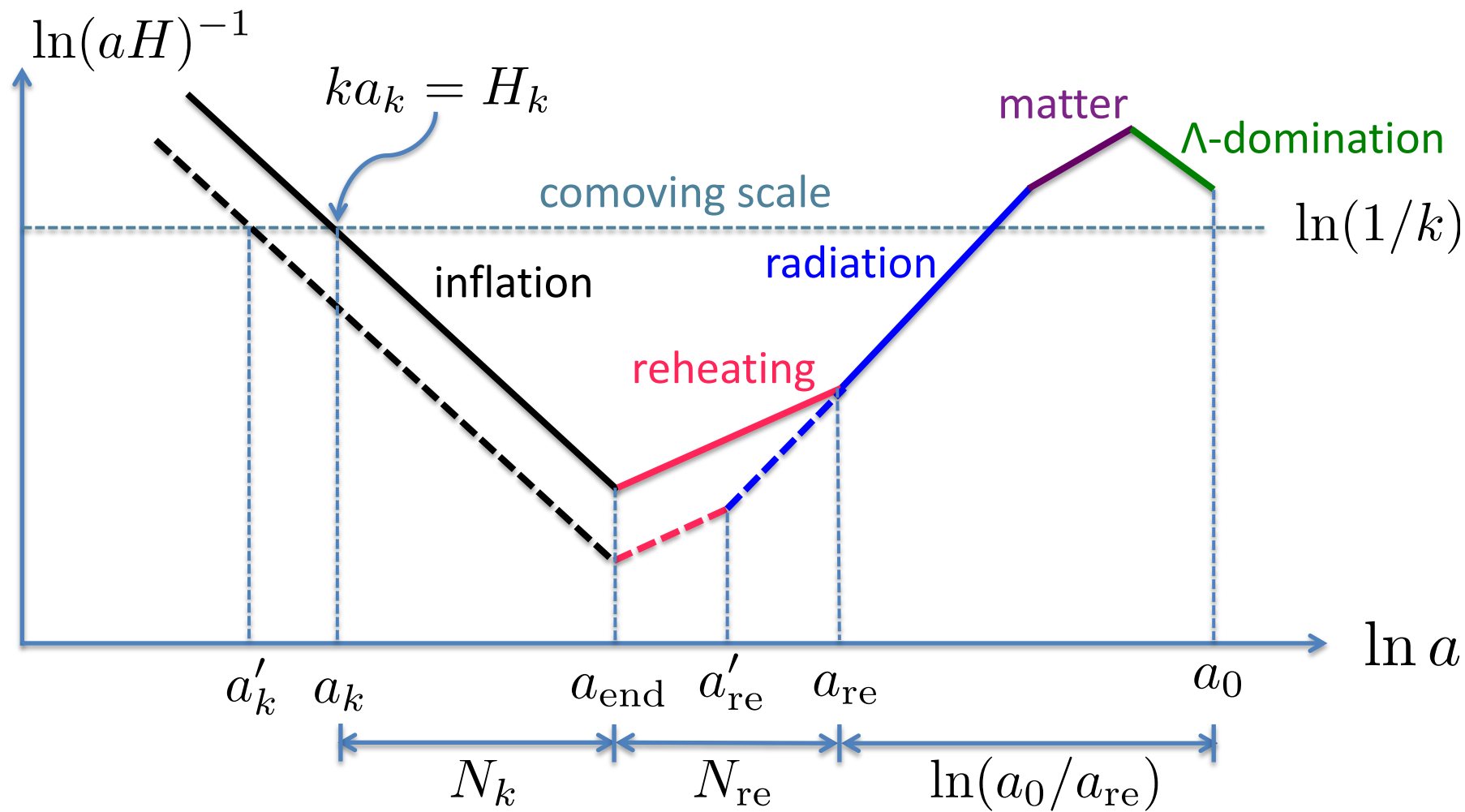
$$\Gamma_{\phi\rightarrow\chi\chi} = H|_{\text{re}} \iff \Gamma_{\phi\rightarrow\chi\chi} = \frac{1}{M_{pl}} \left(\frac{\rho_{\text{end}}}{3} \right)^{1/2} e^{-3N_{\text{re}}/2}$$

$$g^2 = \frac{16\pi m_\phi \Lambda^2}{3M_{pl}} \left(\frac{2n}{2n + \sqrt{3}\alpha} \right)^n \exp\left(\frac{-3nN_{\text{re}}}{1+n} \right) \frac{\left(\exp\left(\frac{m_\phi}{2T_{\text{re}}} \right) - 1 \right)}{\left(1 - \frac{\lambda T_{\text{re}}^2}{6m_\phi^2} \right)^{1/2} \left(\exp\left(\frac{m_\phi}{2T_{\text{re}}} \right) + 1 \right)}$$

Scalar interaction: $g\phi\chi^2$







$$\Gamma' > \Gamma \implies N'_{\text{re}} < N_{\text{re}} \implies N'_k > N_k \implies |\phi'_k| > |\phi_k|$$

$$\implies \epsilon'_k < \epsilon_k, \quad \eta'_k < \eta_k \implies 1 - n'_s < 1 - n_s, \quad r' < r$$

Scalar interaction: $g\phi\chi^2$

Parametric resonances

Kofman, Linde, Starobinsky 97

$$\ddot{\chi}_k + [k^2 + m_\chi^2 + 2\tilde{g}m_\phi\Phi \sin(m_\phi t)]\chi_k = 0$$

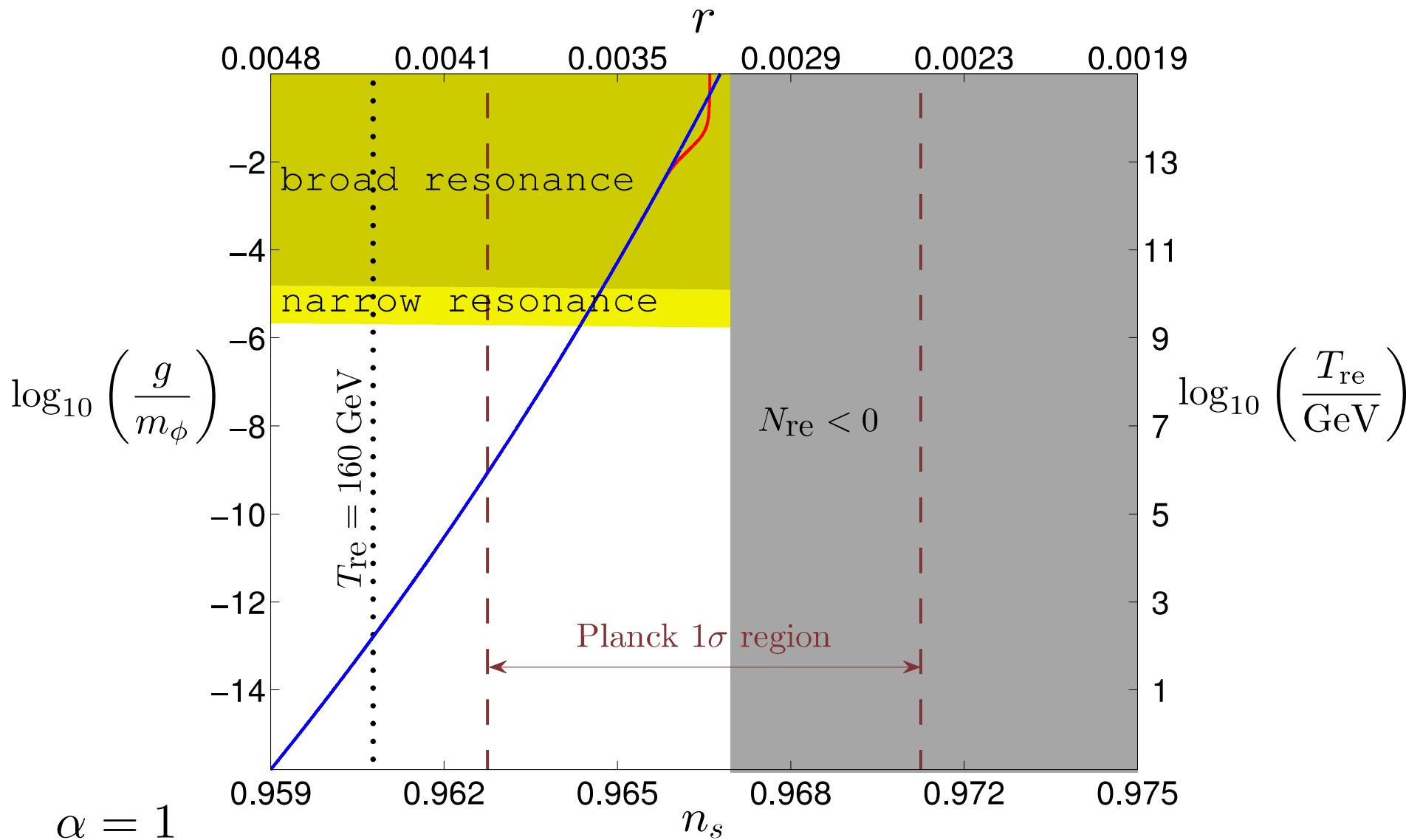
$$\phi = \Phi \sin m_\phi t, \quad \tilde{g} = \frac{g}{m_\phi}, \quad q = 4\tilde{g}\Phi/m_\phi$$

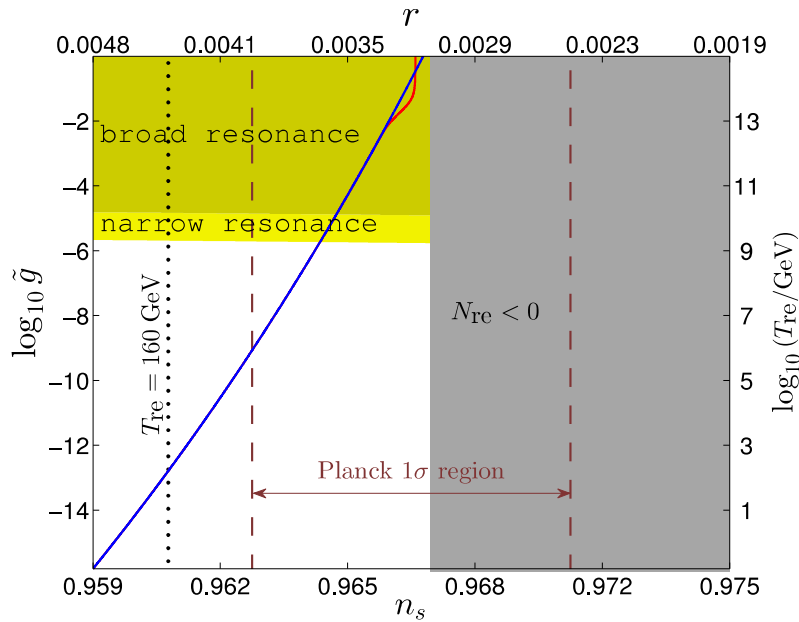
$$\text{broad resonance} \quad \Rightarrow \quad \frac{m_\phi}{\phi_{\text{end}}} < \tilde{g} \\ (1 < q)$$

$$\text{narrow resonance} \quad \Rightarrow \quad \frac{V_{\text{end}}^{1/4}}{\phi_{\text{end}}} \left(\frac{m_\phi}{24M_{pl}} \right)^{1/2} < \tilde{g} < \frac{m_\phi}{\phi_{\text{end}}} \\ \left((H/m_\phi)^{1/2} < q < 1 \right)$$

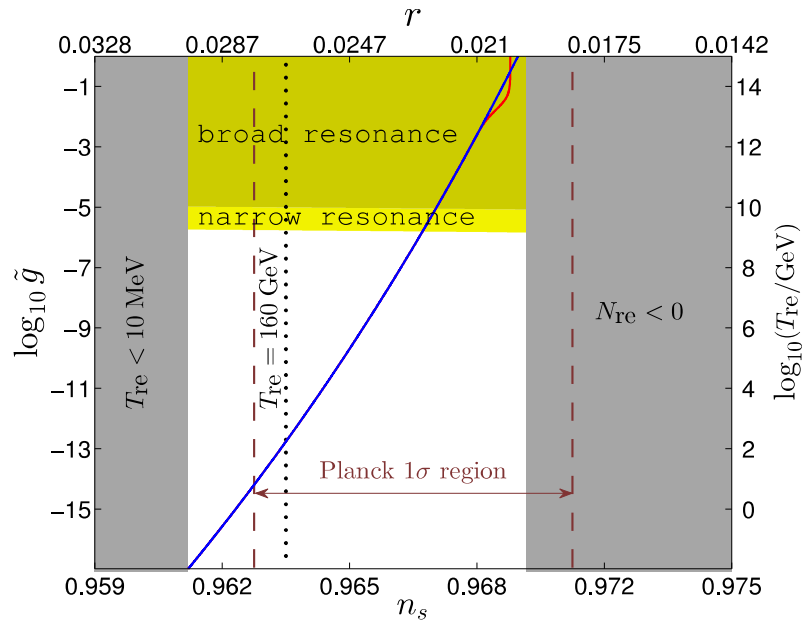
$$\text{perturbative regime} \quad \Rightarrow \quad \tilde{g} < \frac{V_{\text{end}}^{1/4}}{\phi_{\text{end}}} \left(\frac{m_\phi}{24M_{pl}} \right)^{1/2} \\ \left(q < (H/m_\phi)^{1/2} \right)$$

Scalar interaction: $g\phi\chi^2$

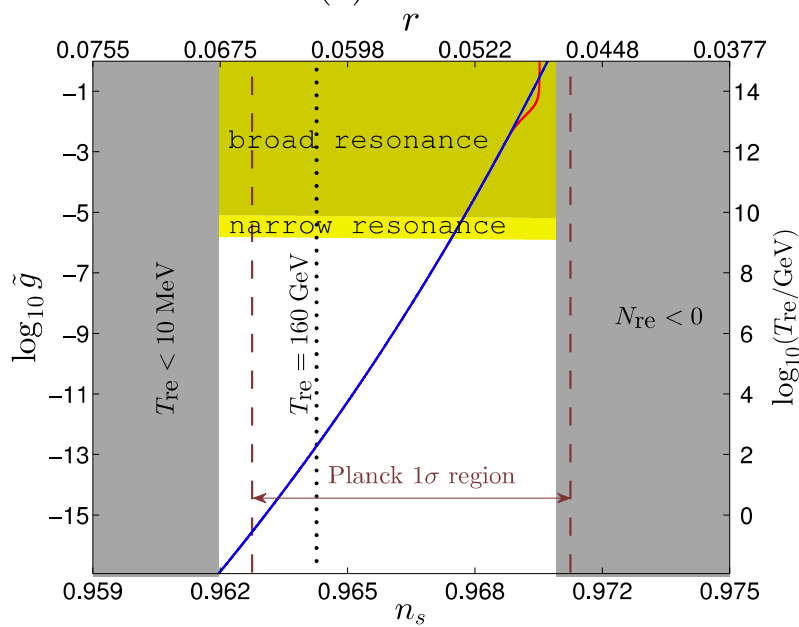




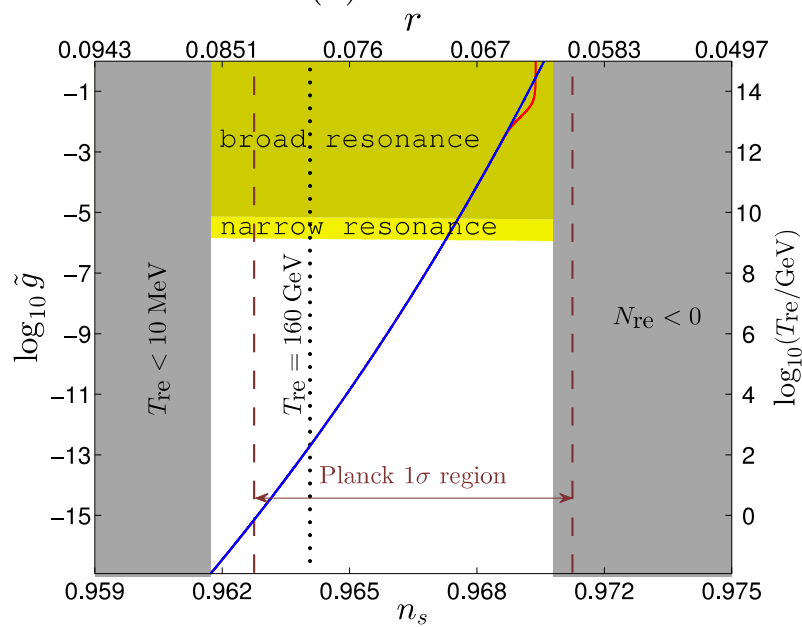
(a) $\alpha = 1$



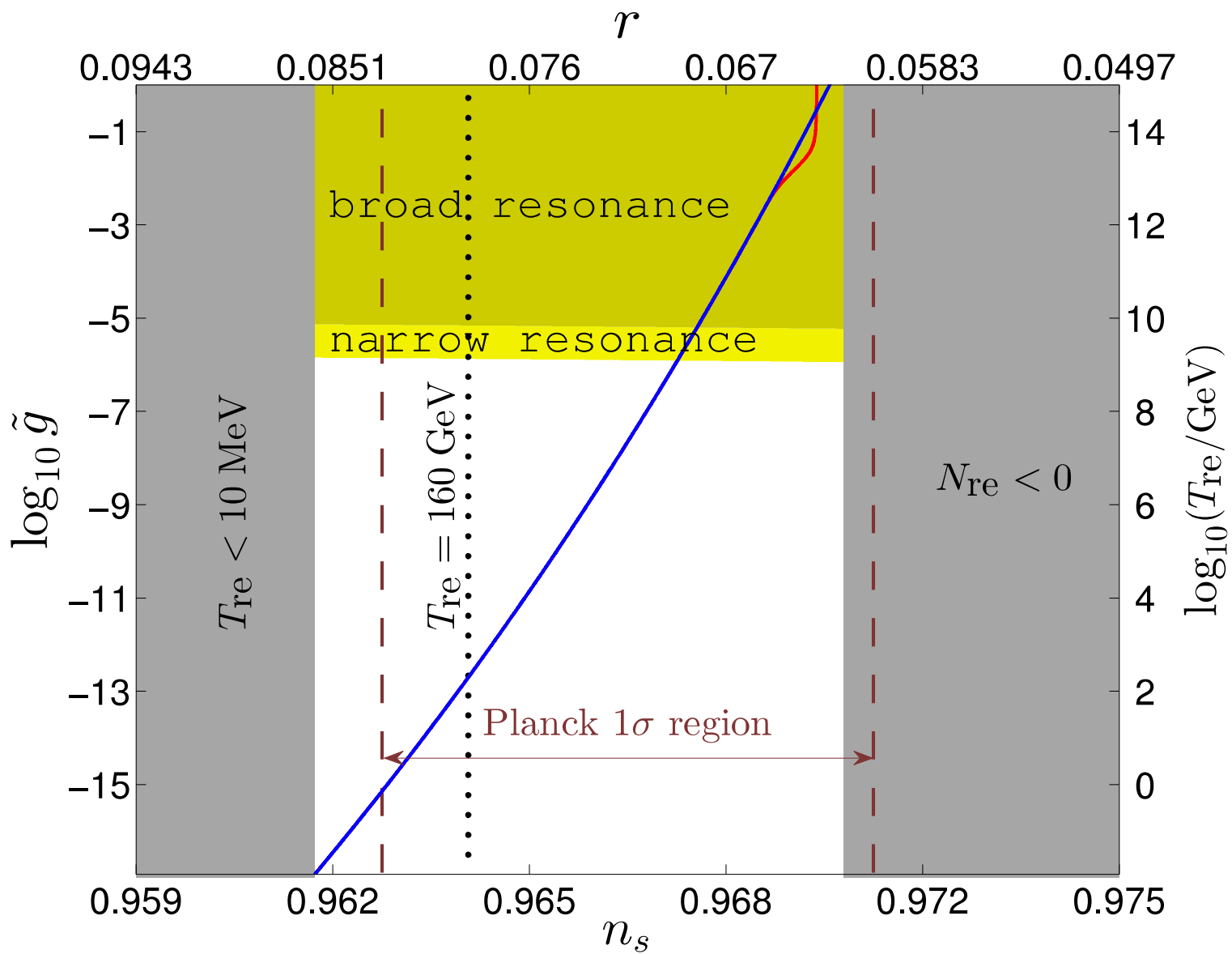
(b) $\alpha = 10$



(c) $\alpha = 50$



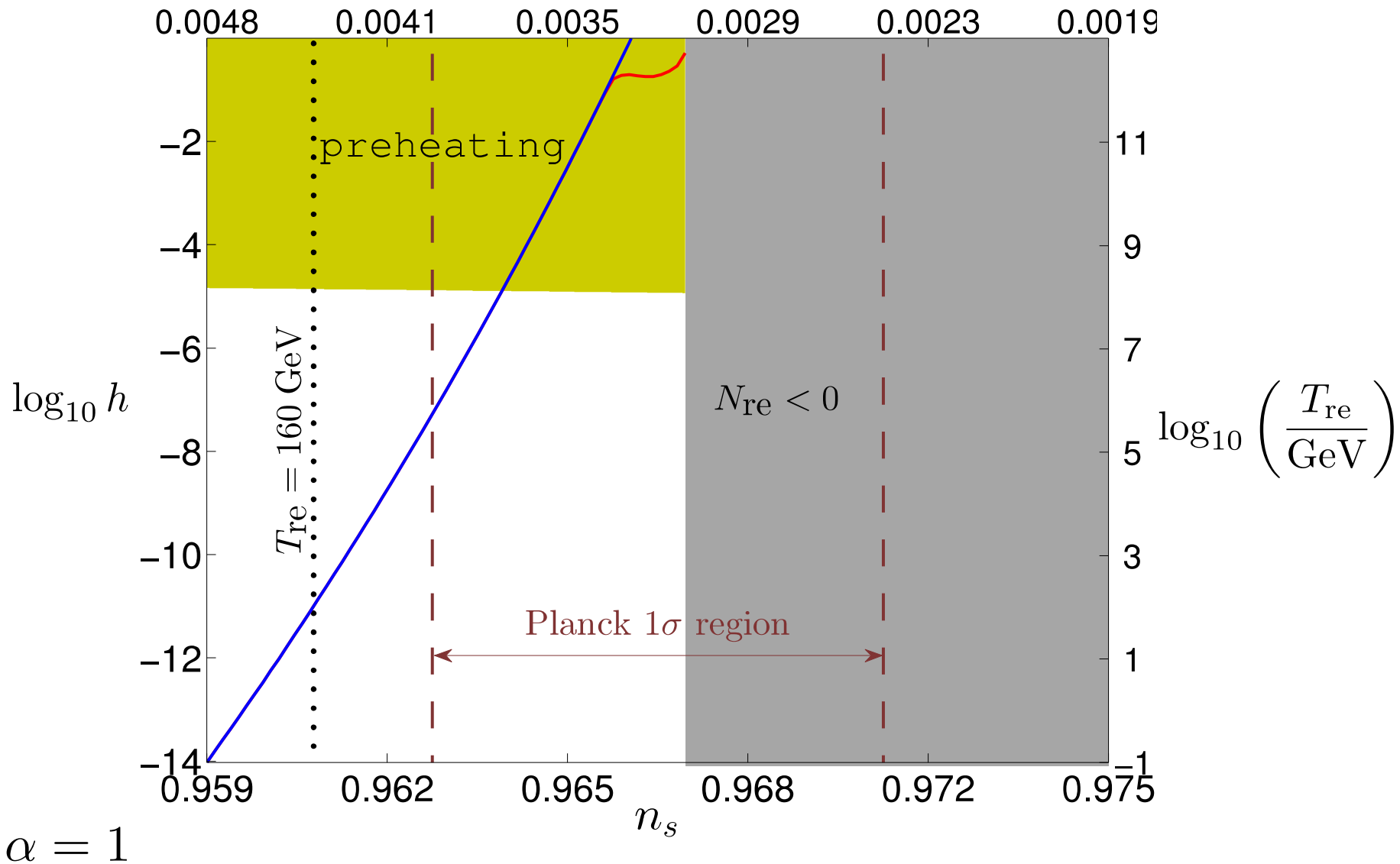
(d) $\alpha = 100$



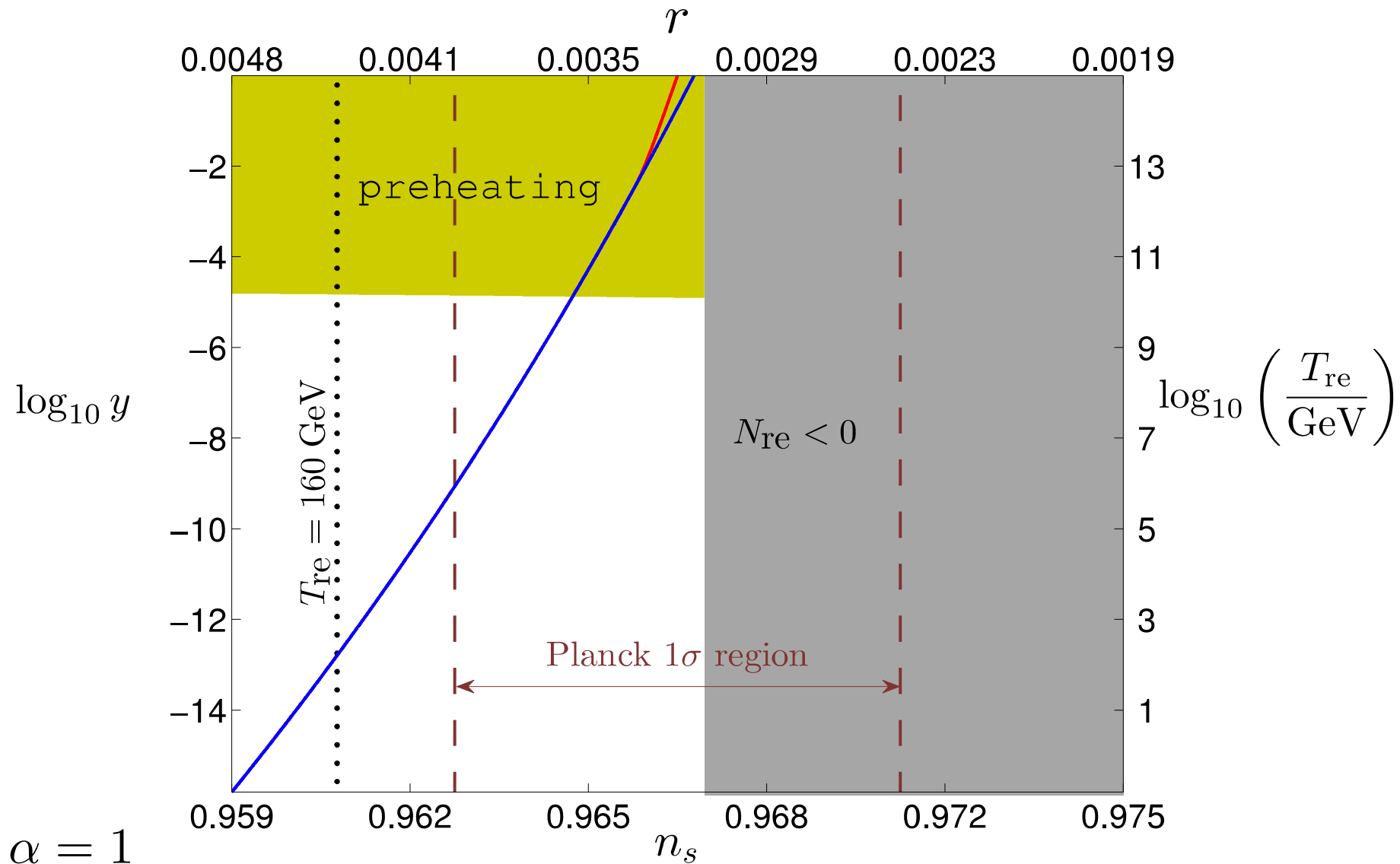
(d) $\alpha = 100$

Scalar interaction: $h\phi\chi^3$

r



Yukawa interaction: $y\phi\bar{\psi}\psi$



Conclusion

- CMB can be used to probe the microphysical properties at reheating, i.e. the inflaton-matter couplings, which may be of importance for the embedding of an inflationary model into more fundamental theory.
- This idea has been worked out in the α -attractor model with inflaton couplings to other scalars and fermions, focusing on the perturbative regime.
 - There exist viable regions in the parameter space in which the inflaton couplings can be “measured” from CMB data, in particular, the spectral tilt.
 - In this model the thermal effects seem to play no significant role in the perturbative regime. [Drewes 2015](#)

Conclusion

- The relation between CMB data and the inflaton coupling is model-dependent, i.e. depend on the potential parameters both in the slow and fast-roll regime.
- The most dominant interaction can be constrained. If there are several comparable interactions (due to symmetry), certain combination of them can be constrained.