Superconducting Qubits

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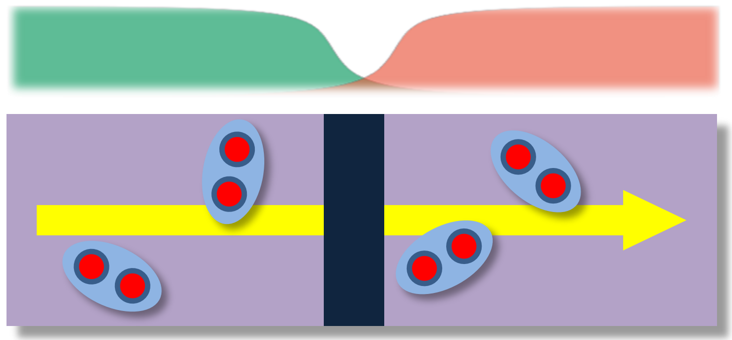
The Josephson's Lego bricks box

Josephson junction

$$\psi_L = \sqrt{\rho_L}\, e^{i\varphi_L} \qquad \psi_R = \sqrt{\rho_R}\, e^{i\varphi_R}$$



Insulating barrier

Phase difference

$$\varphi = \varphi_R - \varphi_L$$

$$\phi_0 = \frac{h}{2e}$$

$$\phi_b = \frac{\hbar}{2e}$$

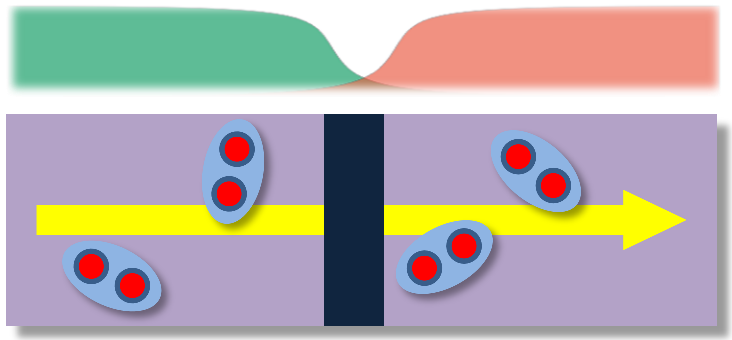
Josephson equations

$$I = I_0 sin\varphi$$

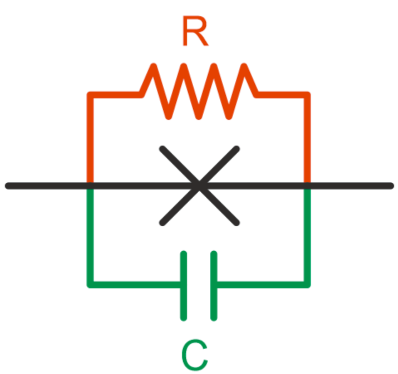
$$V = \frac{\hbar}{2e}\frac{d\varphi}{dt}$$

Symbol

$$\psi_L = \sqrt{\rho_L}\, e^{i\varphi_L} \qquad \psi_R = \sqrt{\rho_R}\, e^{i\varphi_R}$$



Insulating barrier



Phase difference

$$\varphi = \varphi_R - \varphi_L$$

$$\phi_0 = \frac{h}{2e} \qquad \phi_b = \frac{\hbar}{2e}$$

Josephson equations

$$I = I_0 \sin\varphi + C\frac{dV}{dt} + \frac{V}{R}$$

$$V = \frac{\hbar}{2e}\frac{d\varphi}{dt}$$

$$C\phi_b \frac{d^2\varphi}{dt^2} + \frac{\phi_b}{R}\frac{d\varphi}{dt} + I_0 \sin\varphi = I$$

Mechanical equivalent

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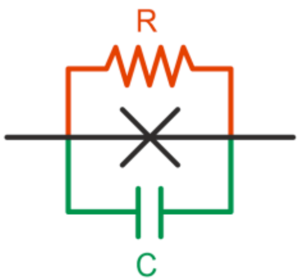


$$C\phi_b \frac{d^2\varphi}{dt^2} + \frac{\phi_b}{R}\frac{d\varphi}{dt} + I_0 sin\varphi = I$$

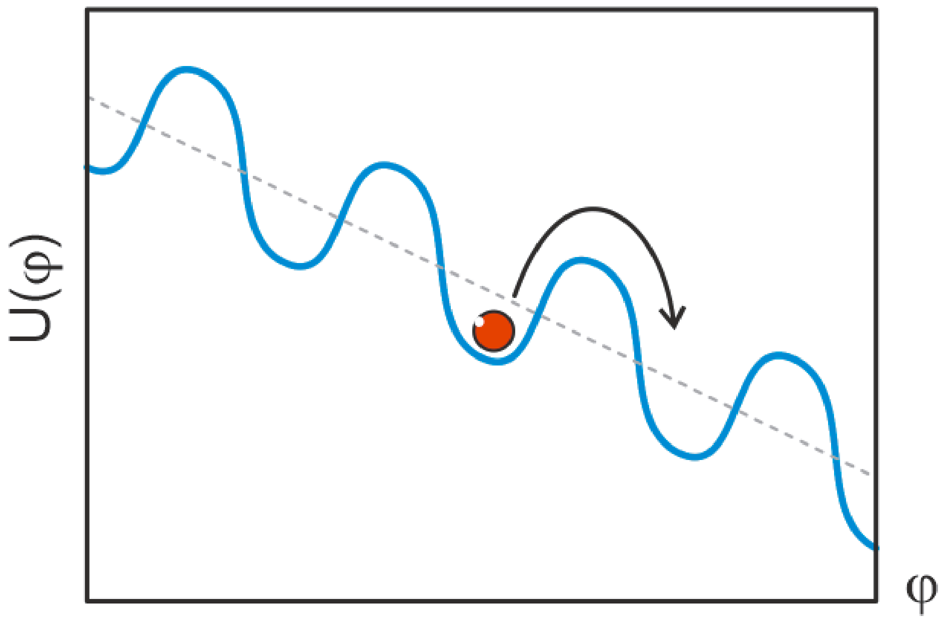
Motion equations

$$M\frac{d^2\varphi}{dt^2} + M\gamma\frac{d\varphi}{dt} = -\frac{dU}{d\varphi}$$

Effective potential

$$U = -E_j(cos\varphi + I/I_0)$$

$$M = C\phi_b^2$$
$$E_j = I_0\phi_b^2$$
$$\gamma = 1/RC$$



$$V = \frac{\hbar}{2e}\frac{d\varphi}{dt}$$

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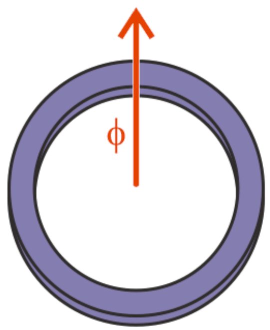

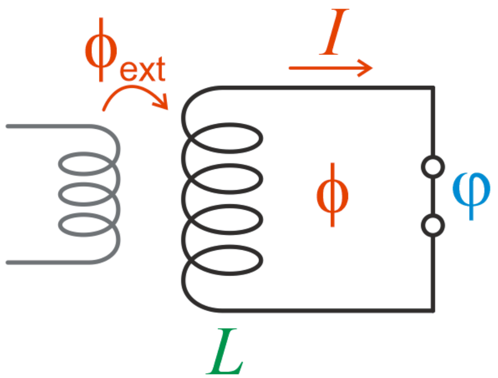

$$\phi = \phi_{ext} - LI$$

$$\phi = \frac{\hbar}{2e}\,\varphi$$

$$\varphi = 2\pi\, n$$

$$\phi = \phi_{ext} - LI = n\phi_0$$

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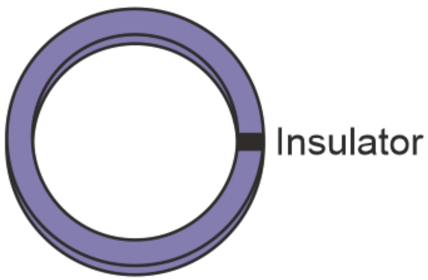

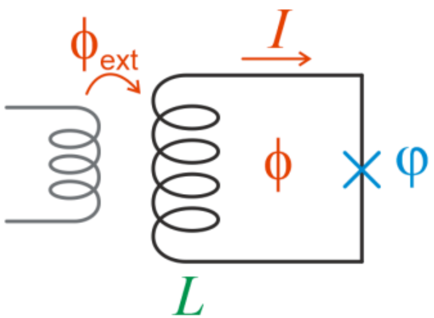

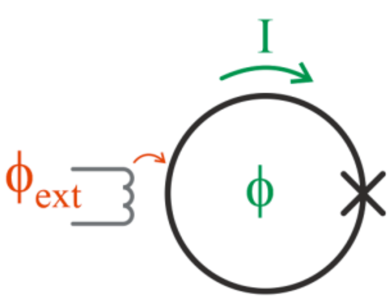

$$C\phi_b \frac{d^2\varphi}{dt^2} + \frac{\phi_b}{R}\frac{d\varphi}{dt} + I_0 sin\varphi = I$$

$$\phi = \phi_{ext} - LI$$

$$\phi = \frac{\hbar}{2e}\varphi$$

Motion equations

$$M\frac{d^2\varphi}{dt^2} + M\gamma\frac{d\varphi}{dt} = -\frac{dU}{d\varphi}$$

Effective potential

$$U = \frac{1}{2}E_L(\varphi - \phi_{ext}/\phi_0)^2 - E_j cos\varphi$$

$$M = C\phi_b^2 \quad \gamma = 1/RC$$
$$E_j = I_0\phi_b^2 \quad E_L = \phi_b^2/L$$

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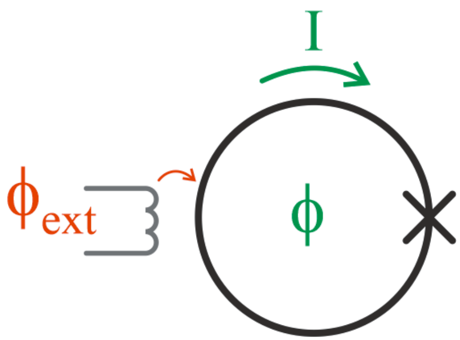

$$M\frac{d^2\varphi}{dt^2} + M\gamma\frac{d\varphi}{dt} = -\frac{dU}{d\varphi}$$

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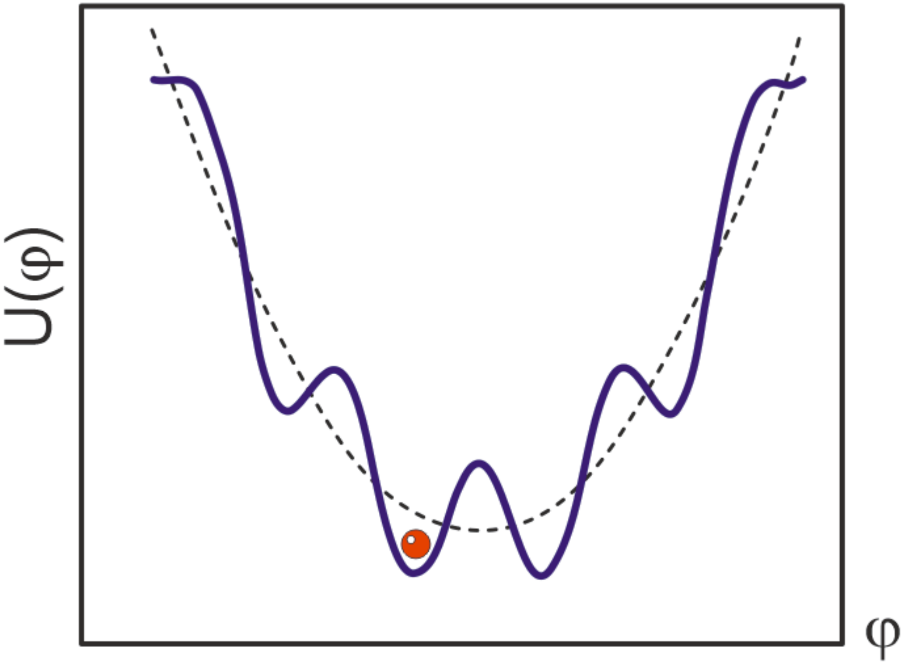
$M = C\phi_b^2 \quad \gamma = 1/RC$

$E_j = I_0\phi_b^2 \quad E_L = \phi_b^2/L$



Torrioli's talk
on SQUIDs at 16:00

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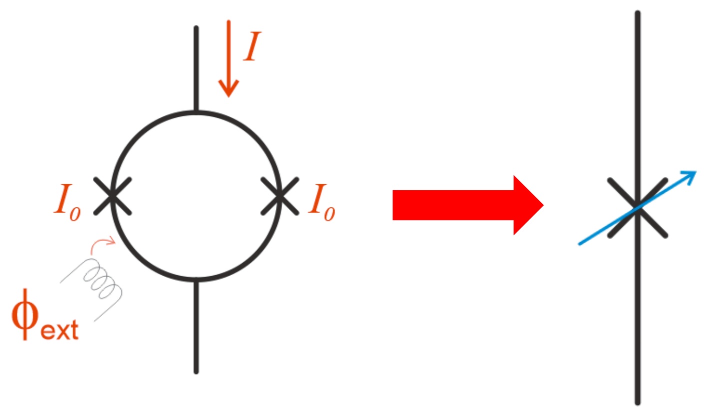
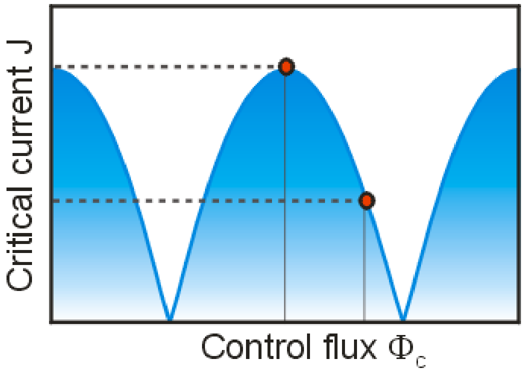
Tunable Josephson element



$$I_0(\phi_{ext}) \cong 2I_0 \cos\left(\pi\phi_{ext}/\phi_0\right)$$

For $LI_0 \ll \phi_b$



Torrioli's talk
on SQUIDs at 16:00

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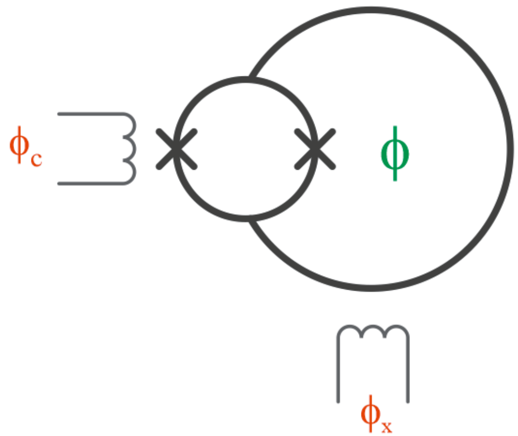
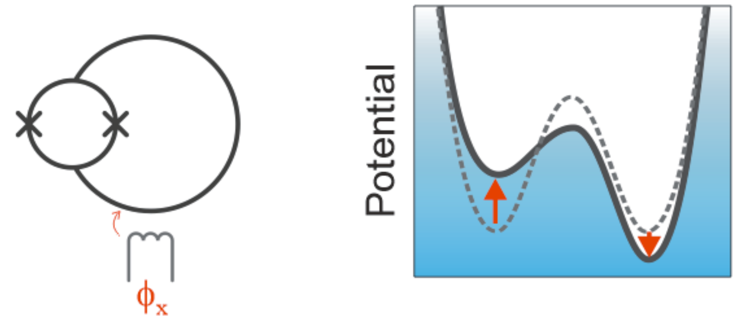
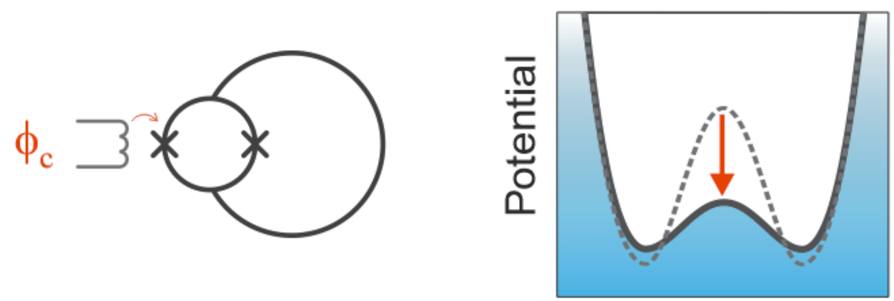
Tunable rf SQUID - two controls



Large loop: symmetry



Small loop: barrier



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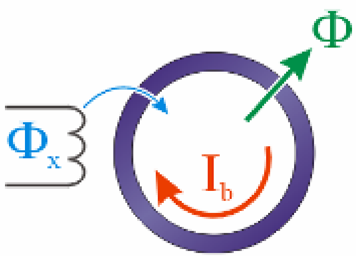
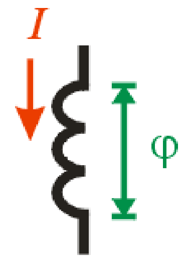
CNR IFN Istituto di Fotonica e Nanotecnologie

Flux quantization



$$\Phi = \Phi_x - LI = n\Phi_0$$

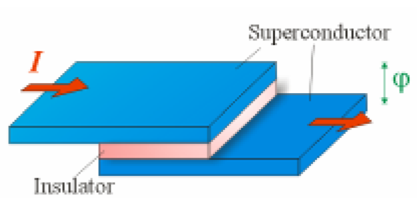
Inductance

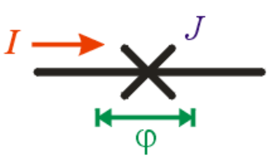


$$I = \Phi_b \varphi / L$$

$$E_L = \frac{\Phi_b^2}{2L}\varphi^2$$

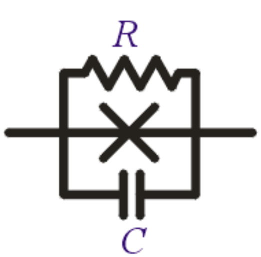
Josephson junction





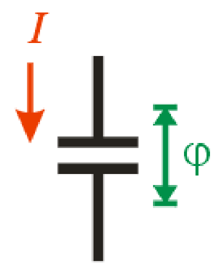
$$\begin{cases} V = \Phi_b \dot{\varphi} \\ I = J \sin\varphi \end{cases}$$

$$E_J = -J\Phi_b \cos\varphi$$



$$I = J\sin\varphi + C\Phi_b\ddot{\varphi} + \frac{\Phi_b}{R}\dot{\varphi}$$

Capacitance



$$I = C\Phi_b\dot{\varphi}$$

$$E_C = \frac{1}{2}C\Phi_b^2\dot{\varphi}^2 = \frac{p^2}{2M}$$

$$M = C\Phi_b^2 \qquad p = M\dot{\varphi}$$

$$\Phi_0 = h/2e \cong 2.068\times 10^{-15}\,Wb$$

$$\Phi_b = \Phi_0/2\pi = \hbar/2e \cong 3.291\times 10^{-16}\,Wb$$

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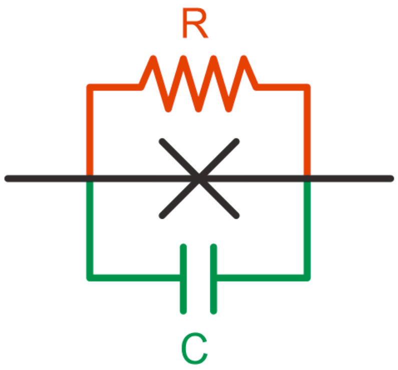

$$M\frac{d^2\varphi}{dt^2} + M\gamma\frac{d\varphi}{dt} = -\frac{dU}{d\varphi}$$

$$U = -E_j(\cos\varphi + I/I_0)$$

$$M = C\phi_b^2$$
$$E_j = I_0\phi_b^2$$
$$\gamma = 1/RC$$

$$H = E_c n^2 + U$$

$$Q = CV = C\phi_b d\varphi/dt$$
$$n = Q/(2e) = Cooper\ pairs\ on\ C$$
$$E_c = 2e^2/C$$

For $E_c \gg E_j$ control on single Cooper pair crossing (small junctions)

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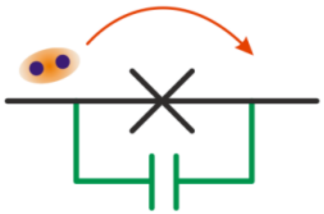
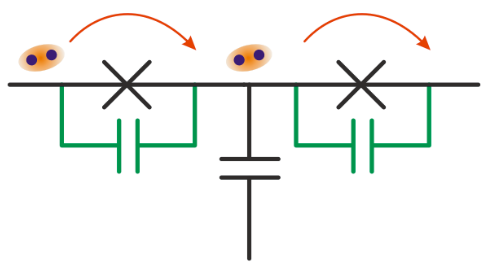
For $E_c \gg E_j$ control on single Cooper pair crossing

$$H = E_c n^2 + U$$

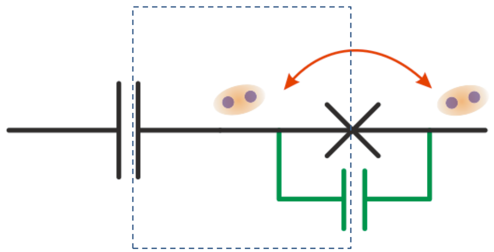


SET Transistor



Flux of Cooper pairs controlled one by one

Cooper Pair Box



Can store a single Cooper pair

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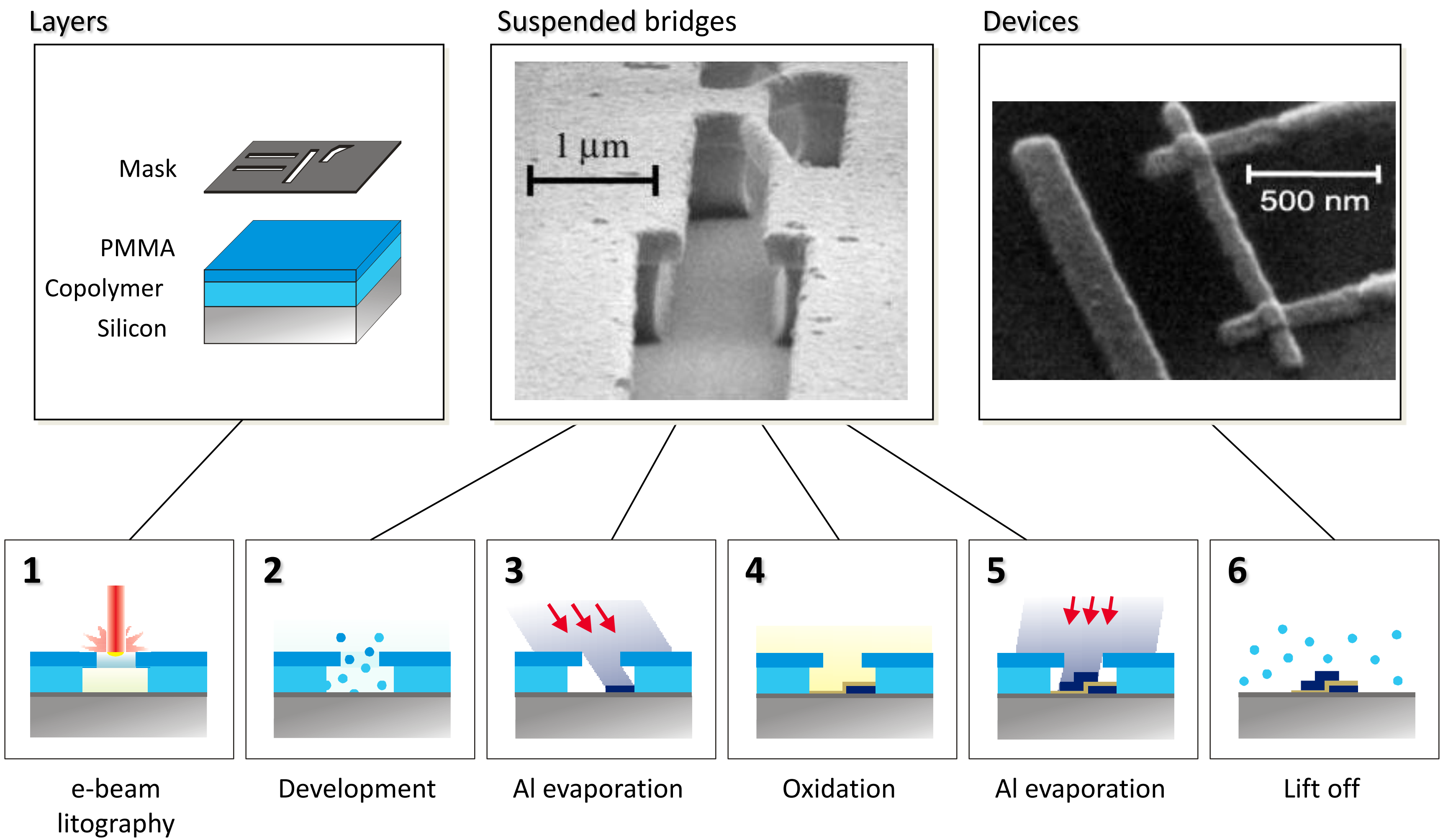
Small Al junctions fabrication



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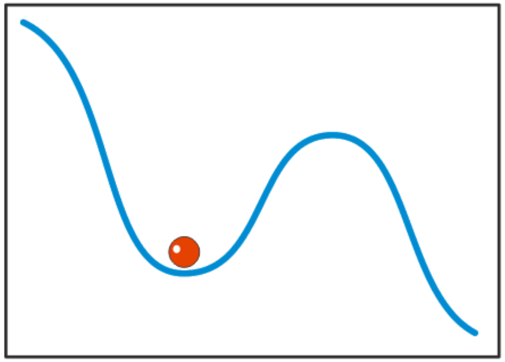
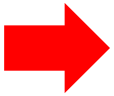
Quantum behavior

Is it possible a quantum description of the equivalent mechanical model?

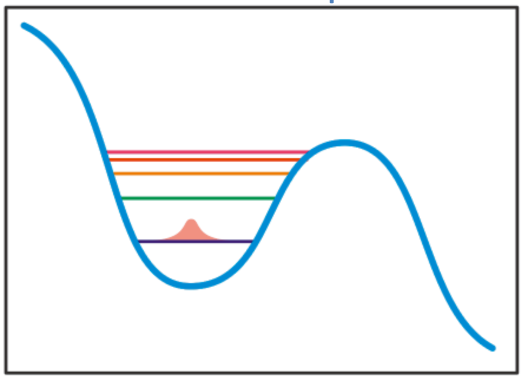
Classical description



$$M\frac{d^2\varphi}{dt^2} = -\frac{dU}{d\varphi}$$



Quantum description



$$i\hbar\frac{\partial}{\partial t}\psi = H\psi$$

$$H = E_c n^2 + U$$

$$[n, \varphi] = -i$$

$$n = Cooper\ pairs\ on\ C$$

$$E_c = 2e^2/C$$

P.W. Anderson, in: E.R. Caianiello (Ed.), Lectures on the Many Body Problem, Vol. 2, Academic Press, New York, 1964.

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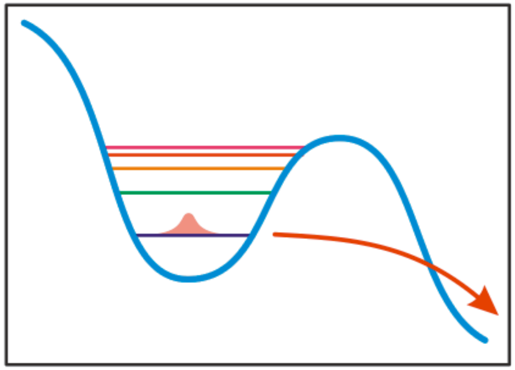
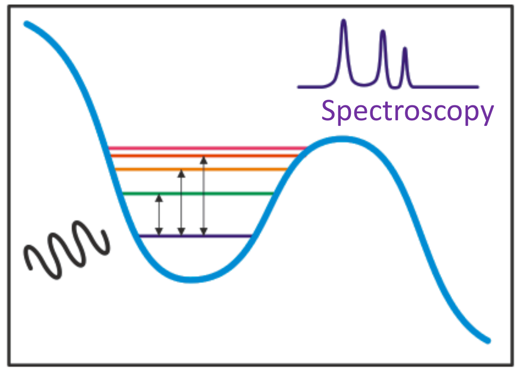
Quantum effects (observed)

Observed quantum effects in Josephson systems

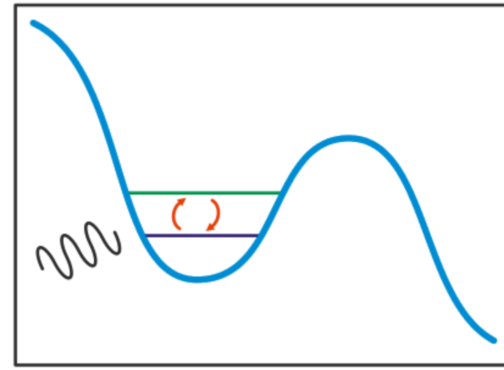
Tunnel Effect



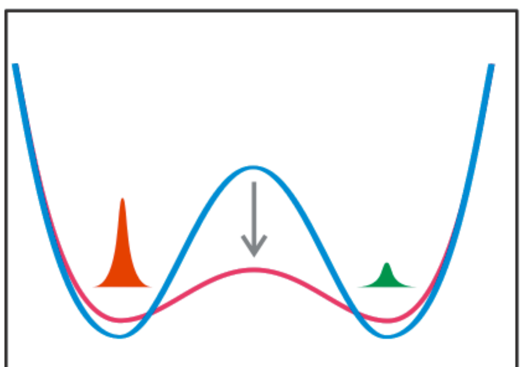
Energy Level Quantization



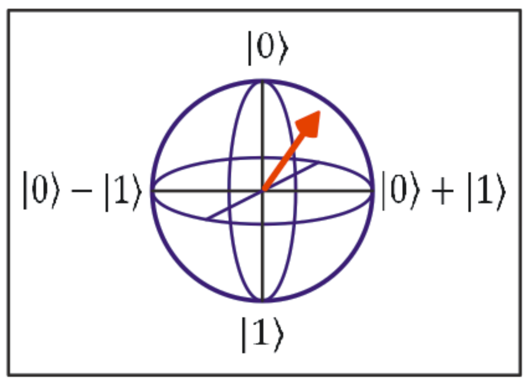
Rabi Oscillations



Nonadiabatic manipulation

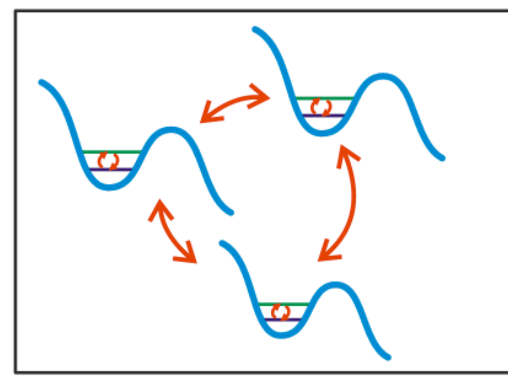


Block Sphere manipulation



(Ramsey fringes,
Spin Echo, …)

Entagled Systems



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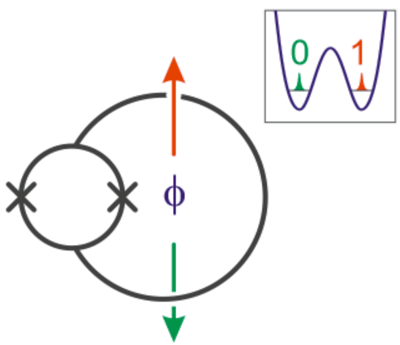
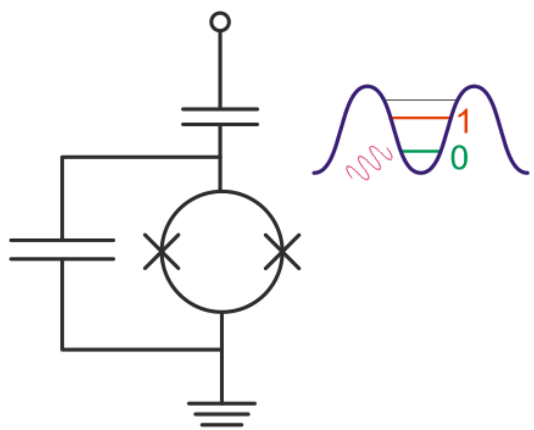
Qubit: quantum two state systems
which can be manipulated and coupled

Flux Qubit



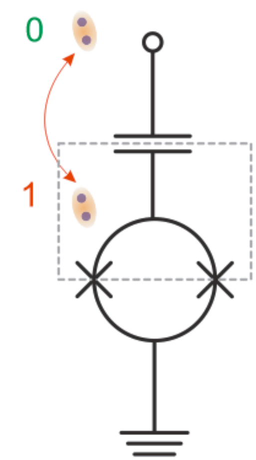
$$E_j > E_L \gg E_c$$

Transmon Qubit



$$E_j > E_c$$

Cooper Pair Box Qubit



$$E_j < E_c$$

Single junction
3 junction flux qubit
Quantronium
...

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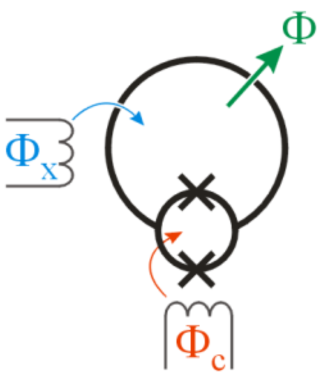
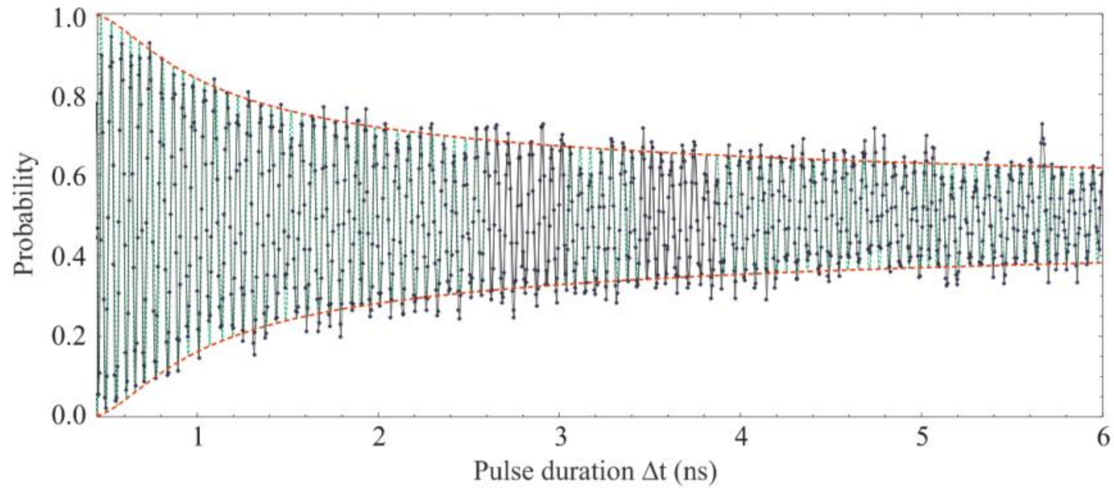
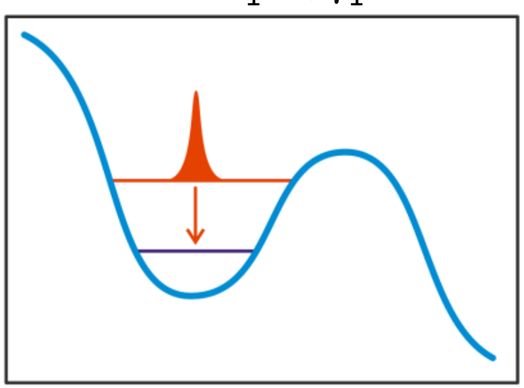
Effect of noise from different sources

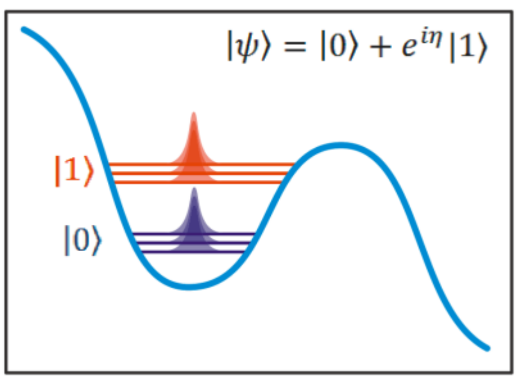




Relaxation $T_1=1/\gamma_1$



Dephasing $T_2=1/\gamma_2$



Ohmic noise

$$\gamma = \frac{\gamma_1}{2} + \gamma_2$$

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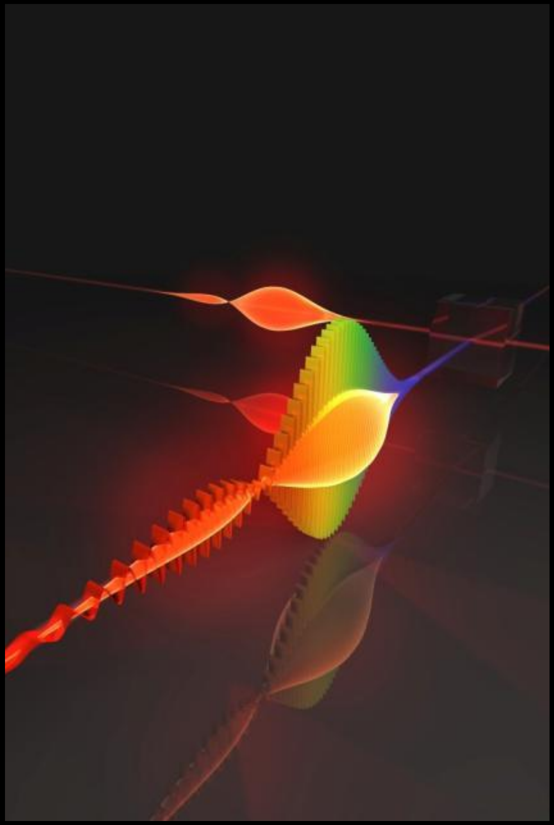
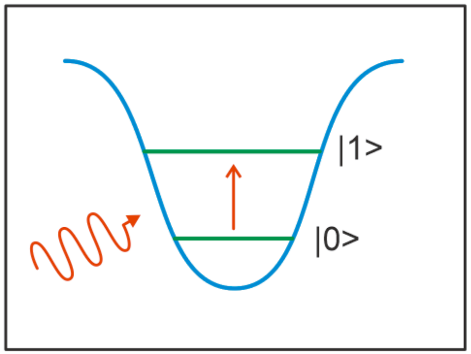
Single photon detection

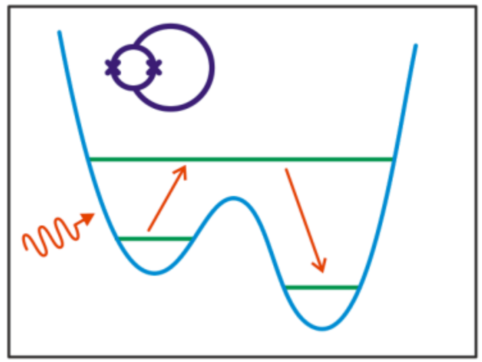


Absorption of a single (GHz) photon
Detection of the qubit state

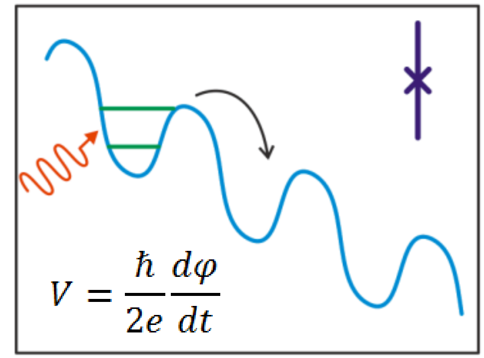
Artificial atom



Lambda transition



Activation in voltage state



$$\Delta f \sim \frac{\sqrt{E_j E_c}}{h} \sim 10\ GHz$$

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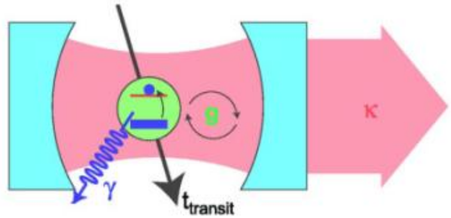
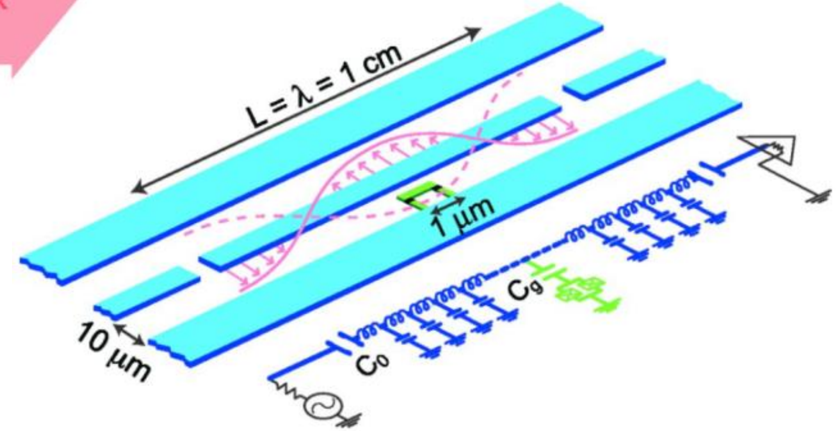
Cavity: coplanar waveguide
Atom: Cooper pair box qubit

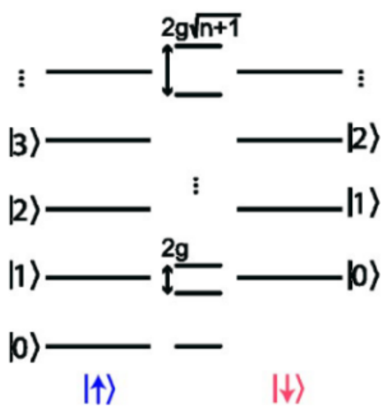




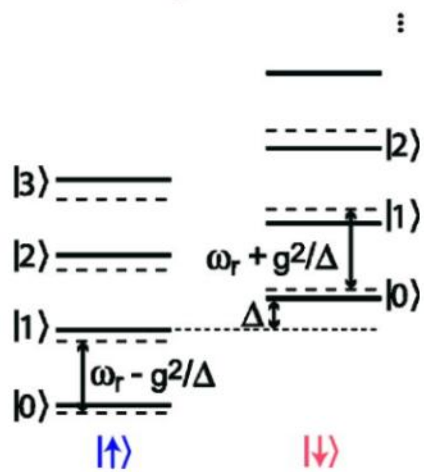
Jaynes-Cummings Hamiltonian

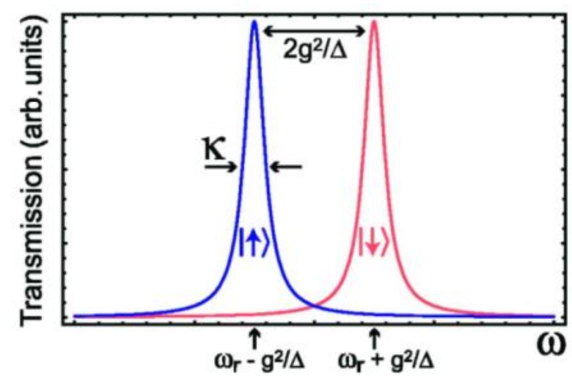
$$H = \hbar\omega_r\left(a^\dagger a + \frac{1}{2}\right) + \frac{\hbar\Omega}{2}\sigma^z + \hbar g(a^\dagger\sigma^- + \sigma^+ a)$$

Tuned $\omega_r = \Omega$



Detuned $\omega_r = \Omega - \Delta$





*A. Blais, et al., Phys. Rev. A **69**, 62320 (2004).*

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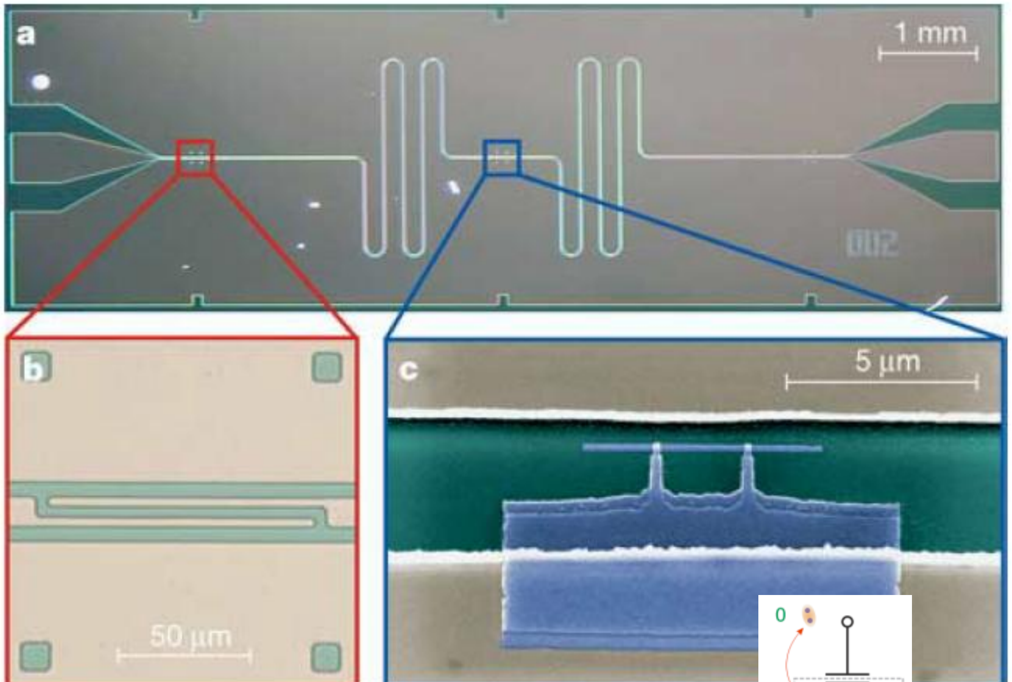
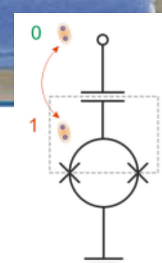
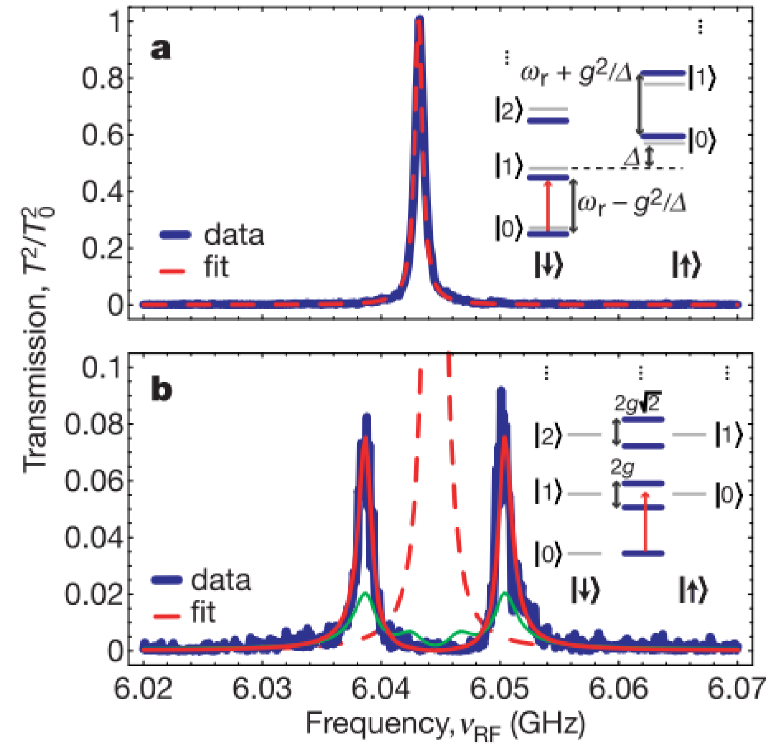
Coupling between an "artificial atom" and a "cavity"







Cavity: coplanar waveguide
Atom: Cooper pair box qubit

Q ≈ 10 000
T_k ≈ 200 ns
T_γ ≈ 230 ns

*A. Wallraff et al., Nature **431**, 162–167 (2004).*

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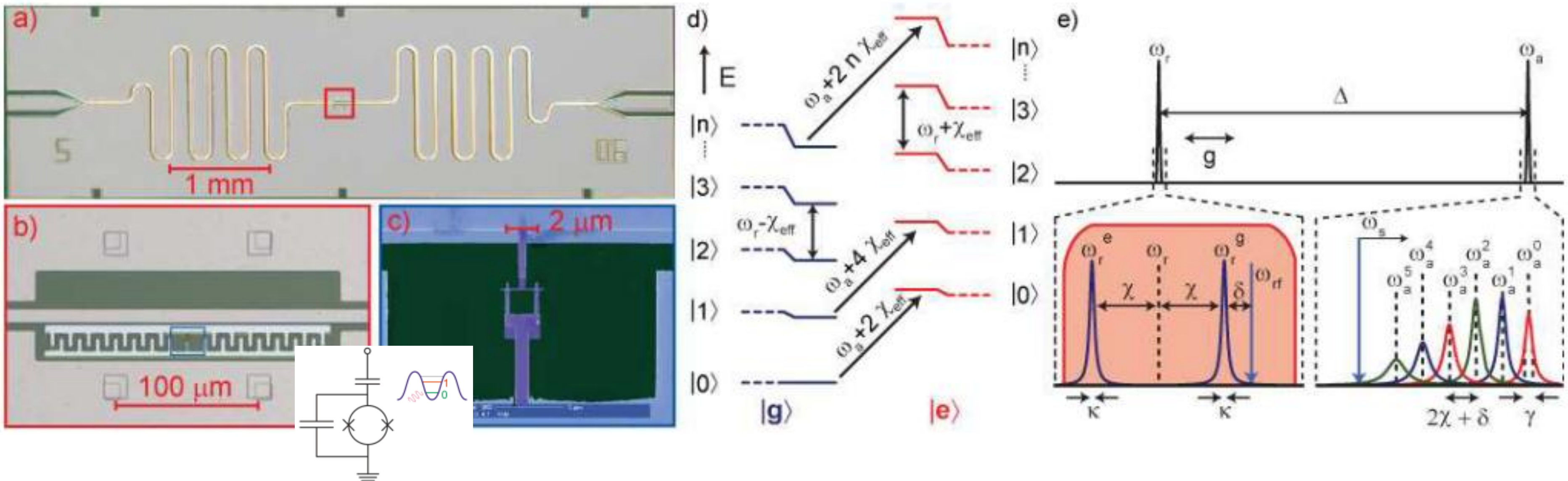
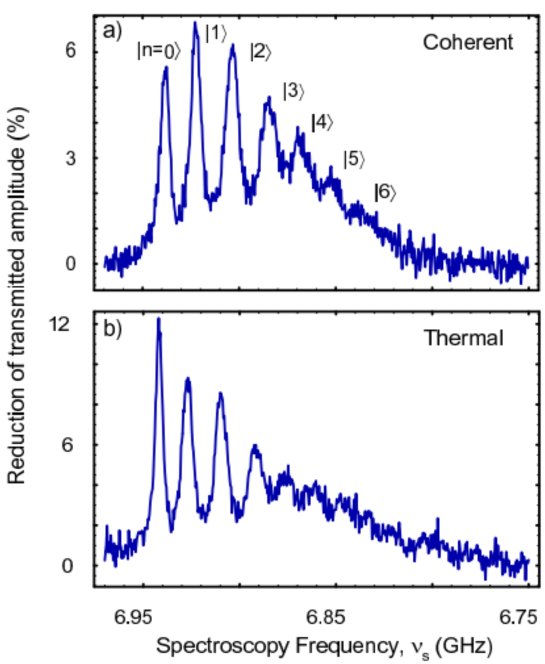
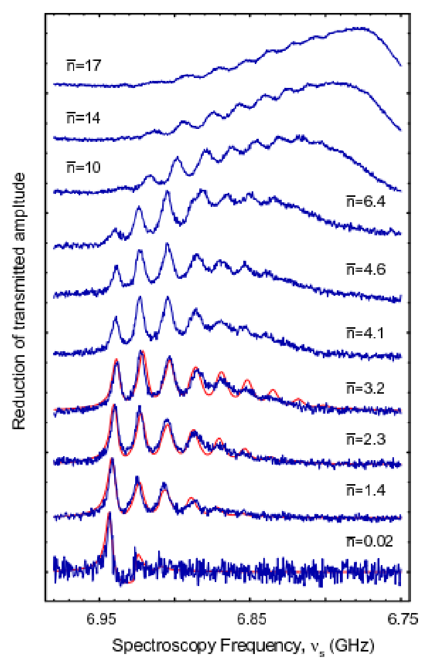




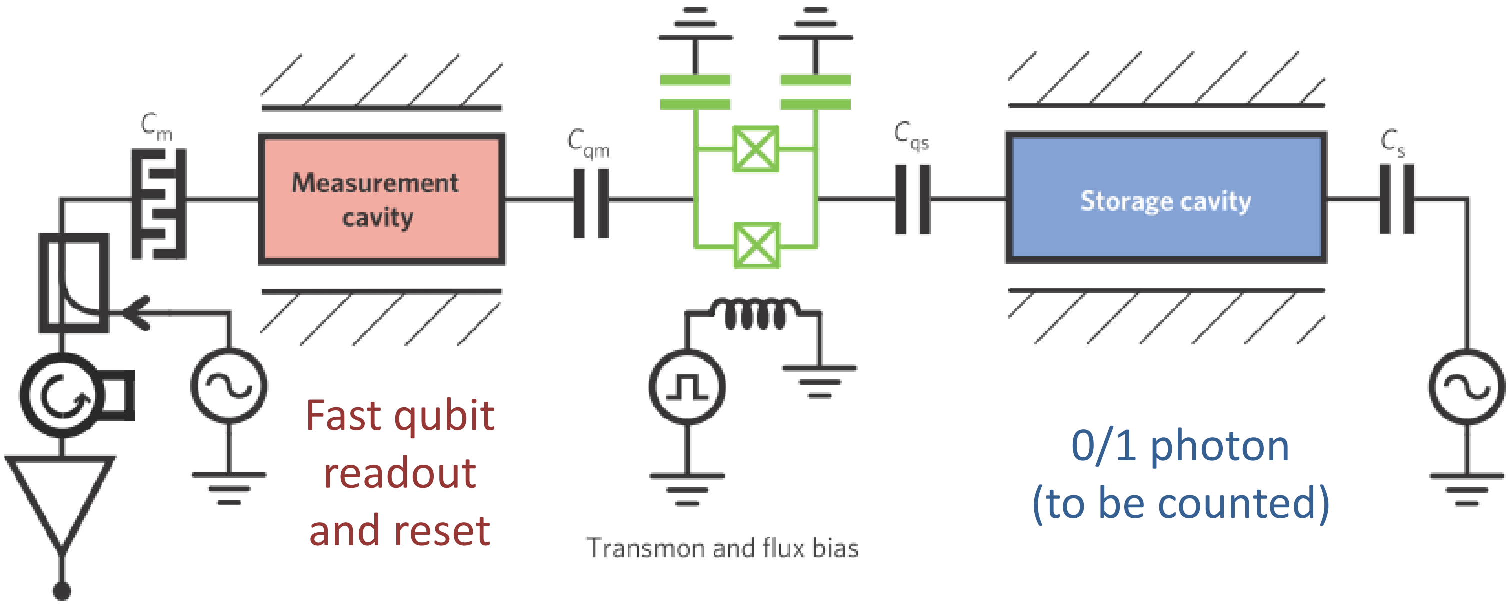


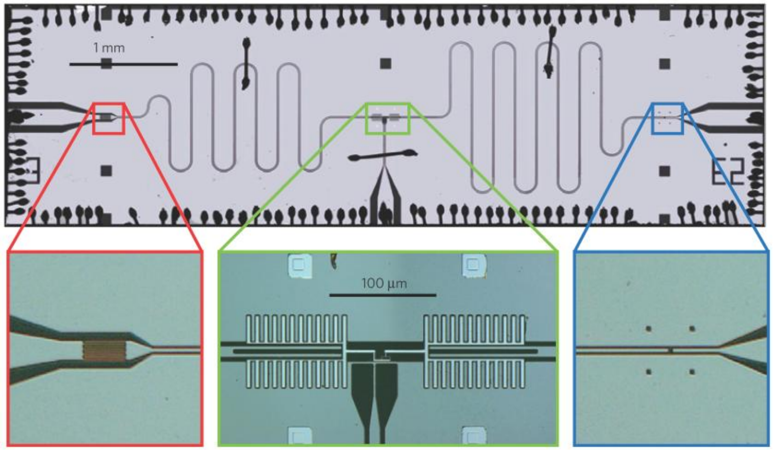
$Q \approx 30\,000$

$T_k \approx 640$ ns

$T_\gamma \approx 84$ ns

D. I. Schuster et al., Nature ***445****, 515–518 (2007).*





90% QND

B. R. Johnson et al., Quantum non-demolition detection of single microwave photons in a circuit, Nat Phys ***6****, 663–667 (2010).*

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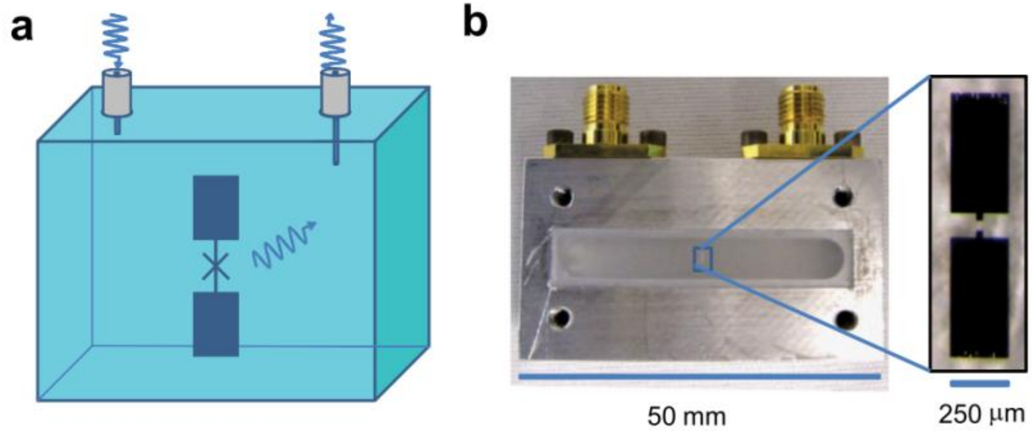
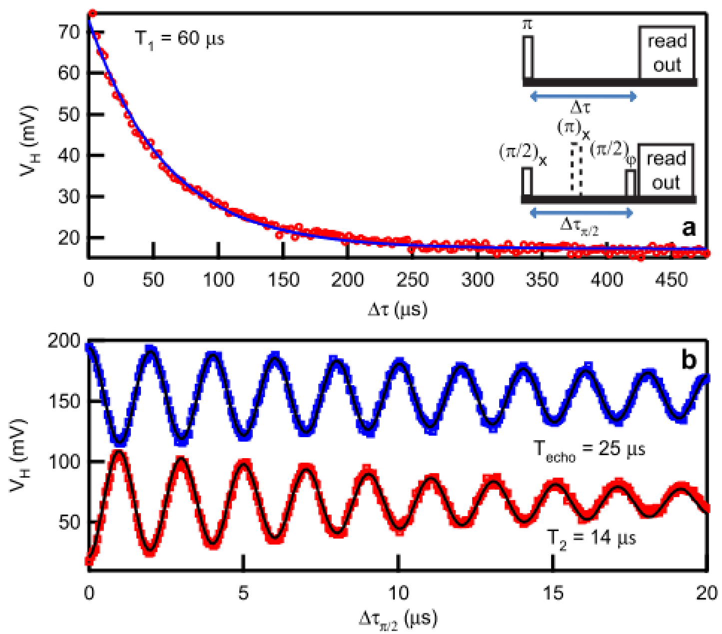
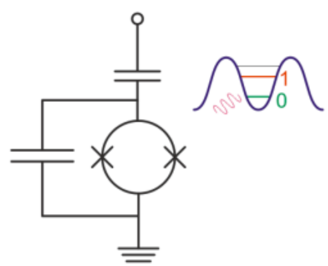
Qubit in a 3D cavity





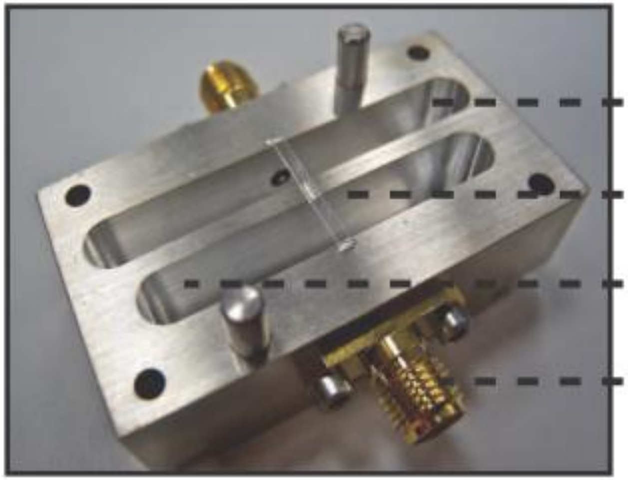


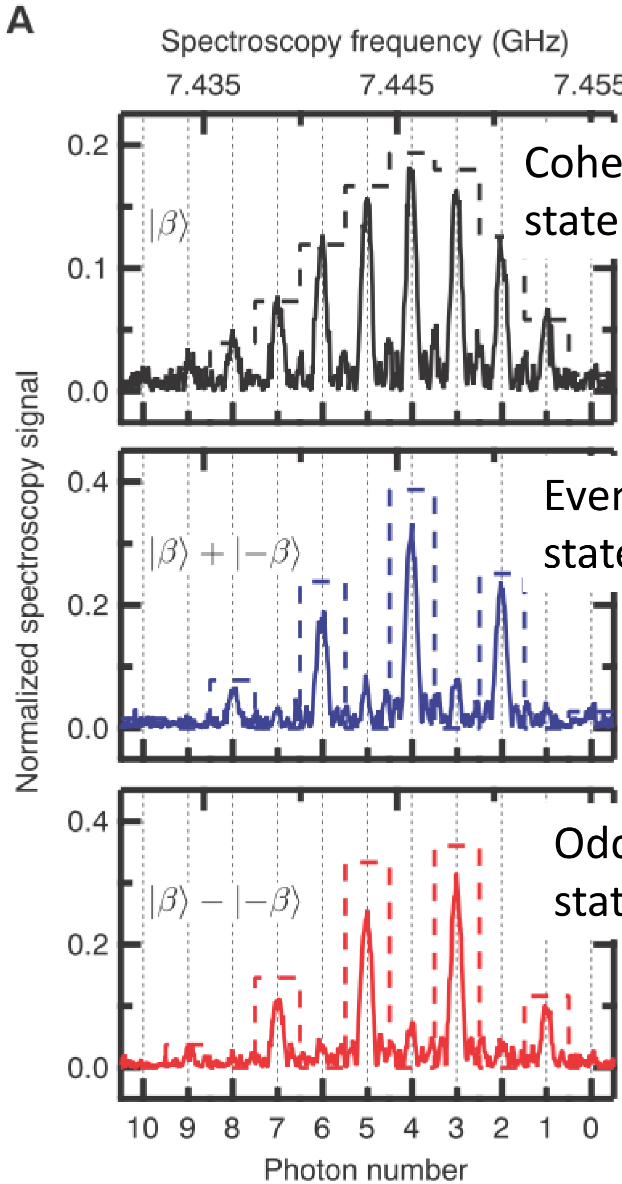
$Q \approx$ 1 000 000
$T_k \approx 20$ μs
$T_\gamma \approx 20$ μs

*H. Paik, Phys. Rev. Lett. **107** (2011).*

First efforts to use a similar system for axions

Akash Dixit
Aaron Chou
Dave Schuster
University of Chicago





*B. Vlastakis et al., Deterministically Encoding Quantum Information Using 100-Photon Schrödinger Cat States, Science **342**, 607–610 (2013).*

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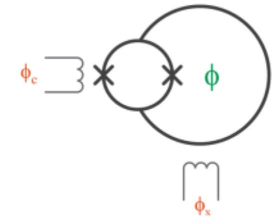
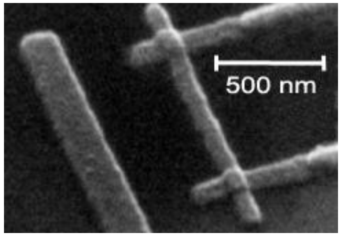
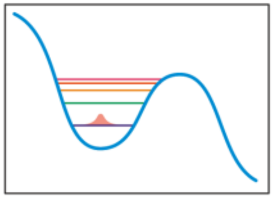
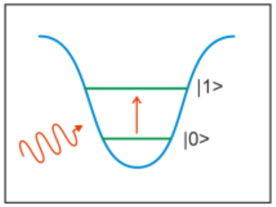
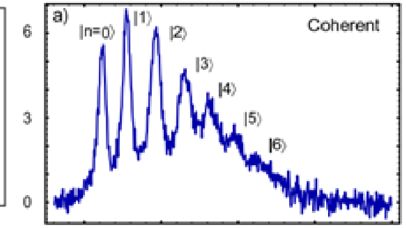
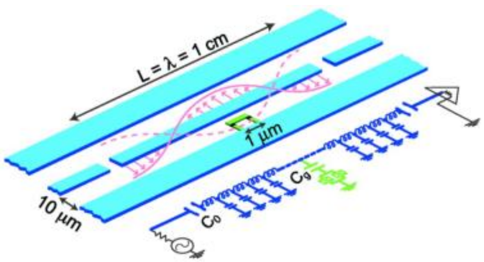
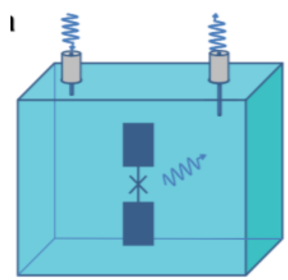
Josephson devices:
- Realization of flexible systems
- Quantum behavior
- Detection of single photons at ~ 10 GHz
- Coupling with cavities at ~ 10 GHz







Thank you!

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