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# SuperConducting Resonant Cavities

## (Temperature, Frequency and Magnetic Field dependence of the Surface Resistance of Superconductors)

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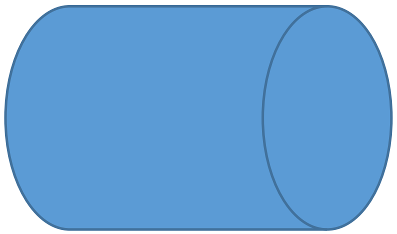
# Resonant Cavities



$$Q = \omega_0 \frac{W}{P_d}$$

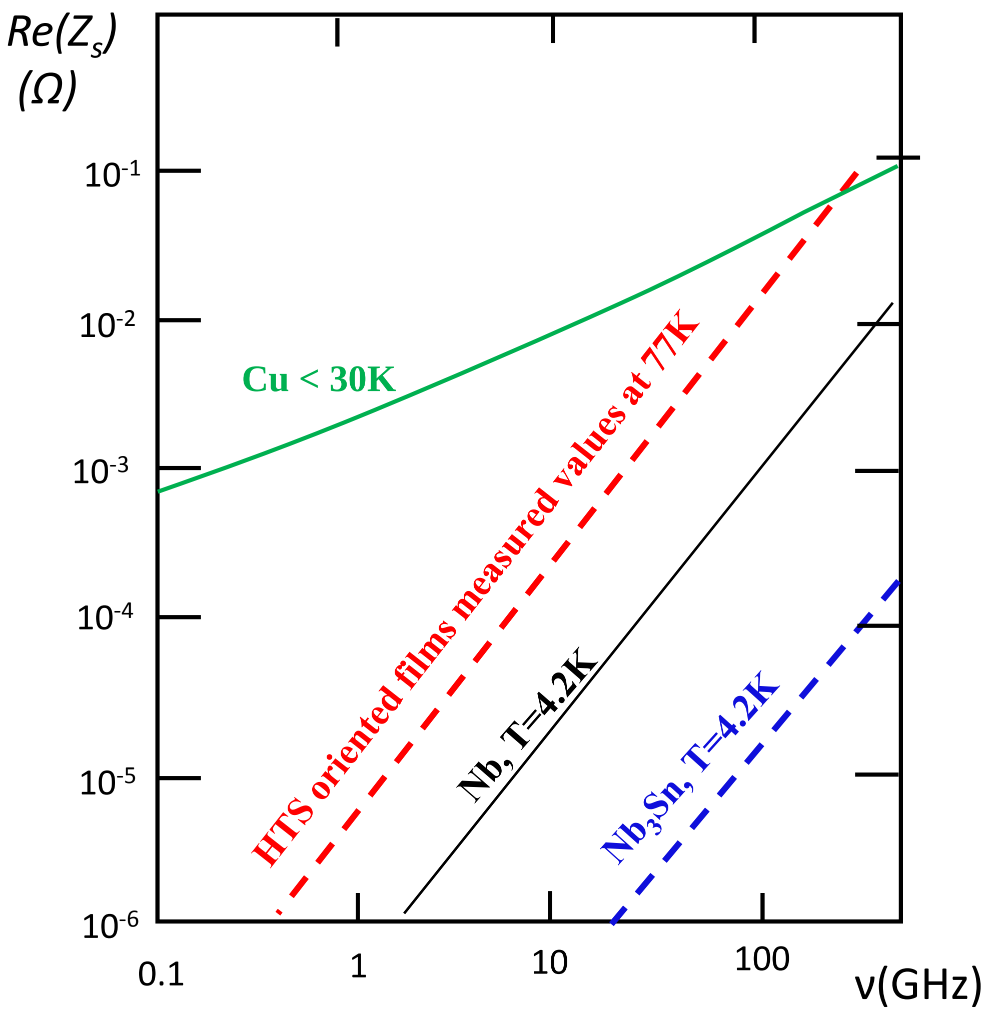
$$Q = \frac{\mu_0 \omega_0 \int_V H_{rf}^2 dV}{\int_S R_S H_{rf}^2 dS}$$

$$R_s = const \Rightarrow Q = \frac{\Gamma}{R_S}$$

**High *Q* corresponds to low $R_s$ !**

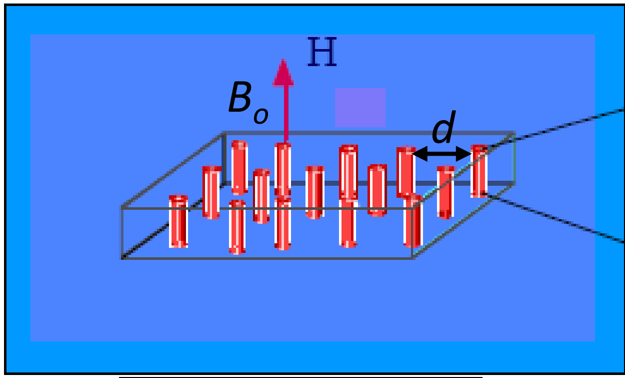
$$\Gamma = \frac{\mu_0 \omega_0 \int_V H^2 dV}{\int_S H^2 dS}$$

# Superconductor at low fields present a Surface Resistance lower than Copper at low temperatures



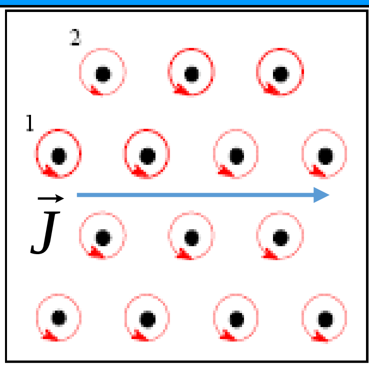
# Superconductors in magnetic fields: Abrikosov Vortices

Type II superconducting sample in a magnetic field




$$B_o = \frac{\phi_o}{d^2} = n\phi_o$$

$$B_{c2} = \frac{\phi_o}{\pi\xi^2}$$




$$\left|\vec{J} \times \vec{B}_o\right| \leq \alpha_c$$

**$\alpha_c$ = pinning force per unit volume**


In d.c. operation the «critical current» $\vec{J}_c$ is reached when $\left|\vec{J}_c \times \vec{B}_o\right| = \alpha_c$

If , $\vec{J} \geq \vec{J}_c$ vortex «flux flow» regime is activated, the sample dissipates power with an equivalent resistivity :

$$\rho_f = \rho_n \frac{B_o}{B_{c2}}$$

# R.f behavior for applied d.c. magnetic field $B_o >> B_{c1}$

**Gittleman and Rosenblum: - Phys Rev. Lett. 16, 734 (1966)**

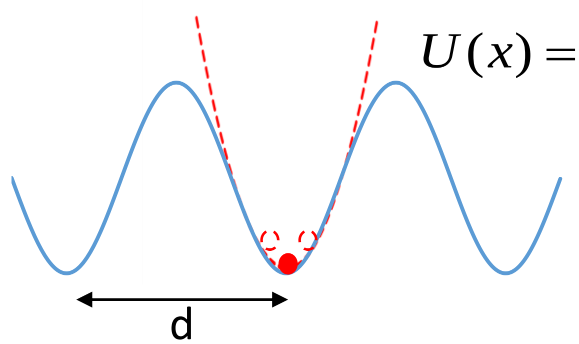
**- J. Appl. Phys. 39, 2617(1968)**

At $B_o >> B_{c1}$ repulsion forces between fluxon lines are higher in respect to the pinning forces. The fluxon array moves rigidly and feels a periodic force of the form :

$$f_p = -J_c \phi_o \sin\left(\frac{2\pi x}{d}\right)$$ (pinning force per unit lenght)

$$U(x) = \frac{1}{2} k x^2$$

$$(J_{rf} << J_c)$$




$$f_p = -kx$$

$$(k = \frac{2\pi}{d} J_c \phi_o)$$

# Motion equation for the fluxon lattice

$$m\ddot{x} + \eta\dot{x} + kx = J_{rf}\phi_o$$

$m$ : fluxon mass per unit lenght

$\eta = \dfrac{\phi_o B_{c2}}{\rho_n}$ :fluxon viscosity per unit lenght

$(m \cong 0)$ $\quad J_{rf} = J_{rfo}e^{i\omega t},\ \dot{x} = v = v_o e^{i\omega t}$

$$\eta v_o\left(1 - i\frac{\omega_o}{\omega}\right) = J_{rfo}\phi_o \quad \left(\omega_o = \frac{k}{\eta}\right)$$

$$v_o = \frac{J_{rfo}\phi_o}{\eta}\left(\frac{\omega^2}{\omega^2 + \omega_o^2} + i\frac{\omega\omega_o}{\omega^2 + \omega_o^2}\right)$$

# S. Calatroni, R.Vaglio : IEEE Trans. Superconductivity

$$\vec{J}_{rf} = (\sigma_1 - i\sigma_2)(\vec{E}_{rf} - \vec{v}\times\vec{B}_o)$$

( $\vec{v}\times\vec{B}_o$ is the «Lorentz field» )

$$\rho_{eff} = \frac{\vec{E}_{rf}}{\vec{J}_{rf}} = \frac{1}{\sigma_1 - i\sigma_2} + \frac{\vec{v}\times\vec{B}_o}{\vec{J}_{rf}} = \frac{1}{\sigma_1 - i\sigma_2} + \rho_n \frac{B_o}{B_{c2}}\left(\frac{\omega^2}{\omega^2+\omega_o^2} + i\frac{\omega\omega_o}{\omega^2+\omega_o^2}\right)$$

$\rho_{eff} = \rho_s + \rho_f$ : resistivity of a superconductor in the presence of an oscillating fluxon array

$$\rho_s = \frac{1}{\sigma_1 - i\sigma_2} \cong \frac{\sigma_1}{\sigma_2^2} + i\frac{1}{\sigma_2}$$

$(T < T_c/2\,,\ \sigma_1 << \sigma_2)$

$$\rho_f = \rho_n \frac{B_o}{B_{c2}}\left[\alpha(\omega) + i\beta(\omega)\right]$$

$$\alpha(\omega) = \frac{\omega^2}{\omega^2+\omega_o^2}\ ;\ \beta(\omega) = \frac{\omega\omega_o}{\omega^2+\omega_o^2}$$

$\omega >> \omega_o\,,\ \alpha = 1\ ,\ \beta = 0 \Rightarrow$ $\rho_f = \rho_n \frac{B_o}{B_{c2}}$ $\omega_o = \frac{k}{\eta}$ «depinning frequency»

( $\rho_f$ is the same as that of a superconductor in flux-flow regime )

$$Z_{sf} = (1+i)\sqrt{\frac{\mu_0\omega}{2}\rho_{eff}} = (1+i)R_n\sqrt{A+iB} \qquad (R_n = \sqrt{\frac{\mu_o\omega}{2}\rho_n}\ )$$

$$(R_{sf} = \mathrm{Re}\left|Z_{sf}\right| = R_n\,\mathrm{Re}\left|(1+i)\sqrt{A+iB}\right|)$$

$$R_{sf} = R_n\sqrt{\sqrt{A^2+B^2}-B}$$

$$A = \frac{\sigma_1/\sigma_n}{\left(\sigma_2/\sigma_n\right)^2} + \frac{B_o}{B_{c2}}\alpha(\omega)$$

$$B = \frac{1}{\sigma_2/\sigma_n} + \frac{B_o}{B_{c2}}\beta(\omega)$$

**General expression for the surface resistance of a superconductor in presence of a rigid vortex array, with the rf current perpendicular to the magnetic field**

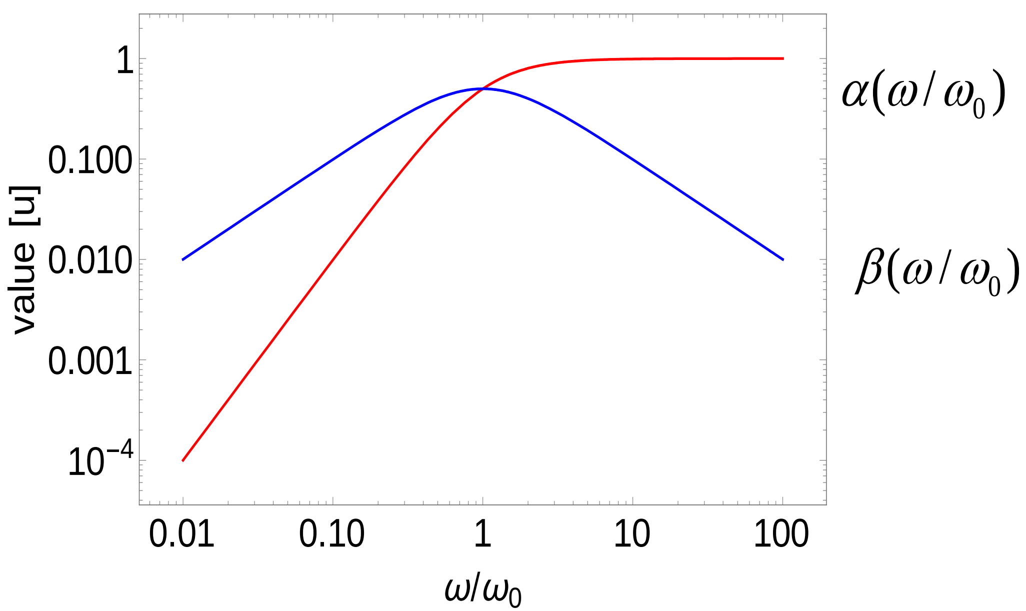
# Surface Impedance in the Large Field, High Frequency limit

**In large magnetic fields :**

$$R_{sf} = R_n \sqrt{\sqrt{A^2 + B^2} - B}$$

$$A = \frac{B_o}{B_{c2}} \alpha(\omega)$$

$$B = \frac{B_o}{B_{c2}} \beta(\omega)$$



# At high frequency :

$$\omega >> \omega_o \ , \alpha = 1 \ , \ \beta = 0$$

$$A = B_o / B_{c2}$$
$$B = 0$$

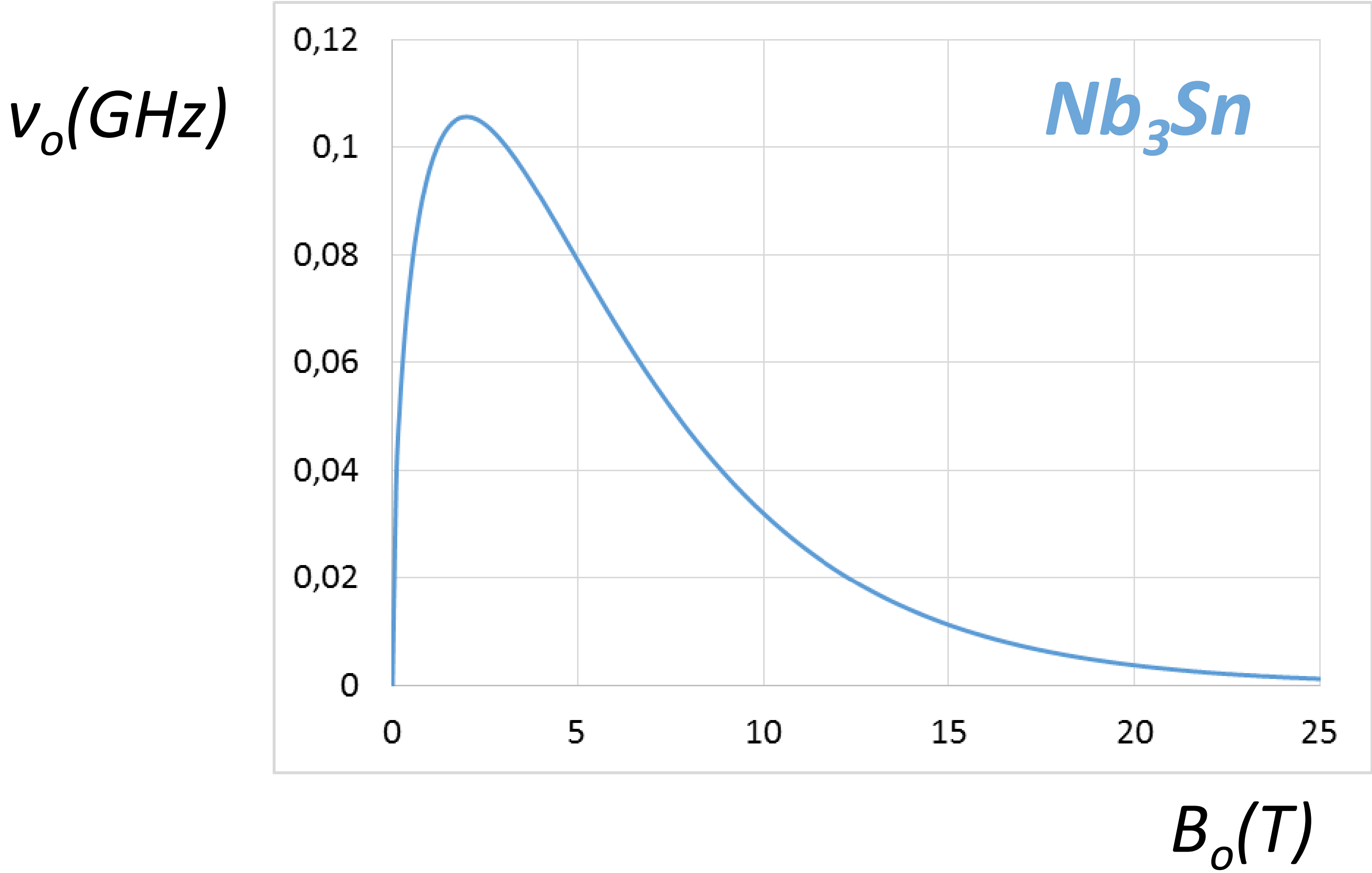
$$R_{sf} = R_n \sqrt{\frac{B_o}{B_{c2}}} = \sqrt{\frac{\mu_o \omega \rho_n B_o}{2B_{c2}}}$$

$$\omega_o = \frac{k}{\eta} = \frac{2\pi J_c \rho_n}{dB_{c2}} = 2\pi \frac{\sqrt{B_o} J_c(B_o)\rho_n}{\sqrt{\phi_o} B_{c2}}$$

# Relevant parameters of selected superconductors

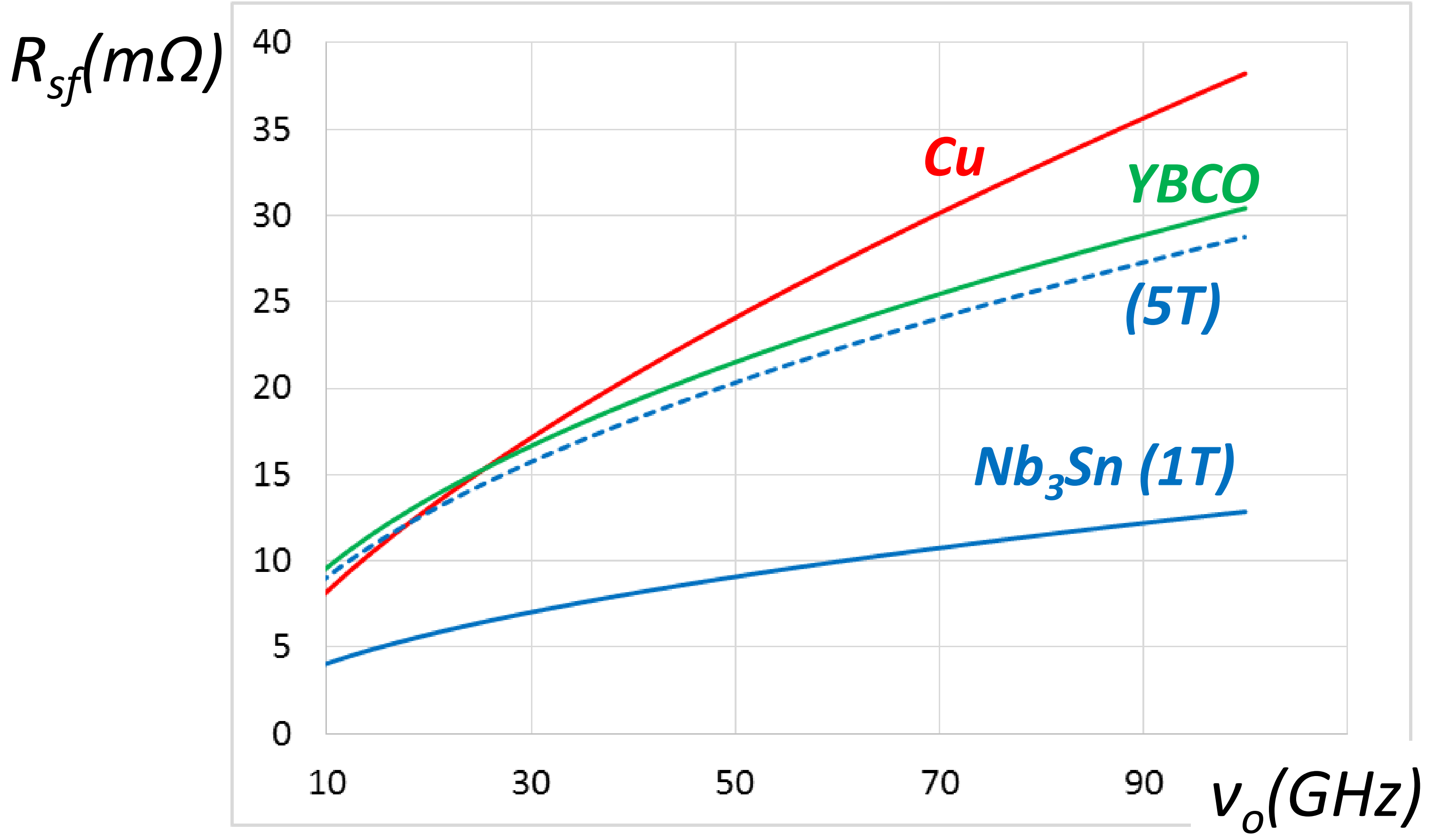
| *Superconductor* *(well oriented films)* | *$T_c$ (K)* | *$\rho_n$ (μΩcm)* | *$B_{c2}$ (T)* |
|---|---|---|---|
| **Nb** | **9.2** | **<0.5** | **0.25** |
| **$Nb_{45}Ti_{55}$** | **9.0** | **≈80** | **14** |
| **NbN** | **17.0** | **>10** | **30** |
| **$Nb_3Sn$** | **18.0** | **<1** | **25** |
| **YBCO** | **90** | **<40** | **>120** |

# Depinning Frequency vs Field




$$\nu_o = \frac{\omega_o}{2\pi} = \frac{\sqrt{B_o}\,J_c(B_o)\rho_n}{\sqrt{\phi_o}\,B_{c2}}$$

# Surface Resistance vs Frequency/Magnetic Field
## (field perpendicular to the superconductor film surface)

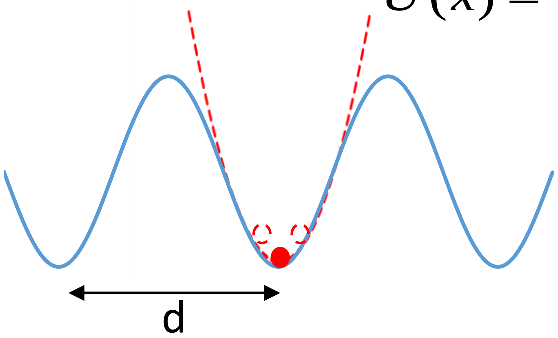



$$\nu >> \nu_o \quad ; \quad R_{sf} = \sqrt{\frac{\pi \mu_o \nu \rho_n B_o}{B_{c2}}}$$

# A further possible issue: Non-linear effects

$$U(x) = \frac{1}{2}kx^2 + \frac{1}{4}\Gamma x^4$$

$$(J_{rf} \leq J_c)$$




$$f_p = -(kx + \Gamma x^3)$$

$$\eta\dot{x} + kx + \Gamma x^3 = J_{rfo}e^{i\omega t}\phi_o$$ $$(m \cong 0)$$

Duffing approximation $(\Gamma \to 0)$

$$v = \dot{x} = \frac{J_{rfo}\phi_o}{\eta}e^{i\omega t} - \omega_o x - \frac{\Gamma}{\eta}x^3$$ , with $x = x_o e^{i\omega t}$ (as in the linear case)

$$v = v_{o1}e^{i\omega t} + v_{o3}e^{i3\omega t}$$ $$(v_{o3} \propto \Gamma)$$