

Axion phenomenology from Lattice QCD

C. Bonati

Dipartimento di Fisica e Astronomia & INFN, Firenze

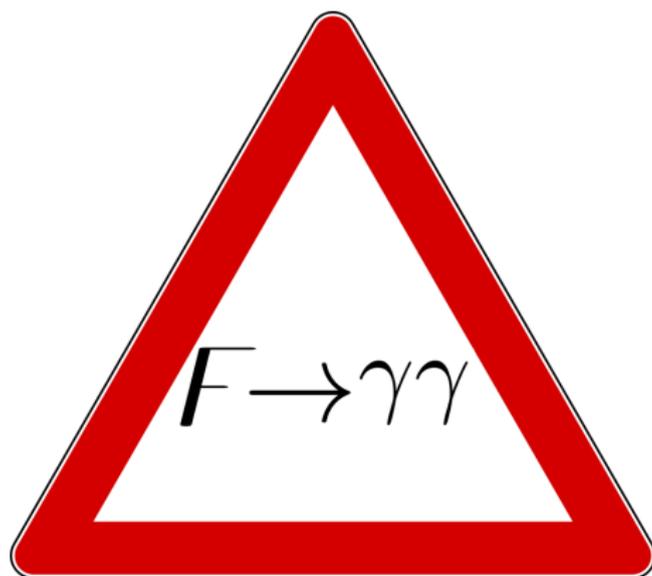
Workshop on Axion Physics and Experiments,
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Based on work in collaboration with M. D'Elia, M. Mariti, G. Martinelli,
M. Mesiti, F. Negro, F. Sanfilippo and G. Villadoro

Outline

- 1 QCD, strong CP and axions
- 2 How to constraint axion properties
- 3 Recent lattice works
- 4 Conclusions

Important premise: beware of theorists!



In theory one tends to use simplifying assumptions that often can only be verified *a posteriori* by comparing with experiments. Without experimental results something can get out of control.

Take home message: read the small print!

θ term and the strong CP problem

The most general QCD lagrangian compatible with the fundamental symmetries of the Standard Model is:

$$\mathcal{L}_{QCD} = -\frac{1}{4} G_{\mu\nu}^a G^{a\mu\nu} + \sum_f \bar{\psi}_f^a (iD_\mu^{ab} \gamma^\mu - m_f) \psi_f^b + \theta q(x)$$
$$q(x) = \frac{g^2}{64\pi^2} \epsilon_{\mu\nu\rho\sigma} G^{a\mu\nu} G^{a\rho\sigma} \quad (\sim \text{"E} \cdot \text{B"})$$

where $\theta \in [0, 2\pi)$ is a **dimensionless, RG invariant parameter**.

For $\theta \neq 0, \pi$ the QCD lagrangian is **not invariant under P and CP** (and for $\theta = \pi$ spontaneous CP symmetry breaking is expected).

Signal of CP violation: electric dipole moment. From neutron EDM measures one gets $|\theta| \lesssim 10^{-10}$ (in going from $|d_n|$ to $|\theta|$ one need the θ -polarizability, that is not well known, hence the \lesssim)

Why should $|\theta|$ be so small? **Strong CP problem**

Possible solutions of the strong CP problem

- 1 $m_u = 0$ (in this case θ can be removed by a change of variables)
- 2 promote CP to be a fundamental symmetry of SM and explain all the CP violation in SM by spontaneous symmetry breaking
- 3 promote θ to be a dynamical variable

Realization of mechanism 3: add to SM a pseudoscalar field “ a ” with coupling $\frac{a}{f_a}q(x)$ and **only derivative interactions**. Since the energy has a minimum at $\theta = 0$, a will acquire a VEV such that $\theta + \frac{\langle a \rangle}{f_a} = 0$.

Goldstone bosons have only derivatives couplings, so the simplest possibility is to think of “ a ” as the GB of some $U(1)$ axial symmetry (**Peccei-Quinn symmetry**). The effective low-energy lagrangian is thus

$$\mathcal{L} = \mathcal{L}_{QCD}^{\theta=0} + \frac{1}{2} \partial_\mu a \partial^\mu a + \frac{a(x)}{f_a} q(x) + \frac{1}{f_a} \left(\text{model dependent terms} \right)$$

Several models for UV completion exist and the most famous are KSVZ and DFSZ.

Axion mass and quartic coupling

θ dependence of the QCD vacuum and axion properties are two names for the same thing. Classical results (Di Vecchia, Veneziano 1980) for the θ dependence obtained in LO chiral perturbation theory give ($f_\pi \simeq 92 \text{ MeV}$)

$$m_a = \frac{\sqrt{m_u m_d}}{m_u + m_d} \frac{m_\pi f_\pi}{f_a}; \quad \lambda_a = - \left(\frac{m_a^2}{f_a^2} \right) \frac{m_u^2 - m_u m_d + m_d^2}{(m_u + m_d)^2}$$

For NLO corrections and some of the model dependent contributions see G. Grilli di Cortona, E. Hardy, J. Pardo Vega, G. Villadoro 1511.02867

$$m_a = 5.70(7) \mu\text{eV} \left(\frac{10^{12} \text{ GeV}}{f_a} \right).$$

Since all the couplings are proportional to $1/f_a$, absence of direct detection gives lower bounds on the value of f_a , that turns out to be huge, typical values being $f_a \gtrsim 10^8 \div 10^9 \text{ GeV}$.

Axions as dark matter

Since f_a is so large, axions are light, weakly interacting and natural candidates for dark matter

Cosmological sources of axions:

thermal production common to all particles, strongly dependent on dynamical details and on model dependent parameters

decay of topological objects both properties and existence of topological objects are model dependent
(if $U(1)_{PQ} \rightarrow \emptyset$ no top. obj. at all)

misalignment mechanism typical of axions, almost (see later) model independent

Overclosure bound: axion density \leq dark matter density
stronger form
axion density from misalignment \leq dark matter density

Misalignment mechanism

The EoM of the axion is ($H = \dot{R}/R$ Hubble “constant”)

$$\ddot{a}(t) + 3H(t)\dot{a}(t) + m_a^2(T)a(t) = 0$$

for $T \gg \Lambda_{QCD}$ the second term dominates and we have $a(t) \sim \text{const}$ (assuming $\dot{a} \ll H$ initially);

when $m_a \sim 3H$ the field start oscillating around the minimum;

when $m_a \gg H$ a WKB-like approximation can be used

$$a(t) \sim A(t) \cos \int^t m_a(\tilde{t}) d\tilde{t}; \quad \frac{d}{dt}(m_a A^2) = -3H(t)(m_a A^2)$$

and thus the **number of axions in the comoving frame** $N_a = m_a A^2 / R^3$ is **conserved in the late evolution**.

The temperature for which $m_a(T) = 3H(T)$ is called the oscillation temperature T_{osc} . All the nontrivial dynamics happens for $T \simeq T_{osc}$.

Misalignment mechanism

The nice feature of the overclosure bound for misalignment is that it provides an upper bound for f_a .

What we need to estimate this upper bound?

- 1 $H(t)$: we need to solve the Friedmann equations, thus we need the equation of state (Lattice QCD can help)
- 2 $m_a(T)$: we need the “temperature dependence” of the axion mass (Lattice QCD can help)
- 3 we need the initial value of the misalignment angle
(nobody can help)

For the last point two possibilities exist depending on when the $U(1)_{PQ}$ symmetry breaking takes place

- **before inflation**: a single angle has to be used in the initial conditions and its value is a new parameter
- **after inflation**: we can expect the initial angle not to be the same in different regions and we average on it

Lattice studies of $\chi(T) = f_a^2 m_a^2(T)$

Recent studies aimed at phenomenology are:

- A. Trunin, F. Burger, E. M. Ilgenfritz, M. P. Lombardo and M. Müller-Preussker,
J. Phys. Conf. Ser. **668**, 012123 (2016) [arXiv:1510.02265 [hep-lat]]
(and more recent unpublished results, see QM2017 conference)
- C. Bonati, M. D'Elia, M. Mariti, G. Martinelli, M. Mesiti, F. Negro, F. Sanfilippo and G. Villadoro,
JHEP **1603**, 155 (2016) [arXiv:1512.06746 [hep-lat]].
- P. Petreczky, H. P. Schadler and S. Sharma,
Phys. Lett. B **762**, 498 (2016) [arXiv:1606.03145 [hep-lat]].
- Sz. Borsanyi, Z. Fodor, K. H. Kampert, S. D. Katz, T. Kawanai, T. G. Kovacs, S. W. Mages, A. Pasztor, F. Pittler, J. Redondo, A. Ringwald, K. K. Szabo,
Nature **539** no.7627, 69 (2016) [arXiv:1606.07494 [hep-lat]].

Lattice results

All lattice studies agree on the following ($T_c \simeq 155$ MeV)

- for $T \lesssim T_c$ we have $\chi(T) \simeq \chi(T=0) = [75.5(5) \text{ MeV}]^2$
- for T slightly larger than T_c a transient behaviour is present
- for larger T the behaviour is well described by $\chi(T) = b/T^\alpha$

There is no agreement (so far) on the values of b, α . In particular α ranges from 3 to 8. This is not as tremendous as it could seem but still introduce some almost $\mathcal{O}(10)$ discrepancies between the final results for f_a .

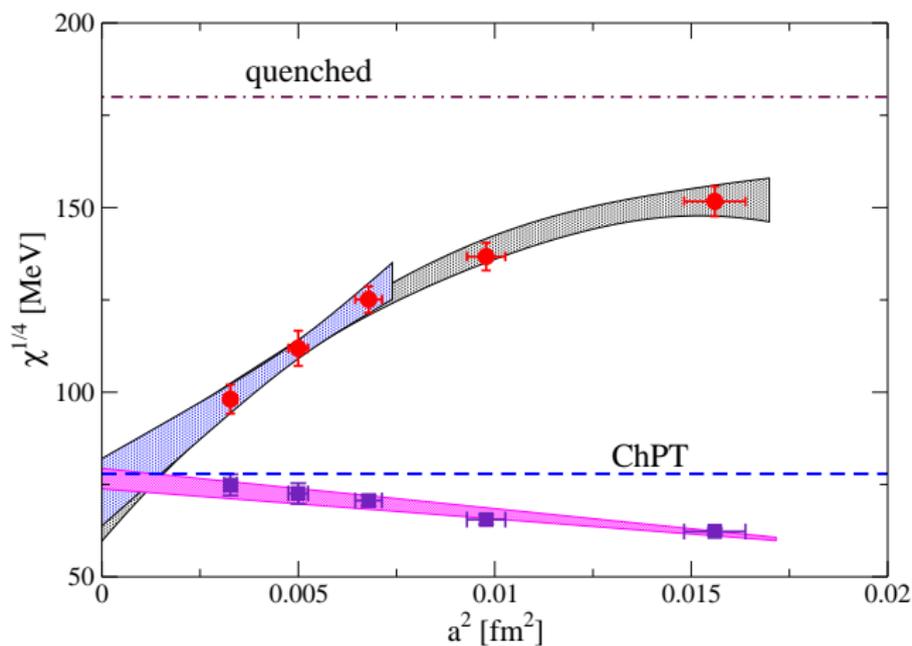
In Lattice QCD $\chi(T)$ is extracted by estimating

$$\chi = \frac{\langle Q^2 \rangle}{V_4}$$

by Markov chain Monte Carlo methods, where Q is the topological charge and V_4 the lattice volume. Q is in the continuum an integer variable, on the lattice is almost integer due to discretization errors.

Problem 1

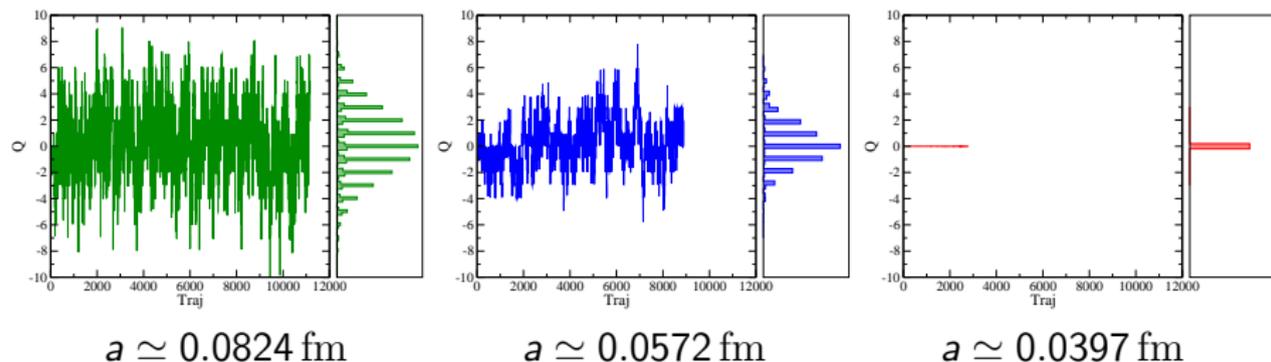
Example of (slow) convergence to the continuum
for the $T = 0$ case (from [Bonati et al.](#))



For $T = 0$ we have a good understanding of the physics from chiral perturbation theory and the convergence can be improved (violet points/strip).

Problem 2

Example of the “freezing” problem at $T = 0$ (from [Bonati et al.](#))



From left to right the autocorrelation time of Q grows until the algorithm samples the distribution $P(Q)$ so badly that it is stuck in the $Q = 0$ sector. In this situation is clearly impossible to reliably estimate χ and we have a lower bound for the usable lattice spacings.

Problem 3

At high temperature states with $|Q| > 0$ are suppressed **in a finite volume**:
 $\lim_{T \rightarrow \infty} \chi(T) \rightarrow 0$ and the size of the typical fluctuation is $\delta \sim \sqrt{\chi V_4}$.

To estimate $\chi(T)$ very large lattices are needed and even in that case almost only the sectors with $|Q| \leq 1$ are explored.

We know that for very large temperatures the distribution of Q is well approximated by a Poisson distribution (i.e. we know everything once we know $P(Q = 1)$), but we do not know the lowest temperature for which this approximation is quantitative accurate.

It is difficult to understand if a distribution is a Poissonian or not by using only $Q = 0$ and $Q = 1$ data.

Terminology: “Poissonian Q distribution” = “dilute instanton gas approx.”

Exit strategies

Slow continuum convergence at finite temperature:

- normalize with the $T = 0$ result, **Bonati et al.** (maybe not enough...).
- introduce *a posteriori* a corrective bias in the analysis **Borsanyi et al.** (“eigenvalue reweighting”, instanton gas inspired, is it reliable?).

Freezing problem:

- avoid it**
- by using coarse enough lattice spacings
 - by using different sampling strategies (open boundary conditions, metadynamics, sub-volume estimates, non-orientable manifolds...).

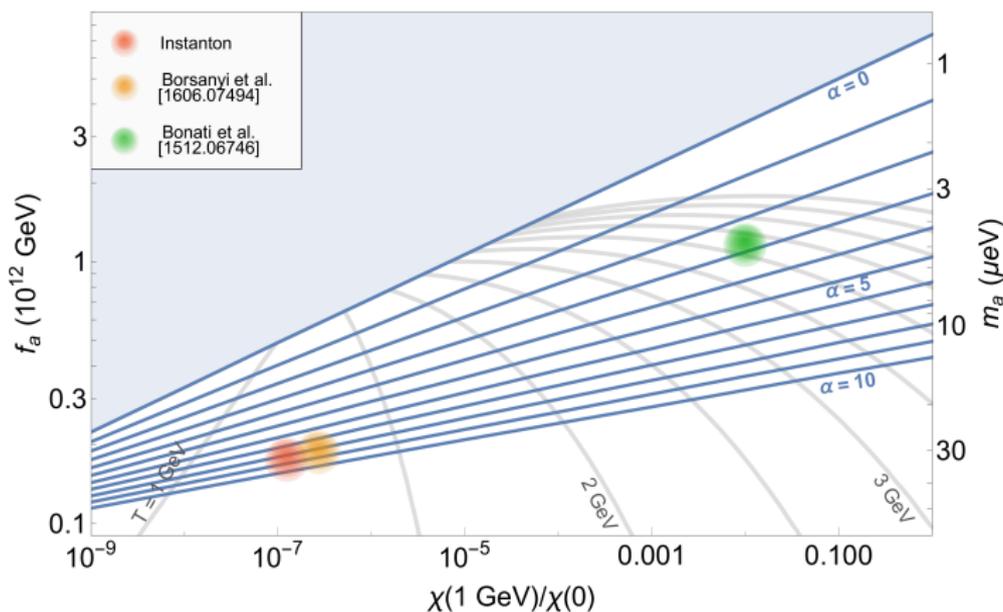
Several tests in toy models but, by now, no real QCD test.

- live with it** by using fixed Q methods (computationally very expensive) or by “thermodynamical integration” **Borsanyi et al.** (again a Poissonian instanton-like behaviour is assumed).

The final results

upper bound for f_a

lower bound for m_a



from [Grilli di Cortona 2016 PhD thesis @ SISSA](#)

Results by other groups come quite close to the ones shown in figure.

Conclusions

Axions are becoming more and more popular as dark matter candidates and Lattice QCD can help in providing bounds for the axion mass and couplings.*

Future works will have to carefully assess the reliability of the procedures adopted and check for possible systematics, in order to provide a solid final result for $\chi(T)$.

*if $U(1)_{PQ}$ breaks after inflation

Thank you for your attention!

Backup slides with something more

Model dependent interactions

$$\mathcal{L} = \mathcal{L}_{QCD}^{\theta=0} + \frac{1}{2} \partial_\mu a \partial^\mu a + \frac{a(x)}{f_a} q(x) + \\ + \frac{1}{4} g_{a\gamma\gamma}^0 a F_{\mu\nu} \tilde{F}^{\mu\nu} + \frac{1}{2f_a} \partial_\mu a \sum_{f=u,d,\dots} c_f^0 \bar{\psi}_f \gamma^\mu \gamma_5 \psi_f$$

where

$$g_{a\gamma\gamma}^0 = \frac{\alpha_{em}}{2\pi f_a} \frac{E}{N} \quad \frac{E}{N} = \frac{\text{em anomaly}}{\text{color anomaly}}$$

The coefficients E/N and c_f^0 are all model dependent.

The low-energy effective $a\gamma\gamma$ interaction contains also a strongly induced model independent electromagnetic term and $g_{a\gamma\gamma}$ is given by

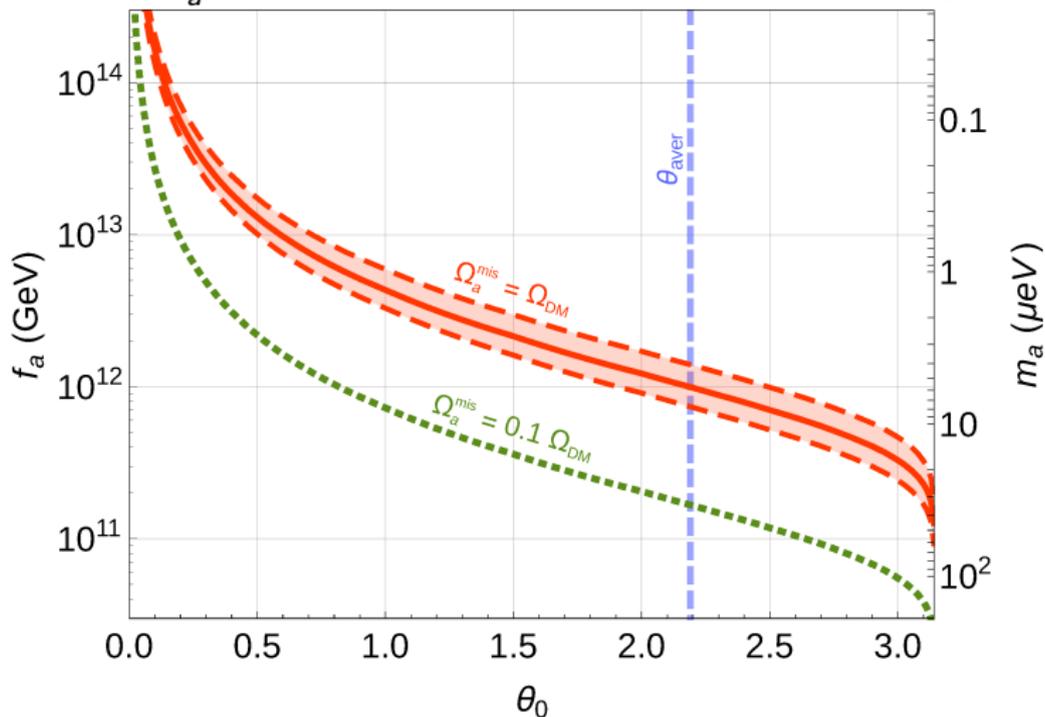
$$g_{a\gamma\gamma} = \frac{\alpha_{em}}{2\pi f_a} \left(\frac{E}{N} - \frac{2}{3} \frac{4m_d + m_u}{m_d + m_u} + \dots \right)$$

from Borsanyi et al.

All DM from misalignment	$m_a = 28(2)\mu eV$
50% from misalignment	$m_a = 40(4)\mu eV$
1% from misalignment	$m_a = 1500\mu eV$

upper bound for f_a

lower bound for m_a



from Bonati et al.