The QCD axion, precisely.

Giovanni Villadoro

The Abdus Salam
International Centre
for Theoretical Physics

based on:

1511.02867 [JHEP 1601, 034]

G. Grilli di Cortona
E. Hardy
J. Pardo Vega

1512.06746 [JHEP 1603., 155]

C.Bonati
M.D'Elia
M.Mariti
G.Martinelli
M.Mesiti
F.Negro
F.Sanfilippo

the problem

The Strong CP problem

$$\delta\mathcal{L} = \frac{\theta_0}{32\pi^2} G\tilde{G} \qquad\qquad \theta = \theta_0 + \arg\det M_q$$

The Strong CP problem

$$\delta\mathcal{L} = \frac{\theta_0}{32\pi^2} G\tilde{G} \qquad\qquad \theta = \theta_0 + \arg\det M_q$$

neutron EDM

$$d_n < 2.9 \cdot 10^{-26}\, e\, cm = 1.5 \cdot 10^{-12}\, e\, \mathrm{GeV}^{-1}$$

The Strong CP problem

$$\delta\mathcal{L} = \frac{\theta_0}{32\pi^2} G\tilde{G} \qquad\qquad \theta = \theta_0 + \arg\det M_q$$

neutron EDM

$$d_n < 2.9 \cdot 10^{-26}\, e\, cm = 1.5 \cdot 10^{-12}\, e\, \mathrm{GeV}^{-1}$$

$$\frac{g}{m_n} \bar{n}_L F_{\mu\nu} \gamma^{\mu\nu} n_R + h.c.$$

The Strong CP problem

$$\delta\mathcal{L} = \frac{\theta_0}{32\pi^2} G\tilde{G} \qquad \theta = \theta_0 + \arg\det M_q$$

neutron EDM

$$d_n < 2.9 \cdot 10^{-26}\, e\, cm = 1.5 \cdot 10^{-12}\, e\, \mathrm{GeV}^{-1}$$

$$\left(1 + c\,\frac{m_q}{m_n} e^{i\theta}\right) \frac{g}{m_n} \bar{n}_L F_{\mu\nu} \gamma^{\mu\nu} n_R + h.c.$$

The Strong CP problem

$$\delta\mathcal{L} = \frac{\theta_0}{32\pi^2} G\tilde{G} \qquad\qquad \theta = \theta_0 + \arg\det M_q$$

neutron EDM

$$d_n < 2.9 \cdot 10^{-26}\, e\, cm = 1.5 \cdot 10^{-12}\, e\, \mathrm{GeV}^{-1}$$

$$\left(1 + c\,\frac{m_q}{m_n} e^{i\theta}\right) \frac{g}{m_n} \bar{n}_L F_{\mu\nu} \gamma^{\mu\nu} n_R + h.c.$$

$$\boxed{10^{-2} \div 10^{-3}}$$

The Strong CP problem

$$\delta\mathcal{L} = \frac{\theta_0}{32\pi^2} G\tilde{G} \qquad\qquad \theta = \theta_0 + \arg\det M_q$$

neutron EDM

$$d_n < 2.9 \cdot 10^{-26}\, e\, cm = 1.5 \cdot 10^{-12}\, e\, \mathrm{GeV}^{-1}$$

$$\left(1 + c\,\frac{m_q}{m_n} e^{i\theta}\right)\frac{g}{m_n}\bar{n}_L F_{\mu\nu}\gamma^{\mu\nu} n_R + h.c.$$

$$\boxed{10^{-2} \div 10^{-3}} \qquad \boxed{e\,\mathrm{GeV}^{-1}} \qquad \Longrightarrow \qquad \theta \lesssim 10^{-10}$$

the axion solution

the QCD axion: *what it is*

$$\mathcal{L}_{SM} + \frac{1}{2}(\partial_\mu a)^2 + \frac{a}{f_a}\frac{\alpha_s}{8\pi}G\tilde{G}$$

the QCD axion: *what it is*

$$a \to a + \delta_{\mathrm{PQ}}$$

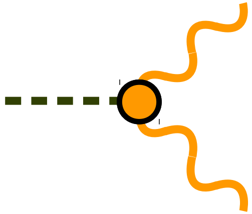
$$\mathcal{L}_{SM} + \frac{1}{2}(\partial_\mu a)^2 + \frac{a}{f_a}\frac{\alpha_s}{8\pi}G\tilde{G}$$

the QCD axion: *what it is*

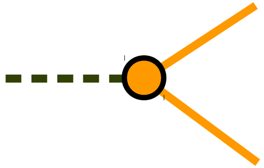
$$a \to a + \delta_{\mathrm{PQ}}$$

$$\mathcal{L}_{SM} + \frac{1}{2}(\partial_\mu a)^2 + \frac{a}{f_a}\frac{\alpha_s}{8\pi}G\tilde{G}$$

$$\frac{1}{4}\, a\, g^0_{a\gamma\gamma} F_{\mu\nu}\tilde{F}^{\mu\nu}$$



$$c^0_q \bar{q}\gamma^\mu\gamma_5 q\,\frac{\partial_\mu a}{2 f_a}$$

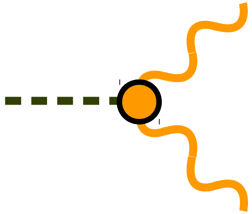


the QCD axion: *what it is*

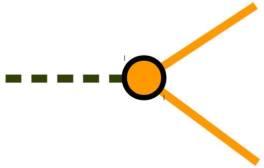
$$a \to a + \delta_{\mathrm{PQ}}$$

$$\mathcal{L}_{SM} + \frac{1}{2}(\partial_\mu a)^2 + \frac{a}{f_a}\frac{\alpha_s}{8\pi}G\tilde{G}$$

$$\frac{1}{4}\, a\, g^0_{a\gamma\gamma} F_{\mu\nu}\tilde{F}^{\mu\nu}$$



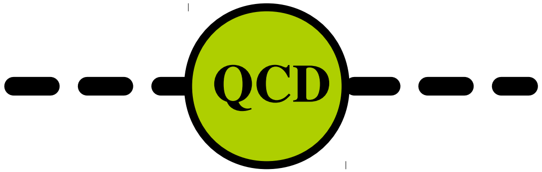
$$c^0_q \bar{q}\gamma^\mu\gamma_5 q\, \frac{\partial_\mu a}{2f_a}$$



$$\bar{q}_L M_q e^{iQ_a a/f_a} q_R + h.c.$$

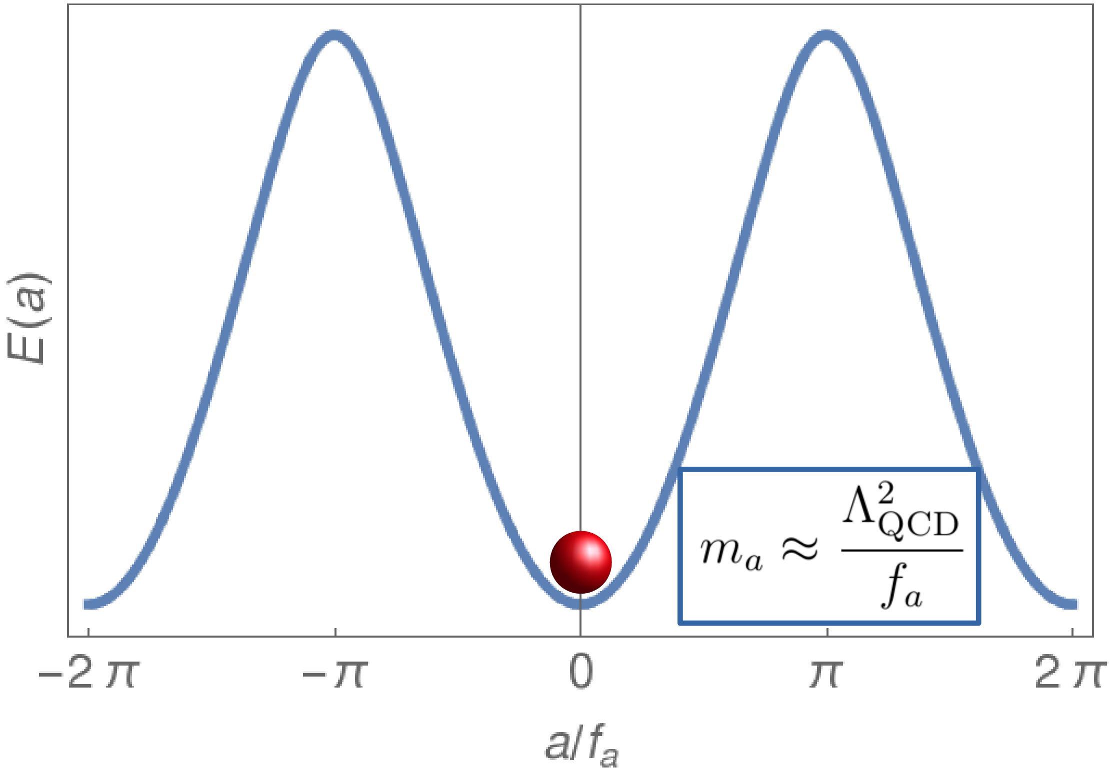
the QCD axion: *how it solves the problem*

$$\boxed{\frac{a}{f_a}\frac{\alpha_s}{8\pi}G_{\mu\nu}\tilde{G}^{\mu\nu}}$$



$$
\begin{aligned}
e^{-V_4 E(\theta)} &= \int \delta[\phi] e^{-S_0 + i\theta Q} \\
&= \left| \int \delta[\phi] e^{-S_0 + i\theta Q} \right| \\
&\leq \int \delta[\phi] \left| e^{-S_0 + i\theta Q} \right| \\
&= e^{-V_4 E(0)}
\end{aligned}
$$

Vafa Witten '84



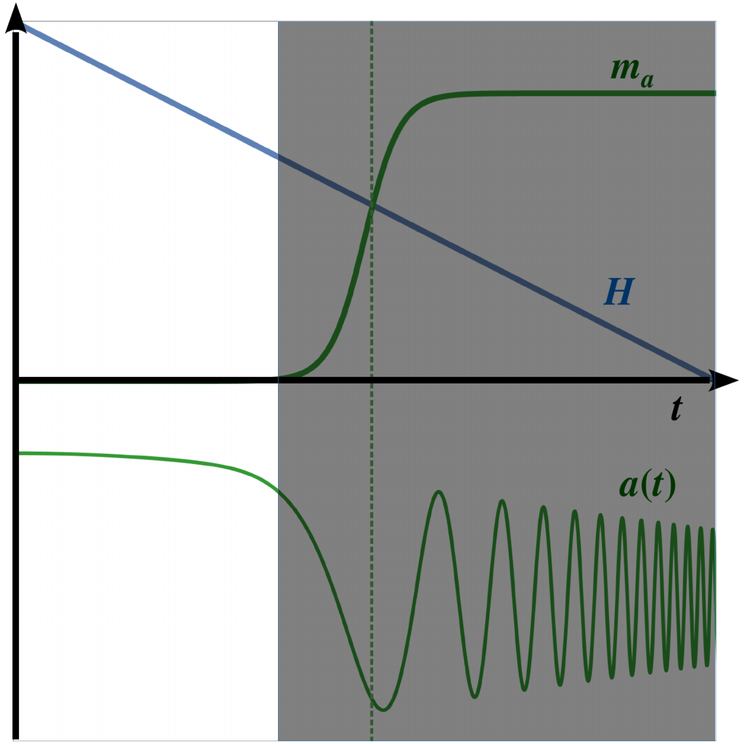
2 birds with 1 stone

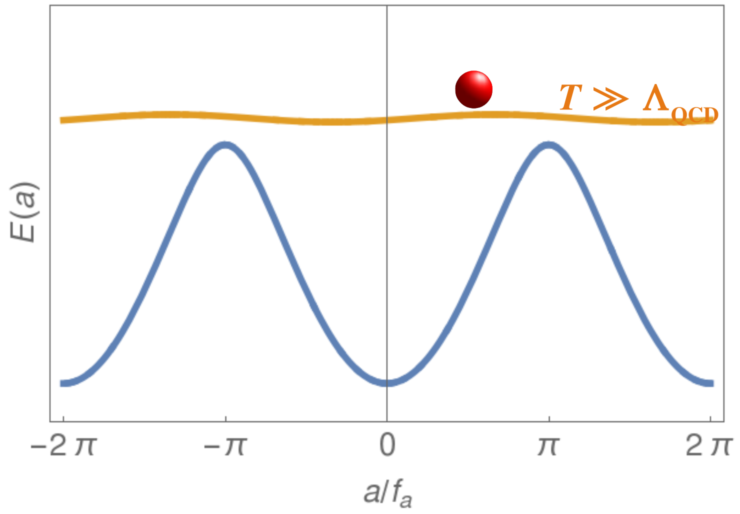
the QCD axion: *relic abundance*

$$\ddot{a} + 3H\dot{a} + m_a^2 a = 0$$

the QCD axion: *relic abundance*

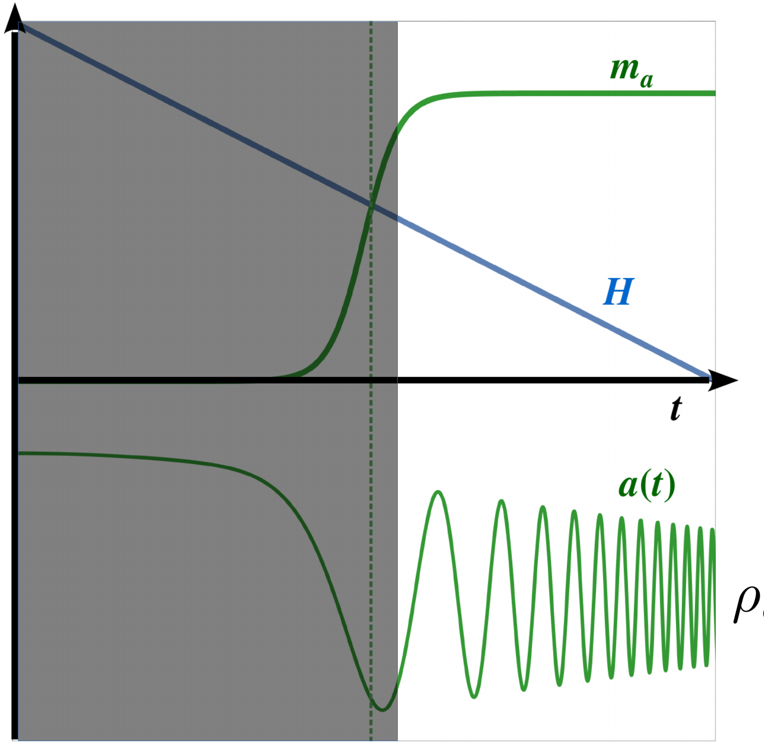
$$\ddot{a} + 3H\dot{a} \boxed{+ m_a^2 a} = 0$$



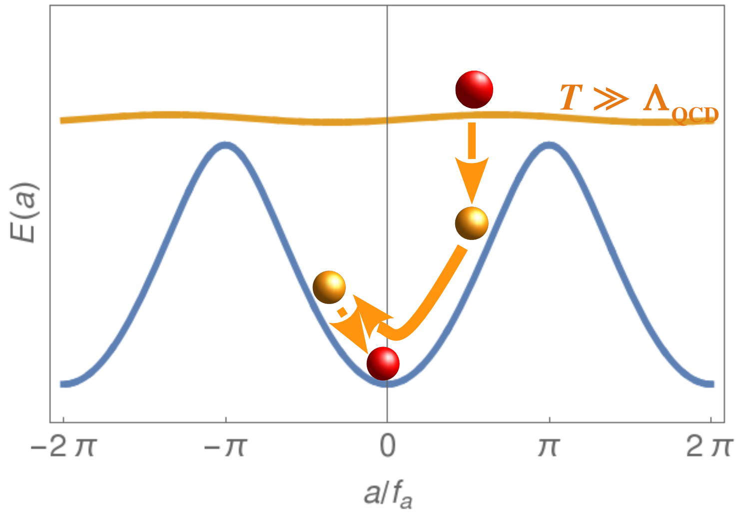


the QCD axion: *relic abundance*

$$\ddot{a} + 3H\dot{a} + m_a^2 a = 0$$

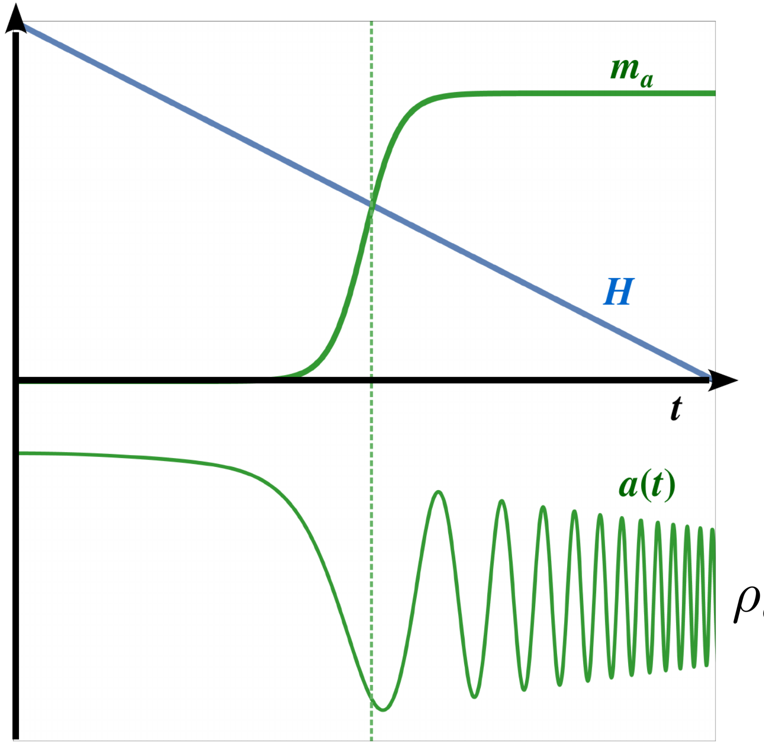


$$\rho_a = m_a^2 a^2$$

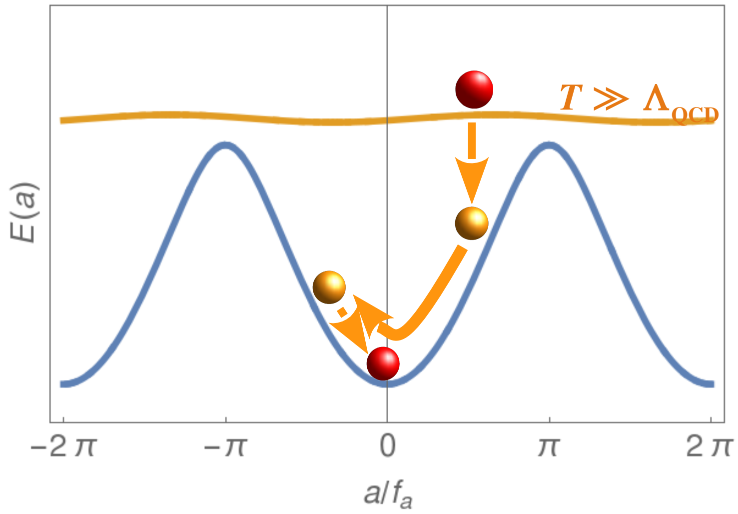


the QCD axion: *relic abundance*

$$\ddot{a} + 3H\dot{a} + m_a^2 a = 0$$



$$\rho_a = m_a^2 a^2$$



$$\boxed{\Omega_a \approx \Omega_{DM}\, k\, \theta_0^2 \left[\frac{f_a}{10^{12}\ \mathrm{GeV}}\right]^{1+\epsilon}}$$

the QCD axion: *relic abundance*

$f_a < \max\{H_I, T_R\}$	$f_a > \max\{H_I, T_R\}$

the QCD axion: *relic abundance*

$f_a < \max\{H_I, T_R\}$	$f_a > \max\{H_I, T_R\}$
$a(t_0) = \text{random}$	
misalignment contribution fixed $$\theta_0^2 \approx \frac{\langle a^2 \rangle}{f_a^2} \approx (2.2)^2$$	

the QCD axion: *relic abundance*

$f_a < \max\{H_I, T_R\}$	$f_a > \max\{H_I, T_R\}$
$a(t_0) = \text{random}$	

misalignment contribution fixed

$$\theta_0^2 \approx \frac{\langle a^2 \rangle}{f_a^2} \approx (2.2)^2$$

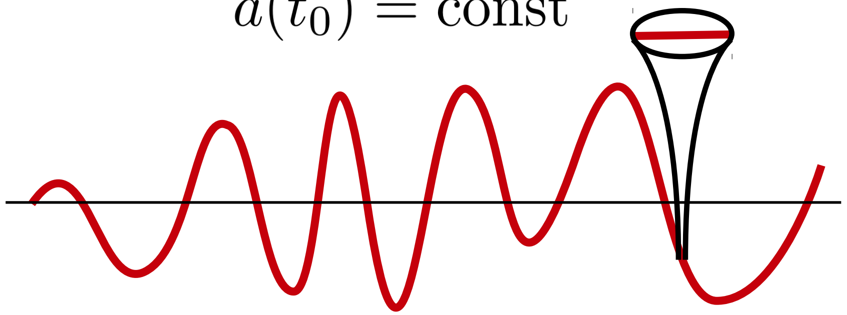
extra axion from strings + domain walls
large uncertainties

the QCD axion: *relic abundance*

$f_a < \max\{H_I, T_R\}$	$f_a > \max\{H_I, T_R\}$
$a(t_0) = \text{random}$	
misalignment contribution fixed $\theta_0^2 \approx \frac{\langle a^2 \rangle}{f_a^2} \approx (2.2)^2$	
extra axion from strings + domain walls large uncertainties	
$\Omega_a > \Omega_a^{mis} \quad \Leftrightarrow \quad f_a < f_a^{mis} \sim 10^{12}\ \text{GeV}$	

the QCD axion: *relic abundance*

$f_a < \max\{H_I, T_R\}$	$f_a > \max\{H_I, T_R\}$
$a(t_0) = \text{random}$	$a(t_0) = \text{const}$



misalignment contribution fixed

$$\theta_0^2 \approx \frac{\langle a^2 \rangle}{f_a^2} \approx (2.2)^2$$

extra axion from strings + domain walls
large uncertainties

$$\Omega_a > \Omega_a^{mis} \quad \Leftrightarrow \quad f_a < f_a^{mis} \sim 10^{12}\ \text{GeV}$$

the QCD axion: *relic abundance*

$f_a < \max\{H_I, T_R\}$	$f_a > \max\{H_I, T_R\}$
$a(t_0) = \text{random}$	$a(t_0) = \text{const}$

$a(t_0) = \text{random}$

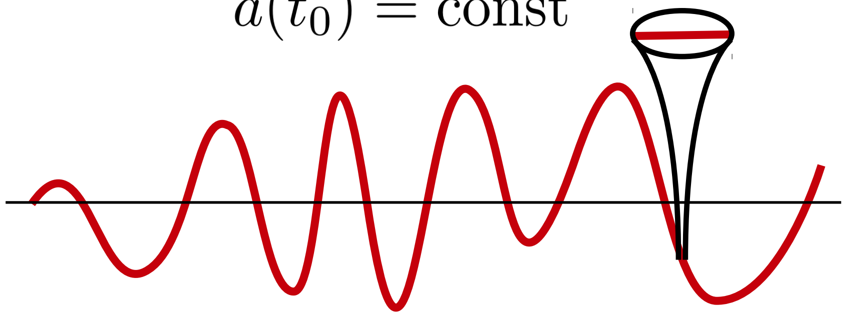
misalignment contribution fixed

$$\theta_0^2 \approx \frac{\langle a^2 \rangle}{f_a^2} \approx (2.2)^2$$

extra axion from strings + domain walls
large uncertainties

$$\Omega_a > \Omega_a^{mis} \quad \Leftrightarrow \quad f_a < f_a^{mis} \sim 10^{12}\ \text{GeV}$$

$a(t_0) = \text{const}$



only contribution from misalignment
but not calculable

the QCD axion: *relic abundance*

$f_a < \max\{H_I, T_R\}$

$$a(t_0) = \text{random}$$

misalignment contribution fixed

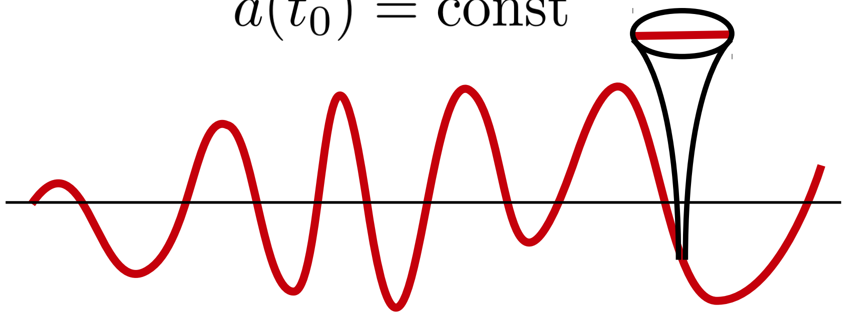
$$\theta_0^2 \approx \frac{\langle a^2 \rangle}{f_a^2} \approx (2.2)^2$$

extra axion from strings + domain walls
large uncertainties

$$\Omega_a > \Omega_a^{mis} \quad \Leftrightarrow \quad f_a < f_a^{mis} \sim 10^{12}\ \text{GeV}$$

$f_a > \max\{H_I, T_R\}$

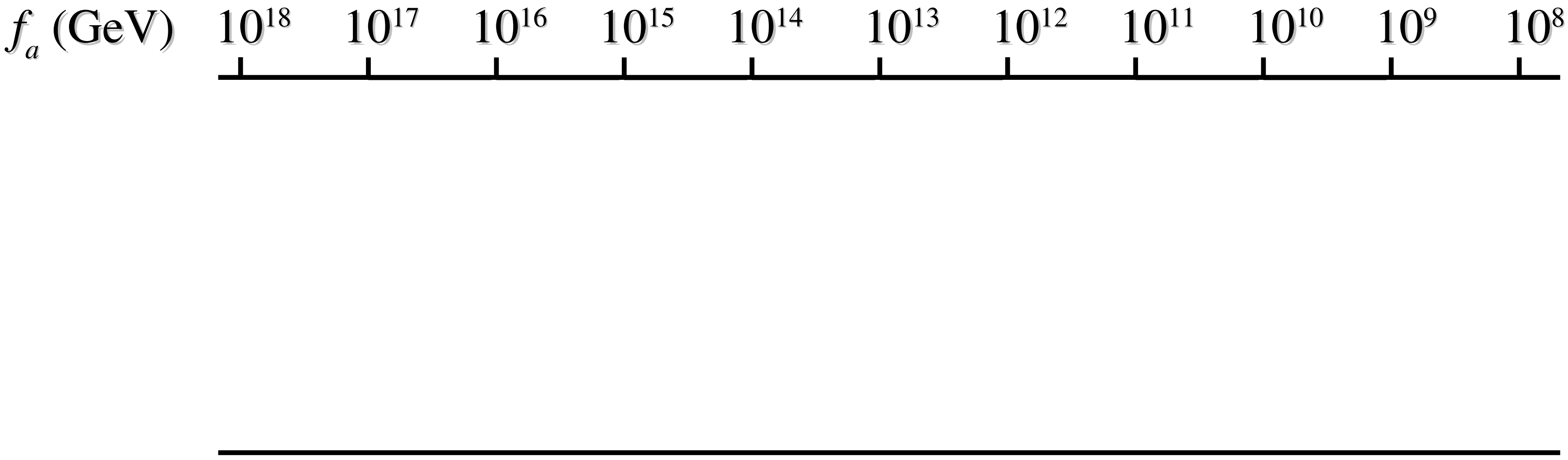
$$a(t_0) = \text{const}$$

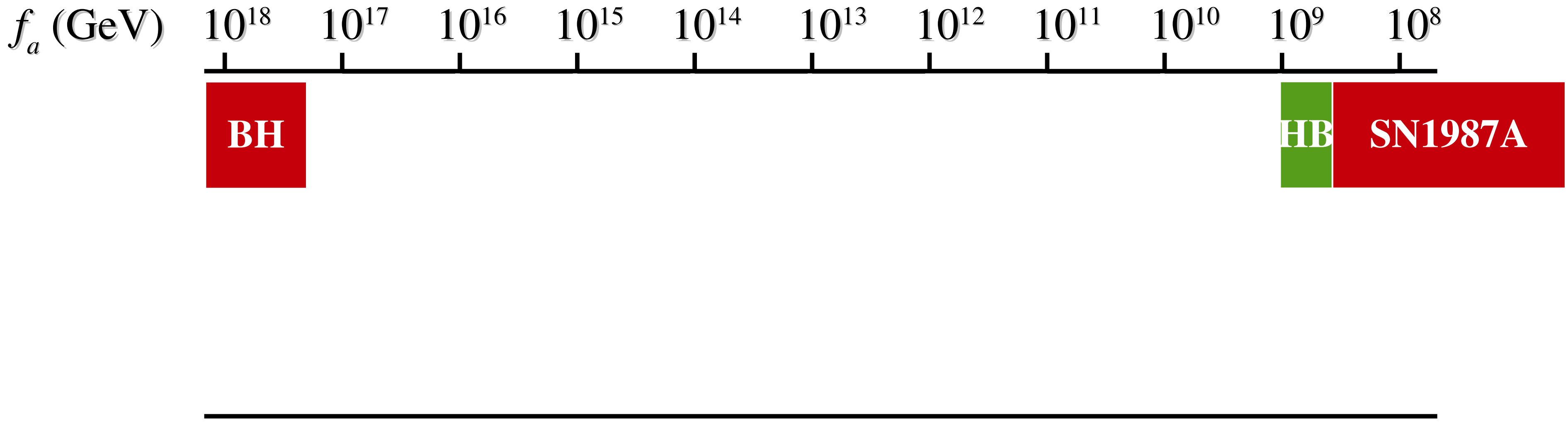


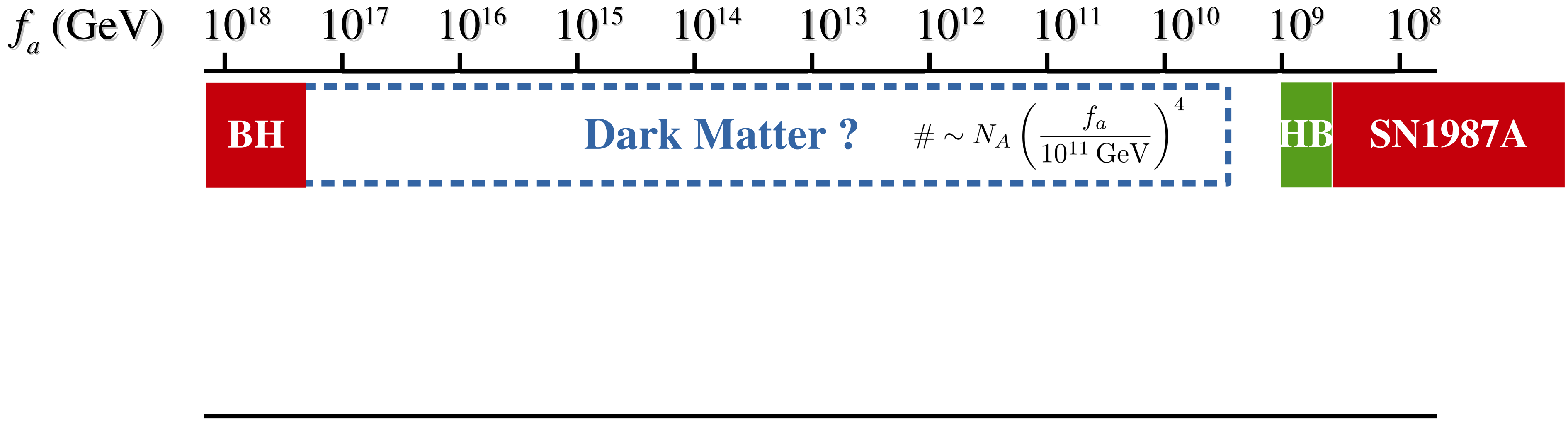
only contribution from misalignment
but not calculable

$$\theta_0 < \pi \quad \Leftrightarrow \quad f_a \gtrsim 10^{10}\ \text{GeV}$$

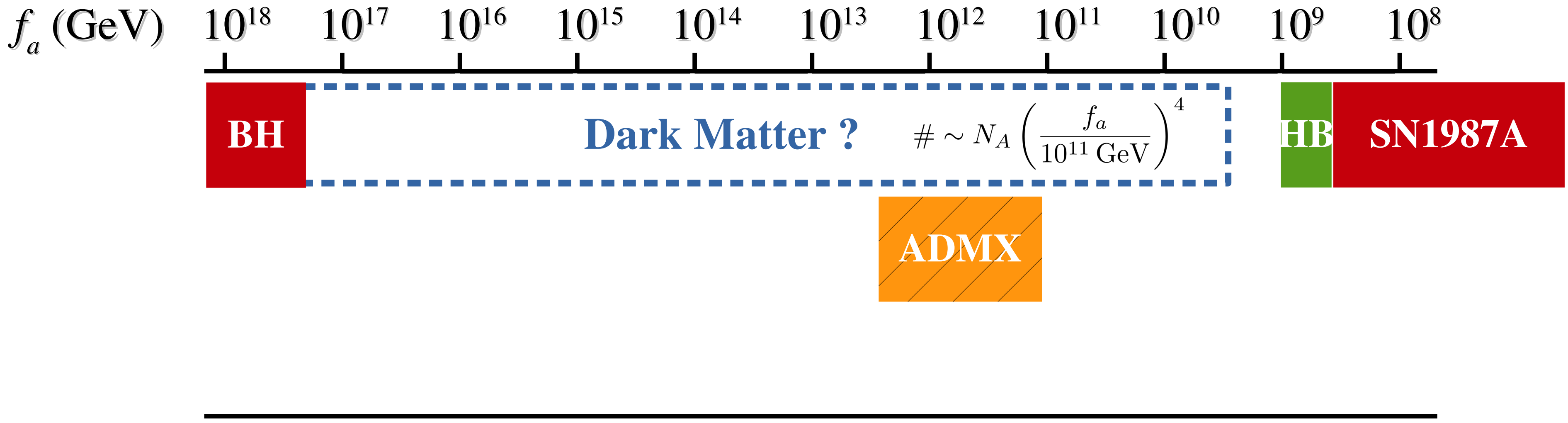
axion hunting


f_a (GeV)
10^{18}
10^{17}
10^{16}
10^{15}
10^{14}
10^{13}
10^{12}
10^{11}
10^{10}
10^{9}
10^{8}

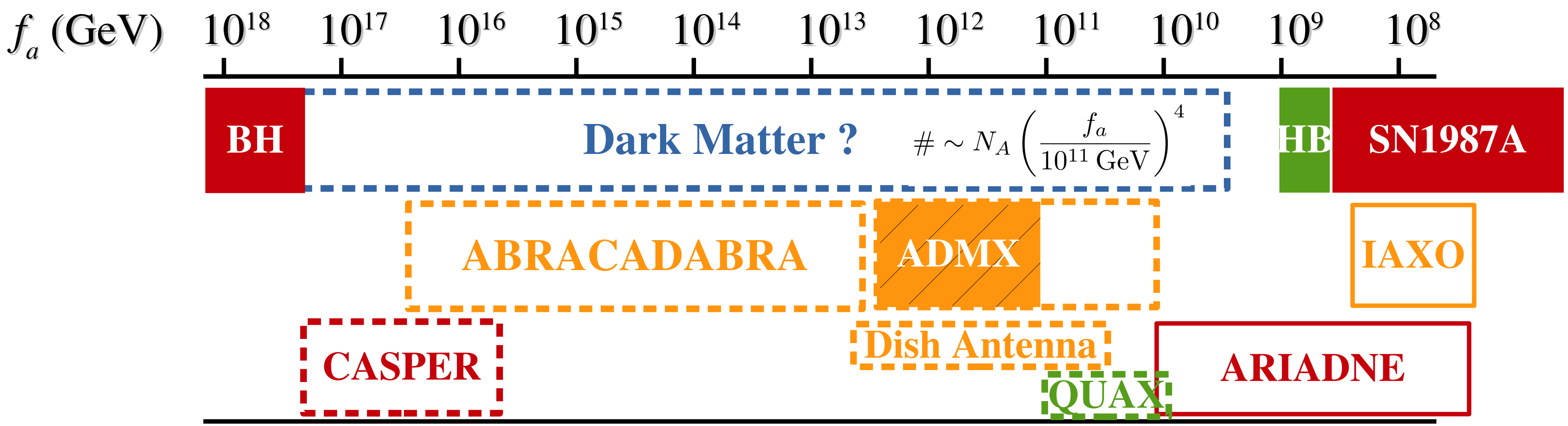

f_a (GeV)
10^{18} 10^{17} 10^{16} 10^{15} 10^{14} 10^{13} 10^{12} 10^{11} 10^{10} 10^{9} 10^{8}
BH
HB
SN1987A



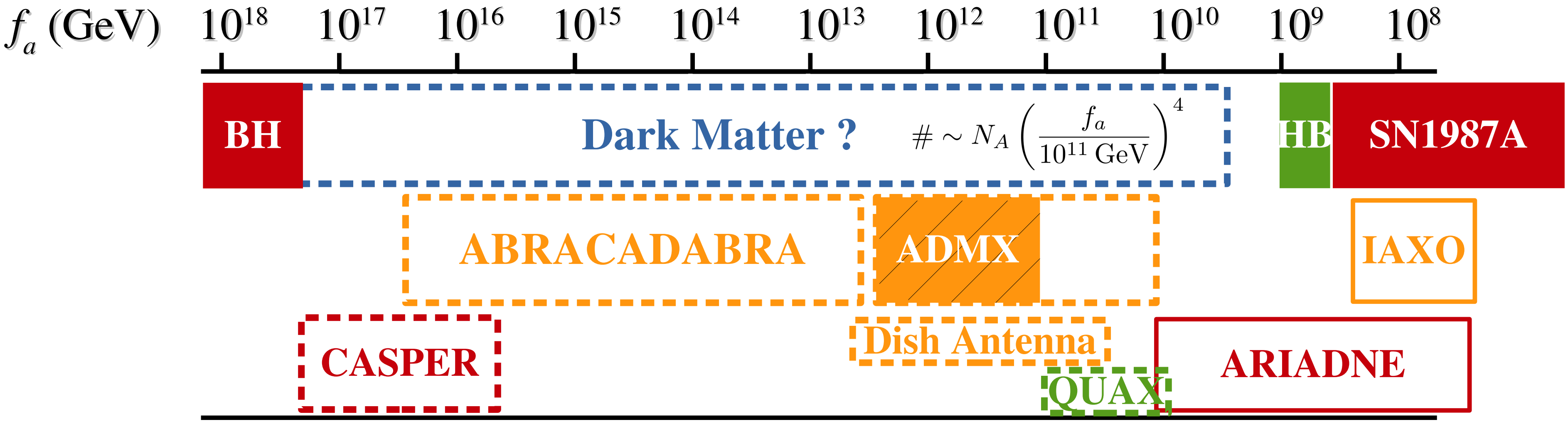
f_a (GeV)
10^{18} 10^{17} 10^{16} 10^{15} 10^{14} 10^{13} 10^{12} 10^{11} 10^{10} 10^{9} 10^{8}
BH
Dark Matter ?
$\# \sim N_A \left(\frac{f_a}{10^{11}\,\mathrm{GeV}} \right)^4$
HB
SN1987A



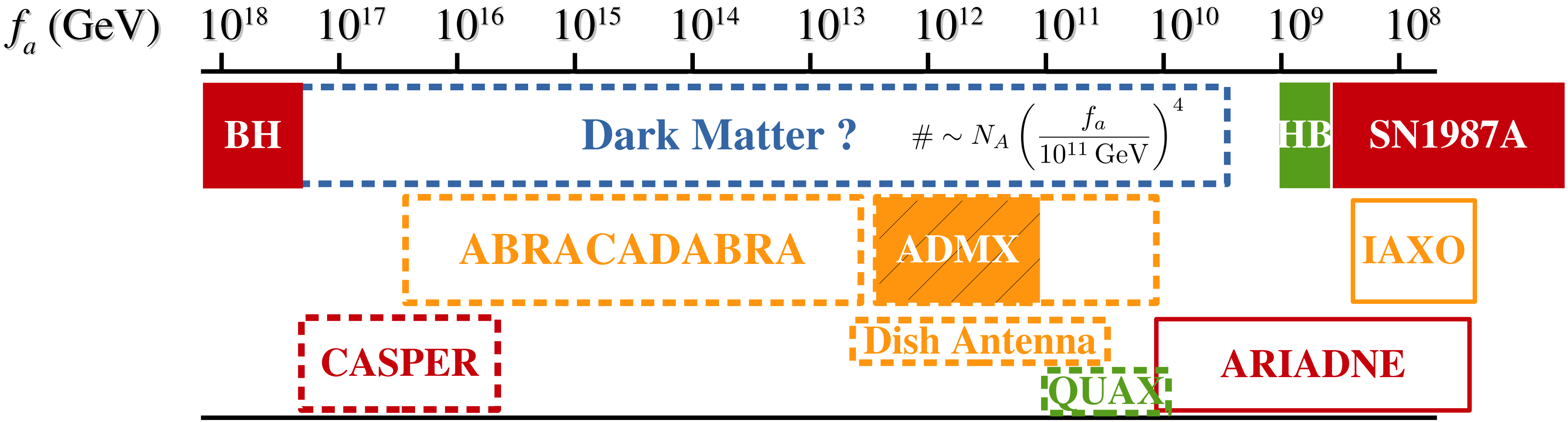
f_a (GeV)
10^{18} 10^{17} 10^{16} 10^{15} 10^{14} 10^{13} 10^{12} 10^{11} 10^{10} 10^{9} 10^{8}
BH
Dark Matter ?
$\# \sim N_A \left(\frac{f_a}{10^{11}\,\mathrm{GeV}}\right)^4$
HB
SN1987A
ADMX



f_a (GeV)
10^{18} 10^{17} 10^{16} 10^{15} 10^{14} 10^{13} 10^{12} 10^{11} 10^{10} 10^{9} 10^{8}
BH
Dark Matter ?
$\# \sim N_A \left(\frac{f_a}{10^{11}\,\mathrm{GeV}}\right)^4$
HB
SN1987A
ABRACADABRA
ADMX
IAXO
CASPER
Dish Antenna
ARIADNE
QUAX



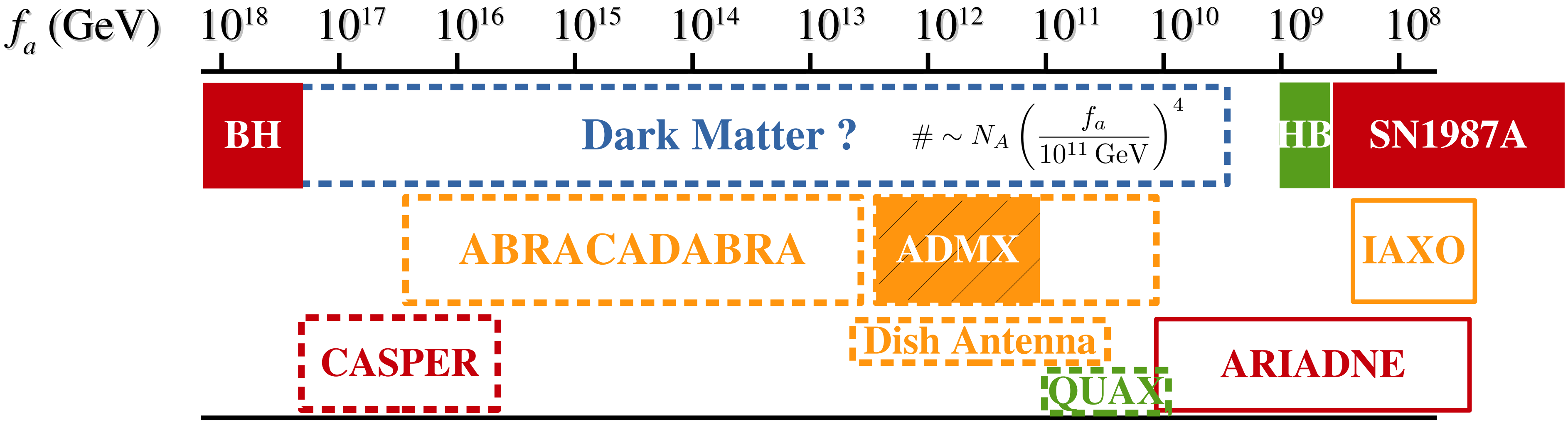
Axion DM: $a(t) \simeq 10^{-19}\, f_a\, \sin(\omega_a t)$



Axion DM:

$$a(t) \simeq 10^{-19} f_a \sin(\boxed{\omega_a} t)$$

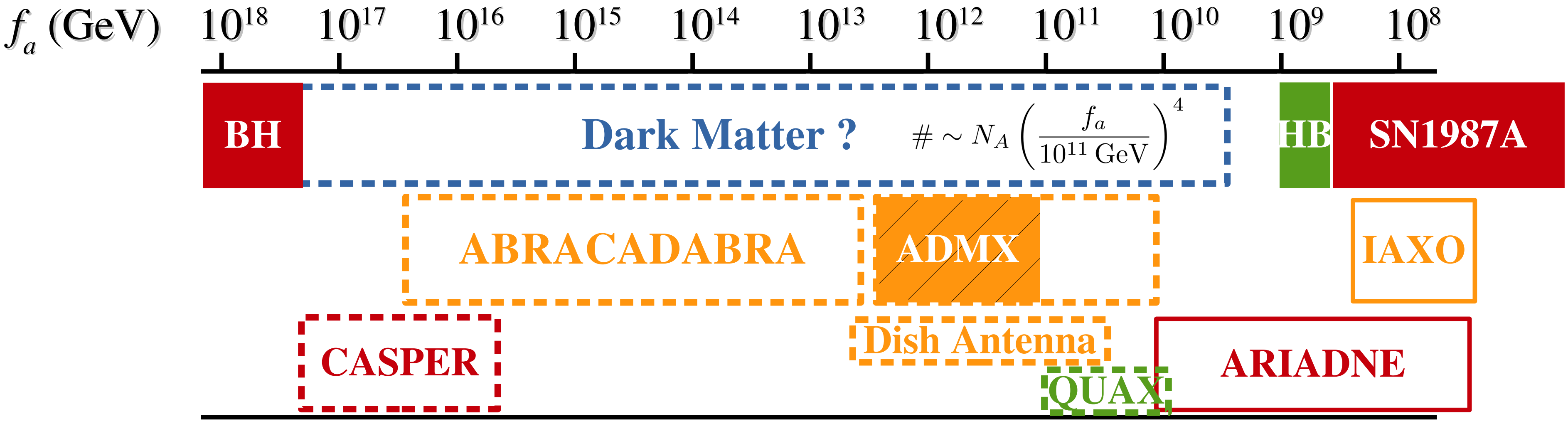
$$\boxed{\omega_a} = m_a + \frac{1}{2} m_a \langle v^2 \rangle = m_a (1 \pm 10^{-6})$$



Axion DM: $a(t) \simeq 10^{-19} f_a \sin(\boxed{\omega_a} t)$

$$\boxed{\omega_a} = m_a + \frac{1}{2} m_a \langle v^2 \rangle = m_a (1 \pm 10^{-6})$$

Resonant Exp.: $\boxed{\delta m_a / m_a \approx 10^{-6}}$ **!**



Axion DM: $a(t) \simeq 10^{-19} f_a \sin(\boxed{\omega_a} t)$

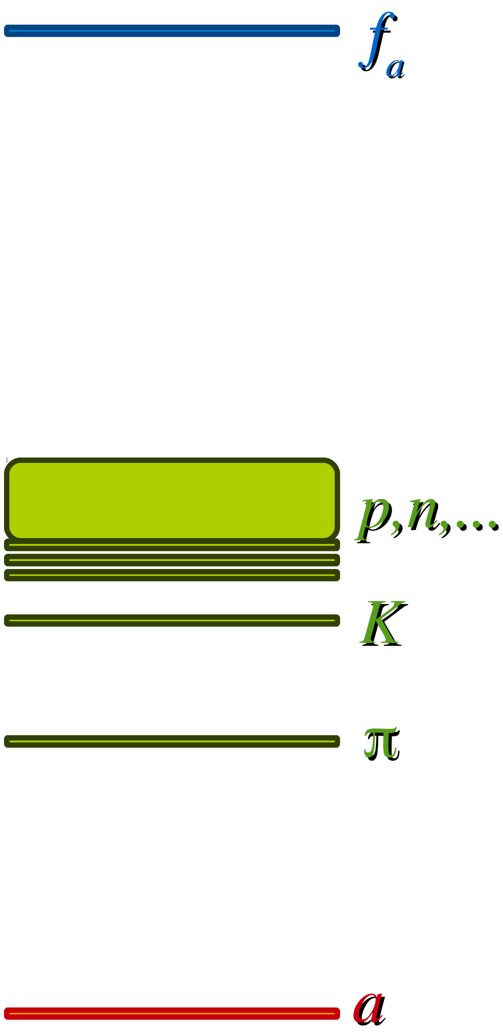
$$\boxed{\omega_a} = m_a + \frac{1}{2} m_a \langle v^2 \rangle = m_a (1 \pm 10^{-6})$$

Resonant Exp.: $\boxed{\delta m_a / m_a \approx 10^{-6}}$ **!**

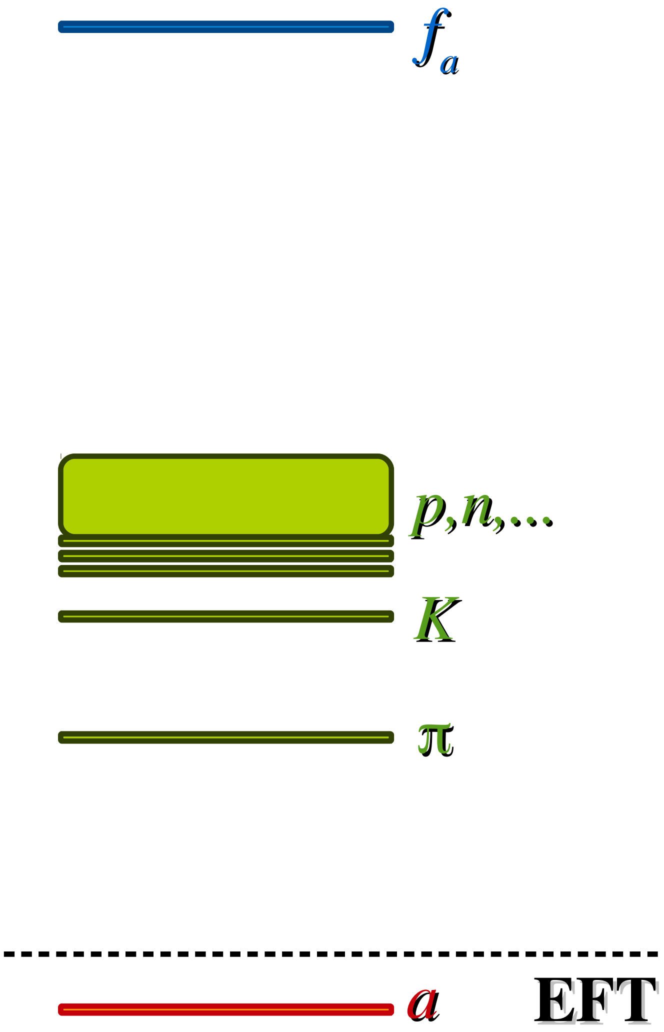
Other couplings?

QCD axion properties

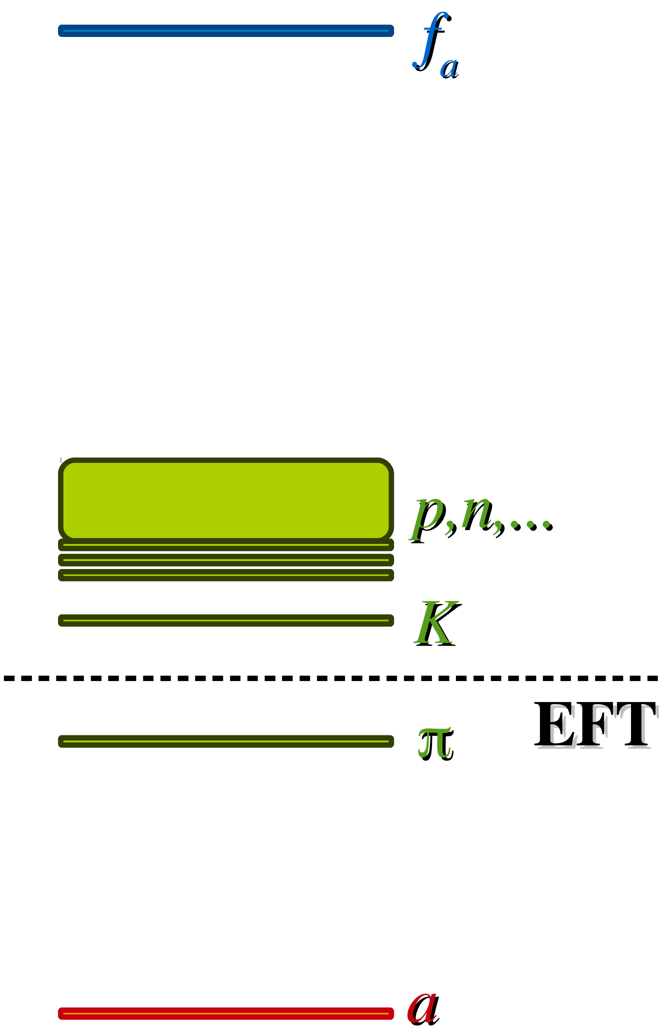
the QCD axion: *and its EFT*



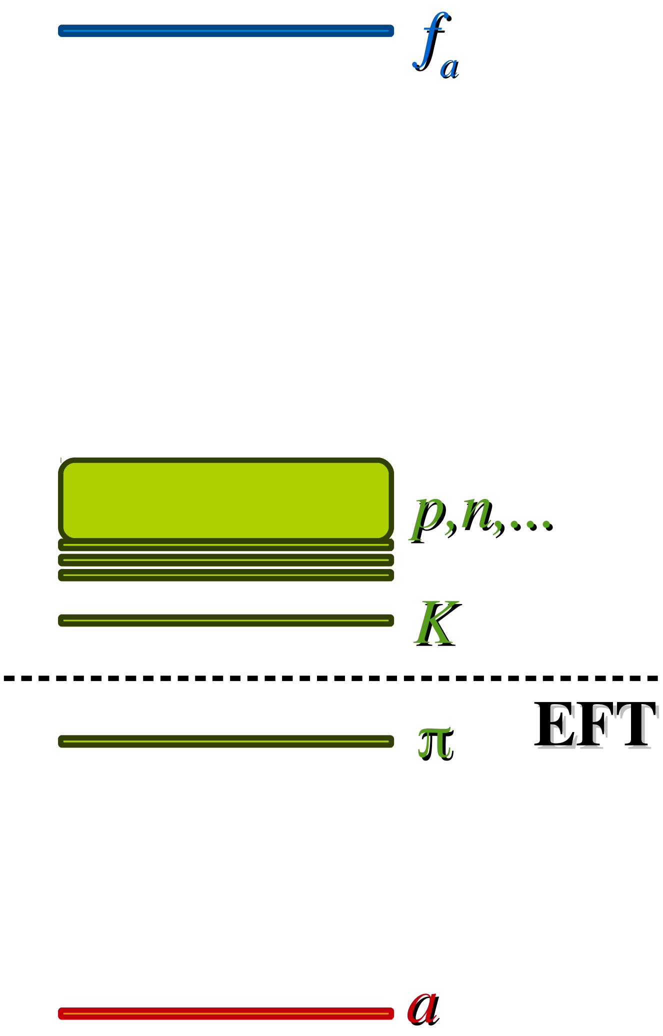
the QCD axion: *and its EFT*



the QCD axion: *and its EFT*

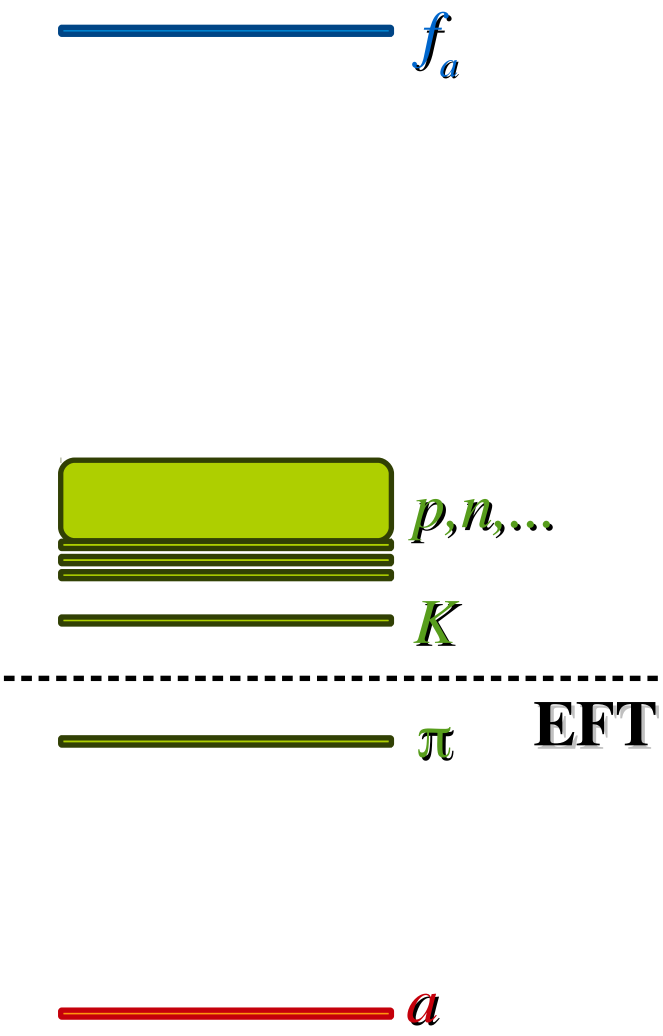


the QCD axion: ***and its EFT***



$$\frac{a}{32\pi^2 f_a} G\tilde{G} \;\to\; \overline{q}_L M_q e^{ia/2f_a} q_R + h.c.$$

the QCD axion: *and its EFT*



$$\frac{a}{32\pi^2 f_a} G\tilde{G} \;\rightarrow\; \bar{q}_L M_q e^{ia/2f_a} q_R + h.c.$$

$$\boxed{\mathcal{L}_{\mathrm{QCD}}(A_\mu, M_q e^{ia/2f_a}) \rightarrow \mathcal{L}_{\mathrm{ChPT}}(A_\mu, M_q e^{ia/2f_a})}$$

$$A_\mu = \partial_\mu a / f_a$$

the axion is an external source
in the EFT at LO in $1/f_a$

the QCD axion: ***potential***

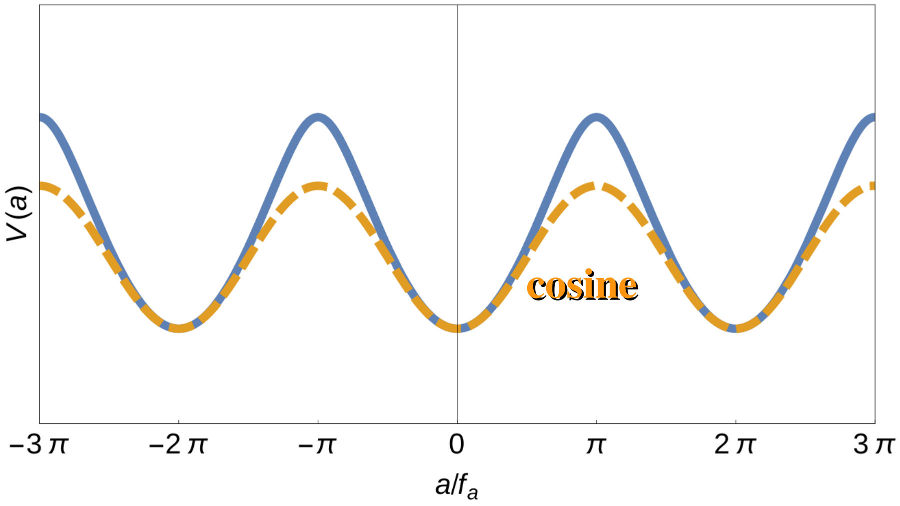
$$V(a,\pi) = -\frac{B_0 f_\pi^2}{2}\langle e^{-i\pi(x)/f_\pi} M_q e^{ia(x)/2f_a} + h.c.\rangle$$

the QCD axion: *potential*

$$V(a,\pi) = -\frac{B_0 f_\pi^2}{2}\langle e^{-i\pi(x)/f_\pi} M_q e^{ia(x)/2f_a} + h.c.\rangle$$

$$\boxed{V(a) = -m_\pi^2 f_\pi^2 \sqrt{1 - \frac{4m_u m_d}{(m_u+m_d)^2}\sin^2\left(\frac{a}{2f_a}\right)}}$$

Di Vecchia Veneziano '80

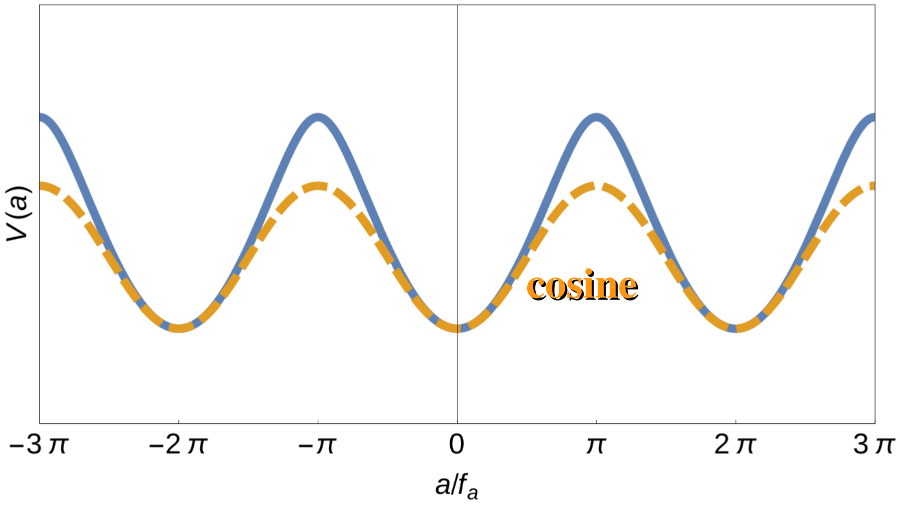


the QCD axion: *potential*

$$V(a,\pi) = -\frac{B_0 f_\pi^2}{2}\langle e^{-i\pi(x)/f_\pi} M_q e^{ia(x)/2f_a} + h.c.\rangle$$

$$\boxed{V(a) = -m_\pi^2 f_\pi^2 \sqrt{1 - \frac{4 m_u m_d}{(m_u+m_d)^2}\sin^2\left(\frac{a}{2f_a}\right)}}$$

Di Vecchia Veneziano '80



$$m_a^2 = \frac{m_u m_d}{(m_u+m_d)^2}\frac{m_\pi^2 f_\pi^2}{f_a^2}$$

Weinberg '78

the QCD axion: *the mass @NLO*

$$m_a^2 = \frac{m_u m_d}{(m_u + m_d)^2} \frac{m_\pi^2 f_\pi^2}{f_a^2}$$

the QCD axion: *the mass @NLO*

$$m_a^2 = \frac{m_u m_d}{(m_u + m_d)^2} \frac{m_\pi^2 f_\pi^2}{f_a^2} \left[1 + 2 \frac{m_\pi^2}{f_\pi^2} \left(h_1^r - h_3^r - l_4^r + \frac{m_u^2 - 6 m_u m_d + m_d^2}{(m_u + m_d)^2} l_7^r \right) \right]$$

the QCD axion: *the mass @NLO*

$$m_a^2 = \frac{m_u m_d}{(m_u+m_d)^2}\frac{m_\pi^2 f_\pi^2}{f_a^2}\left[1 + 2\frac{m_\pi^2}{f_\pi^2}\left(\underbrace{h_1^r - h_3^r - l_4^r}_{4.8\pm1.4\cdot10^{-3}} + \frac{m_u^2 - 6m_u m_d + m_d^2}{(m_u+m_d)^2}\underbrace{l_7^r}_{7(4)\cdot10^{-3}}\right)\right]$$

the QCD axion: *the mass @NLO*

$$m_a^2 = \frac{m_u m_d}{(m_u+m_d)^2}\frac{m_\pi^2 f_\pi^2}{f_a^2}\left[1 + 2\frac{m_\pi^2}{f_\pi^2}\left(\underbrace{h_1^r - h_3^r - l_4^r}_{4.8\pm1.4\cdot10^{-3}} + \frac{m_u^2 - 6m_u m_d + m_d^2}{(m_u+m_d)^2}\underbrace{l_7^r}_{7(4)\cdot10^{-3}}\right)\right]$$

lattice average:

$$\boxed{z \equiv \frac{m_u^{\overline{\mathrm{MS}}}(2\ \mathrm{GeV})}{m_d^{\overline{\mathrm{MS}}}(2\ \mathrm{GeV})} = 0.48(3)}$$

the QCD axion: *the mass @NLO*

$$m_a^2 = \frac{m_u m_d}{(m_u+m_d)^2}\frac{m_\pi^2 f_\pi^2}{f_a^2}\left[1 + 2\frac{m_\pi^2}{f_\pi^2}\left(\underbrace{h_1^r - h_3^r - l_4^r}_{4.8\pm1.4\cdot10^{-3}} + \frac{m_u^2 - 6m_u m_d + m_d^2}{(m_u+m_d)^2}\underbrace{l_7^r}_{7(4)\cdot10^{-3}}\right)\right]$$

lattice average:

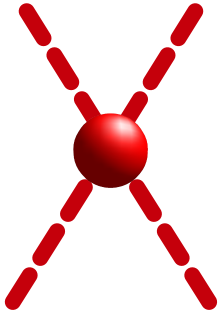
$$\boxed{z \equiv \frac{m_u^{\overline{\mathrm{MS}}}(2\ \mathrm{GeV})}{m_d^{\overline{\mathrm{MS}}}(2\ \mathrm{GeV})} = 0.48(3)}$$

$$\boxed{m_a = 5.70(6)(4)\ \mu\mathrm{eV}\left(\frac{10^{12}\mathrm{GeV}}{f_a}\right)}$$

$$(\chi^{top})^{1/4} = \sqrt{m_a f_a} = 75.5(5)\ \mathrm{MeV}$$

the QCD axion: *potential @NLO*

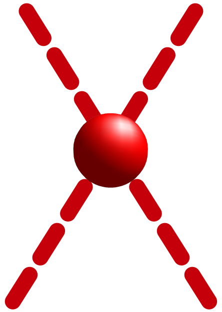
$$\lambda_a = -\frac{m_a^2}{f_a^2} \quad \frac{m_u^2 - m_u m_d + m_d^2}{(m_u + m_d)^2}$$



the QCD axion: *potential @NLO*

$$\lambda_a = -\frac{m_a^2}{f_a^2}\left\{\frac{m_u^2 - m_u m_d + m_d^2}{(m_u+m_d)^2}\right.$$

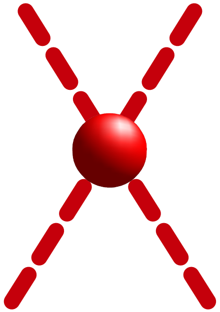
$$\left. +6\frac{m_\pi^2}{f_\pi^2}\frac{m_u m_d}{(m_u+m_d)^2}\left[h_1^r - h_3^r - l_4^r + \frac{4\bar{l}_4 - \bar{l}_3 - 3}{64\pi^2} - 4\frac{m_u^2 - m_u m_d + m_d^2}{(m_u+m_d)^2} l_7^r\right]\right\}$$



$$\boxed{\lambda_a = -0.346(22)\cdot\frac{m_a^2}{f_a^2}}$$

the QCD axion: *potential @NLO*

$$\lambda_a = -\frac{m_a^2}{f_a^2}\left\{\frac{m_u^2 - m_u m_d + m_d^2}{(m_u+m_d)^2}\right.$$
$$\left. +6\frac{m_\pi^2}{f_\pi^2}\frac{m_u m_d}{(m_u+m_d)^2}\left[h_1^r - h_3^r - l_4^r + \frac{4\bar{l}_4 - \bar{l}_3 - 3}{64\pi^2} - 4\frac{m_u^2 - m_u m_d + m_d^2}{(m_u+m_d)^2} l_7^r\right]\right\}$$

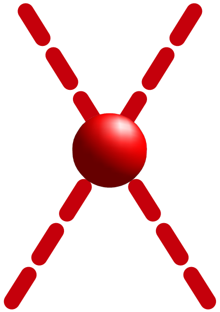


$$\boxed{\lambda_a = -0.346(22)\cdot\frac{m_a^2}{f_a^2}}$$

cosine→1

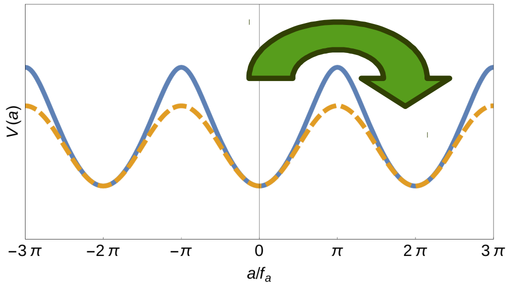
the QCD axion: *potential @NLO*

$$\lambda_a = -\frac{m_a^2}{f_a^2}\left\{\frac{m_u^2 - m_u m_d + m_d^2}{(m_u+m_d)^2}\right.$$
$$\left. +6\frac{m_\pi^2}{f_\pi^2}\frac{m_u m_d}{(m_u+m_d)^2}\left[h_1^r - h_3^r - l_4^r + \frac{4\bar{l}_4 - \bar{l}_3 - 3}{64\pi^2} - 4\frac{m_u^2 - m_u m_d + m_d^2}{(m_u+m_d)^2} l_7^r\right]\right\}$$



$$\boxed{\lambda_a = -0.346(22)\cdot\frac{m_a^2}{f_a^2}}$$

cosine→1

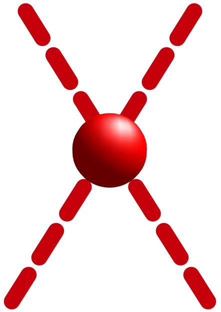


domain wall

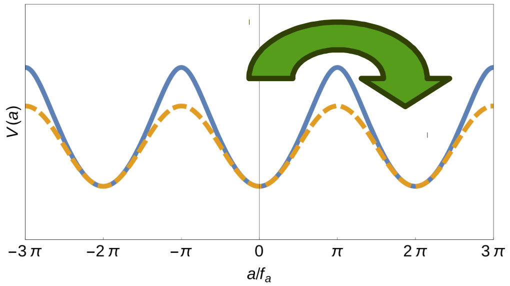
$$\boxed{\sigma = 2f_a\int_0^\pi d\theta\,\sqrt{2[V(\theta)-V(0)]} = 8.97(5)\,m_a f_a^2}$$

the QCD axion: *potential @NLO*

$$\lambda_a = -\frac{m_a^2}{f_a^2}\left\{\frac{m_u^2 - m_u m_d + m_d^2}{(m_u+m_d)^2}\right.$$
$$\left. +6\frac{m_\pi^2}{f_\pi^2}\frac{m_u m_d}{(m_u+m_d)^2}\left[h_1^r - h_3^r - l_4^r + \frac{4\bar{l}_4 - \bar{l}_3 - 3}{64\pi^2} - 4\frac{m_u^2 - m_u m_d + m_d^2}{(m_u+m_d)^2} l_7^r\right]\right\}$$



$$\boxed{\lambda_a = \underbrace{-0.346(22)}_{\text{cosine}\to 1} \cdot \frac{m_a^2}{f_a^2}}$$

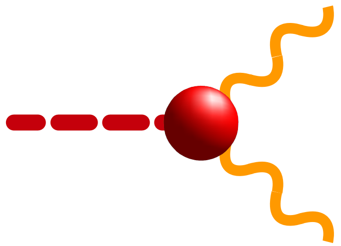


domain wall

$$\boxed{\sigma = 2 f_a \int_0^\pi d\theta \sqrt{2[V(\theta) - V(0)]} = \underbrace{8.97(5)}_{\text{cosine}\to 8}\, m_a f_a^2}$$

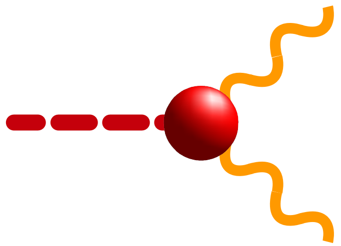
the QCD axion: ***photon coupling @NLO***

$$g_{a\gamma\gamma} = \frac{\alpha_{em}}{2\pi f_a}\left\{\frac{E}{N} - \frac{2}{3}\frac{4m_d + m_u}{m_d + m_u}\right\}$$



the QCD axion: *photon coupling @NLO*

$$g_{a\gamma\gamma} = \frac{\alpha_{em}}{2\pi f_a}\left\{\boxed{\frac{E}{N}} - \frac{2}{3}\frac{4m_d+m_u}{m_d+m_u}\right\}$$

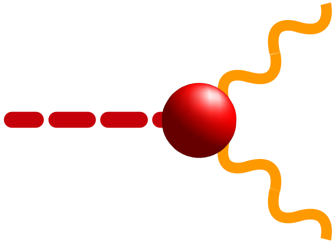


E/N =

0	**(KSVZ,...)**
8/3	**(DFSZ, GUT-KSVZ,...)**
2	**(Unificaxion,...)**

the QCD axion: *photon coupling @NLO*

$$g_{a\gamma\gamma} = \frac{\alpha_{em}}{2\pi f_a}\left\{\boxed{\frac{E}{N}}\boxed{-\frac{2}{3}\frac{4m_d+m_u}{m_d+m_u}}\right\}$$



E/N =

0 (KSVZ,...)

8/3 (DFSZ, GUT-KSVZ,...)

2 (Unificaxion,...)

tree ~ - 2

$a \to \pi \to \gamma\gamma$

the QCD axion: *photon coupling @NLO*

$$g_{a\gamma\gamma} = \frac{\alpha_{em}}{2\pi f_a}\left\{\boxed{\frac{E}{N}}\boxed{-\frac{2}{3}\frac{4m_d+m_u}{m_d+m_u}}+\frac{m_\pi^2}{f_\pi^2}\frac{8m_u m_d}{(m_u+m_d)^2}\left[\frac{8}{9}\left(5\tilde{c}_3^W+\tilde{c}_7^W+2\tilde{c}_8^W\right)-\frac{m_d-m_u}{m_d+m_u}l_7^r\right]\right\}$$

***E/N* =**

0	**(KSVZ,...)**
8/3	**(DFSZ, GUT-KSVZ,...)**
2	**(Unificaxion,...)**

tree ~ - 2

$a \to \pi \to \gamma\gamma$

the QCD axion: *photon coupling @NLO*

$$g_{a\gamma\gamma} = \frac{\alpha_{em}}{2\pi f_a}\left\{\boxed{\frac{E}{N}}\boxed{-\frac{2}{3}\frac{4m_d+m_u}{m_d+m_u}}+\boxed{\frac{m_\pi^2}{f_\pi^2}\frac{8m_u m_d}{(m_u+m_d)^2}\left[\frac{8}{9}\underbrace{\left(5\tilde{c}_3^W+\tilde{c}_7^W+2\tilde{c}_8^W\right)}_{\text{NLO}}-\frac{m_d-m_u}{m_d+m_u}l_7^r\right]}\right\}$$

E/N =

0 (KSVZ,...)

8/3 (DFSZ, GUT-KSVZ,...)

2 (Unificaxion,...)

tree ~ - 2

$a \to \pi \to \gamma\gamma$

NLO $= 0.033(6)$

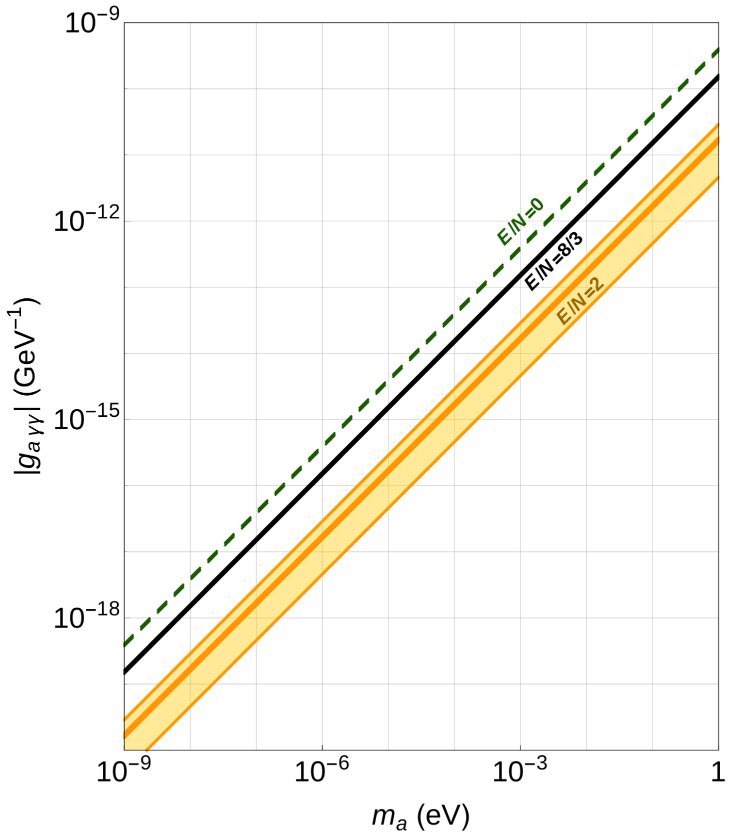
from $\pi \to \gamma\gamma \quad \eta \to \gamma\gamma$

the QCD axion: *photon coupling @NLO*

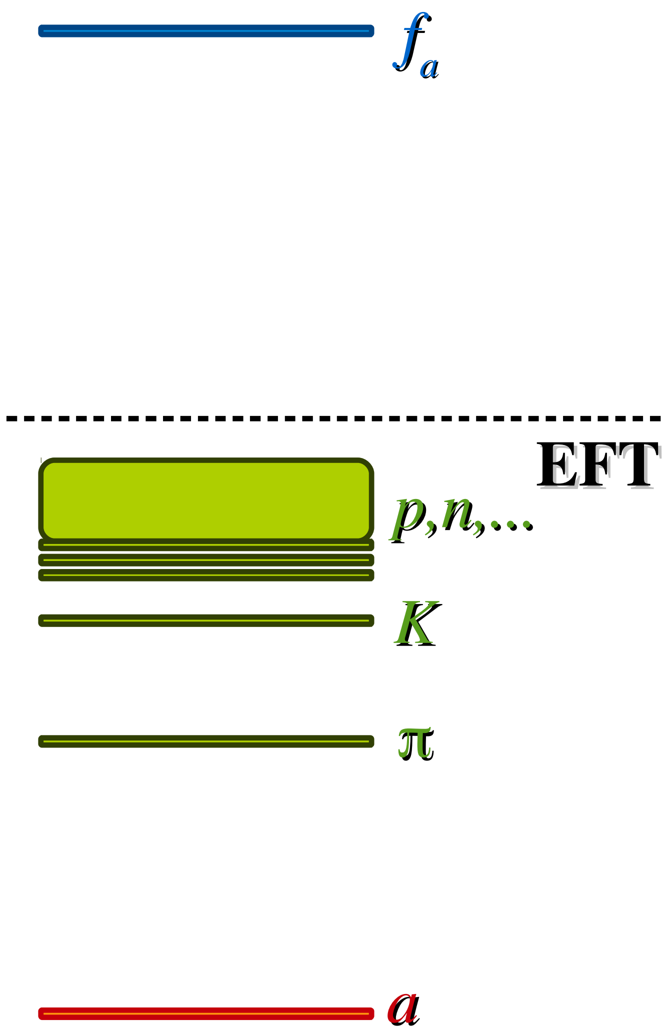
$$g_{a\gamma\gamma} = \frac{\alpha_{em}}{2\pi f_a}\left\{\frac{E}{N} - \frac{2}{3}\frac{4m_d + m_u}{m_d + m_u} + \frac{m_\pi^2}{f_\pi^2}\frac{8m_u m_d}{(m_u+m_d)^2}\left[\frac{8}{9}\left(5\tilde{c}_3^W + \tilde{c}_7^W + 2\tilde{c}_8^W\right) - \frac{m_d - m_u}{m_d + m_u} l_7^r\right]\right\}$$

$$\boxed{g_{a\gamma\gamma} = \frac{\alpha_{em}}{2\pi f_a}\left[\frac{E}{N} - 1.92(4)\right]}$$

$$g_{a\gamma\gamma} = \begin{cases} -2.227(44)\cdot 10^{-3}/f_a & E/N = 0 \\ 0.870(44)\cdot 10^{-3}/f_a & E/N = 8/3 \\ 0.095(44)\cdot 10^{-3}/f_a & E/N = 2 \end{cases}$$

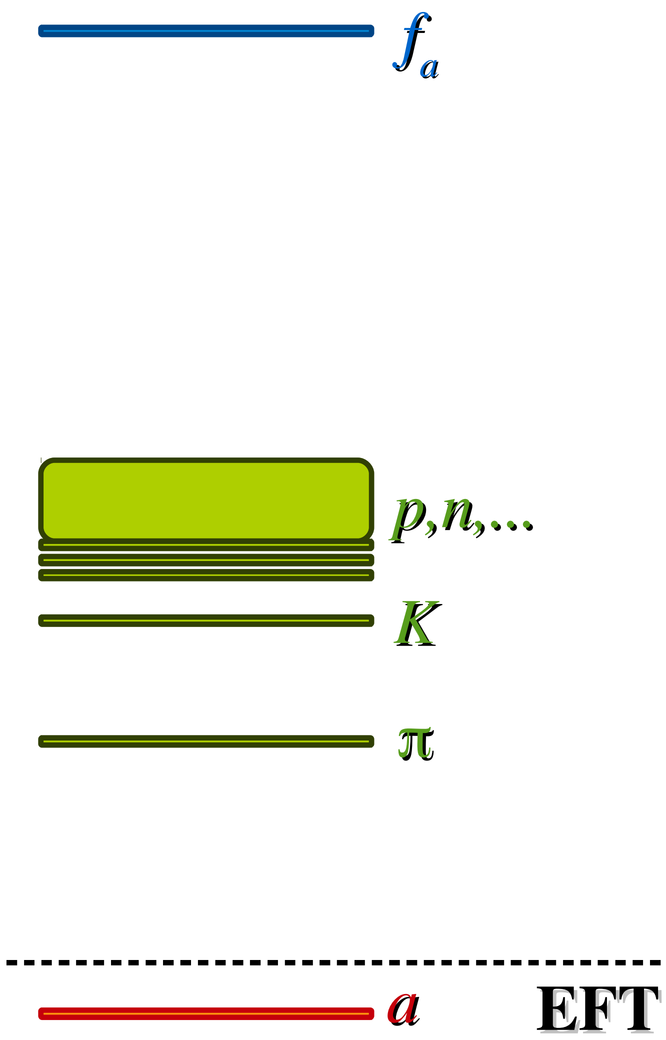


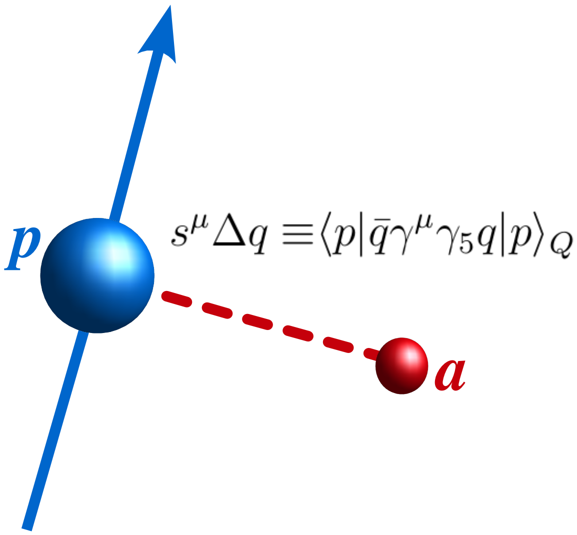
the QCD axion: *matter coupling*



the QCD axion: *matter coupling*

$$\mathcal{L}_N = \bar{N} v^\mu D_\mu N + 2 g_A \bar{N} S^\mu \hat{A}_\mu N + 2 g_0^i \bar{N} S^\mu N \bar{A}_\mu^i$$





the QCD axion: *matter coupling*

from β-decays: $\Delta u - \Delta d = g_A = 1.2723(23)$

from lattice QCD: $g_0^{ud} = \Delta u + \Delta d = 0.541(50)\,, \qquad \Delta s = -0.0227(34)\,, \qquad \Delta c = \pm 0.004$

the QCD axion: *matter coupling*

from β-decays: $\Delta u - \Delta d = g_A = 1.2723(23)$

from lattice QCD: $g_0^{ud} = \Delta u + \Delta d = 0.541(50)\,,\qquad \Delta s = -0.0227(34)\,,\qquad \Delta c = \pm 0.004$

$$\boxed{\frac{\partial_\mu a}{2f_a} c_N \bar{N}\gamma^\mu\gamma_5 N}$$

$$\begin{aligned} c_p &= -0.48(3) + 0.89(2)c_u^0 - 0.38(2)c_d^0 - 0.036(4)c_s^0 \\ &\qquad - 0.013(5)c_c^0 - 0.009(2)c_b^0 - 0.0036(4)c_t^0 \\ c_n &= -0.03(3) + 0.89(2)c_d^0 - 0.38(2)c_u^0 - 0.036(4)c_s^0 \\ &\qquad - 0.013(5)c_c^0 - 0.009(2)c_b^0 - 0.0036(4)c_t^0 \end{aligned}$$

the QCD axion: *matter coupling*

from β-decays: $\Delta u - \Delta d = g_A = 1.2723(23)$

from lattice QCD: $g_0^{ud} = \Delta u + \Delta d = 0.541(50)\,,\qquad \Delta s = -0.0227(34)\,,\qquad \Delta c = \pm 0.004$

$$\boxed{\frac{\partial_\mu a}{2 f_a} c_N \bar{N} \gamma^\mu \gamma_5 N}$$

$$\begin{aligned}
c_p &= \boxed{-0.48(3)} + 0.89(2) c_u^0 - 0.38(2) c_d^0 - 0.036(4) c_s^0 \\
&\quad - 0.013(5) c_c^0 - 0.009(2) c_b^0 - 0.0036(4) c_t^0 \\
c_n &= \boxed{-0.03(3)} + 0.89(2) c_d^0 - 0.38(2) c_u^0 - 0.036(4) c_s^0 \\
&\quad - 0.013(5) c_c^0 - 0.009(2) c_b^0 - 0.0036(4) c_t^0
\end{aligned}$$

model independent couplings

the QCD axion: *matter coupling*

from β-decays: $\Delta u - \Delta d = g_A = 1.2723(23)$

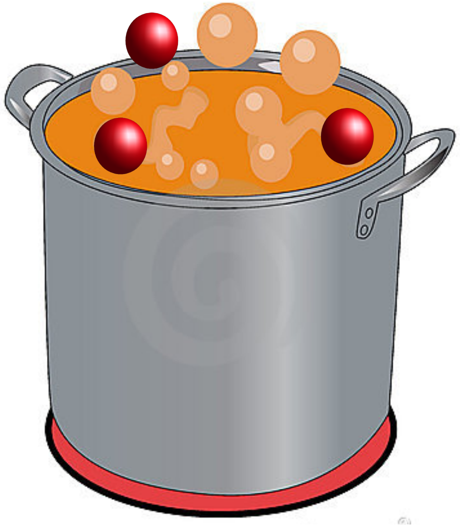
from lattice QCD: $g_0^{ud} = \Delta u + \Delta d = 0.541(50)\,, \qquad \Delta s = -0.0227(34)\,, \qquad \Delta c = \pm 0.004$

$$\boxed{\frac{\partial_\mu a}{2 f_a} c_N \bar{N} \gamma^\mu \gamma_5 N}$$

$$\begin{aligned} c_p &= \boxed{-0.48(3)} + 0.89(2)c_u^0 - 0.38(2)c_d^0 - 0.036(4)c_s^0 \\ &\quad \boxed{- 0.013(5)c_c^0 - 0.009(2)c_b^0 - 0.0036(4)c_t^0} \\ c_n &= \boxed{-0.03(3)} + 0.89(2)c_d^0 - 0.38(2)c_u^0 - 0.036(4)c_s^0 \\ &\quad \boxed{- 0.013(5)c_c^0 - 0.009(2)c_b^0 - 0.0036(4)c_t^0} \end{aligned}$$

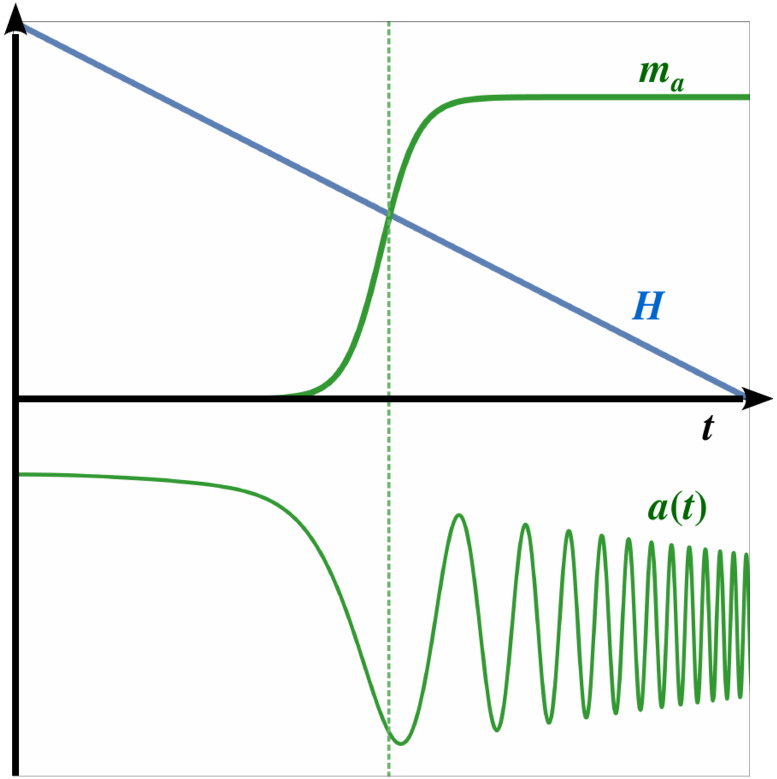
from RGE effects

model independent couplings



the *hot* axion

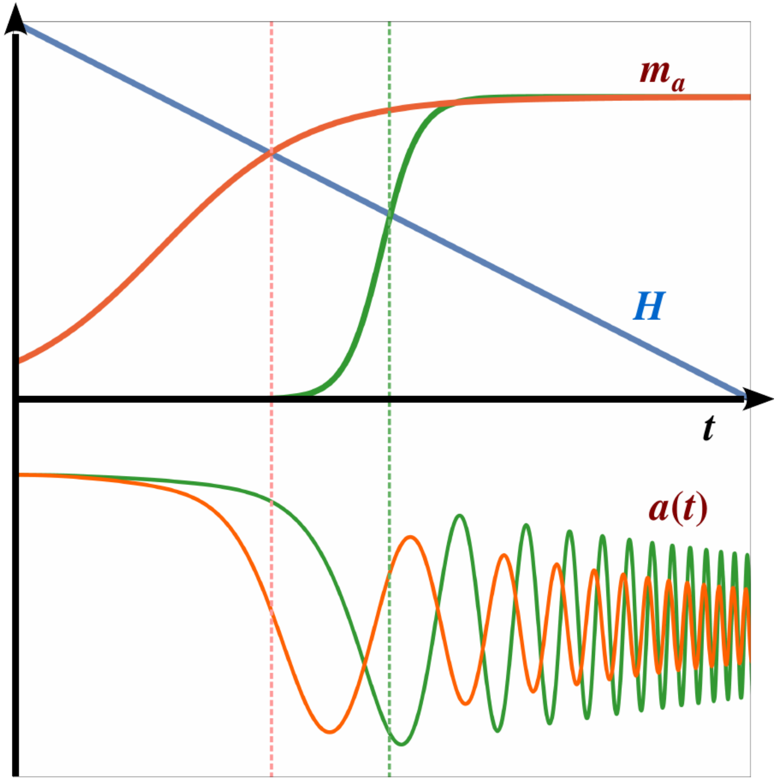
the QCD axion: *relic abundance*



$$\Omega_a \approx \Omega_{DM}\, k\, \theta_0^2 \left[\frac{f_a}{10^{12}\ \mathrm{GeV}}\right]^{1+\epsilon}$$

$$\boxed{\rho_a = m_a^2 a^2}$$

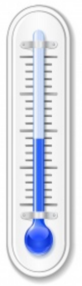
the QCD axion: *relic abundance*



$$\Omega_a \approx \Omega_{DM}\, k\, \theta_0^2 \left[\frac{f_a}{10^{12}\ \mathrm{GeV}}\right]^{1+\epsilon}$$

$$\boxed{\rho_a = m_a^2 a^2}$$

the QCD axion: *@ small temperature*

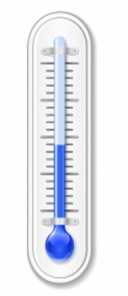


$$\frac{m_a^2(T)}{m_a^2} = 1 - \frac{3}{2}\frac{T^2}{f_\pi^2} J_1\left[\frac{m_\pi^2}{T^2}\right]$$

$$\frac{V(a;T)}{V(a)} = 1 + \frac{3}{2}\frac{T^4}{f_\pi^2 m_\pi^2\left(\frac{a}{f_a}\right)} J_0\left[\frac{m_\pi^2\left(\frac{a}{f_a}\right)}{T^2}\right]$$

$T < T_c \sim 155$ **MeV**

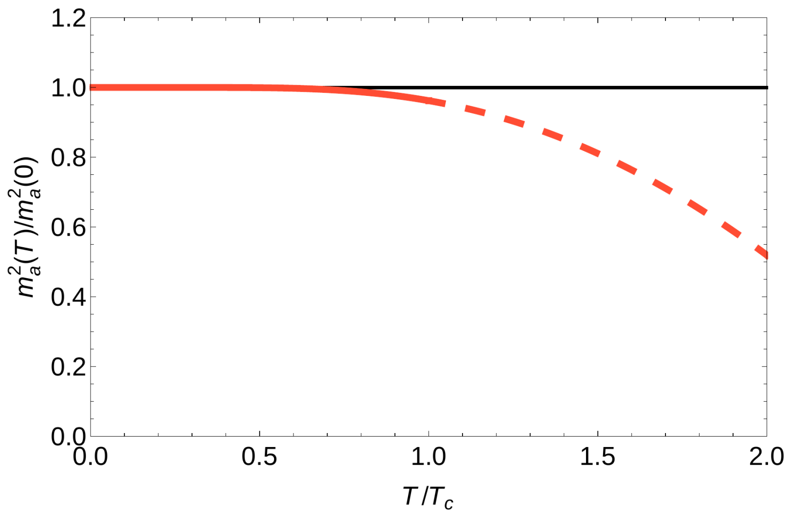
the QCD axion: *@ small temperature*



$$\frac{m_a^2(T)}{m_a^2} = 1 - \frac{3}{2}\frac{T^2}{f_\pi^2} J_1\left[\frac{m_\pi^2}{T^2}\right] \simeq 1 - \frac{3}{2(2\pi)^{3/2}}\frac{m_\pi^2}{f_\pi^2}\left[\frac{T}{m_\pi}\right]^{3/2} e^{-m_\pi/T}$$

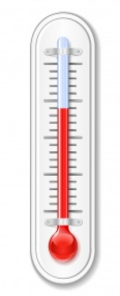
$$\frac{V(a;T)}{V(a)} = 1 + \frac{3}{2}\frac{T^4}{f_\pi^2 m_\pi^2\left(\frac{a}{f_a}\right)} J_0\left[\frac{m_\pi^2\left(\frac{a}{f_a}\right)}{T^2}\right]$$

$T < T_c \sim 155$ MeV



the QCD axion: *@ higher temperature*

Gross Pisarski Yaffe '81

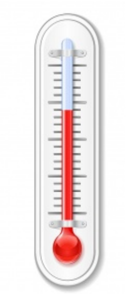


***T >> T**$_c$*

$$f_a^2 m_a^2(T) \simeq 2 \int d\rho\, n(\rho, 0) e^{-\frac{2\pi^2}{g_s^2} m_{D1}^2 \rho^2 + \ldots}$$

the QCD axion: *@ higher temperature*

Gross Pisarski Yaffe '81

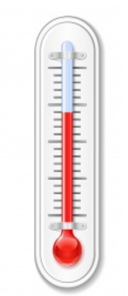


T >> T$_c$

$$f_a^2 m_a^2(T) \simeq 2 \underbrace{\int d\rho}_{\text{integral over instanton sizes}} \, n(\rho, 0) e^{-\frac{2\pi^2}{g_s^2} m_{D1}^2 \rho^2 + \dots}$$

the QCD axion: *@ higher temperature*

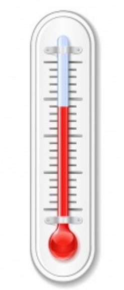
Gross Pisarski Yaffe '81



T >> T$_c$

$$f_a^2 m_a^2(T) \simeq 2 \underbrace{\int d\rho}_{\text{integral over instanton sizes}} \, \overbrace{n(\rho, 0)}^{\propto\, m_u m_d e^{-8\pi^2/g_s^2(\rho)}} e^{-\frac{2\pi^2}{g_s^2} m_{D1}^2 \rho^2 + \ldots}$$

the QCD axion: *@ higher temperature*



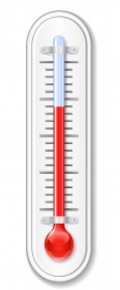
T >> T $_c$

Gross Pisarski Yaffe '81

$$f_a^2 m_a^2(T) \simeq 2 \underbrace{\int d\rho}_{\text{integral over instanton sizes}} \overbrace{n(\rho, 0)}^{\propto\, m_u m_d e^{-8\pi^2/g_s^2(\rho)}} \underbrace{e^{-\frac{2\pi^2}{g_s^2} m_{D1}^2 \rho^2 + \dots}}_{\text{Debye screening cut-off from 1-loop thermal corrections}}$$

the QCD axion: *@ higher temperature*

Gross Pisarski Yaffe '81

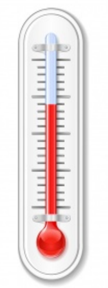


T >> T$_c$

$$f_a^2 m_a^2(T) \simeq 2\int \underbrace{d\rho}_{\boxed{\text{integral over instanton sizes}}}\, \overbrace{n(\rho,0)}^{\boxed{\propto m_u m_d e^{-8\pi^2/g_s^2(\rho)}}}\, \underbrace{e^{-\frac{2\pi^2}{g_s^2}m_{D1}^2\rho^2+\dots}}_{\boxed{\text{Debye screening cut-off from 1-loop thermal corrections}}} \sim T^{-7-n_f/3}$$

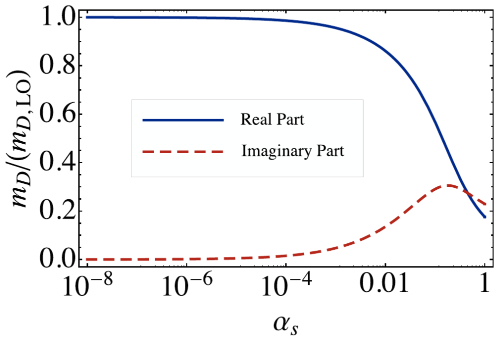
the QCD axion: *@ higher temperature*

Gross Pisarski Yaffe '81



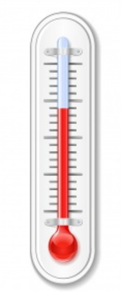
***T >> T**$_c$*

$$f_a^2 m_a^2(T) \simeq 2 \int d\rho\, n(\rho,0) e^{-\frac{2\pi^2}{g_s^2} m_{D1}^2 \rho^2 + \dots} \sim T^{-7-n_f/3}$$



Bad convergence of thermal QCD
good only above T~$10^{5\text{-}6}$ GeV !!

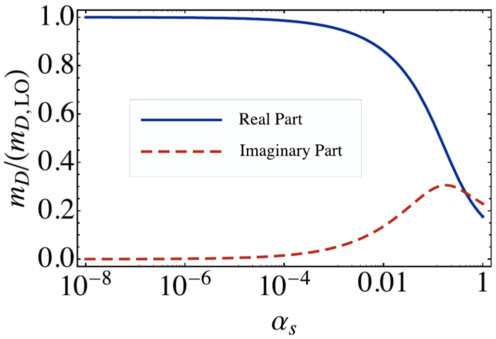
the QCD axion: *@ higher temperature*



T >> T$_c$

$$f_a^2 m_a^2(T) \simeq 2\int d\rho\, n(\rho,0)\, e^{-\frac{2\pi^2}{g_s^2} m_{D1}^2 \rho^2 + \dots} \sim T^{-7-n_f/3}$$

Gross Pisarski Yaffe '81



Bad convergence of thermal QCD
good only above T~10$^{5\text{-}6}$ GeV !!

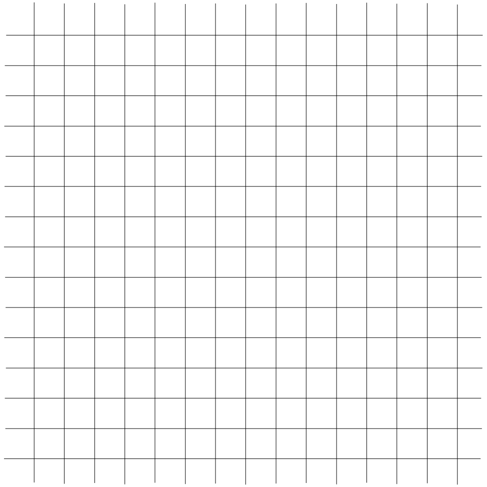
can we trust the instanton approx.?

the QCD axion from Lattice QCD

2+1 flavors with **physical m_q**

$$\chi = \int d^4x \langle q(x) q(0) \rangle_{\theta=0} = \frac{\langle Q^2 \rangle_{\theta=0}}{\mathcal{V}}$$

$$b_2 = -\frac{\langle Q^4 \rangle_{\theta=0} - 3\langle Q^2 \rangle^2_{\theta=0}}{12 \langle Q^2 \rangle_{\theta=0}}$$



the QCD axion from Lattice QCD

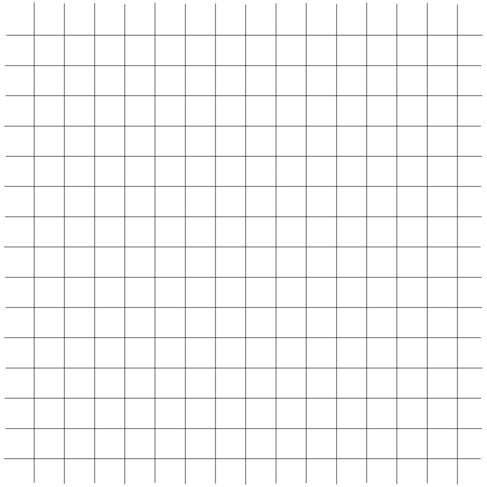
$$\chi = \int d^4x \langle q(x) q(0) \rangle_{\theta=0} = \frac{\langle Q^2 \rangle_{\theta=0}}{\mathcal{V}}$$

$$b_2 = -\frac{\langle Q^4 \rangle_{\theta=0} - 3\langle Q^2 \rangle^2_{\theta=0}}{12 \langle Q^2 \rangle_{\theta=0}}$$

2+1 flavors with **physical $\boldsymbol{m_q}$**

Temp. up to ~ 600 MeV (~ 4 T_c)

$48^3 \times 24 \div 48^3 \times 6$



the QCD axion from Lattice QCD

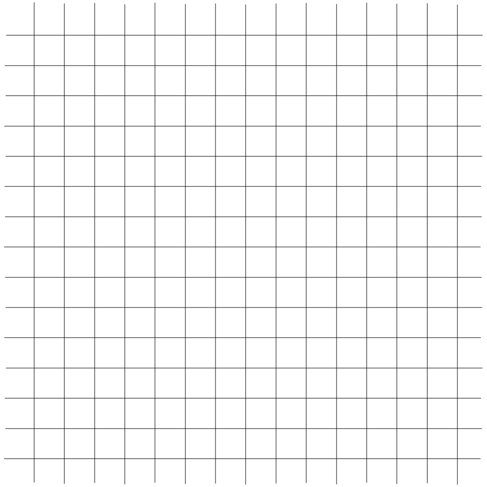
$$\chi = \int d^4x \langle q(x)q(0)\rangle_{\theta=0} = \frac{\langle Q^2\rangle_{\theta=0}}{\mathcal{V}}$$

$$b_2 = -\frac{\langle Q^4\rangle_{\theta=0} - 3\langle Q^2\rangle^2_{\theta=0}}{12\langle Q^2\rangle_{\theta=0}}$$

2+1 flavors with **physical m_q**

Temp. up to ~ 600 MeV (~ 4 T_c)

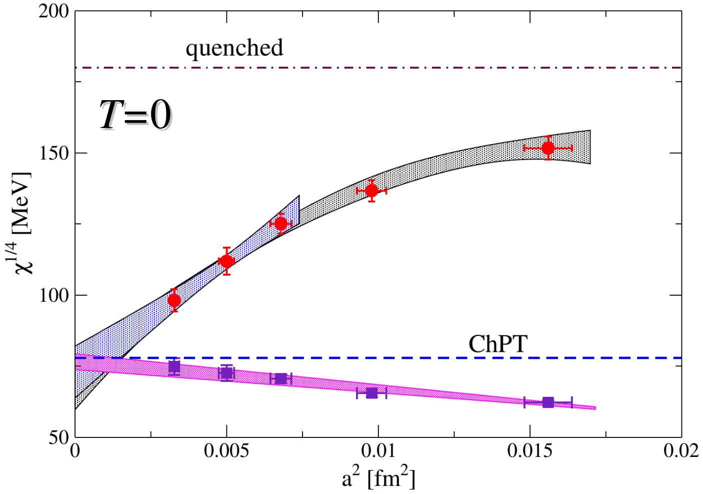
$48^3\times24 \div 48^3\times6$



a = 0.082 ÷ 0.057 *fm*

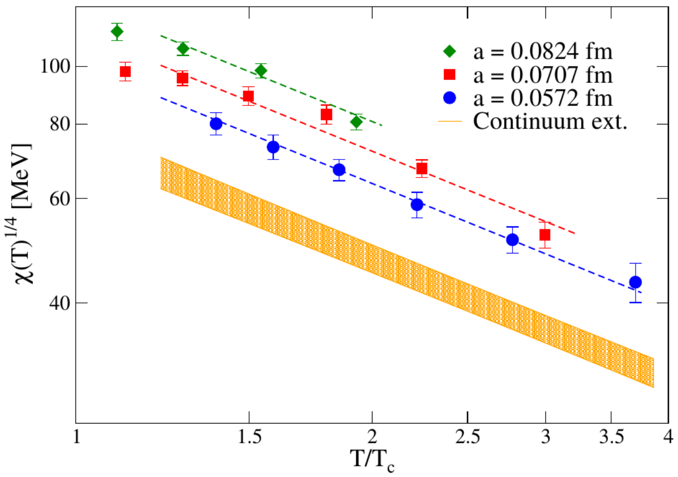
a^{-1} ~ 2.4 ÷ 3.5 GeV

Importance of continuum limit



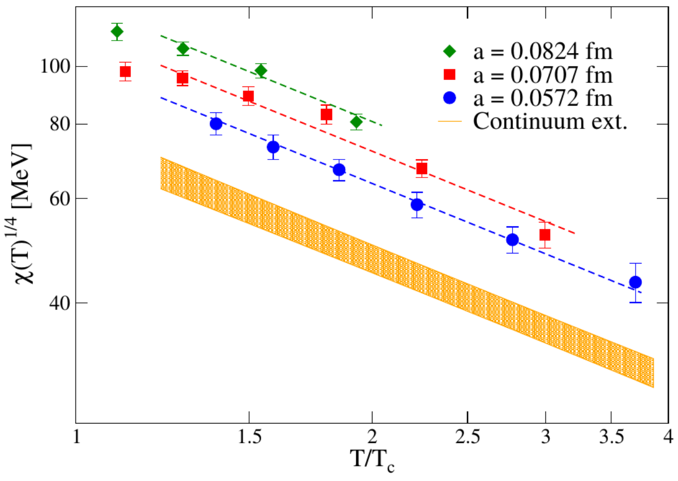
the QCD axion from Lattice QCD

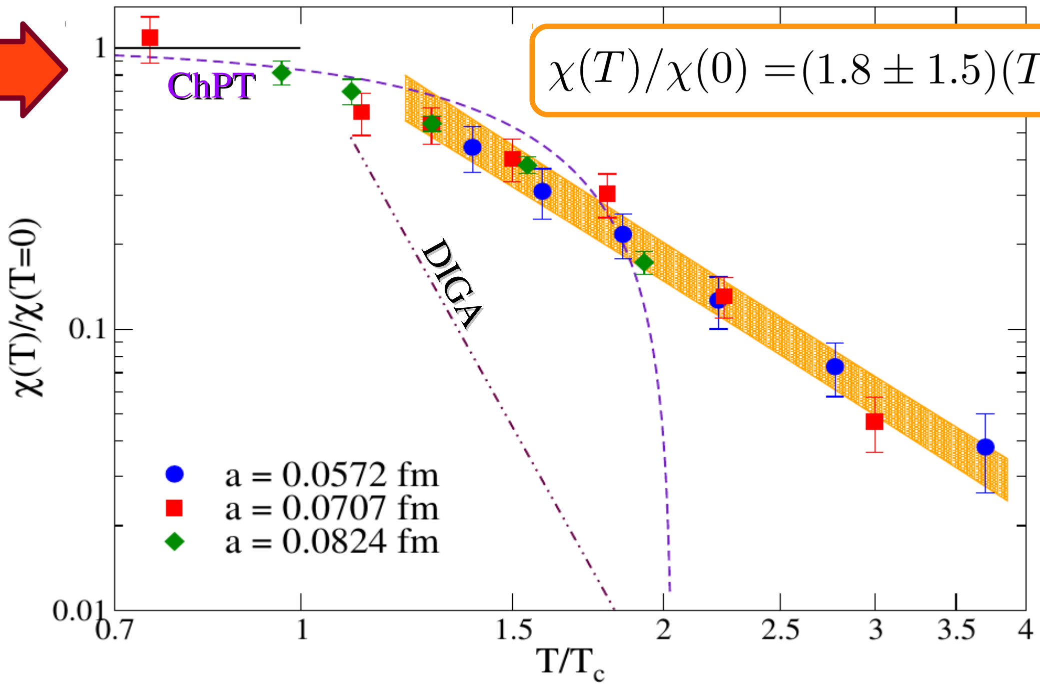
$$\chi = \int d^4x \langle q(x)q(0)\rangle_{\theta=0} = \frac{\langle Q^2\rangle_{\theta=0}}{\mathcal{V}} = m_a^2 f_a^2$$



the QCD axion from Lattice QCD

$$\chi = \int d^4x \langle q(x)q(0)\rangle_{\theta=0} = \frac{\langle Q^2\rangle_{\theta=0}}{\mathcal{V}} = m_a^2 f_a^2$$

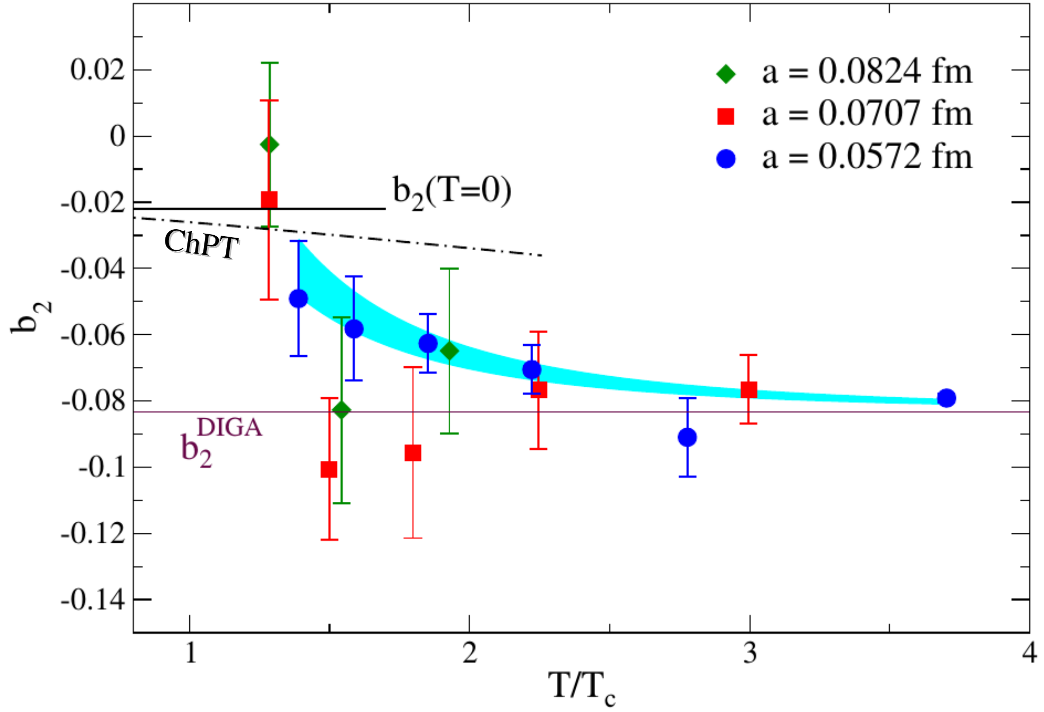




$$\chi(T)/\chi(0) = (1.8 \pm 1.5)(T_c/T)^{2.90\pm0.65}$$

the QCD axion from Lattice QCD

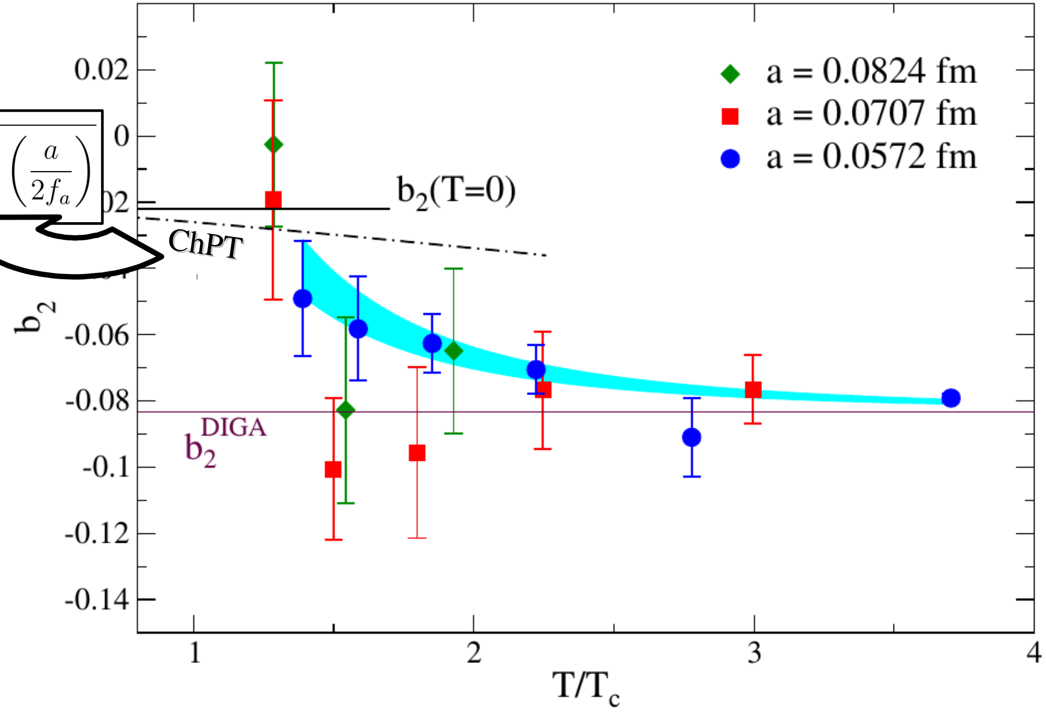
$$b_2 = -\frac{\langle Q^4\rangle_{\theta=0} - 3\langle Q^2\rangle^2_{\theta=0}}{12\langle Q^2\rangle_{\theta=0}} = \frac{\lambda_a}{12}\frac{f_a^2}{m_a^2}$$



the QCD axion from Lattice QCD

$$b_2 = -\frac{\langle Q^4\rangle_{\theta=0} - 3\langle Q^2\rangle^2_{\theta=0}}{12\langle Q^2\rangle_{\theta=0}} = \frac{\lambda_a}{12}\frac{f_a^2}{m_a^2}$$

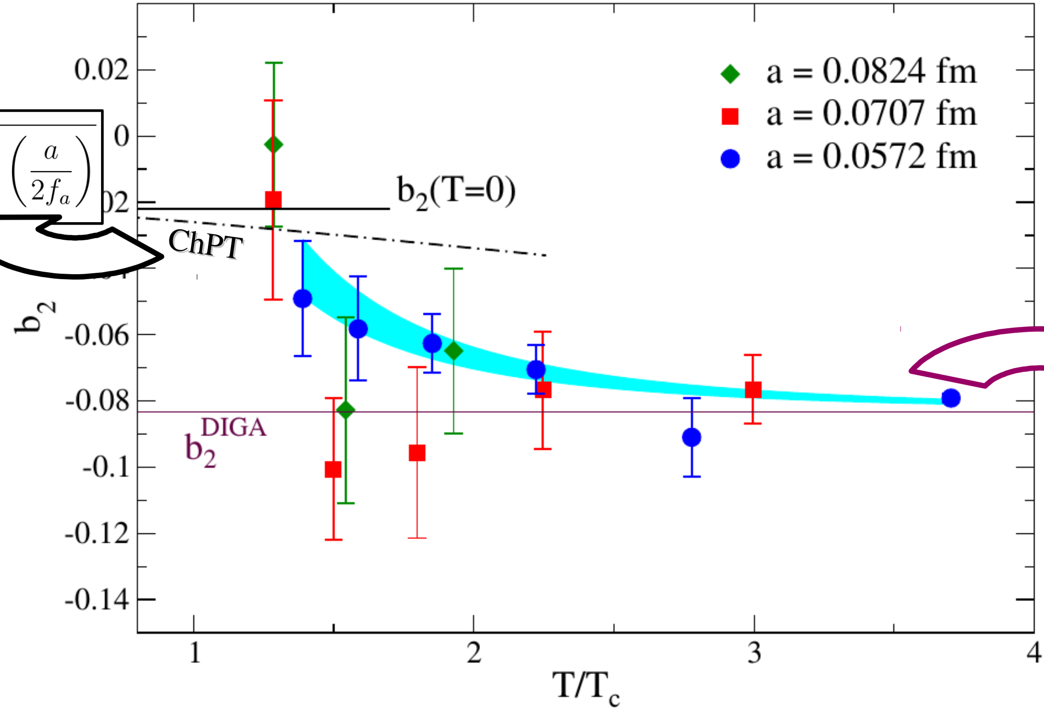
$$\boxed{V(a) = -m_\pi^2 f_\pi^2 \sqrt{1 - \frac{4m_u m_d}{(m_u+m_d)^2}\sin^2\left(\frac{a}{2f_a}\right)}}$$



the QCD axion from Lattice QCD

$$b_2 = -\frac{\langle Q^4\rangle_{\theta=0} - 3\langle Q^2\rangle^2_{\theta=0}}{12\langle Q^2\rangle_{\theta=0}} = \frac{\lambda_a}{12}\frac{f_a^2}{m_a^2}$$

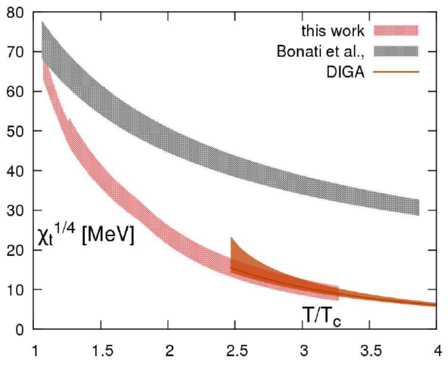
$$V(a) = -m_\pi^2 f_\pi^2 \sqrt{1 - \frac{4m_u m_d}{(m_u+m_d)^2}\sin^2\left(\frac{a}{2f_a}\right)}$$



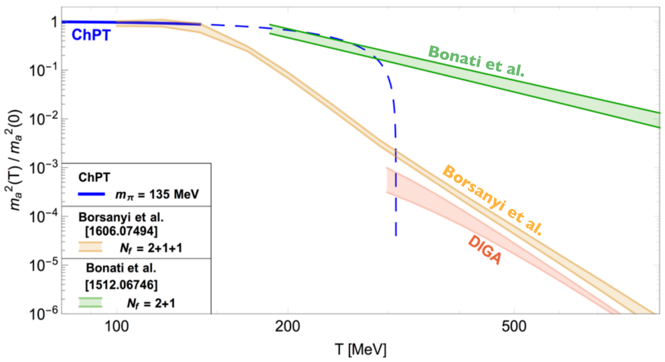
single-*cosine* potential

More Lattice Results:

Petreczky et al. [1606.03145]

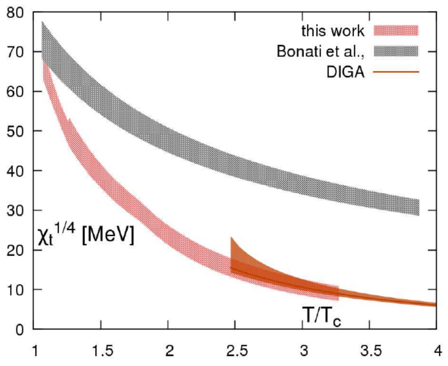


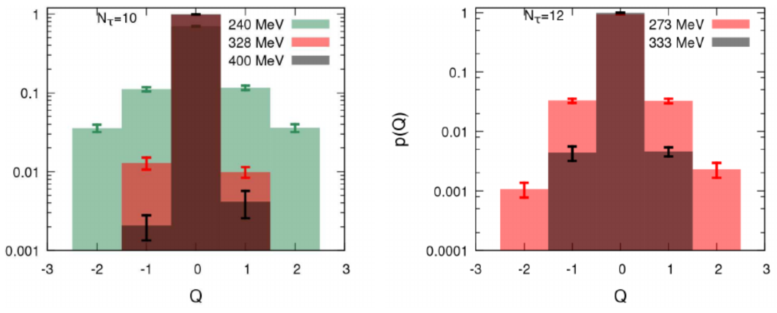
Borsanyi et al. [1606.07494]



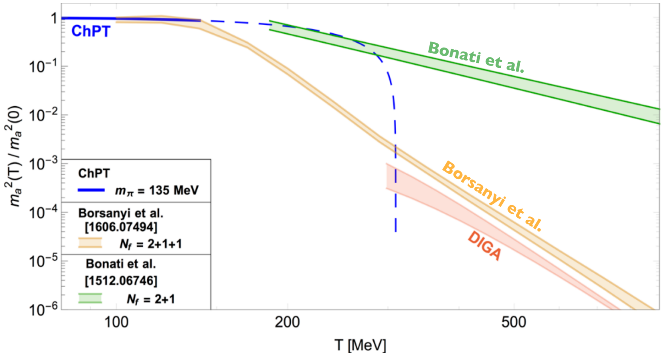
More Lattice Results:

Petreczky et al. [1606.03145]



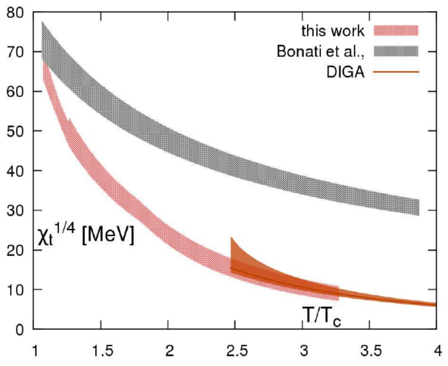


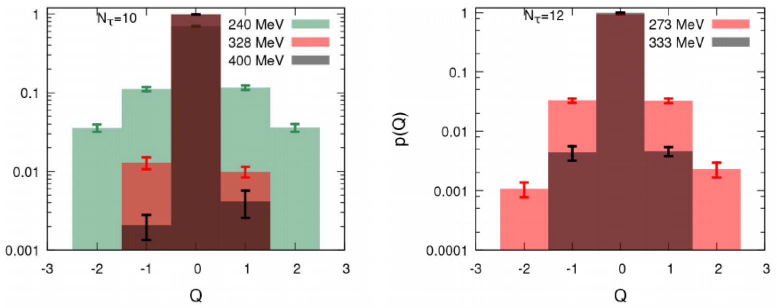
Borsanyi et al. [1606.07494]



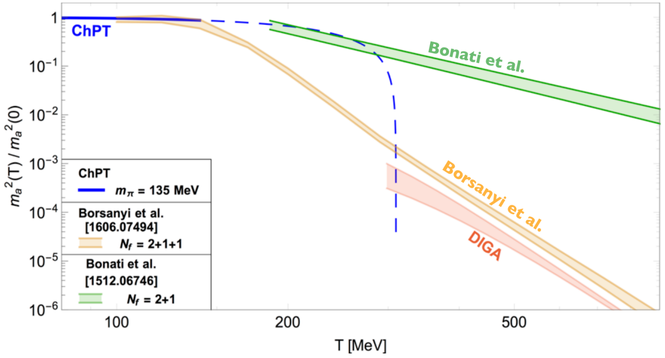
More Lattice Results:

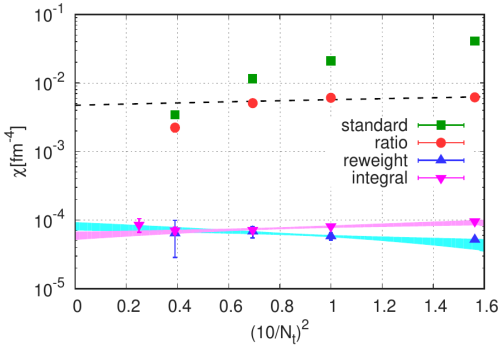
Petreczky et al. [1606.03145]

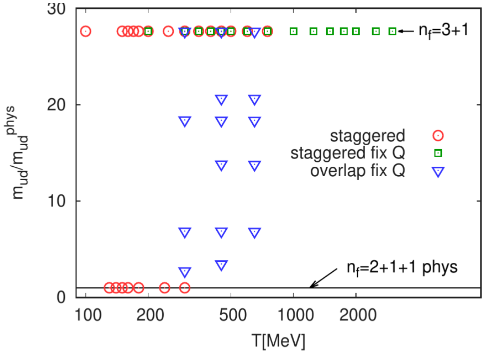




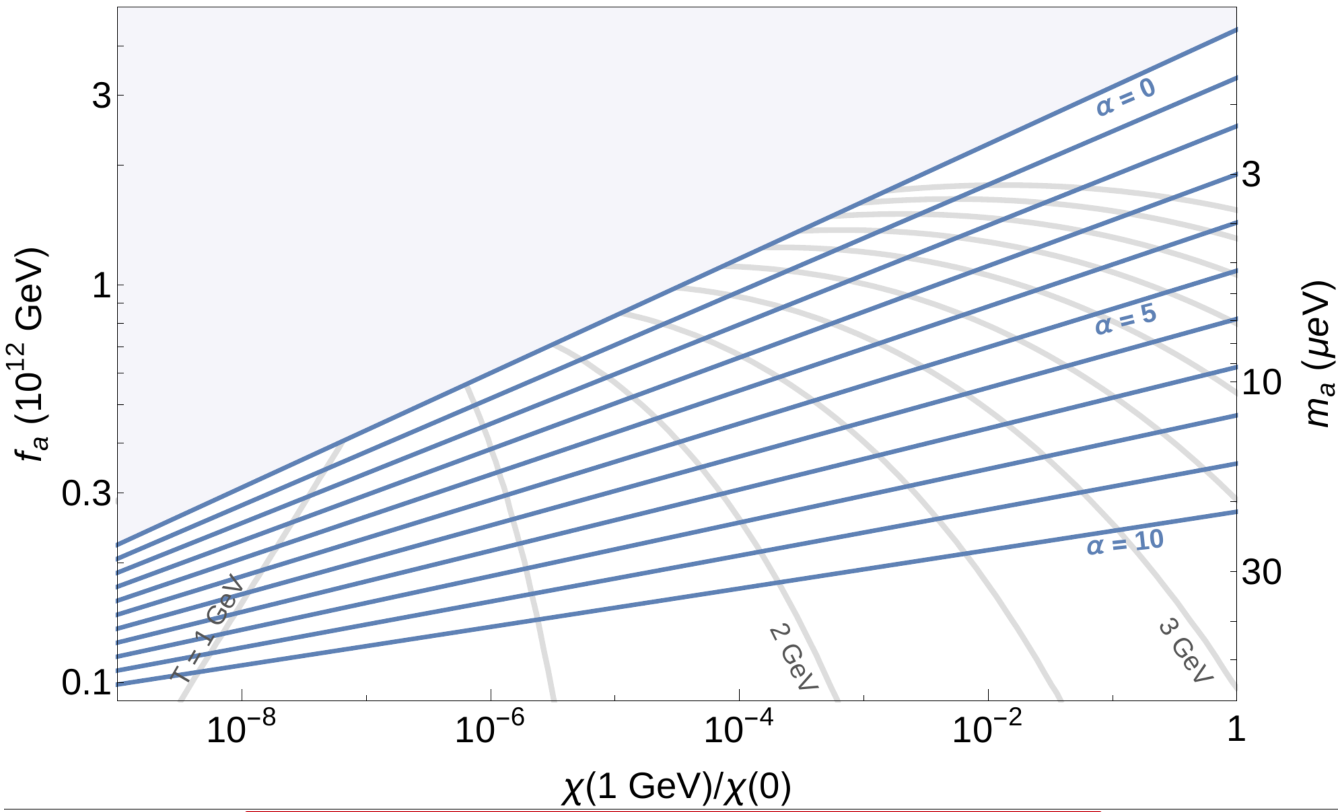
Borsanyi et al. [1606.07494]





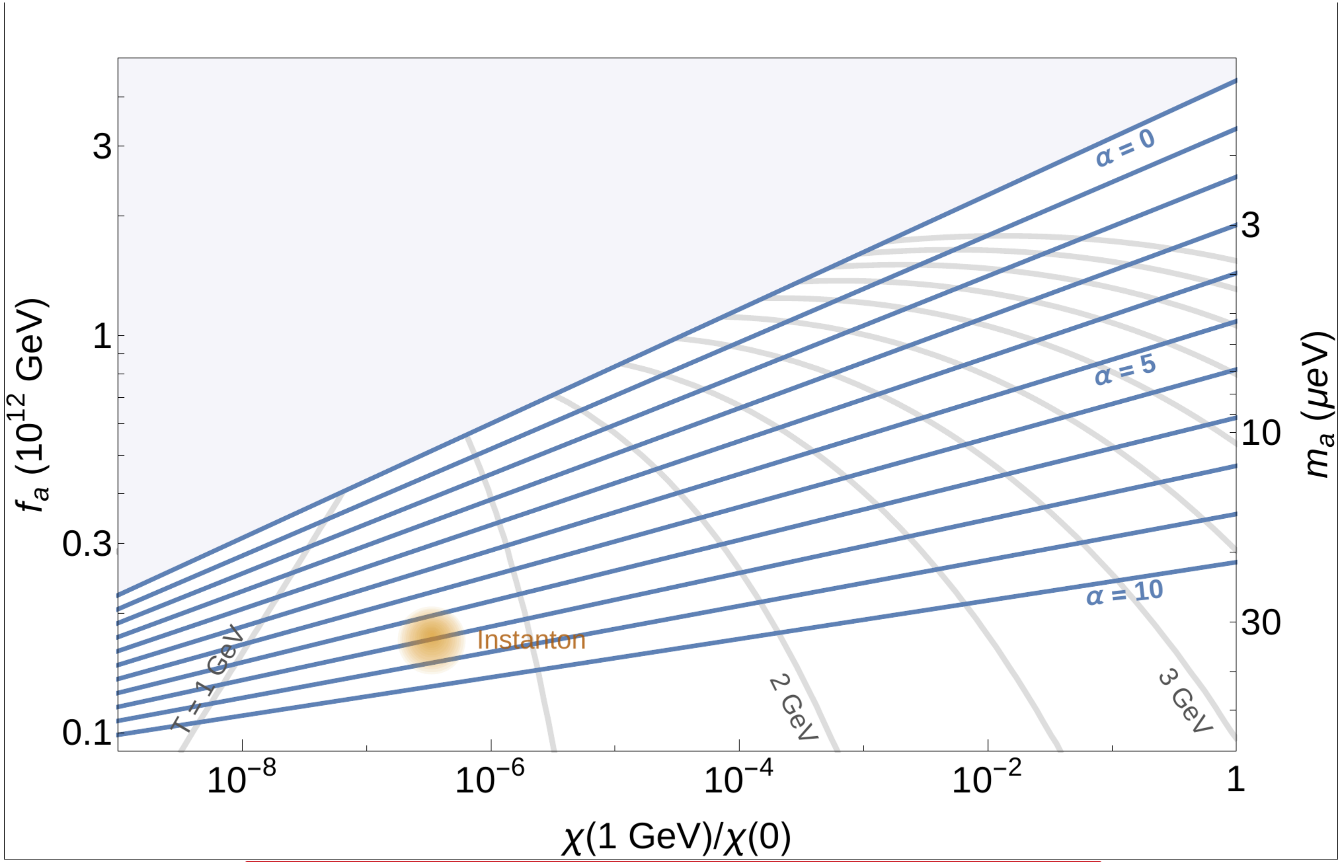


the QCD axion: *relic abundance*



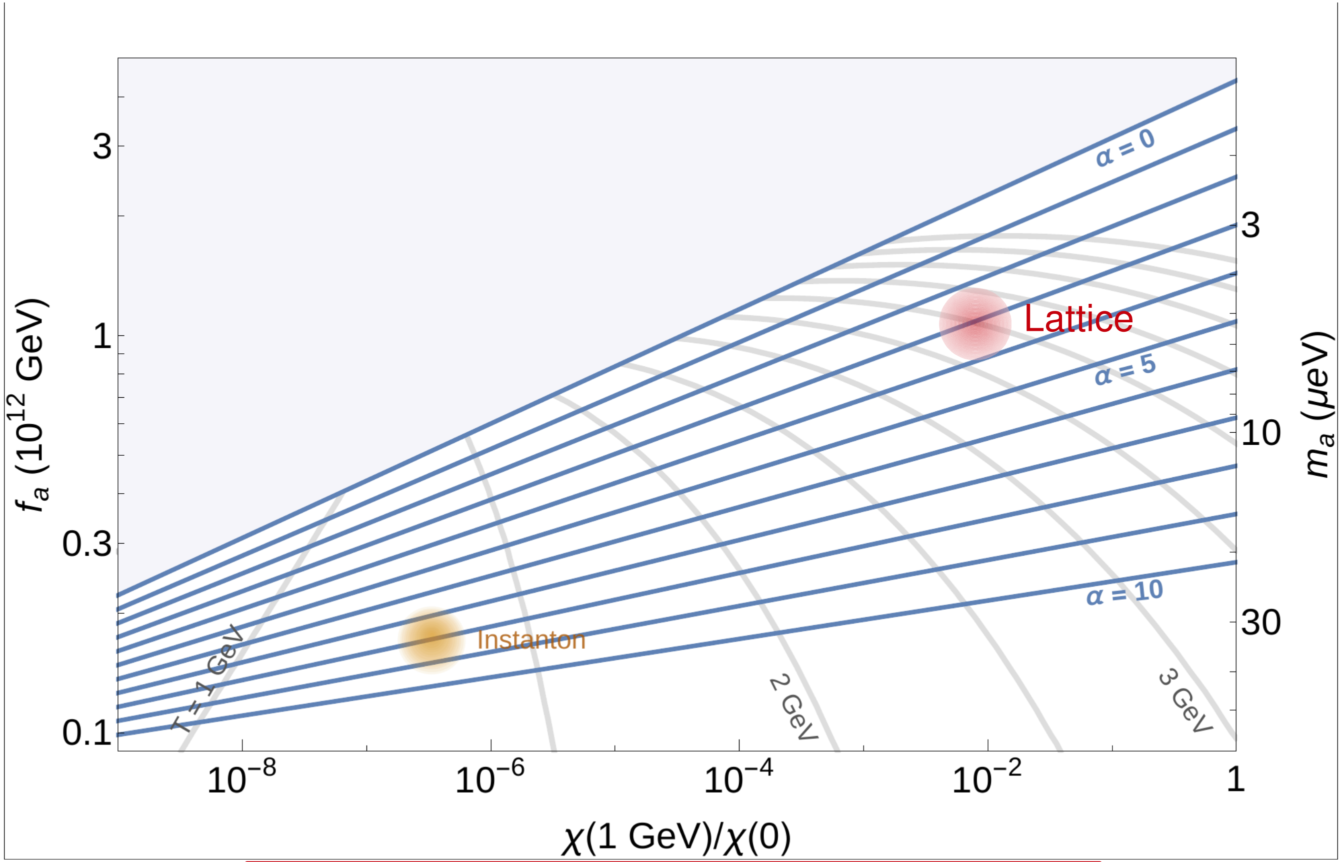
$$\boxed{m_a^2(T) = m_a^2(1\ \mathrm{GeV})\left(\frac{\mathrm{GeV}}{T}\right)^{\alpha} = m_a^2\frac{\chi(1\ \mathrm{GeV})}{\chi(0)}\left(\frac{\mathrm{GeV}}{T}\right)^{\alpha}}$$

the QCD axion: *relic abundance*



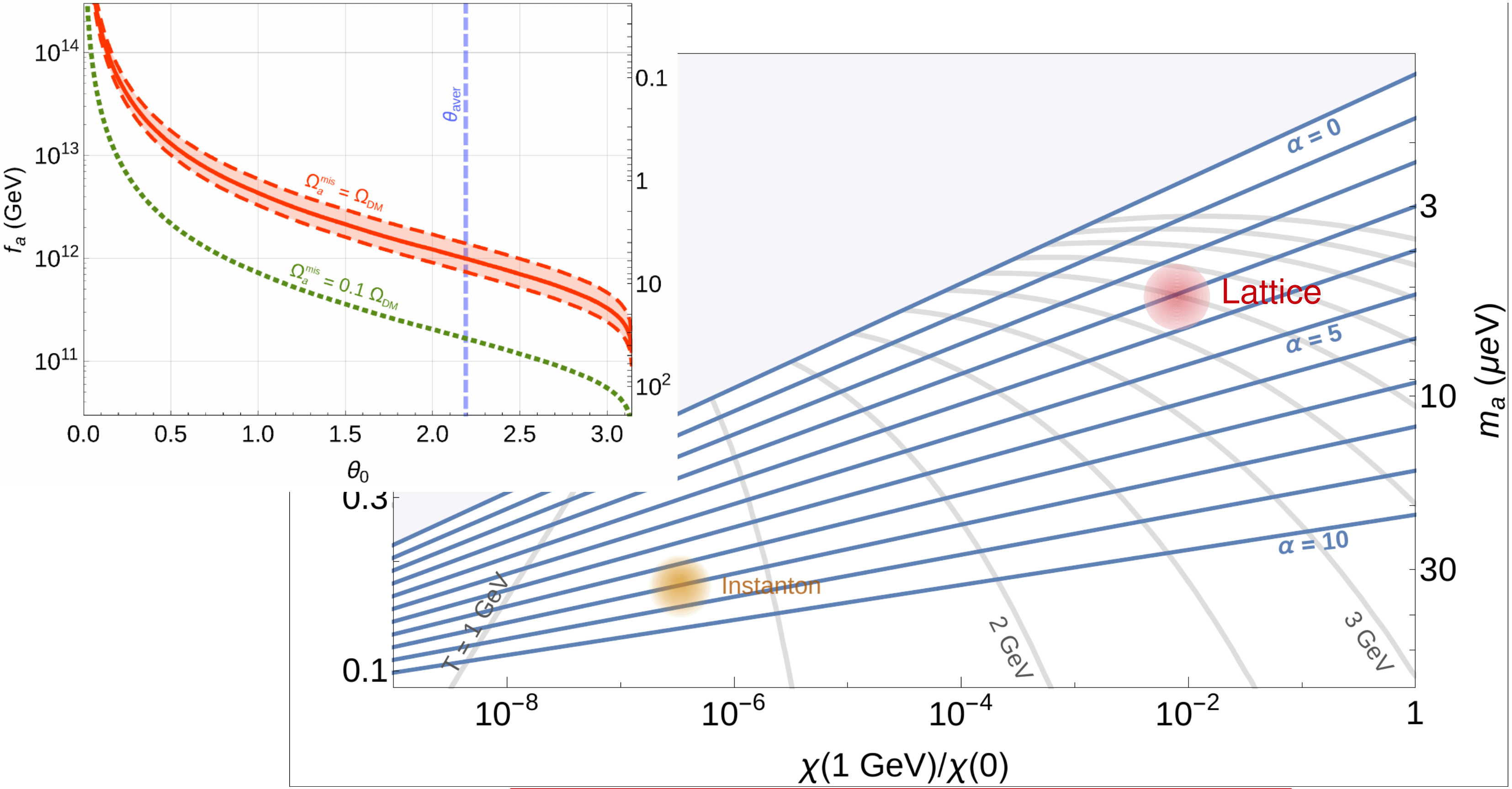
$$\boxed{m_a^2(T) = m_a^2(1\ \mathrm{GeV})\left(\frac{\mathrm{GeV}}{T}\right)^{\alpha} = m_a^2\frac{\chi(1\ \mathrm{GeV})}{\chi(0)}\left(\frac{\mathrm{GeV}}{T}\right)^{\alpha}}$$

the QCD axion: *relic abundance*



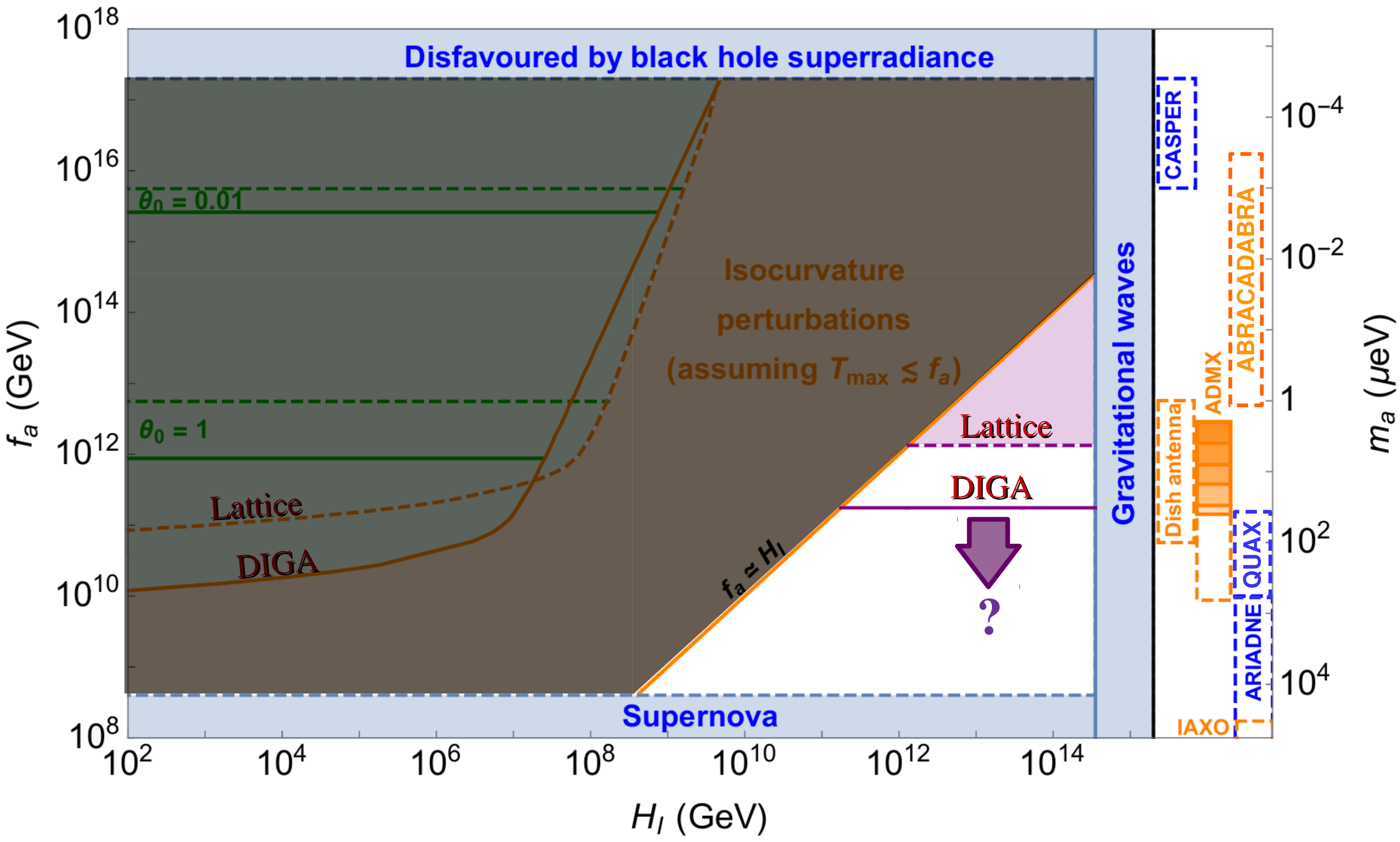
$$\boxed{m_a^2(T) = m_a^2(1\ \mathrm{GeV})\left(\frac{\mathrm{GeV}}{T}\right)^{\alpha} = m_a^2 \frac{\chi(1\ \mathrm{GeV})}{\chi(0)}\left(\frac{\mathrm{GeV}}{T}\right)^{\alpha}}$$

the QCD axion: *relic abundance*

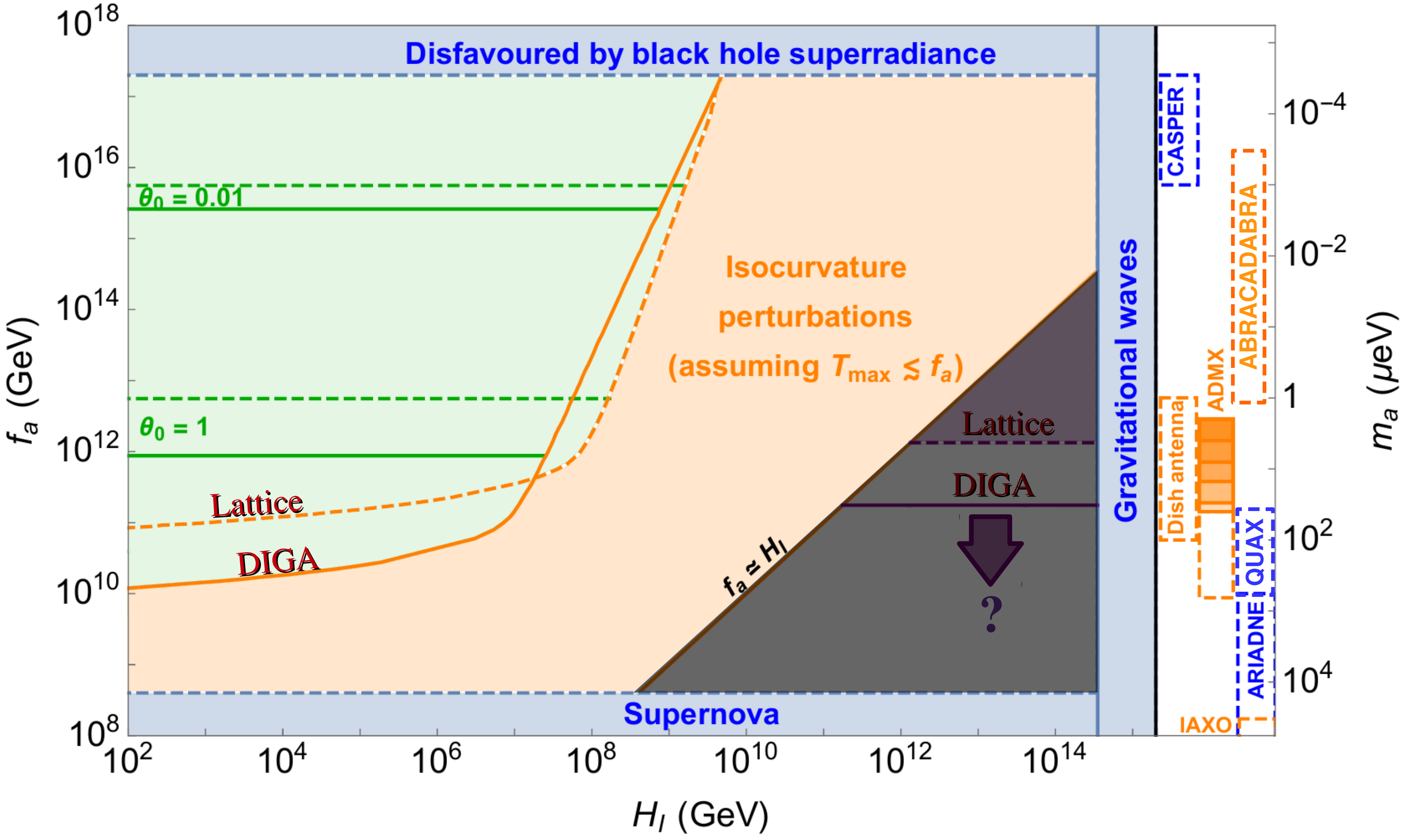


$$\boxed{m_a^2(T) = m_a^2(1\ \mathrm{GeV})\left(\frac{\mathrm{GeV}}{T}\right)^{\alpha} = m_a^2\frac{\chi(1\ \mathrm{GeV})}{\chi(0)}\left(\frac{\mathrm{GeV}}{T}\right)^{\alpha}}$$

the QCD axion: *parameter space*



the QCD axion: *parameter space*



Conclusions:

Precision QCD axion physics:

already @ 1% - 10% accuracy
(room for improvement)

High temperature:

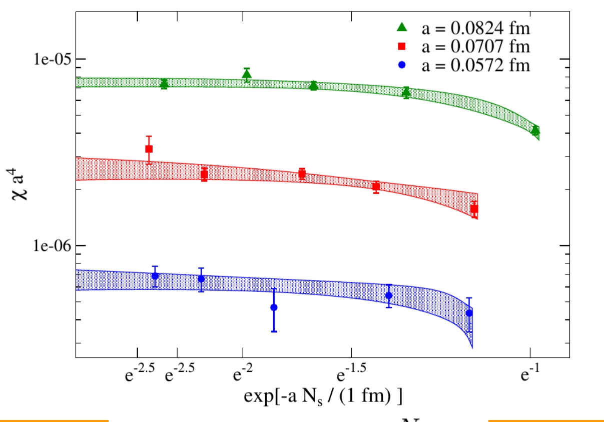
instantons unreliable → first lattice computations available
further studies required

To Do:

- CP violating couplings
- relic abundance from topological defects?

Backup

Volume dependence

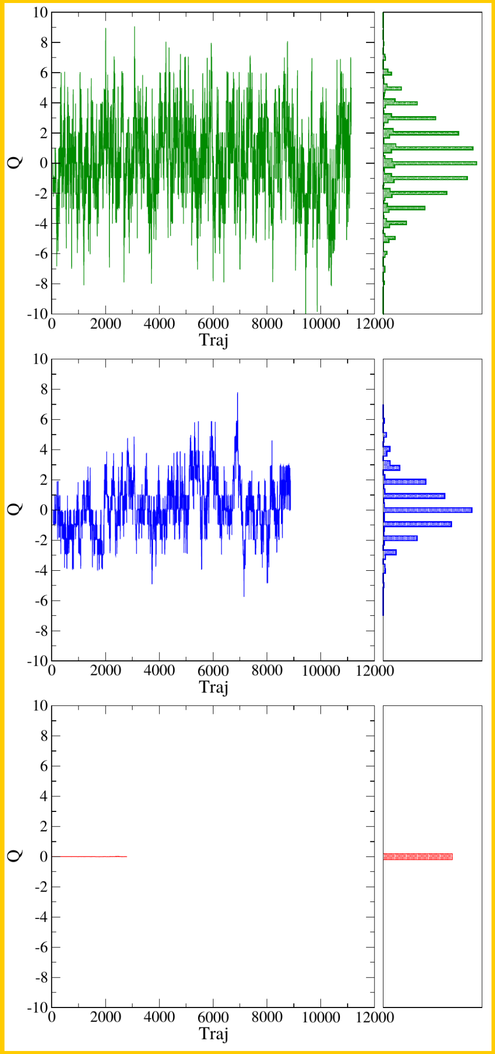


$$\chi_{N_s} \sim \chi_\infty + C e^{-a N_s m_{\eta'}}$$

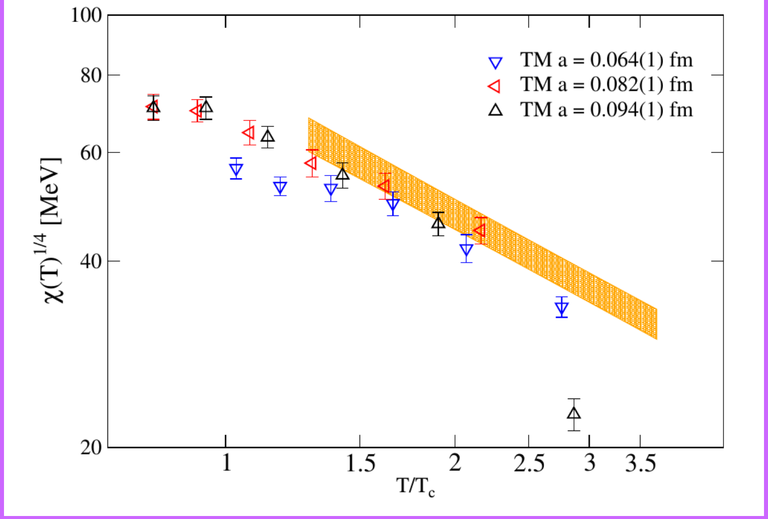
$$\chi = \int d^4x \langle q(x) q(0) \rangle_{\theta=0} = \frac{\langle Q^2 \rangle_{\theta=0}}{\mathcal{V}}$$

$$b_2 = -\frac{\langle Q^4 \rangle_{\theta=0} - 3\langle Q^2 \rangle_{\theta=0}^2}{12 \langle Q^2 \rangle_{\theta=0}}$$

freezing of topological charge



Comparisons with Trunin et al. '15



the QCD axion: *potential @ NLO*

$$V(a)^{\mathrm{NLO}} = -\, m_\pi^2\left(\tfrac{a}{f_a}\right) f_\pi^2 \left\{1 - 2\frac{m_\pi^2}{f_\pi^2}\left[l_3^r + l_4^r - \frac{(m_d - m_u)^2}{(m_d + m_u)^2} l_7^r - \frac{3}{64\pi^2}\log\left(\frac{m_\pi^2}{\mu^2}\right)\right]\right.$$

$$\left. + \frac{m_\pi^2\left(\frac{a}{f_a}\right)}{f_\pi^2}\left[h_1^r - h_3^r + l_3^r + \frac{4m_u^2 m_d^2}{(m_u + m_d)^4}\frac{m_\pi^8 \sin^2\left(\frac{a}{f_a}\right)}{m_\pi^8\left(\frac{a}{f_a}\right)} l_7^r - \frac{3}{64\pi^2}\left(\log\left(\frac{m_\pi^2\left(\frac{a}{f_a}\right)}{\mu^2}\right) - \frac{1}{2}\right)\right]\right\}$$

$$m_\pi^2(\theta) = m_\pi^2 \sqrt{1 - \frac{4m_u m_d}{(m_u + m_d)^2}\sin^2\left(\frac{\theta}{2}\right)}$$

the QCD axion: *relic abundance*

$$\Omega_a = \frac{86}{33}\frac{\Omega_\gamma}{T_\gamma}\frac{n_a^\star}{s^\star}m_a$$

the QCD axion: *relic abundance*

$$\Omega_a = \frac{86}{33}\frac{\Omega_\gamma}{T_\gamma}\frac{n_a^\star}{s^\star}m_a$$

$$\boxed{n_a = \langle m_a a^2 \rangle}$$

the QCD axion: *relic abundance*

$$\Omega_a = \frac{86}{33}\frac{\Omega_\gamma}{T_\gamma}\frac{n_a^\star}{s^\star}m_a$$

$$\boxed{n_a = \langle m_a a^2 \rangle}$$

$$\boxed{\ddot{a} + 3H\dot{a} + m_a^2(T)\, f_a \sin\left(\frac{a}{f_a}\right) = 0}$$

the QCD axion: *relic abundance*

$$\Omega_a = \frac{86}{33}\frac{\Omega_\gamma}{T_\gamma}\frac{n_a^\star}{s^\star}m_a$$

$$\boxed{n_a = \langle m_a a^2 \rangle}$$

$$\boxed{\ddot{a} + 3H\dot{a} + m_a^2(T) f_a \sin\left(\frac{a}{f_a}\right) = 0}$$

$$\boxed{m_a^2(T) = m_a^2(1\ \mathrm{GeV})\left(\frac{\mathrm{GeV}}{T}\right)^\alpha = m_a^2\frac{\chi(1\ \mathrm{GeV})}{\chi(0)}\left(\frac{\mathrm{GeV}}{T}\right)^\alpha}$$