Nuclear Shapes

Lecture IV

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Selected Topics in Nuclear and Atomic Physics
Fiera di Primiero
2.-6. October 2017


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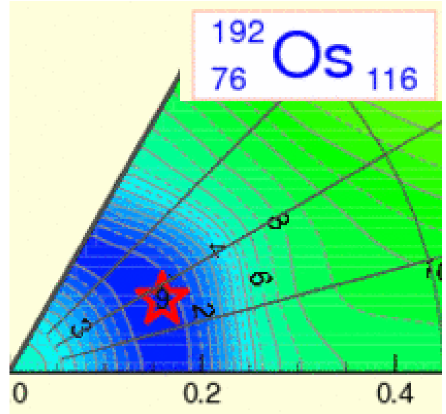
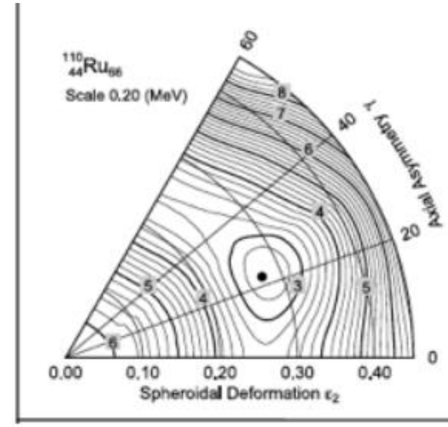
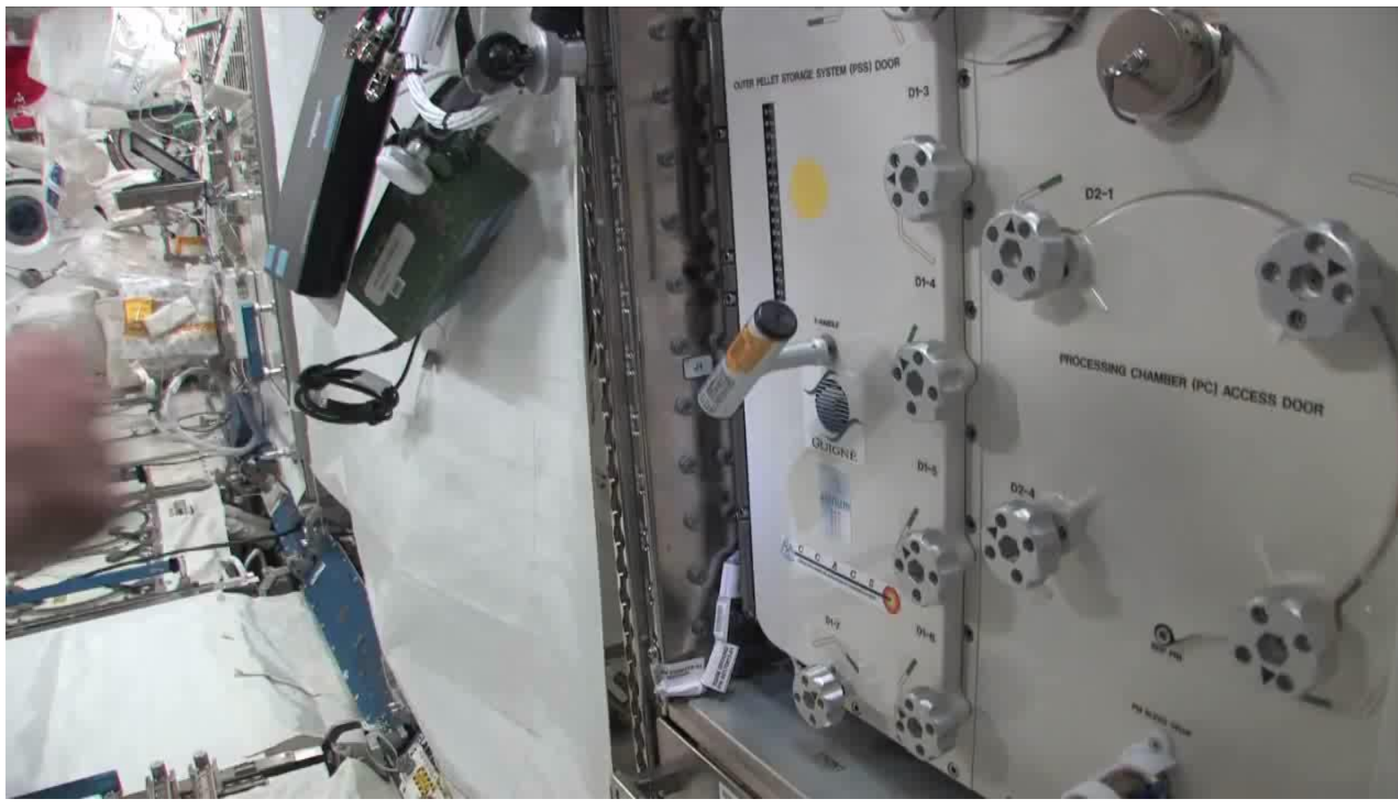


gamma-soft



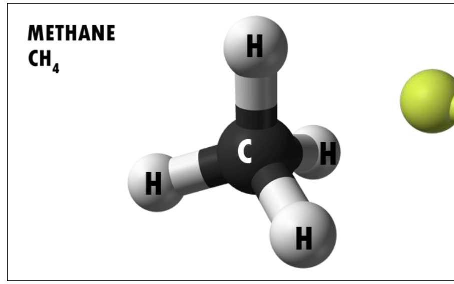
more gamma-rigid

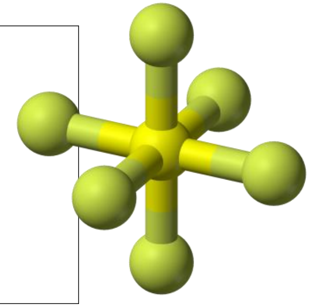
what happens if we rotate a triaxial body?



Molecular spectroscopy

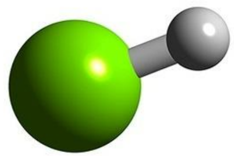
spherical rotor: CH_4, SF_6

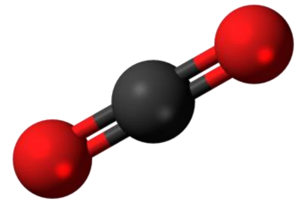




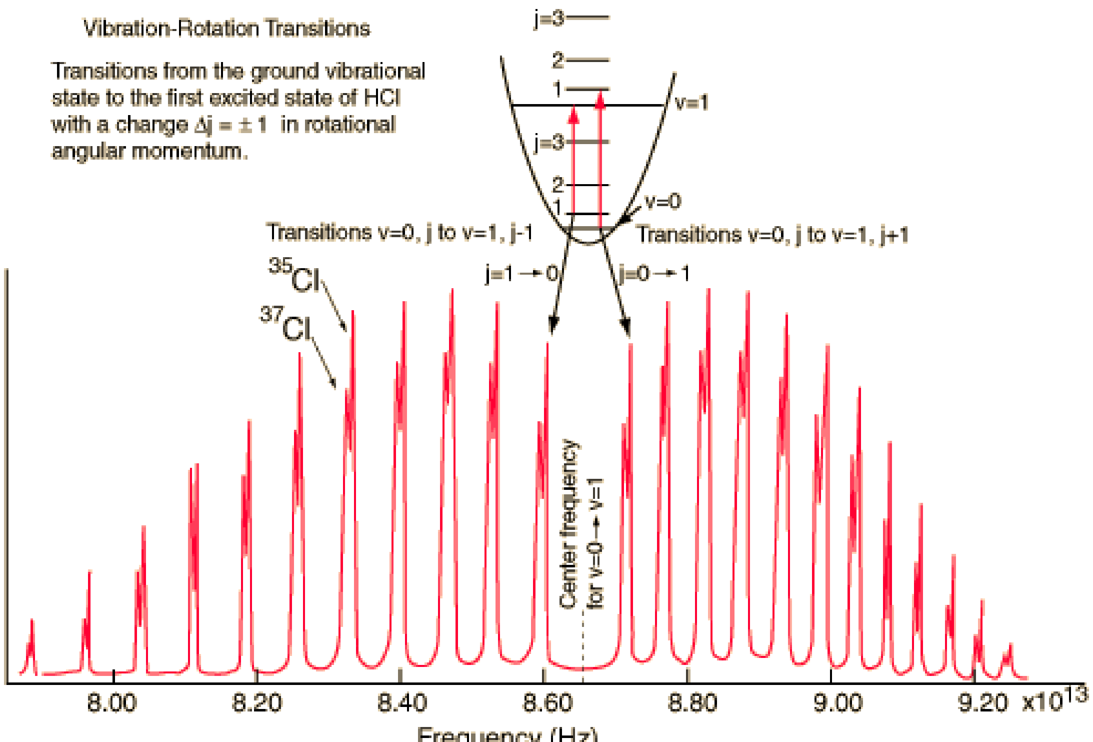
$$\mathfrak{I}_a = \mathfrak{I}_b = \mathfrak{I}_c$$

linear rotor: HCl, CO_2



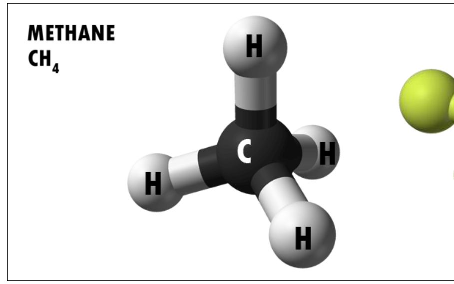


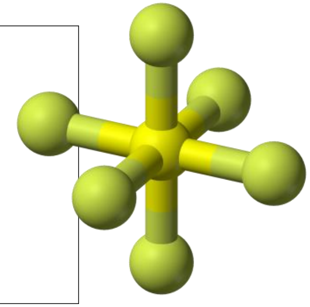
$$\mathfrak{I}_a = 0, \quad \mathfrak{I}_b = \mathfrak{I}_c$$



Molecular spectroscopy

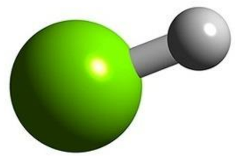
spherical rotor: CH_4, SF_6

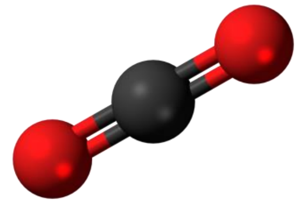




$$\mathfrak{I}_a = \mathfrak{I}_b = \mathfrak{I}_c$$

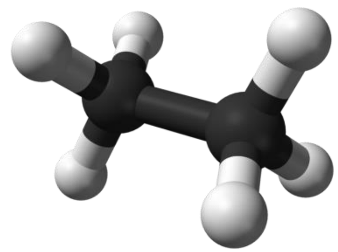
linear rotor: HCl, CO_2





$$\mathfrak{I}_a = 0, \quad \mathfrak{I}_b = \mathfrak{I}_c$$

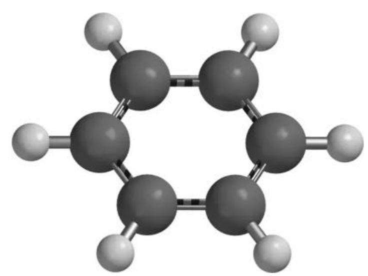
symmetric prolate rotor: C_2H_6



$$\mathfrak{I}_a < \mathfrak{I}_b = \mathfrak{I}_c$$

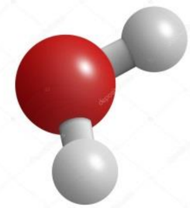
$$E_{J,K} = \frac{J(J+1)\hbar^2}{2\mathfrak{I}_b} + \left(\frac{1}{2\mathfrak{I}_a} - \frac{1}{2\mathfrak{I}_b}\right) K^2 \hbar^2$$

symmetric oblate rotor: C_6H_6

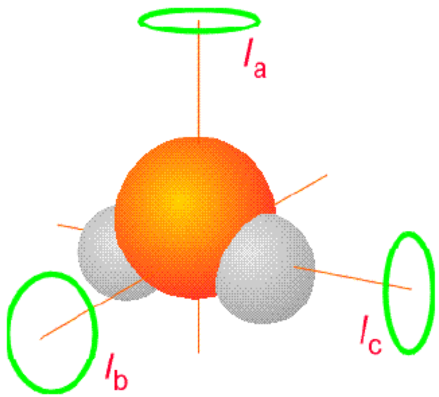


$$\mathfrak{I}_a = \mathfrak{I}_b < \mathfrak{I}_c$$

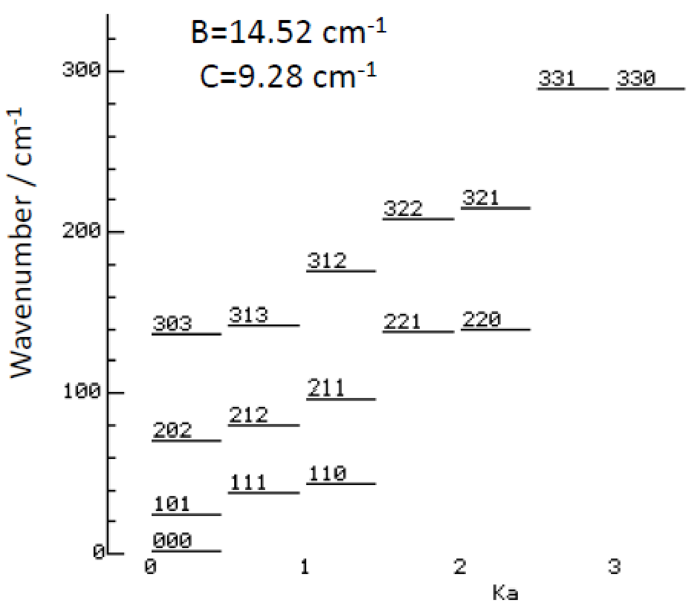
asymmetric rotor: H_2O

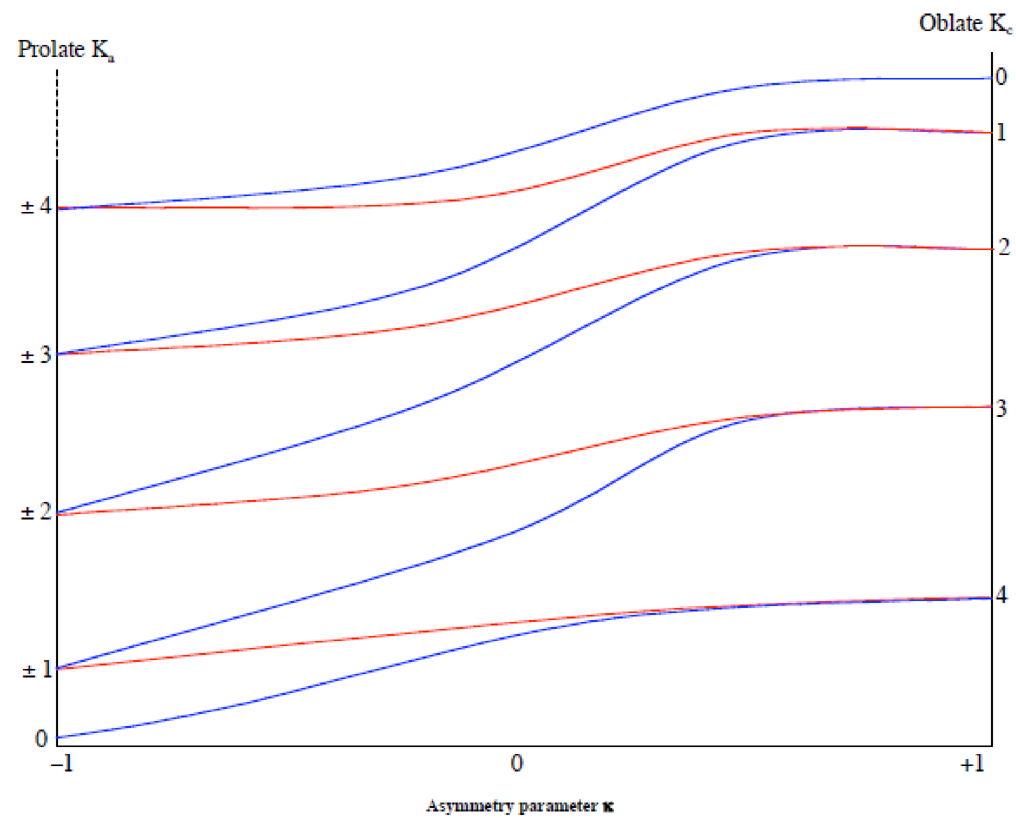


$$\Im_a < \Im_b < \Im_c$$



states classified by $(J K_a K_c)$





asymmetry parameter: $\kappa = \frac{2B-A-C}{A-C}$

Triaxial nuclei and wobbling

Bohr and Mottelson apply this for triaxial nuclei:

$$\mathfrak{J}_a > \mathfrak{J}_b > \mathfrak{J}_c$$

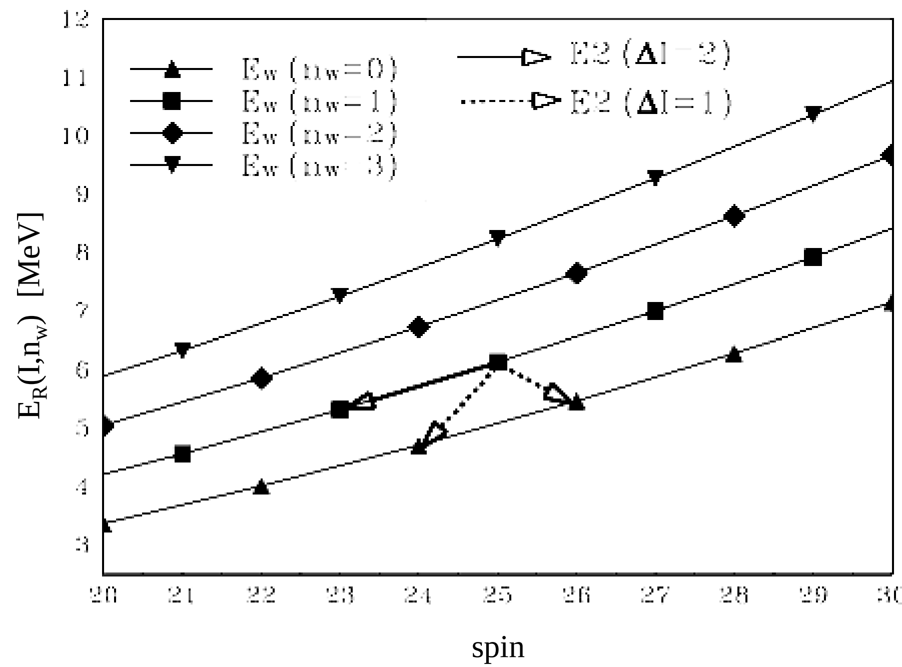
high spin: $I \approx I_x \gg 1$

$$E(I, n_w) = \frac{I(I+1)}{2\mathfrak{J}_x} + \hbar\omega_w(n_w + \tfrac{1}{2})$$

n_w wobbling phonon number
ω_w wobbling frquency

$$\hbar\omega_w = \frac{I}{\mathfrak{J}_a}\sqrt{\frac{(\mathfrak{J}_a - \mathfrak{J}_b)(\mathfrak{J}_a - \mathfrak{J}_c)}{\mathfrak{J}_b \mathfrak{J}_c}}$$

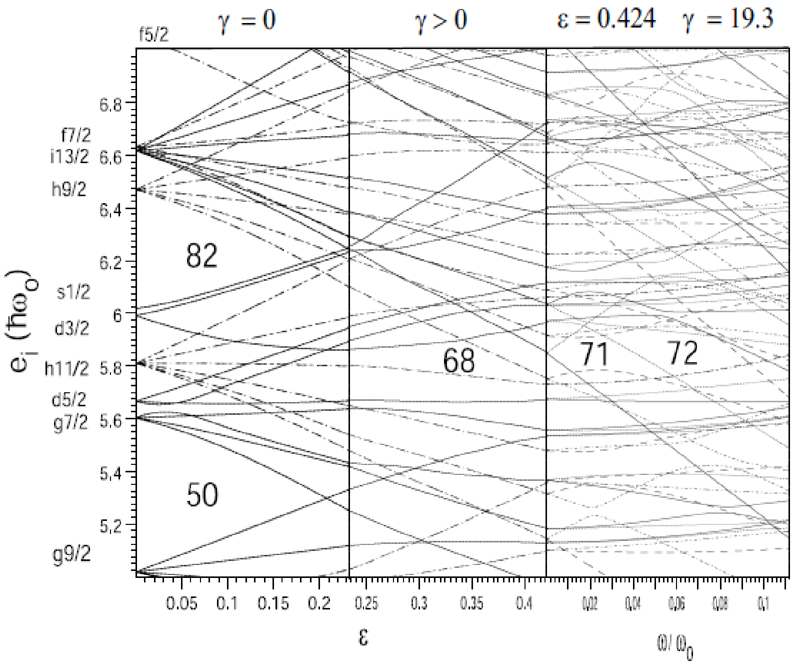
Bohr & Mottelson, Vol. 2, p.190 ff



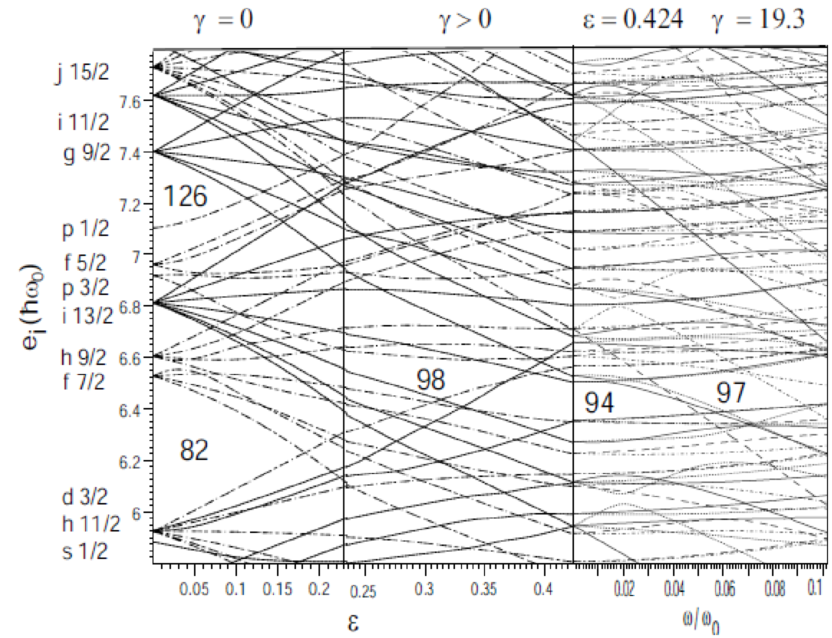
- Family of bands with similar rotational properties.
- Each band characterized by the wobbling phonon number n_w.
- Collective E2 inter-band decay compete with in-band transitions.

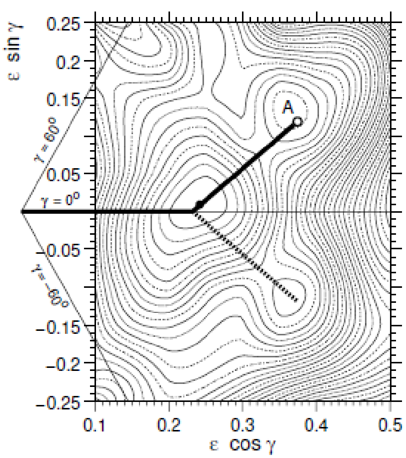
Where can we find nuclei with triaxial shape (at high spin) ?

Single Proton Levels



Single Neutron Levels

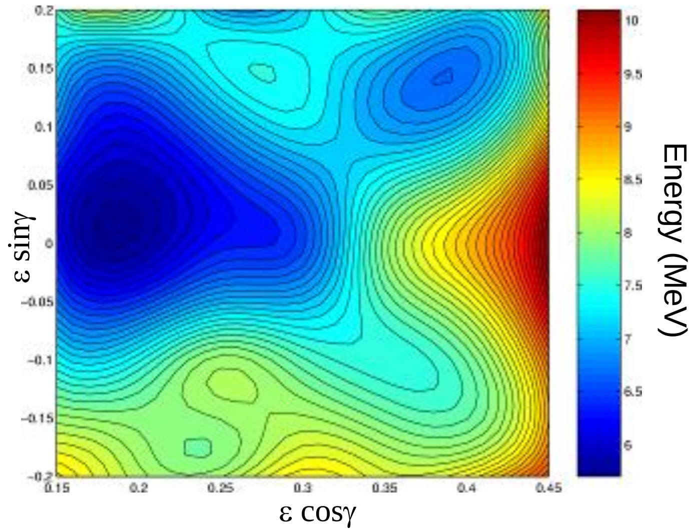




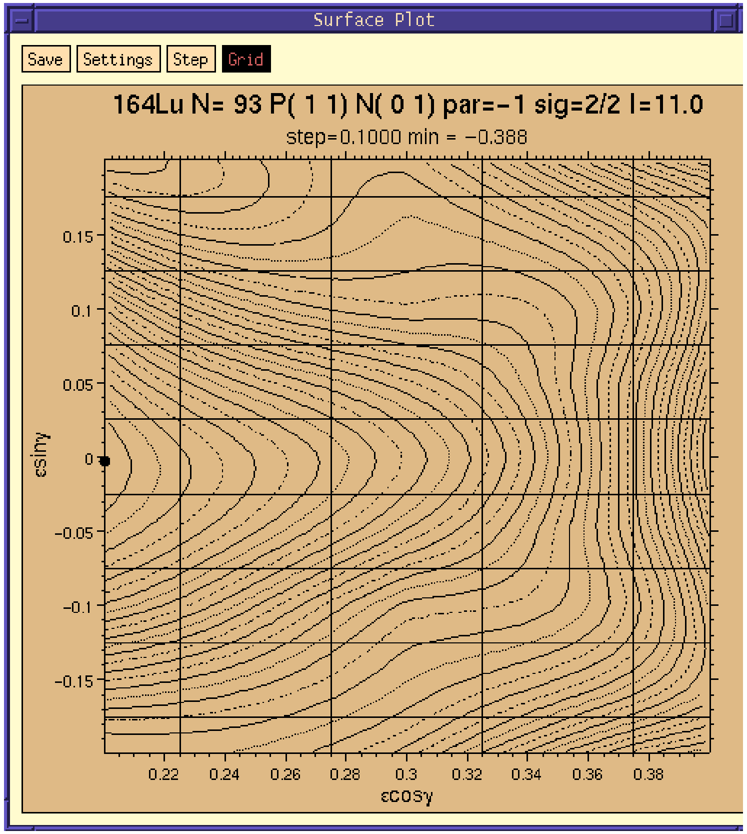
R. Bengtsson, H. Ryde
Eur. Phys. J. A 22, 355 (2004)

Strongly deformed triaxial shapes stabilized by shell gaps in the region $Z \approx 71$ and $N \approx 94$ (^{165}Lu)

Triaxial superdeformation



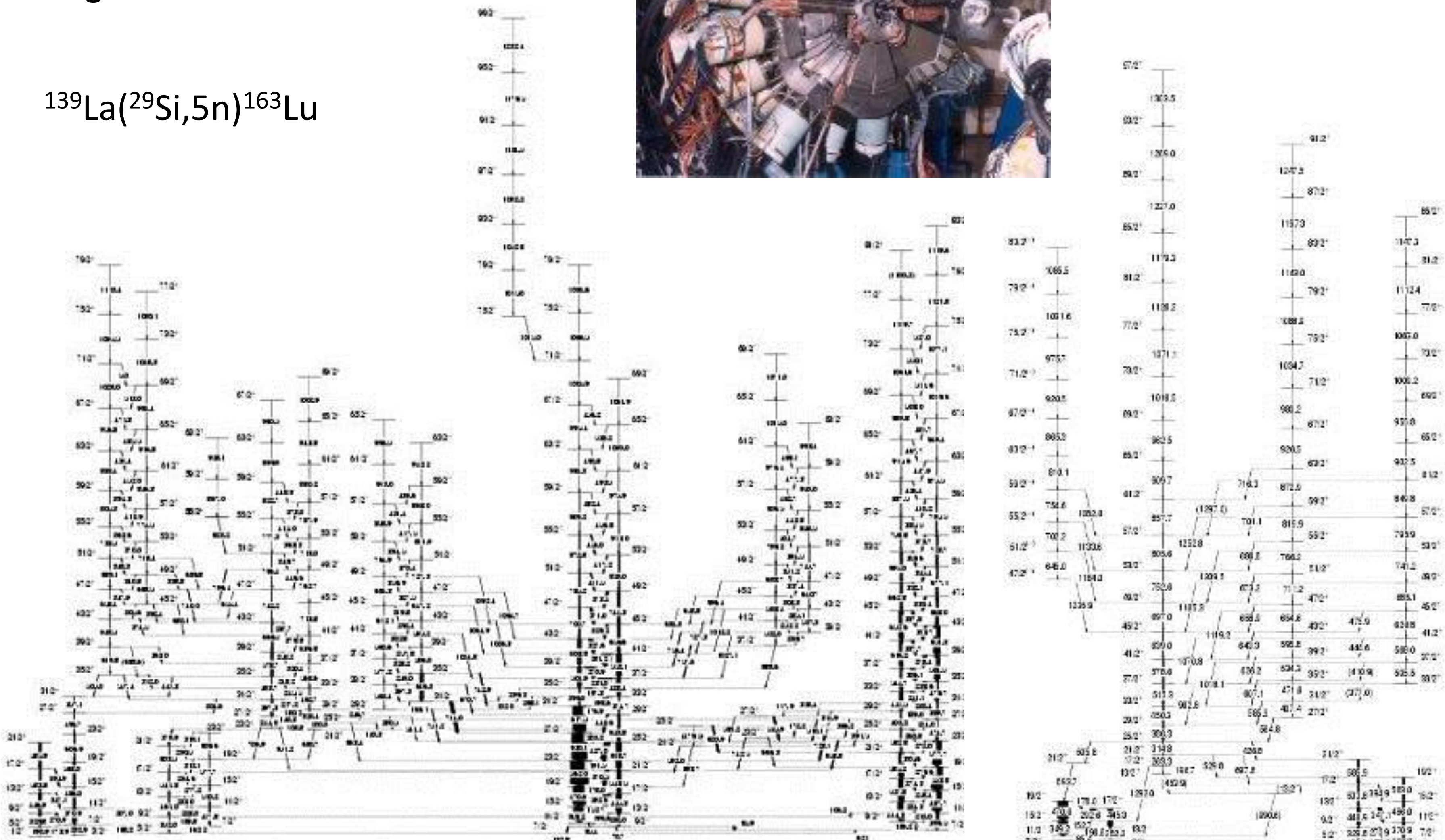
Potential energy surface for ^{163}Lu at I=57/2



Level scheme of ^{163}Lu

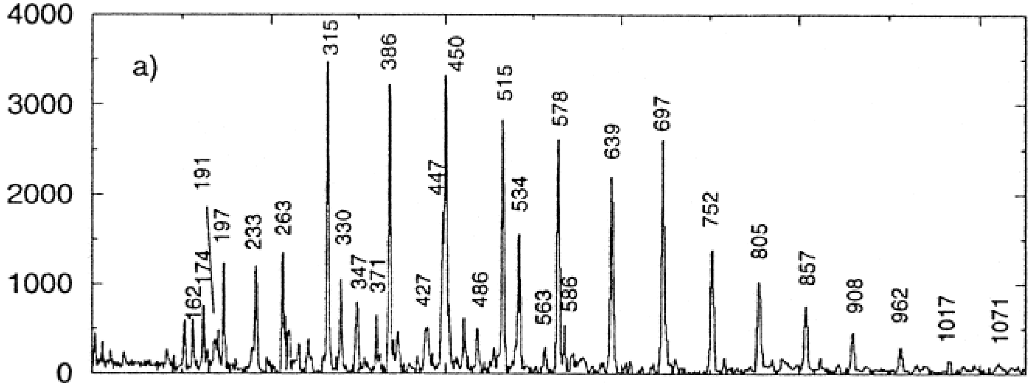
277 levels
489 gammas

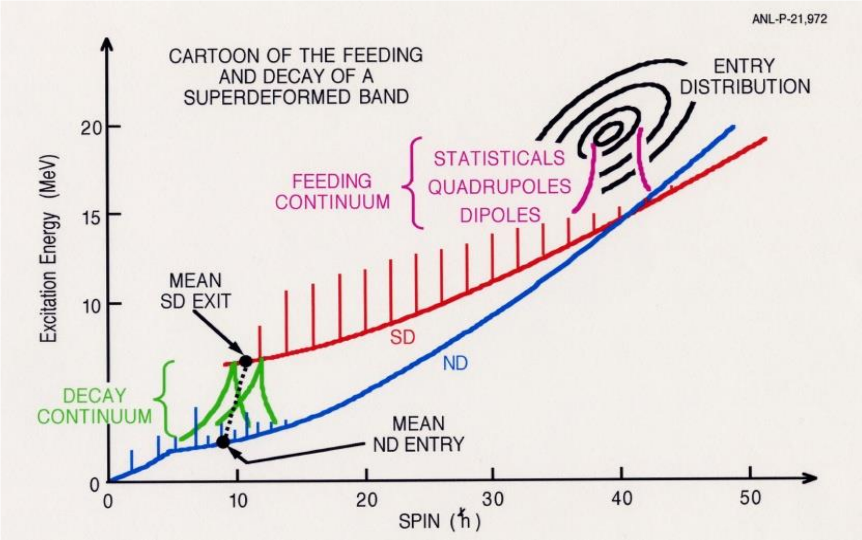
$^{139}La(^{29}Si,5n)^{163}Lu$



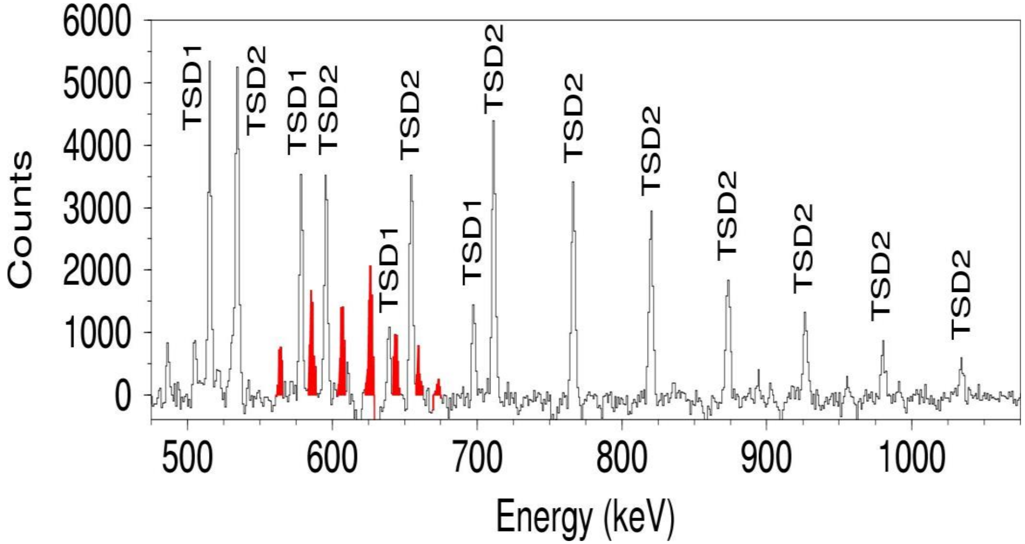
D.R. Jensen et al., Phys. Rev. Lett. 89, 142503 (2002)
Nucl. Phys. A 703, 3 (2002)

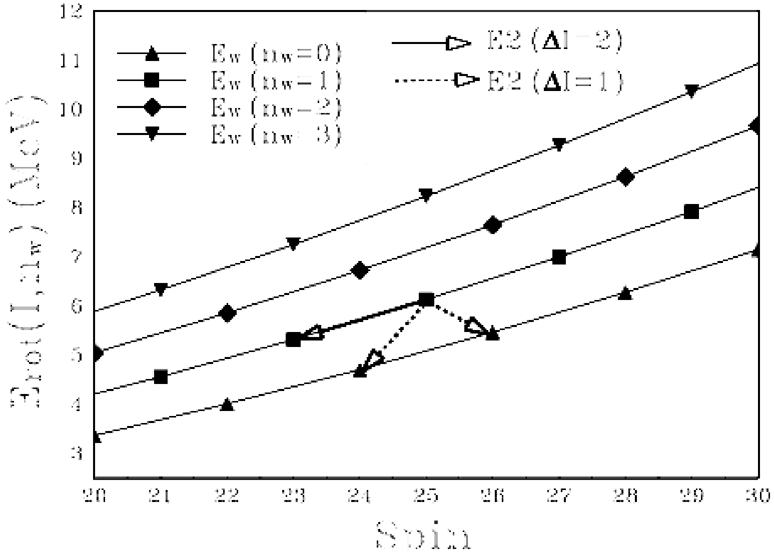
Triaxial superdeformed (TSD) bands in ^{163}Lu





J.Domscheit et al., Nucl. Phys. A 660, 381 (1999)





D.R.Jensen et al., Nucl. Phys. A 703, 3 (2002)

Level scheme of ^{163}Lu

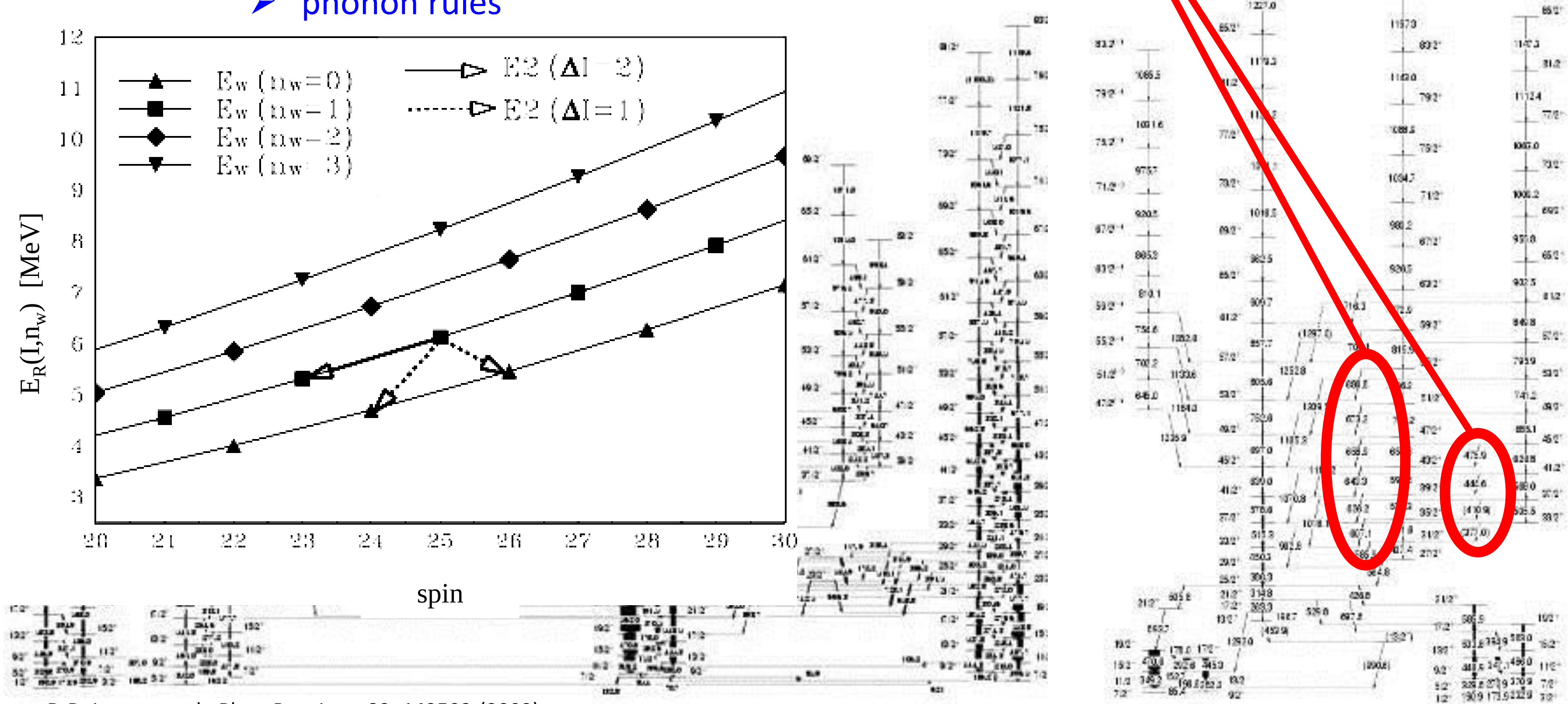
triaxial superdeformed wobbling bands

277 levels
489 gammas

Wobbling scenario requires inter-band transitions with:

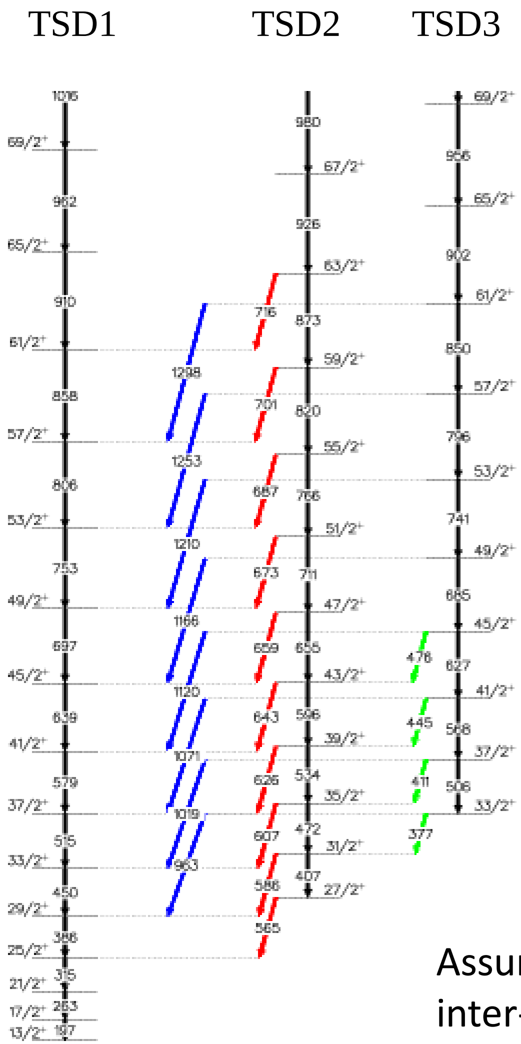
- $\Delta I = 1$
- E2 character
- phonon rules

properties of inter-band transitions

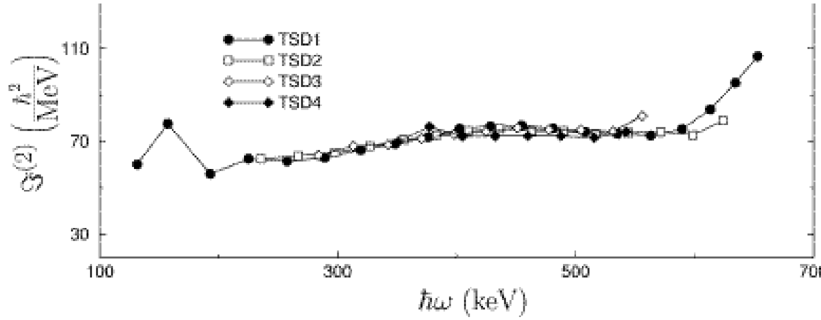


D.R. Jensen et al., Phys. Rev. Lett. 89, 142503 (2002)
Nucl. Phys. A 703, 3 (2002)

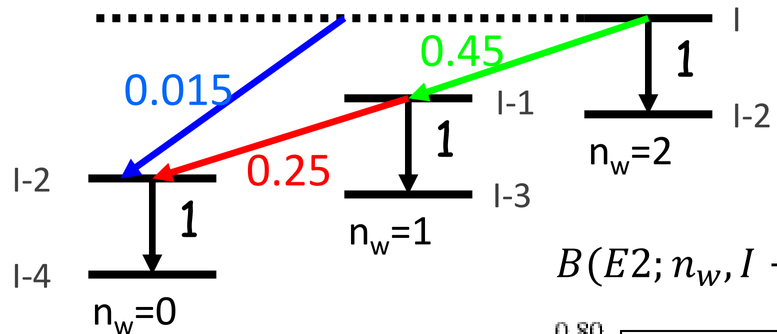
Evidence for the wobbling mode in ^{163}Lu



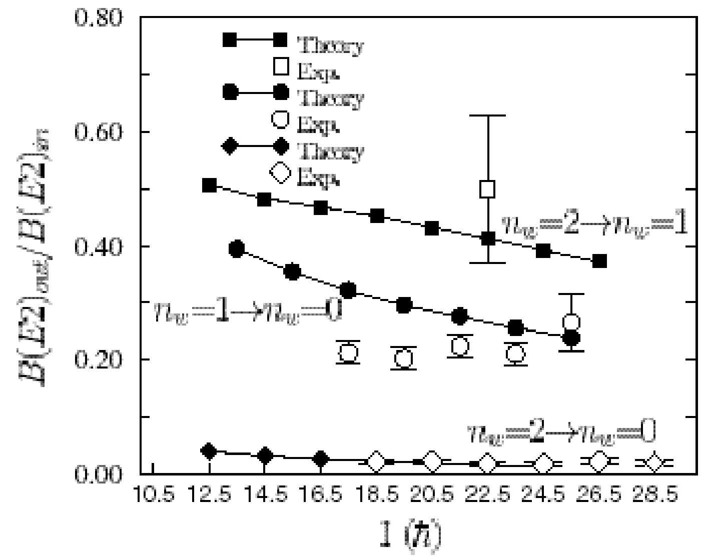
Family of bands
with very similar
rotational properties.



Particle Rotor B(E2)



$$B(E2; n_w, I \rightarrow n_w - 1, I - 1) \propto \frac{n_w}{I}$$

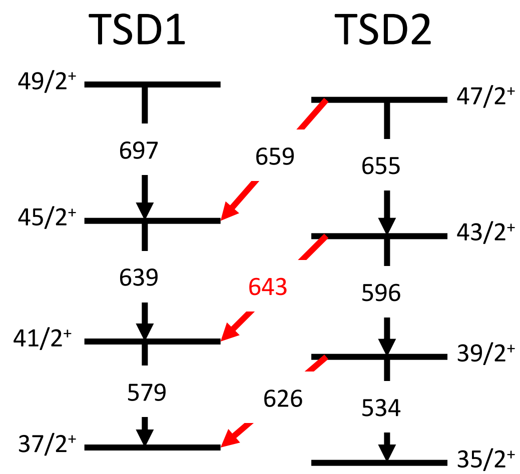


Assuming E2 character for the inter-band transitions, they follow the phonon rule.

D.R. Jensen et al.,
Phys. Rev. Lett. 89, 142503 (2002)

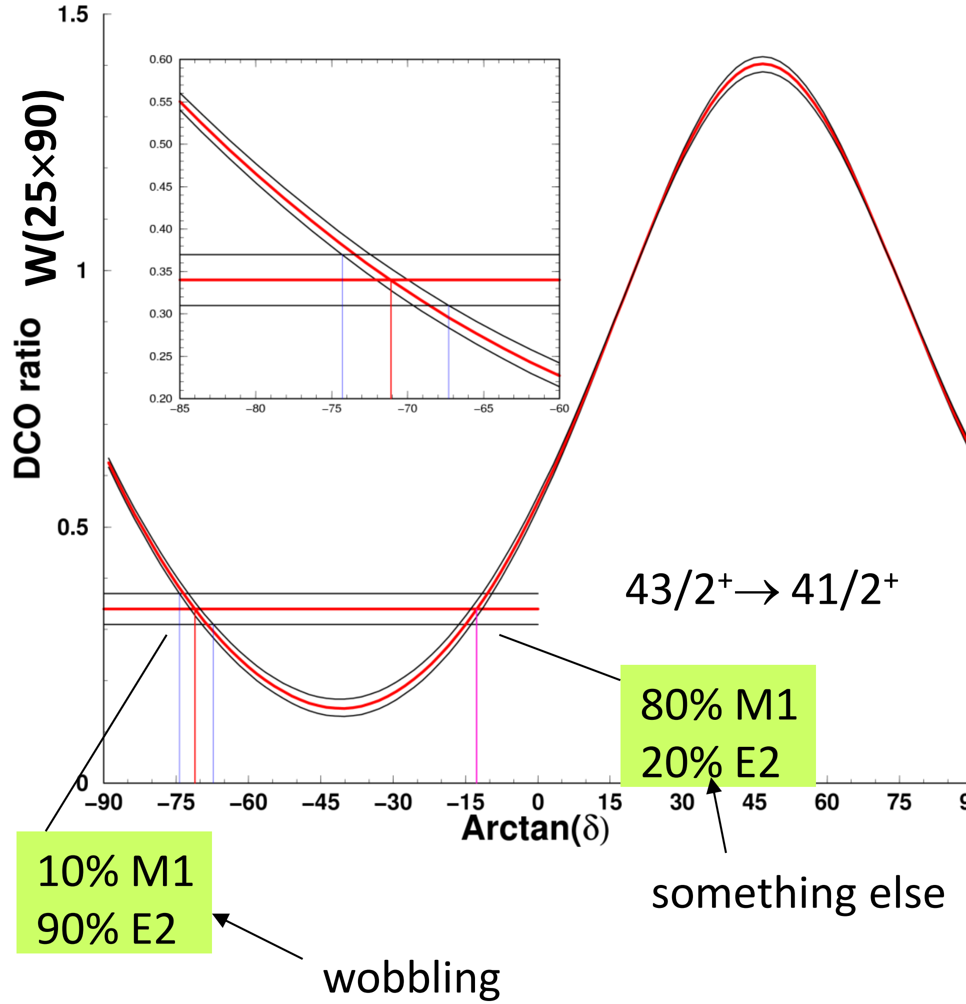
Are they E2?

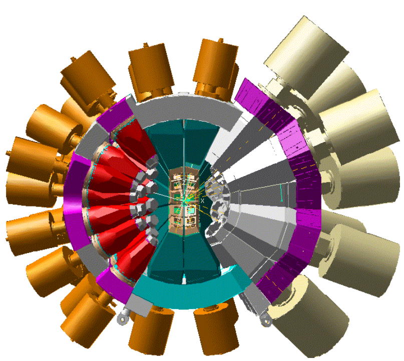
Properties of inter-band transitions



angular distribution (DCO ratio)
for inter-band transitions

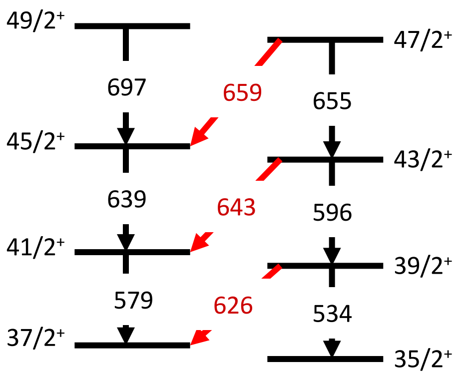
Two possible solutions



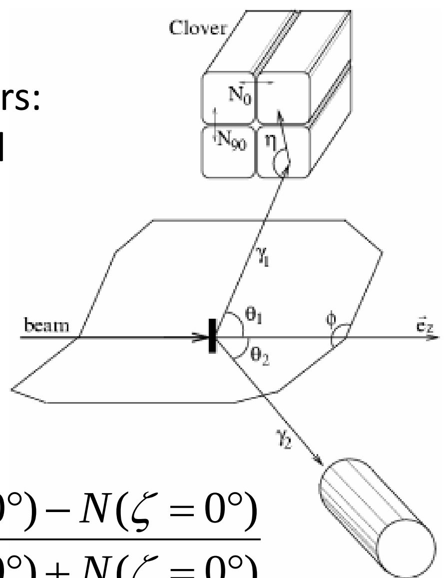


beam axis as quantization axis
probability of emission forward/backward
relative to emission at 90°

Polarization measurement in ^{163}Lu

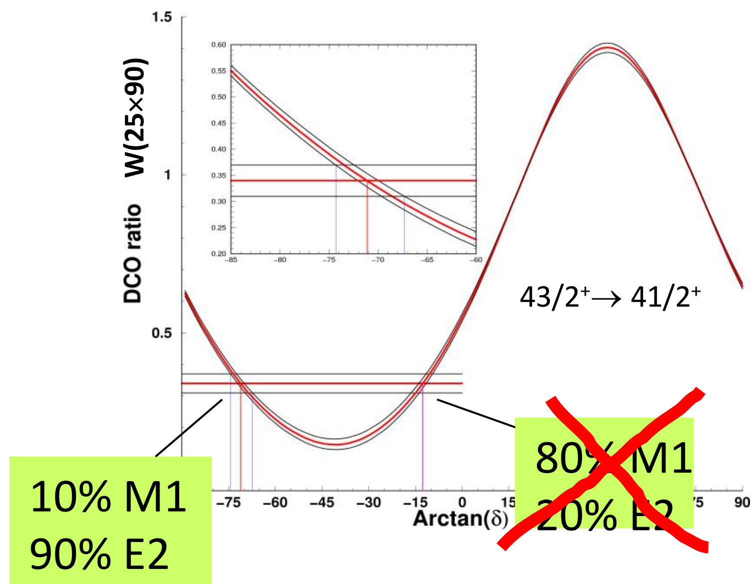


Clover detectors as Compton polarimeters: horizontal vs. vertical scattering



$$P = \frac{A}{Q} = \frac{1}{Q} \frac{N(\zeta = 90°) - N(\zeta = 0°)}{N(\zeta = 90°) + N(\zeta = 0°)}$$

	Eγ	$A = \frac{N(90°) - N(0°)}{N(90°) + N(0°)}$	
E2	579	**0.10 ± 0.03**	positive
	697	**0.13 ± 0.03**	
	386	**0.06 ± 0.05**	
	534	**0.05 ± 0.04**	
M1	349	**-0.11 ± 0.05**	negative
inter-band	607	**0.05 ± 0.05**	positive ⇒ electric
	626	**0.12 ± 0.05**	
	643	**0.11 ± 0.05**	
	659	**0.17 ± 0.09**	
	673	**0.18 ± 0.09**	



Evidence for the wobbling mode in nuclei
⇒ triaxial deformation

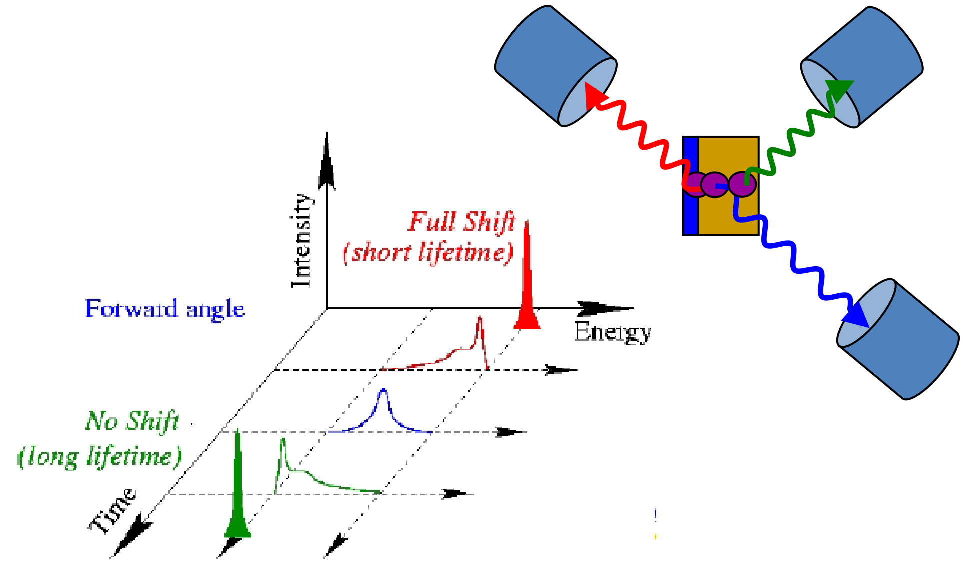
S.W. Ødegård et al., Phys. Rev. Lett. 86, 5866 (2001)

Can we say more about the shape?
$\Rightarrow$ DSAM lifetime measurement

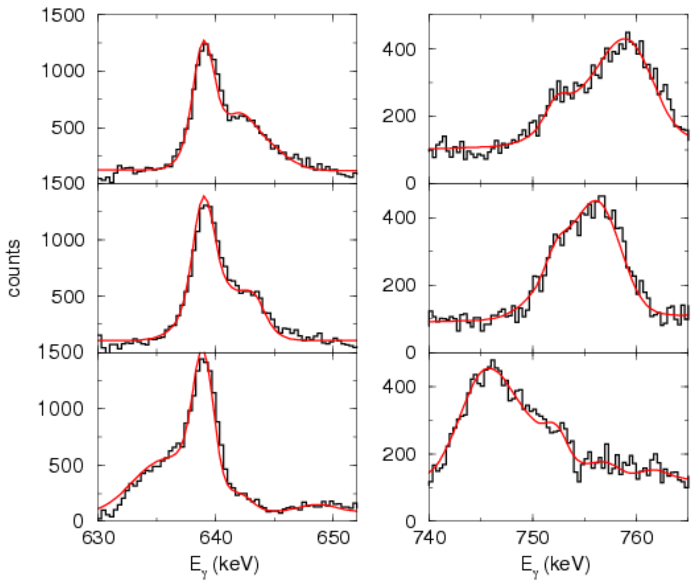
target with backing

gamma rays are emitted

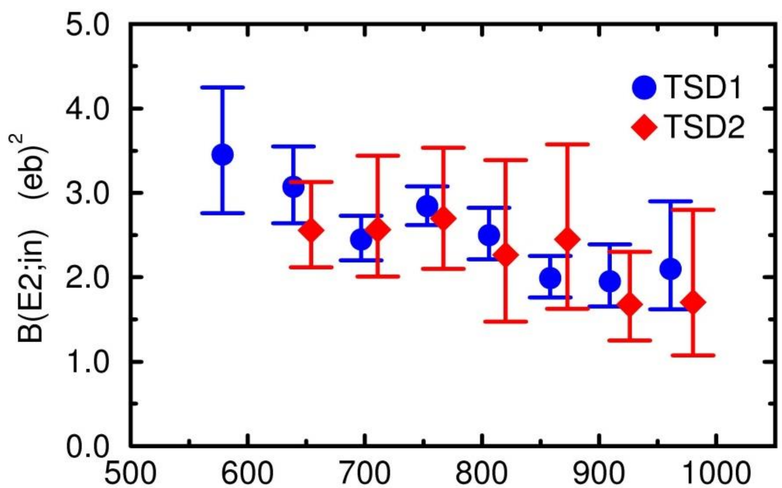
- with full recoil velocity
- slowed down
- finally stopped



characteristic lineshape

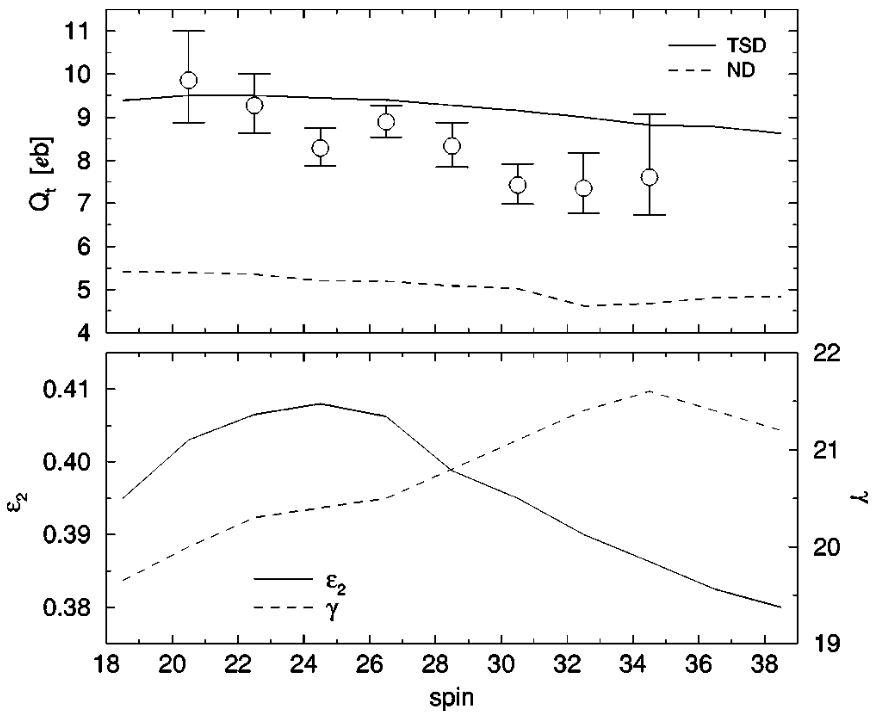


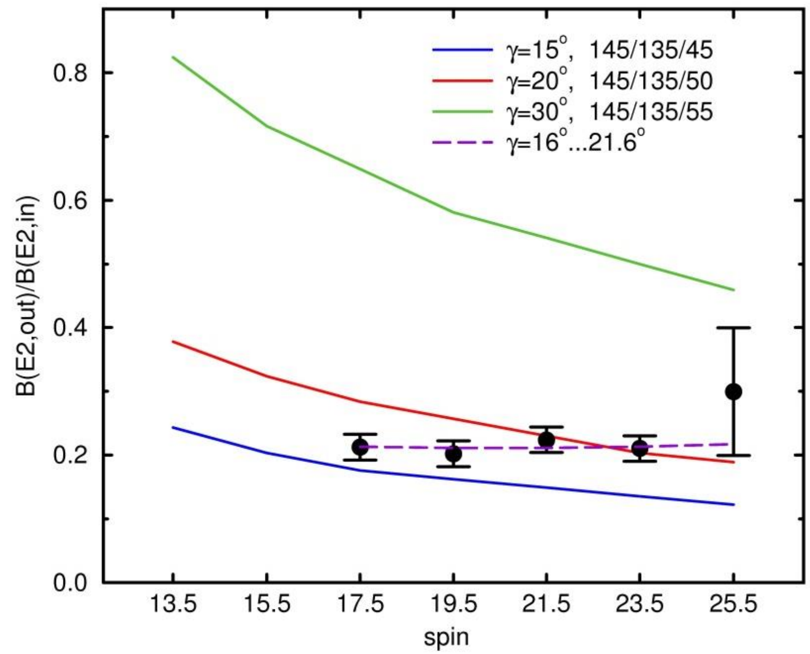
A. Görgen et al.,
Phys. Rev. C 69, 031301(R) (2004)



very similar in-band B(E2) values
for $n_w = 0$ and $n_w = 1$ bands

Deformation of the wobbling bands: triaxial superdeformation



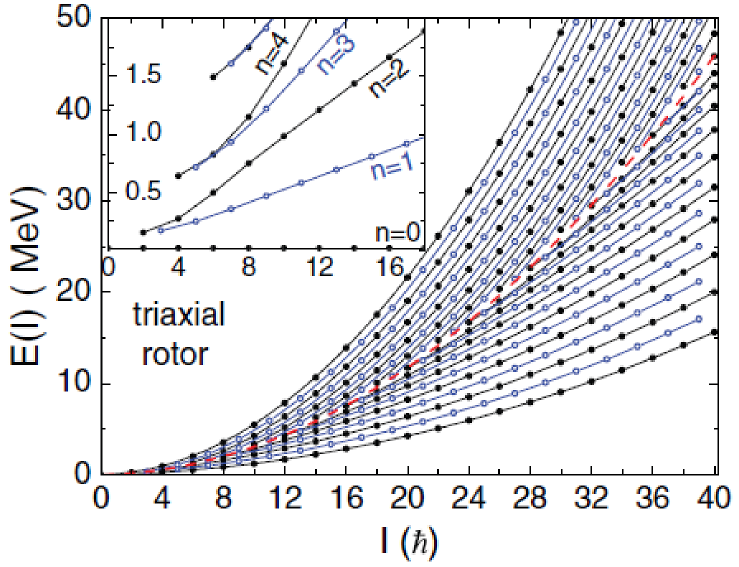


in-band B(E2) values consistent with second minimum in PES

inter-band transition strengths suggest increase in γ deformation with spin

A. Görgen et al.,
Phys. Rev. C 69, 031301(R) (2004)

Wobbling frequency



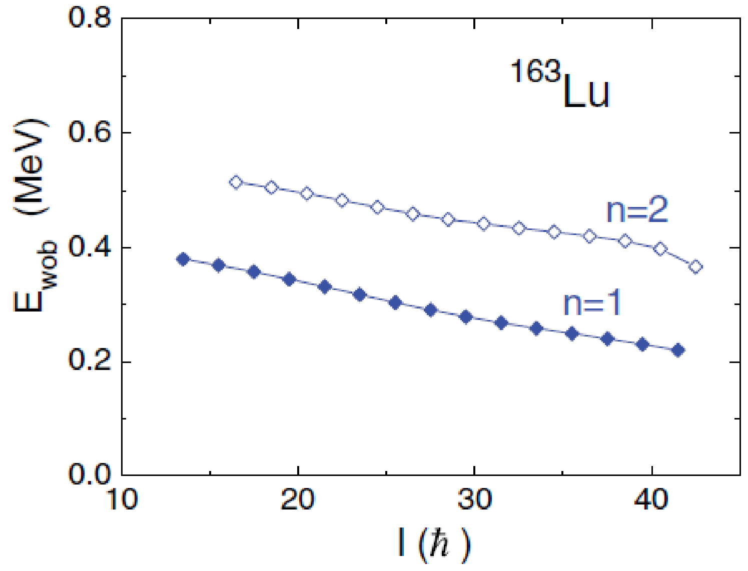
band structure for a triaxial rotor

S.Frauendorf and F.Dönau
PRC 89, 014322 (2014)

wobbling frequency should increase with spin

$$\hbar\omega_w = \frac{I}{\mathfrak{J}_a}\sqrt{\frac{(\mathfrak{J}_a - \mathfrak{J}_b)(\mathfrak{J}_a - \mathfrak{J}_c)}{\mathfrak{J}_b \mathfrak{J}_c}}$$

wobbling frequency decreases experimentally



what about the odd particle?

if the odd particle aligns with the largest MoI:

$$E = \frac{(I-j)^2}{2\mathfrak{J}} + \hbar\omega_w(n_w + \tfrac{1}{2})$$

energies are shifted, but wobbling frequency should still increase with spin

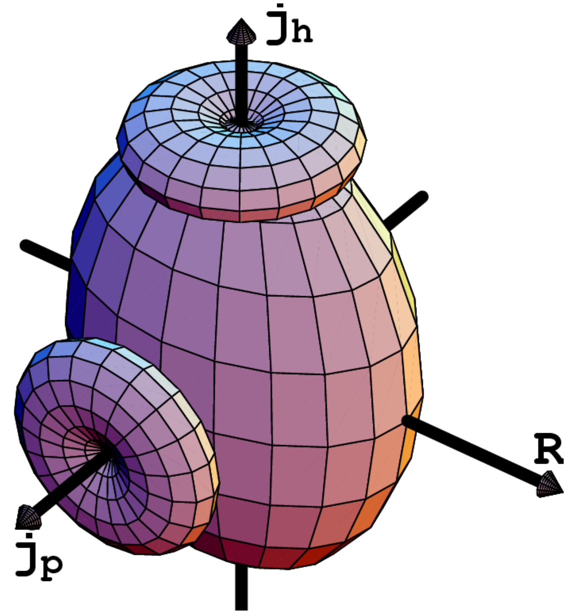
⇒ longitudinal wobbling

Wobbling motion

triaxial nucleus with short, medium, and long axis

typically it is: $\Im_m > \Im_s > \Im_l$

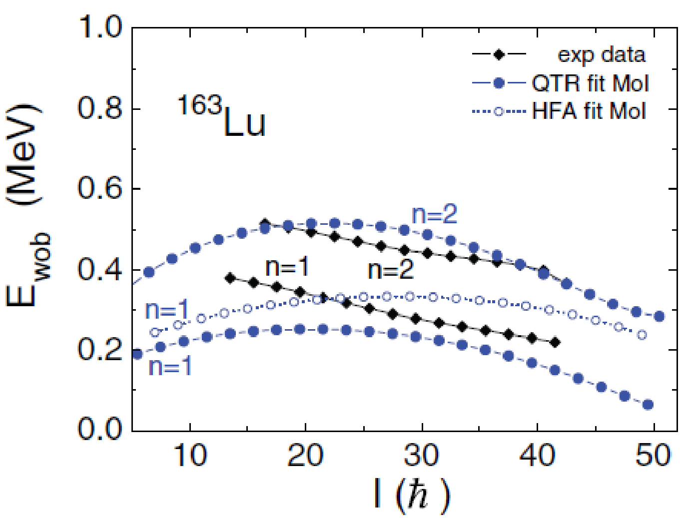
Nucleus	ε	γ(deg)	Model	$\mathcal{J}_m$	$\mathcal{J}_s$	$\mathcal{J}_l$
^{163}Lu	0.4	20	fit	64	56	13
	0.4	20	hydrodyn	68	29	8
	0.4	20	cranking	59	51	13



- high-j particle aligns with short axis (maximum overlap for density distribution)
- high-j hole aligns with long axis
- collective rotation about medium axis

⇒ transverse wobbling

quasi-particle triaxial rotor calculation (with *frozen alignment* approximation)



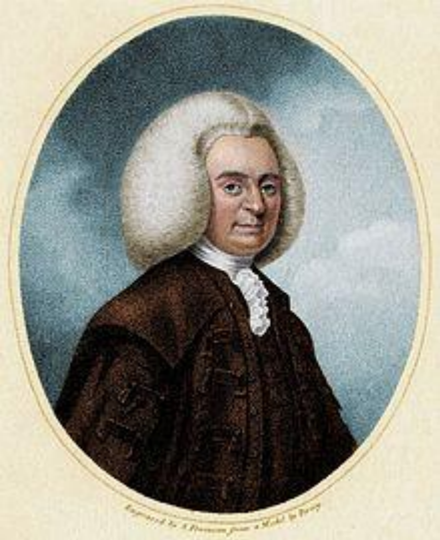
MacLaurin shapes

What happens if we spin a liquid drop ?

- equatorial diameter 12756 km
- polar diameter 12714 km

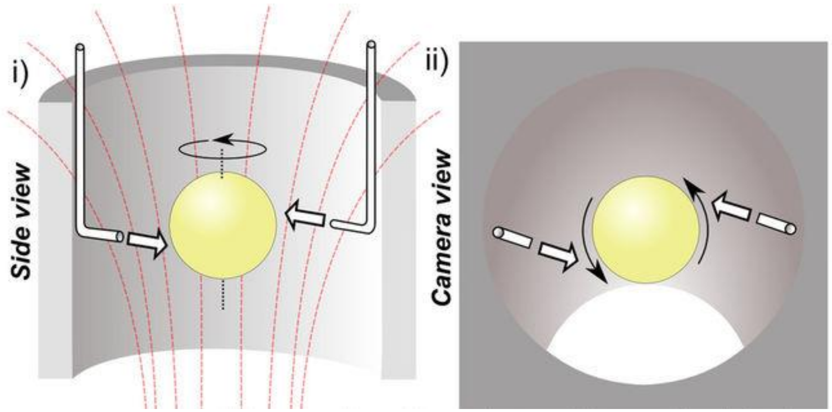
It becomes oblate !

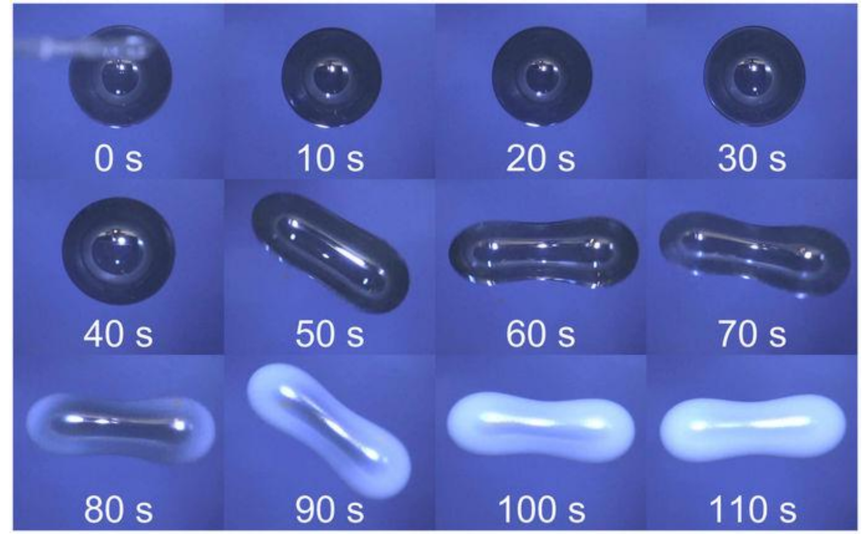
MacLaurin shape
after Colin MacLaurin
(1698-1746)



But what if we spin really fast ?

Toy model: spinning wax levitated in magnetic field

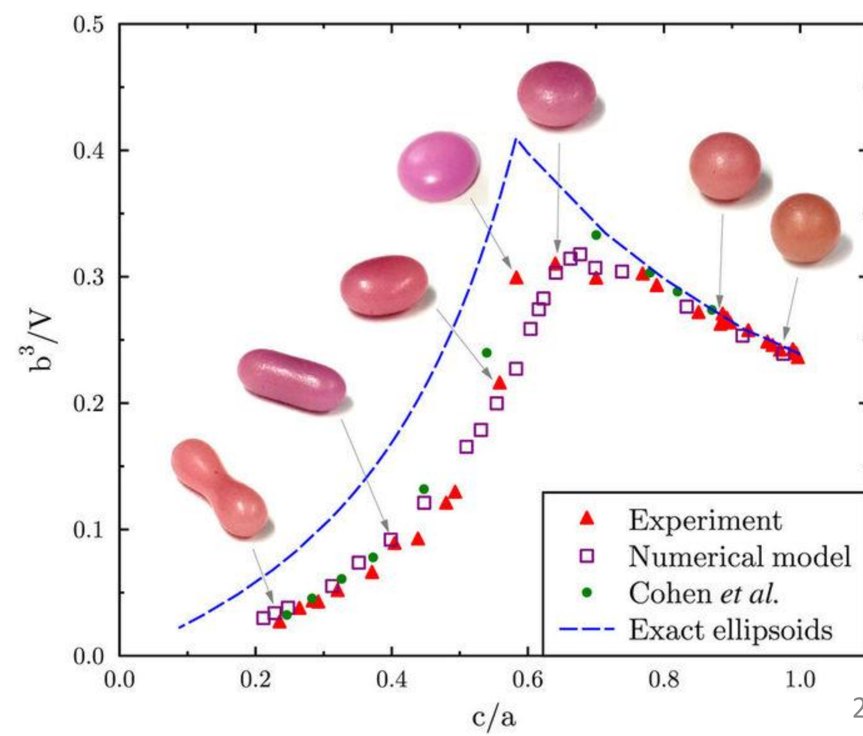




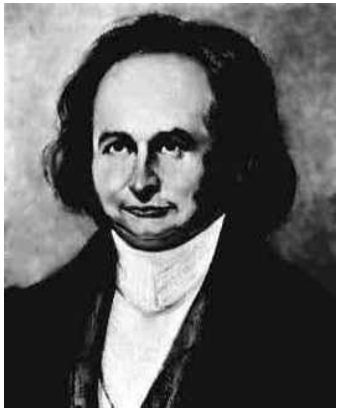
diamagnetic levitation in 18.5 T magnetic field
rotated by air flow from two nozzles

K.A. Baldwin et al., Sci. Rep. 5, 7660 (2015)

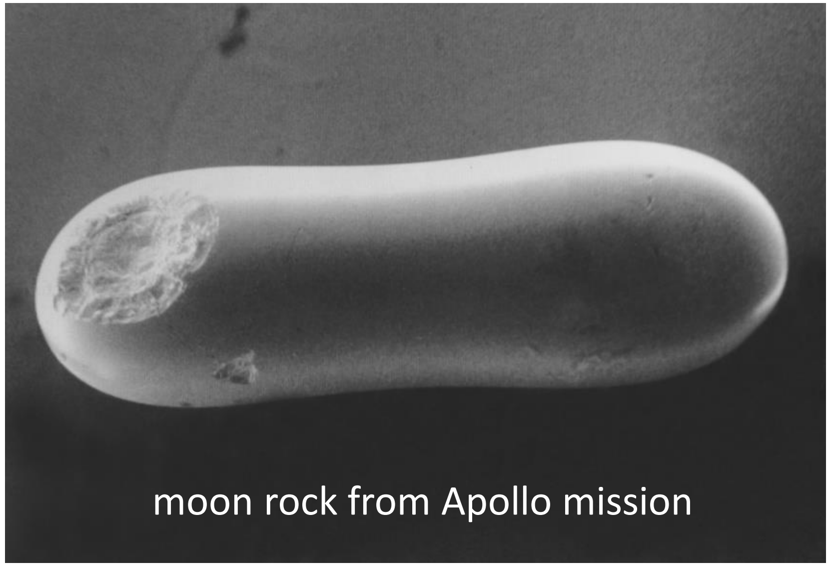
b) increasing angular momentum



Jacobi shapes



Carl Gustav Jacob Jacobi
(1804 - 1851)

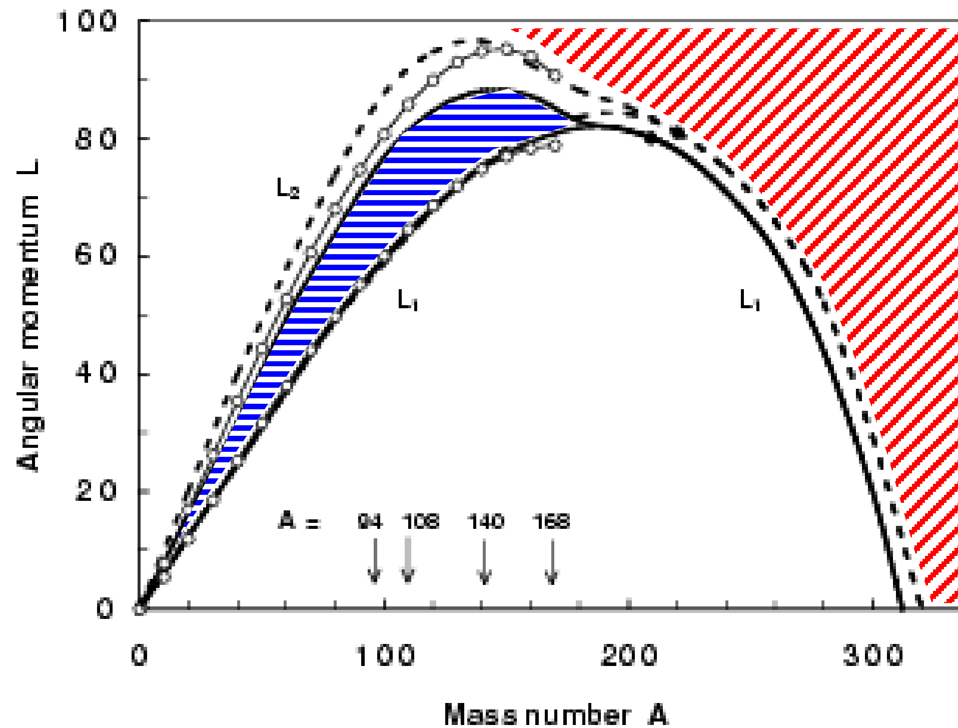


What happens if you spin even faster?

Fission!

Nuclear liquid drop model
(no shell effects)

- heavy nuclei: fission for $L > L_1$
- Jacobi transition for $L_1 < L < L_2$
  largest spin window around A ≈ 100



W.D. Myers and W.J. Swiatecki
Acta Phys. Pol. B 32, 1033 (2001)

What is the signature of a Jacobi transition in nuclei ?

- sharp decrease of frequency with increasing angular momentum (giant backbend of the moment of inertia)
- frequency of collective rotation is related to the E2 γ-ray energy: $\hbar\omega = \frac{1}{2}E_\gamma$
- many rotational bands at high spin quasi-continuous transitions
- measure the energy of the quasi-continuous 'E2 bump' as a function of angular momentum
- series of experiments with Gammasphere

$^{48}Ca + ^{50}Ti$

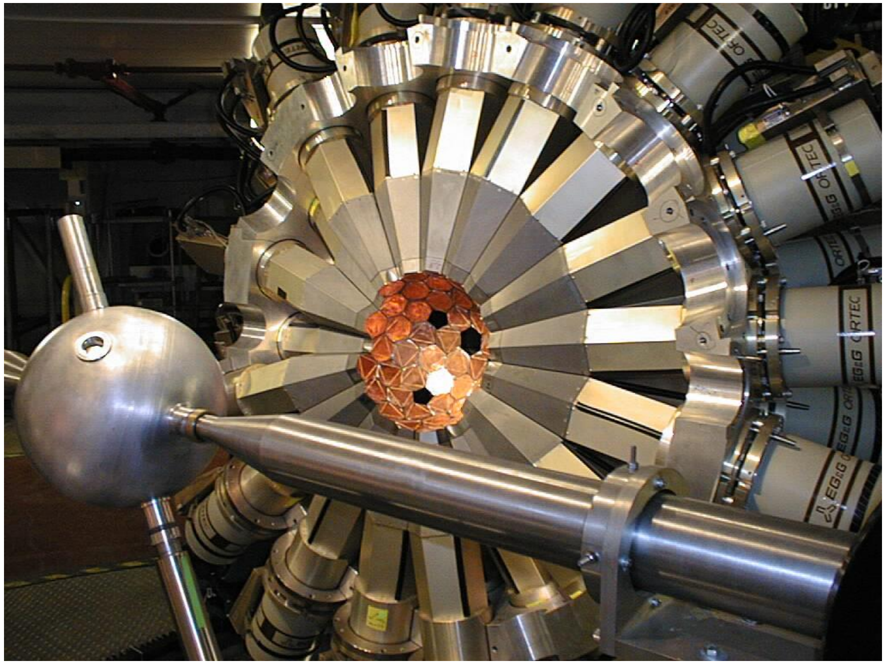
$^{48}Ca + ^{64}Ni$

$^{48}Ca + ^{96}Zr$

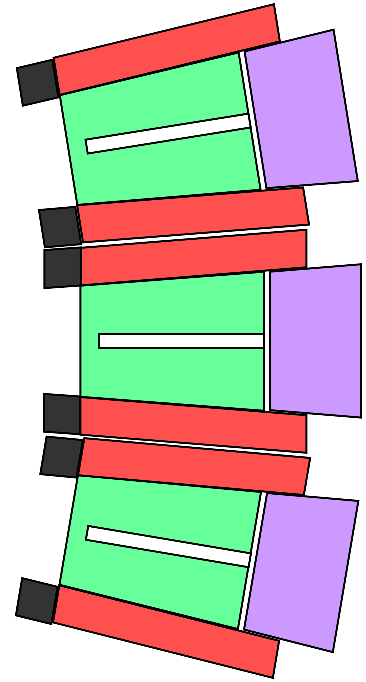
$^{48}Ca + ^{124}Sn$

as neutron rich as possible:
$\Rightarrow$ higher fission barrier

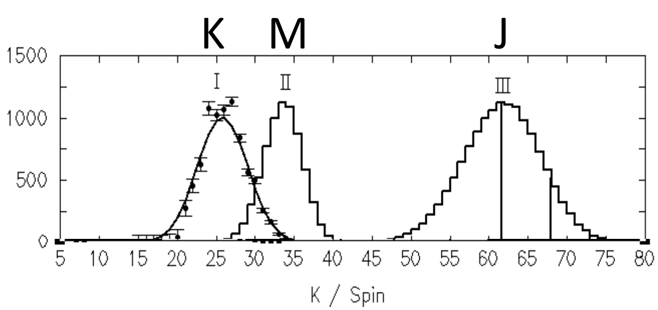
Measuring angular momentum with Gammasphere



108 Compton-suppressed
HPGe detectors



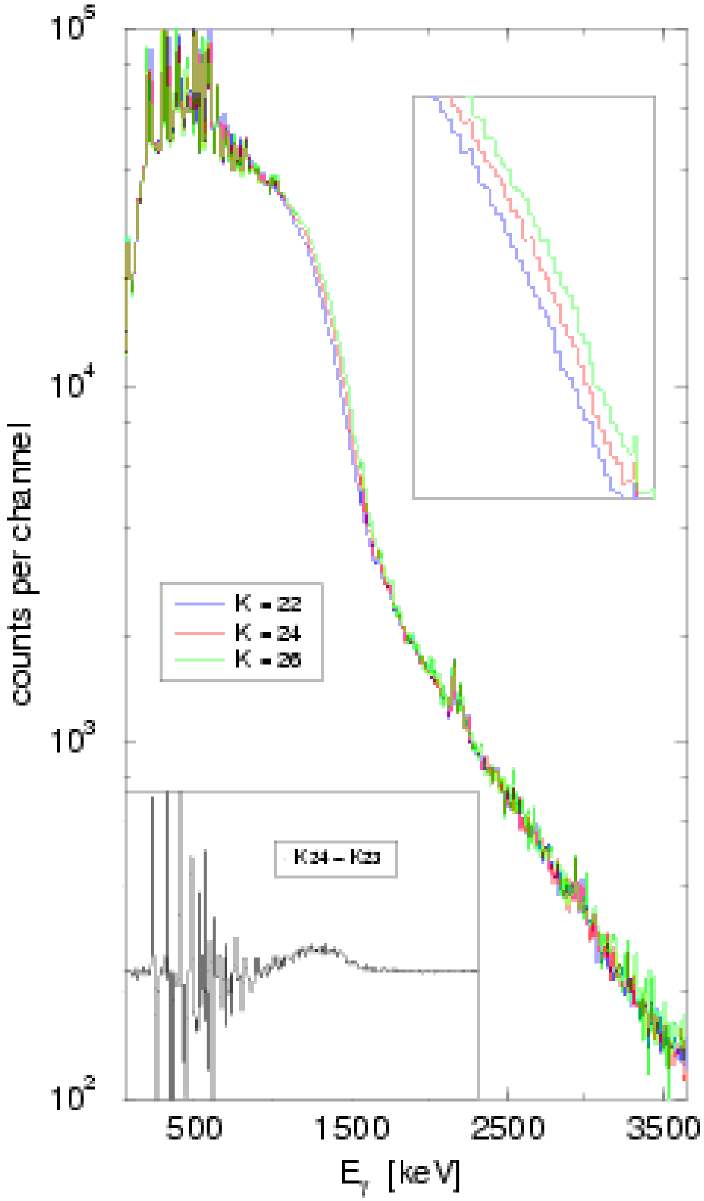
108 Ge detectors
6 x 108 = 648 BGO detectors



increase in false veto signals
reduced Ge efficiency
but very high granularity

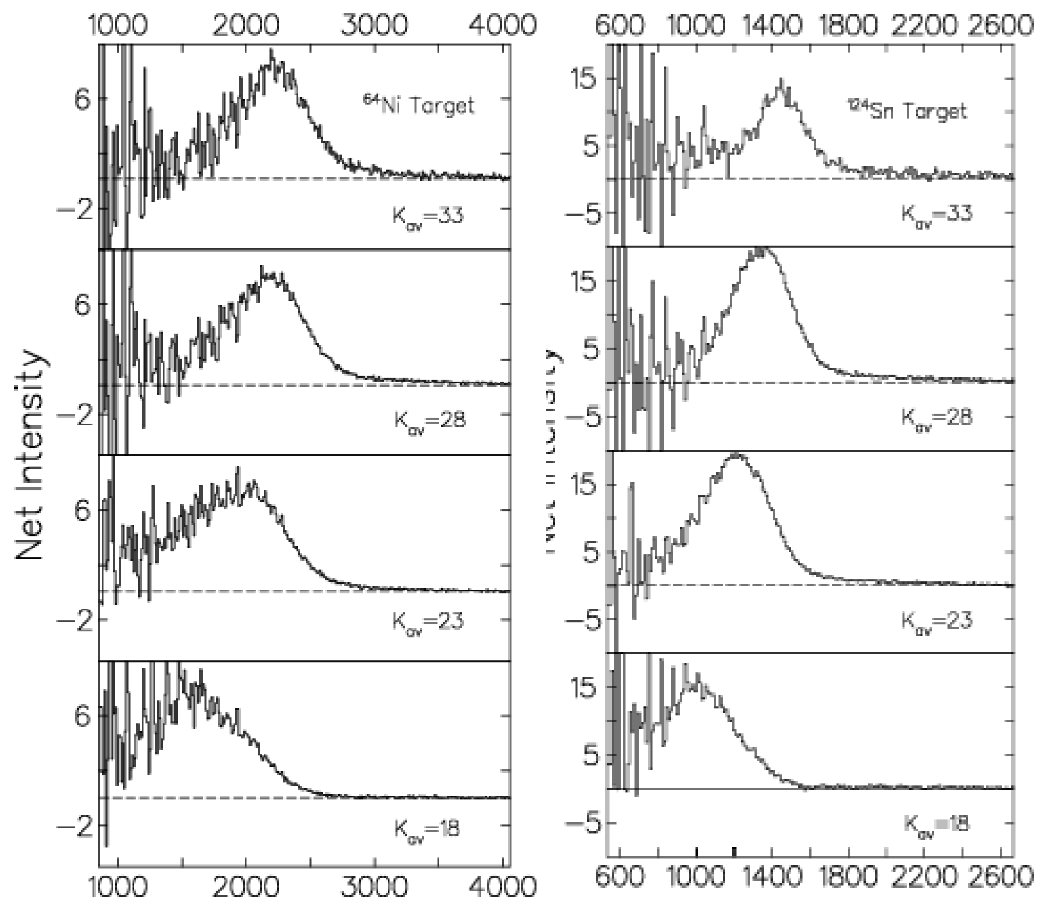
K = number of hits = fold
M = γ rays emitted = multiplicity (from response function)
J = initial angular momentum (from angular distribution)

The E2 bump



Incremental spectra:

Multiplicity (K_{av}-1) gated spectrum subtracted from (K_{av}+1) spectrum

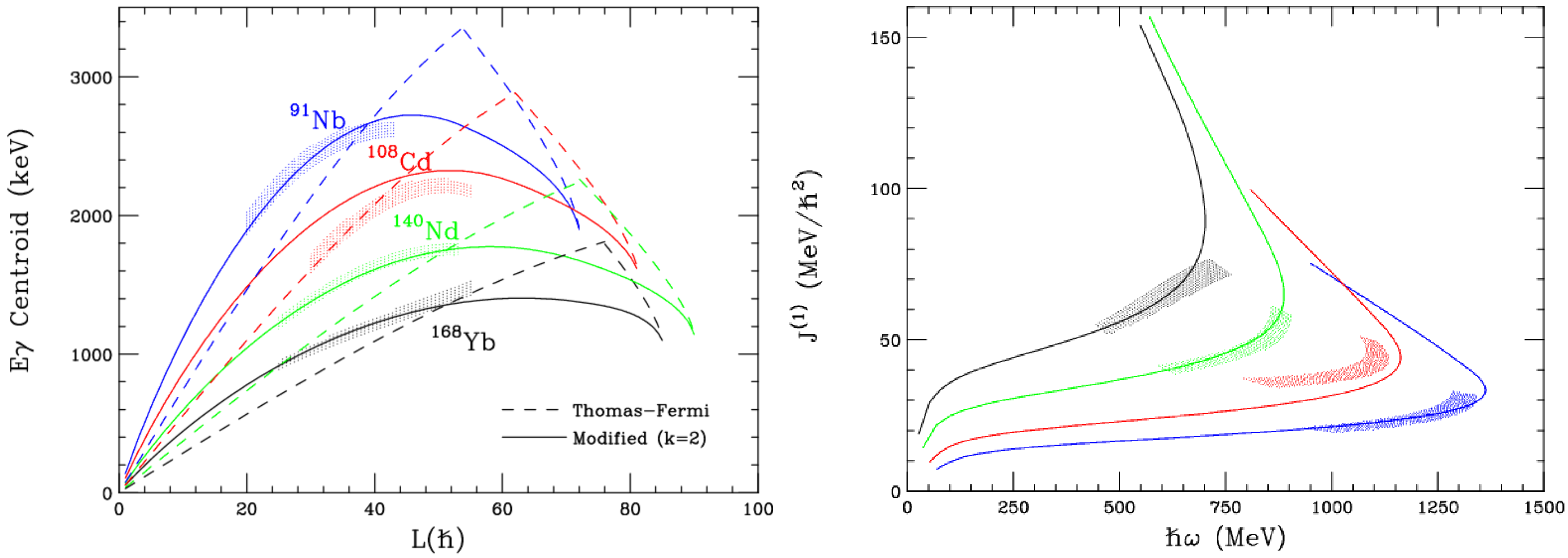


K measures the angular momentum

E2 bump measures rotational frequency

Comparison to liquid drop calculations

D. Ward et al., Phys. Rev. C 66, 024317 (2002)



two modifications:

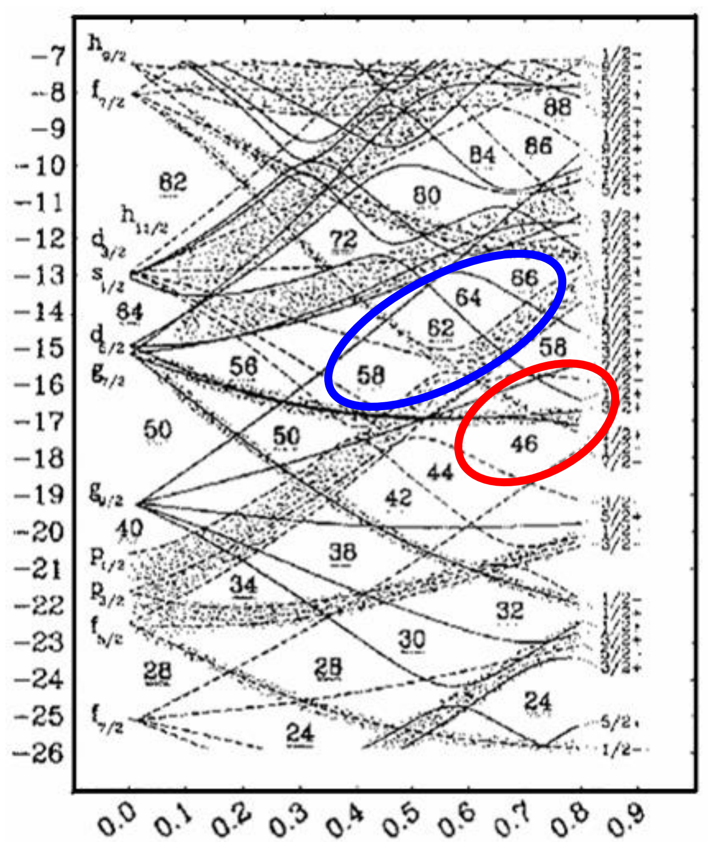
- lower effective moment of inertia at low spin due to pairing
- no collective rotation about axially symmetric (MacLaurin) shapes in nuclei,
  instead, collective rotations are associated with (mostly) prolate shapes
  → no sharp transition caused by breaking of axial symmetry, but smooth transition

shell effects?

Extreme deformation in ^{108}Cd?

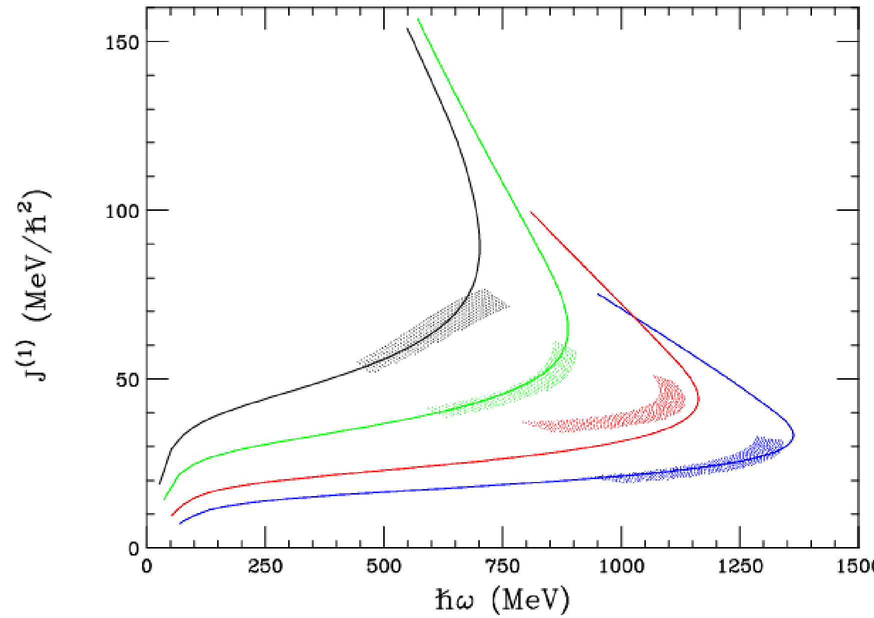
- indication of Jacobi transition at very high spin ($I > 50\ \hbar$)

single-particle energy (Woods-Saxon)



quadrupole deformation

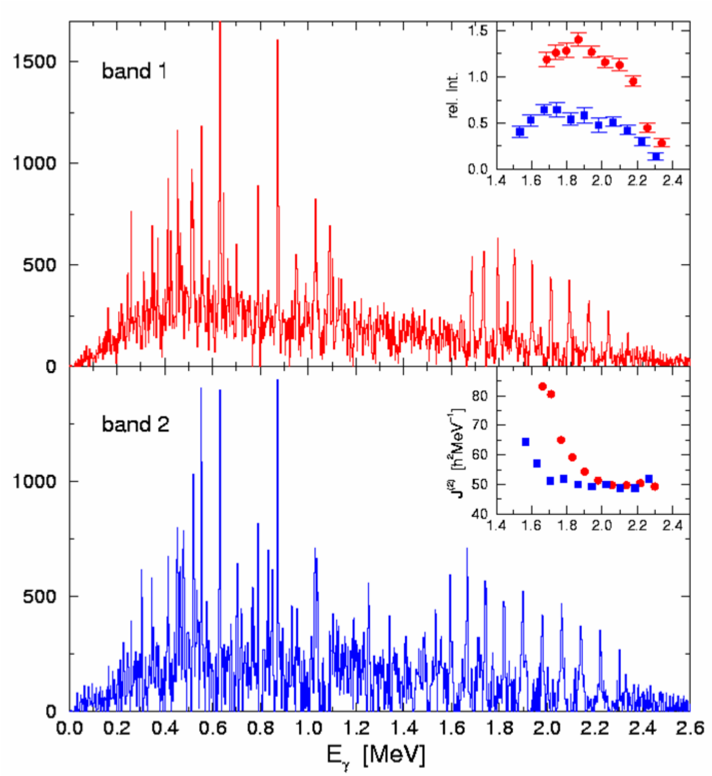
D. Ward et al., Phys. Rev. C 66, 024317 (2002)



- shell gaps at large deformation for $Z \approx 48$ and $N \approx 60$

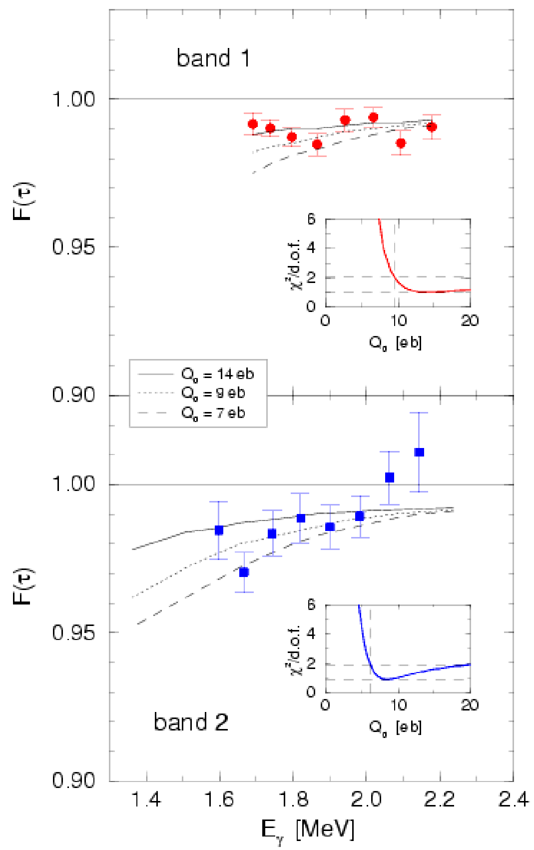
Very deformed structures in ^{108}Cd

$^{64}Ni(^{48}Ca,4n)^{108}Cd$
Gammasphere @ LBNL / ANL

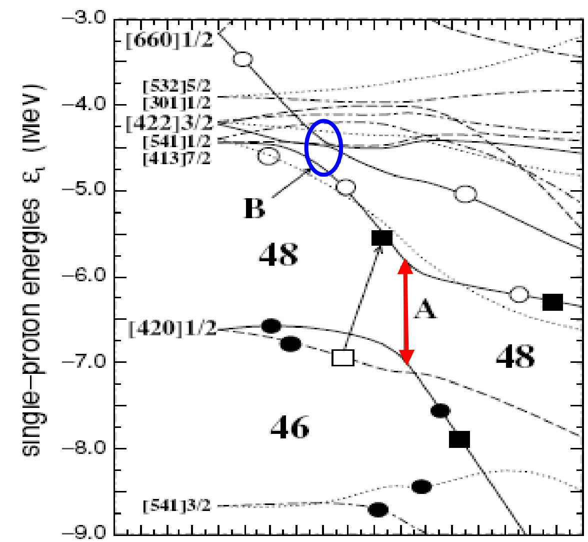


γ-ray multiplicity
➢ spin range 40-60 ħ

R.M. Clark et al., PRL 87, 202502 (2001)
A. Görgen et al., PRC 65, 027302 (2002)



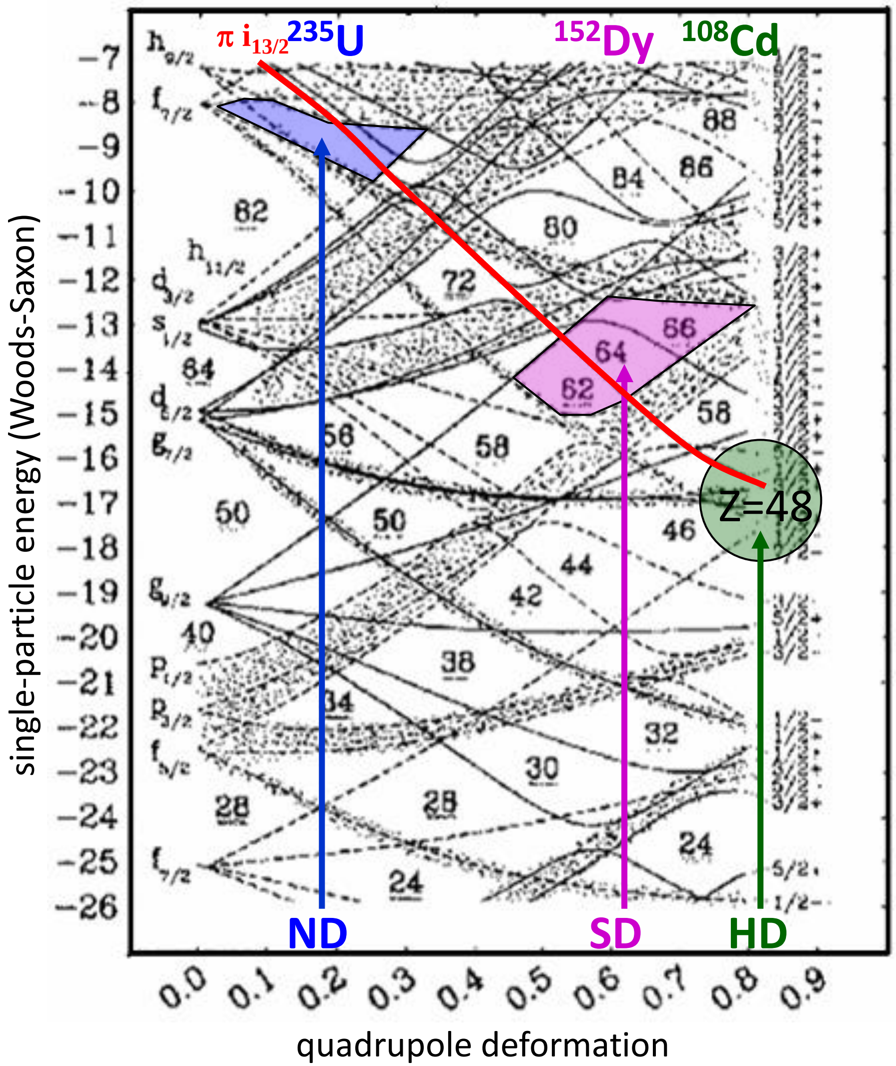
residual Doppler shifts
➢ lower limit on Q_t, $\beta_2 \geq 0.6$

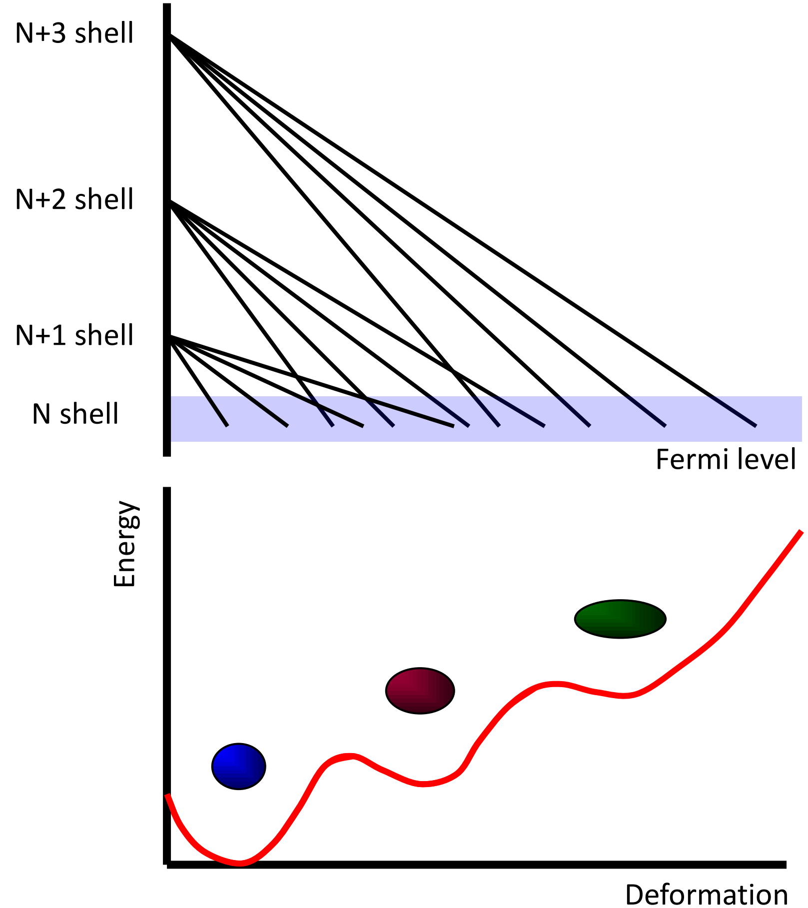


A.V. Afanasjev, S. Frauendorf, PRC 72, 031301 (2005)

cranked RMF calculations
➢ **evidence for occupation of proton $i_{13/2}$ orbital**

Intruder orbitals





- (N+1) *intruder*
  ⇒ normal deformed, e.g. ^{235}U
- (N+2) *super-intruder*
  ⇒ Superdeformation, e.g. ^{152}Dy
- (N+3) *hyper-intruder* occupied in ^{108}Cd
  ⇒ Hyperdeformation ?