

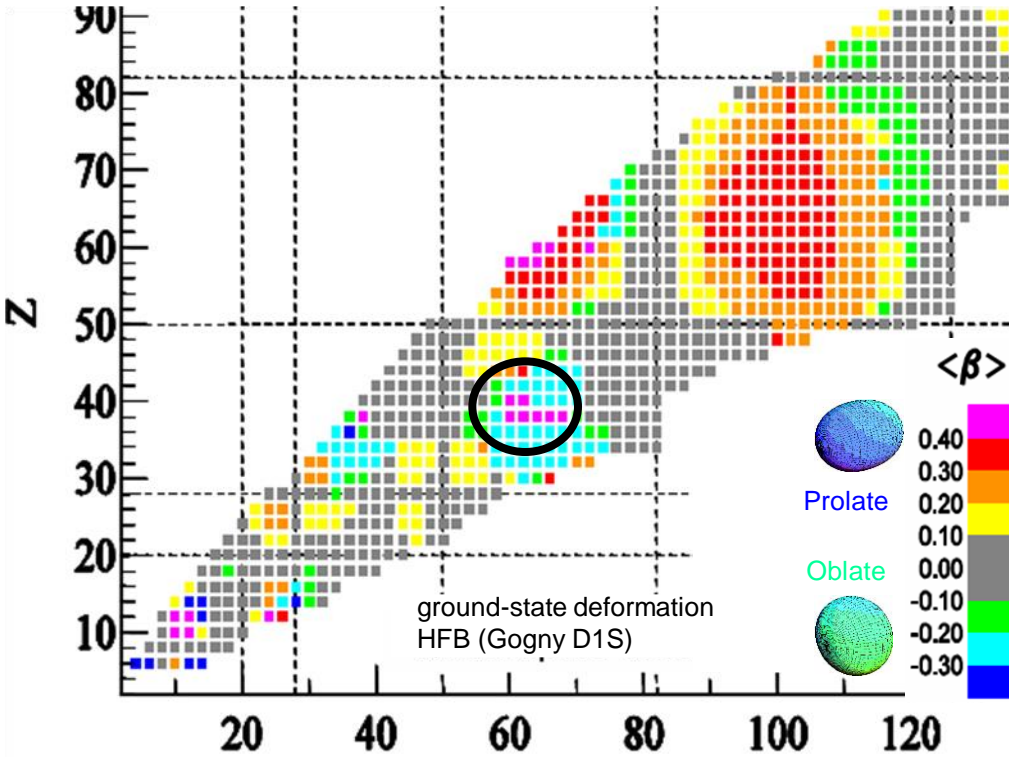
Nuclear Shapes

Lecture III

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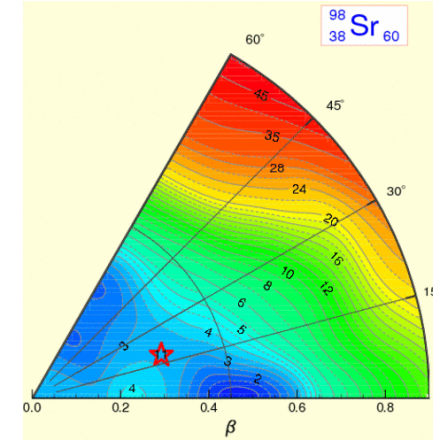
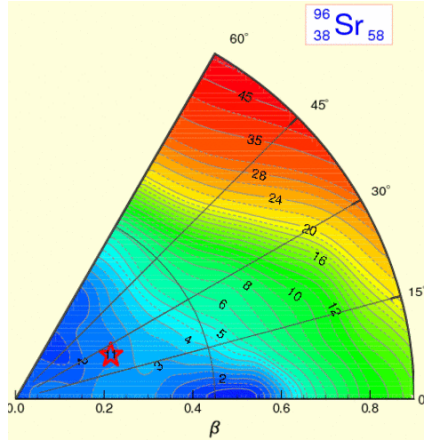
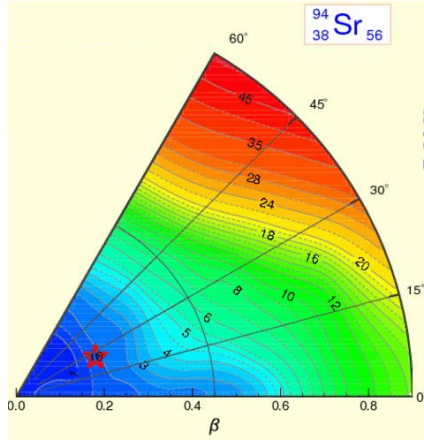
Selected Topics in Nuclear and Atomic Physics
Fiera di Primiero
2.-6. October 2017

Shapes and shape transitions in the nuclear chart

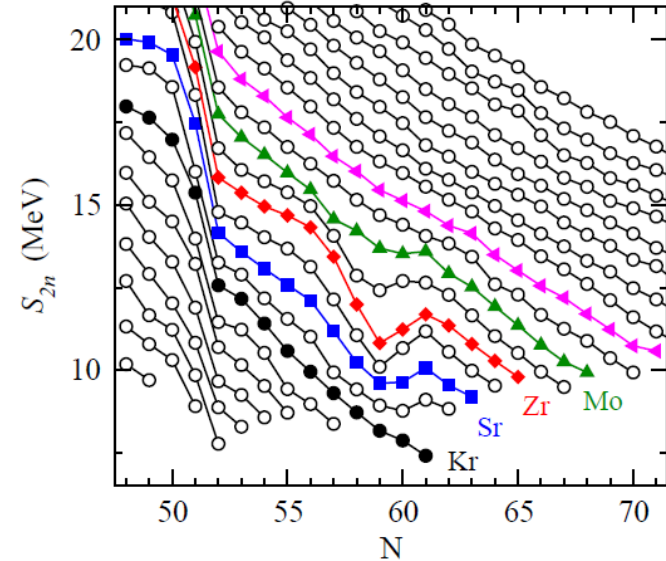
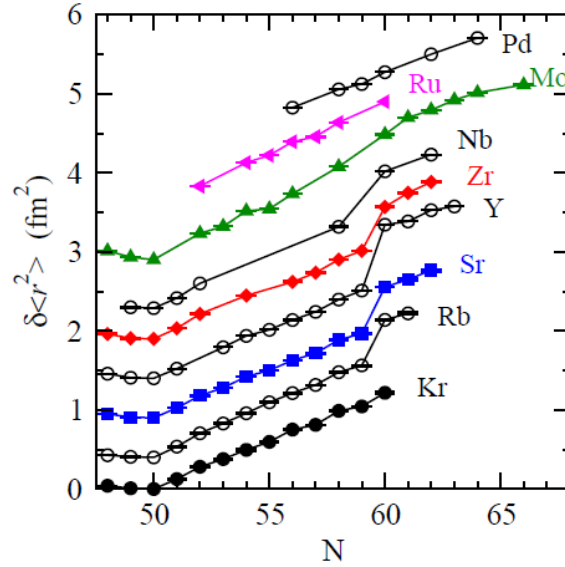
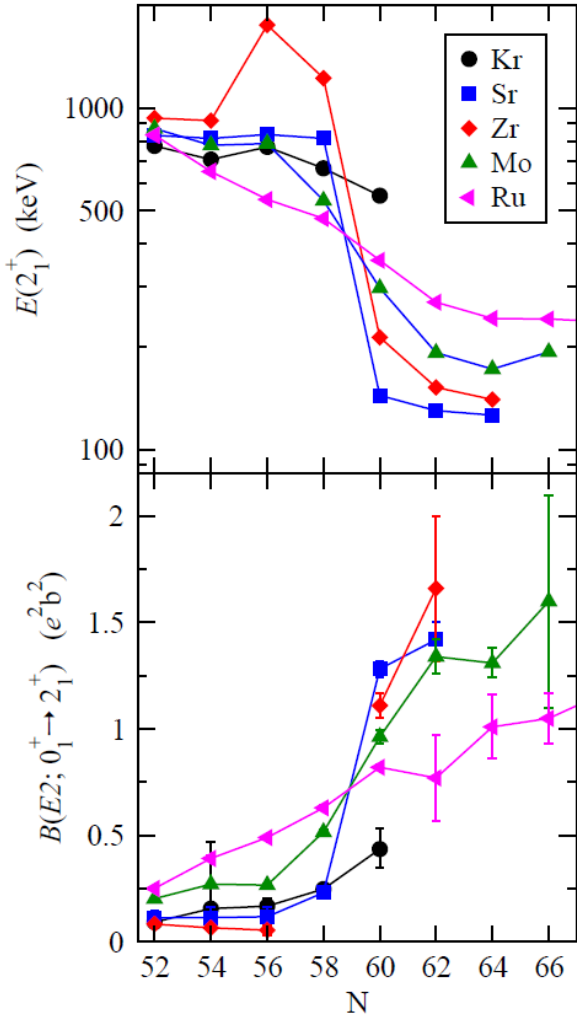


- rapid changes in the ground-state deformation for $Z \approx 40$ and $N \approx 60$
- shape coexistence in the regions of shape transition
- observables related to nuclear shape: quadrupole moments, $B(E2)$ values
- benchmarks for theory

<http://www-phynu.cea.fr/>
 S.Hilaire, M.Girod, Eur. Phys. J. A 33, 237 (2007)



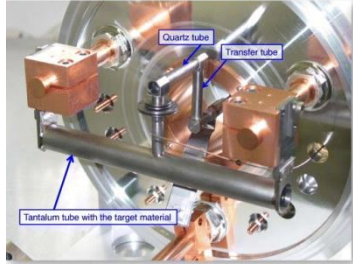
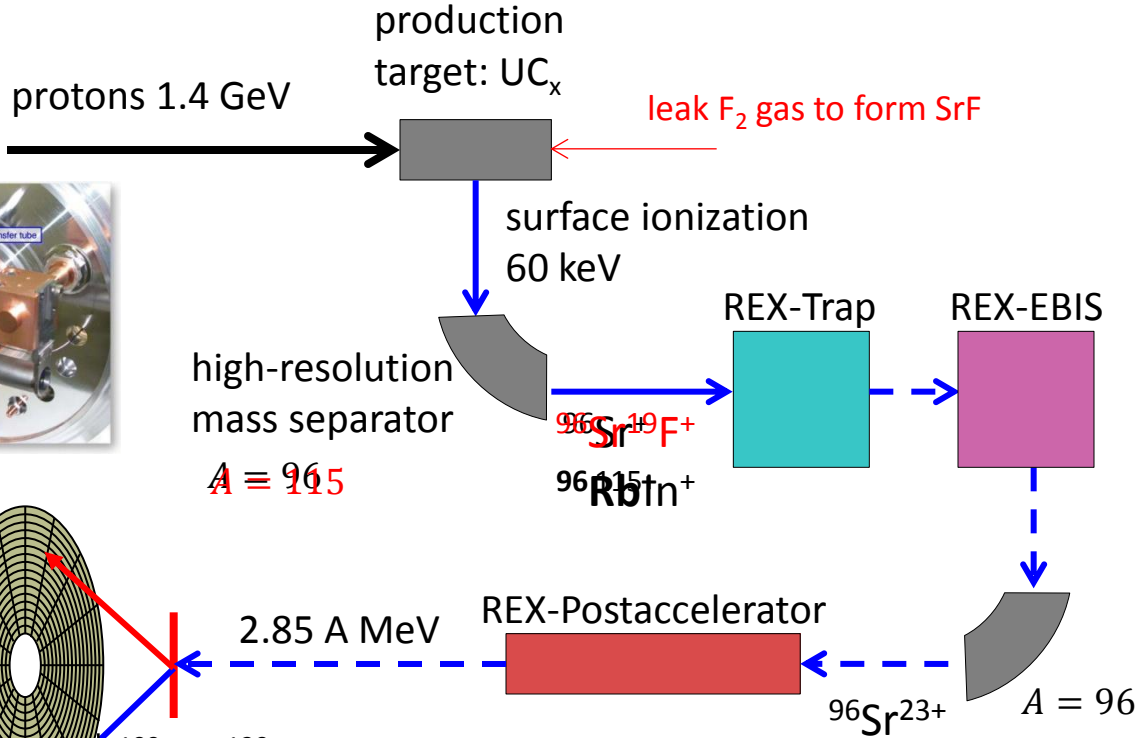
Shape transition at $N = 60$



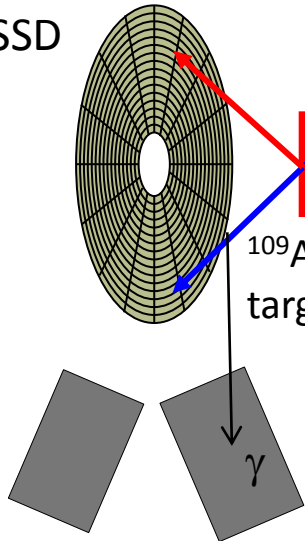
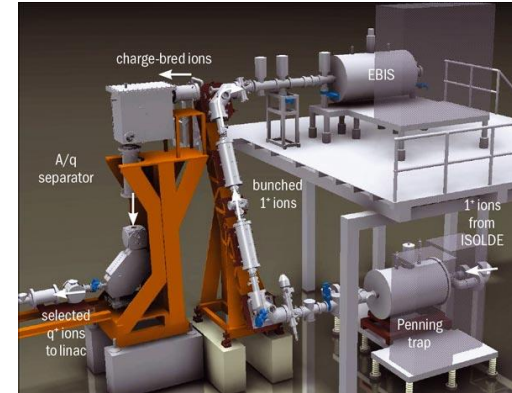
At $N = 60$:

- drop in energy of first 2^+ state
- step increase of $B(E2; 0^+ \rightarrow 2^+)$
- discontinuity in charge radii
- irregularity in two-neutron binding energy

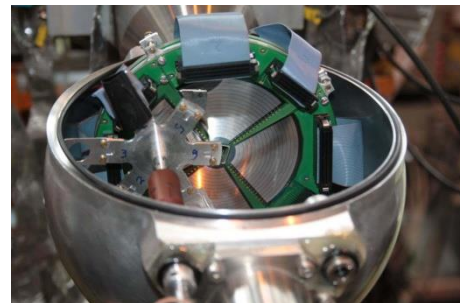
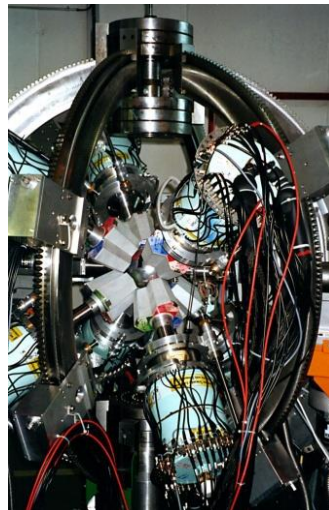
Isotope separation on-line and postacceleration at CERN-ISOLDE



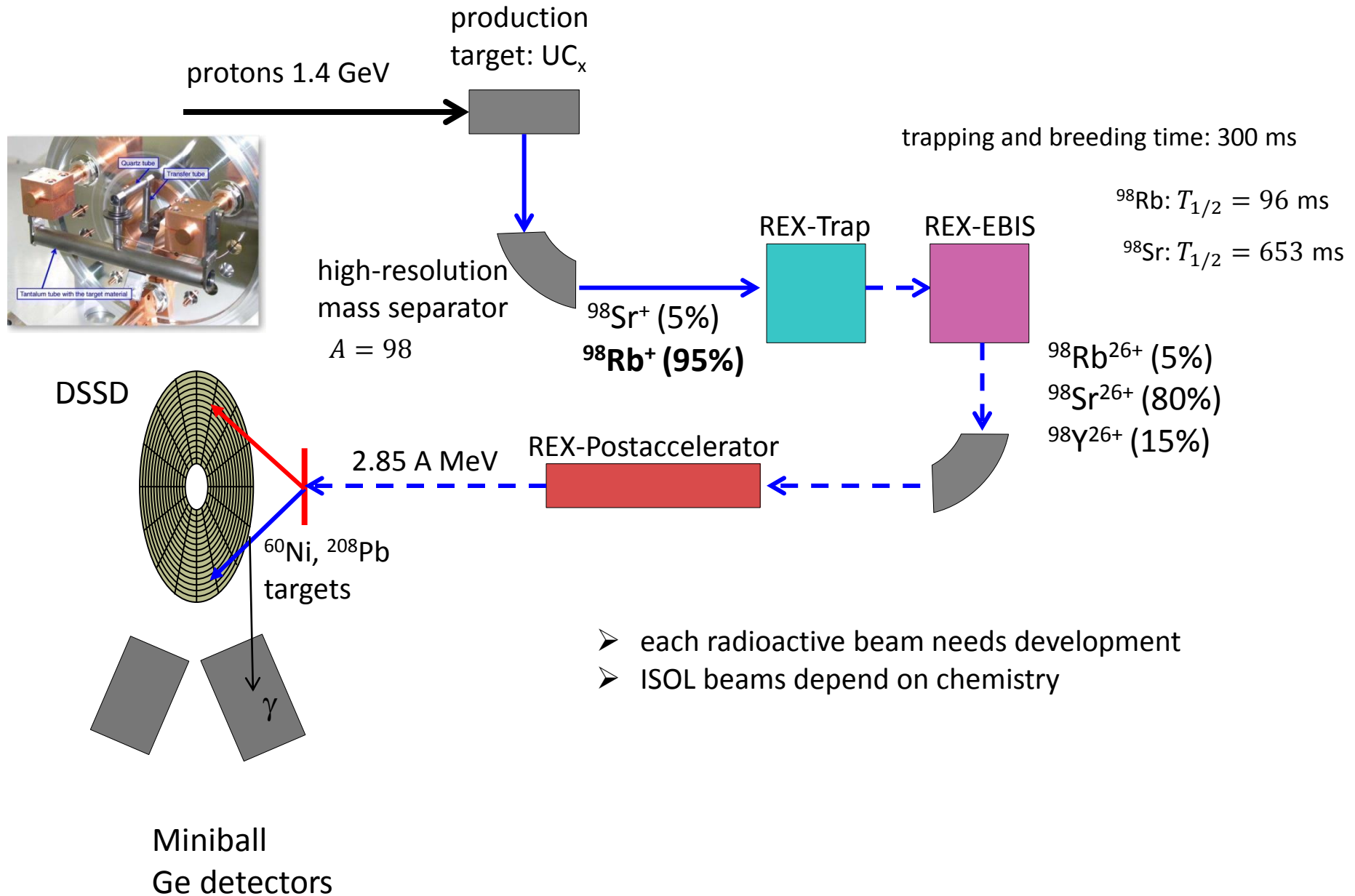
increase charge state
break SrF molecule



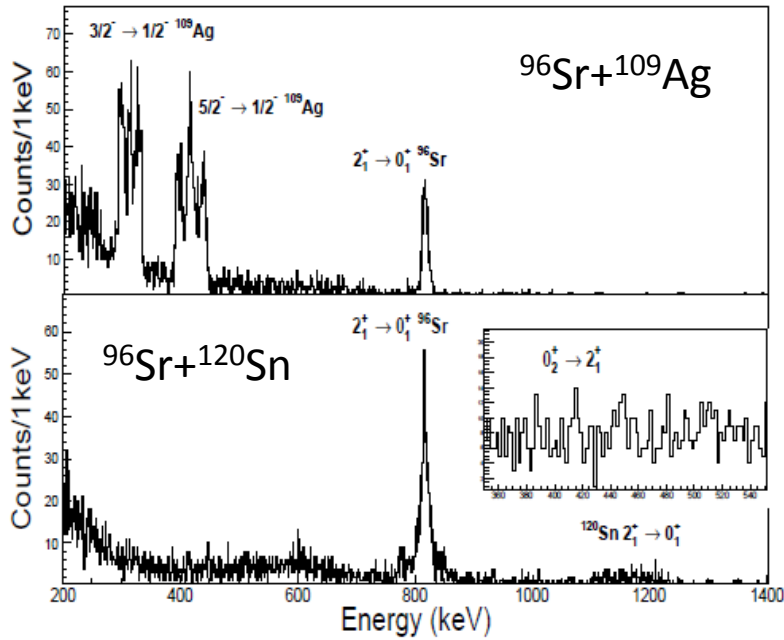
Miniball
Ge detectors



Isotope separation on-line and postacceleration at CERN-ISOLDE



Coulomb excitation of $^{96,98}\text{Sr}$ at CERN – ISOLDE



Doppler correction for projectiles

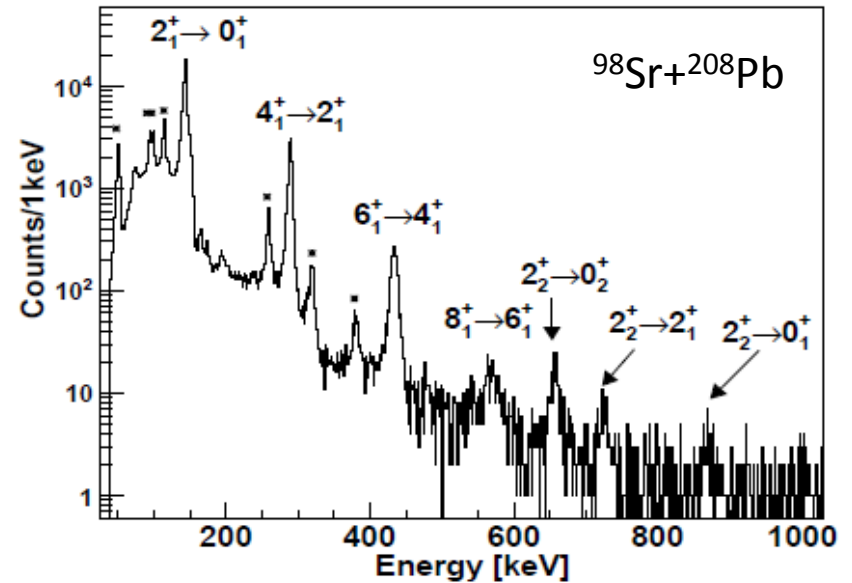
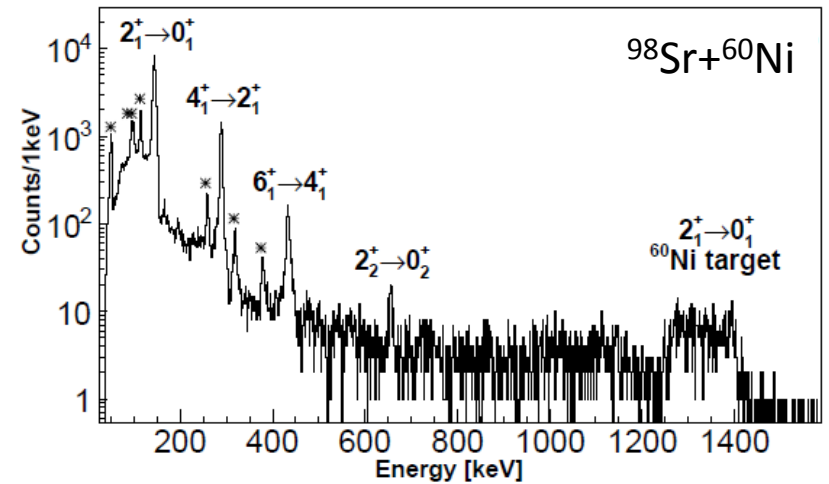
normalization to known $B(E2)$ values in target

Coulomb excitation probabilities:

(as function of θ and Z)

⇒ $B(E2)$ values

⇒ spectroscopic quadrupole moments Q_s
(reorientation effect)



E. Clément et al.

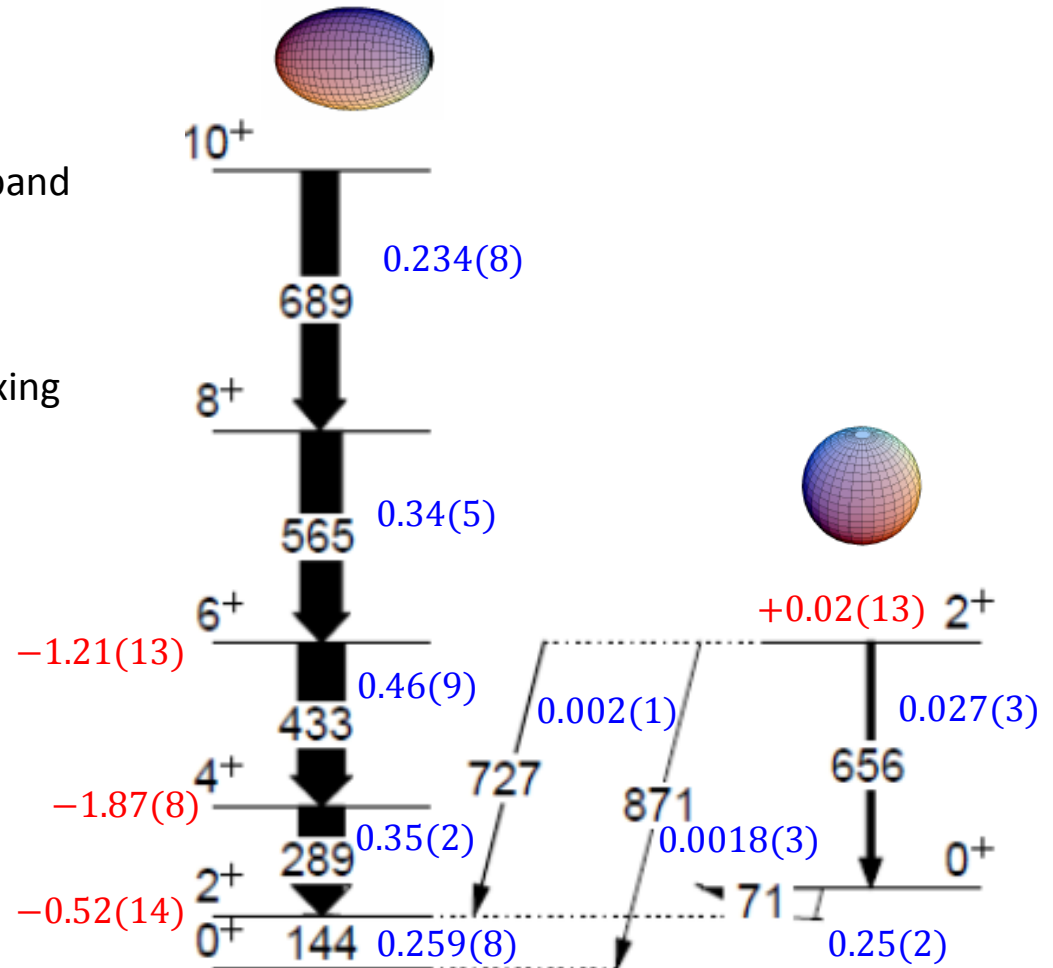
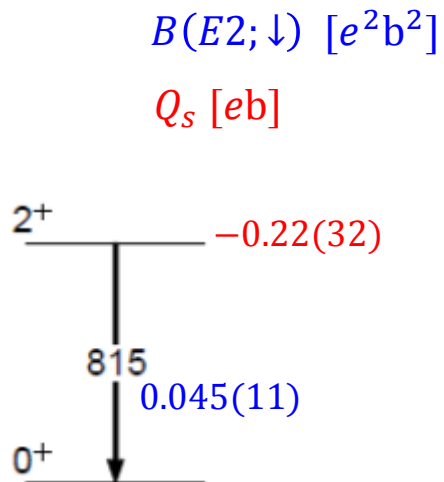
Phys.Rev.Lett. 116, 022701 (2016)

Phys.Rev. C 94, 054326 (2016)

Coulomb excitation of ^{98}Sr at CERN – ISOLDE

^{98}Sr

- rotational behavior of ground-state band
- excited configuration similar to ^{96}Sr
- quadrupole moments confirm shape coexistence
- $B(E2; 2_2^+ \rightarrow 0_1^+)$ indicates strong mixing

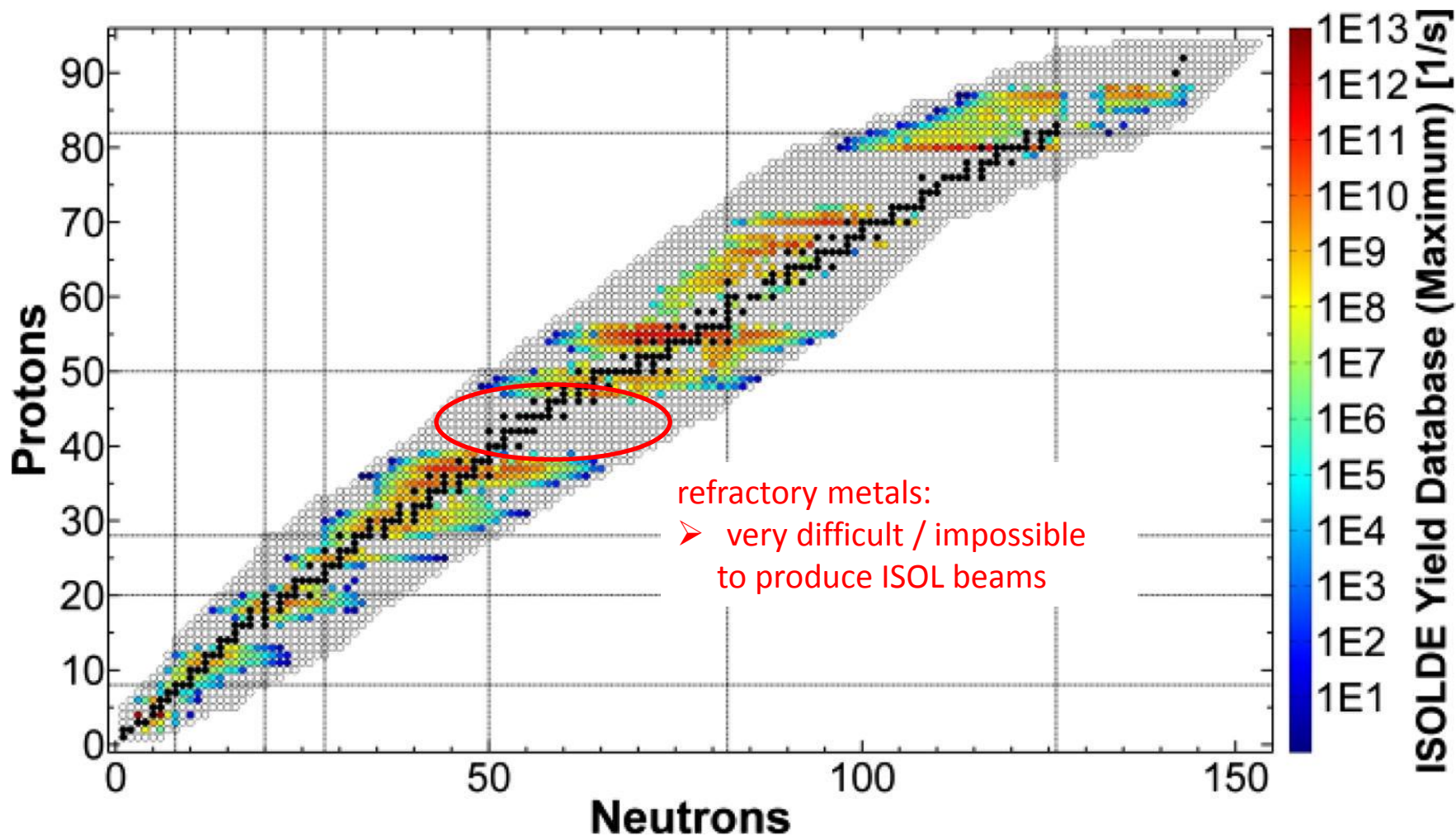


^{96}Sr

- $Q_s(2_1^+) \approx 0$
⇒ no static quadrupole deformation
- sizeable $B(E2)$
⇒ vibrational character of 2_1^+

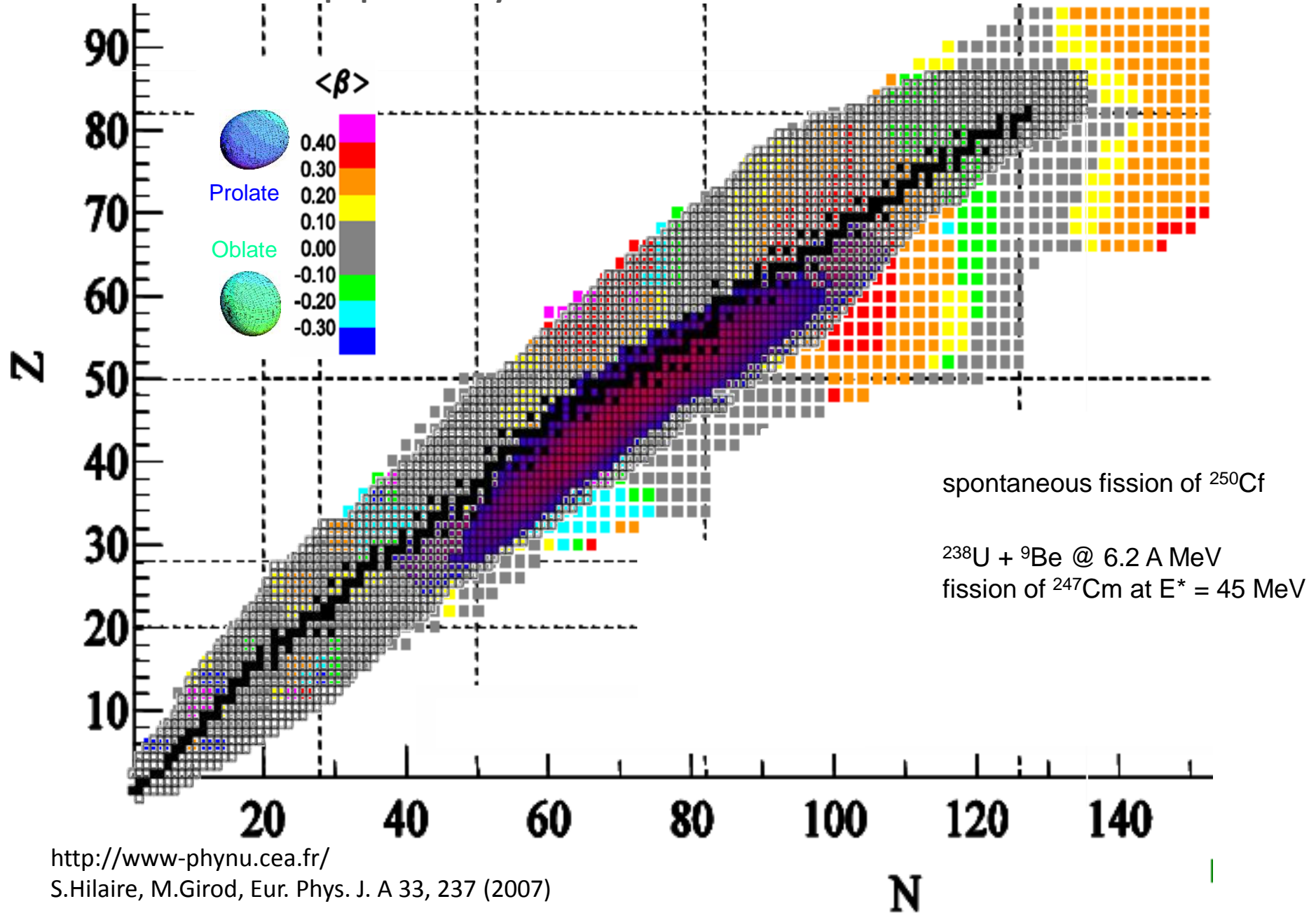
E. Clément et al.
 Phys.Rev.Lett. 116, 022701 (2016)
 Phys.Rev. C 94, 054326 (2016)

ISOL beams produced at CERN-ISOLDE



this is a region of dramatic shape transitions
and shape coexistence !

Neutron-rich nuclei populated by fission

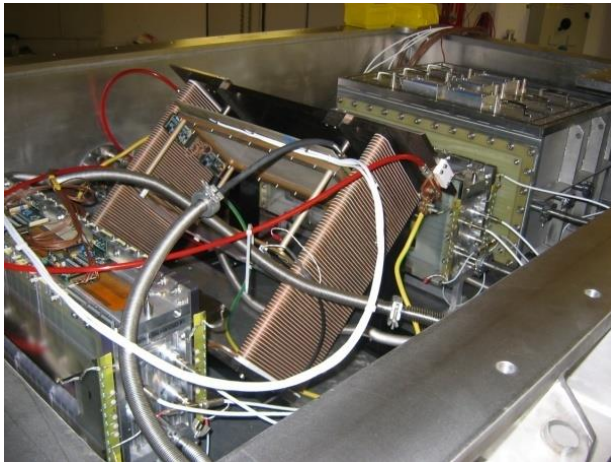
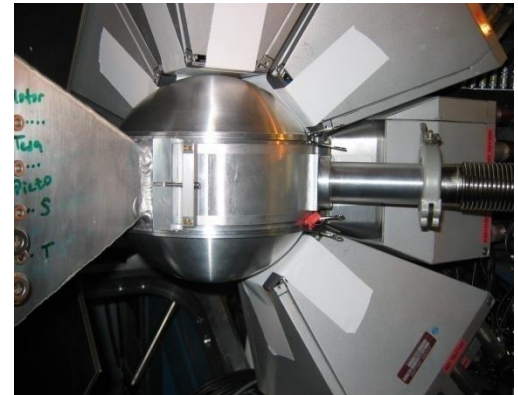
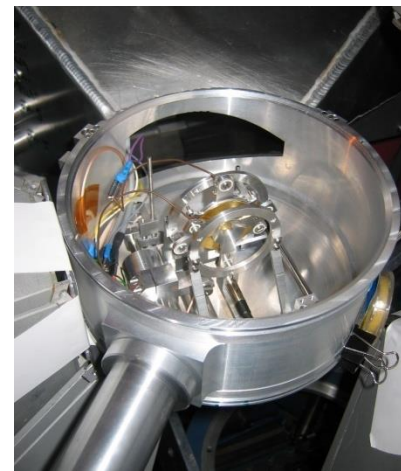
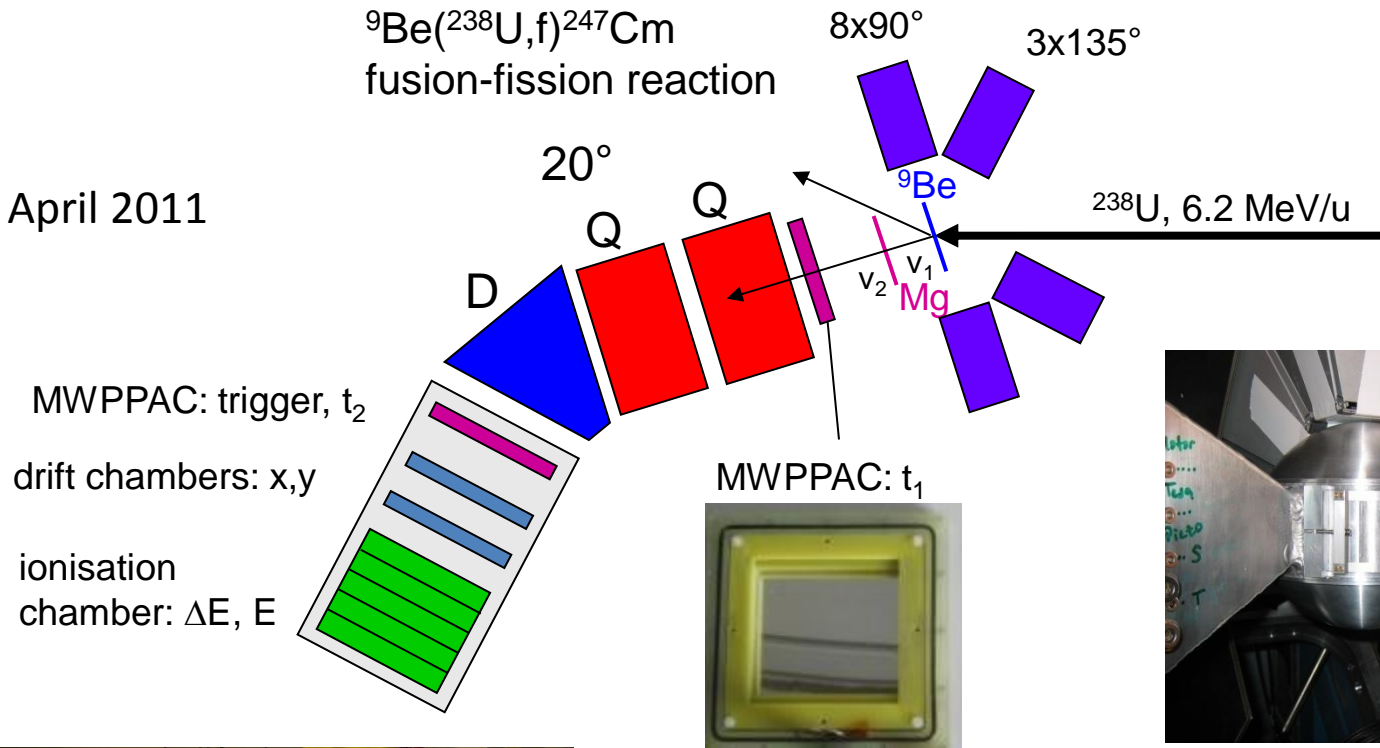


<http://www-phynu.cea.fr/>
 S.Hilaire, M.Girod, Eur. Phys. J. A 33, 237 (2007)

Heavy-ion induced fission experiments with VAMOS

${}^9\text{Be}({}^{238}\text{U}, f){}^{247}\text{Cm}$
fusion-fission reaction

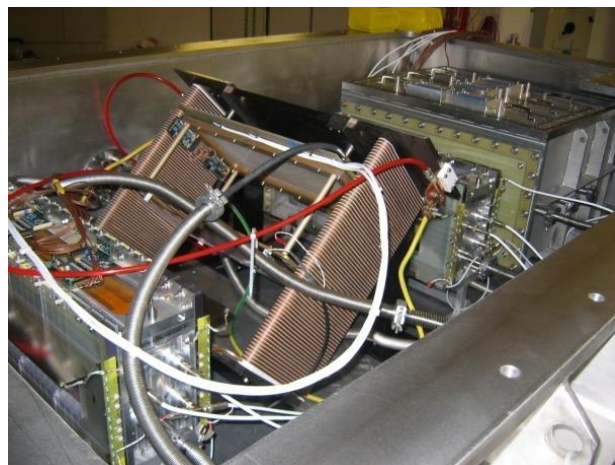
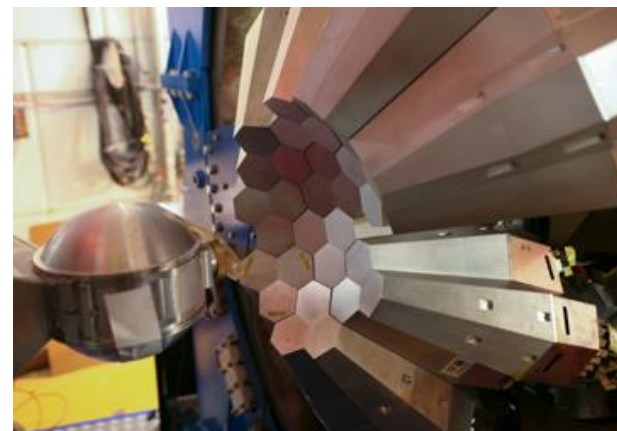
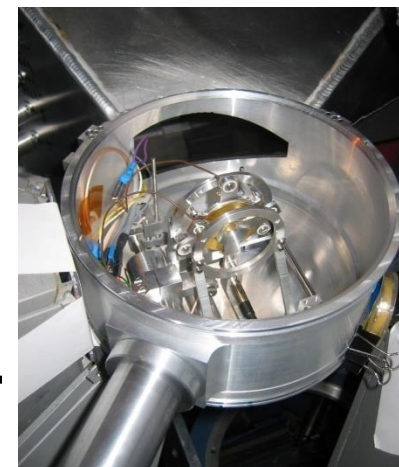
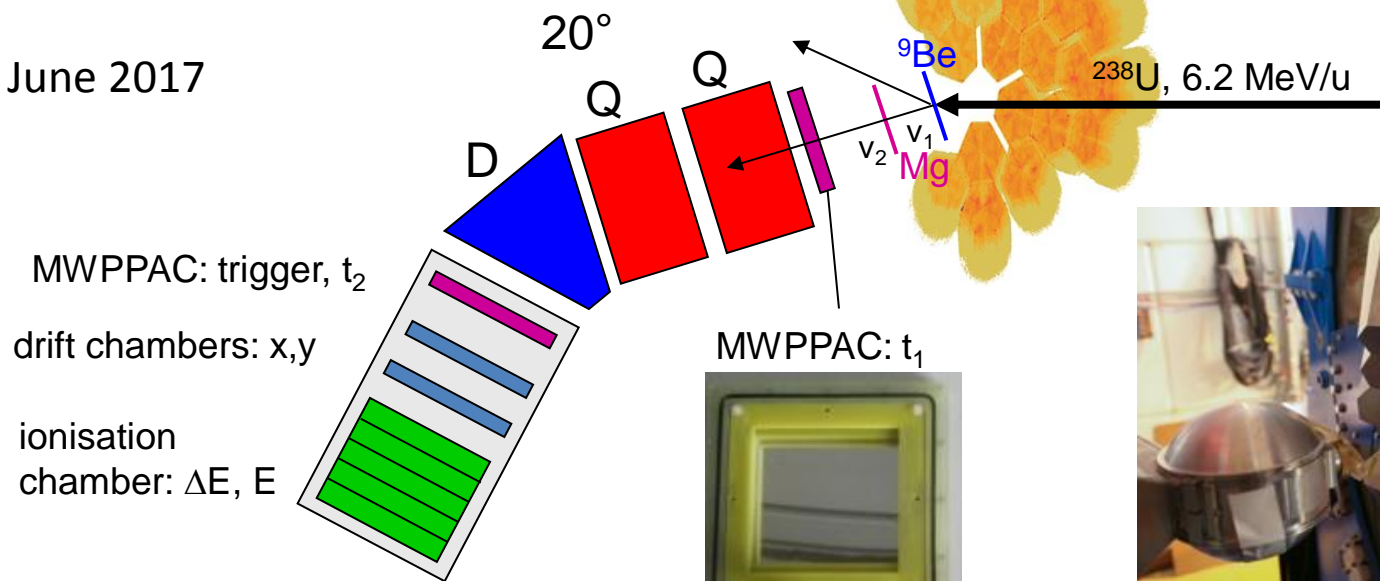
April 2011



Heavy-ion induced fission experiments with VAMOS

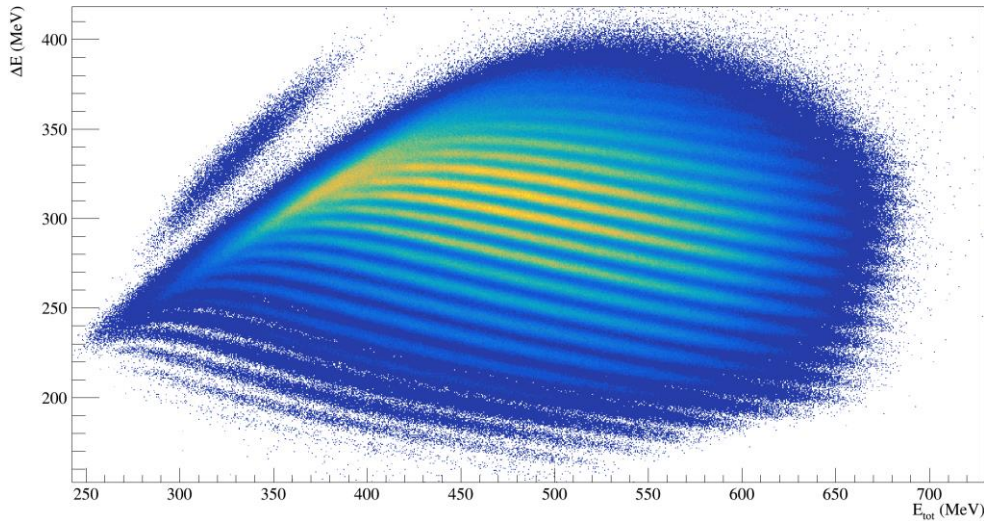
${}^9\text{Be}({}^{238}\text{U}, f){}^{247}\text{Cm}$
fusion-fission reaction

June 2017

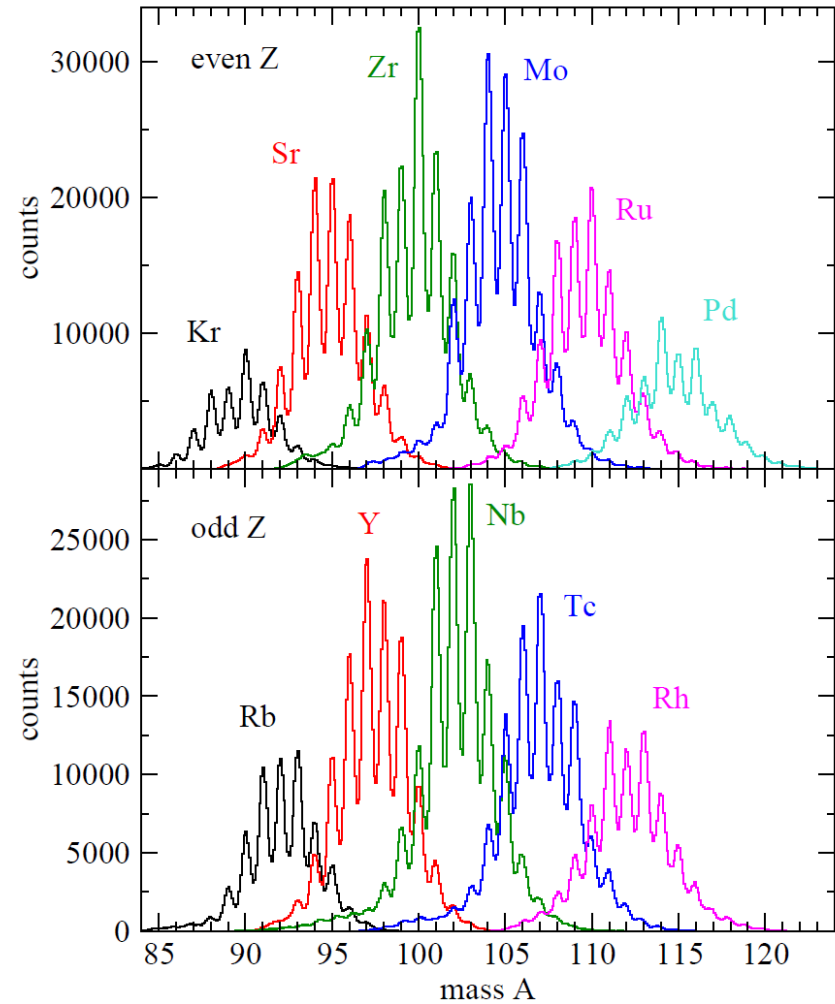
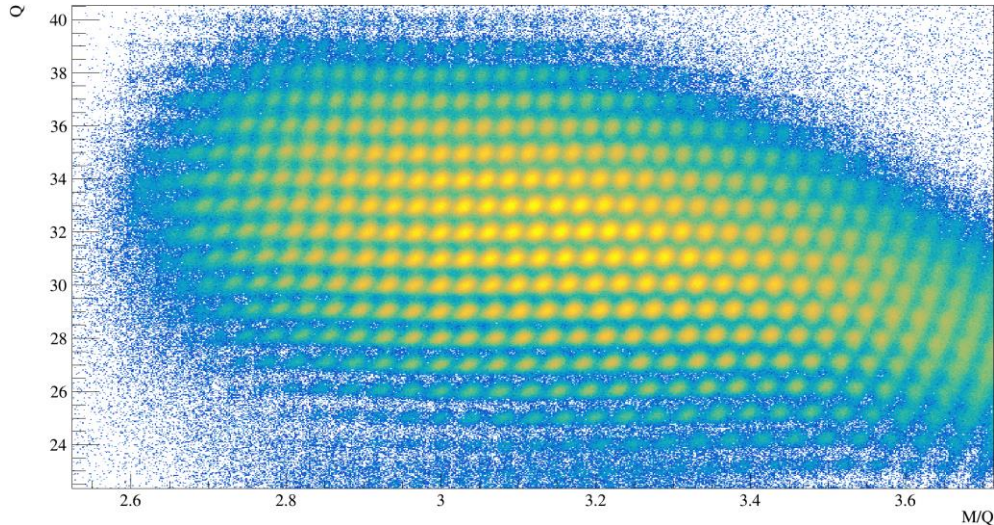


Identification of fission fragments in VAMOS

energy loss and total energy in ionization chamber $\Rightarrow Z$

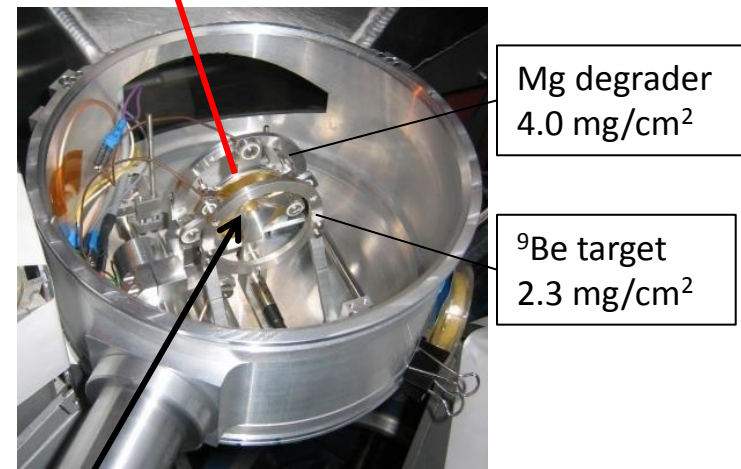
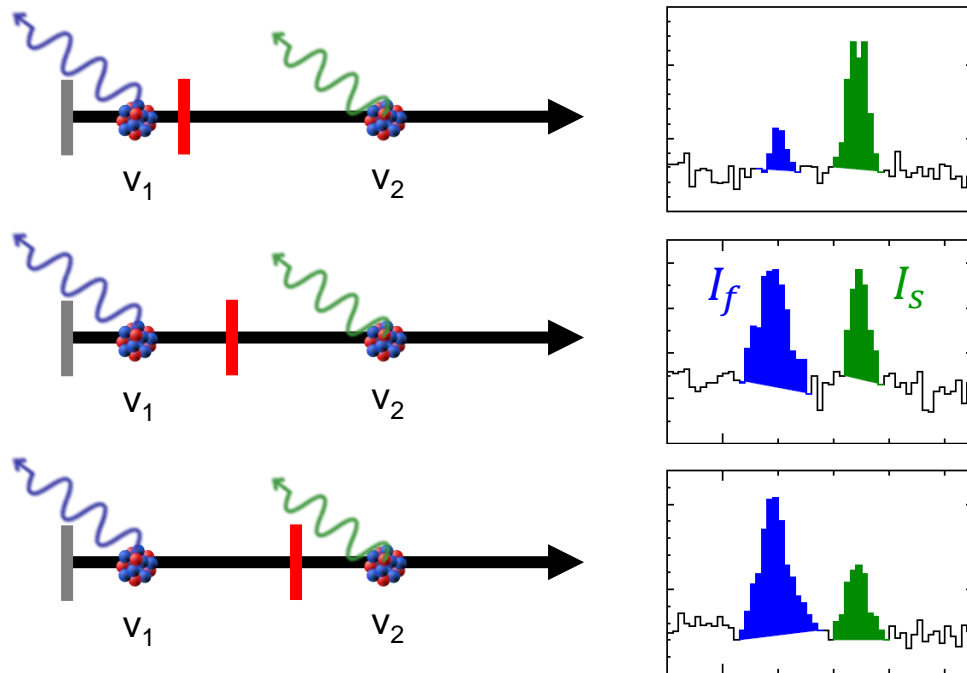


reconstructed trajectory, magnetic rigidity
time of flight, total energy \Rightarrow mass M and M/Q



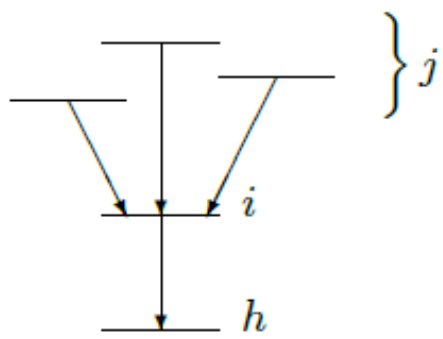
event-by-event identification
in Z and A of more than 200
fission fragments

Recoil distance Doppler shift (RDDS) method



²³⁸U beam

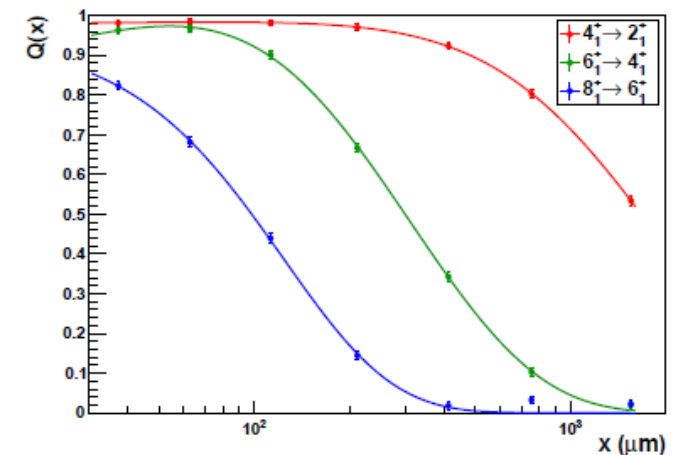
10 distances: 30 – 2650 μm
18 h per distance



decay curve:

$$Q_i(x) = \frac{I_{i,s}(x)}{I_{i,s}(x) + I_{i,f}(x)}$$

$$\tau_i(x) = - \frac{Q_i(x) - \sum_j \alpha_{ij} Q_j(x)}{v_1 \frac{dQ_i(x)}{dx}}$$



A. Dewald et al., Z. Phys. A. 334, 163 (1989)

Lifetime results in even-even nuclei

- Lifetimes in ps
- adopted values
 - **this work**

^{112}Pd 2 ⁺ 84 (14) 4 ⁺ 5.4 (1.7)	^{114}Pd 2 ⁺ 82 (14) 4 ⁺ 5.7 (9)	^{116}Pd 2 ⁺ 110 (30) 4 ⁺ 8.7 (12) 6 ⁺ 2.6 (9)
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^{108}Ru 2 ⁺ 360 (30) 4 ⁺ 13.4 (10) 6 ⁺ 13.6 (9) 2.9 (3)	^{110}Ru 2 ⁺ 320 (20) 4 ⁺ 15.4 (17) 6 ⁺ 2.4 (10) 15.1 (9) 3.2 (5)	^{112}Ru 2 ⁺ 320 (30) 4 ⁺ 14.6 (21)
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^{102}Mo 2 ⁺ 125 (4) 4 ⁺ 12.5 (25) 6 ⁺ 9.4 (10) 3.4 (6)	^{104}Mo 2 ⁺ 970 (80) 4 ⁺ 26.1 (3) 6 ⁺ 4.73 (15) 18.6 (9) 2.8 (2)	^{106}Mo 2 ⁺ 1250 (30) 4 ⁺ 25.4 (51) 6 ⁺ 4.2 (18) 28.0 (13) 3.1 (3)	^{108}Mo 2 ⁺ 500 (300) 4 ⁺ 23.3 (51)
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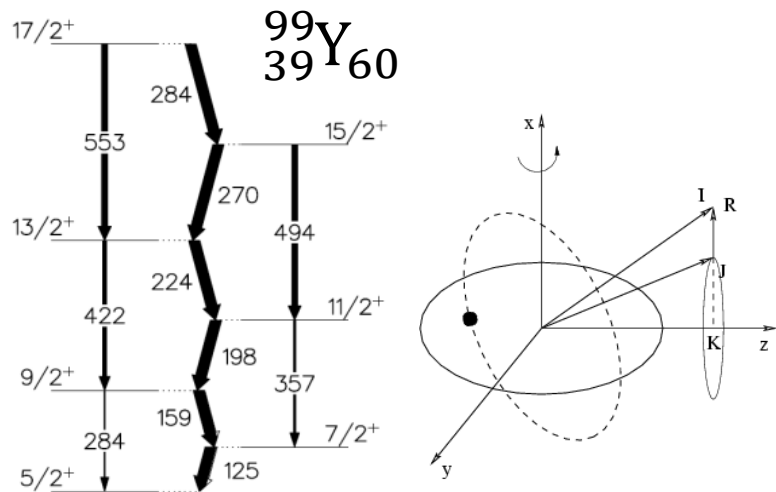
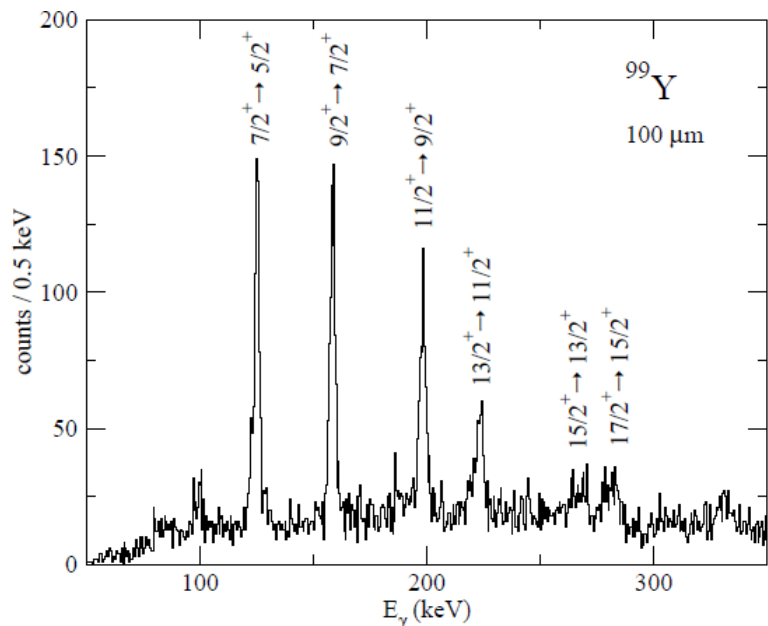
L. Grente, PhD Univ. Paris Sud, 2014
and to be published

^{98}Zr 2 ⁺ <11ps 4.9 (26) 4 ⁺ 28 (3)	^{100}Zr 2 ⁺ 590 (30) 4 ⁺ 37 (3) 6 ⁺ 4.9 (11) 18.1 (14) 3.1 (3)	^{102}Zr 2 ⁺ 1800 (400) 4 ⁺ 32.1 (34) 6 ⁺ 4.7 (5)
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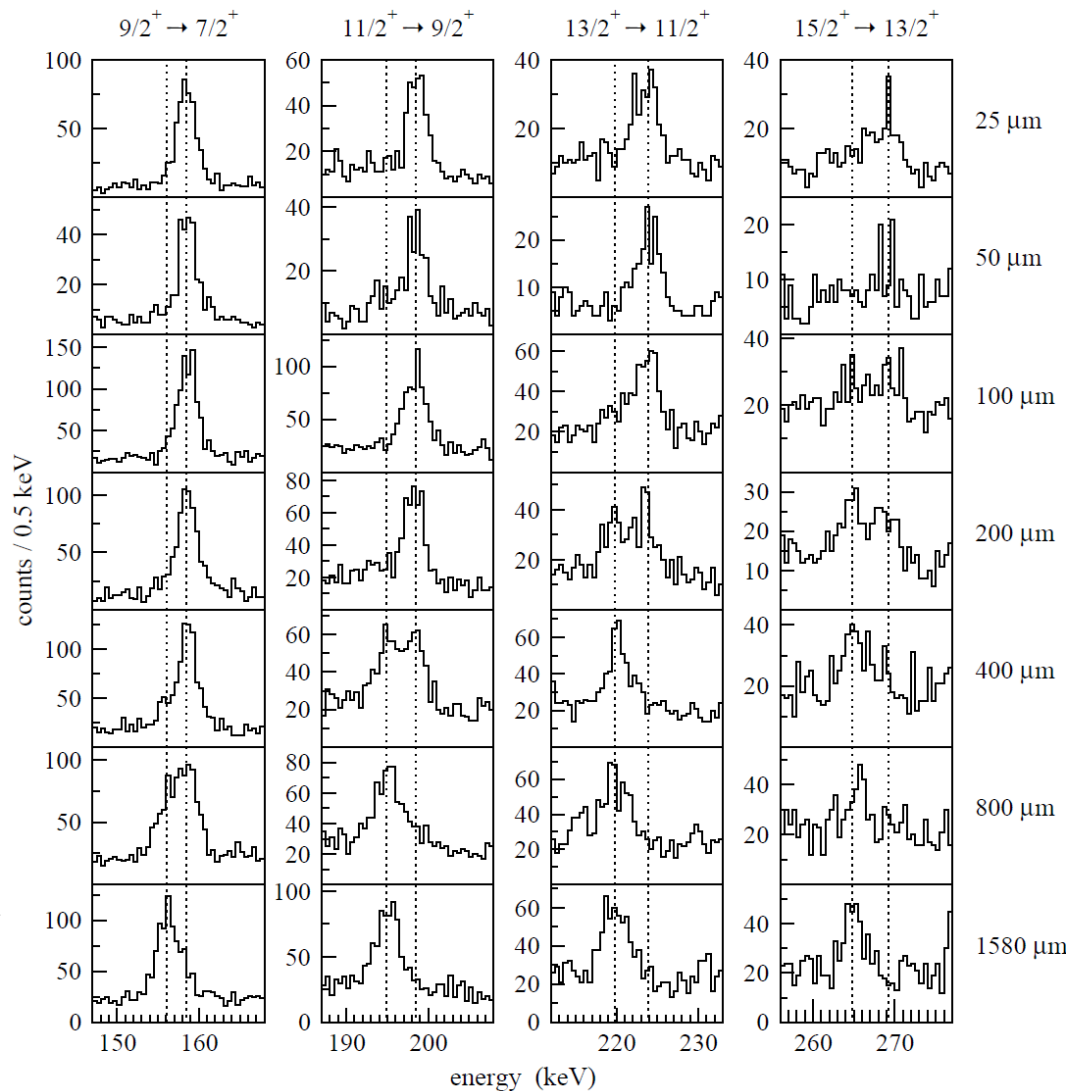
Pd104 0 ⁺ 11.14	Pd105 5/2 ⁺ 22.33	Pd106 0 ⁺ 27.33	Pd107 6.8E-6 s 5/2 ⁺	Pd108 0 ⁺ 26.46	Pd109 13.7011 h 5/2 ⁺	Pd110 0 ⁺ 11.72	Pd111 23.4 m 5/2 ⁺	Pd112 21.03 h 0 ⁺	Pd113 93 s (8/2) ⁺	Pd114 2.42 m 0 ⁺	Pd115 25 s (5/2) ⁺	Pd116 11.8 s 0 ⁺	Pd117 4.3 s (5/2) ⁺	Pd118 1.9 s 0 ⁺	Pd119 0.92 s 0 ⁺
Rh103 1/2 ⁻ 100	Rh104 42.3 s 1 ⁺ EC,β	Rh105 35.36 h 7/2 ⁺	Rh106 29.80 s 1 ⁺	Rh107 21.7 m 7/2 ⁺	Rh108 6.0 m (5 ⁺)	Rh109 30 s 7/2 ⁺	Rh110 3.2 s 1 ⁺	Rh111 11 s (7/2) ⁺	Rh112 1.1 s 1 ⁺	Rh113 2.80 s	Rh114 1.85 s 1 ⁺	Rh115 0.99 s (7/2) ⁺	Rh116 0.68 s 1 ⁺	Rh117 0.44 s (7/2) ⁺	Rh118
Ru102 0 ⁺ 31.6	Ru103 39.26 d 3/2 ⁺	Ru104 0 ⁺ 18.7	Ru105 4.44 h 3/2 ⁺	Ru106 373.59 d 0 ⁺	Ru107 3.75 m (5/2) ⁺	Ru108 4.55 m 0 ⁺	Ru109 34.5 s (5/2) ⁺	Ru110 14.6 s 0 ⁺	Ru111 2.12 s	Ru112 1.75 s 0 ⁺	Ru113 0.80 s	Ru114 0.53 s 0 ⁺	Ru115 0.40 s 0 ⁺	Ru116 0 ⁺	Ru117
Tc101 14.21 m (9/2) ⁺	Tc102 5.28 s 1 ⁺	Tc103 54.2 s 5/2 ⁺	Tc104 18.3 m (3 ⁺)	Tc105 7.6 m (3/2) ⁻	Tc106 35.6 s (1,2)	Tc107 21.2 s	Tc108 517 s (2 ⁺)	Tc109 0.87 s	Tc110 0.92 s (1 ⁺ ,2 ⁺)	Tc111 0.30 s	Tc112 0.28 s	Tc113 130 ms	Tc114	Tc115	
Mo100 1.2E19 y 0 ⁺ ββ	Mo101 14.61 m 12 ⁺	Mo102 11.3 m 0 ⁺	Mo103 97.5 s (3/2) ⁺	Mo104 60 s 0 ⁺	Mo105 35.8 s (5/2) ⁻	Mo106 34 s 0 ⁺	Mo107 3.5 s	Mo108 1.09 s 0 ⁺	Mo109 0.33 s	Mo110 0.30 s	Mo111	Mo112 0 ⁺	Mo113		
Nb99 15.0 s 9/2 ⁺	Nb100 15 s 1 ⁺	Nb101 7.1 s +	Nb102 1.3 s 1 ⁺	Nb103 15 s (5/2) ⁺	Nb104 4.8 s (1 ⁺)	Nb105 2.95 s (5/2) ⁺	Nb106 1.02 s	Nb107 330 ms	Nb108 0.19 s (2)	Nb109 0.19 s	Nb110 0.17 s				
Zr98 36.7 s 0 ⁺	Zr99 21 s (1/2) ⁺	Zr100 7.1 s 0 ⁺	Zr101 1.3 s (3/2) ⁺	Zr102 2.9 s 0 ⁺	Zr103 1.3 s (5/2) ⁻	Zr104 1.2 s 0 ⁺	Zr105 0.6 s	Zr106 0 ⁺	Zr107	Zr108 0 ⁺					

⇒ comparison of B(E2) values
with beyond-mean field calculations

Lifetime measurement for odd-mass Y isotopes with EXOGAM

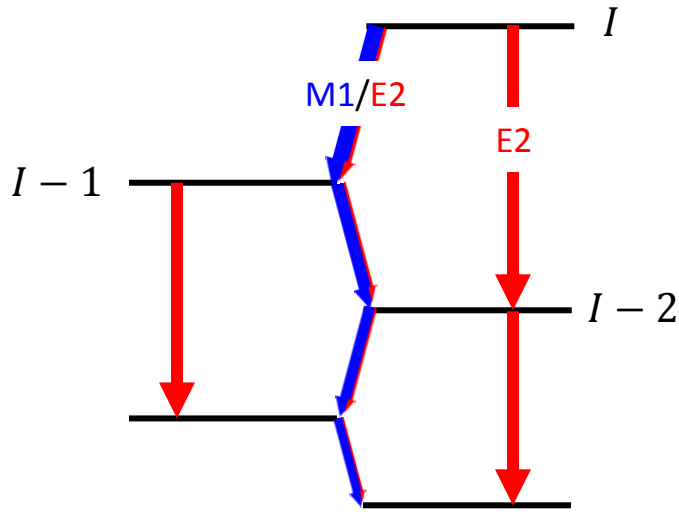


deformed ^{98}Sr plus extra proton
 \Rightarrow strongly coupled band



T.W.Hagen et al.,
 PRC 95, 034302 (2017)

Rotational bands in the particle – rotor model



branching ratio:
$$\frac{I_{\gamma}(\Delta I = 1)}{I_{\gamma}(\Delta I = 2)} \Rightarrow \frac{B(M1)}{B(E2)}$$

lifetime:
$$\frac{1}{\tau} = \lambda_{tot} = \lambda_{M1} + \lambda_{E2, \Delta I=1} + \lambda_{E2, \Delta I=2}$$

\Rightarrow absolute $B(M1)$ and $B(E2)$ values

\Rightarrow comparison with theoretical calculations

Particle – rotor model:
$$B(M1; I_i K \rightarrow I_f K) = \frac{3}{4\pi} e^2 (g_K - g_R)^2 K^2 |\langle I_i 1 K 0 | I_f K \rangle|^2$$

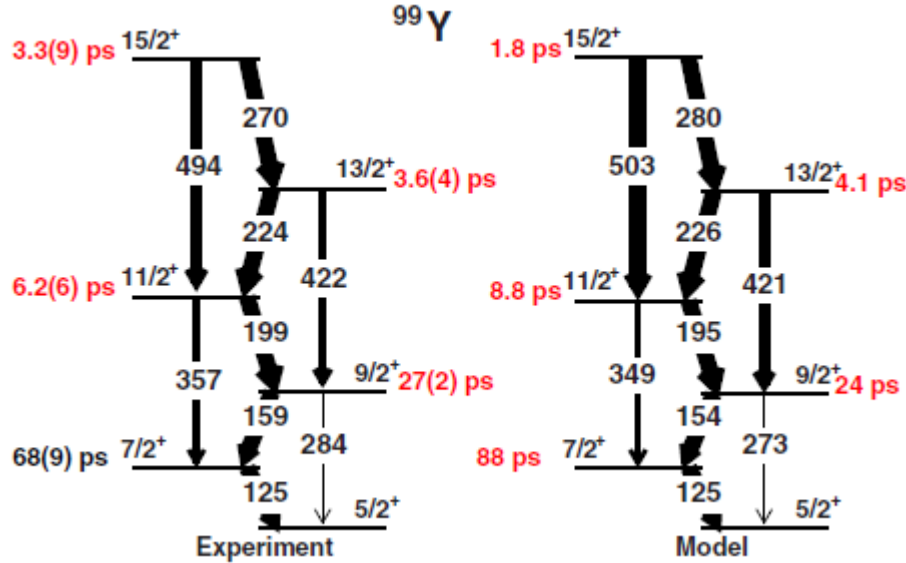
$$B(E2; I_i K \rightarrow I_f K) = \frac{5}{16\pi} e^2 Q_0^2 |\langle I_i 2 K 0 | I_f K \rangle|^2$$

branching ratio:
$$\frac{B(M1)}{B(E2)} \Rightarrow \frac{(g_K - g_R)}{Q_0}$$

lifetime: $\triangleright (g_K - g_R) \Rightarrow$ configuration of the odd particle

$\triangleright Q_0 \Rightarrow$ deformation of the core

Results for ^{99}Y



microscopic article-rotor calculations
with $\varepsilon_2 = 0.41$ and $\gamma = 0^\circ$

T.W.Hagen et al.,
PRC 95, 034302 (2017)

I_i	$b_{E2/M1}$	τ [ps]	$g_K - g_R$	Q_0 [eb]
9/2	0.18	27	0.97	5.0
11/2	0.35	6.2	1.25	5.9
13/2	0.45	3.6	1.27	5.0
15/2	0.64	3.3	0.92	3.7

Laser spectroscopy of $5/2^+$ **ground state**

B. Cheal et al., Phys. Lett. B 645, 133 (2007)

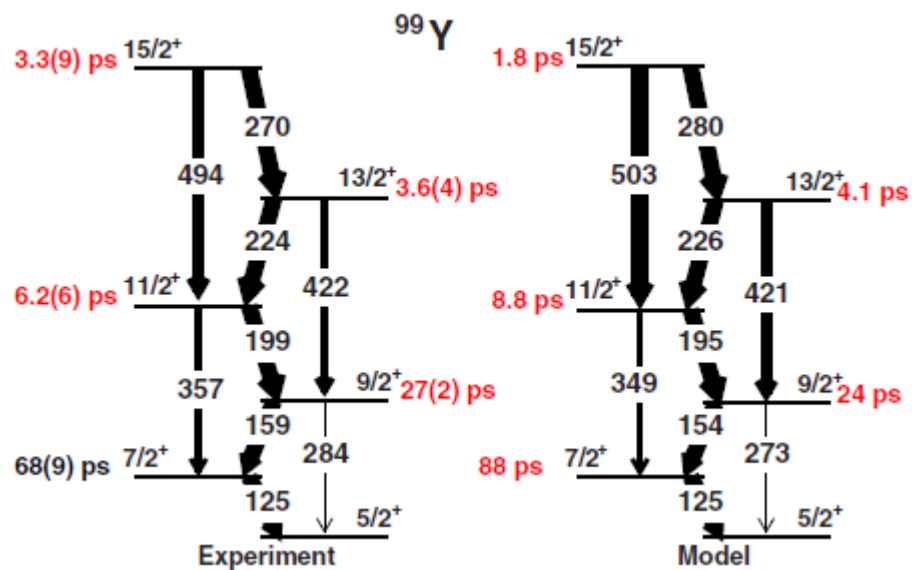
- $\mu = 3.18(2) \mu_N$
with $g_R = Z/A \Rightarrow (g_K - g_R) = 1.23$
consistent with $[422]5/2^+$ configuration
- $Q_s = +1.55(17) \text{ eb}$
rotational model: $Q_s = \frac{3K^2 - I(I+1)}{(I+1)(2I+3)} Q_0$
 $\Rightarrow Q_0 = 4.34(48) \text{ eb}$

Coulomb excitation of ^{98}Sr :

E. Clément et al., PRL 116, 022701 (2016)

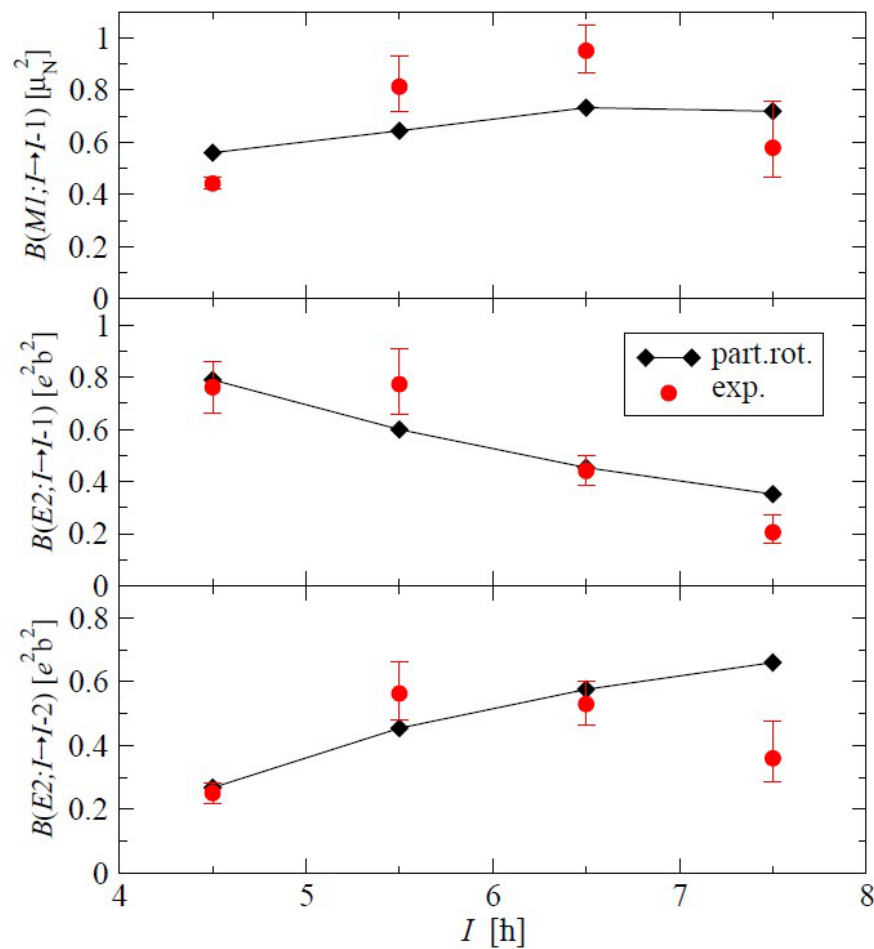
- $Q_s(4^+) = -1.87_{-25}^{+14} \text{ eb}$
 $\Rightarrow Q_0 = 5.14_{-0.69}^{+0.39} \text{ eb}$

Results for ^{99}Y

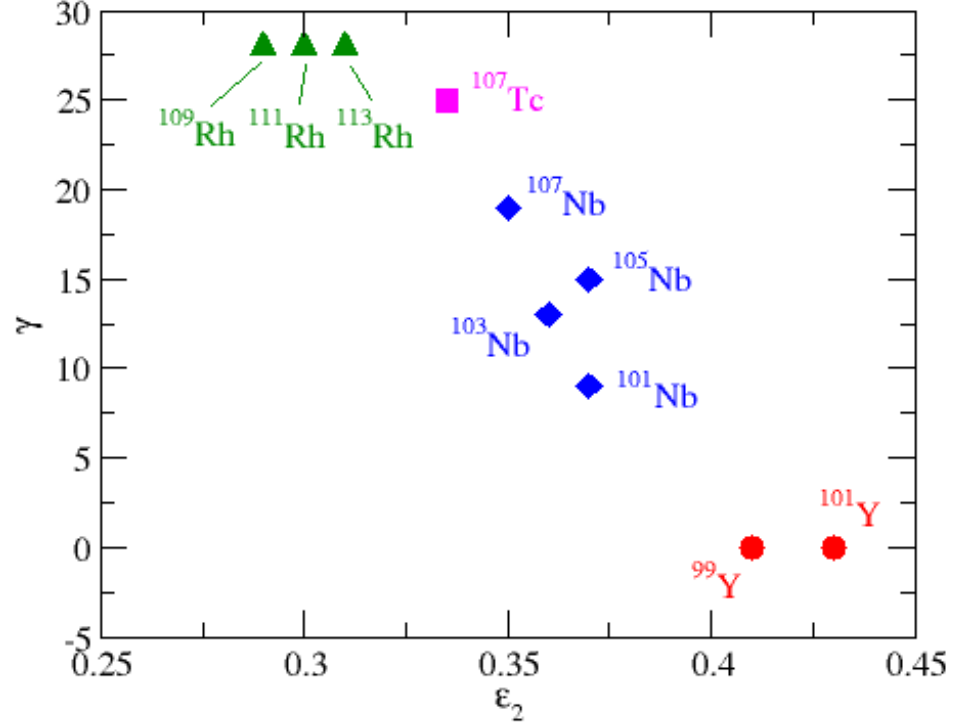
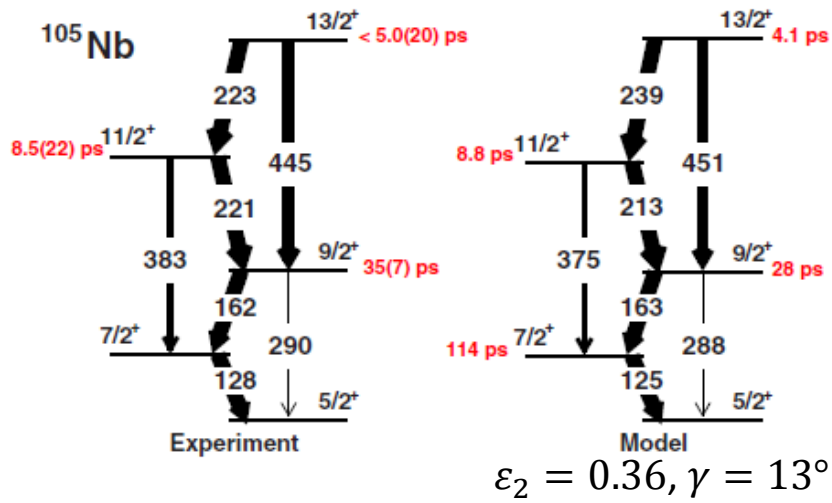


microscopic article-rotor calculations
with $\varepsilon_2 = 0.41$ and $\gamma = 0^\circ$

T.W.Hagen et al.,
PRC 95, 034302 (2017)



Many more results from the same experiment



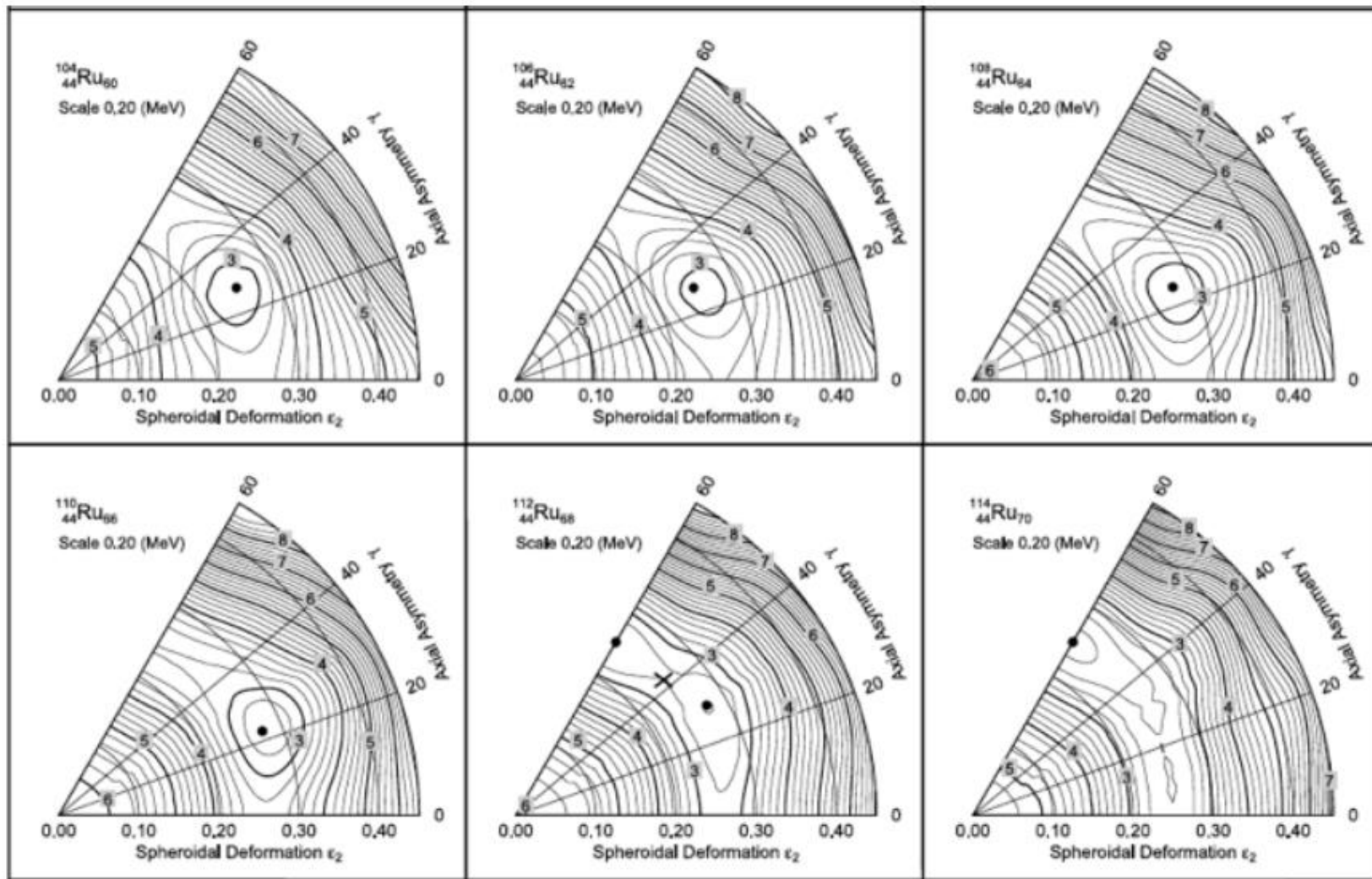
Results for $_{39}\text{Y}$, $_{41}\text{Nb}$, $_{43}\text{Tc}$, $_{45}\text{Rh}$:
with increasing Z :

- increase of triaxiality γ
- decrease of deformation ε_2

94	Rh95	Rh96	Rh97	Rh98	Rh99	Rh100	Rh101	Rh102	Rh103	Rh104	Rh105	Rh106	Rh107	Rh108	Rh109	Rh110	Rh111	Rh112	Rh113	Rh114	Rh115	Rh116	Rh117	Rh118	Rh119	Rh120	Rh121
93	Ru94	Ru95	Ru96	Ru97	Ru98	Ru99	Ru100	Ru101	Ru102	Ru103	Ru104	Ru105	Ru106	Ru107	Ru108	Ru109	Ru110	Ru111	Ru112	Ru113	Ru114	Ru115	Ru116	Ru117	Ru118		
92	Tc93	Tc94	Tc95	Tc96	Tc97	Tc98	Tc99	Tc100	Tc101	Tc102	Tc103	Tc104	Tc105	Tc106	Tc107	Tc108	Tc109	Tc110	Tc111	Tc112	Tc113	Tc114	Tc115				
91	Mo92	Mo93	Mo94	Mo95	Mo96	Mo97	Mo98	Mo99	Mo100	Mo101	Mo102	Mo103	Mo104	Mo105	Mo106	Mo107	Mo108	Mo109	Mo110	Mo111	Mo112	Mo113					
90	Nb91	Nb92	Nb93	Nb94	Nb95	Nb96	Nb97	Nb98	Nb99	Nb100	Nb101	Nb102	Nb103	Nb104	Nb105	Nb106	Nb107	Nb108	Nb109	Nb110							
89	Zr90	Zr91	Zr92	Zr93	Zr94	Zr95	Zr96	Zr97	Zr98	Zr99	Zr100	Zr101	Zr102	Zr103	Zr104	Zr105	Zr106	Zr107	Zr108								
88	Y89	Y90	Y91	Y92	Y93	Y94	Y95	Y96	Y97	Y98	Y99	Y100	Y101	Y102	Y103	Y104	Y105										
87	Sr88	Sr89	Sr90	Sr91	Sr92	Sr93	Sr94	Sr95	Sr96	Sr97	Sr98	Sr99	Sr100	Sr101	Sr102	Sr103	Sr104										

T.W. Hagen
PhD Univ. Oslo (2016)

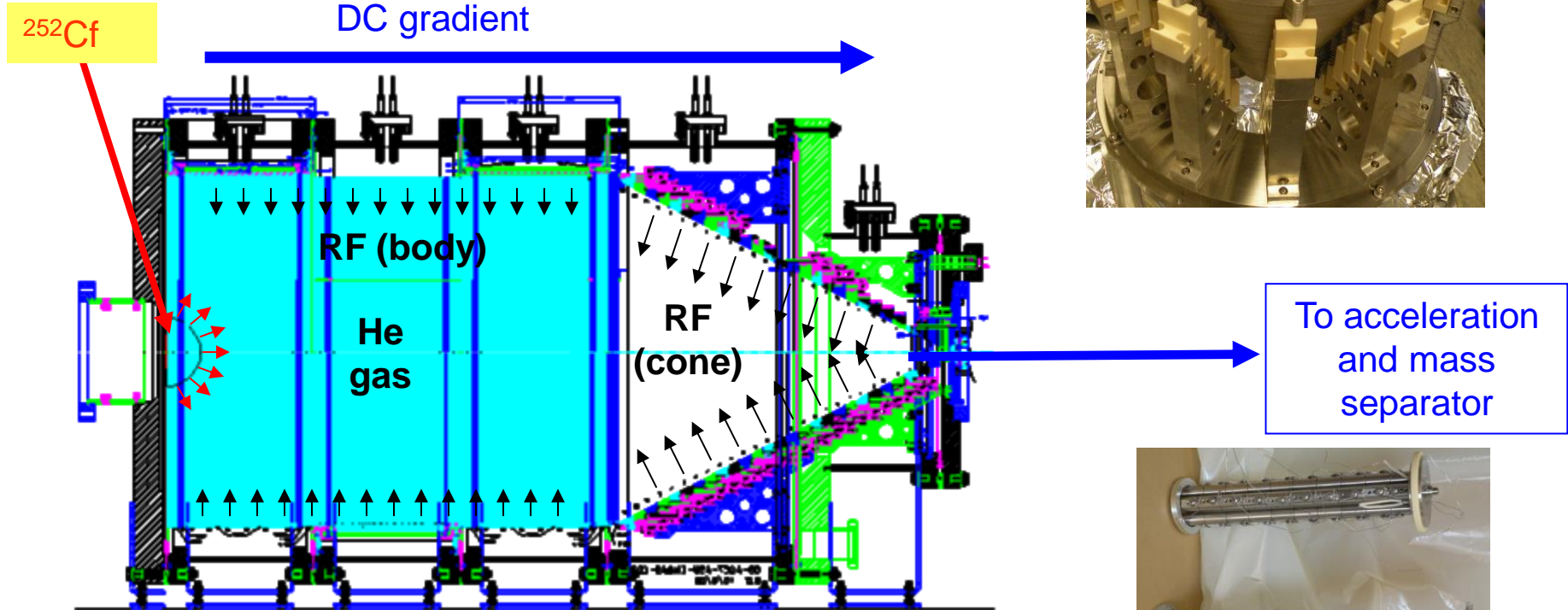
Triaxiality predicted for $Z \approx 44$ and $N \approx 64$



P. Möller et al., At. Data Nucl. Data Table 94 (2008)

CARIBU (CALifornium Rare Isotope Breeder Upgrade) at Argonne

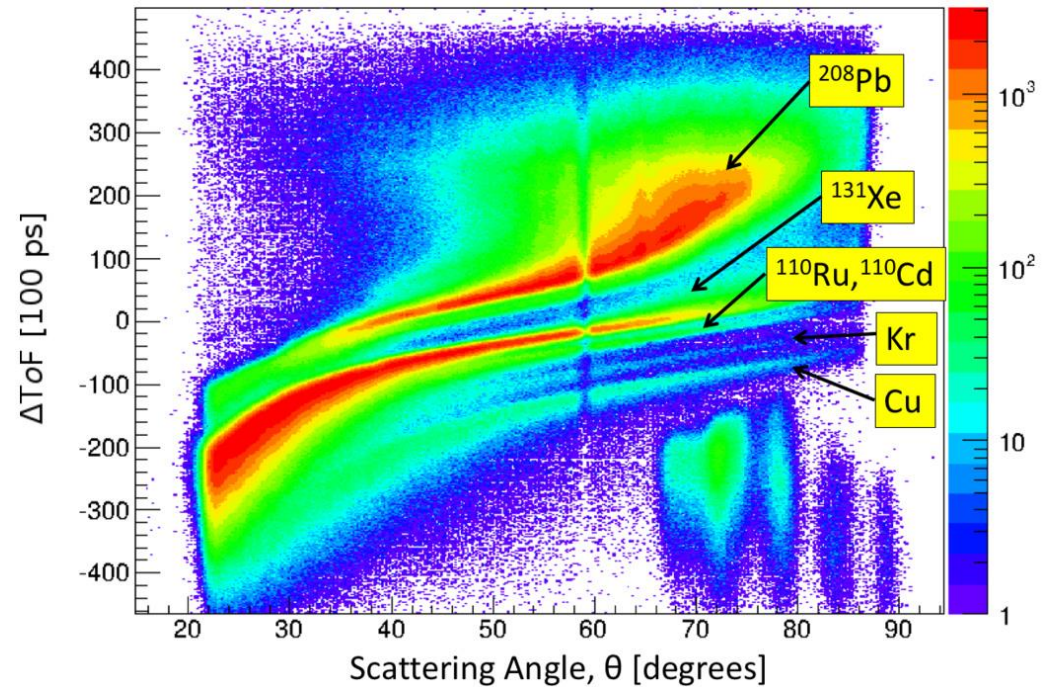
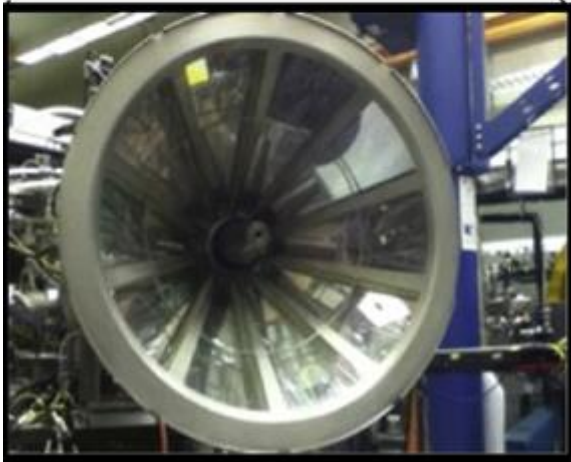
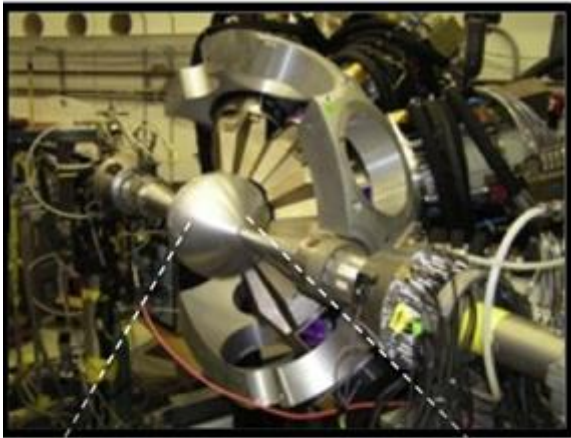
- 1.7 Ci ^{252}Cf source
- recoiling fission fragments stopped in Helium gas
- ion transport by RF field + DC field + gas flow
- Fast and independent of chemistry
- Extraction in 2 RFQ sections



G.Savard et al., Nucl. Instr. Meth. B 266, 4086 (2008)

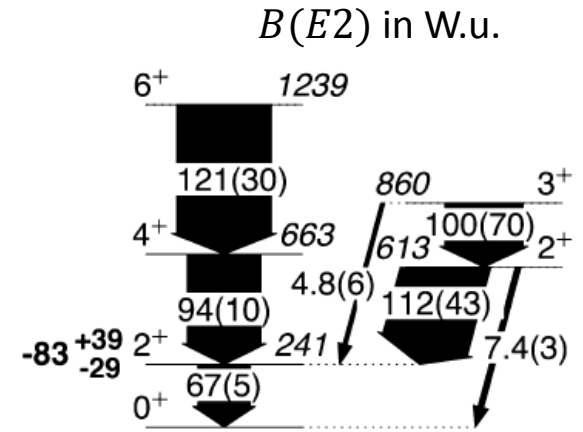
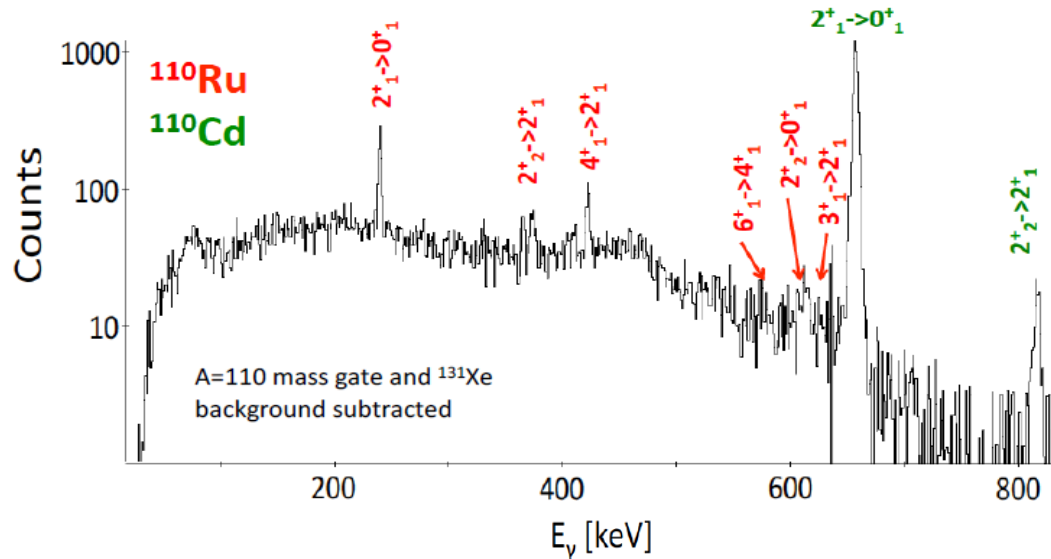


Coulomb excitation of ^{110}Ru with GRETINA and CHICO2



- ToF in CHICO2:
⇒ some mass identification: $\frac{\Delta A}{A} \approx 10\%$
- position resolution in GRETINA and CHICO2:
⇒ Doppler correction
- isobaric contaminants problematic

Coulomb excitation of ^{110}Ru with GREINA and CHICO2



D.Doherty et al.,
Phys.Lett.B 766, 334 (2017)

γ	$\frac{\epsilon_1(2)}{\epsilon_1(2)}$	$b(E2; 22 \rightarrow 0)$	$b(E2; 22 \rightarrow 21)$	$\frac{b(E2; 22 \rightarrow 21)}{b(E2; 22 \rightarrow 0)}$
0	∞	0	0	1.43
5	64.2	0.0074	0.011	1.49
10	15.9	0.028	0.051	1.70
15	6.85	0.053	0.143	2.70
20	3.73	0.067	0.357	5.35
22.5	2.93	0.0625	0.563	19.02
24	2.59	0.052	0.782	15.1
25	2.41	0.0425	0.865	20.6
26	2.26	0.0324	1.01	31.2
28	2.07	0.010	1.28	126
29	2.01	0.004	1.41	363
30	2.00	0	1.43	∞

A.S.Davydov and G.F.Filippov, Nucl. Phys. 8, 237 (1958)

$$\frac{E(2^+_2)}{E(2^+_1)} = 2.54$$

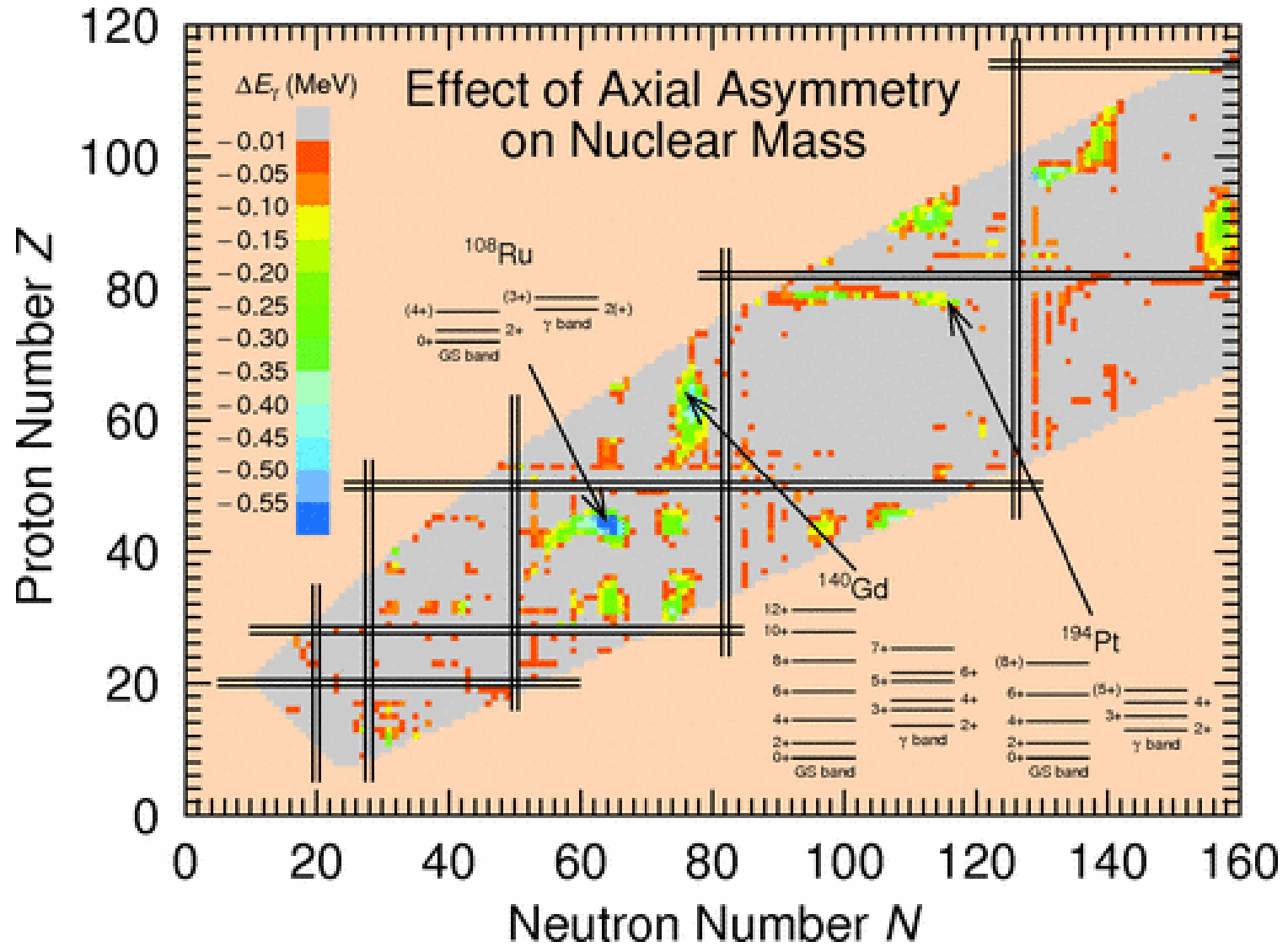
$$\frac{B(E2; 2^+_2 \rightarrow 0^+_1)}{B(E2; 2^+_1 \rightarrow 0^+_1)} = 0.11(1)$$

$$\frac{B(E2; 2^+_2 \rightarrow 2^+_1)}{B(E2; 2^+_1 \rightarrow 0^+_1)} = 1.67(65)$$

$$\frac{B(E2; 2^+_2 \rightarrow 2^+_1)}{B(E2; 2^+_2 \rightarrow 0^+_1)} = 15(6)$$

$B(E2)$ values compatible with $\gamma \approx 25^\circ$

Regions of triaxiality near the ground state

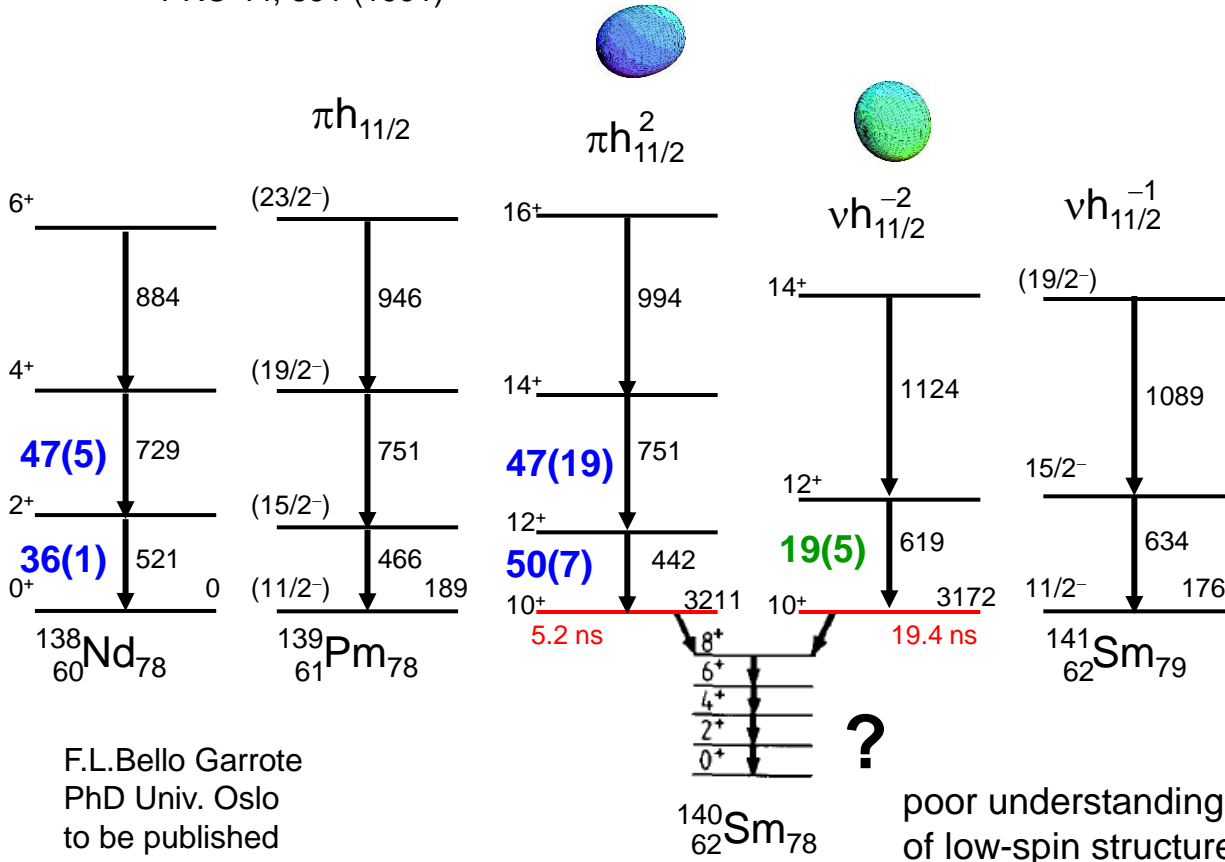


P. Möller et al., Phys. Rev. Lett. 97, 162502 (2006)

Shape coexistence in ^{140}Sm

B(E2) [W.u.]

M.A. Cardona et al.
PRC 44, 891 (1991)

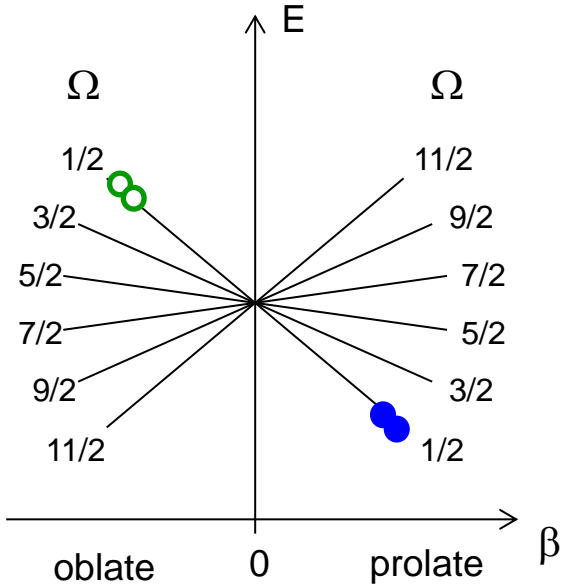


F.L. Bello Garrote
PhD Univ. Oslo
to be published

poor understanding
of low-spin structure

need B(E2) and Q_s

\Rightarrow Coulomb excitation

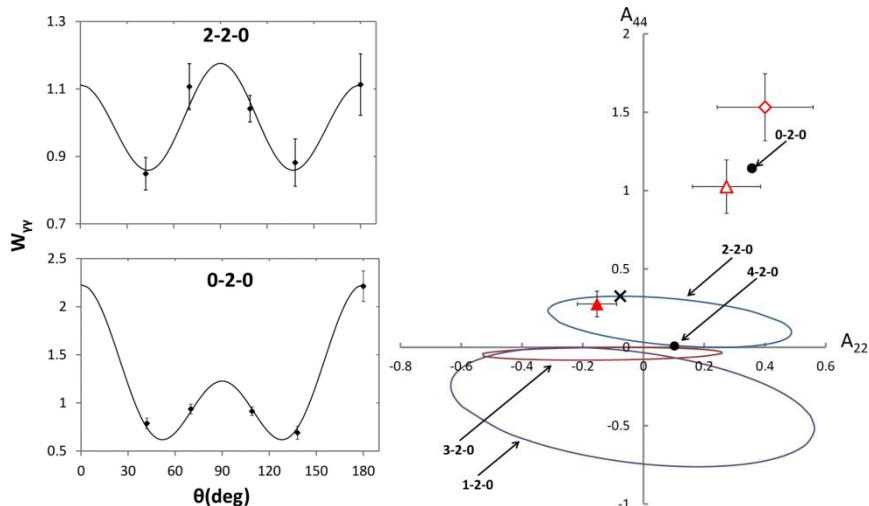
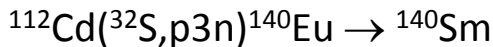


rotationally aligned 2qp bands
built on $\pi(h_{11/2})^2$ and $\nu(h_{11/2})^{-2}$

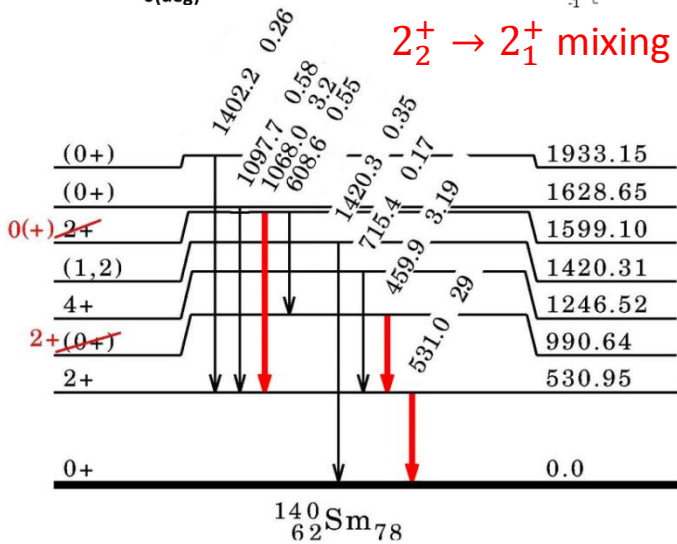
indirect evidence for
shape coexistence
at intermediate spins

Shape coexistence also at low spin?

angular correlation measurement



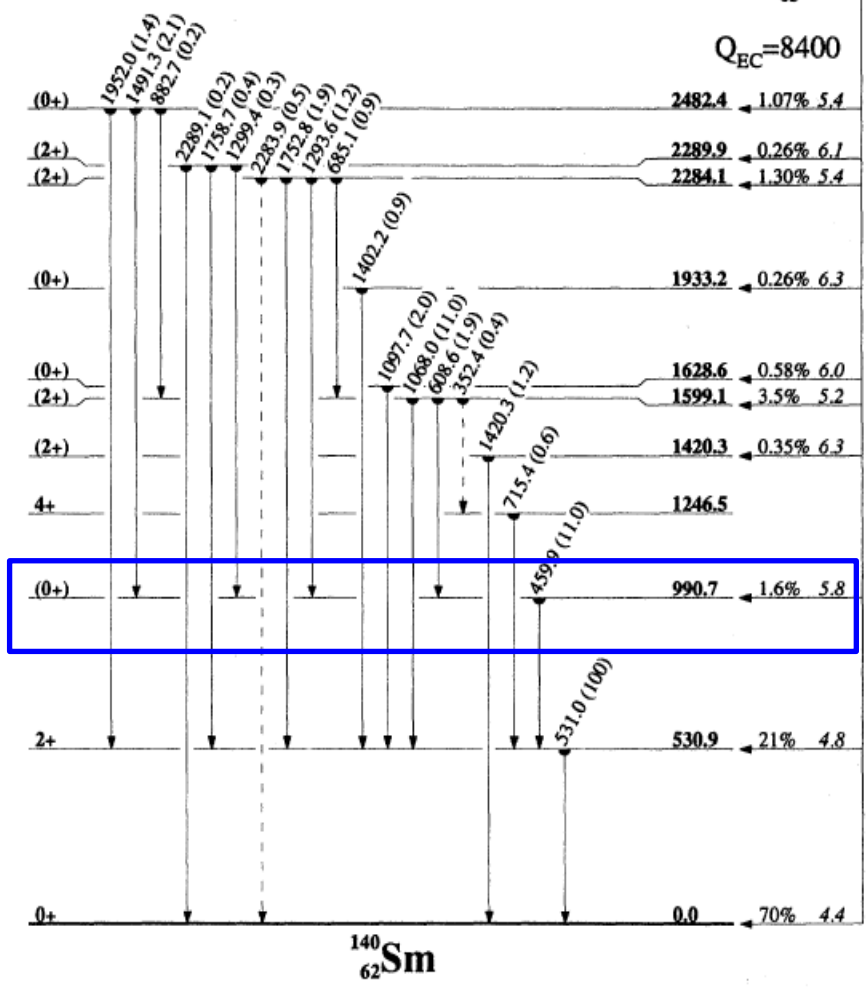
$2_2^+ \rightarrow 2_1^+$ mixing ratio: 98% E2



β decay of ^{140}Eu

$^{140}_{63}\text{Eu}$

$Q_{EC} = 8400$



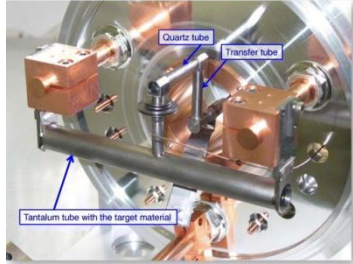
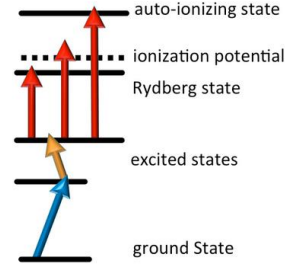
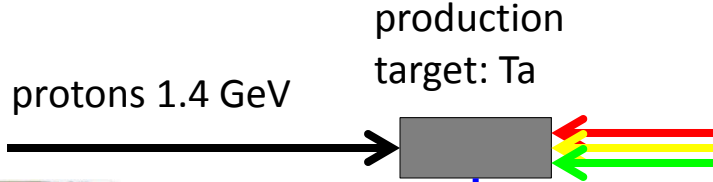
low-lying (0+) state ?

R.B. Firestone et. al. PRC 43, 1066 (1991)

J.Samorajczyk et al., PRC 92, 044322 (2015)

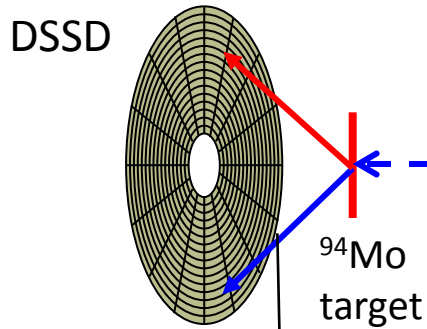
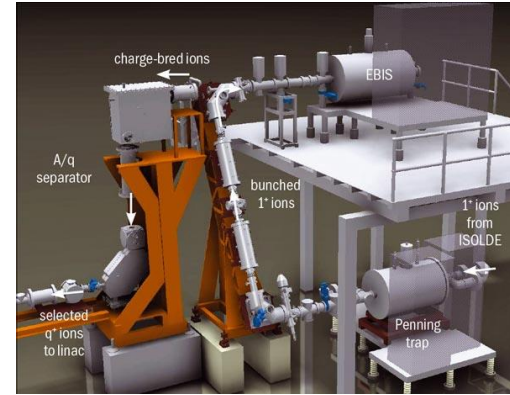
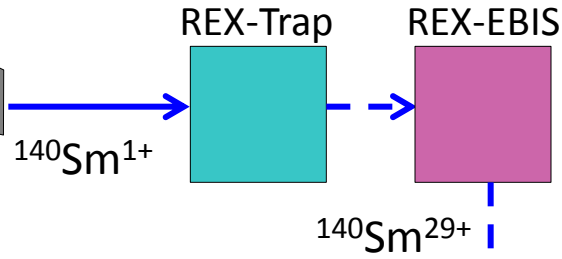
Isotope separation on-line and postacceleration at ISOLDE

resonant laser ionization



Sm^{1+}
60 keV

high-resolution
mass separator
 $A = 140$

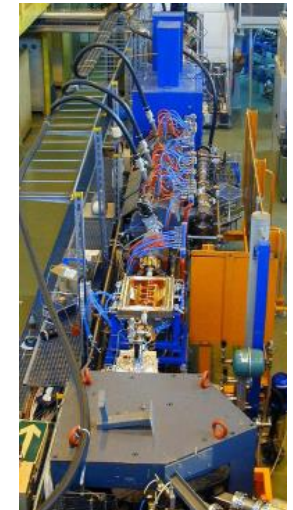
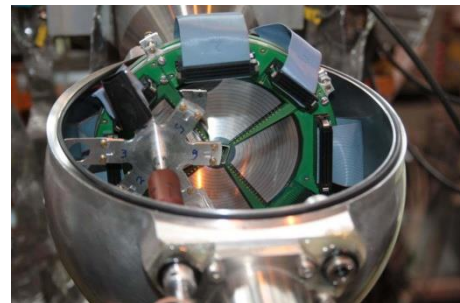
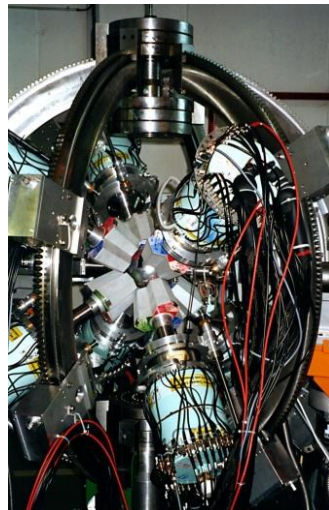


Miniball
Ge detectors

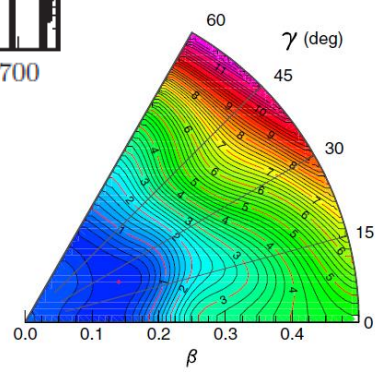
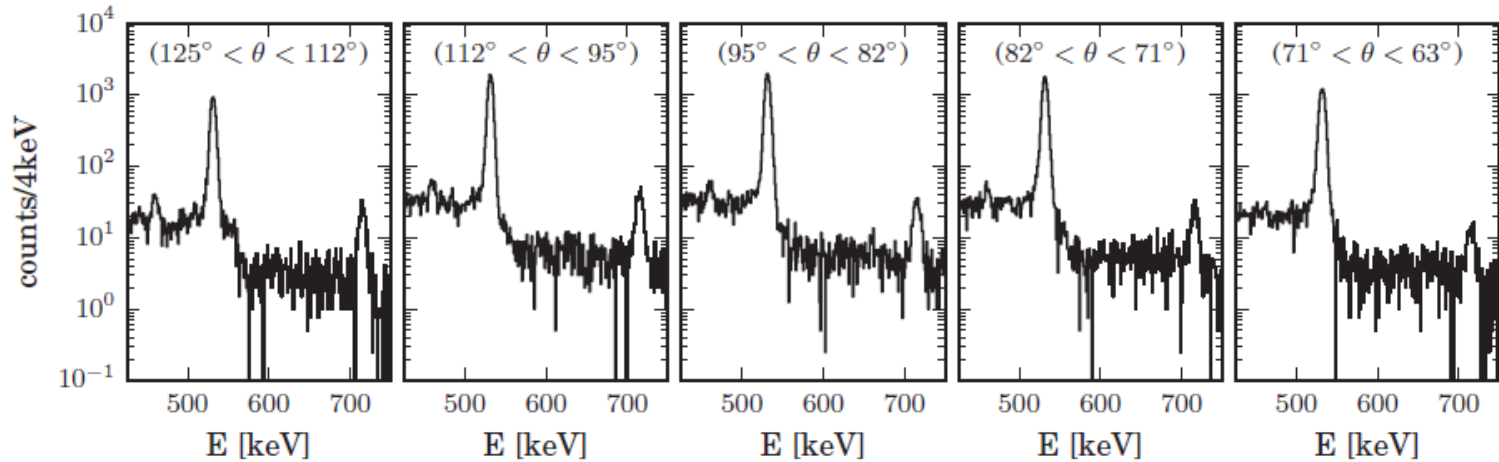
^{140}Sm
2.85 A MeV

REX-Postaccelerator

mass
separator

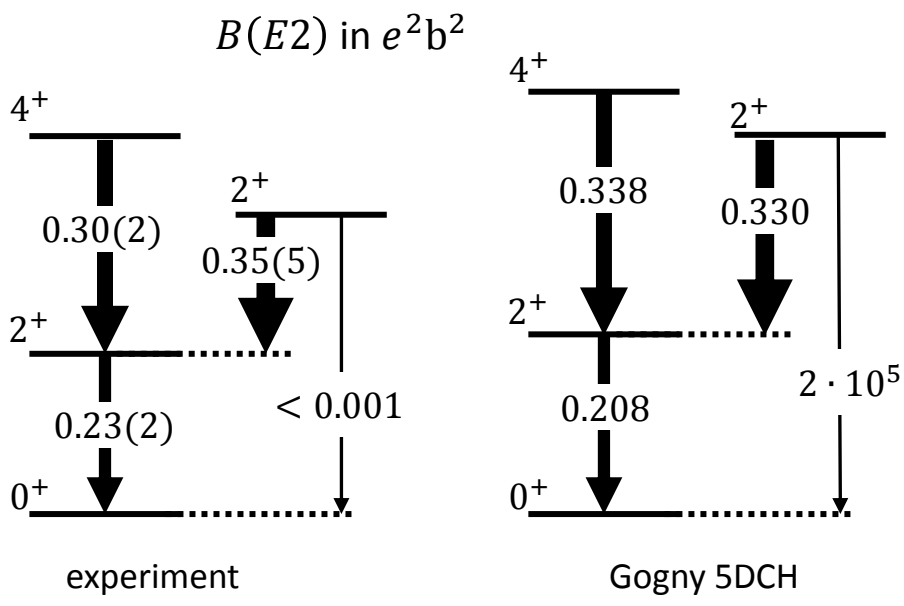


Coulomb excitation of ^{140}Sm



$Q_s(2_1^+) = -0.06^{+0.41}_{-0.15} \text{ eb}$

γ rigid?
 γ soft?



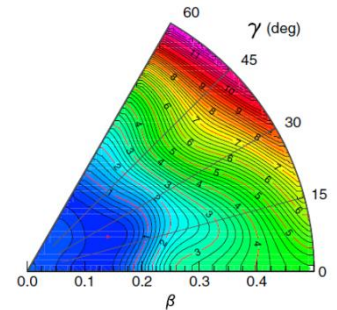
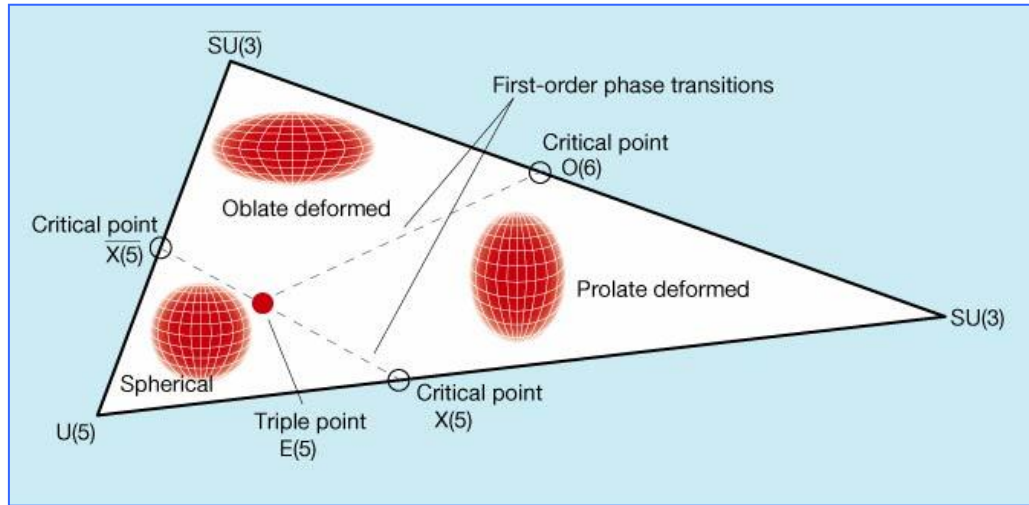
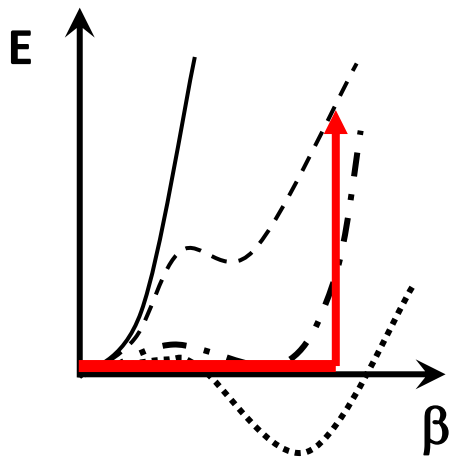
$Q_s(2_2^+) = +0.11 \text{ eb}$
 $Q_s(2_1^+) = -0.12 \text{ eb}$

M. Klintefjord et al. Phys. Rev. C 93, 054303 (2016)

Phase diagram of nuclear shapes

Quantum phase transitions in equilibrium shapes of nuclei

control parameters: N, Z



approximation: infinite square-well potential

- analytically solvable
- certain characteristics of 1. order phase transition
- $X(5)$ critical point symmetry

square well that is also independent of γ

- also analytically solvable
- characteristics of 2. order phase transition
- $E(5)$ critical point symmetry

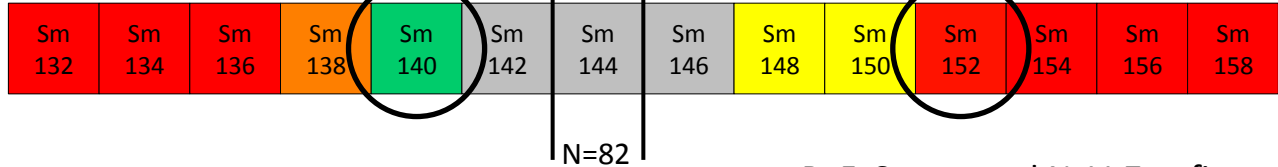
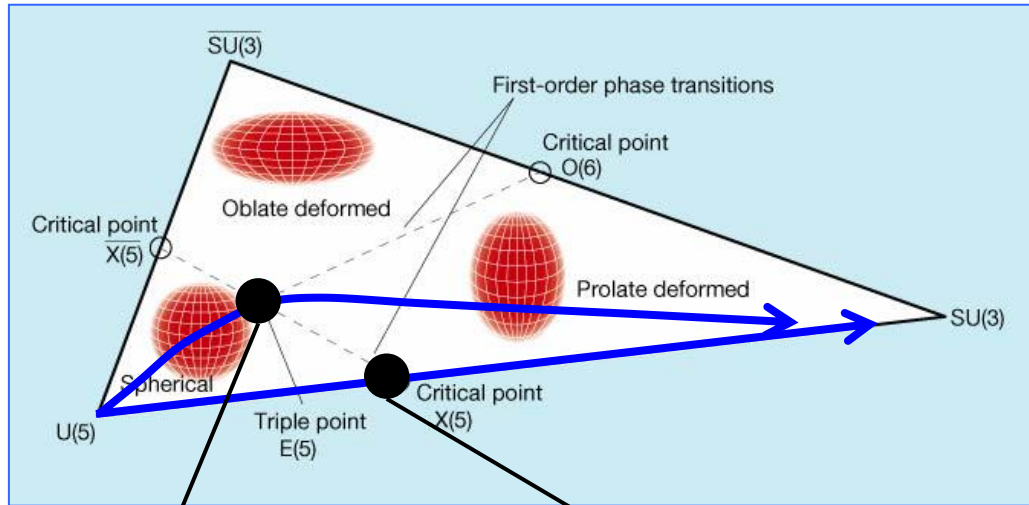
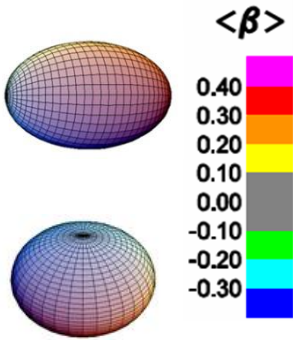
F. Iachello, Phys. Rev. Lett. 85, 3580 (2000)

F. Iachello, Phys. Rev. Lett. 87, 052502 (2001)

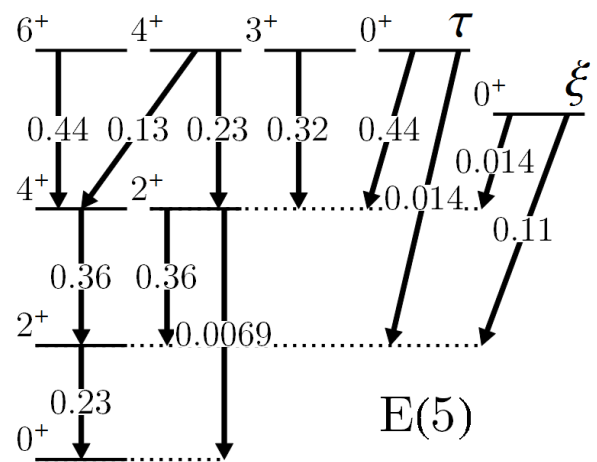
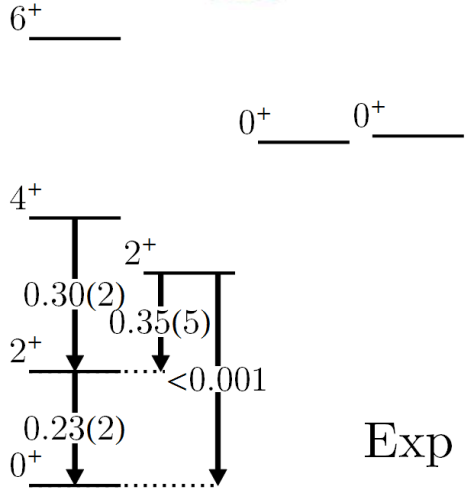
Phase diagram of nuclear shapes

Quantum phase transitions in equilibrium shapes of nuclei

control parameters: N, Z



R. F. Casten and N. V. Zamfir
PRL 87, 052503 (2001)



new HIE-ISOLDE experiment
August 2017

M. Klintefjord, PhD Univ. Oslo 2016
M. Klintefjord et al. Phys. Rev. C 93, 054303 (2016)