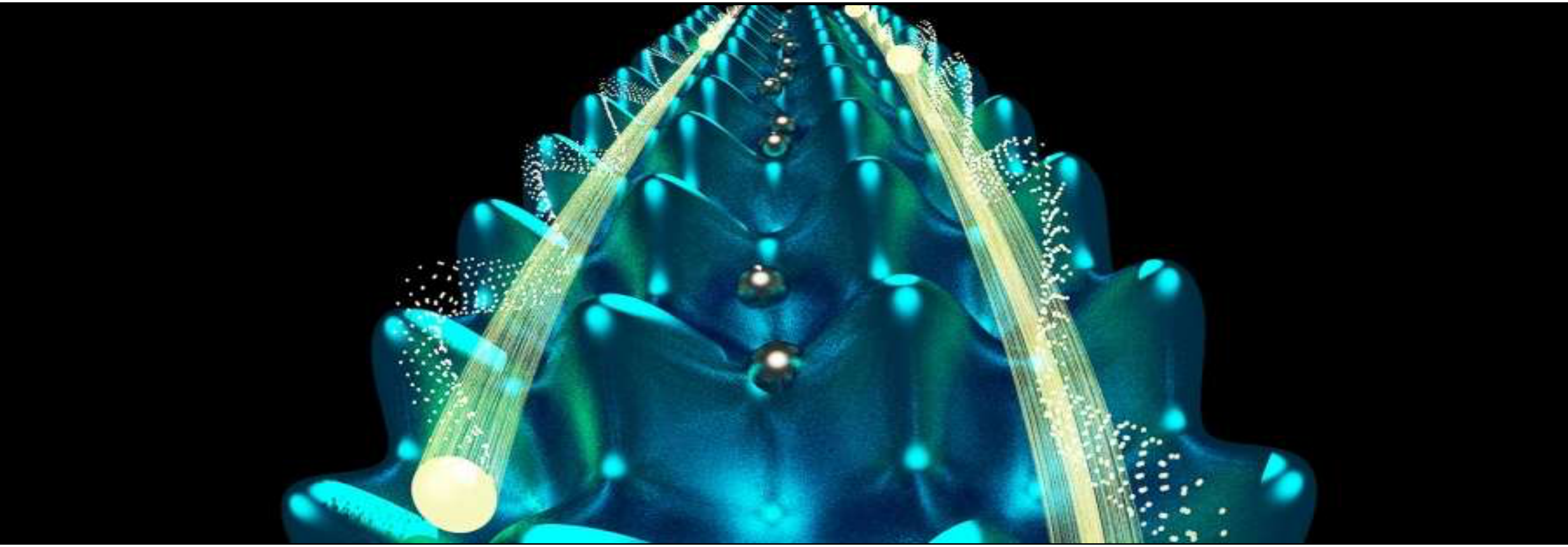
NON-EQUILIBRIUM DYNAMICS AND CHARGE FRACTIONALIZATION IN INTEGER QH EDGE STATES



Mirco Milletarì

Genova, 23/01/2017

NUS National University of Singapore | Centre for Advanced 2D Materials

National Research Foundation
Prime Minister's Office
Singapore

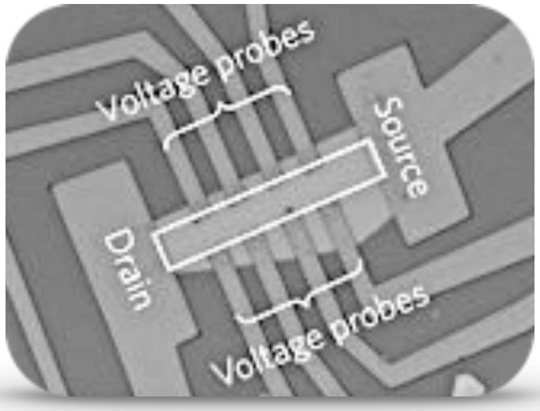
COLLABORATORS

Bernd Rosenow
ITP, Leipzig (Germany)

Alexander Schneider
PhD student in economics

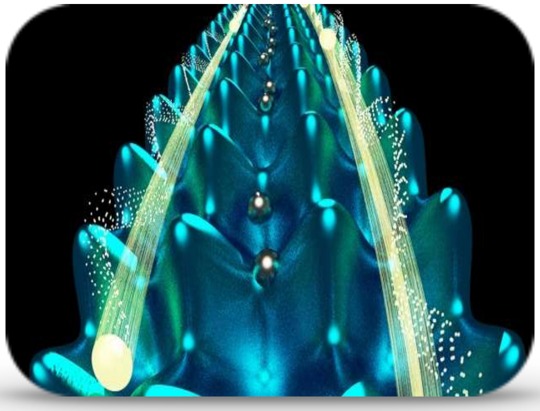
ACKNOWLEDGEMENTS *H. Inoue and M. Heiblum, Weizmann (Israel)*

OUTLINE



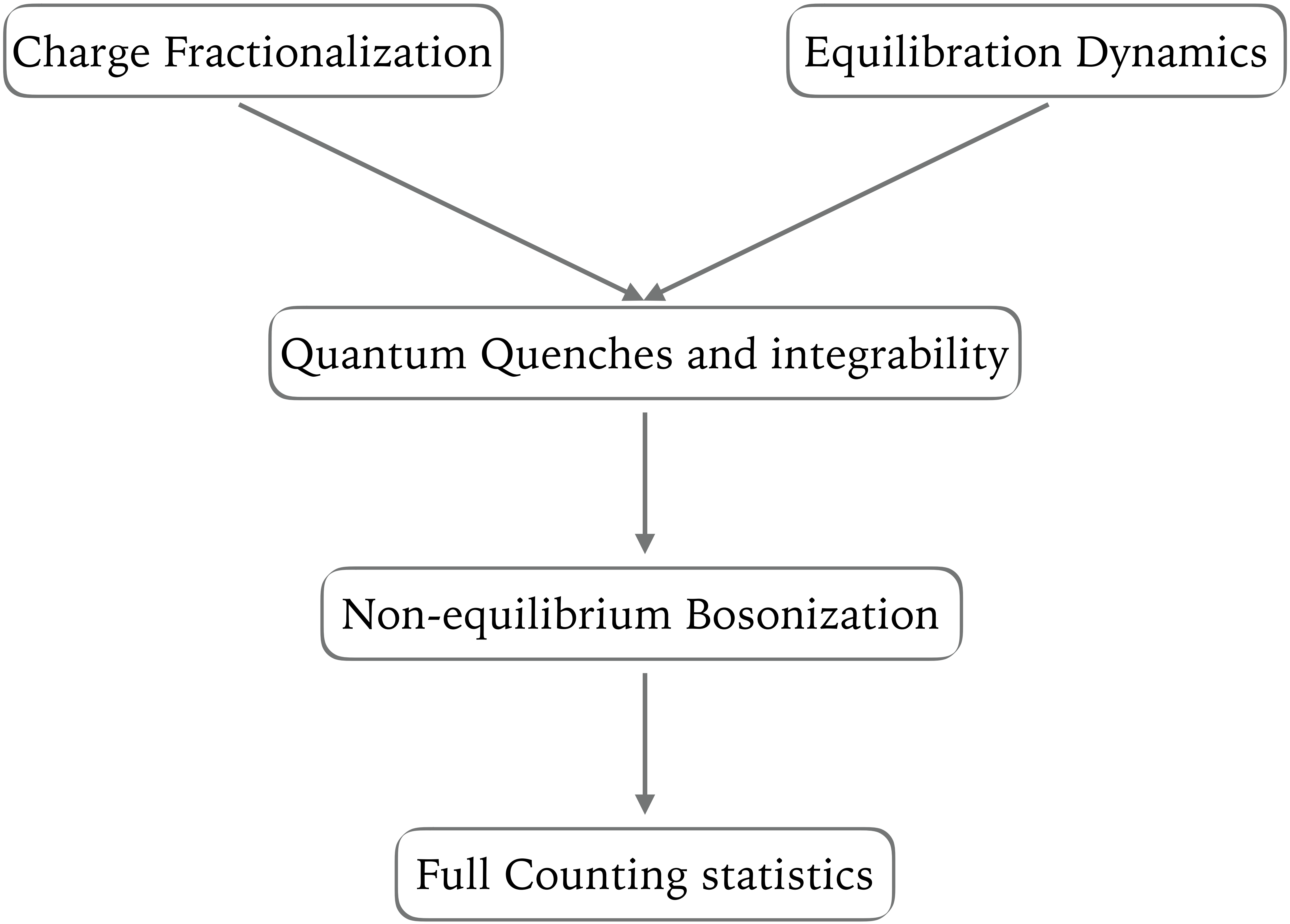
- The QHE and edge states
- Fractional charges
- Overview on noise and FCS

- Equilibrium v.s. non-equilibrium bosonization



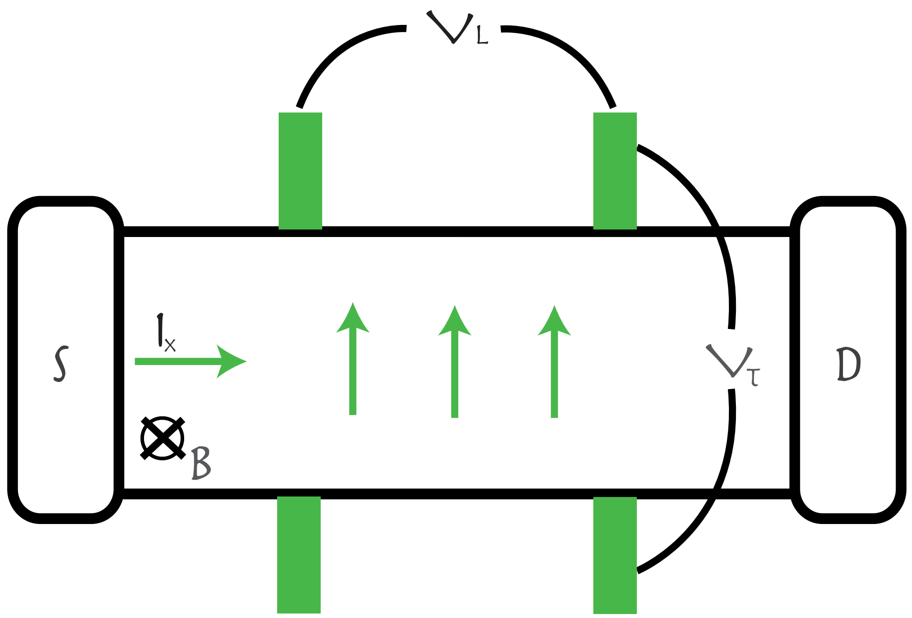
- Quantum quenches in chiral Luttinger Liquids
- Non-equilibrium driven correlations
- Prethermalization and fractional charges

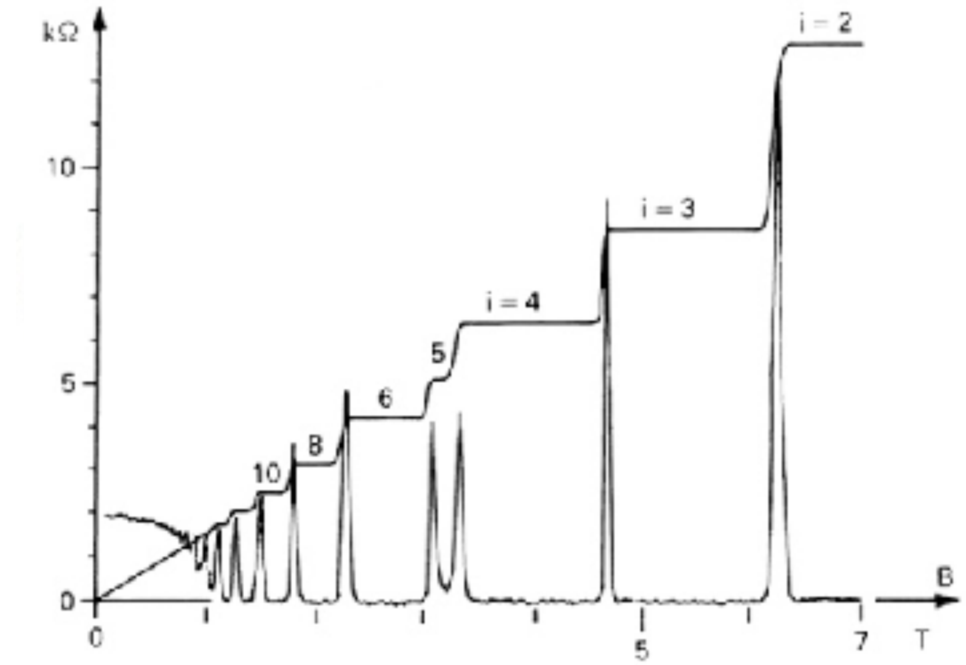
MAIN CONCEPTS IN THIS TALK



THE QUANTUM HALL EFFECT IN A (TINY) NUTSHELL

Von Klitzing (1980). Tsui,Stormer and Gossard (1982)





The quantum Hall state is an incompressible state characterised by

$$\sigma_{xx} = 0 \qquad \sigma_H = \nu \frac{e^2}{h}$$

Incompressibility

$$\nu = N_e \frac{\Phi_0}{\Phi(B)} = \frac{\text{\# particles}}{\text{\# flux quanta}}$$

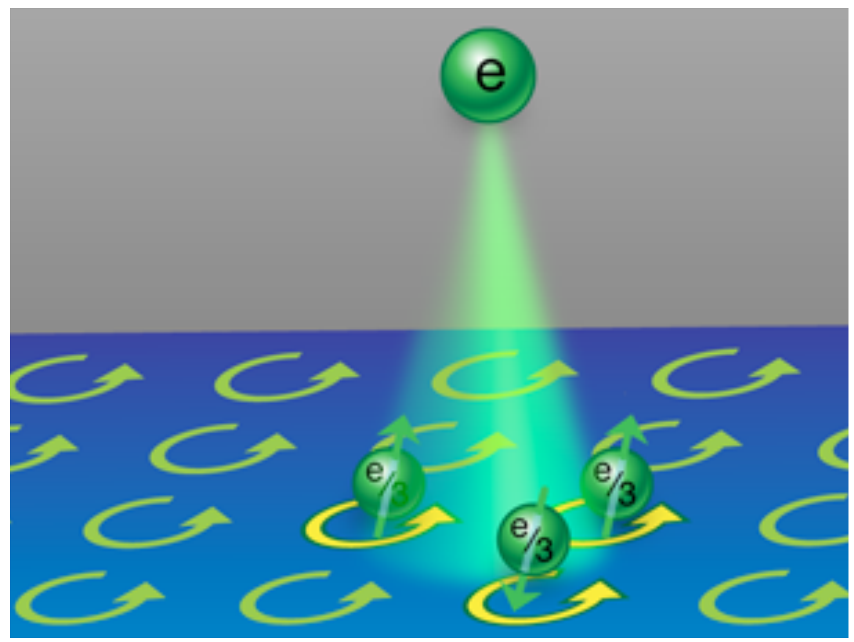
ν is a topological invariant related to the fist Chern character.

Niu, Thouless & Wu : Phys. Rev. B **31**,3372

Transitions between different fillings do not break ANY symmetry: **Topological phase transition.**

CHARGE FRACTIONALIZATION

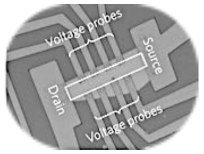
As a result of strong interactions + topology, quasiparticle excitations carry a fraction of the electron charge



Fractional charges cannot be directly measured. Fractionalization signatures appear in the **Shot Noise**.

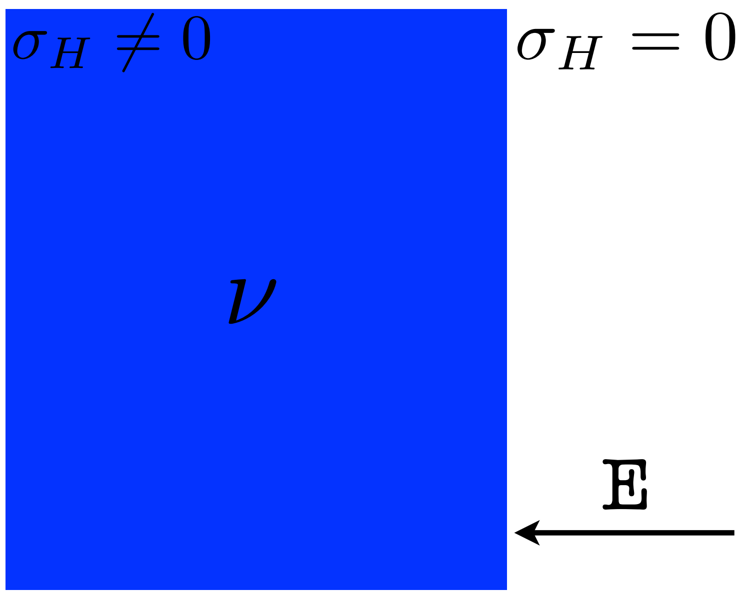
Kane & Fisher, PRL 72 (1994)

de-Picciotto et al., Nature 389 (1997)



GAPLESS EDGE STATES

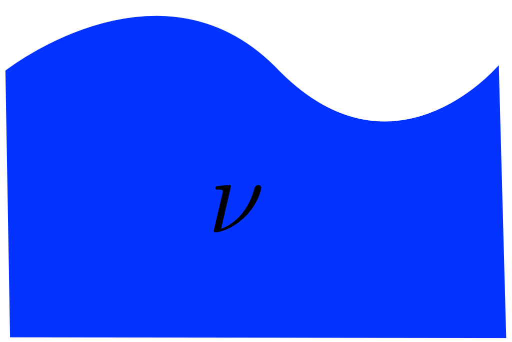
Halperin, PRB **25**.4 (1982)
Wen, Int.J. Mod. Phys. B **6**, 1711(1992)



Conservation of the edge current in 1d implies

$$(\partial_t + u_x \partial_x)\rho(x) = 0$$

1d wave propagating along the edge



$$S_{\pm} = \frac{1}{4\pi\nu} \int_{x,t} (\pm\partial_t + u_x\, \partial_x \phi_{\pm})\, \partial_x \phi_{\pm}$$

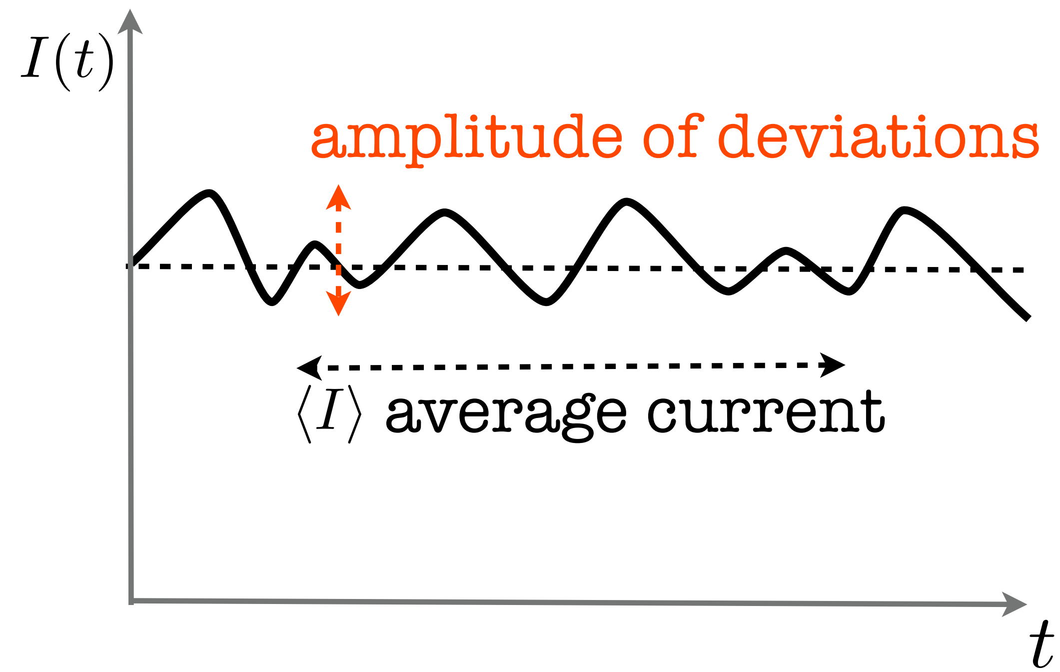
$$\rho(x) = \frac{1}{2\pi} \partial_x\, \phi(x) \qquad [\phi_\eta(x), \phi_\xi(y)] = \imath\pi\, \nu\, \delta_{\eta,\xi}\, sgn(x-y)$$

This is known as a **chiral Luttinger Liquid**

It is a strongly correlated state of matter, not belonging to the Fermi liquid universality class. It is robust against impurity scattering.

CURRENT AND NOISE (CLASSICAL)

Schottky, W. (1918). Annalen der Physik **57**: 541-567



Average number of transmitted particles and variance :

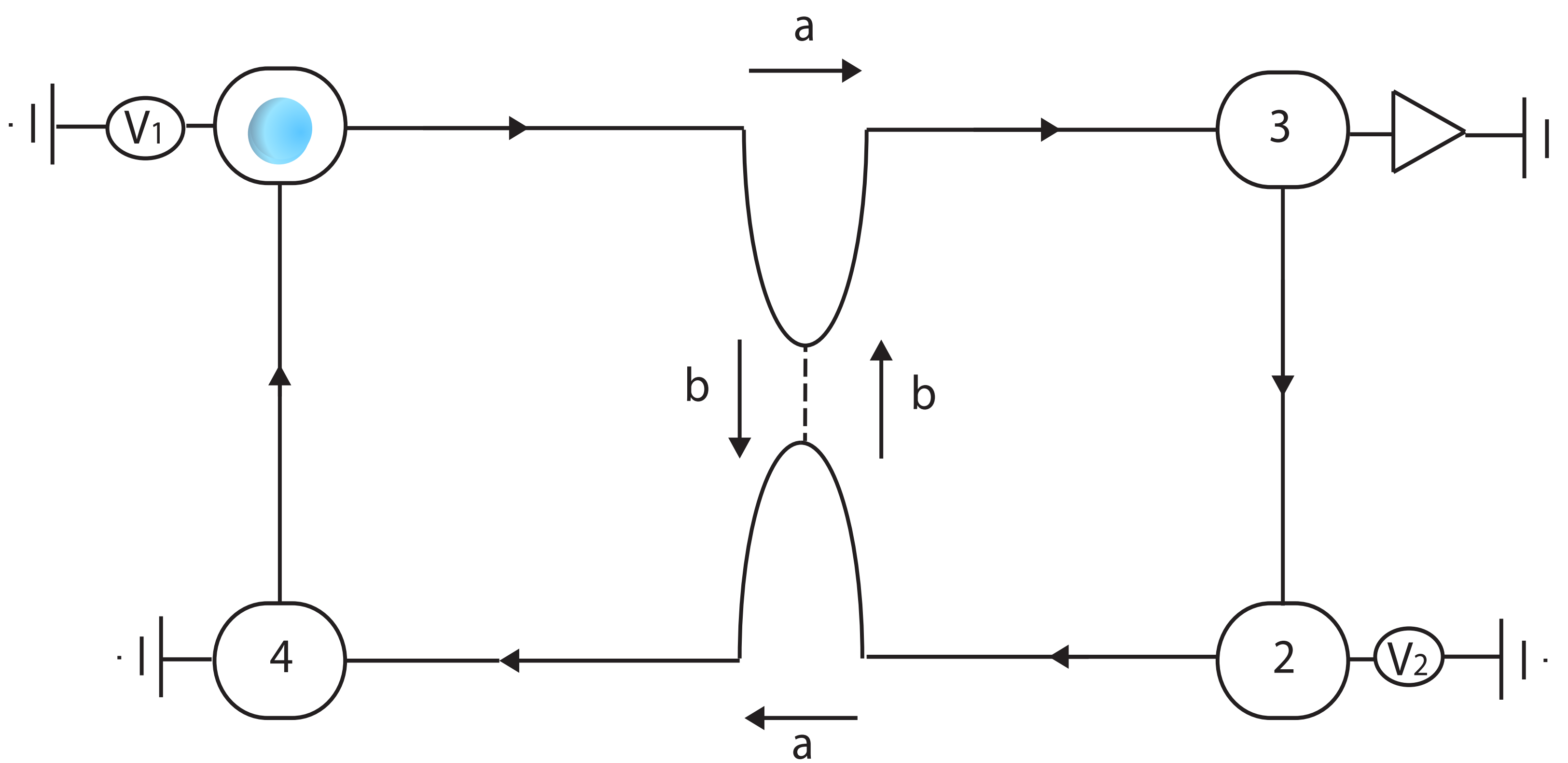
$$\langle N \rangle = t/\tau \qquad \langle N^2 \rangle - \langle N \rangle^2 = \langle N \rangle$$

$$\langle I \rangle = e\langle N \rangle / t$$

$$S = 2e^2(\langle N^2 \rangle - \langle N \rangle^2)/t = 2e\langle I \rangle$$

MESOSCOPIC COUNTING STATISTICS

How many electrons arrive at contact 3 ?

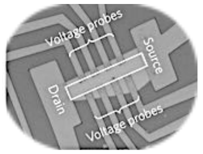


# of electrons

0

Measured average number of electrons

$$\langle m \rangle = Na$$



FULL COUNTING STATISTICS (QUANTUM)

Levitov, Lee, Lesovik: J.Math.Phys. 37.10 (1996), Klich cond-mat/0209642v1

Current Characteristic function

$$\Delta(\bar{\lambda}, \tau) = \sum_{\bar{\alpha}, \bar{\beta}} P[\bar{\alpha}(t=0), \bar{\beta}(t=\tau)] e^{\imath \frac{q}{2} \sum_i \lambda_i (\beta_i - \alpha_i)},$$

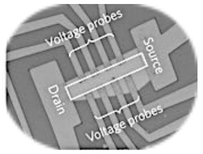
transferred electrons (m)

Final state of the electron

$$|\beta(t=\tau)\rangle = U(\tau, 0)|\beta(t=0)\rangle$$

This can be manipulated into

$$\Delta(\bar{\lambda}, \tau) = \det\left[1 + \left(U^{\dagger} e^{\imath q \lambda/2} U e^{-\imath q \lambda/2} - 1\right) F(\epsilon)\right].$$



FULL COUNTING STATISTICS II

If the scattering time is small compared to the entire evolution, we can introduce scattering states. For n=2 channels

$$S = \begin{pmatrix} r & t \\ t & r \end{pmatrix} \quad F(\epsilon) = \begin{pmatrix} f_1(\epsilon) & 0 \\ 0 & f_2(\epsilon) \end{pmatrix}.$$

Zero temperature characteristic function: $\Delta(\lambda) = (1 - a + a\, e^{\imath\lambda})^N \;, \quad N = \dfrac{q\Delta V \tau}{h}$

Tunnelling current: $\langle I \rangle = q\langle m \rangle / \tau = a\dfrac{e^2}{h}\Delta V$ Quantum of conductance

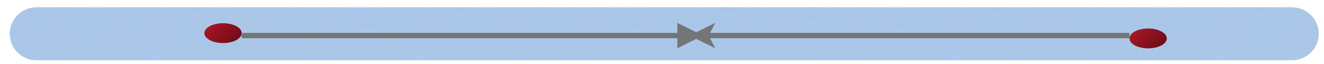
Quantum Shot Noise: $S(\omega \to 0) = 2e^2\langle (m - \langle m \rangle)^2 \rangle = 2e\langle I \rangle (1-a)$

Quantum statistics

Define the **Fano Factor** as: $F \equiv \dfrac{S(\omega \to 0)}{2e\langle I \rangle}$ Reference Noise

BOSONIZATION OUT OF EQUILIBRIUM

BOSONIZATION IN A NUTSHELL



In 1D it is impossible to disentangle the statistical properties from the interacting ones. Distinction between Fermions and Bosons is not well defined

Statistics transmutation is at the very heart of the Bosonization procedure

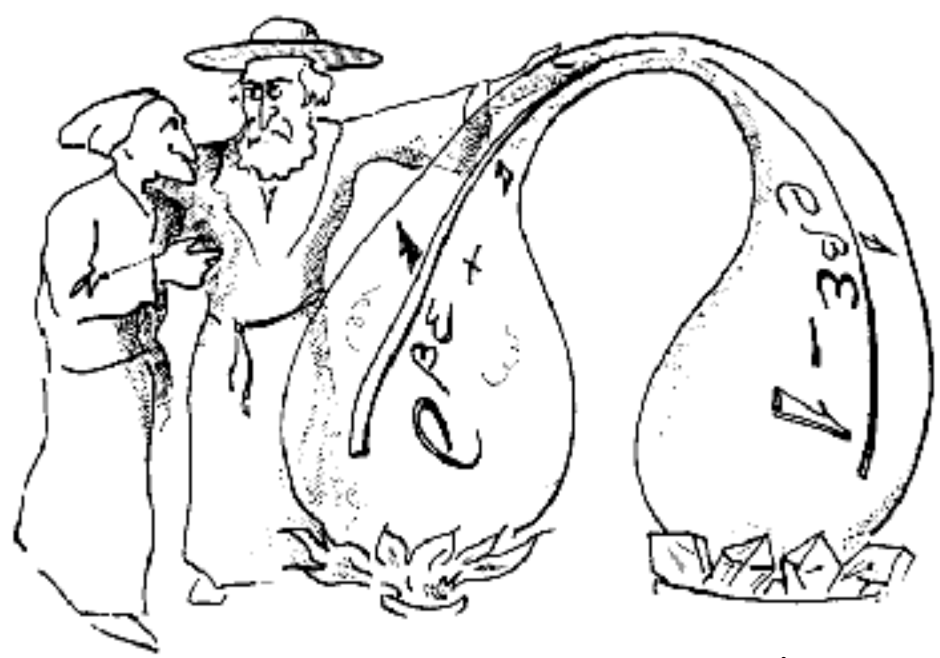
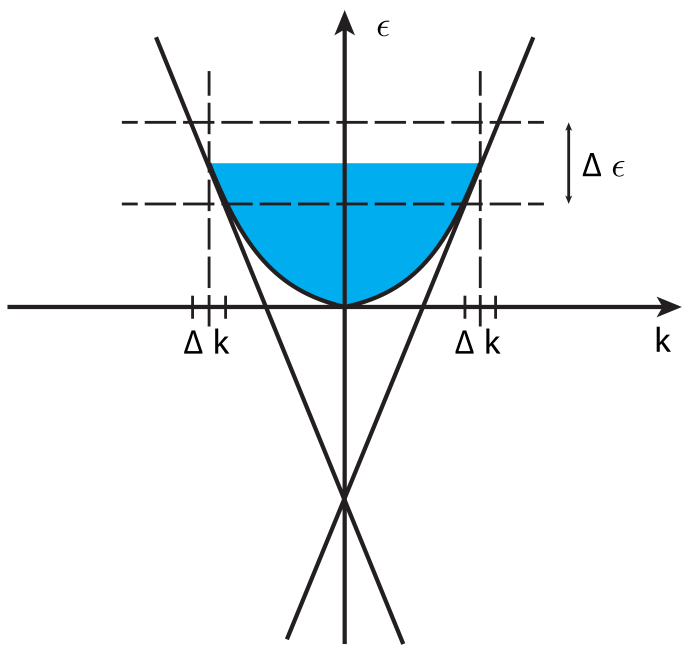


image from Tsvelik : Quantum Field theory in Condensed matter physics.

I. TOMONAGA–LUTTINGER MODEL

$$\mathscr{H}_0 = \psi^\dagger(x)\left(-\frac{\hbar^2}{2m}\partial_x^2 - \mu\right)\psi(x) \quad + \mathscr{H}_I = \psi^\dagger(x)\psi(x)U(x-x')\psi^\dagger(x')\psi(x')$$

Linearize around the two Fermi points



$$\lambda \sim \Delta k$$

$$\Lambda \sim \Delta\epsilon$$

$$\mathscr{H}_0 = -\imath u_F \Psi^\dagger(x)\sigma_3\partial_x\Psi(x), \quad \Psi(x) = \begin{pmatrix} \psi_R(x) \\ \psi_L(x) \end{pmatrix}, \quad \sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

Exact Mapping to a bosonic Hamiltonian

$$\mathscr{H}_0 = \frac{u_F}{4\pi}\left\{[\partial_x\phi_R(x)]^2 + [\partial_x\phi_L(x)]^2\right\}$$

No mention about statistical properties → holds out of equilibrium.

II. FERMIONIC EXPECTATION VALUES

Consider for example the equal time correlation function

$$\langle \psi^\dagger_\eta(x,t)\psi_\eta(0,t)\rangle_{th} = Tr(\hat\rho_F\, \psi^\dagger_\eta(x,t)\psi_\eta(0,t)) \qquad \rho_F = Z_F^{-1} e^{-\beta H_F}$$

Fix $\eta = R$, and consider the T=0 $\lim_{T\to 0} \langle \psi^\dagger_R(x,t)\psi_R(0,t)\rangle_{th} = \frac{\imath}{2\pi}\frac{1}{x+\imath\alpha}$

In equilibrium we can map $\hat\rho_F \xrightarrow{\text{B}} \hat\rho_B$ and compute the same quantity

$$\langle \psi^\dagger_R(x,t)\psi_R(0,t)\rangle_{th} = \frac{1}{2\pi\alpha} e^{[\phi_R(x),\phi_R(0)]/2} Tr(\hat\rho_B e^{-\imath(\phi_R(x)-\phi_R(0))})$$

In equilibrium, only the gaussian term is finite, higher order cumulants are identically zero! Dzyaloshinskii-Larkin Sov.Phys.JETP 38,202, (1973)

$$Tr(\hat\rho_B\, e^{-\imath(\phi_R(x)-\phi_R(0))}) = e^{-\frac{1}{2}\langle(\phi_R(x)-\phi_R(0))^2\rangle_{th}}$$

Is the simple mapping $\hat\rho_F \xrightarrow{\text{B}} \hat\rho_B$ valid out of equilibrium?

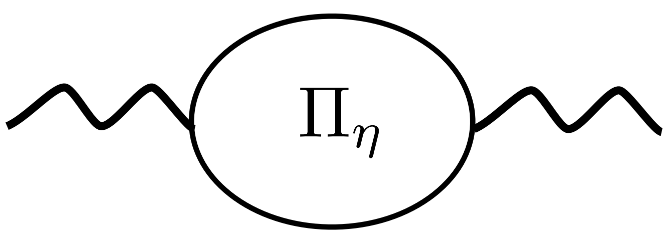
… GENERALLY NOT!

I. P. Levkivskyi, E. V. Sukhorukov, Phys. Rev. Lett. **103**, 036801 (2009)

D. B. Gutman, Yuval Gefen and A.D. Mirlin, Phys. Rev. B **81** 085436 (2010)

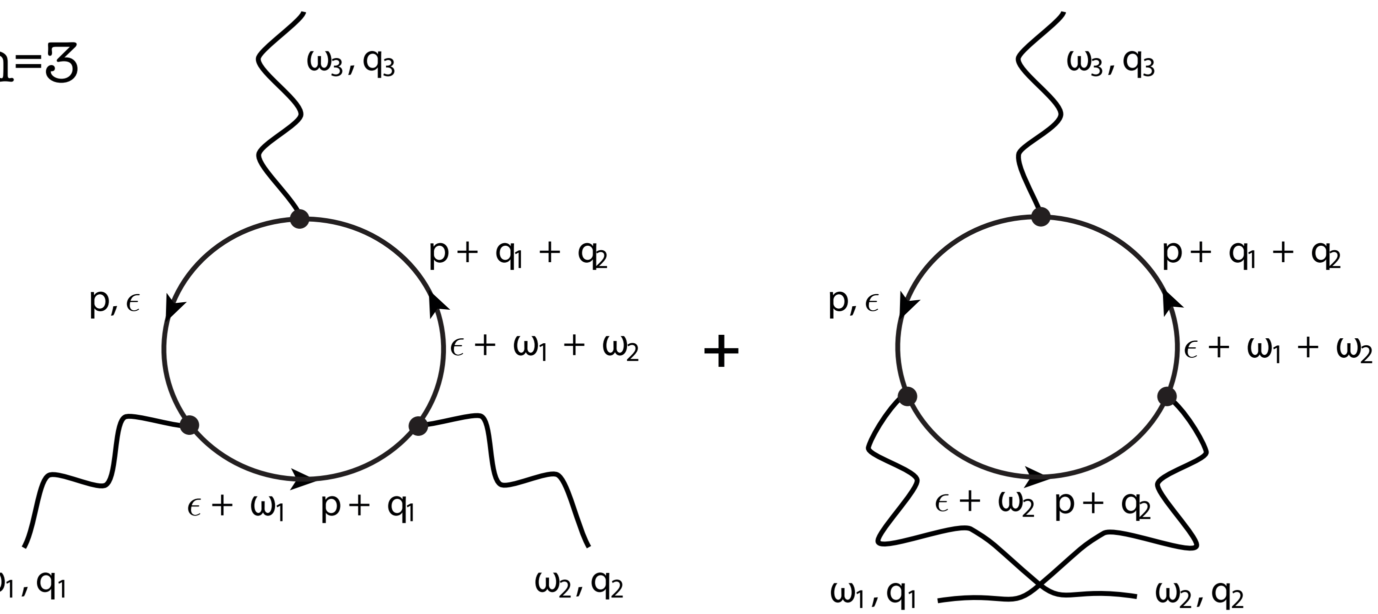
Wee need to look again at the cumulant expansion

n=2



+

n=3



In equilibrium, the two (n=3) diagrams cancel each other. This is not so for an initial out of equilibrium distribution!

However, re-summing higher order cumulants is equivalent to the problem of full counting statistics!

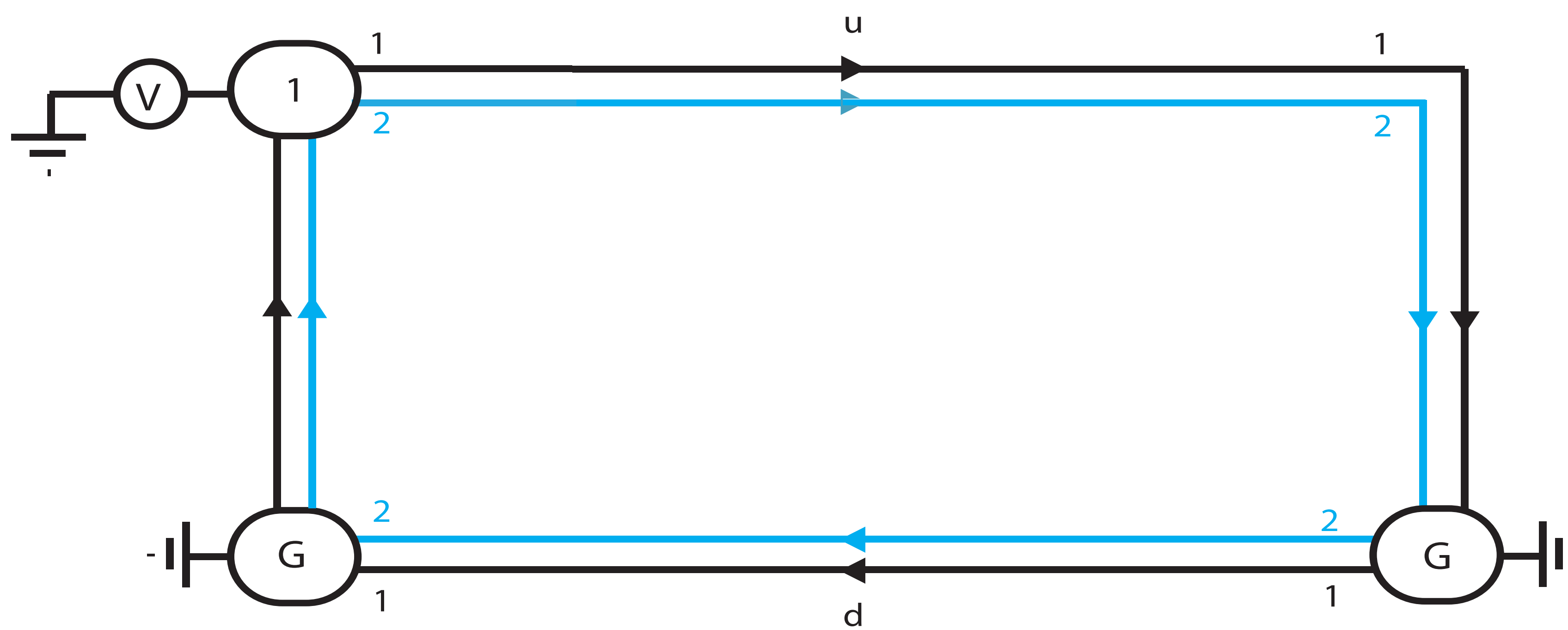
NON-EQUILIBRIUM DRIVEN CHARGE FRACTIONALIZATION

M. Milletari , B . Rosenow, PRL ***111****, 136807 (2013)*

A. Schneider, M. Milletari , B . Rosenow, arXiv:1610.02036 (2016)

EDGE STATES AT $\nu = 2$ (FULLY POLARIZED)

Two co-propagating chiral Fermi Liquids



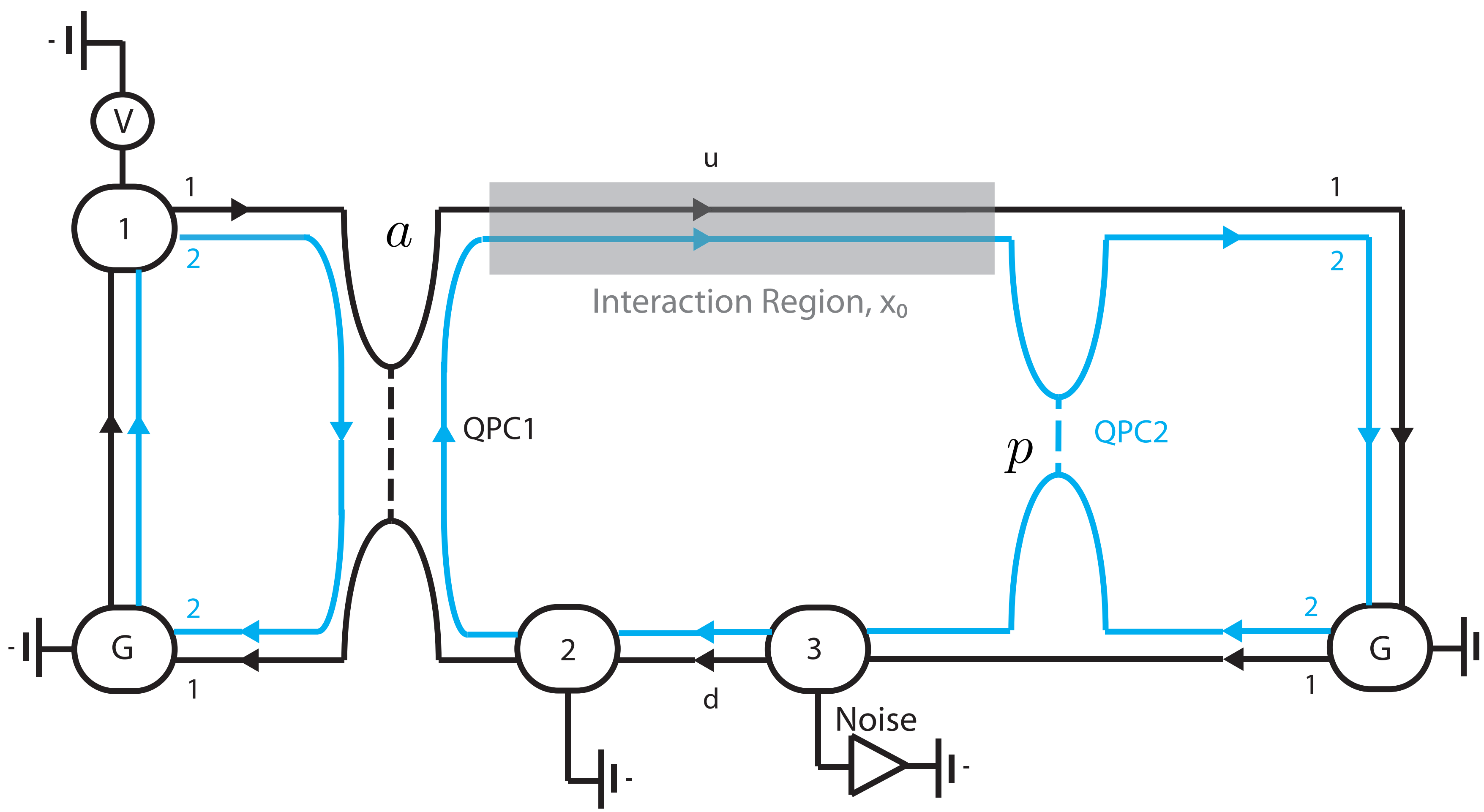
$$\mathscr{H} = \mathscr{H}_{0,\eta} + \mathscr{H}_{I,\eta}$$

$$\mathscr{H}_{0,\eta} = 2\pi \int_x (v_1 \rho_{1,\eta}^2(x) + v_2 \rho_{2,\eta}^2(x))$$ Edge modes

$$\mathscr{H}_{I,\eta} = 2\pi \int_x v_{12} \rho_{1,\eta}(x) \rho_{2,\eta}(x)$$ Inter-mode interaction

$$\psi_\eta(x) = \frac{1}{\sqrt{2\pi\alpha}} e^{\imath \phi_\eta(x)}$$ bosonic representation of the fermionic field

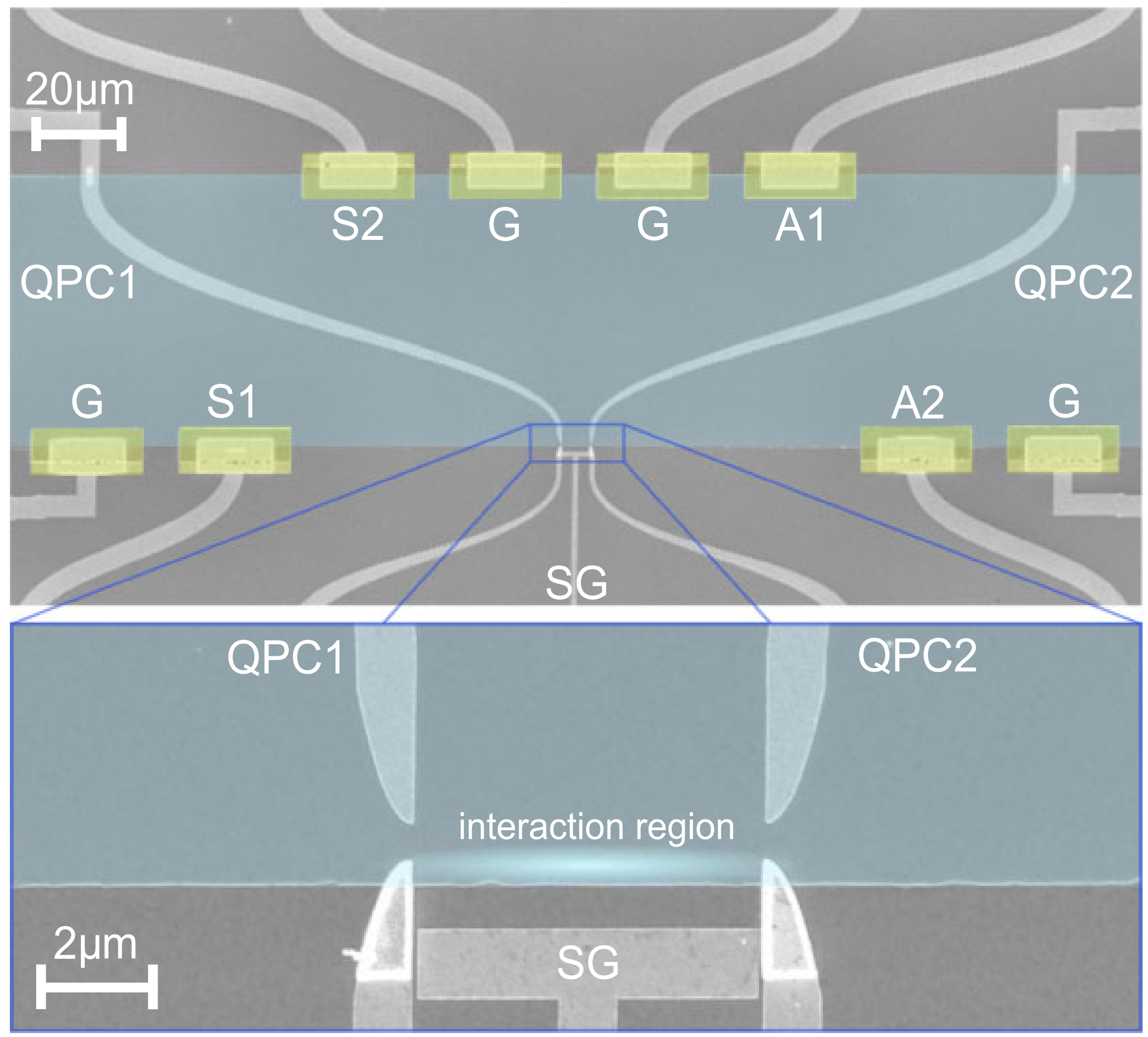
SETUP see also Neder, PRL **108**, 186404 (2012), Bocquillon et al, Nat. Comm. 4 1839 (2013)



- The two modes are in equilibrium
- **QPC1**: **Mode 2** completely reflected, **Mode 1** partially transmitted with probability a
- The two modes are not in equilibrium: Interactions
- **QPC2**: **Mode 1** completely transmitted, **Mode 2** partially reflected with probability p

YES, SUCH A DEVICE EXISTS !

H. Inoue, A, Grivnin, N. Ofek, I. Neder, M. Heiblum, V. Umansky and D. Mahalu PRL **112**, 166801 (2014)



$$l = 8\,\mu m$$

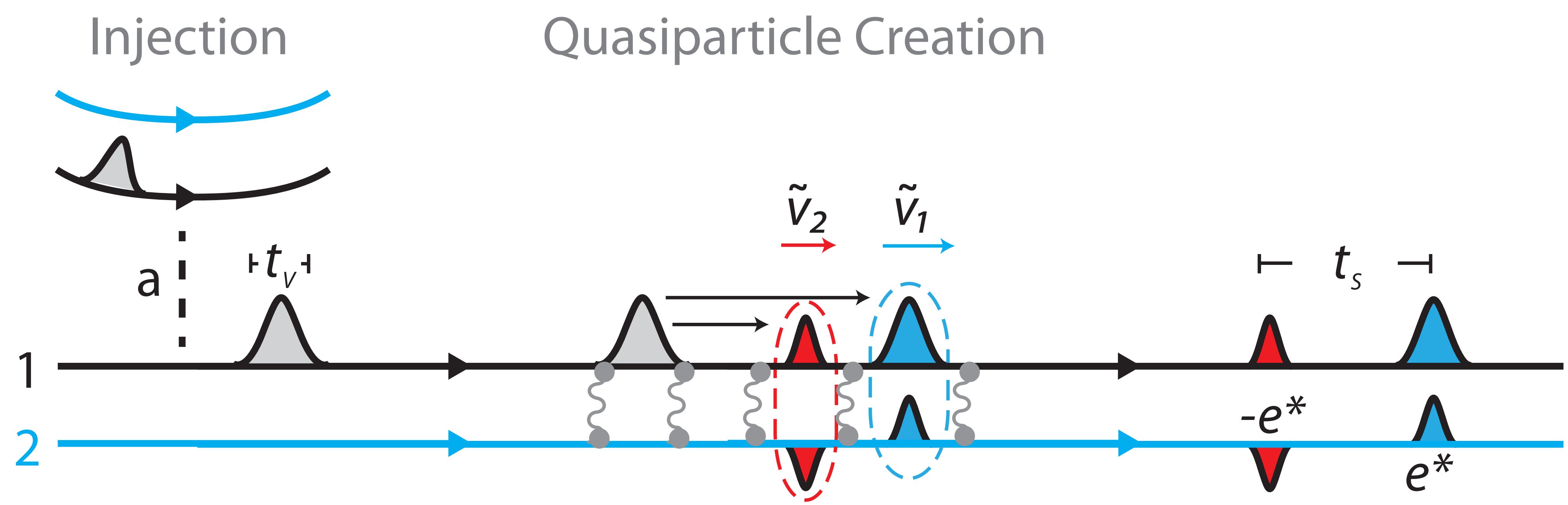
T= 4.2 K $\quad \mu = 4.2\,10^6\,cm^2/V s \quad B = 1.7\,T$

CHARGE FRACTIONALIZATION

Berg et al. PRL **102**, 236402 (2009)
Horsdal et al. PRB **84**, 115313 (2011)

Inject a charge "e" on edge mode 1...

...due to interactions it fractionalizes as:



Fractional charges (equilibrium):

$$e^* = e \sin 2\theta / 2$$

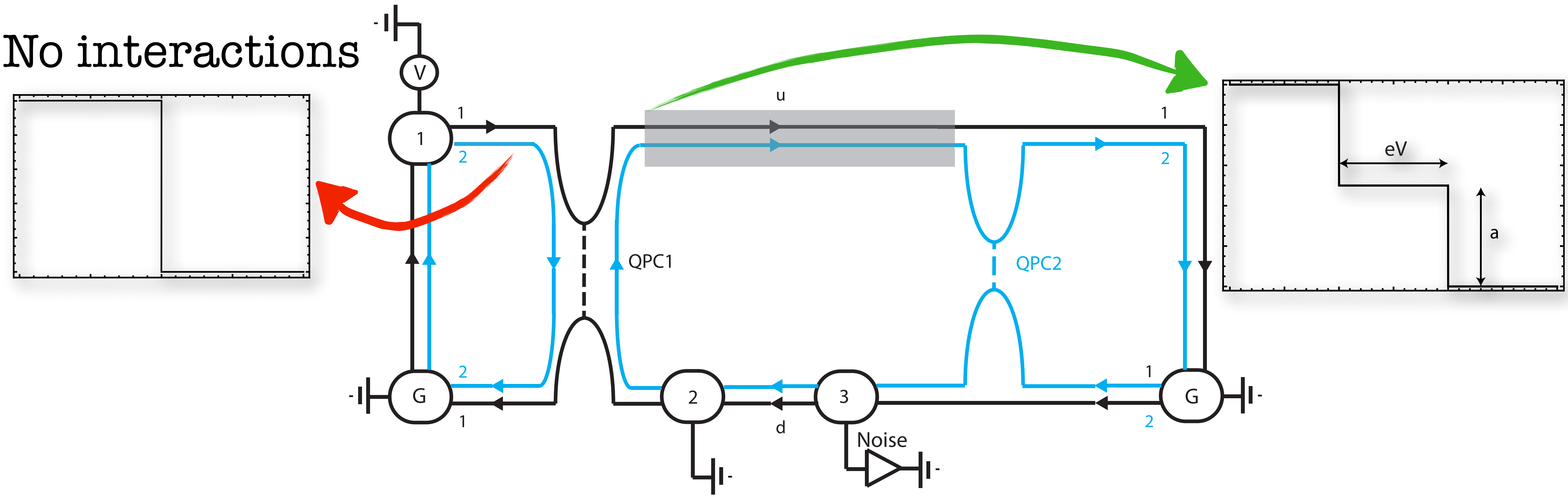
$$e_\pm = e/2 \pm \sqrt{e^2/4 - (e^*)^2}$$

$$\tan 2\theta = \frac{v_{12}}{(v_1 - v_2)}$$

However, this is not what we measure!

QUANTUM QUENCHES

No interactions



Interactions are switched on via a **Quantum Quench**

Cazalilla, PRL **97**,156403 (2006) & Kovrizhin, Chalker PRB **84**, 085105 (2012)

QUANTUM QUENCH

$t = 0$	$t > 0$		
H_0 $\hat{\rho}$ → Q → no interactions	$H \longrightarrow \tilde{H}$ Diagonalize $\beta_{i,q} = \sum_j M_{ij} b_{j,q}$	$\beta_{iq}(t) = e^{-\imath q \tilde{v}_i t} \beta_{iq}(t=0)$ Time evolution in the diagonal basis	$b_{1q}(t) = u_q(t) b_{1q} + s_q(t) b_{2q}$ $b_{2q}(t) = s_q(t) b_{1q} + v_q(t) b_{2q}$ Undo the transformation: connection between t=0 and t>0

$$M = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}$$

$$\tilde{v}_{1(2)} = v_{1(2)} \cos^2\theta + v_{2(1)} \sin^2\theta \pm \frac{1}{2} v_{12} \sin 2\theta$$

$$u_q(t_0) = \cos^2\theta \, e^{-\imath q \tilde{v}_1 t_0} + \sin^2\theta \, e^{-\imath q \tilde{v}_2 t_0}$$

$$v_q(t_0) = \cos^2\theta \, e^{-\imath q \tilde{v}_2 t_0} + \sin^2\theta \, e^{-\imath q \tilde{v}_1 t_0}$$

$$s_q(t_0) = \frac{1}{2} \sin 2\theta (e^{-\imath q \tilde{v}_1 t_0} - e^{-\imath q \tilde{v}_2 t_0})$$

$$\tan 2\theta = \frac{v_{12}}{(v_1 - v_2)}$$

EFFECT ON THE GREEN'S FUNCTIONS

Consider the Gaussian case first

$$Tr(\hat{\rho}_F \tilde{\psi}_2(x,t_0)\tilde{\psi}_2(0,t_0)) \propto e^{-Tr(\hat{\rho}_B(\tilde{\phi}_2(x,t_0)-\tilde{\phi}(0,t_0)^2)/2}.$$

$$\mathrm{Tr}\left[\hat{\rho}\left(\tilde{\phi}_2(x,t)-\tilde{\phi}_2(0,t)\right)^2\right] \propto \left[|s_q(t_0)|^2(1+2n_{1,B}(\epsilon_q)) + \left[|v_q(t_0)|^2(1+2n_{2,B}(\epsilon_q))\right]\right]$$

In equilibrium, interactions between co-propagating states do not change the free nature of the system!

$$|s_q(t_0)|^2 + |v_q(t_0)|^2 = 1$$

One obtains the non-interacting result, i.e. **no** fractionalization

$$Tr(\hat{\rho}_B \tilde{\psi}_2(x,t_0)\tilde{\psi}_2(0,t_0)) = \frac{\imath}{2\pi\alpha}\frac{1}{x+\imath\alpha}$$

FOR OUR SETUP

$$S(\omega \to 0) = \frac{2e^2}{h}\frac{|t_2|^2}{2\pi}\int_\epsilon G^<_{2u}(\epsilon)G^>_{2d}(\epsilon) + G^<_{2d}(\epsilon)G^>_{2u}(\epsilon)$$

Noise at QPC2

$$G^<_{2u}(\tau) = \langle \psi^\dagger_{2u}(t+\tau, x_0)\psi_{2u}(t, x_0)\rangle$$

use vertex representation:

$$\psi_\eta(x) = \frac{1}{\sqrt{2\pi\alpha}} e^{\imath\phi_\eta(x)}$$

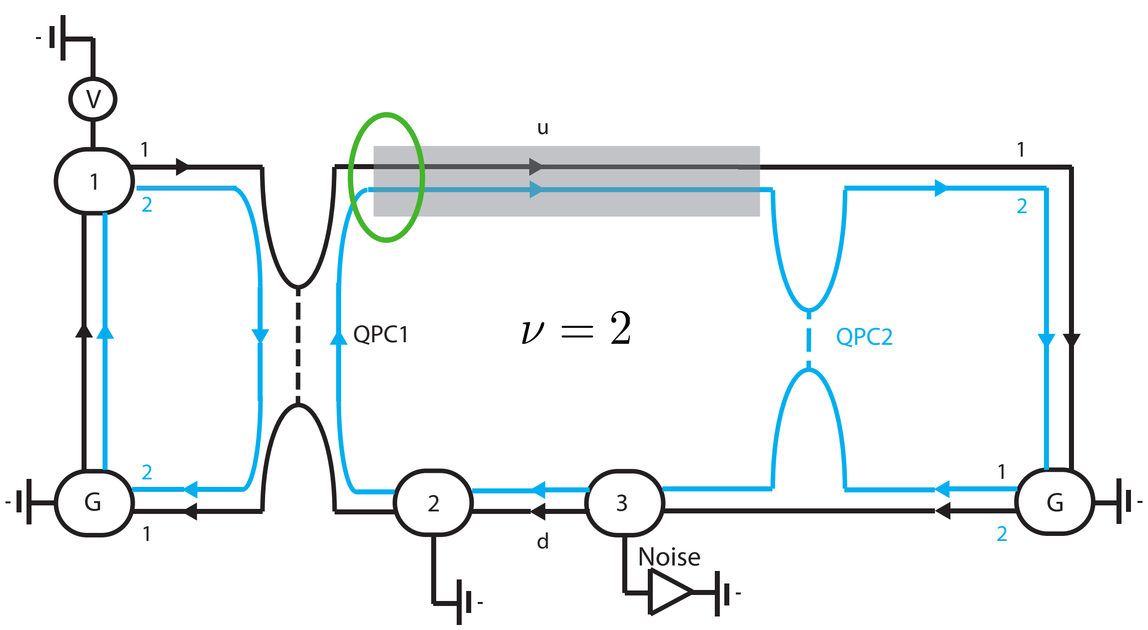
$$= G^<_0(\tau)\langle e^{\sum_q \lambda^\star_{1u}(q,t,\tau)b^\dagger_{1u,q}} e^{-\sum_q \lambda_{1u}(q,t,\tau)b_{1u,q}}\rangle_1$$

$$G^<_0(\tau) = \frac{1}{2\pi}\frac{1}{(-\imath\,\tilde{v}_1\tau + \alpha)^{\sin^2\theta}}\frac{1}{(-\imath\,\tilde{v}_2\tau + \alpha)^{\cos^2\theta}}$$

$$\lambda_{1u,q}(t,\tau) = \imath\sqrt{2\pi/qL}\, e^{\imath q(x_0 - \tilde{u}_2 t)}\left[e^{-\imath\tilde{u}_2 q\tau} s_q(t+\tau) - s_q(t)\right]$$

STEADY STATE DENSITY MATRIX

After QPC1, free electrons have a double step distribution



$$\hat{\rho}_3 = Z_{1u}^{-1} e^{-\beta(\epsilon - \tilde{\mu}_1)\hat{N}_{1u}} Z_{1d}^{-1} e^{-\beta(\epsilon - \tilde{\mu}_2)\hat{N}_{1d}}$$

$$\tilde{\mu}_1 = (1-a)\Delta\mu \quad \tilde{\mu}_2 = -a\Delta\mu$$

CONNECT INCOMING AND OUTGOING STATES

We can now compute the non-Eq. bosonic distribution in RPA

$$\omega B_\eta(\omega) = (a^2 + (1-a)^2)\omega n_B(\omega) + a(1-a)[(\omega + \Delta\mu)n_B(\omega + \Delta\mu) + (\omega - \Delta\mu)n_B(\omega - \Delta\mu)]$$

In agreement with the Keldysh approach

EFFECTIVE TEMPERATURE

Using the non-equilibrium Bosonic distribution

$$E_B = \int_0^\infty \frac{d\omega_q}{\hbar v_1} \omega_q B(\omega_q) \xrightarrow{T \to 0} \frac{\Delta\mu^2}{2\hbar v_1} a(1-a)$$

We define an effective temperature by considering the energy of a 1d equilibrium system

$$E_{B,eq} = \int_0^\infty \frac{d\omega_q}{\hbar v_1} \omega_q B(\omega_q) = \frac{(\pi K_B T_b)^2}{6\hbar v_1}$$

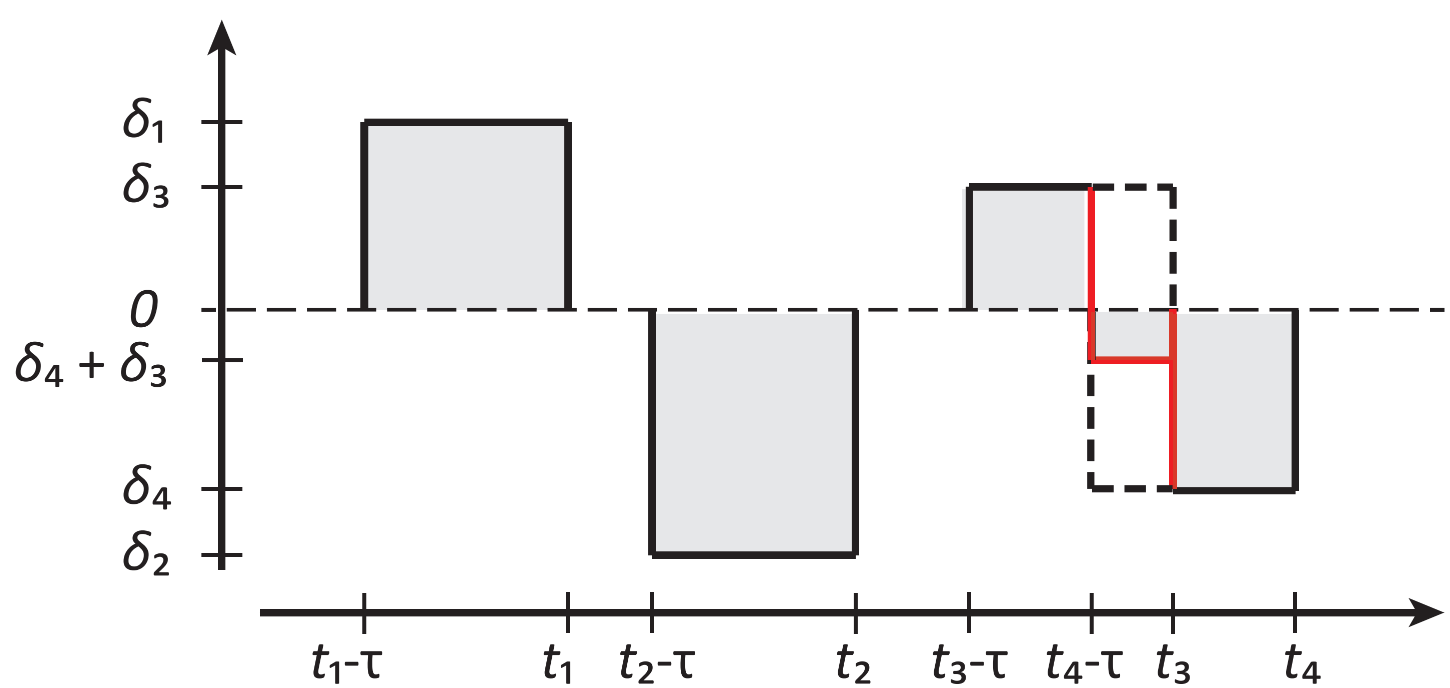
$$T^* = \frac{eV}{\pi\, k_B} \sqrt{3/2\, a(1-a)}$$ Interaction independent

SUMMING OVER ALL CUMULANTS

$$\langle e^{\sum_q \lambda^\star_{1u}(q,t,\tau) b^\dagger_{1u,q}} e^{-\sum_q \lambda_{1u}(q,t,\tau) b_{1u,q}} \rangle = \frac{\det\left[1 + (e^{-i\delta_\tau(t)} - 1) f(\epsilon)\right]}{\det\left[1 + (e^{-i\delta_\tau(t)} - 1)\theta(-\epsilon)\right]} = \bar{\Delta}_\tau(\delta)$$

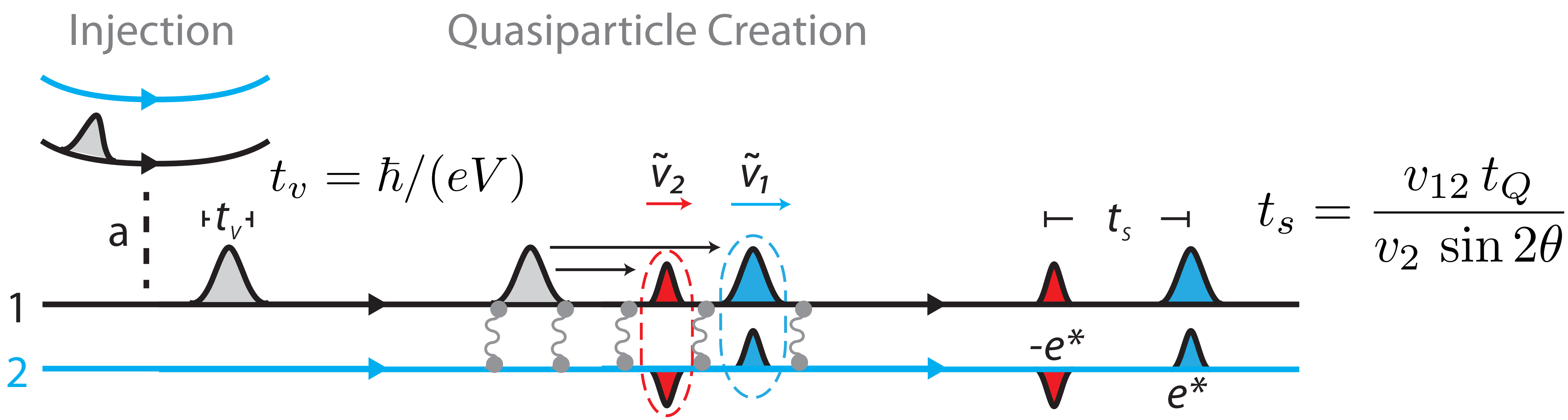
scattering phase $\delta_\tau = 2\pi \frac{e^*}{e} \omega_\tau(t, x_0)$ window function

interactions $e^* = e/2 \sin(2\theta)$



*M. M. , B . Rosenow, PRL **111**, 136807 (2013)*

FUNCTIONAL DETERMINANTS AND REGIMES

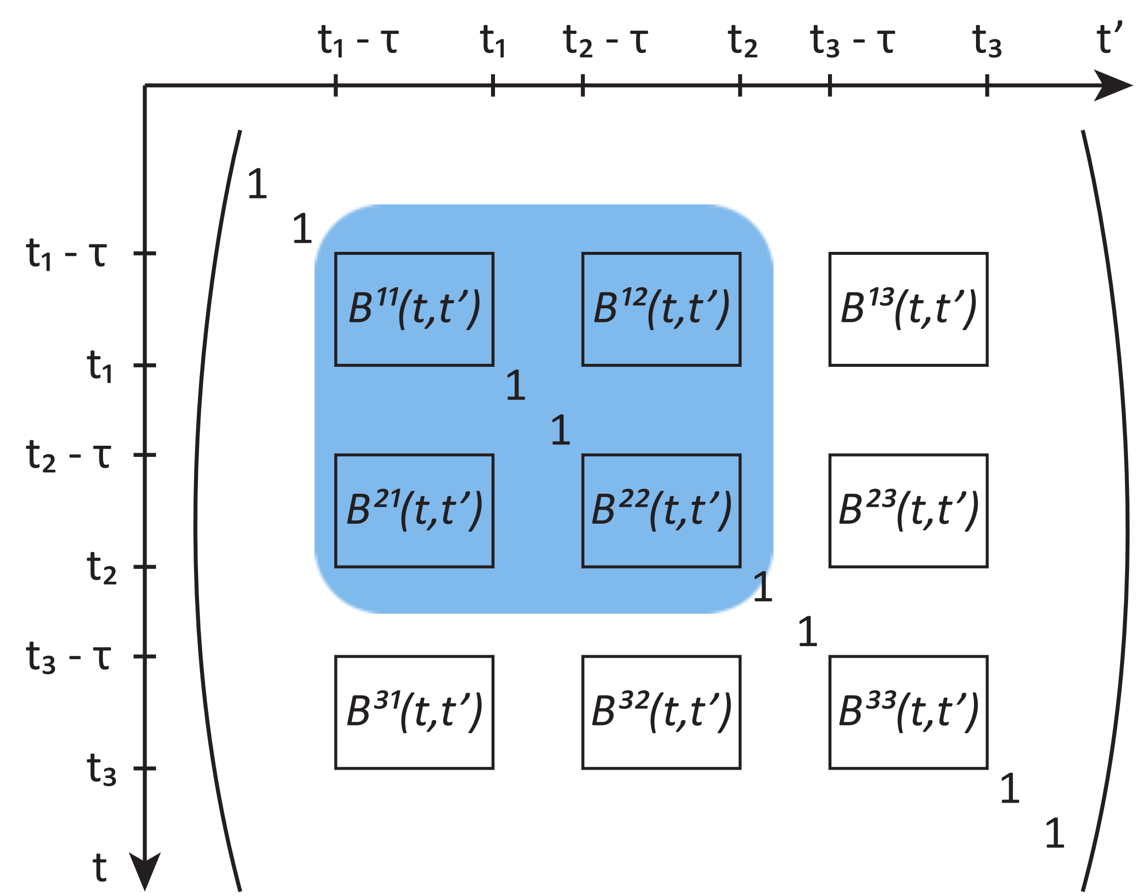


In general, we need to treat time and energy as operators $[\hat{t}, \hat{\epsilon}] = i\hbar$

The determinant has a block Toeplitz form

$$B^{lm}(t,t') = w_\tau(t,t_l)\, b^l(t-t')\, w_\tau(t',t_m),$$

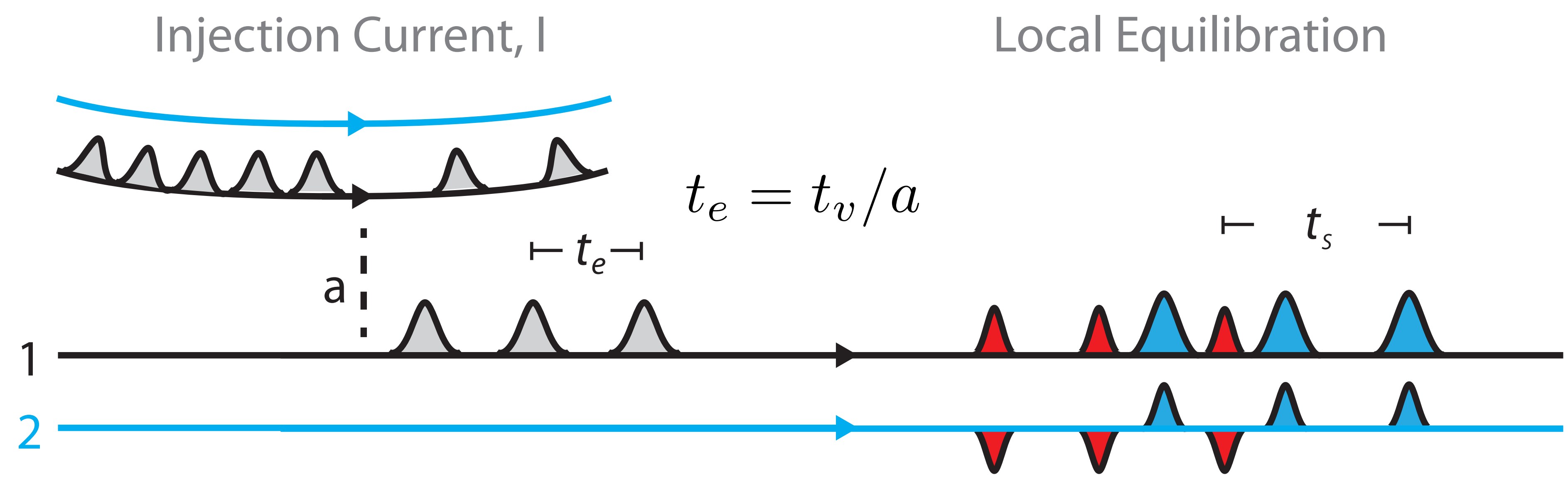
$$b^l(t-t') = \int d\epsilon \, \frac{e^{i\epsilon(t-t')}}{2\pi} \left(1 + (e^{-i\delta_l} - 1) f(\epsilon)\right)$$



A. Schneider, M. M. , B . Rosenow, arXiv:1610.02036 (2016)

REGIMES II

A. Schneider, M. M. , B . Rosenow, arXiv:1610.02036 (2016)



Regime A: $t_s \leq t_v$(overlapping)

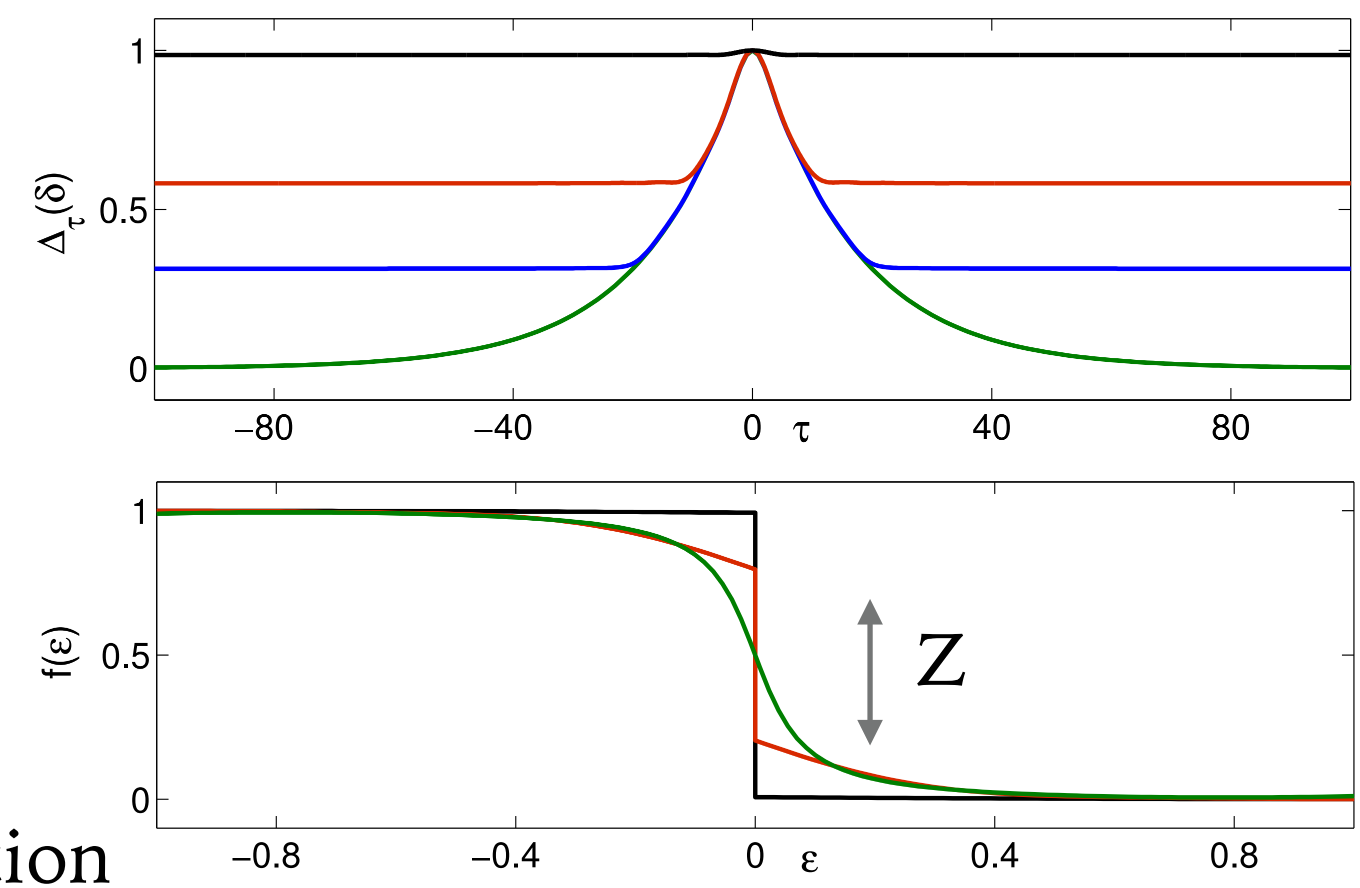
The two modes are nearly free

Regime B: $t_s \gg t_v$

separated quasiparticles

Regime C: $t_s \gg t_e$

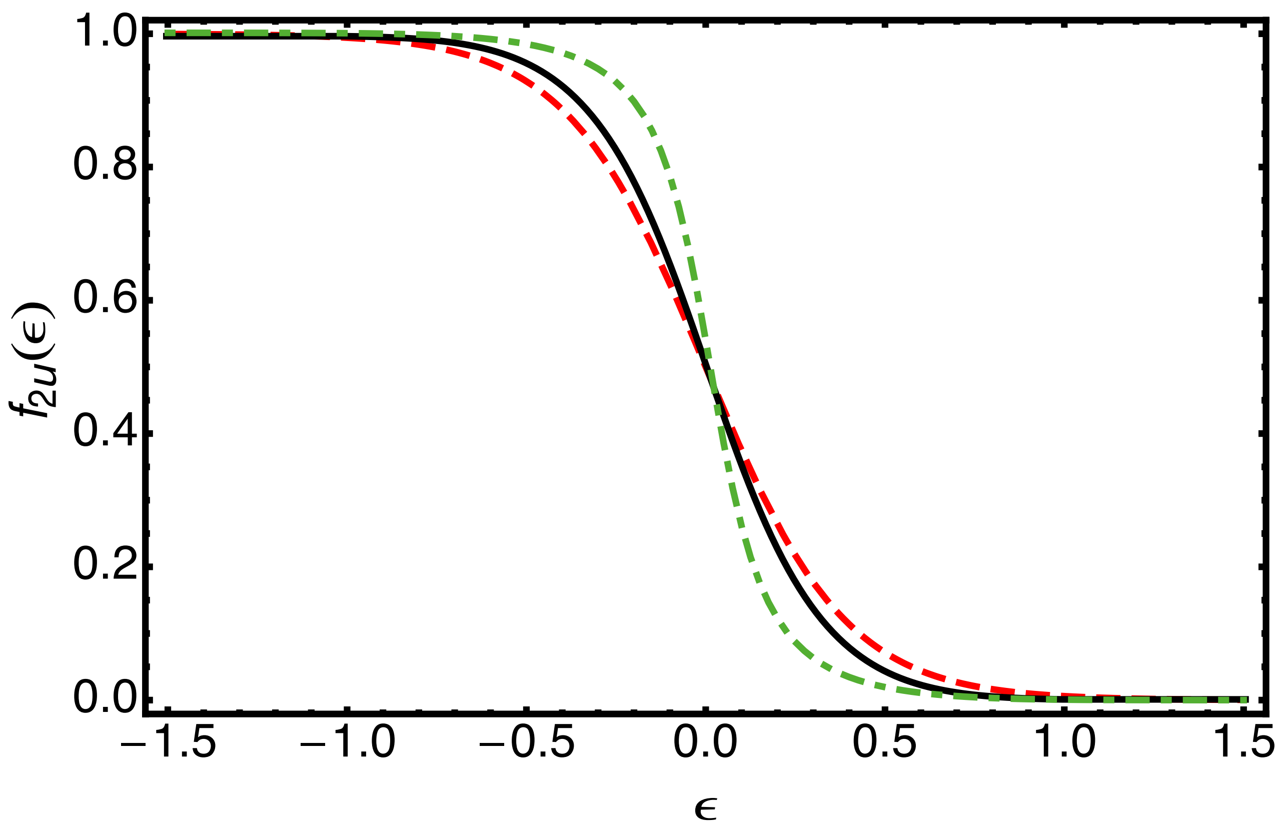
quasiparticles mixing, prethermalization



LONG TIME LIMIT (REGIME C) *M. M , B . Rosenow, PRL **111**, 136807*

The determinant factorises as a product of single pulse determinants

$$G^{<}_{2u}(\tau) = G^{<}_{0}(\tau)\,|\bar{\Delta}_\tau|^2$$



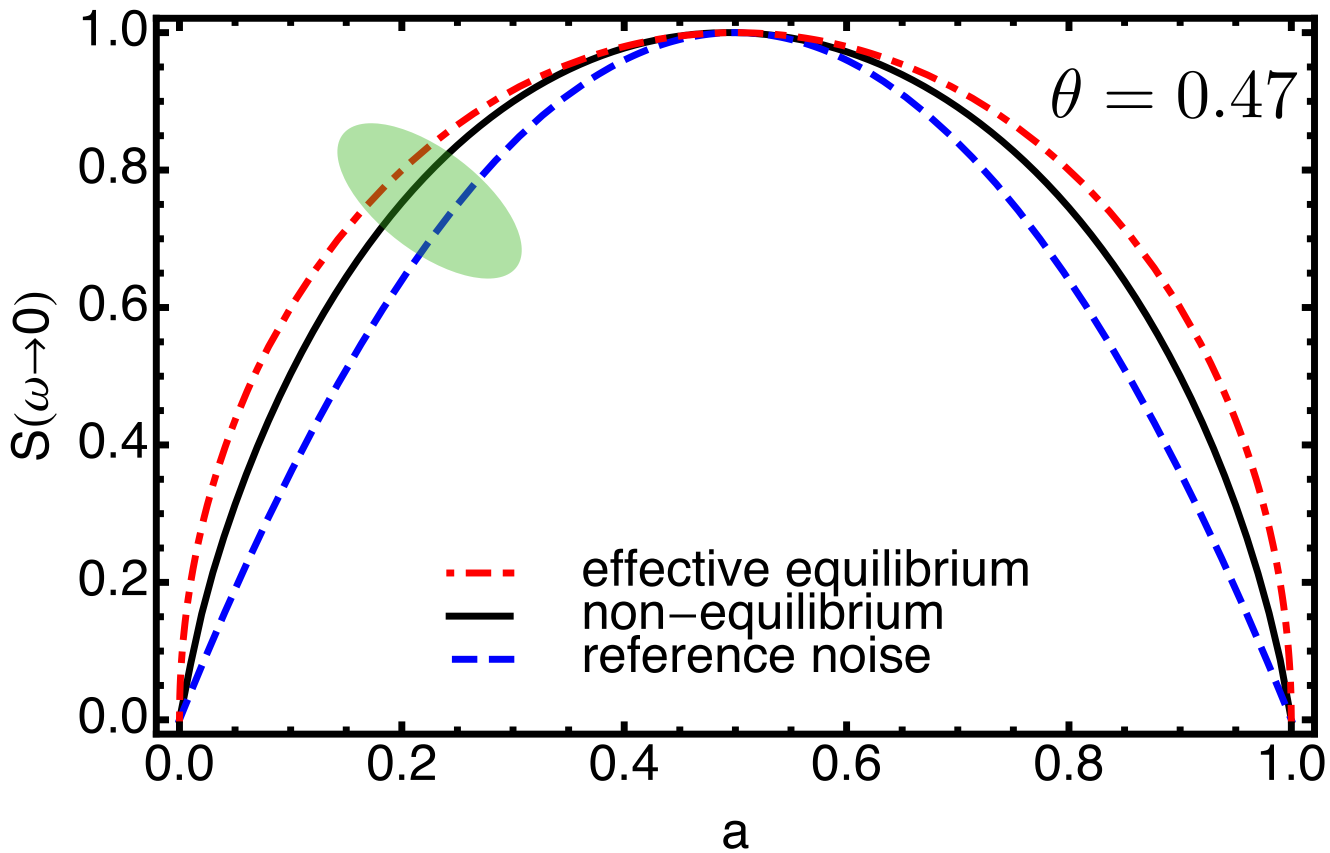
Only Gaussian contribution.

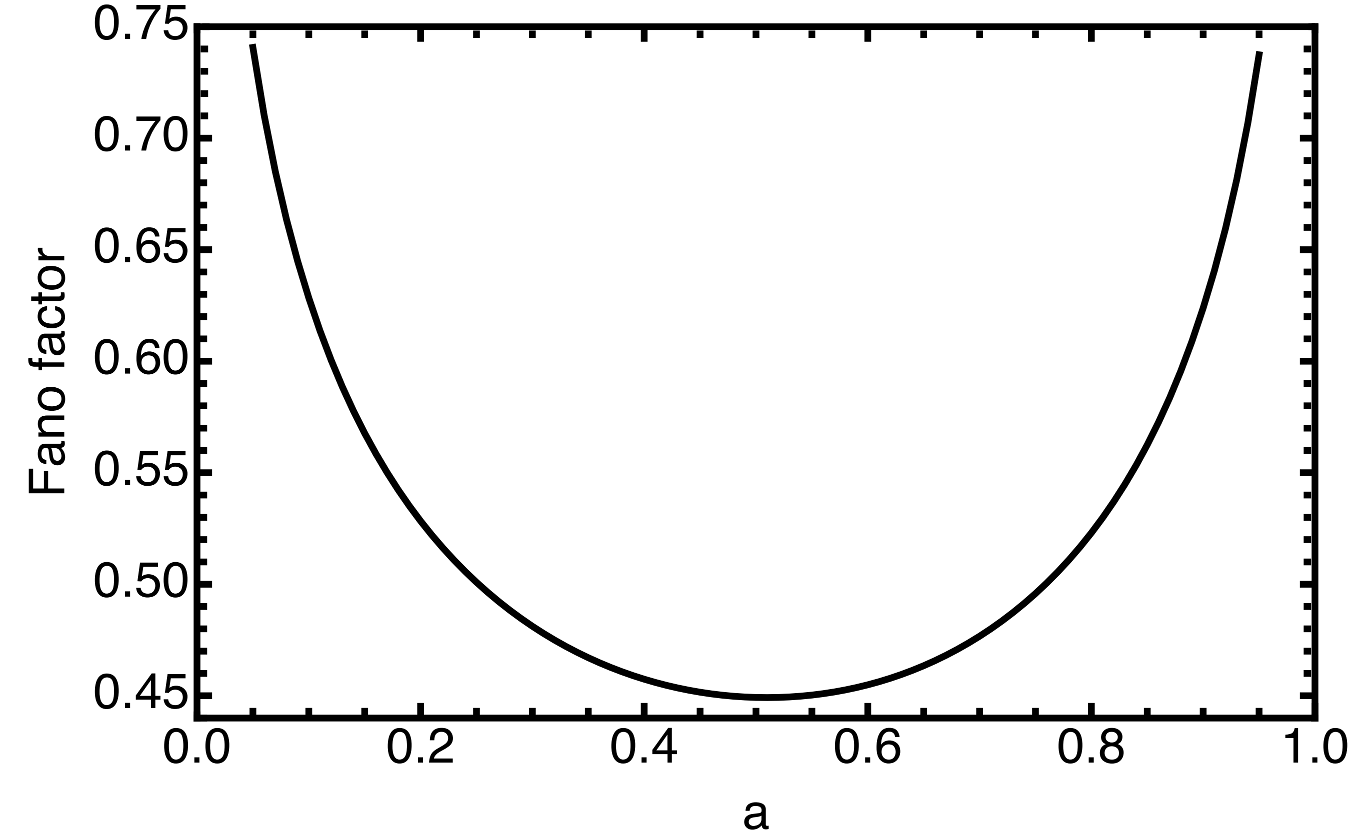
Non-equilibrium bosonization.

Effective equilibrium at

$$T^* = \frac{eV}{\pi}\sqrt{(3/2)a(1-a)}$$

RESULTS: SHOT-NOISE AT QPC2





Reference Noise

$$S_{\text{ref}}(\omega \to 0) = 4\, e\, p\, a(1-a)\frac{e^2}{h}V$$

non-interacting electrons
along a single edge.

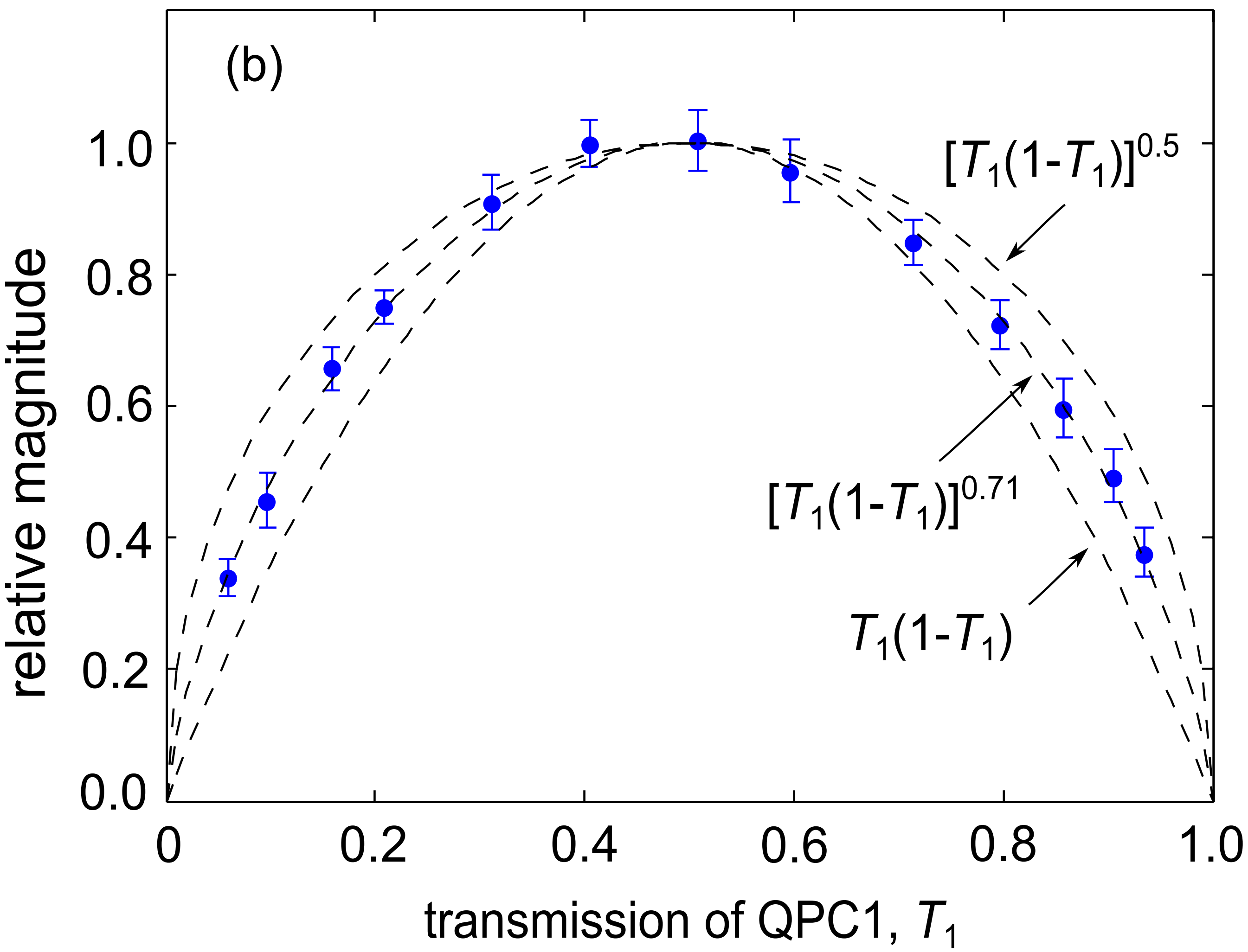
$Fano = 0.45$ at $\theta = 0.47$ $a = 1/2$

Form factor $[a(1-a)]^{\gamma_1}$

$\gamma_1 = 1$	Non interacting
$\gamma_1 = 0.5$	Thermal
$\gamma_1 = 0.71$	prethermalized

INTERACTIONS AND FORM FACTOR ($a = 1/2$)

Experiment

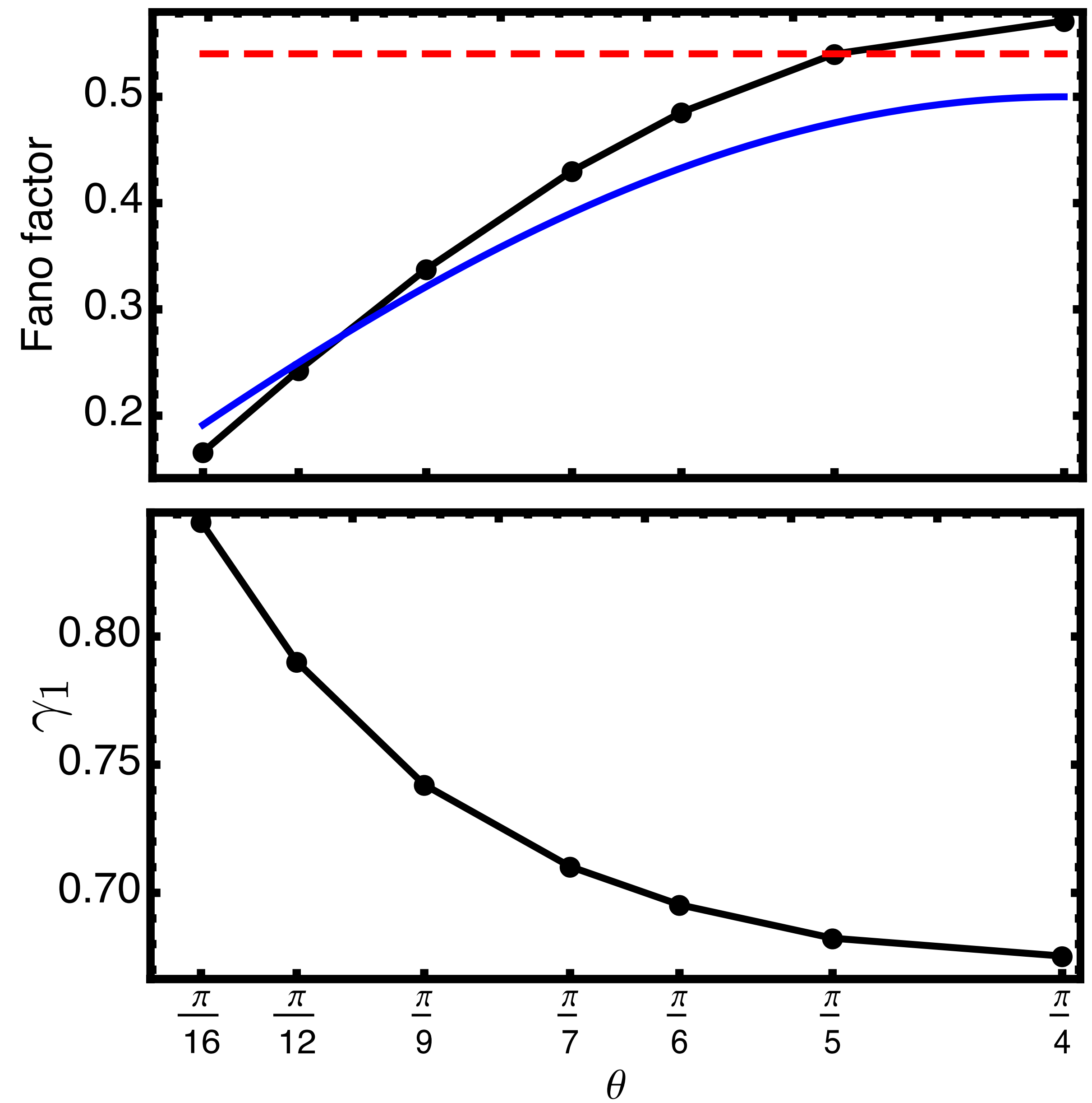


$\gamma_1 = 0.71 \pm 0.01$

$F = 0.44 \pm 0.02$

H. Inoue et al. PRL **112**, 166801 (2014)

Theory



BREAK-DOWN OF PERTURBATION THEORY

Would perturbation theory for a<<1 and p<<1 have been a valid alternative?

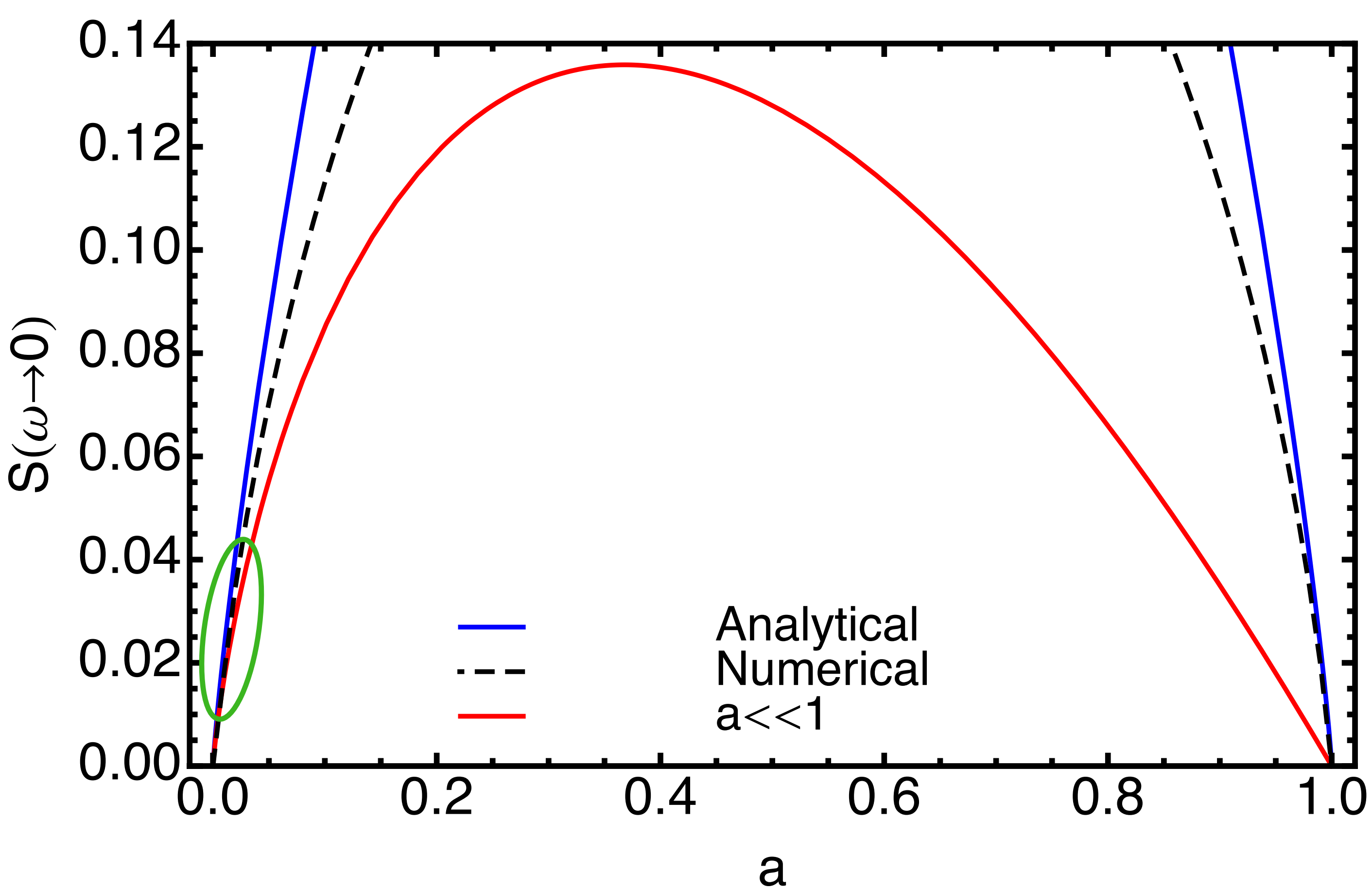
Consider the $eV\tau \gg 1$ regime, then we can approximate the determinant as :

$$\bar{\Delta}_\tau(\delta) \simeq e^{-|\tau|/(2\tau_\phi)}$$

$$\tau_\phi^{-1} = -eV/(2\pi)\log(1-4a(1-a)\sin^2(\delta/2))$$

where only leading terms have been kept. The noise is then given by :

$$S(\omega \to 0) \simeq 8peV(e^2/h)\sin^2(\delta/2)/\pi^2 \, a\log(1/a)$$



Neder, PRL **108**, 186404 (2012)

finds a diverging noise as a function of time

Shot Noise depends in a singular way on “a”!

MEASURING EQUILIBRATION FROM NOISE

Equilibration of edge mode 2u is characterised by transitions between different power laws of shot-noise signatures

Regime A: $S(\omega \to 0) \propto a \sin(2\theta)^2\, t_s^2 (eV)^3$

Regime B: $S(\omega \to 0) \propto a\, eV\, \log(t_s\, eV)$

Regime C: $S(\omega \to 0) \propto a\, eV\, \log(1/a)$

SUMMARY MESSAGES

Non-equilibrium driven charge fractionalization

Relation between fractionalization and equilibration regimes

Non-equilibrium bosonization is needed to access the steady state

Equilibration can be probed via shot noise measurements

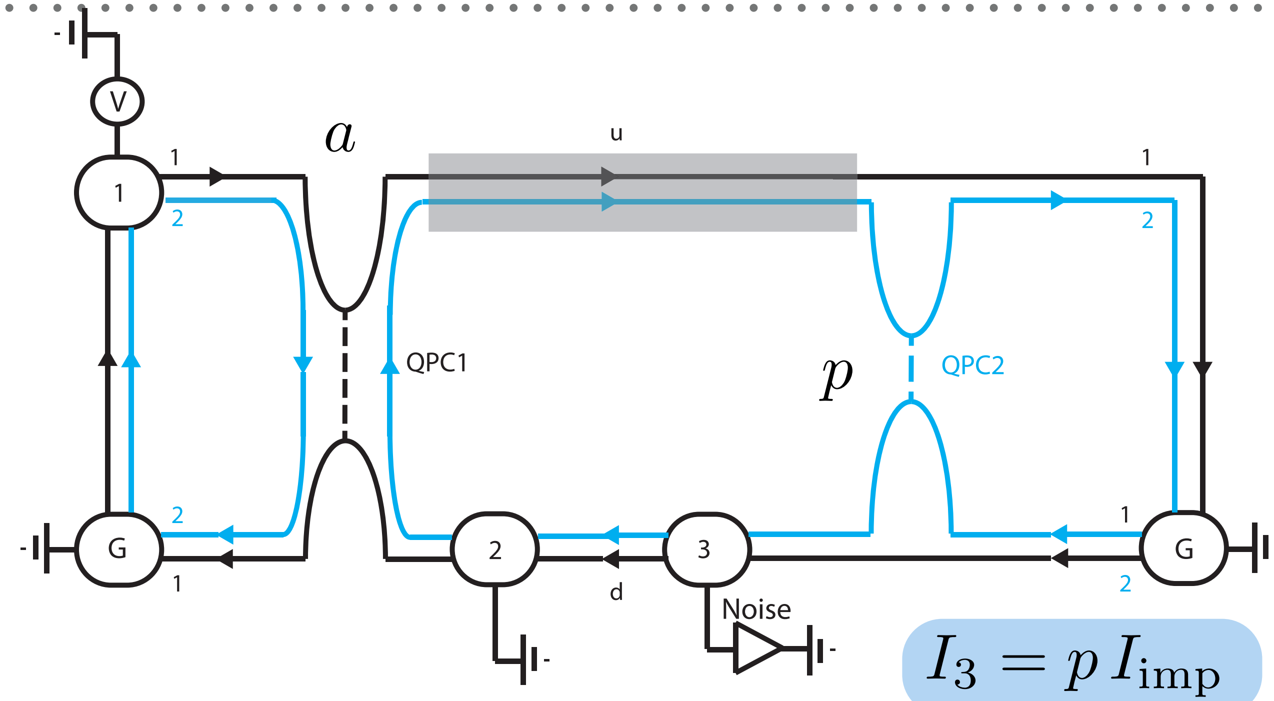
REFERENCES

*M. Milletari , B . Rosenow, PRL **111**, 136807 (2013)*

A. Schneider, M. Milletari , B . Rosenow, arXiv:1610.02036 (2016)

*H. Inoue et al. PRL **112**, 166801 (2014)*

SHOT NOISE FROM A SIMPLE POISSONIAN MODEL



$I_3 = p\, I_{\text{imp}}$

1. Assumption: fractional charges are well separated

Impinging current: $I_{\text{imp}} = e^* \frac{\langle N_{2u,e^*} \rangle}{\Delta t}$

Measured current: $I_3 = e \frac{\langle N_{2d,e} \rangle}{\Delta t}$

$$\langle N_{2d,e} \rangle = \left(p \frac{e^*}{e} \right) \langle N_{2u,e^*} \rangle$$

$$p \rightarrow p \frac{e^*}{e}$$

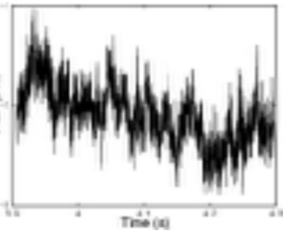
2. Assumption: $\forall\, e$ added on $1u$ $\exists\, e^*$ on $2u$

$$\langle N_{1u,e} \rangle = \langle N_{2u,e^*} \rangle$$

$$I_{1u} = e \frac{\langle N_{1u,e} \rangle}{\Delta t} = a \frac{e^2}{h} V \Rightarrow \langle N_{1u,e} \rangle = a \frac{eV}{h} \Delta t.$$

3. Assumption: Poissonian distribution

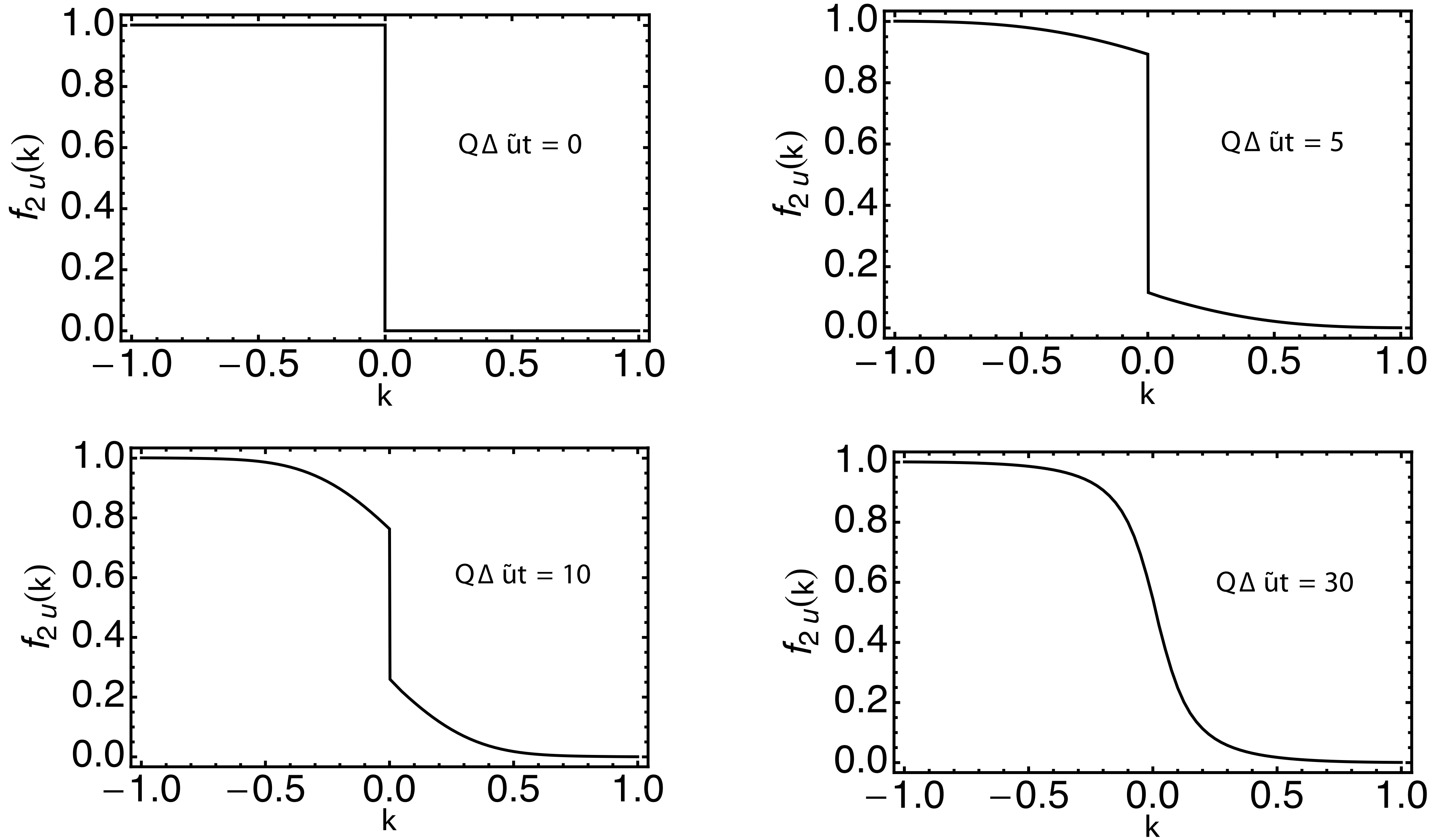
$$S_{\text{frac}} = 2e^2 \frac{\langle N_{2d,e}^2 \rangle - \langle N_{2d,e} \rangle^2}{\Delta t} = 2e^2 \frac{\langle N_{2d,e} \rangle}{\Delta t} = 2e^* a\, p\, I$$



Gaussian approximation (RPA)

$$\left\langle e^{\sum_q \lambda^\star_{1u}(q,t,\tau) b^\dagger_{1u,q}} e^{-\sum_q \lambda_{1u}(q,t,\tau) b_{1u,q}} \right\rangle = e^{-\sum_q \lambda^\star_{1u,q} \lambda_{1u,q} \langle b^\dagger_{1u,q} b_{1u,q} \rangle}$$

Taking the convolution in energy space with $G_0^<(\tau)$
we obtain the time evolution of the momentum distribution function



$Q = eV/u_1 (eV > 0)$ $\Delta\tilde{u} = \tilde{u}_1 - \tilde{u}_2$