Supersymmetric Meson-Baryon Properties of QCD from Light-Front Holography and Superconformal Algebra









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# Goal: An analytic first approximation to QCD

- As Simple as Schrödinger Theory in Atomic Physics
- Relativistic, Frame-Independent, Color-Confining
- Confinement in QCD -- What is the analytic form of the confining interaction?
- What sets the QCD mass scale?
- QCD Running Coupling at all scales
- Hadron Spectroscopy-Regge Trajectories
- Light-Front Wavefunctions
- Form Factors, Structure Functions, Hadronic Observables
- Constituent Counting Rules
- Hadronization at the Amplitude Level
- Insights into QCD Condensates
- Chiral Symmetry
- Systematically improvable



Supersymmetric Features of QCD from LF Holography





QCD Lagrangian

$$\mathcal{L}_{QCD} = -\frac{1}{4} Tr(G^{\mu\nu}G_{\mu\nu}) + \sum_{f=1}^{n_f} i\bar{\Psi}_f D_{\mu}\gamma^{\mu}\Psi_f + \sum_{f=1}^{n_f} i_f \bar{\Psi}_f \Psi_f$$

$$iD^{\mu} = i\partial^{\mu} - gA^{\mu} \qquad G^{\mu\nu} = \partial^{\mu}A^{\mu} - \partial^{\nu}A^{\mu} - g[A^{\mu}, A^{\nu}]$$

#### **Classical Chiral Lagrangian is Conformally Invariant**

## Where does the QCD Mass Scale come from?

QCD does not know what MeV units mean! Only Ratios of Masses Determined

ode Alfaro, Fubini, Furlan:

Scale can appear in Hamiltonian and EQM without affecting conformal invariance of action!

## Unique confinement potential!

## Invariance Principles of Quantum Field Theory

- Polncarè Invariance: Physical predictions must be independent of the observer's Lorentz frame: Front Form
- Causality: Information within causal horizon: Front Form
- Gauge Invariance: Physical predictions of gauge theories must be independent of the choice of gauge
- Scheme-Independence: Physical predictions of renormalizable theories must be independent of the choice of the renormalization scheme — Principle of Maximum Conformality (PMC)
- Mass-Scale Invariance: Conformal Invariance of the Action (DAFF)



Supersymmetric Features of QCD from LF Holography



P.A.M Dirac, Rev. Mod. Phys. 21, Dírac's Amazing Idea: 392 (1949) The "Front Form" **Evolve in Evolve in** ordinary time light-front time!  $\tau = t + z/c$  $\sigma = ct - z$ ct ct Ζ Ζ y У **Front Form Instant Form** Casual, Boost Invariant! Satisfies Poincarè Invariance

Each element of flash photograph illuminated at same LF time

$$\tau = t + z/c$$

**Causal, frame-independent** *Evolve in LF time* 

$$P^- = i \frac{d}{d\tau}$$

Eigenstate -- independent of au

$$H_{LF} = P^+ P^- - \vec{P}_{\perp}^2$$
$$H_{LF}^{QCD} |\Psi_h \rangle = \mathcal{M}_h^2 |\Psi_h \rangle$$



#### HELEN BRADLEY - PHOTOGRAPHY

#### **Bound States in Relativistic Quantum Field Theory:**

Light-Front Wavefunctions Dirac's Front Form: Fixed  $\tau = t + z/c$ 

Fixed 
$$\tau = t + z/c$$
  
 $\psi(x_i, \vec{k}_{\perp i}, \lambda_i)$   
 $x = \frac{k^+}{P^+} = \frac{k^0 + k^3}{P^0 + P^3}$ 

Invariant under boosts. Independent of  $P^{\mu}$ 

$$\mathbf{H}_{LF}^{QCD}|\psi\rangle = M^2|\psi\rangle$$

**Direct connection to QCD Lagrangian** 

# **Off-shell in invariant mass**

Remarkable new insights from AdS/CFT, the duality between conformal field theory and Anti-de Sitter Space



Causal, Frame-independent. Creation Operators on Simple Vacuum, 

Polncarè Invariance
Current Matrix Elements are Overlaps of LFWFS



## Advantages of the Dirac's Front Form for Hadron Physics Poincare' Invariant

# Physics Independent of Observer's Motion

- Measurements are made at fixed τ
- Causality is automatic
- Structure Functions are squares of LFWFs
- Form Factors are overlap of LFWFs
- LFWFs are frame-independent: no boosts, no pancakes!
- Same structure function measured at an e p collider and the proton rest frame
- No dependence of hadron structure on observer's frame
- LF Holography: Dual to AdS space
- LF Vacuum trivial -- no vacuum condensates!
- Profound implications for Cosmological Constant



## Terrell, Penrose

Light-Front QCD

#### Physical gauge: $A^+ = 0$

(c)

mma

Exact frame-independent formulation of nonperturbative QCD!

$$\begin{split} L^{QCD} &\to H_{LF}^{QCD} \\ H_{LF}^{QCD} &= \sum_{i} [\frac{m^{2} + k_{\perp}^{2}}{x}]_{i} + H_{LF}^{int} \\ H_{LF}^{int}: \text{ Matrix in Fock Space} \\ H_{LF}^{QCD} |\Psi_{h} \rangle &= \mathcal{M}_{h}^{2} |\Psi_{h} \rangle \\ |p, J_{z} \rangle &= \sum_{n=3}^{\infty} \psi_{n}(x_{i}, \vec{k}_{\perp i}, \lambda_{i}) |n; x_{i}, \vec{k}_{\perp i}, \lambda_{i} \rangle \end{split}$$

Eigenvalues and Eigensolutions give Hadronic Spectrum and Light-Front wavefunctions

### LFWFs: Off-shell in P- and invariant mass



Yukawa Híggs coupling of confined quark to Híggs zero mode gives

$$\bar{u}u \ g_q < h > = \frac{m_q}{x_q} m_q = \frac{m_q^2}{x_q}$$

$$H_{LF} = \sum_{q} \frac{k_{\perp q}^2 + m_q^2}{x_q}$$

$$|p, S_z\rangle = \sum_{n=3}^{QCD} |\Psi_h\rangle = \mathcal{M}_h^2 |\Psi_h\rangle$$

sum over states with n=3, 4, ... constituents

The Light Front Fock State Wavefunctions

$$\Psi_n(x_i, \vec{k}_{\perp i}, \lambda_i)$$

are boost invariant; they are independent of the hadron's energy and momentum  $P^{\mu}$ .

The light-cone momentum fractions

$$x_i = \frac{k_i^+}{p^+} = \frac{k_i^0 + k_i^z}{P^0 + P^z}$$

are boost invariant.

$$\sum_{i}^{n} k_{i}^{+} = P^{+}, \ \sum_{i}^{n} x_{i} = 1, \ \sum_{i}^{n} \vec{k}_{i}^{\perp} = \vec{0}^{\perp}.$$

Intrinsic heavy quarks s(x), c(x), b(x) at high x !

$$\overline{\bar{s}(x) \neq s(x)}$$
$$\overline{\bar{u}(x) \neq \bar{d}(x)}$$



Fixed LF time



 $= 2p^+F(q^2)$ 

# Front Form



Drell, sjb



Must include vacuum-induced currents to compute form factors and other current matrix elements in instant form

Boost are dynamical in instant form

Calculation of Form Factors in Equal-Time Theory



Need vacuum-induced currents

Calculation of Form Factors in Light-Front Theory



Exact LF Formula for Paulí Form Factor

$$\frac{F_{2}(q^{2})}{2M} = \sum_{a} \int [dx][d^{2}\mathbf{k}_{\perp}] \sum_{j} e_{j} \frac{1}{2} \times Drell, sjb$$

$$\begin{bmatrix} -\frac{1}{q^{L}}\psi_{a}^{\uparrow *}(x_{i}, \mathbf{k}'_{\perp i}, \lambda_{i}) \psi_{a}^{\downarrow}(x_{i}, \mathbf{k}_{\perp i}, \lambda_{i}) + \frac{1}{q^{R}}\psi_{a}^{\downarrow *}(x_{i}, \mathbf{k}'_{\perp i}, \lambda_{i}) \psi_{a}^{\uparrow}(x_{i}, \mathbf{k}_{\perp i}, \lambda_{i}) \end{bmatrix}$$

$$\mathbf{k}'_{\perp i} = \mathbf{k}_{\perp i} - x_{i}\mathbf{q}_{\perp} \qquad \mathbf{k}'_{\perp j} = \mathbf{k}_{\perp j} + (1 - x_{j})\mathbf{q}_{\perp}$$

$$\mathbf{q}_{R,L} = q^{x} \pm iq^{y}$$

$$\mathbf{p}, \mathbf{S}_{z} = \frac{1}{2} - \frac{1}{2} \qquad \mathbf{p} + \mathbf{q}, \mathbf{S}_{z} = \frac{1}{2} - \frac{1}{2}$$

#### Must have $\Delta \ell_z = \pm 1$ to have nonzero $F_2(q^2)$

Nonzero Proton Anomalous Moment --> Nonzero orbítal quark angular momentum

- LF wavefunctions play the role of Schrödinger wavefunctions in Atomic Physics
- LFWFs=Hadron Eigensolutions: Direct Connection to QCD Lagrangian
- Relativistic, frame-independent: no boosts, no disc contraction, Melosh built into LF spinors
- Hadronic observables computed from LFWFs: Form factors, Structure Functions, Distribution Amplitudes, GPDs, TMDs, Weak Decays, .... modulo `lensing' from ISIs, FSIs
- Cannot compute current matrix elements using instant form from eigensolutions alone -- need to include vacuum currents!
- •Hadron Physics without LFWFs is like Biology without DNA!



Supersymmetric Features of QCD from LF Holography





 $\Psi_n(x_i, \vec{k}_{\perp i}, \lambda_i)$ 



Sign reversal in DY!

Light-Front Perturbation Theory for pQCD

$$T = H_I + H_I \frac{1}{\mathcal{M}_{initial}^2 - \mathcal{M}_{intermediate}^2 + i\epsilon} H_I + \cdots$$

- "History": Compute any subgraph only once since the LFPth numerator does not depend on the process — only the denominator changes!
- Wick Theorem applies, but few amplitudes since all  $k^+ > 0$ .
- J<sub>z</sub> Conservation at every vertex  $\left|\sum_{initial} S^{z} \sum_{final} S_{z}\right| \leq n$  at order  $g^{n}$  K. Chiu, sjb
- Unitarity is explicit
- Loop Integrals are 3-dimensional

$$\int_0^1 dx \int d^2 k_\perp$$

• hadronization: coalesce comoving quarks and gluons to hadrons using light-front wavefunctions  $\Psi_n(x_i, \vec{k}_{\perp i}, \lambda_i)$ 

Need a First Approximation to QCD

Comparable in simplicity to Schrödinger Theory in Atomic Physics

**Relativistic, Frame-Independent, Color-Confining** 

**Origin of hadronic mass scale** 

AdS/QCD Líght-Front Holography Superconformal Algebra



$$\begin{split} & \mathcal{L}ight\text{-}Front\ \mathcal{Q}CD \\ & \mathcal{L}_{QCD} \longrightarrow H_{QCD}^{LF} \\ & (H_{LF}^{0} + H_{LF}^{I})|\Psi >= M^{2}|\Psi > \\ & (H_{LF}^{0} + H_{LF}^{I})|\Psi >= M^{2}|\Psi > \\ & (H_{LF}^{0} + H_{LF}^{I})|\Psi >= M^{2}\psi_{LF}(x,\vec{k}_{\perp}) \\ & (I_{\chi(1-x)}^{\ell} + V_{\text{eff}}^{LF})\psi_{LF}(x,\vec{k}_{\perp}) = M^{2}\psi_{LF}(x,\vec{k}_{\perp}) \\ & (I_{\chi(1-x)}^{\ell} + \frac{1-4L^{2}}{4\zeta^{2}} + U(\zeta)]\psi(\zeta) = \mathcal{M}^{2}\psi(\zeta) \\ & \mathbf{AdS}/\mathbf{QCD}: \\ & (U(\zeta) = \kappa^{4}\zeta^{2} + 2\kappa^{2}(L+S-1)) \end{split}$$

Semiclassical first approximation to QCD

Fixed  $\tau = t + z/c$ 



Coupled Fock states

Elímínate hígher Fock states and retarded interactions

Effective two-particle equation

Azimuthal Basis  $\zeta, \phi$  $m_q=0$ Single variable  $\zeta$ 

Confining AdS/QCD potential!

Sums an infinite # diagrams

de Tèramond, Dosch, sjb

AdS/QCD Soft-Wall Model

 $e^{\varphi(z)} = e^{+\kappa^2 z^2}$ 



 $\zeta^2 = x(1-x)\mathbf{b}^2_{\perp}$ 

$$\left[-\frac{d^2}{d\zeta^2} + \frac{1-4L^2}{4\zeta^2} + U(\zeta)\right]\psi(\zeta) = \mathcal{M}^2\psi(\zeta)$$



Light-Front Schrödinger Equation  $U(\zeta) = \kappa^4 \zeta^2 + 2\kappa^2 (L + S - 1)$ Single variable  $\zeta$ Confinement scale:  $\kappa \simeq 0.5 \ GeV$ 

Unique Confinement Potential!

Conformal Symmetry of the action

de Alfaro, Fubini, Furlan:
Fubini, Rabinovici

Scale can appear in Hamiltonian and EQM without affecting conformal invariance of action!

#### • de Alfaro, Fubini, Furlan



Retains conformal invariance of action despite mass scale!  $4uw-v^2=\kappa^4=[M]^4$ 

Identical to LF Hamiltonian with unique potential and dilaton!

Dosch, de Teramond, sjb

$$\left[-\frac{d^2}{d\zeta^2} + \frac{1-4L^2}{4\zeta^2} + U(\zeta)\right]\psi(\zeta) = \mathcal{M}^2\psi(\zeta)$$
$$U(\zeta) = \kappa^4\zeta^2 + 2\kappa^2(L+S-1)$$

dAFF: New Time Variable

$$\tau = \frac{2}{\sqrt{4uw - v^2}} \arctan\left(\frac{2tw + v}{\sqrt{4uw - v^2}}\right)$$

- Identify with difference of LF time  $\Delta x^+/P^+$ between constituents
- Finite range
- Measure in Double-Parton Processes

Retains conformal invariance of action despite mass scale!

Genoa Feb 8, 2017



Supersymmetric Features of QCD from LF Holography





#### Meson Spectrum in Soft Wall Model

$$m_{\pi} = 0$$
 if  $m_q = 0$ 

Pion: Negative term for J=0 cancels positive terms from LFKE and potential

Massless pion!

• Effective potential:  $U(\zeta^2) = \kappa^4 \zeta^2 + 2\kappa^2 (J-1)$ 

LF WE

$$\left(-rac{d^2}{d\zeta^2}-rac{1-4L^2}{4\zeta^2}+\kappa^4\zeta^2+2\kappa^2(J-1)
ight)\phi_J(\zeta)=M^2\phi_J(\zeta)$$

• Normalized eigenfunctions  $\;\langle \phi | \phi 
angle = \int d\zeta \, \phi^2(z)^2 = 1\;$ 

$$\phi_{n,L}(\zeta) = \kappa^{1+L} \sqrt{\frac{2n!}{(n+L)!}} \zeta^{1/2+L} e^{-\kappa^2 \zeta^2/2} L_n^L(\kappa^2 \zeta^2)$$
$$\mathcal{M}_{n,J,L}^2 = 4\kappa^2 \left(n + \frac{J+L}{2}\right)$$

G. de Teramond, H. G. Dosch, sjb

Eigenvalues

$$m_u = m_d = 0$$

#### de Tèramond, Dosch, sjb



$$M^{2}(n, L, S) = 4\kappa^{2}(n + L + S/2)$$

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Supersymmetric Features of QCD from LF Holography

Stan Brodsky





## Prediction from AdS/QCD: Meson LFWF



week ending 24 AUGUST 2012



#### AdS/QCD Holographic Wave Function for the $\rho$ Meson and Diffractive $\rho$ Meson Electroproduction

• Light Front Wavefunctions:  $\Psi_n(x_i, \vec{k}_{\perp i}, \lambda_i)$ off-shell in  $P^-$  and invariant mass  $\mathcal{M}^2_{q\bar{q}}$ 



**Boost-invariant LFWF connects confined quarks and gluons to hadrons** 



Changes in physical length scale mapped to evolution in the 5th dimension z

AdS<sub>5</sub>

- Truncated AdS/CFT (Hard-Wall) model: cut-off at  $z_0 = 1/\Lambda_{QCD}$  breaks conformal invariance and allows the introduction of the QCD scale (Hard-Wall Model) Polchinski and Strassler (2001).
- Smooth cutoff: introduction of a background dilaton field  $\varphi(z)$  usual linear Regge dependence can be obtained (Soft-Wall Model) Karch, Katz, Son and Stephanov (2006).







• Isomorphism of SO(4,2) of conformal QCD with the group of isometries of AdS space

$$ds^2 = \frac{R^2}{r^2} (\eta_{\mu\nu} dx^{\mu} dx^{\nu} - dz^2),$$
 invariant measure

 $x^{\mu} \rightarrow \lambda x^{\mu}, \ z \rightarrow \lambda z$ , maps scale transformations into the holographic coordinate z.

- AdS mode in z is the extension of the hadron wf into the fifth dimension.
- Different values of z correspond to different scales at which the hadron is examined.

$$x^2 \to \lambda^2 x^2, \quad z \to \lambda z.$$

 $x^2 = x_\mu x^\mu$ : invariant separation between quarks

- The AdS boundary at  $z \to 0$  correspond to the  $Q \to \infty,$  UV zero separation limit.

# AdS/CFT

# Dílaton-Modífied AdS/QCD

$$ds^{2} = e^{\varphi(z)} \frac{R^{2}}{z^{2}} (\eta_{\mu\nu} x^{\mu} x^{\nu} - dz^{2})$$



- Soft-wall dilaton profile breaks conformal invariance  $e^{\varphi(z)} = e^{+\kappa^2 z^2}$
- Color Confinement in z
- ullet Introduces confinement scale  $\kappa$
- Uses AdS<sub>5</sub> as template for conformal theory



Supersymmetric Features of QCD from LF Holography




$e^{\varphi(z)} = e^{+\kappa^2 z^2}$ 

Ads Soft-Wall Schrödinger Equation for bound state of two scalar constituents:

$$\left[-\frac{d^2}{dz^2} - \frac{1 - 4L^2}{4z^2} + U(z)\right]\Phi(z) = \mathcal{M}^2\Phi(z)$$

$$U(z) = \kappa^4 z^2 + 2\kappa^2 (L + S - 1)$$

Derived from variation of Action for Dilaton-Modified AdS<sub>5</sub> **Identical to Single-Variable Light-Front Bound State Equation in**  $\zeta$ !



**Light-Front Holography**: Unique mapping derived from equality of LF and AdS formula for EM and gravitational current matrix elements and identical equations of motion

de Tèramond, Dosch, sjb

AdS/QCD Soft-Wall Model

 $e^{\varphi(z)} = e^{+\kappa^2 z^2}$ 



 $\zeta^2 = x(1-x)\mathbf{b}^2_{\perp}$ 

$$\left[-\frac{d^2}{d\zeta^2} + \frac{1-4L^2}{4\zeta^2} + U(\zeta)\right]\psi(\zeta) = \mathcal{M}^2\psi(\zeta)$$



Light-Front Schrödinger Equation  $U(\zeta) = \kappa^4 \zeta^2 + 2\kappa^2 (L + S - 1)$ Single variable  $\zeta$ Confinement scale:  $\kappa \simeq 0.5 \ GeV$ 

Unique Confinement Potential!

Conformal Symmetry of the action

de Alfaro, Fubini, Furlan:
Fubini, Rabinovici

Scale can appear in Hamiltonian and EQM without affecting conformal invariance of action!

### Introduce "Dílaton" to símulate confinement analytically

• Nonconformal metric dual to a confining gauge theory

$$ds^2 = \frac{R^2}{z^2} e^{\varphi(z)} \left( \eta_{\mu\nu} dx^{\mu} dx^{\nu} - dz^2 \right)$$

where  $\varphi(z) \to 0$  at small z for geometries which are asymptotically  ${\rm AdS}_5$ 

• Gravitational potential energy for object of mass m

$$V = mc^2 \sqrt{g_{00}} = mc^2 R \, \frac{e^{\varphi(z)/2}}{z}$$

- Consider warp factor  $\exp(\pm\kappa^2 z^2)$
- Plus solution: V(z) increases exponentially confining any object in modified AdS metrics to distances  $\langle z\rangle\sim 1/\kappa$



Klebanov and Maldacena

$$e^{\varphi(z)} = e^{+\kappa^2 z^2}$$

### **Positive-sign dilaton**

de Teramond, sjb

# Uniqueness of Dilaton

$$\varphi_p(z) = \kappa^p z^p$$



Dosch, de Tèramond, sjb

Haag, Lopuszanski, Sohnius (1974)

Superconformal Quantum Mechanics  $\{\psi,\psi^+\} = 1$   $B = \frac{1}{2}[\psi^+,\psi] = \frac{1}{2}\sigma_3$  $\psi = \frac{1}{2}(\sigma_1 - i\sigma_2), \quad \psi^+ = \frac{1}{2}(\sigma_1 + i\sigma_2)$  $Q = \psi^{+}[-\partial_{x} + \frac{f}{x}], \quad Q^{+} = \psi[\partial_{x} + \frac{f}{x}], \quad S = \psi^{+}x, \quad S^{+} = \psi x$  $\{Q, Q^+\} = 2H, \{S, S^+\} = 2K$  $\{Q, S^+\} = f - B + 2iD, \ \{Q^+, S\} = f - B - 2iD$ generates conformal algebra [H,D] = i H, [H, K] = 2 i D, [K, D] = - i K $Q \simeq \sqrt{H}, \quad S \simeq \sqrt{K}$ 

## Superconformal Quantum Mechanics

# **Baryon Equation** $Q \simeq \sqrt{H}, S \simeq \sqrt{K}$

Consider  $R_w = Q + wS;$ 

w: dimensions of mass squared

$$G = \{R_w, R_w^+\} = 2H + 2w^2K + 2wfI - 2wB \qquad 2B = \sigma_3$$

**Retains Conformal Invariance of Action** 

Fubini and Rabinovici

New Extended Hamíltonían G ís díagonal:

$$G_{11} = \left(-\partial_x^2 + w^2 x^2 + 2wf - w + \frac{4(f + \frac{1}{2})^2 - 1}{4x^2}\right)$$

$$G_{22} = \left(-\partial_x^2 + w^2 x^2 + 2wf + w + \frac{4(f - \frac{1}{2})^2 - 1}{4x^2}\right)$$
  
Identify  $f - \frac{1}{2} = L_B$ ,  $w = \kappa^2$ 

Eigenvalue of G:  $M^2(n, L) = 4\kappa^2(n + L_B + 1)$ 

# LF Holography

**Baryon Equation** 

Superconformal Quantum Mechanics

$$\left(-\partial_{\zeta}^{2} + \kappa^{4}\zeta^{2} + 2\kappa^{2}(L_{B}+1) + \frac{4L_{B}^{2}-1}{4\zeta^{2}}\right)\psi_{J}^{+} = M^{2}\psi_{J}^{+}$$

$$\left(-\partial_{\zeta}^{2} + \kappa^{4}\zeta^{2} + 2\kappa^{2}L_{B} + \frac{4(L_{B}+1)^{2} - 1}{4\zeta^{2}}\right)\psi_{J}^{-} = M^{2}\psi_{J}^{-}$$

$$M^{2}(n, L_{B}) = 4\kappa^{2}(n + L_{B} + 1)$$
 S=1/2, P=+

both chiralities

## **Meson Equation**

$$\left(-\partial_{\zeta}^{2} + \kappa^{4}\zeta^{2} + 2\kappa^{2}(J-1) + \frac{4L_{M}^{2} - 1}{4\zeta^{2}}\right)\phi_{J} = M^{2}\phi_{J}$$

$$M^2(n, L_M) = 4\kappa^2(n + L_M) \qquad Same_{\varkappa}!$$

## S=0, I=1 Meson is superpartner of S=1/2, I=1 Baryon Meson-Baryon Degeneracy for L<sub>M</sub>=L<sub>B</sub>+1

#### **Fermionic Modes and Baryon Spectrum**

[Hard wall model: GdT and S. J. Brodsky, PRL **94**, 201601 (2005)] [Soft wall model: GdT and S. J. Brodsky, (2005), arXiv:1001.5193]



From Nick Evans

• Nucleon LF modes

$$\psi_{+}(\zeta)_{n,L} = \kappa^{2+L} \sqrt{\frac{2n!}{(n+L)!}} \zeta^{3/2+L} e^{-\kappa^{2}\zeta^{2}/2} L_{n}^{L+1} \left(\kappa^{2}\zeta^{2}\right)$$
$$\psi_{-}(\zeta)_{n,L} = \kappa^{3+L} \frac{1}{\sqrt{n+L+2}} \sqrt{\frac{2n!}{(n+L)!}} \zeta^{5/2+L} e^{-\kappa^{2}\zeta^{2}/2} L_{n}^{L+2} \left(\kappa^{2}\zeta^{2}\right)$$

• Normalization

$$\int d\zeta \,\psi_+^2(\zeta) = \int d\zeta \,\psi_-^2(\zeta) = 1$$

Quark Chíral Symmetry of Eígenstate!

• Eigenvalues

$$\mathcal{M}_{n,L,S=1/2}^2 = 4\kappa^2 \,(n+L+1)$$

• "Chiral partners"

$$\frac{\mathcal{M}_{N(1535)}}{\mathcal{M}_{N(940)}} = \sqrt{2}$$

## Nucleon: Equal Probability for L=0, I



#### Dosch, de Teramond, Lorce, sjb

$$m_u = m_d = 46 \text{ MeV}, m_s = 357 \text{ MeV}$$



Fit to the slope of Regge trajectories, including radial excitations

Same Regge Slope for Meson, Baryons: Supersymmetric feature of hadron physics



de Tèramond, Dosch, sjb



### Solid line: $\chi = 0.53$ GeV



Superconformal meson-nucleon partners

de Tèramond, Dosch, sjb

#### **Space-Like Dirac Proton Form Factor**

• Consider the spin non-flip form factors

$$F_{+}(Q^{2}) = g_{+} \int d\zeta J(Q,\zeta) |\psi_{+}(\zeta)|^{2},$$
  
$$F_{-}(Q^{2}) = g_{-} \int d\zeta J(Q,\zeta) |\psi_{-}(\zeta)|^{2},$$

where the effective charges  $g_+$  and  $g_-$  are determined from the spin-flavor structure of the theory.

- Choose the struck quark to have  $S^z = +1/2$ . The two AdS solutions  $\psi_+(\zeta)$  and  $\psi_-(\zeta)$  correspond to nucleons with  $J^z = +1/2$  and -1/2.
- For SU(6) spin-flavor symmetry

$$F_1^p(Q^2) = \int d\zeta J(Q,\zeta) |\psi_+(\zeta)|^2,$$
  

$$F_1^n(Q^2) = -\frac{1}{3} \int d\zeta J(Q,\zeta) \left[ |\psi_+(\zeta)|^2 - |\psi_-(\zeta)|^2 \right],$$

where  $F_1^p(0) = 1$ ,  $F_1^n(0) = 0$ .

• Compute Dirac proton form factor using SU(6) flavor symmetry

$$F_1^p(Q^2) = R^4 \int \frac{dz}{z^4} V(Q, z) \Psi_+^2(z)$$

Nucleon AdS wave function

$$\Psi_{+}(z) = \frac{\kappa^{2+L}}{R^2} \sqrt{\frac{2n!}{(n+L)!}} z^{7/2+L} L_n^{L+1} \left(\kappa^2 z^2\right) e^{-\kappa^2 z^2/2}$$

• Normalization  $(F_1^p(0) = 1, V(Q = 0, z) = 1)$ 

$$R^4 \int \frac{dz}{z^4} \, \Psi_+^2(z) = 1$$

• Bulk-to-boundary propagator [Grigoryan and Radyushkin (2007)]

$$V(Q,z) = \kappa^2 z^2 \int_0^1 \frac{dx}{(1-x)^2} x^{\frac{Q^2}{4\kappa^2}} e^{-\kappa^2 z^2 x/(1-x)}$$

• Find

$$F_1^p(Q^2) = \frac{1}{\left(1 + \frac{Q^2}{\mathcal{M}_{\rho}^2}\right)\left(1 + \frac{Q^2}{\mathcal{M}_{\rho'}^2}\right)}$$

with  $\mathcal{M}_{\rho_n}^2 \to 4\kappa^2(n+1/2)$ 



Using SU(6) flavor symmetry and normalization to static quantities





#### **Nucleon Transition Form Factors**

- Compute spin non-flip EM transition  $N(940) \rightarrow N^*(1440)$ :  $\Psi^{n=0,L=0}_+ \rightarrow \Psi^{n=1,L=0}_+$
- Transition form factor

$$F_{1N \to N^*}^{p}(Q^2) = R^4 \int \frac{dz}{z^4} \Psi_+^{n=1,L=0}(z) V(Q,z) \Psi_+^{n=0,L=0}(z)$$

• Orthonormality of Laguerre functions  $(F_1^p_{N \to N^*}(0) = 0, V(Q = 0, z) = 1)$ 

$$R^4 \int \frac{dz}{z^4} \Psi_+^{n',L}(z) \Psi_+^{n,L}(z) = \delta_{n,n'}$$

• Find

with  $\mathcal{M}_{\rho_n}^2$ 

$$F_{1N\to N^{*}}^{p}(Q^{2}) = \frac{2\sqrt{2}}{3} \frac{\frac{Q^{2}}{M_{P}^{2}}}{\left(1 + \frac{Q^{2}}{M_{\rho}^{2}}\right)\left(1 + \frac{Q^{2}}{M_{\rho'}^{2}}\right)\left(1 + \frac{Q^{2}}{M_{\rho''}^{2}}\right)} \to 4\kappa^{2}(n+1/2)$$

de Teramond, sjb

#### Consistent with counting rule, twist 3

Predict hadron spectroscopy and dynamics







## Flavor Dependence of $Q^6 F_2(Q^2)$

Sufian, de Teramond, Deur, Dosch, sjb

Dressed soft-wall current brings in higher Fock states and more vector meson poles



#### **Current Matrix Elements in AdS Space (SW)**

• Propagation of external current inside AdS space described by the AdS wave equation

$$\left[z^2\partial_z^2 - z\left(1 + 2\kappa^2 z^2\right)\partial_z - Q^2 z^2\right]J_{\kappa}(Q, z) = 0.$$

• Solution bulk-to-boundary propagator

$$J_{\kappa}(Q,z) = \Gamma\left(1 + \frac{Q^2}{4\kappa^2}\right) U\left(\frac{Q^2}{4\kappa^2}, 0, \kappa^2 z^2\right),$$

where U(a, b, c) is the confluent hypergeometric function

$$\Gamma(a)U(a,b,z) = \int_0^\infty e^{-zt} t^{a-1} (1+t)^{b-a-1} dt.$$

• Form factor in presence of the dilaton background  $\varphi = \kappa^2 z^2$ 

$$F(Q^2) = R^3 \int \frac{dz}{z^3} e^{-\kappa^2 z^2} \Phi(z) J_{\kappa}(Q, z) \Phi(z).$$

• For large  $Q^2 \gg 4\kappa^2$ 

$$J_{\kappa}(Q,z) \to zQK_1(zQ) = J(Q,z),$$

the external current decouples from the dilaton field.

de Tèramond & sjb Grigoryan and Radyushkin

Dressed Current ín Soft-Wall Model

$$e^{\varphi(z)} = e^{+\kappa^2 z}$$

## Timelike Pion Form Factor from AdS/QCD and Light-Front Holography





# Superconformal Algebra

## 2X2 Hadronic Multiplets

Bosons, Fermions with Equal Mass!



Proton: quark + scalar diquark |q(qq) >(Equal weight: L = 0, L = 1)

# Features of Supersymmetric Equations

 J =L+S baryon simultaneously satisfies both equations of G with L, L+1 with same mass eigenvalue

• 
$$J^z = L^z + 1/2 = (L^z + 1) - 1/2$$
  $S^z = \pm 1/2$ 

- Proton spin carried by quark L<sup>z</sup>  $< J^z >= \frac{1}{2}(S_q^z = \frac{1}{2}, L^z = 0) + \frac{1}{2}(S_q^z = -\frac{1}{2}, L^z = 1) = < L^z >= \frac{1}{2}$ 
  - Mass-degenerate meson "superpartner" with L<sub>M</sub>=L<sub>B</sub>+1. "Shifted meson-baryon Duality"

Mesons and baryons have same  $\kappa$  !

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### Solid line: $\chi = 0.53$ GeV



Superconformal meson-nucleon partners

de Tèramond, Dosch, sjb

E. Klempt and B. Ch. Metsch



The leading Regge trajectory:  $\Delta$  resonances with maximal J in a given mass range. Also shown is the Regge trajectory for mesons with J = L+S.



 $\chi(mesons) = -1$   $\chi(baryons, tetraquarks) = +1$ 

# Universal Hadronic Features

• Universal quark light-front kinetic energy

$$\Delta \mathcal{M}_{LFKE}^2 = \kappa^2 (1 + 2n + L)$$

Equal: Virial Universal quark light-front potential energy  $\Delta M_{LFPE}^2 = \kappa^2 (1 + 2n + L)$ 

# Universal Constant Term

$$\mathcal{M}_{spin}^2 = 2\kappa^2 (S + L - 1 + 2n_{diquark})$$

$$M^{2} = \Delta \mathcal{M}_{LFKE}^{2} + \Delta \mathcal{M}_{LFPE}^{2} + \Delta \mathcal{M}_{spin}^{2}$$
$$+ < \sum_{i} \frac{m_{i}^{2}}{x_{i}} >$$

**New World of Tetraquarks** 

$$3_C \times 3_C = \overline{3}_C + 6_C$$
  
Bound!

- Diquark: Color-Confined Constituents: Color  $3_C$
- Diquark-Antidiquark bound states  $\overline{3}_C \times 3_C = 1_C$

$$\sigma(TN) \simeq 2\sigma(pN) - \sigma(\pi N)$$

 $2\big[\sigma([\{qq\}N) + \sigma(qN)\big] - [\sigma(qN) + \sigma(\bar{q}N)] = [\sigma(\{qq\}N) + \sigma(\{qq\}N)]$ 

Candidates  $f_0(980)I = 0, J^P = 0^+$ , partner of proton

 $a_1(1260)I = 0, J^P = 1^+$ , partner of  $\Delta(1233)$ 

#### Dosch, de Teramond, sjb

Supersymmetry across the light and heavy-light spectrum



Heavy charm quark mass does not break supersymmetry

#### Dosch, de Teramond, sjb

Supersymmetry across the light and heavy-light spectrum



Heavy bottom quark mass does not break supersymmetry

# Foundations of Light-Front Holography

- The QCD Lagrangian for  $m_q = 0$  has no mass scale.
- What determines the hadron mass scale?
- DAFF principle: add terms linear in D and K to Conformal Hamiltonian: Mass scale k appears, but action remains scale invariant —> unique harmonic oscillator potential
- Apply DAFF to the Poincare' invariant LF Hamiltonian: Unique color-confining potential
- Fixes AdS<sub>5</sub> dilaton: predicts Spin and Spin-Orbit Interactions
- Apply DAFF to Superconformal representation of the Lorentz group
- Predicts Meson, Baryon, Tetraquark spectroscopy, dynamics
- Supersymmetric Features of Spectrum

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# Chíral Features of Soft-Wall AdS/QCD Model

- Boost Invariant
- Trivial LF vacuum! No condensate, but consistent with GMOR
- Massless Pion
- Hadron Eigenstates (even the pion) have LF Fock components of different L<sup>z</sup>

• Proton: equal probability  $S^z=+1/2, L^z=0; S^z=-1/2, L^z=+1$ 

$$J^z = +1/2 :< L^z >= 1/2, < S^z_q >= 0$$

- Self-Dual Massive Eigenstates: Proton is its own chiral partner.
- Label State by minimum L as in Atomic Physics
- Minimum L dominates at short distances
- AdS/QCD Dictionary: Match to Interpolating Operator Twist at z=0.
   No mass -degenerate parity partners!
## Running Coupling from Modified Ads/QCD

#### Deur, de Teramond, sjb

• Consider five-dim gauge fields propagating in AdS $_5$  space in dilaton background  $arphi(z)=\kappa^2 z^2$ 

$$S = -\frac{1}{4} \int d^4x \, dz \, \sqrt{g} \, e^{\varphi(z)} \, \frac{1}{g_5^2} \, G^2$$

• Flow equation

$$\frac{1}{g_5^2(z)} = e^{\varphi(z)} \frac{1}{g_5^2(0)} \quad \text{or} \quad g_5^2(z) = e^{-\kappa^2 z^2} g_5^2(0)$$

where the coupling  $g_5(z)$  incorporates the non-conformal dynamics of confinement

- YM coupling  $\alpha_s(\zeta) = g_{YM}^2(\zeta)/4\pi$  is the five dim coupling up to a factor:  $g_5(z) \to g_{YM}(\zeta)$
- $\bullet\,$  Coupling measured at momentum scale Q

$$\alpha_s^{AdS}(Q) \sim \int_0^\infty \zeta d\zeta J_0(\zeta Q) \,\alpha_s^{AdS}(\zeta)$$

 $\alpha_s^{AdS}(Q^2) = \alpha_s^{AdS}(0) e^{-Q^2/4\kappa^2}$  from dilaton  $e^{\kappa^2 z^2}$ 

Solution

where the coupling  $\alpha_s^{AdS}$  incorporates the non-conformal dynamics of confinement

Bjorken sum rule defines effective charge 
$$\alpha_{g1}(Q^2)$$
$$\int_0^1 dx [g_1^{ep}(x,Q^2) - g_1^{en}(x,Q^2)] \equiv \frac{g_a}{6} [1 - \frac{\alpha_{g1}(Q^2)}{\pi}]$$

- Can be used as standard QCD coupling
- Well measured
- Asymptotic freedom at large Q<sup>2</sup>
- Computable at large Q<sup>2</sup> in any pQCD scheme
- Universal  $\beta_0$ ,  $\beta_1$

## Running Coupling from Modified AdS/QCD

#### Deur, de Teramond, sjb

• Consider five-dim gauge fields propagating in AdS $_5$  space in dilaton background  $arphi(z)=\kappa^2 z^2$ 

$$S = -\frac{1}{4} \int d^4x \, dz \, \sqrt{g} \, e^{\varphi(z)} \, \frac{1}{g_5^2} \, G^2$$

• Flow equation

$$\frac{1}{g_5^2(z)} = e^{\varphi(z)} \frac{1}{g_5^2(0)} \quad \text{or} \quad g_5^2(z) = e^{-\kappa^2 z^2} g_5^2(0)$$

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- $\bullet\,$  Coupling measured at momentum scale Q

$$\alpha_s^{AdS}(Q) \sim \int_0^\infty \zeta d\zeta J_0(\zeta Q) \, \alpha_s^{AdS}(\zeta)$$

Solution

 $\alpha_s^{AdS}(Q^2)=\alpha_s^{AdS}(0)\,e^{-Q^2/4\kappa^2}.$  where the coupling  $\alpha_s^{AdS}$  incorporates the non-conformal dynamics of confinement



#### Analytic, defined at all scales, IR Fixed Point

AdS/QCD dilaton captures the higher twist corrections to effective charges for Q < 1 GeV

$$e^{\varphi} = e^{+\kappa^2 z}$$

 $\mathbf{2}$ 

Deur, de Teramond, sjb







#### **Process-independent strong running coupling**

Daniele Binosi,<sup>1</sup> Cédric Mezrag,<sup>2</sup> Joannis Papavassiliou,<sup>3</sup> Craig D. Roberts,<sup>2</sup> and Jose Rodríguez-Quintero<sup>4</sup>

# Features of LF Holographic QCD

- Regge spectroscopy—same slope in n,L for mesons, baryons
- Chiral features for  $m_q=0$ :  $m_{\pi}=0$ , chiral-invariant proton
- Hadronic LFWFs
- Counting Rules
- Connection between hadron masses and  $\Lambda_{\overline{MS}}$

Superconformal AdS Light-Front Holographic QCD (LFHQCD)

Meson-Baryon Mass Degeneracy for L<sub>M</sub>=L<sub>B</sub>+1



Supersymmetric Features of QCD from LF Holography





# **Tony Zee**

# "Quantum Field Theory in a Nutshell"

# Dreams of Exact Solvability

"In other words, if you manage to calculate  $m_P$  it better come out proportional to  $\Lambda_{QCD}$  since  $\Lambda_{QCD}$  is the only quantity with dimension of mass around.

Light-Front Holography:

Similarly for  $m_{\rho}$ .

$$m_p \simeq 3.21 \ \Lambda_{\overline{MS}}$$

$$m_{\rho} \simeq 2.2 \ \Lambda_{\overline{MS}}$$

Put in precise terms, if you publish a paper with a formula giving  $m_{\rho}/m_{P}$  in terms of pure numbers such as 2 and  $\pi$ , the field theory community will hail you as a conquering hero who has solved QCD exactly."

$$\frac{\Lambda_{\overline{MS}}}{m_{\rho}} = 0.455 \pm 0.031$$

### Fundamental Hadronic Features of Hadrons

Virial Theorem Partition of the Proton's Mass: Potential vs. Kinetic Contributions Color Confinement  $U(\zeta^2) = \kappa^4 \zeta^2$   $\begin{aligned} \Delta \mathcal{M}^2_{LFKE} &= \kappa^2 (1+2n+L) \\ \Delta \mathcal{M}^2_{LFPE} &= \kappa^2 (1+2n+L) \end{aligned}$ Role of Quark Orbital Angular Momentum in the Proton Equal L=0, I Quark-Diquark Structure Quark Mass Contribution  $\Delta M^2 = < rac{m_q^2}{r} > from the Yukawa coupling to the Higgs zero mode$ Baryonic Regge Trajectory  $M_{\rm p}^2(n, L_B) = 4\kappa^2(n + L_B + 1)$ Mesonic Supersymmetric Partners  $L_M = L_R + 1$ Proton Light-Front Wavefunctions and Dynamical Observables  $\psi_M(x,k_{\perp}) = \frac{4\pi}{\kappa_{\sqrt{x(1-x)}}}e^{-\frac{k_{\perp}^2}{2\kappa^2x(1-x)}}$ Form Factors, Distribution Amplitudes, Structure Functions Non-Perturbative - Perturbative OCD Transition  $Q_0 = 0.87 \pm 0.08~GeV~\overline{MS}~scheme$  $m_p \simeq 3.21 \ \Lambda_{\overline{MS}}$  $m_{
ho} \simeq 2.2 \ \Lambda_{\overline{MS}}$ Dimensional Transmutation: Supersymmetric Features of QCD Genoa **Stan Brodsky** INFN Feb 8, 2017 from LF Holography Istituto Nazional

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## Connection to the Linear Instant-Form Potential



#### A.P.Trawinski, S.D. Glazek, H. D. Dosch, G. de Teramond, sjb

### Connection to the Linear Instant-Form Potential

• Compare invariant mass in the instant-form in the hadron center-of-mass system  ${f P}=0,$ 

$$M_{q\overline{q}}^2 = 4\,m_q^2 + 4\mathbf{p}^2$$

with the invariant mass in the front-form in the constituent rest frame,  ${f k}_q+{f k}_{\overline{q}}=0$ 

$$M_{q\overline{q}}^2 = \frac{\mathbf{k}_{\perp}^2 + m_q^2}{x(1-x)}$$

obtain

$$U = V^2 + 2\sqrt{\mathbf{p}^2 + m_q^2} \, V + 2 \, V \sqrt{\mathbf{p}^2 + m_q^2}$$

where  $\mathbf{p}_{\perp}^2 = \frac{\mathbf{k}_{\perp}^2}{4x(1-x)}$ ,  $p_3 = \frac{m_q(x-1/2)}{\sqrt{x(1-x)}}$ , and V is the effective potential in the instant-form

• For small quark masses a linear instant-form potential V implies a harmonic front-form potential U and thus linear Regge trajectories

A.P.Trawinski, S.D. Glazek, H. D. Dosch, G. de Teramond, sjb

# Fundamental Hadronic Features of the Proton

Supersymmetric Features of QCD

from LF Holography

- Partition of the Proton's Mass: Potential vs. Kinetic Contributions
- Color Confinement
- Role of Quark Orbital Angular Momentum in the Proton
- Quark-Diquark Structure
- Quark Mass Contribution
- Baryonic Regge Trajectory

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- Mesonic Supersymmetric Partners
- Proton Light-Front Wavefunctions and Dynamical Observables
- Form Factors, Distribution Amplitudes, Structure Functions
- Non-Perturbative Perturbative QCD Transition
- Dimensional Transmutation:  $M_p/\Lambda_{\overline{MS}}$





# Future Directions

- Hadronization at the Amplitude Level: LFWFs
- Running Coupling at all Q<sup>2</sup>
- Factorization Scale for ERBL, DGLAP evolution: Qo
- Calculate Sivers Effect including FSI and ISI
- Eliminate renormalizations scale ambiguity: PMC
- Compute Tetraquark Spectroscopy: Sequential Clusters
- Update SU(6) spin-flavor symmetry
- Heavy Quark States: Supersymmetry, not conformal
- Compute higher Fock states; e.g. Intrinsic Heavy Quarks
- Nuclear States Hidden Color
- Basis LF Quantization

de Tèramond, Dosch, Wu, Vary, sjb

Remarkable símílarítíes wíth DSE approach of Roberts et al.



- Flavor-Dependent Anti-Shadowing
- LFVacuum and Cosmological Constant: No QCD condensates
- Principle of Maximum Conformality (PMC): Eliminate renormalization anomaly; scheme independent
- Match Perturbative and Non-Perturbative Domains
- Hadronization at Amplitude Level
- Intrinsic Heavy Quarks from AdS/QCD: Higgs at high x<sub>F</sub>
- Ridge from flux tube collisions
- Baryon-to-meson anomaly at high pT



Supersymmetric Features of QCD from LF Holography







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"One of the gravest puzzles of theoretical physics"

DARK ENERGY AND THE COSMOLOGICAL CONSTANT PARADOX

A. ZEE

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$$(\Omega_{\Lambda})_{QCD} \sim 10^{45}$$
  

$$(\Omega_{\Lambda})_{EW} \sim 10^{56}$$
  

$$\Omega_{\Lambda} = 0.76(expt)$$

**Extraordinary conflict between the conventional definition of the vacuum in** quantum field theory and cosmology

Elements of the solution: (A) Light-Front Quantization: causal, frame-independent vacuum (B) New understanding of QCD "Condensates" (C) Higgs Light-Front Zero Mode

## Light-Front vacuum can símulate empty universe

#### Shrock, Tandy, Roberts, sjb

- Independent of observer frame
- Causal
- Lowest invariant mass state M= o.
- Trivial up to k+=o zero modes-- already normal-ordering
- Higgs theory consistent with trivial LF vacuum (Srivastava, sjb)
- QCD and AdS/QCD: "In-hadron"condensates (Maris, Tandy Roberts) -- GMOR satisfied.
- QED vacuum; no loops
- Zero cosmological constant from QED, QCD, EW



Supersymmetric Features of QCD from LF Holography







- Test QCD to maximum precision at the LHC
- Maximize sensitivity to new physics
- High precision determination of fundamental parameters
- Determine renormalizations scales without ambiguity
- Eliminate scheme dependence

Predictions for physical observables cannot depend on theoretical conventions such as the renormalization scheme

# Myths concerning scale setting

- Renormalization scale "unphysical": No optimal physical scale
- Can ignore possibility of multiple physical scales
- Accuracy of PQCD prediction can be judged by taking arbitrary guess  $\mu_R = Q$  with an arbitrary range  $Q/2 < \mu_R < 2Q$
- Factorization scale should be taken equal to renormalization scale  $\mu_F = \mu_R$

These assumptions are untrue in QED and thus they cannot be true for QCD

**Clearly heuristic. Wrong in QED. Scheme dependent!** 

Electron-Electron Scattering in QED





#### **Gell-Mann--Low Effective Charge**

## Electron-Electron Scattering in QED

$$\mathcal{M}_{ee \to ee}(++;++) = \frac{8\pi s}{t} \alpha(t) + \frac{8\pi s}{u} \alpha(u)$$

- Two separate physical scales: t, u = photon virtuality
- Gauge Invariant. Dressed photon propagator
- Sums all vacuum polarization, non-zero beta terms into running coupling. This is the purpose of the running coupling!



- If one chooses a different initial scale, one must sum an infinite number of graphs -- but always recover same result!
- Number of active leptons correctly set
- Analytic: reproduces correct behavior at lepton mass thresholds
- No renormalization scale ambiguity!

# Lessons from QED

In the (physical) Gell Mann-Low scheme, the momentum scale of the running coupling is the virtuality of the exchanged photon; independent of initial scale.

$$\alpha(t) = \frac{\alpha(t_0)}{1 - \Pi(t, t_0)} \qquad \Pi(t, t_0) = \frac{\Pi(t) - \Pi(t_0)}{1 - \Pi(t_0)}$$



For any other scale choice an infinite set of diagrams must be taken into account to obtain the correct result!

In any other scheme, the correct scale displacement must be used

$$\log \frac{\mu_{\overline{MS}}^2}{m_{\ell}^2} = 6 \int_0^1 dx \, x(1-x) \log \frac{m_{\ell}^2 + Q^2 x(1-x)}{m_{\ell}^2}, \quad Q^2 \gg m_{\ell}^2 \log \frac{Q^2}{m_{\ell}^2} - \frac{5}{3}$$
$$\alpha_{\overline{MS}}(e^{-5/3}q^2) = \alpha_{GM-L}(q^2).$$

#### S

#### Systematic All-Orders Method to Eliminate Renormalization-Scale and Scheme Ambiguities in Perturbative QCD

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We introduce a generalization of the conventional renormalization schemes used in dimensional regularization, which illuminates the renormalization scheme and scale ambiguities of perturbative QCD predictions, exposes the general pattern of nonconformal  $\{\beta_i\}$  terms, and reveals a special degeneracy of the terms in the perturbative coefficients. It allows us to systematically determine the argument of the running coupling order by order in perturbative QCD in a form which can be readily automatized. The new method satisfies all of the principles of the renormalization group and eliminates an unnecessary source of systematic error.



Supersymmetric Features of QCD from LF Holography





## $\delta$ -Renormalization Scheme ( $\mathcal{R}_{\delta}$ scheme)

In dim. reg.  $1/\epsilon$  poles come in powers of [Bollini & Gambiagi, 't Hooft & Veltman, '72]

$$\ln\frac{\mu^2}{\Lambda^2} + \frac{1}{\epsilon} + c$$

In the modified minimal subtraction scheme (MS-bar) one subtracts together with the pole a constant [Bardeen, Buras, Duke, Muta (1978) on DIS results]:

$$\ln(4\pi) - \gamma_E$$

This corresponds to a shift in the scale:

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$$\mu_{\overline{\mathrm{MS}}}^2 = \mu^2 \exp(\ln 4\pi - \gamma_E)$$

A finite subtraction from infinity is arbitrary. Let's make use of this!

Subtract an arbitrary constant and keep it in your calculation:  $\mathcal{R}_{\delta}$ -scheme **M. Mojaza, Xing-Gang Wu, sjb**   $\ln(4\pi) - \gamma_E - \delta,$   $\mu_{\delta}^2 = \mu_{\overline{MS}}^2 \exp(-\delta) = \mu^2 \exp(\ln 4\pi - \gamma_E - \delta)$  **Genoa Feb 8, 2017 Genoa Feb 8, 2017 Genoa Feb 8, 2017 Supersymmetric Features of QCD from LF Holography Stan Brodsky** 



# Exposing the Renormalization Scheme Dependence

### Observable in the $\mathcal{R}_{\delta}$ -scheme:

 $\rho_{\delta}(Q^2) = r_0 + r_1 a(\mu) + [r_2 + \beta_0 r_1 \delta] a(\mu)^2 + [r_3 + \beta_1 r_1 \delta + 2\beta_0 r_2 \delta + \beta_0^2 r_1 \delta^2] a(\mu)^3 + \cdots$ 

 $\mathcal{R}_0 = \overline{\mathrm{MS}}$ ,  $\mathcal{R}_{\ln 4\pi - \gamma_E} = \mathrm{MS}$   $\mu^2 = \mu_{\overline{\mathrm{MS}}}^2 \exp(\ln 4\pi - \gamma_E)$ ,  $\mu_{\delta_2}^2 = \mu_{\delta_1}^2 \exp(\delta_2 - \delta_1)$ 

Note the divergent 'renormalon series'  $n!\beta^n \alpha_s^n$ 

Renormalization Scheme Equation

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$$\frac{d\rho}{d\delta} = -\beta(a)\frac{d\rho}{da} \stackrel{!}{=} 0 \longrightarrow \text{PMC}$$

$$\begin{split} \rho_{\delta}(Q^2) = & r_0 + r_1 a_1(\mu_1) + (r_2 + \beta_0 r_1 \delta_1) a_2(\mu_2)^2 + [r_3 + \beta_1 r_1 \delta_1 + 2\beta_0 r_2 \delta_2 + \beta_0^2 r_1 \delta_1^2] a_3(\mu_3)^3 \\ \text{The } \delta_k^p a^n \text{-term indicates the term associated to a diagram with } 1/\epsilon^{n-k} \text{ divergence for any } p. \text{ Grouping the different } \delta_k \text{-terms, one recovers in the } N_c \to 0 \\ \text{Abelian limit the dressed skeleton expansion.} \end{split}$$

Supersymmetric Features of QCD from LF Holography





## Special Degeneracy in PQCD

There is nothing special about a particular value for  $~\delta$  , thus for any  $\delta$ 

$$\begin{split} \rho(Q^2) = & r_{0,0} + r_{1,0}a(Q) + [r_{2,0} + \beta_0 r_{2,1}]a(Q)^2 + [r_{3,0} + \beta_1 r_{2,1} + 2\beta_0 r_{3,1} + \beta_0^2 r_{3,2}]a(Q)^3 \\ &+ [r_{4,0} + \beta_2 r_{2,1} + 2\beta_1 r_{3,1} + \frac{5}{2}\beta_1 \beta_0 r_{3,2} + 3\beta_0 r_{4,1} + 3\beta_0^2 r_{4,2} + \beta_0^3 r_{4,3}]a(Q)^4 \end{split}$$

According to the principal of maximum conformality we must set the scales such to absorb all 'renormalon-terms', i.e. non-conformal terms

$$\rho(Q^{2}) = r_{0,0} + r_{1,0}a(Q) + (\beta_{0}a(Q)^{2} + \beta_{1}a(Q)^{3} + \beta_{2}a(Q)^{4} + \cdots)r_{2,1} + (\beta_{0}^{2}a(Q)^{3} + \frac{5}{2}\beta_{1}\beta_{0}a(Q)^{4} + \cdots)r_{3,2} + (\beta_{0}^{3} + \cdots)r_{4,3} + r_{2,0}a(Q)^{2} + 2a(Q)(\beta_{0}a(Q)^{2} + \beta_{1}a(Q)^{3} + \cdots)r_{3,1} + \cdots + \cdots + r_{1,0}a(Q_{1}) = r_{1,0}a(Q) - \beta(a)r_{2,1} + \frac{1}{2}\beta(a)\frac{\partial\beta}{\partial a}r_{3,2} + \cdots + \frac{(-1)^{n}}{n!}\frac{d^{n-1}\beta}{(d\ln\mu^{2})^{n-1}}r_{n+1,n} + r_{2,0}a(Q_{2})^{2} = r_{2,0}a(Q)^{2} - 2a(Q)\beta(a)r_{3,1} + \cdots + r_{2,0}a(Q)^{2} + r_{2,0}a(Q)^{2} + r_{2,0}a(Q)\beta(a)r_{3,1} + \cdots + r_{2,0}a(Q)\beta(a)r_{3,1} + \cdots + r_{2,0}a(Q)\beta(a)r_{3,1} + \cdots + r_{2,0}a(Q)\beta(a)r_{3,1} + r_{2,0}a(Q)\beta(a)r_{3,1} + \cdots + r_{2,0}a(Q)\beta(a)r_{3,1} + \cdots + r_{2,0}a(Q)\beta(a)r_{3,1} + \cdots + r_{2,0}a(Q)\beta(a)r_{3,1} + r_{2,0}a(Q)\beta(a)r_{3,1} + \cdots + r_{2,0}a(Q)\beta(a)r_{3,1} + r_{2,0}a(Q)\beta(a)r_{3,1}a(Q)\beta(a)r_{3,1} + r_{2,0}a(Q)\beta(a)r_{3,1}a(Q)\beta(a)r_{3,1}a(Q)\beta(a)r_{3,1}a(Q)\beta(a)r_{3,1}a(Q)\beta(a)r_{3,1}a(Q)\beta(a)r_{3,1}a(Q)\beta(a)r_{3,1}a(Q)\beta(a)r_{3,1}a(Q)\beta(a)r_{3,1}a(Q)\beta(a)r_{3,1}a(Q)\beta(a)r_{3,1}a(Q)\beta(a)r_{3,1}a(Q)\beta(a)r_{3,1}a(Q)\beta(a)r_{3,1}a(Q)\beta(a)r_{3,1}a(Q)\beta(a)r_{3,1}a(Q)\beta(a)r_{3,1}a(Q$$

#### M. Mojaza, Xing-Gang Wu, sjb

General result for an observable in any  $\mathcal{R}_{\delta}$  renormalization scheme:

$$\begin{split} \rho(Q^2) = & r_{0,0} + r_{1,0}a(Q) + [r_{2,0} + \beta_0 r_{2,1}]a(Q)^2 \\ &+ [r_{3,0} + \beta_1 r_{2,1} + 2\beta_0 r_{3,1} + \beta_0^2 r_{3,2}]a(Q)^3 \\ &+ [r_{4,0} + \beta_2 r_{2,1} + 2\beta_1 r_{3,1} + \frac{5}{2}\beta_1 \beta_0 r_{3,2} + 3\beta_0 r_{4,1} \\ &+ 3\beta_0^2 r_{4,2} + \beta_0^3 r_{4,3}]a(Q)^4 + \mathcal{O}(a^5) \end{split}$$

## **PMC** scales thus satisfy

$$r_{1,0}a(Q_{1}) = r_{1,0}a(Q) - \beta(a)r_{2,1}$$

$$r_{2,0}a(Q_{2})^{2} = r_{2,0}a(Q)^{2} - 2a(Q)\beta(a)r_{3,1}$$

$$r_{3,0}a(Q_{2})^{3} = r_{3,0}a(Q)^{3} - 3a(Q)^{2}\beta(a)r_{4,1}$$

$$\vdots$$

$$r_{k,0}a(Q_k)^k = r_{k,0}a(Q)^2 - k \ a(Q)^{k-1}\beta(a)r_{k+1,1}$$

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Feb 8, 2017

Supersymmetric Features of QCD from LF Holography





### Important Example: Top-Quark FB Asymmetry

Brodsky, Wu, Phys.Rev.Lett. 109, [arXiv:1203.5312]



Implications for the  $\bar{p}p \rightarrow ttX$  asymmetry at the Tevatron



#### Interferes with Born term.

Small value of renormalization scale increases asymmetry, just as in QED

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Supersymmetric Features of QCD from LF Holography

Xing-Gang Wu, sjb





The Renormalization Scale Ambiguity for Top-Pair Production Eliminated Using the 'Principle of Maximum Conformality' (PMC)



Top quark forward-backward asymmetry predicted by pQCD NNLO within 1  $_{\rm 0}$  of CDF/D0 measurements using PMC/BLM scale setting



Predictions for the cumulative front-back asymmetry.

# Reanalysis of the Higher Order Perturbative QCD corrections to Hadronic Z Decays using the Principle of Maximum Conformality

S-Q Wang, X-G Wu, sjb

P.A. Baikov, K.G. Chetyrkin, J.H. Kuhn, and J. Rittinger, Phys. Rev. Lett. 108, 222003 (2012).



The values of  $r_{\text{NS}}^{(n)} = 1 + \sum_{i=1}^{n} C_i^{\text{NS}} a_s^i$  and their errors  $\pm |C_n^{\text{NS}} a_s^n|_{\text{MAX}}$ . The diamonds and the crosses are for conventional (Conv.) and PMC scale settings, respectively. The central values assume the initial scale choice  $\mu_r^{\text{init}} = M_Z$ .

#### Set multiple renormalization scales --Lensing, DGLAP, ERBL Evolution ...



# Features of BLM/PMC

- Predictions are scheme-independent
- Matches conformal series
- Commensurate Scale Relations between observables: Generalized Crewther Relation (Kataev, Lu, Rathsman, sjb)
- No n! Renormalon growth
- New scale at each order; n<sub>F</sub> determined at each order
- Multiple Physical Scales Incorporated
- Rigorous: Satisfies all Renormalization Group Principles
- Realistic Estimate of Higher-Order Terms
- Eliminates unnecessary theory error

# Elímination of QCD Scale Ambiguíties

The Principle of Maximum Conformality (PMC)

Applications of PMC renormalization-scale-setting for top, Higgs production, and other processes at the LHC


S-Q Wang, X-G Wu, sjb



 $\sigma_{
m fid}(p$ 

20

10

0



Comparison of the PMC predictions for the fiducial cross section  $\sigma_{\rm fid}(pp \rightarrow H \rightarrow \gamma \gamma)$  with the ATLAS measurements at various collision energies. The LHC-XS predictions are presented as a comparison.

$\sigma_{\rm fid}(pp \to H \to \gamma\gamma)$	$7 { m TeV}$	$8 { m TeV}$	$13 { m TeV}$
ATLAS data [48]	$49\pm18$	$42.5^{+10.3}_{-10.2}$	$52^{+40}_{-37}$
LHC-XS $[3]$	$24.7\pm2.6$	$31.0\pm3.2$	$66.1_{-6.6}^{+6.8}$
PMC prediction	$30.1^{+2.3}_{-2.2}$	$38.4^{+2.9}_{-2.8}$	$85.8^{+5.7}_{-5.3}$



Collins, Ellis, Gunion, Mueller, sjb Polyakov, et al. Hoyer, Vogt, et al



Two Components (separate evolution):

 $c(x, Q^2) = c(x, Q^2)_{\text{extrinsic}} + c(x, Q^2)_{\text{intrinsic}}$ 

week ending 15 MAY 2009



Consistent with EMC measurement of charm structure function at high x

Goldhaber, Kopeliovich, Schmidt, Soffer sjb

Intrínsic Charm Mechanism for Inclusive High-X<sub>F</sub> Higgs Production



Also: intrinsic strangeness, bottom, top

**Higgs can have > 80% of Proton Momentum!** New production mechanism for Higgs

QCD Myths

- Anti-Shadowing is Universal: Nuclear PDF Sum Rules!
- ISI and FSI are higher twist effects and universal
- High transverse momentum hadrons arise only from jet fragmentation -- baryon anomaly!
- heavy quarks only from gluon splitting
- renormalization scale cannot be fixed
- QCD condensates are vacuum effects
- Infrared Slavery
- Nuclei are composites of nucleons only
- Real part of DVCS arbitrary



Supersymmetric Features of QCD from LF Holography





Supersymmetric Meson-Baryon Properties of QCD from Light-Front Holography and Superconformal Algebra









with Guy de Tèramo C. Lorce, K. Chiu

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