

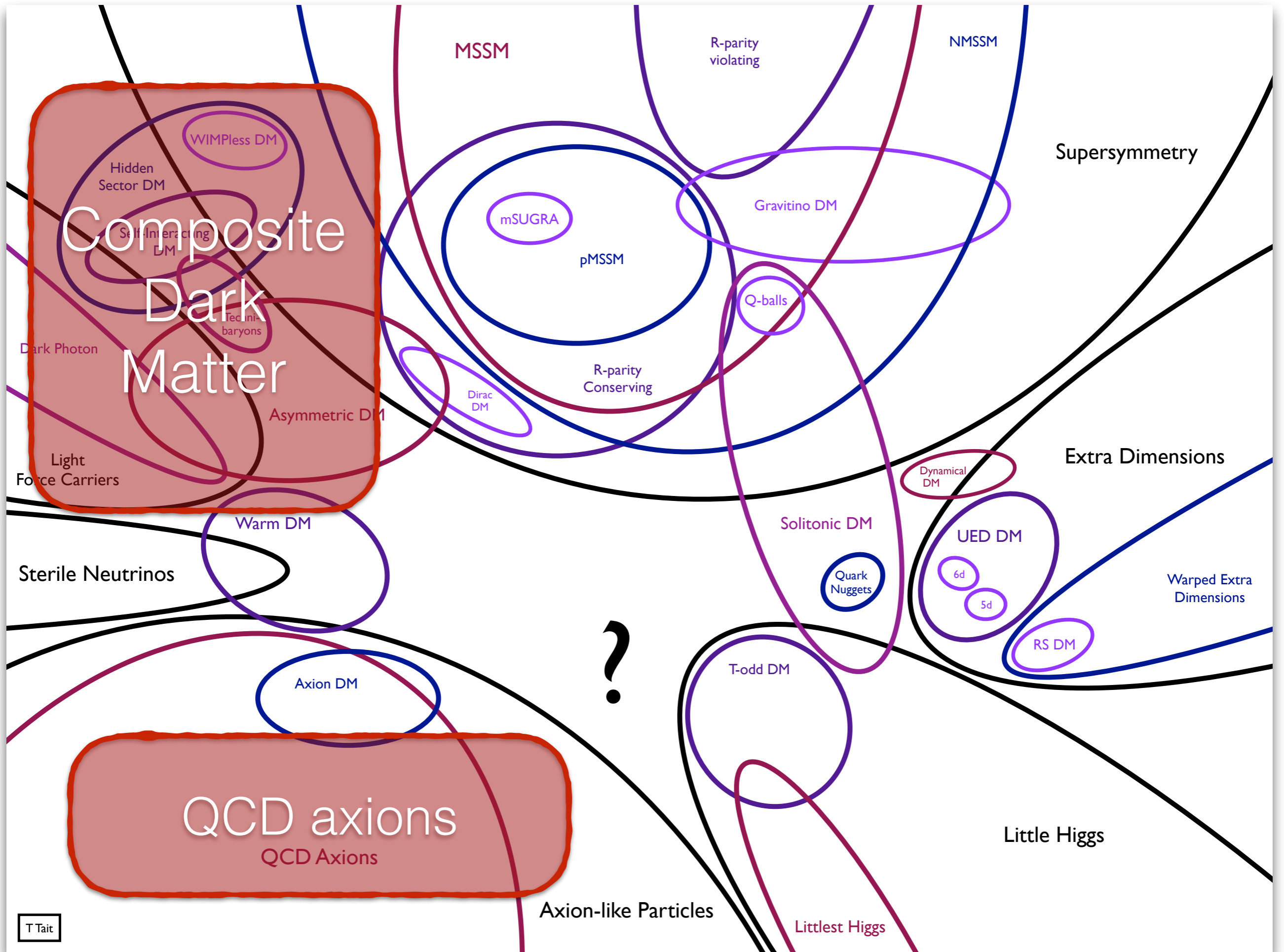


What lattice gauge theories can do for Dark Matter searches

Enrico Rinaldi



RIKEN BNL Research Center



Composite Dark Matter

QCD axions

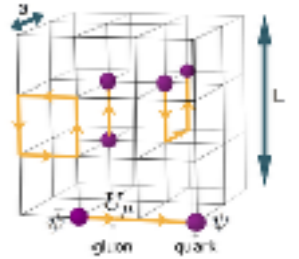
QCD Axions

WIMPless DM
 Hidden Sector DM
 Self-Interacting DM
 Technibaryons
 Dark Photon
 Light Force Carriers
 Asymmetric DM

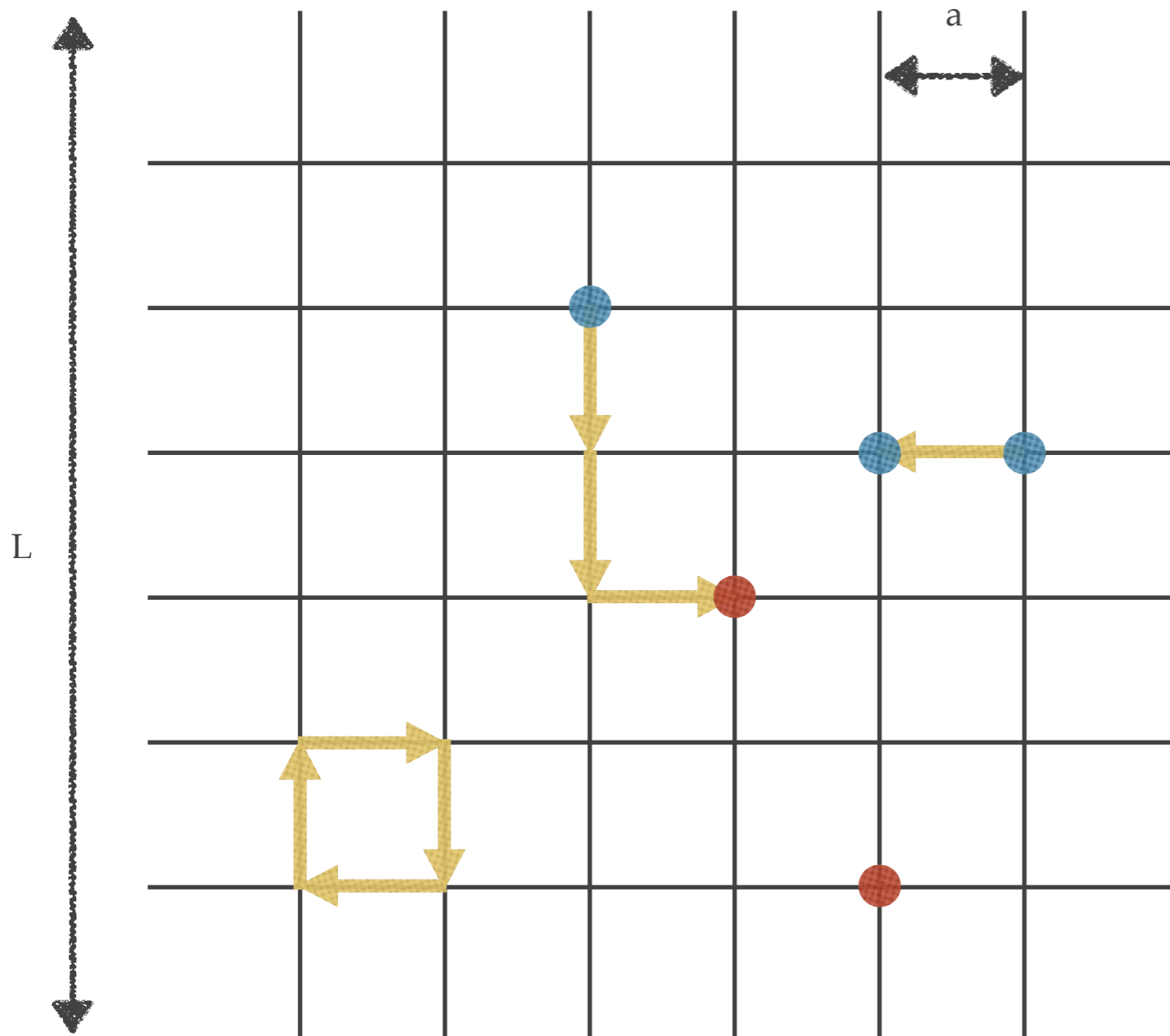
Warm DM
 Sterile Neutrinos
 Axion DM

MSSM
 R-parity violating
 NMSSM
 Supersymmetry
 Gravitino DM
 mSUGRA
 pMSSM
 R-parity Conserving
 Dirac DM
 Q-balls
 Solitonic DM
 Quark Nuggets
 UED DM
 6d
 5d
 RS DM
 Dynamical DM
 Extra Dimensions
 Warped Extra Dimensions
 T-odd DM
 Littlest Higgs
 Axion-like Particles

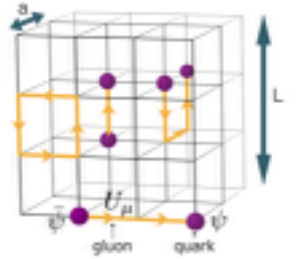
?



Lattice Gauge Theory - basics

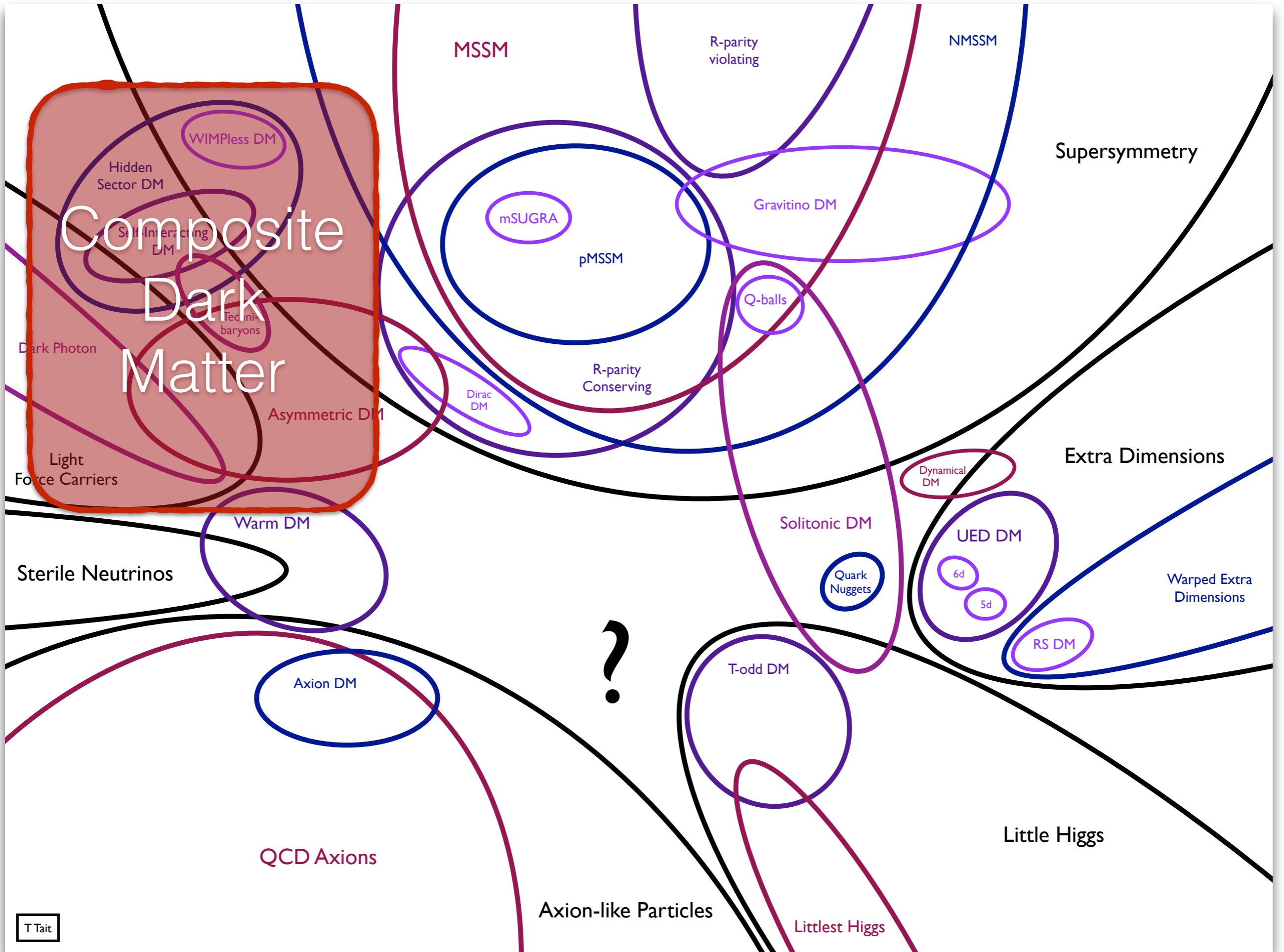


- Discretize space and time
 - lattice spacing “a”
 - lattice size “L”
- Keep all d.o.f. of the theory
 - not a model!
 - no simplifications
- Amenable to numerical methods
 - Monte Carlo sampling
 - use supercomputers
- Precisely quantifiable and improvable errors
 - Systematic
 - Statistical



Importance of lattice field theory simulations

- ◆ *lattice simulations are needed to numerically solve strong dynamics*
- ◆ controllable systematic errors and room for improvement
- ◆ Naive dimensional analysis and EFT approaches can miss important non-perturbative contributions
- ◆ NDA is not precise enough when confronting experimental results and might not work for certain situations: there are uncontrolled theoretical errors

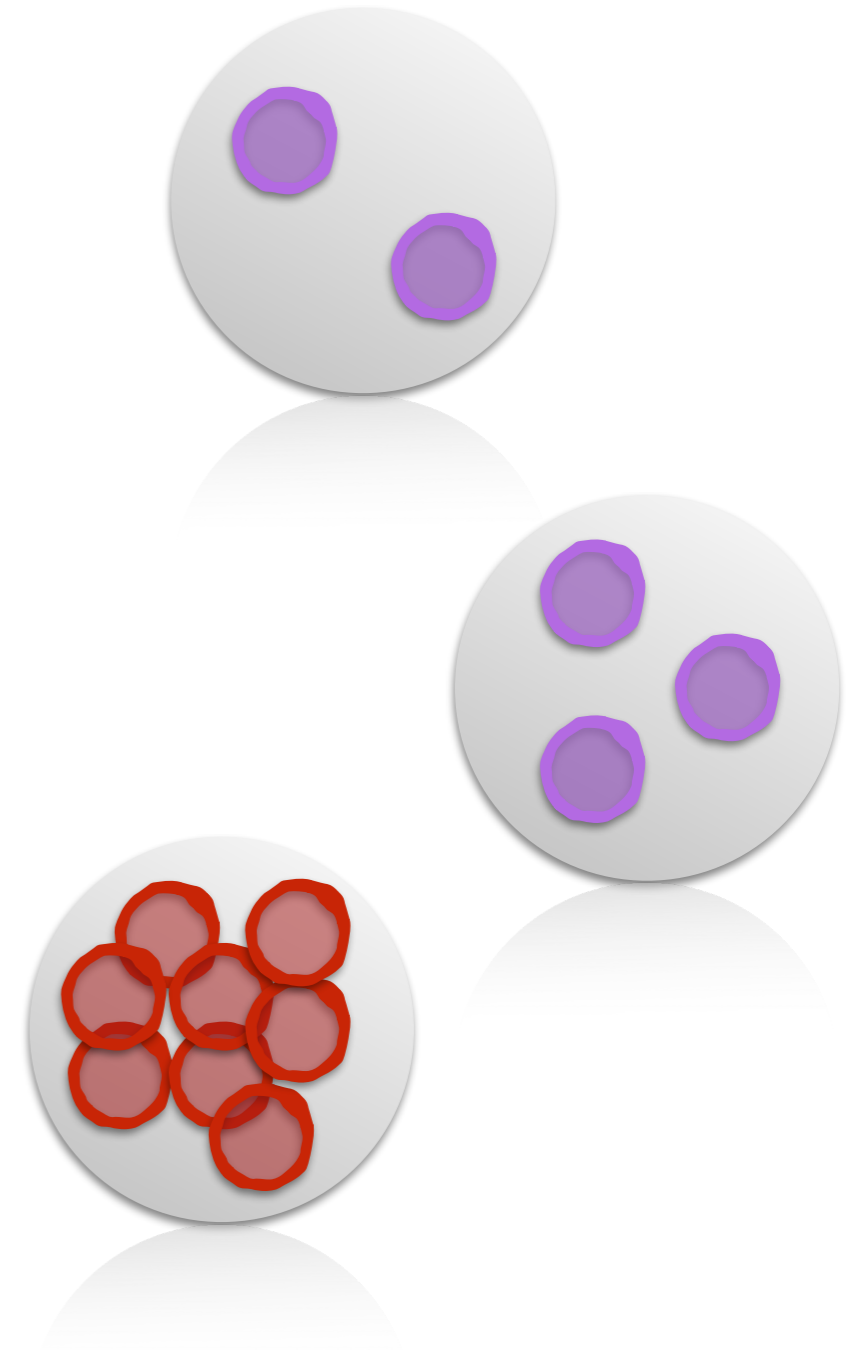


A very familiar picture

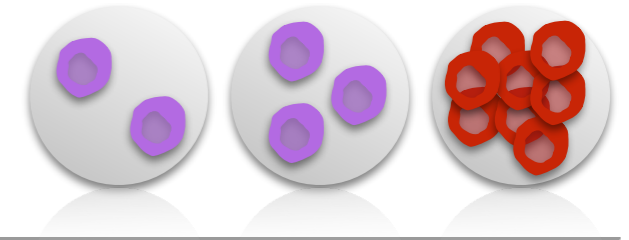
The Standard Model of particles

	mass →	charge →	spin →					
QUARKS	$\approx 2.3 \text{ MeV}/c^2$	$2/3$	$1/2$	u up	$\approx 1.275 \text{ GeV}/c^2$	$2/3$	$1/2$	c charm
					$\approx 173.07 \text{ GeV}/c^2$	$2/3$	$1/2$	t top
					0	0	1	g gluon
								H Higgs boson
	$\approx 4.8 \text{ MeV}/c^2$	$-1/3$	$1/2$	d down	$\approx 95 \text{ MeV}/c^2$	$-1/3$	$1/2$	s strange
					$\approx 4.18 \text{ GeV}/c^2$	$-1/3$	$1/2$	b bottom
					0	0	1	γ photon
LEPTONS	$0.511 \text{ MeV}/c^2$	-1	$1/2$	e electron	$105.7 \text{ MeV}/c^2$	-1	$1/2$	μ muon
					$1.777 \text{ GeV}/c^2$	-1	$1/2$	τ tau
					$91.2 \text{ GeV}/c^2$	0	1	Z Z boson
	$< 2.2 \text{ eV}/c^2$	0	$1/2$	ν_e electron neutrino	$< 0.17 \text{ MeV}/c^2$	0	$1/2$	ν_μ muon neutrino
					$< 15.5 \text{ MeV}/c^2$	0	$1/2$	ν_τ tau neutrino
					$80.4 \text{ GeV}/c^2$	± 1	1	W W boson

Mesons, Baryons and Glueballs



Composite Dark Matter



- ◆ Dark Matter is a **composite** object

e.g. **techni-baryon** or
hidden **glueball**

- ◆ Interesting and complicated internal **structure**

Lattice Field Theory methods

- ◆ Properties dictated by **strong dynamics**

Similar to **QCD**

- ◆ **Self-interactions** are natural

- ◆ Composite object is **neutral**

Chance to **observe them**
in experiments and give the
correct **relic abundance**

- ◆ Constituents **interact with Standard Model**
particles

Natural features of Composite Dark Matter

Stability is a direct consequence of accidental **symmetries**

Neutrality follows naturally from **confinement** into singlet objects wrt. SM charges

Small **interactions** with SM particles arise from form factor **suppression** (higher dim. operators)

Self-interactions are included due to **strongly coupled** dynamics

Models for Composite Dark Matter

★ Pion-like (dark quark-antiquark)

- ◆ pNGB DM [Hietanen et al., 1308.4130]
- ◆ Quirky DM [Kribs et al., 0909.2034]
- ◆ Ectocolor DM [Buckley&Neil, 1209.6054]
- ◆ SIMP [Hochberg et al., 1411.3727]
- ◆ Minimal SU(2) [Lewis, 1610.10068]

★ Glueball-like (only gluons)

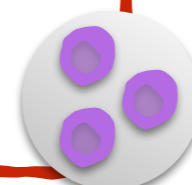
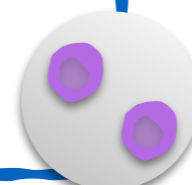
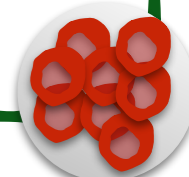
- ◆ SUNonia [Boddy et al., 1402.3629]
[Soni, 1602.00714]

★ Dark Nuclei [Detmold et al., 1406.2276-1406.4116]

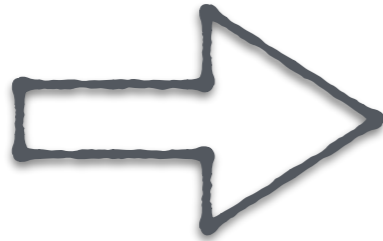
★ Baryon-like (multiple quarks)

- ◆ “Technibaryons” [LSD, 1301.1693]
- ◆ Stealth DM [LSD, 1503.04203-1503.04205]
- ◆ One-family WTC [LatKMI, 1510.07373]
- ◆ Sextet CH [LatHC, 1601.03302]

★ Atoms, Molecules [Cline et al., 1312.3325]

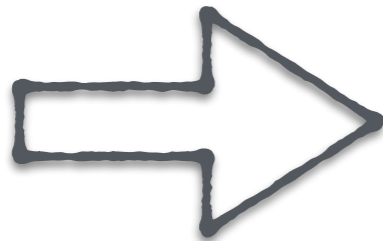


The darkness of Composite Dark Matter



Relevant if the constituents have SM color charges

[Chivukula et al., hep-ph/9210274] [Godbole et al., 1506.01408] [Bay&Osborne, 1506.07110]

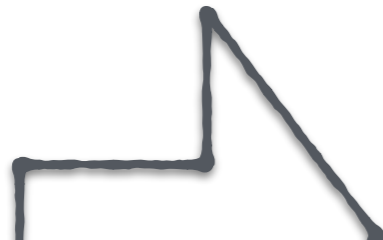


Lowest dimensional operators:

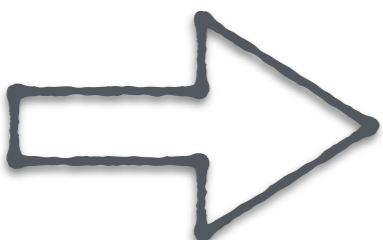
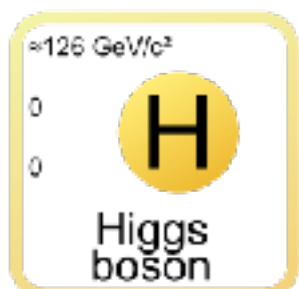
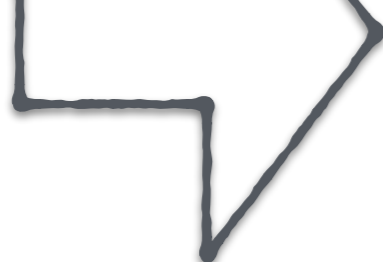
★ magnetic dipole (5)

★ charge radius (6)

★ polarizability (7)



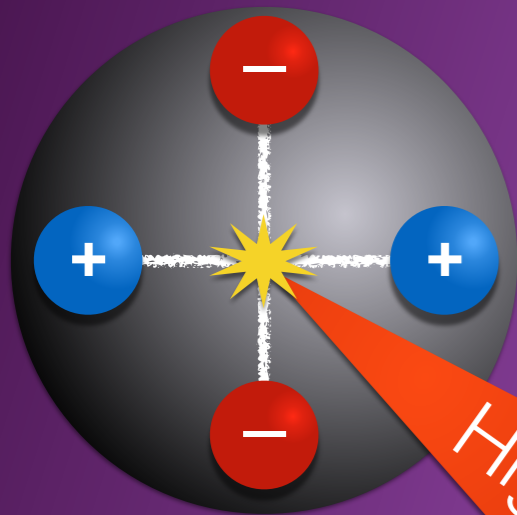
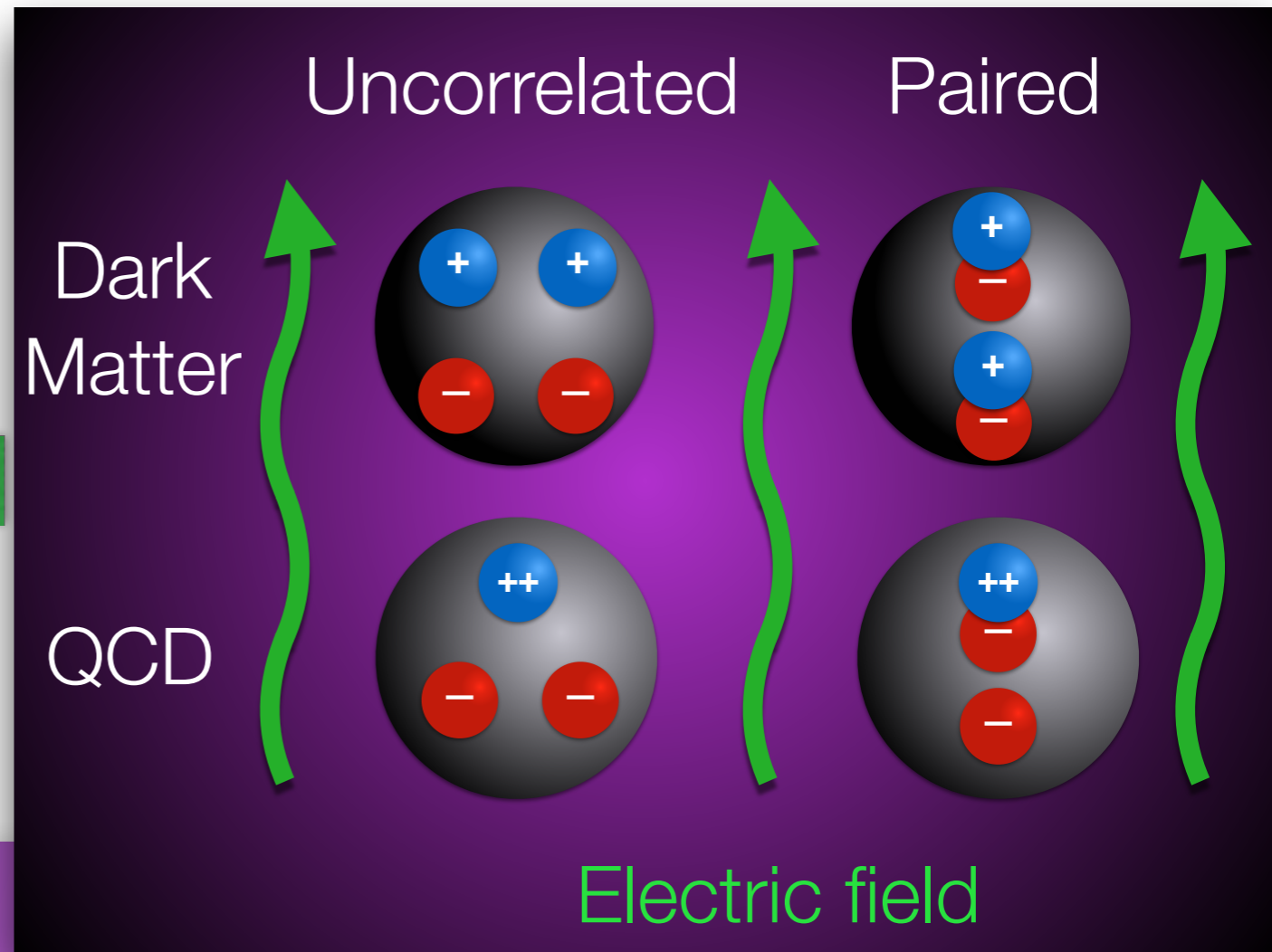
Same as γ but suppressed due to heavy mass



Most relevant interaction if constituents have Yukawa couplings!

PRL Editors' Suggestion: Polarizability

[LSD collab., Phys. Rev. Lett. 115 (2015) 171803]



PRD Editors' Suggestion: Higgs exchange

[LSD collab., Phys. Rev. D92 (2015) 075030]

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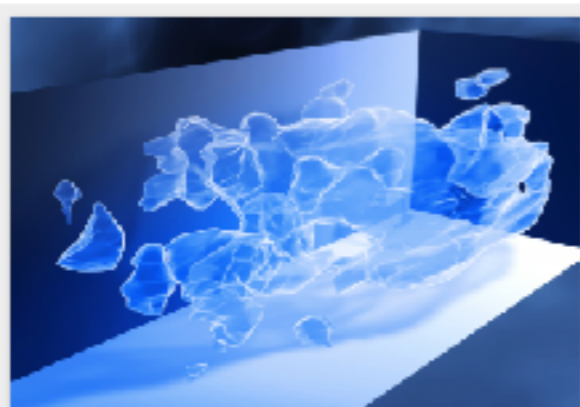
Materia oscura "stealth"

Quark oscuri tenuti insieme da un'interazione forte a sua volta oscura. Ecco come la dark matter riuscirebbe a eludere a ogni tentativo d'incastriarla. Enrico Rinaldi (LLNL): «Esiste la possibilità che questo "mondo oscuro", con le sue nuove particelle, possa essere rivelato dagli esperimenti in corso al Large Hadron Collider al CERN di Ginevra»

di Marco Malaspina [Segui @malaspina](#)

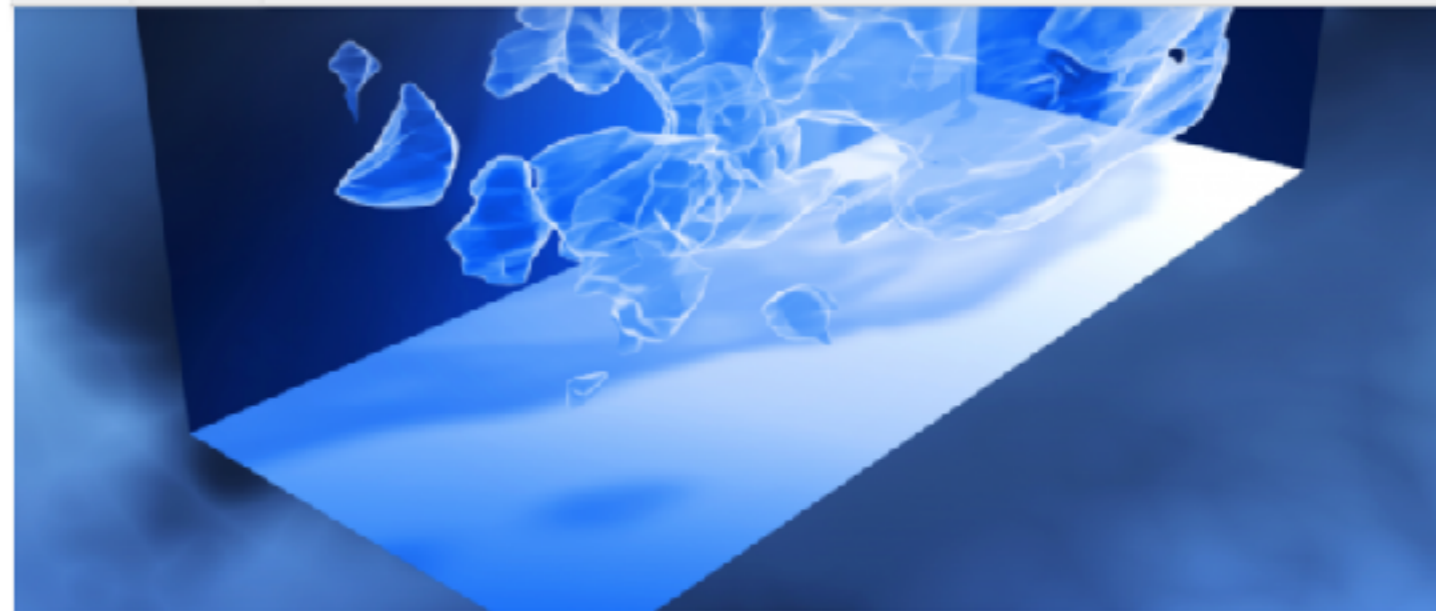
venerdì 25 settembre 2015 @ 16:15

Stealth come furtiva. *Stealth* come imprevedibile. *Stealth* come quei minacciosi aerei da guerra dal profilo sagomato così da essere invisibili al radar. Da quanto emerge dai calcoli dei fisici dell'LLNL, il Lawrence Livermore National Laboratory californiano, e dai modelli dati in pasto a Vulcan (un supercomputer per il calcolo parallelo in grado di masticare numeri al ritmo del *peraflop*), sarebbe questa la natura della materia oscura: *stealthy*, appunto. Per forza non c'è ancora esperimento che sia riuscito a incastrarla.



Mappa 3D della distribuzione su larga scala della materia oscura ricostruita da misure di lente gravitazionale debole utilizzando il telescopio spaziale Hubble

Di cosa è fatta, questa materia della cui



This 3D map illustrates the large-scale distribution of dark matter, reconstructed from measurements of weak gravitational lensing by using the Hubble Space Telescope. [\(Download Image\)](#)

New 'stealth dark matter' theory may explain mystery of the universe's missing mass



Lawrence Livermore National Laboratory (LLNL) scientists have come up with a new theory that may identify why dark matter has evaded direct detection in Earth-based experiments.

Anne M Stark
stark8@llnl.gov
925-422-9799

Detecting Stealth Dark Matter Directly through Electromagnetic Polarizability.

Overview of attention for article published in Physical Review Letters, October 2015



About this score

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Title Detecting Stealth Dark Matter Directly through Electromagnetic Polarizability.
Published In Physical Review Letters, October 2015
DOI 10.1103/physrevlett.115.171803
Pubmed ID 26551103
Authors T. Appelquist, E. Berkowitz, R.C. Brower, M.I. Buchoff, G.T. Fleming, X.-Y. Jin, J. Kiskis, G.D...
Abstract We calculate the spin-independent scattering cross section for direct detection that results from...

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http://www.lescienze.it/news/2015/09/28/news/materia_oscura_stealth_matter_lhc-2779983

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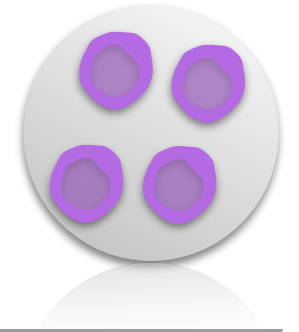
28 settembre 2015

Un nuovo modello per la materia oscura



Questo è il Lawrence Livermore National Laboratory

Questa forma misteriosa di materia potrebbe avere una struttura composita come la materia ordinaria, con "quark oscuri" aggregati e tenuti insieme da un analogo della forza che permette ai normali nuclei di rimanere stabili. I componenti di questo tipo di materia oscura, definita *stealth matter*, potrebbero essere studiati in modo indiretto dal collisore Large Hadron Collider del CERN di Ginevra. *(red)*



“Stealth Dark Matter” Model

- ◆ **New strongly-coupled SU(4) gauge sector** “like” QCD with a **plethora of composite states** in the spectrum: all mass scales are technically natural for hadrons
- ◆ New **Dark fermions**: have **dark color** and also have **electroweak charges** ($W/Z, \gamma$)
- ◆ Dark fermions have **electroweak breaking masses** (Higgs) and **electroweak preserving masses** (not-Higgs)
- ◆ A global symmetry naturally stabilizes the **dark lightest baryonic** composite states (e.g. *dark neutral proton*)

“Stealth Dark Matter” model

EW interactions

- The field content of the model consists in **8 Weyl fermions**
- Dark fermions interact with the SM Higgs and obtain **current/chiral masses**
- Introduce **vector-like masses** for dark fermions that do not break EW symmetry
- Diagonalizing in the mass eigenbasis gives **4 Dirac fermions**
- Assume **custodial SU(2) symmetry** arising when **$u \leftrightarrow d$**

Field	$SU(N)_D$	$(SU(2)_L, Y)$	Q
$F_1 = \begin{pmatrix} F_1^u \\ F_1^d \end{pmatrix}$	\mathbf{N}	$(\mathbf{2}, 0)$	$\begin{pmatrix} +1/2 \\ -1/2 \end{pmatrix}$
$F_2 = \begin{pmatrix} F_2^u \\ F_2^d \end{pmatrix}$	$\overline{\mathbf{N}}$	$(\mathbf{2}, 0)$	$\begin{pmatrix} +1/2 \\ -1/2 \end{pmatrix}$
F_3^u	\mathbf{N}	$(\mathbf{1}, +1/2)$	$+1/2$
F_3^d	\mathbf{N}	$(\mathbf{1}, -1/2)$	$-1/2$
F_4^u	$\overline{\mathbf{N}}$	$(\mathbf{1}, +1/2)$	$+1/2$
F_4^d	$\overline{\mathbf{N}}$	$(\mathbf{1}, -1/2)$	$-1/2$

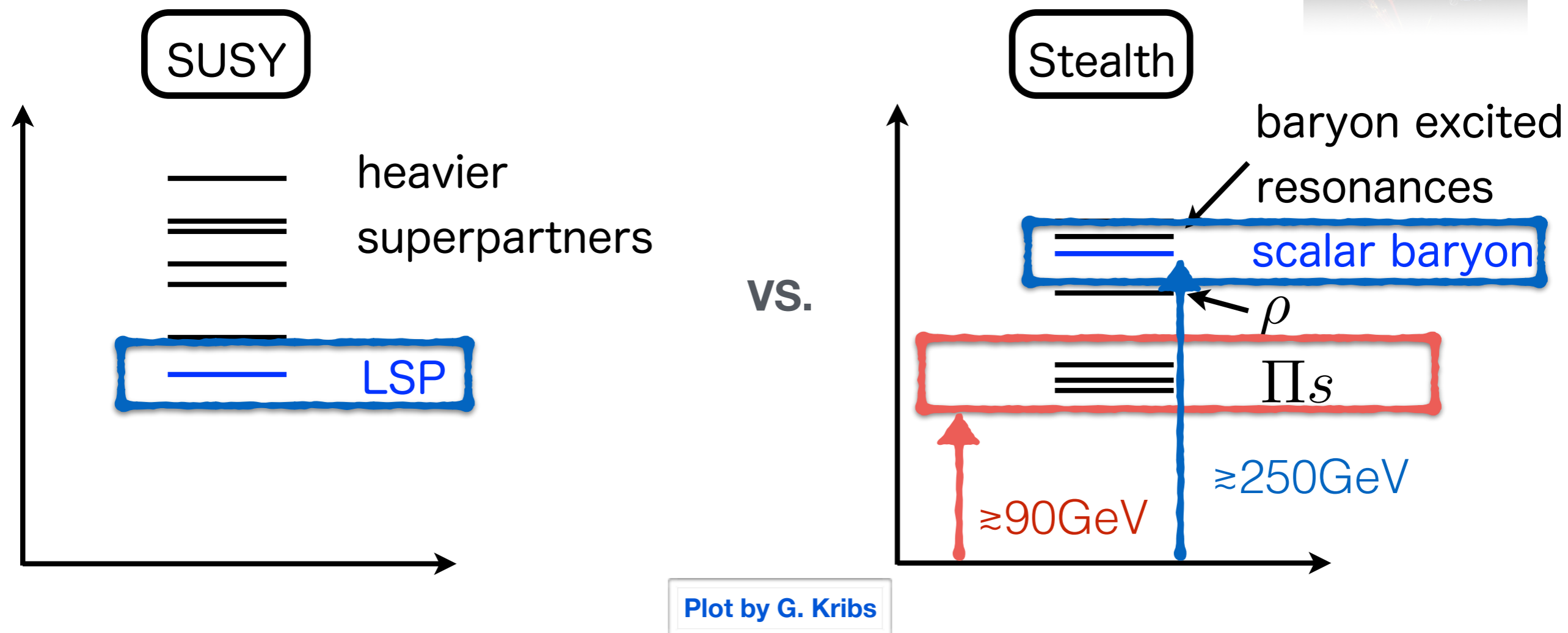
$$\mathcal{L} \supset -y_{14}^u \epsilon_{ij} F_1^i H^j F_4^d + y_{14}^d F_1 \cdot H^\dagger F_4^u - y_{23}^d \epsilon_{ij} F_2^i H^j F_3^d - y_{23}^u F_2 \cdot H^\dagger F_3^u + h.c.$$

$$\mathcal{L} \supset M_{12} \epsilon_{ij} F_1^i F_2^j - M_{34}^u F_3^u F_4^d + M_{34}^d F_3^d F_4^u + h.c.$$

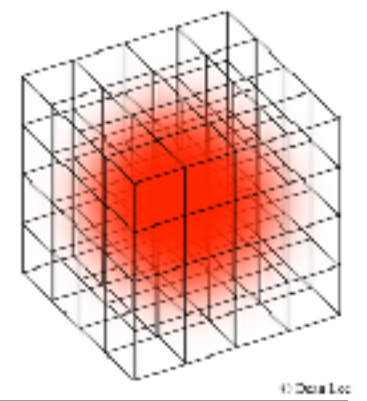
$$y_{14}^u = y_{14}^d \quad y_{23}^u = y_{23}^d \quad M_{34}^u = M_{34}^d$$



Stealth Dark Matter at colliders

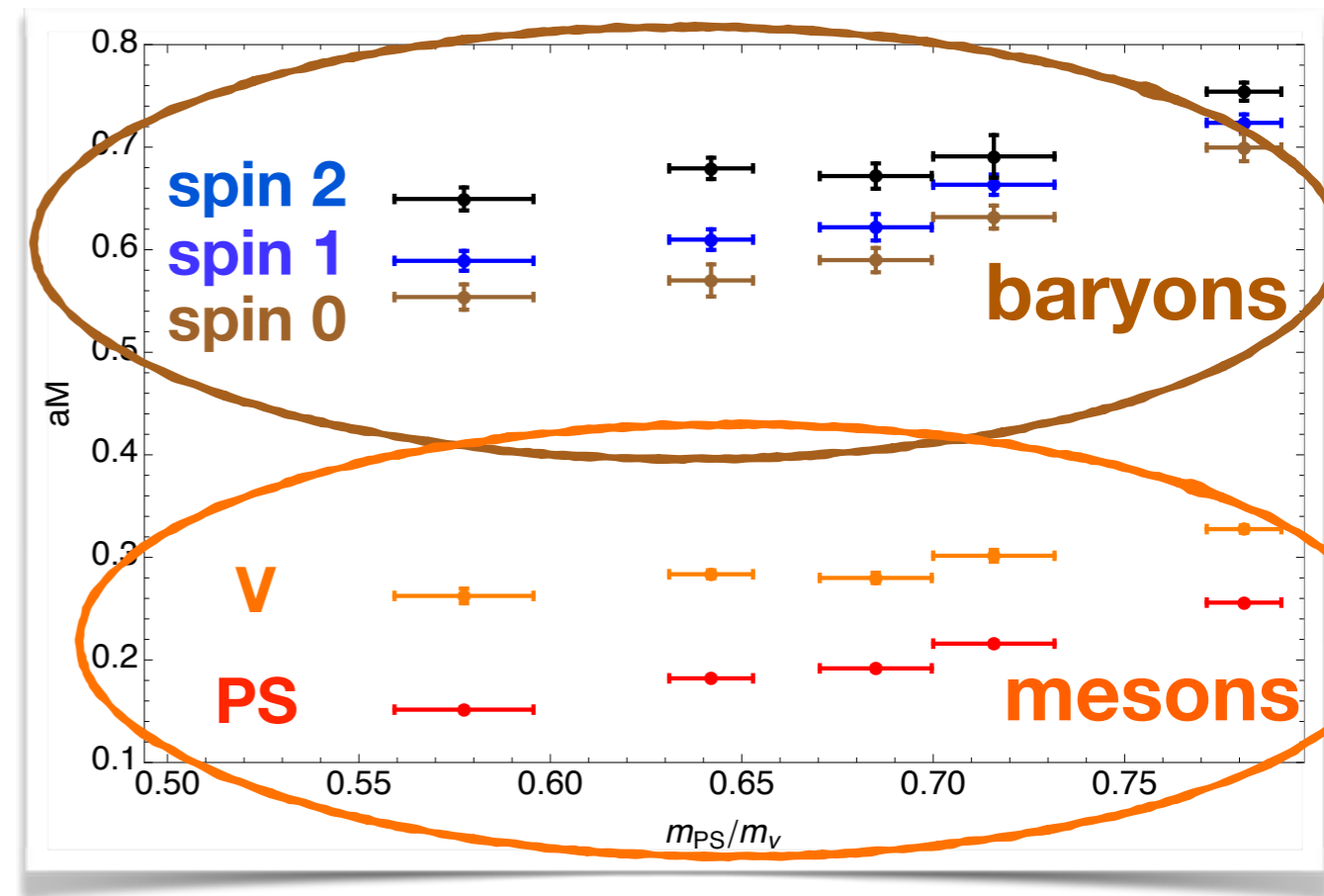


- ◆ Signatures are not dominated by missing energy: **DM is not the lightest particle!** The interactions are suppressed (form factors)
- ◆ Dark mesons production and decay give interesting signatures: **the model can be constrained by collider limits!**



Lattice Stealth Dark Matter

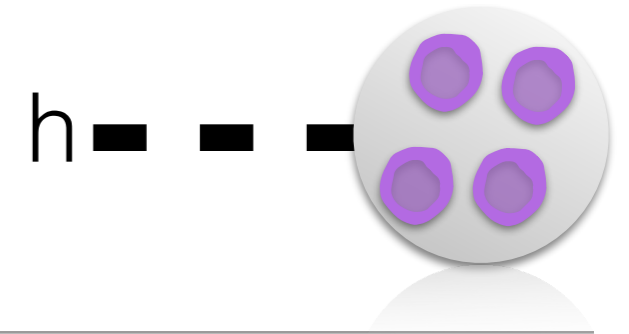
- Non-perturbative lattice calculations of the spectrum confirm that **lightest baryon has spin zero**
- The ratio of **pseudoscalar (PS)** to **vector (V)** is used as probe for different dark fermion masses



- ★ The meson to baryon mass ratio allows us to translate LEP II bounds on charged meson to **LEP II bounds on composite bosonic dark matter**

- ◆ Study **systematic effects** due to lattice discretization and finite volume due to the relative unfamiliar nature of the system

Computing Higgs exchange



◆ Need to evaluate the dark σ -term (non-perturbative)

◆ Effective Higgs coupling non-trivial with mixed chiral and vector-like masses

◆ Model-dependent answer for the cross-section

◆ **Lattice input** is necessary: compute mass and form factor (using Feynman-Hellmann)

$$\mathcal{M}_a = \frac{y_f y_q}{2m_h^2} \sum_f \langle B | \bar{f} f | B \rangle \sum_q \langle a | \bar{q} q | a \rangle$$

1. effective Higgs coupling with dark fermions and quark Yukawa coupling
2. dark baryon scalar form factor: need lattice input for generic DM models!
3. nucleon scalar form factor: ChPT and lattice input

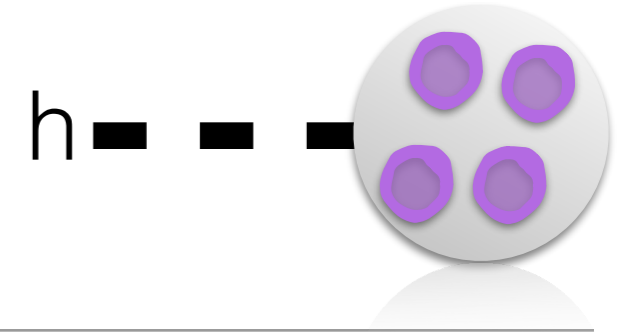
$$y_f \langle B | \bar{f} f | B \rangle = \frac{m_B}{v} \sum_f \left. \frac{v}{m_f} \frac{\partial m_f(h)}{\partial h} \right|_{h=v} f_f^{(B)}$$

$$m_f(h) = m + \frac{y_f h}{\sqrt{2}}$$

$$\alpha \equiv \left. \frac{v}{m_f} \frac{\partial m_f(h)}{\partial h} \right|_{h=v} = \frac{y_f v}{\sqrt{2}m + y_f v}$$

Lattice!

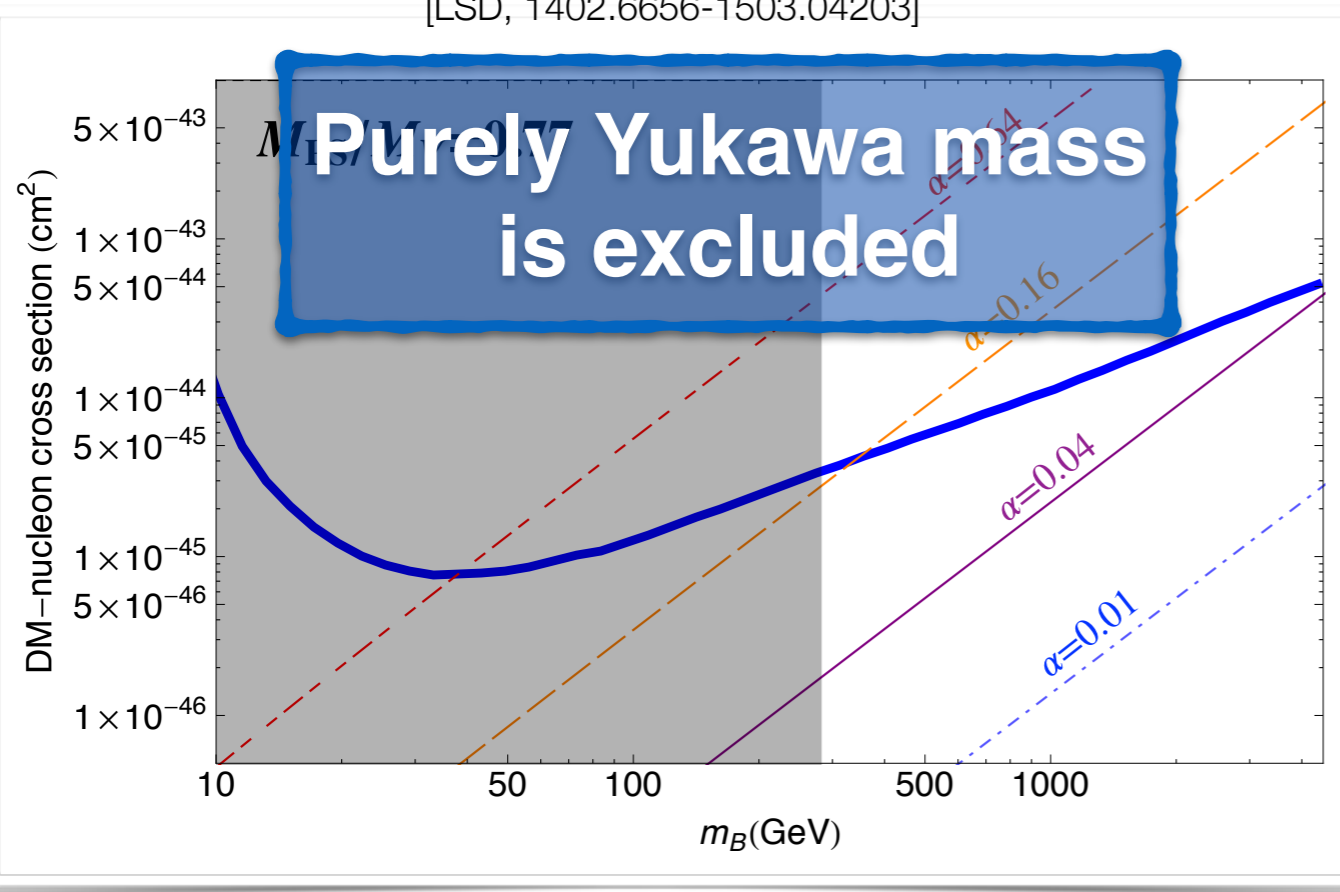
Bounds from Higgs exchange



- ◆ **Lattice results** for the cross-section are compared to **experimental** bounds
- ◆ Coupling space in specific “models” can be vastly constrained

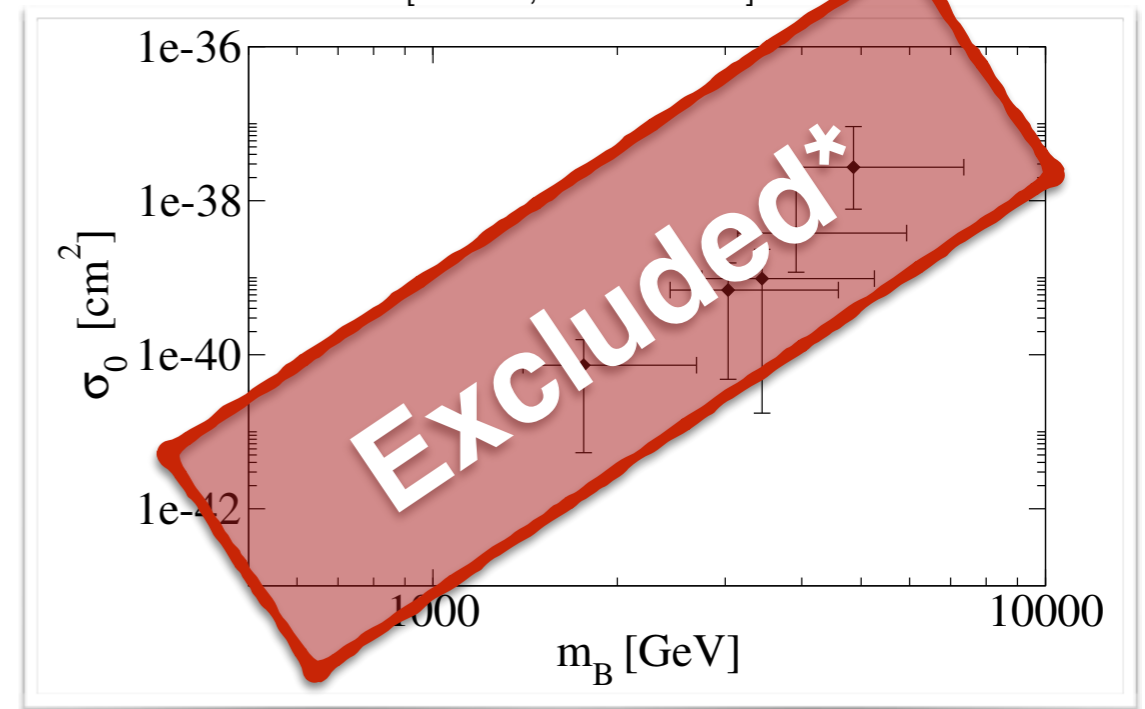
SU(4) $N_f=4$ Stealth DM

[LSD, 1402.6656-1503.04203]



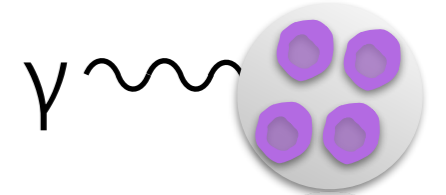
SU(3) $N_f=8$ “technibaryon”

[LatKMI, 1510.07373]



- ◆ Some candidates can be excluded as *dominant sources of dark matter
- ◆ There is **lattice evidence** for universality of dark scalar form factors: includes $N_c=2,3,4,5,7$

Photon interactions



$$\langle \chi(p') | j_{\text{EM}}^\mu | \chi(p) \rangle = F(q^2) q^\mu$$

Expansion at low momentum through effective operators

◆ dimension 5 \rightarrow magnetic

spin 0

$$\frac{(\bar{\chi} \sigma^{\mu\nu} \chi) F_{\mu\nu}}{\Lambda_{\text{dark}}}$$

◆ dimension 6 \rightarrow custodial

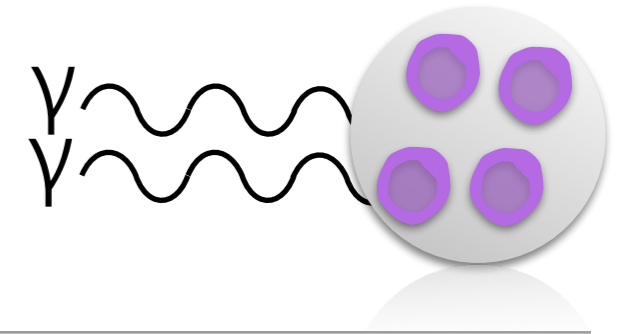
custodial SU(2)

$$\frac{(\bar{\chi} \chi) v_\mu \partial_\nu F^{\mu\nu}}{\Lambda_{\text{dark}}^2}$$

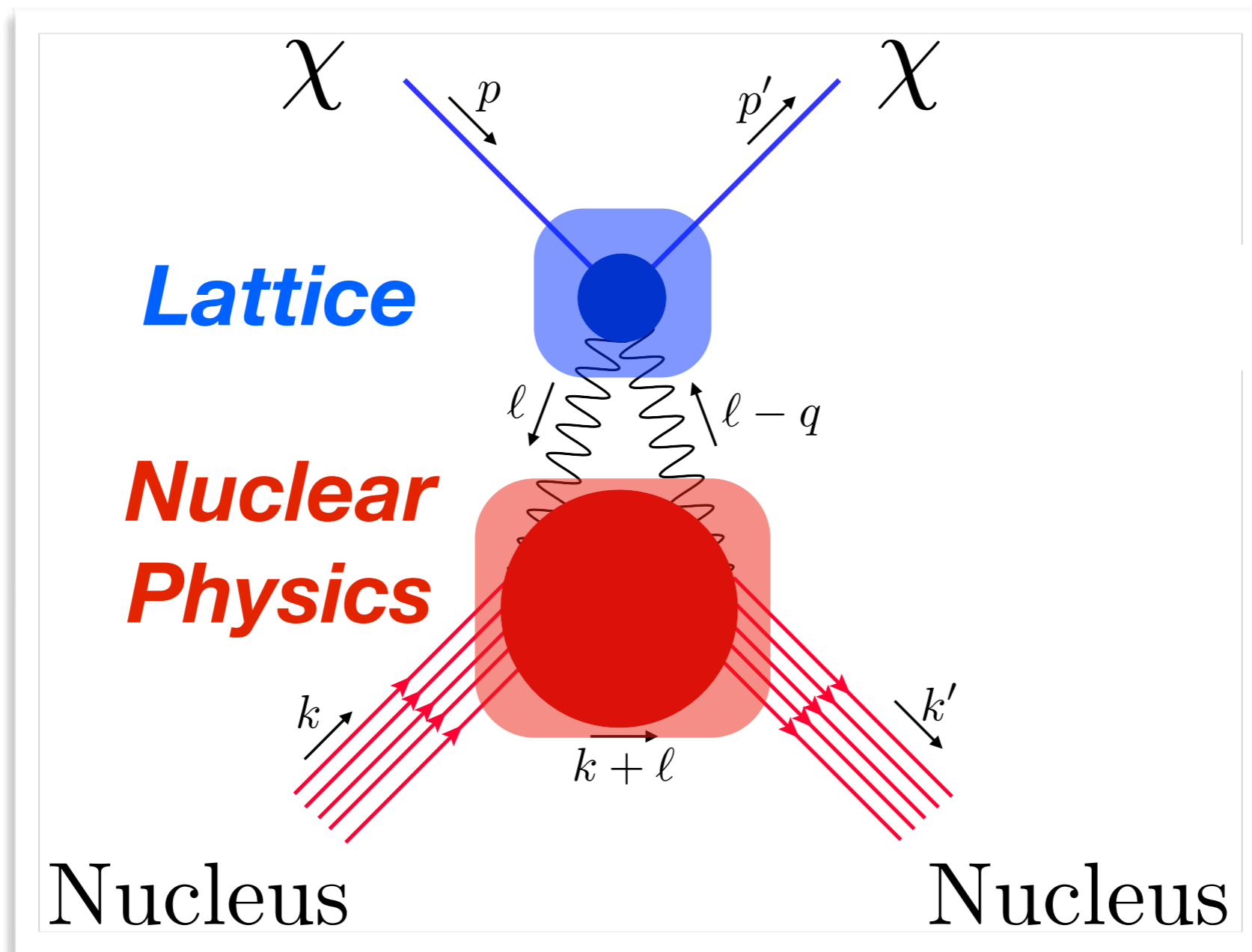
◆ dimension 7 \rightarrow polarizability

$$\frac{(\bar{\chi} \chi) F_{\mu\nu} F^{\mu\nu}}{\Lambda_{\text{dark}}^3}$$

Computing polarizability



$$\frac{c_F e^2}{m_\chi^3} \chi^* \chi F^{\mu\alpha} F_{\alpha}^{\nu} v_\mu v_\nu$$



Lattice: Polarizability of Dark Matter

- **Background field method:**
response of neutral baryon to external electric field \mathcal{E}

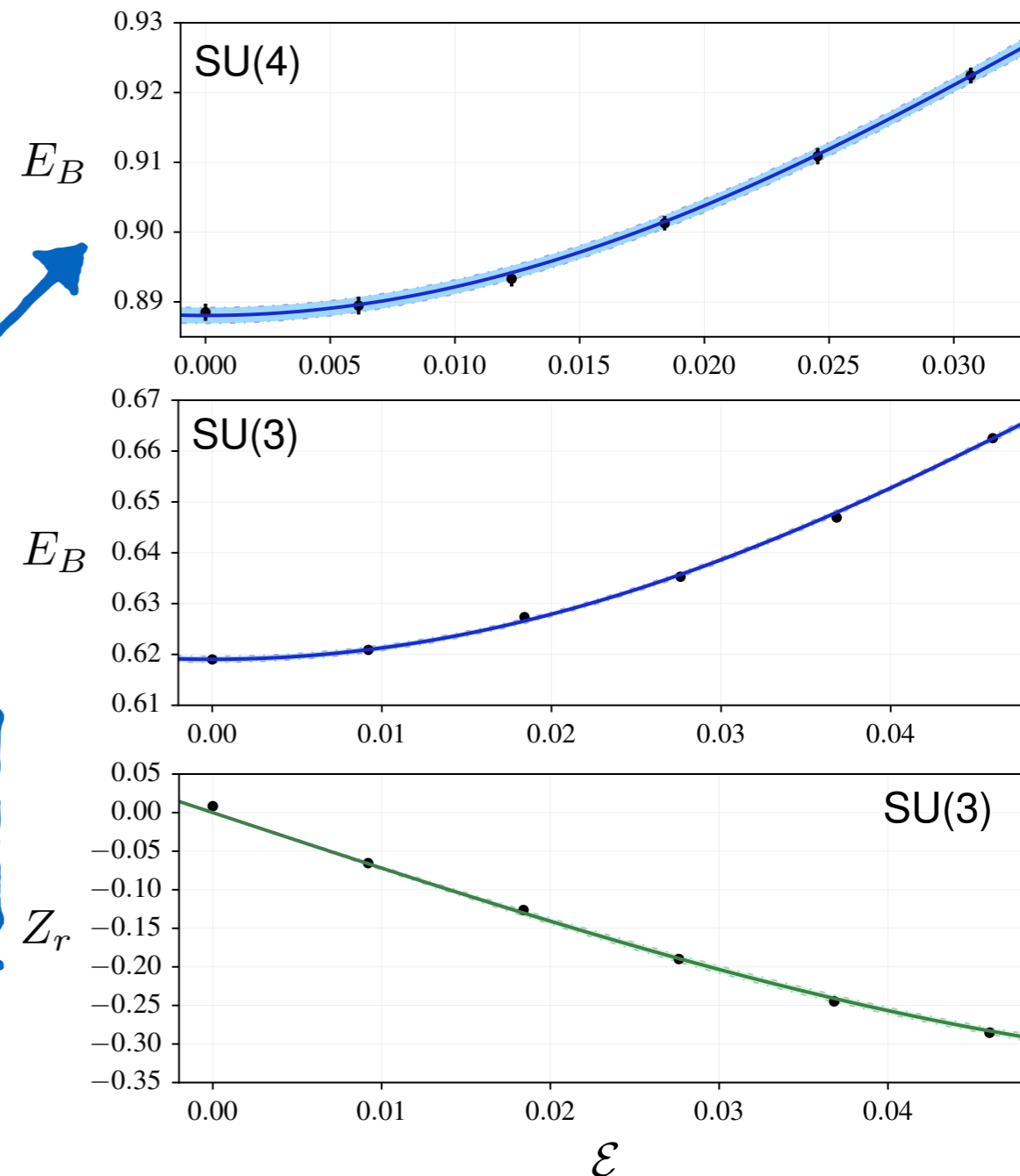
- Measure the shift of the baryon mass as a function of \mathcal{E}

$$E_{B,4c} = m_B + 2C_F |\mathcal{E}|^2 + \mathcal{O}(\mathcal{E}^4)$$

$$E_{B,3c} = m_B + \left(2C_F - \frac{\mu_B^2}{8m_B^3} \right) |\mathcal{E}|^2 + \mathcal{O}(\mathcal{E}^4)$$

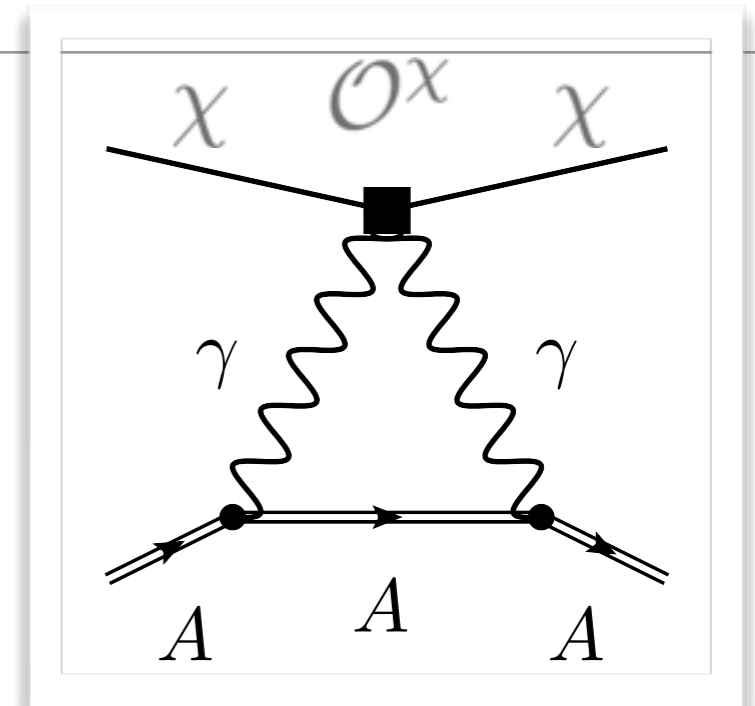
$$Z_r = \frac{\mathcal{E} \mu_B(\mathcal{E})}{2m_B^2}$$

*32³×64 quenched lattices (large volume)
one lattice spacing and two masses (matched)
40 sources on 200 independent configurations
multi-exponential fits with 3 states for the baryon*



Nuclear: Rayleigh scattering

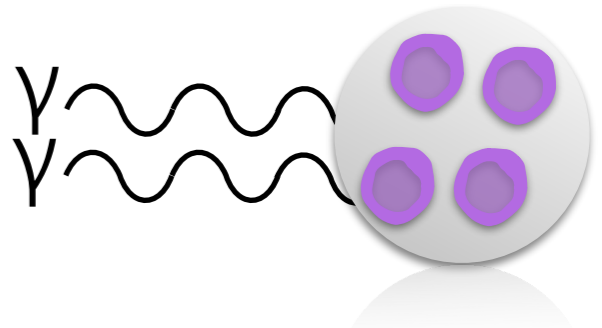
- it is hard to extract the momentum dependence of this nuclear form factor
- similarities with the double-beta decay nuclear matrix element could suggest large uncertainties \sim orders of magnitude
- to assess the impact of uncertainties on the total cross section we start from naive dimensional analysis
- we allow a “magnitude” factor M_F^A to change from 0.3 to 3



$$f_F^A = \langle A | F^{\mu\nu} F_{\mu\nu} | A \rangle$$

$$f_F^A \sim 3 Z^2 \alpha \frac{M_F^A}{R}$$

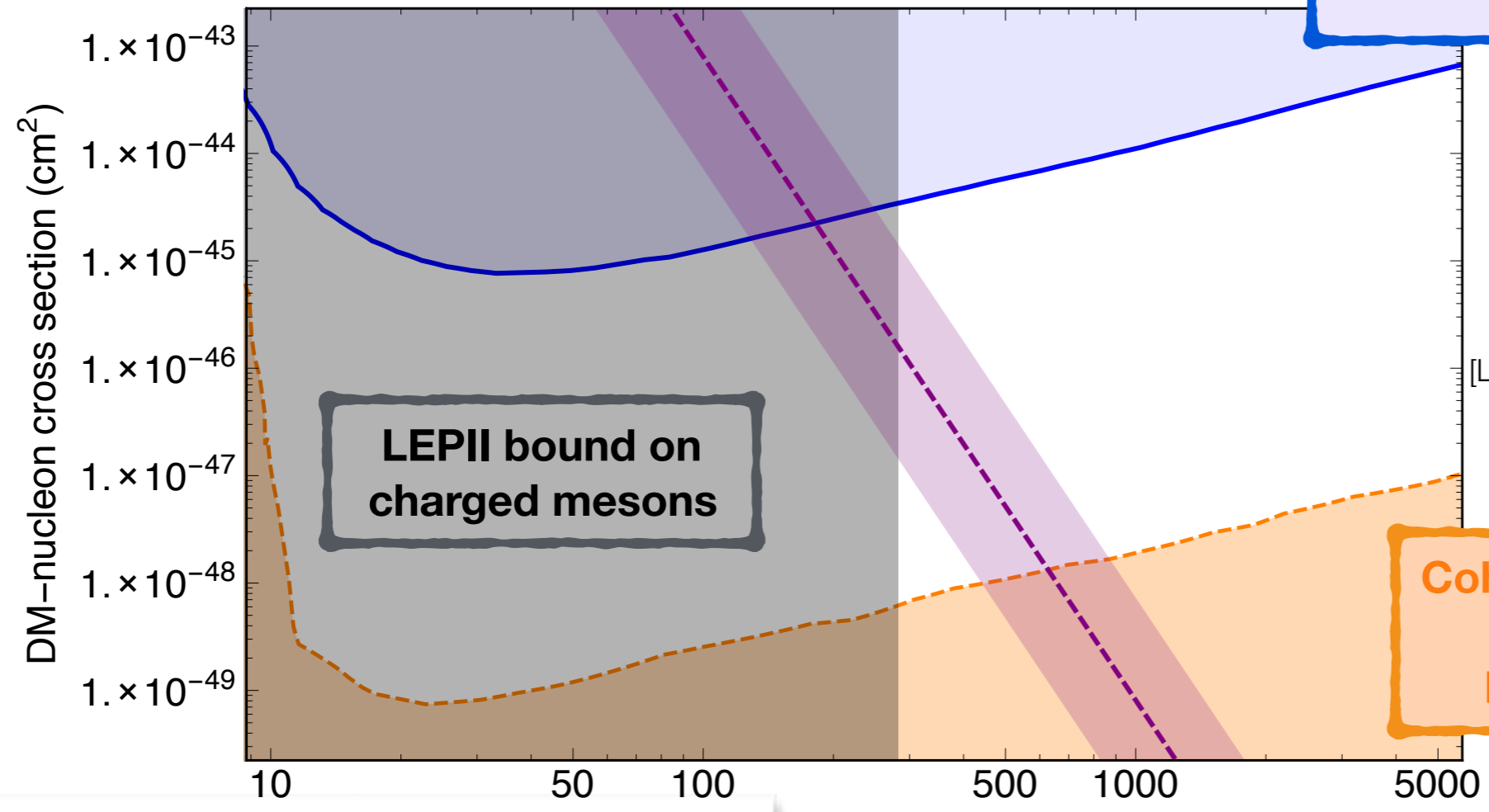
$$\sigma \simeq \frac{\mu_{n\chi}^2}{\pi A^2} \left\langle \left| \frac{c_F e^2}{m_\chi^3} f_F^A \right|^2 \right\rangle$$



Lowest bound from EM polarizability

Electric polarizability from lattice simulations with background fields

LUX exclusion bound for spin-independent cross section



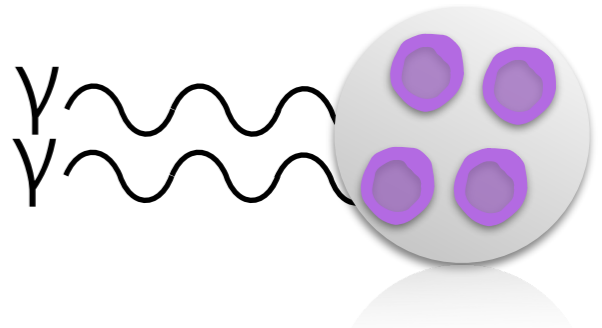
SU(4) N_f=4 Stealth DM
 [LSD collab., Phys. Rev. Lett. 115 (2015) 171803]

Coherent neutrino scattering background

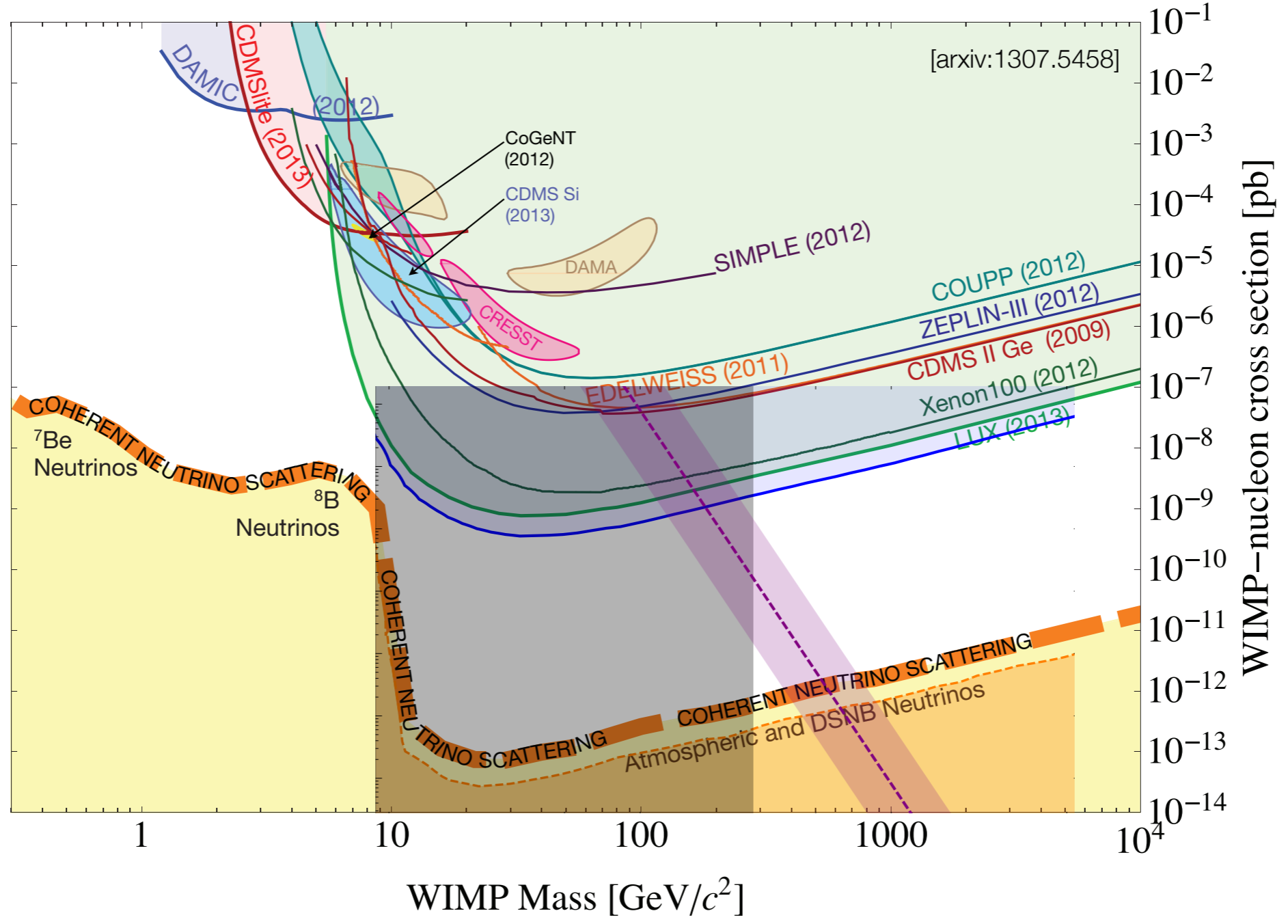
$$\sigma_{\text{nucleon}}(Z, A) = \frac{Z^4}{A^2} \frac{144\pi\alpha^4 \mu_{n\chi}^2 (M_F^A)^2}{m_\chi^6 R^2} [c_F]^2$$

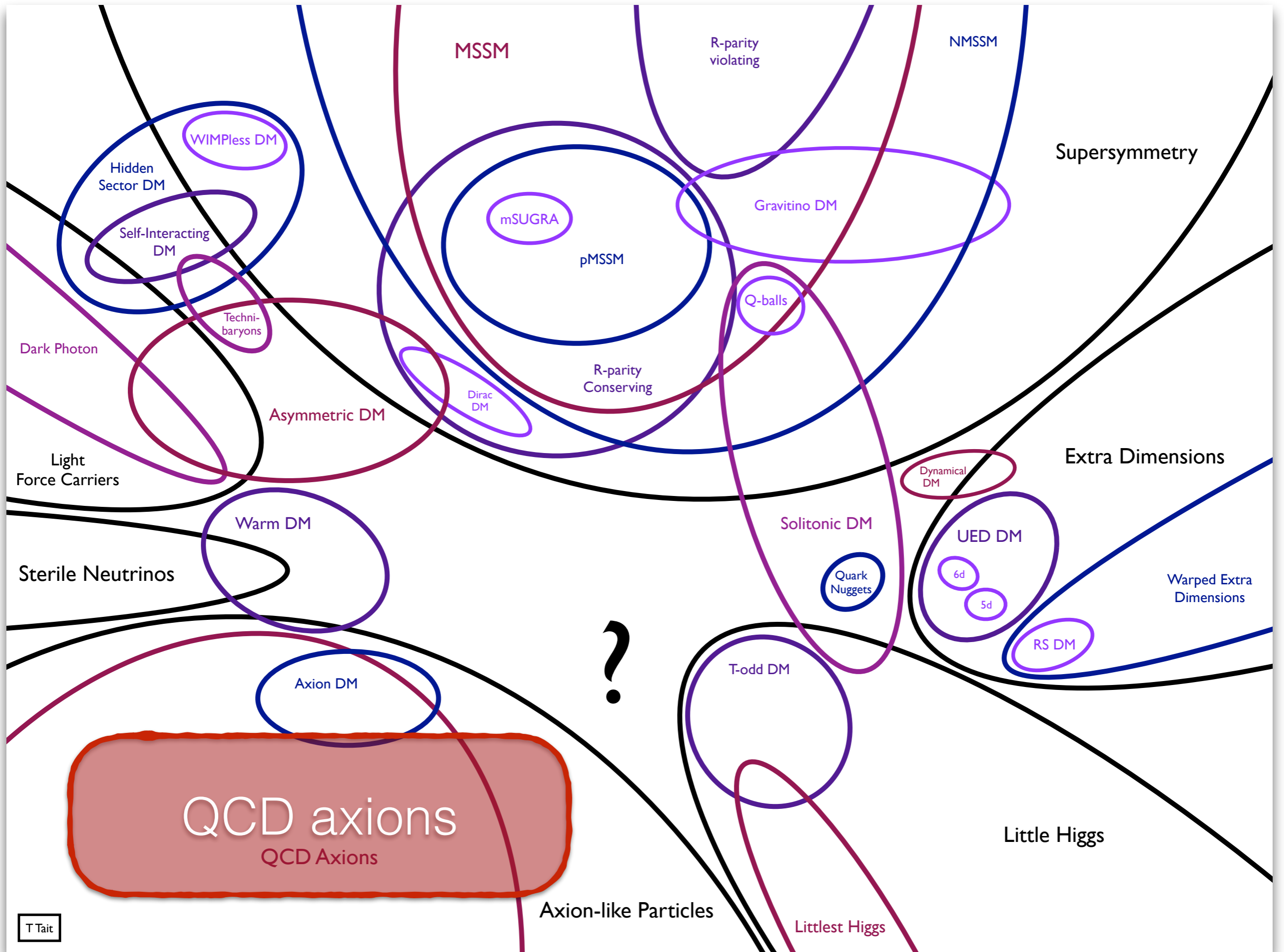
M_χ (GeV) [with LUX, PRL (2013)]

lowest allowed direct detection cross-section for composite dark matter theories with EW charged constituents



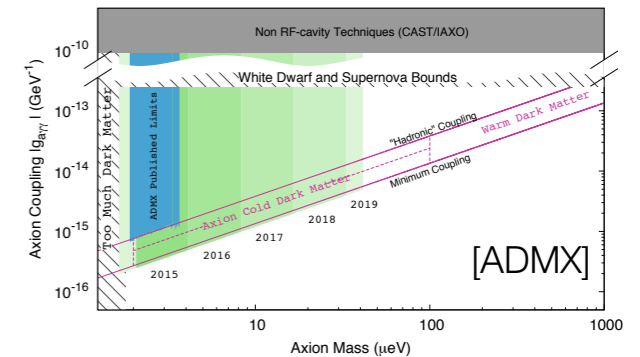
Bound from EM polarizability





T Tait

Axion Dark Matter



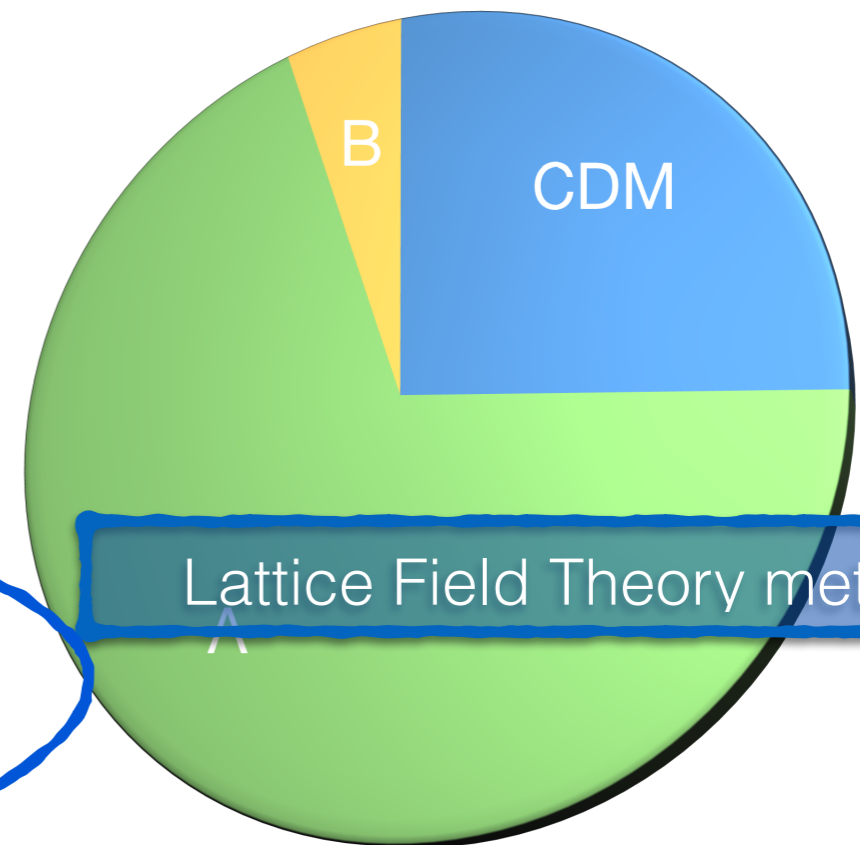
- Axions were originally proposed to deal with the Strong-CP problem

- They also form a plausible DM candidate

- The axion energy density requires **non-perturbative QCD input**

- Being sought in ADMX (LLNL, UW) & CAST-IAXO (CERN) with **large discovery potential** in the next few years

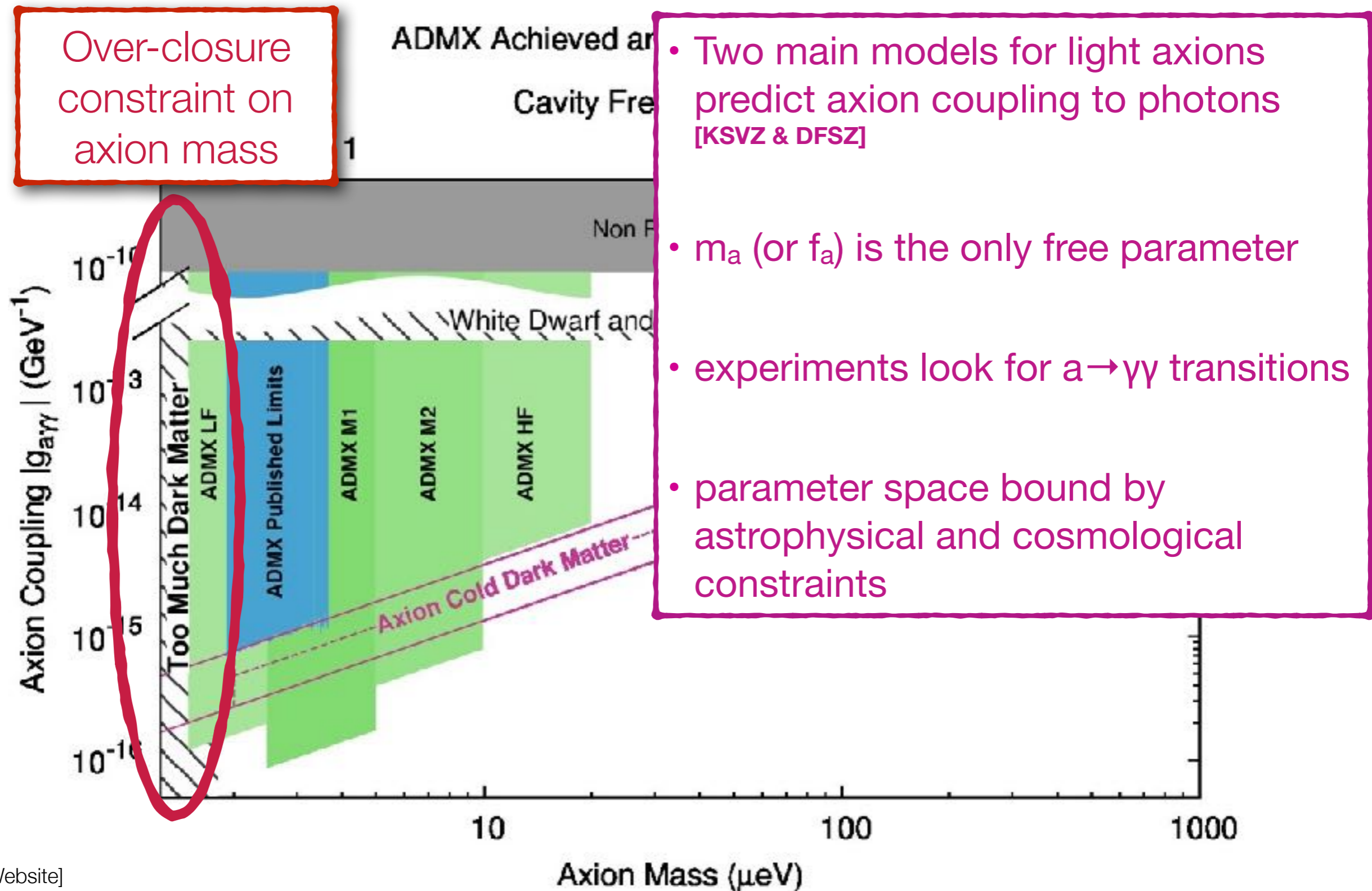
- Requiring $\Omega_{\text{axion}} \leq \Omega_{\text{CDM}}$ yields a **lower bound on the axion mass today**



$$\Omega_{\text{tot}} = 1.000(7)$$

PDG 2014

Current axion constraints



$$m_a^2 f_a^2 = \left. \frac{\partial^2 F}{\partial \theta^2} \right|_{\theta=0}$$

Axion energy density

$$\rho(T_\gamma) = \frac{\rho(t) R^3 m_a(T_\gamma)}{m_a(t) m_a(T_1)} \left(\frac{R(T_1)}{R(T_\gamma)} \right)^3$$

axions in a fixed comoving volume $T_\gamma = 2.73\text{K}$

$$T_1 = T_1(f_a)$$

$$m_a(T_1) = \frac{\sqrt{\chi}}{f_a}$$

$$m_a(T_\gamma) = \frac{1}{f_a} \frac{\sqrt{m_u m_d}}{m_u + m_d} f_\pi m_\pi \quad \text{from } \chi\text{PT}$$

$$R(T) \quad \text{from cosmology}$$

$$\rho(T_1) = \frac{1}{2} m_a^2 f_a^2 \theta_1^2$$

θ_1 is the initial misalignment angle: effectively another free parameter, but can be averaged over in the post-inflationary PQsb scenario

Rely on controllable lattice calculations

$$m_a^2 f_a^2 = \left. \frac{\partial^2 F}{\partial \theta^2} \right|_{\theta=0}$$

Axion mass from lattice simulations

Non-perturbative calculation of QCD topology at finite temperature

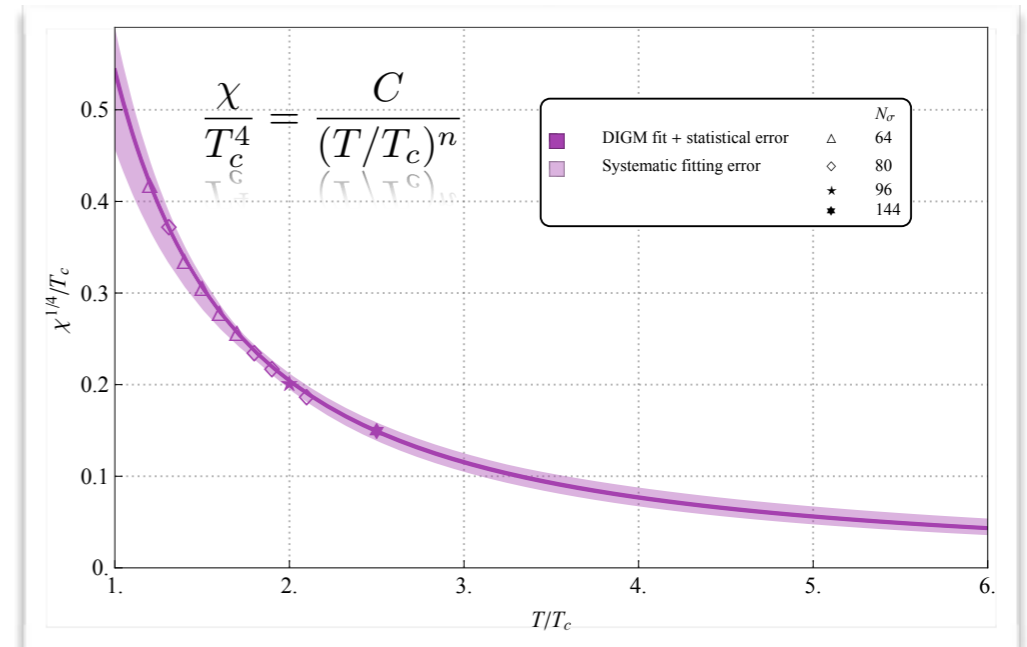
- Pure gauge SU(3) topological susceptibility
 ↪ compatible with model predictions (DIGM/IILM), but **important non-perturbative effects**

[Kitano&Yamada,1506.00370][Borsanyi et al.,1508.06917][Frison et al.,1606.07175]

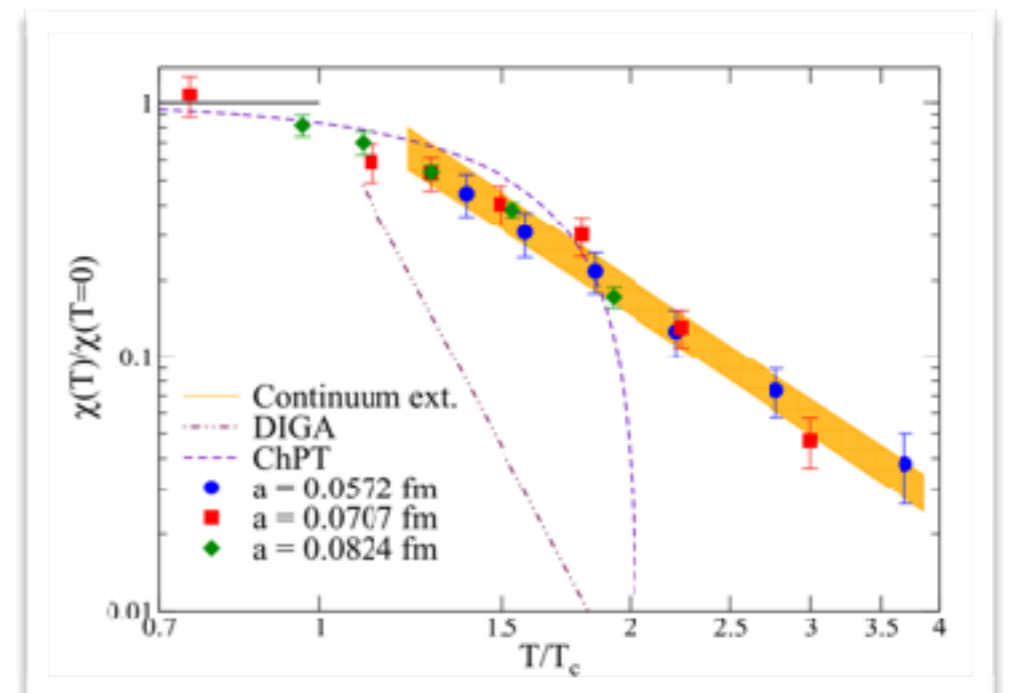
- is QCD topological susceptibility at high-T **well described by models?** ↪ light fermions importantly affect the vacuum

[Trunin et al.,1510.02265][Petreczky et al.,1606.03145][Borsanyi et al.,1606.07494]

Great effort to control all systematic lattice effects in order to impact experiments. Near-final results reached in about a year!



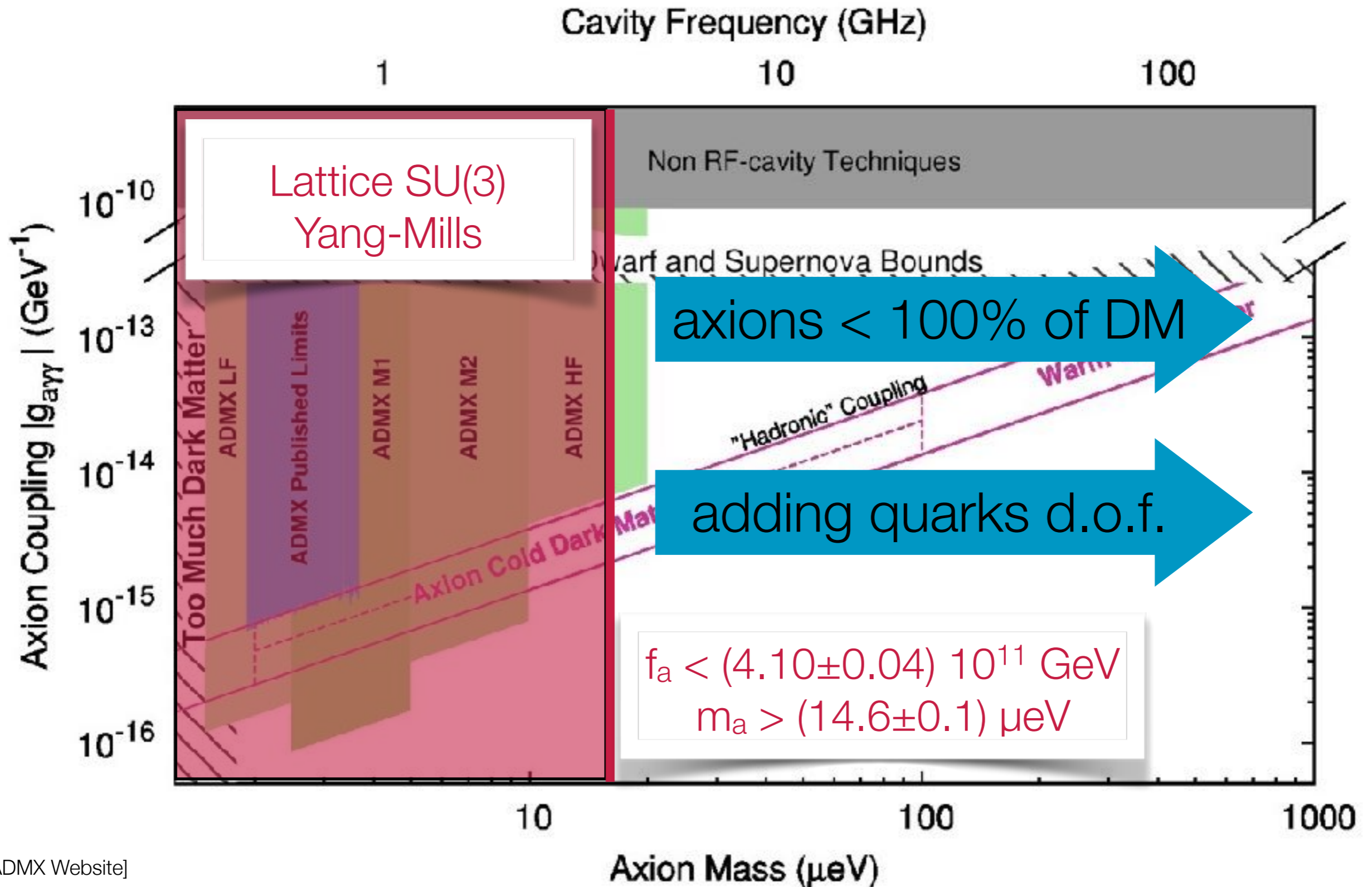
[Berkowitz, Buchoff, ER., 1505.07455]



[Bonati et al., 1512.06746]

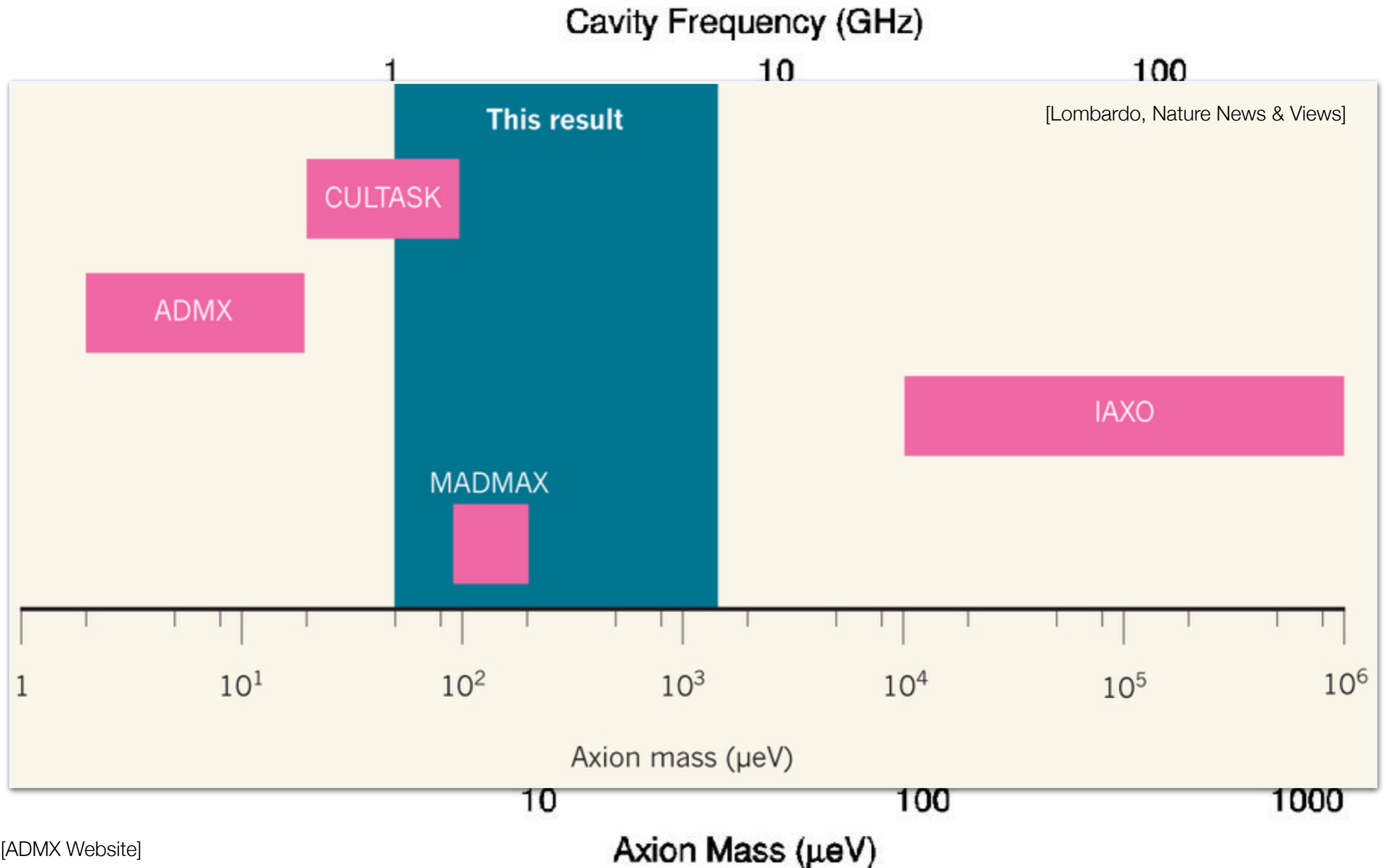
$$m_a^2 f_a^2 = \left. \frac{\partial^2 F}{\partial \theta^2} \right|_{\theta=0}$$

Axion mass lower bound



$$m_a^2 f_a^2 = \left. \frac{\partial^2 F}{\partial \theta^2} \right|_{\theta=0}$$

Axion mass lower bound



Concluding remarks

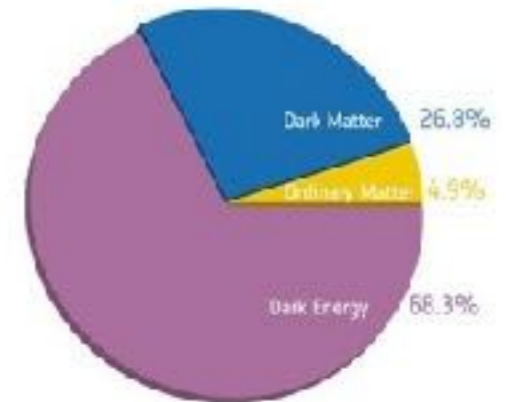
- ★ **QCD ideas** and lattice QCD techniques can be borrowed when exploring the DM landscape (**BSM physics**)
- ★ **Composite** dark matter is a viable interesting possibility with rich **phenomenology**.
- ★ **Axion-like dark matter** has a strong discovery potential
- ★ **Lattice methods** can help in calculating direct detection cross sections, production rates at colliders, and self-interaction cross sections for composite dark matter
- ★ **Lattice methods** are now the preferred tool to study the evolution of the axion mass density in the early universe

extra

Asymmetric dark matter

- It is an **observational fact** that the number density for dark matter and baryonic matter are of the same order of magnitude

$$\Omega_{\text{DM}} \approx 5 \Omega_{\text{B}}$$



[Planck and ESA]

- This can be explained in Technicolor theories where dark matter is a baryon of a new strongly-coupled sector which shares **asymmetry** with standard baryonic matter

requires coupling of DM to SM particles!

higher dimensional operators

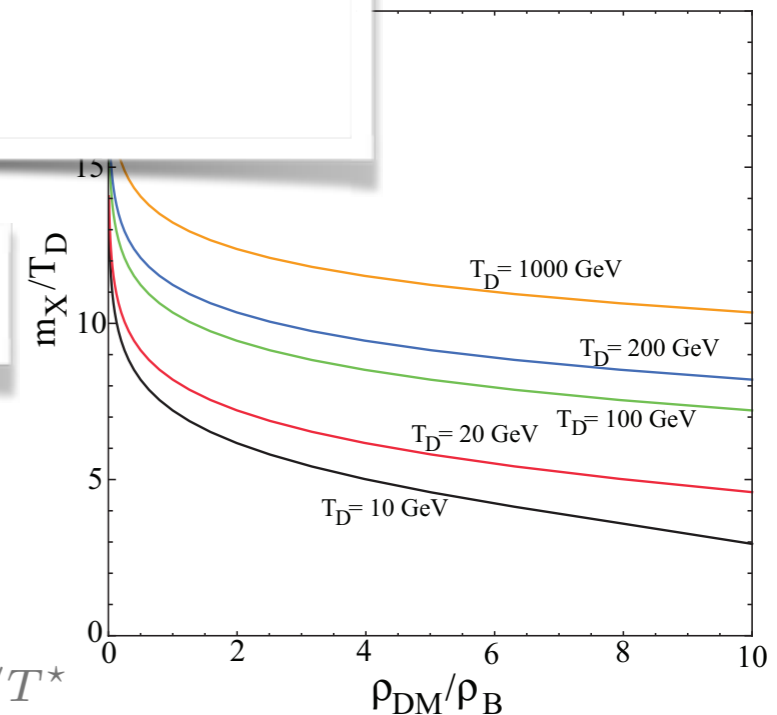
sphaleron processes



$$n_{\text{DM}} \approx n_{\text{B}} \rightarrow M_{\text{DM}} \approx 5 M_{\text{B}}$$



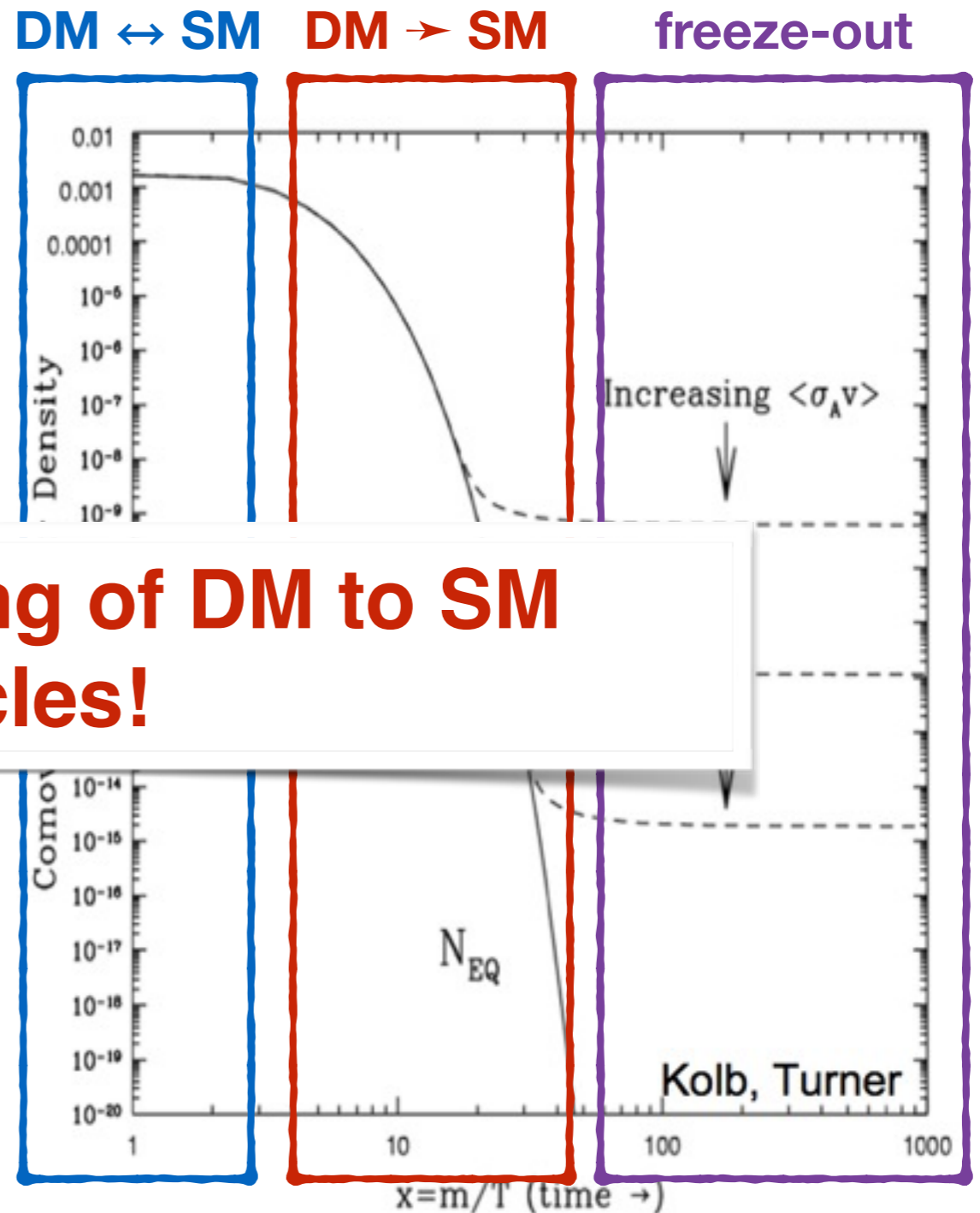
$$M_{\text{DM}} \gg M_{\text{B}} \rightarrow n_{\text{B}} \gg n_{\text{DM}} \approx e^{-M_{\text{DM}}/T^*}$$



[Buckley & Randall, JHEP1109 (2011)]

Thermal Dark Matter

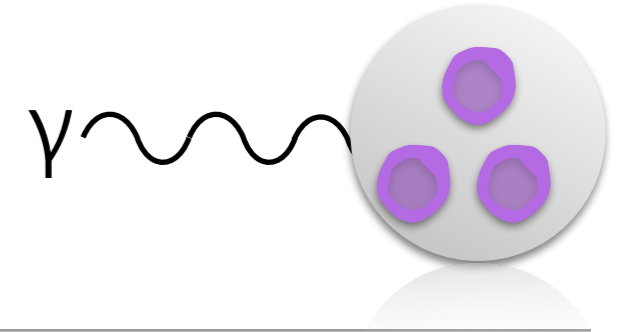
- Dark matter starts in **thermal equilibrium** with ordinary matter
- Dark matter cannot be created in equilibrium when the **temperature drops** below m_χ



requires coupling of DM to SM particles!

- Increases relic density
- Observed relic density:

$$\Omega_\chi \approx \frac{1}{\langle\sigma_a v\rangle} \approx \frac{m_\chi^2}{g_\chi^4}$$

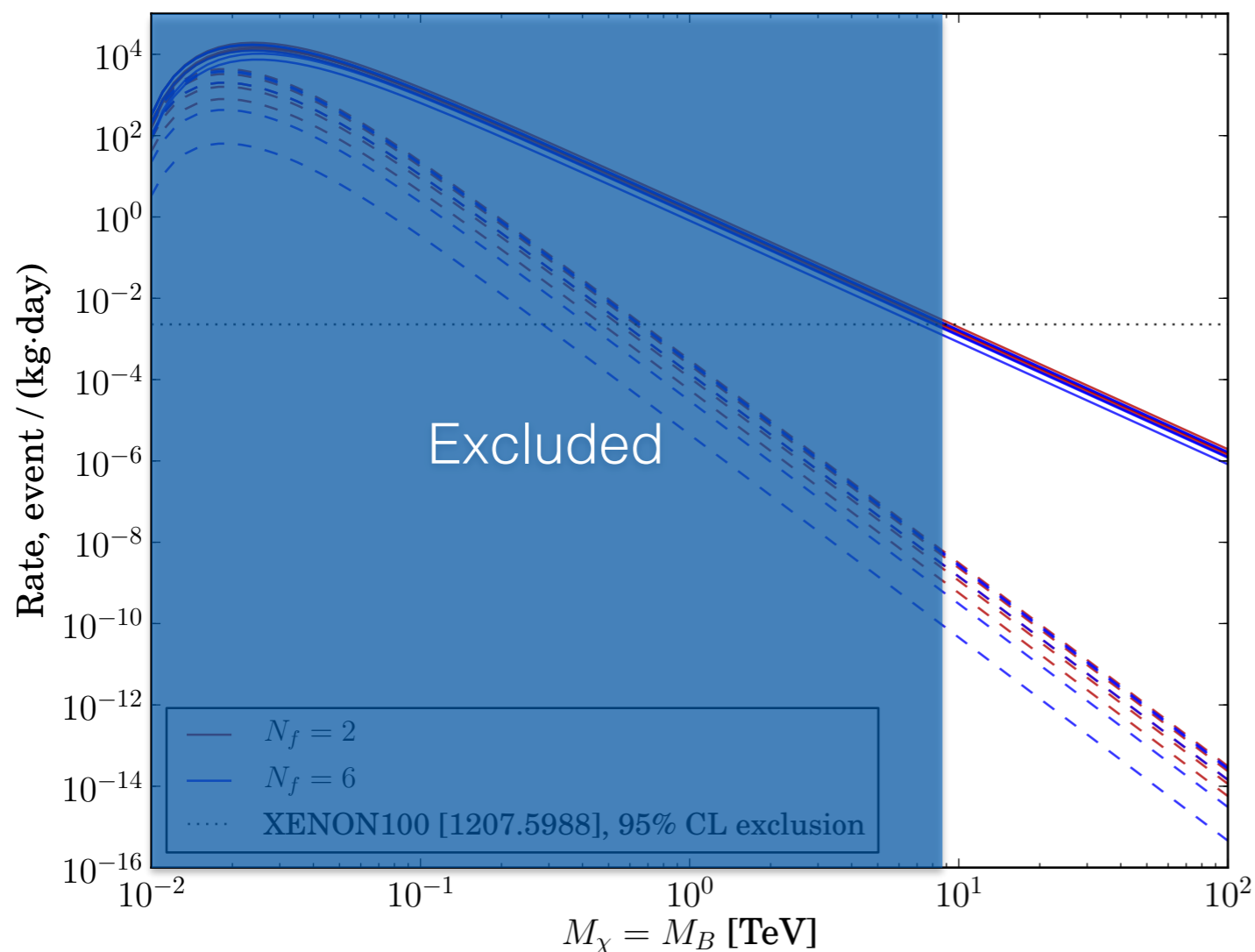


Bounds from EM moments

Mesonic and Baryonic EM form factors directly from lattice simulations

SU(3) $N_f=2,6$ dark fermionic baryon

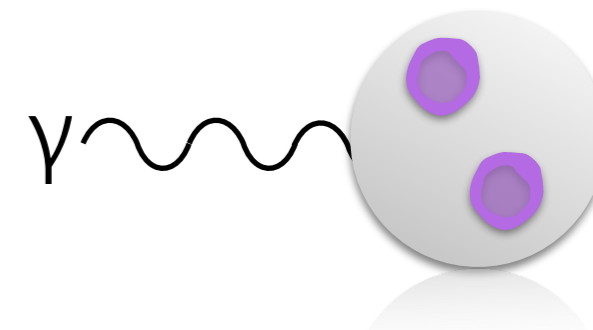
[LSD, 1301.1693]



- ★ baryon similar to QCD neutron
- ★ dark quarks with $Q=Y$
- ★ calculate connected 3pt
- ★ scale set by DM mass
- ★ magnetic moment dominates
- ★ results independent of N_f

$M_B > \sim 10$ TeV

pushed to ~ 100 TeV
with new LUX



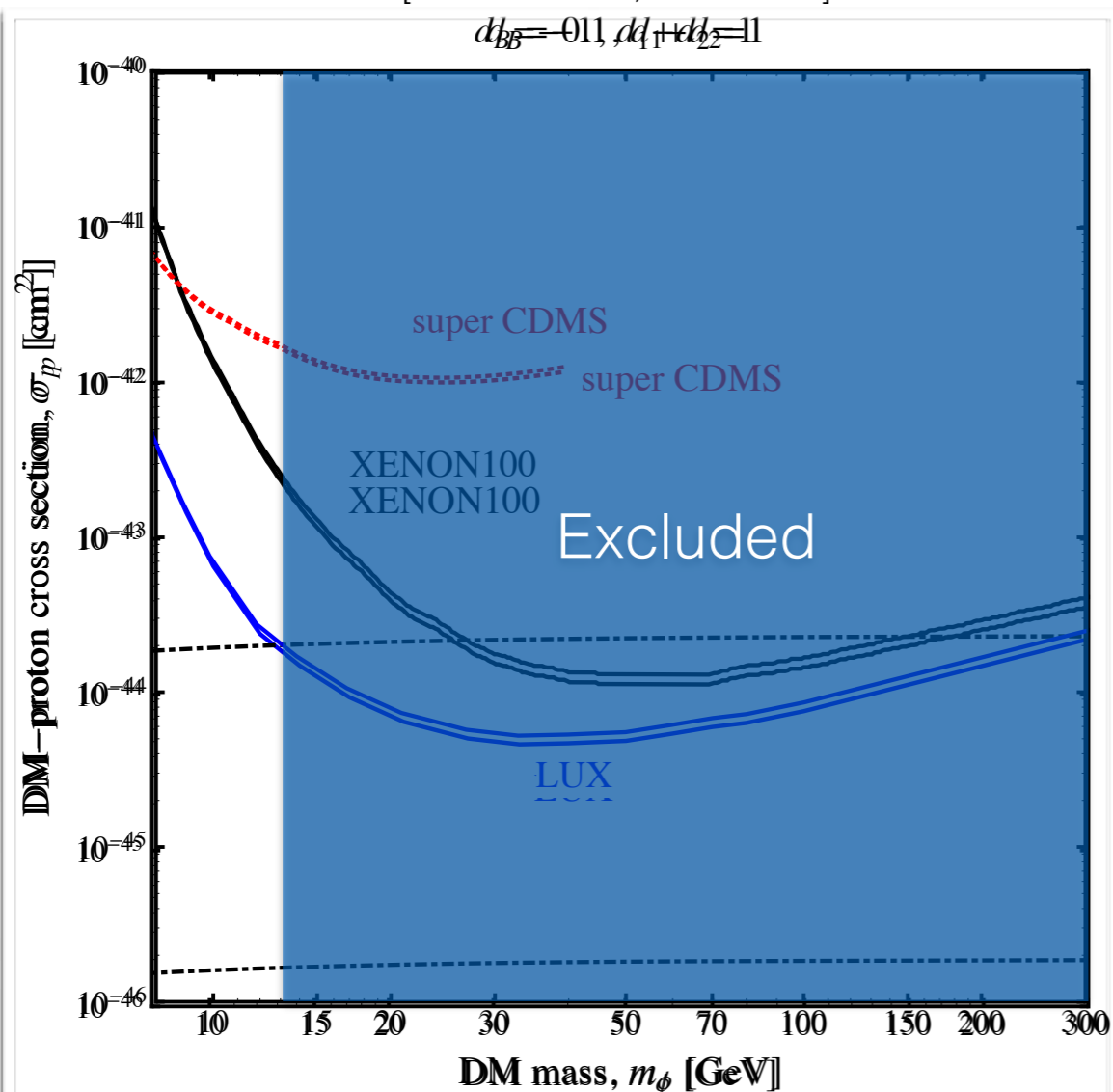
Bounds from EM moments

Mesonic and Baryonic EM form factors directly from lattice simulations

SU(2) $N_f=2$ pNGB DM

[Hietanen et al., 1308.4130]

$$d_B = -0.11, d_1 + d_2 = 1$$



- ★ DM is “mesonic” pNGB
- ★ calculate connected 3pt
- ★ use VMD with lattice ρ mass
- ★ scale set by $F_\pi=256$ GeV
- ★ depends on **isospin breaking d_B**
- ★ also couples to Higgs (d_1+d_2)

$M_B \sim < 13$ GeV
depends on d_B