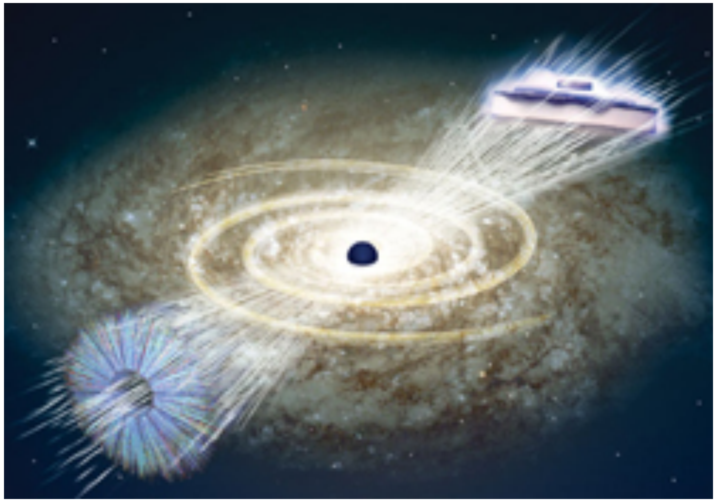
INFN, Firenze, January 31 2017.

The holographic correspondence



Francesco Bigazzi

INFN, Firenze

Istituto Nazionale di Fisica Nucleare

Plan

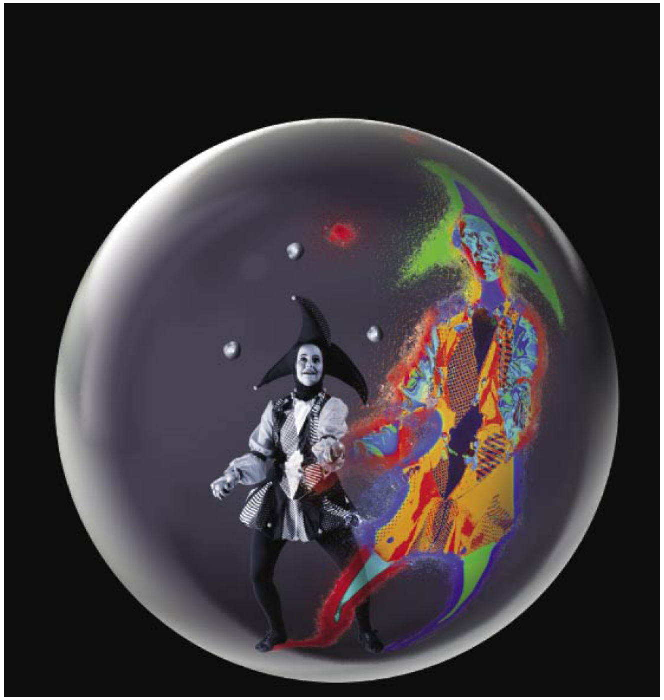
- What is it ?
- How does it work ?
- What is it for ?

Plan

- What is it ?
- How does it work ?
- What is it for ?

The statement

Quantum Gravity in D+1 dimensions	=	Quantum Field Theory in D dimensions



The statement

Quantum Gravity in D+1 dimensions	=	Quantum Field Theory in D dimensions

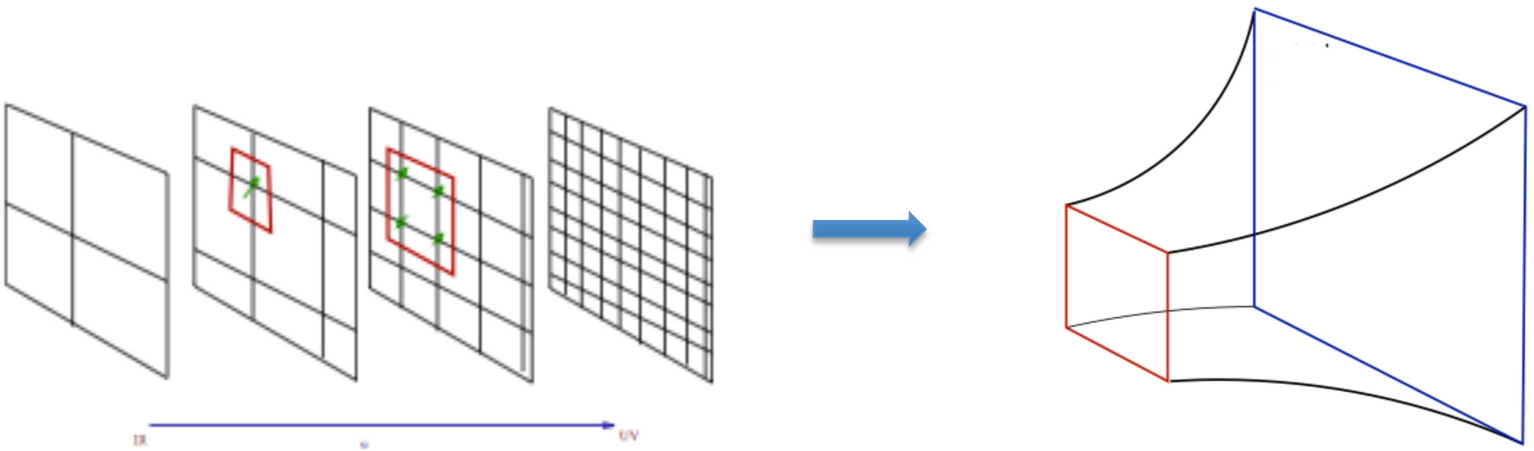
Is this reasonable?
It is not, but…

Heuristic Hint 1: RG flow

- Renormalization Group equations in QFT are local in the energy scale u

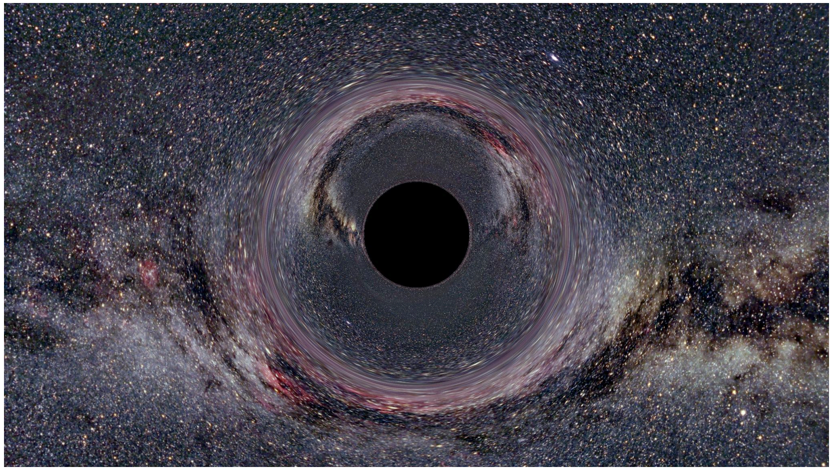
$$u\frac{dg}{du} = \beta(g)$$

- Idea: RG flow of a D-dim QFT as "foliation" in D+1 dims .
- RG scale u = Extra dimension

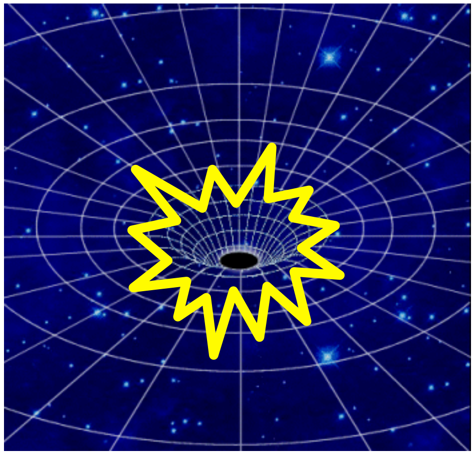


Heuristic Hint 2: black holes

- Model in D+1 must have same number of d.o.f. as the QFT in D-dims
- Gravity is a good candidate: it is "holographic"
- See black hole physics



Black holes… are not so black



- Quantum effects: emit thermal radiation.
- Obey laws of thermodynamics
- Entropy scales like horizon area
  not as the enclosed volume!
  [Bekenstein, Hawking 1974]

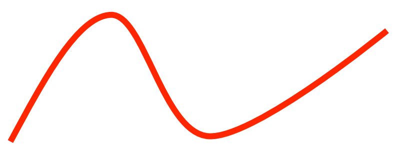
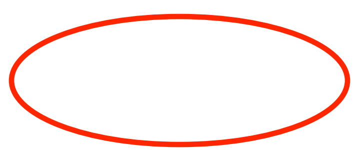
The holographic principle [‘t Hooft, Susskind 1994]

Quantum gravity, whatever it is, is holographic.

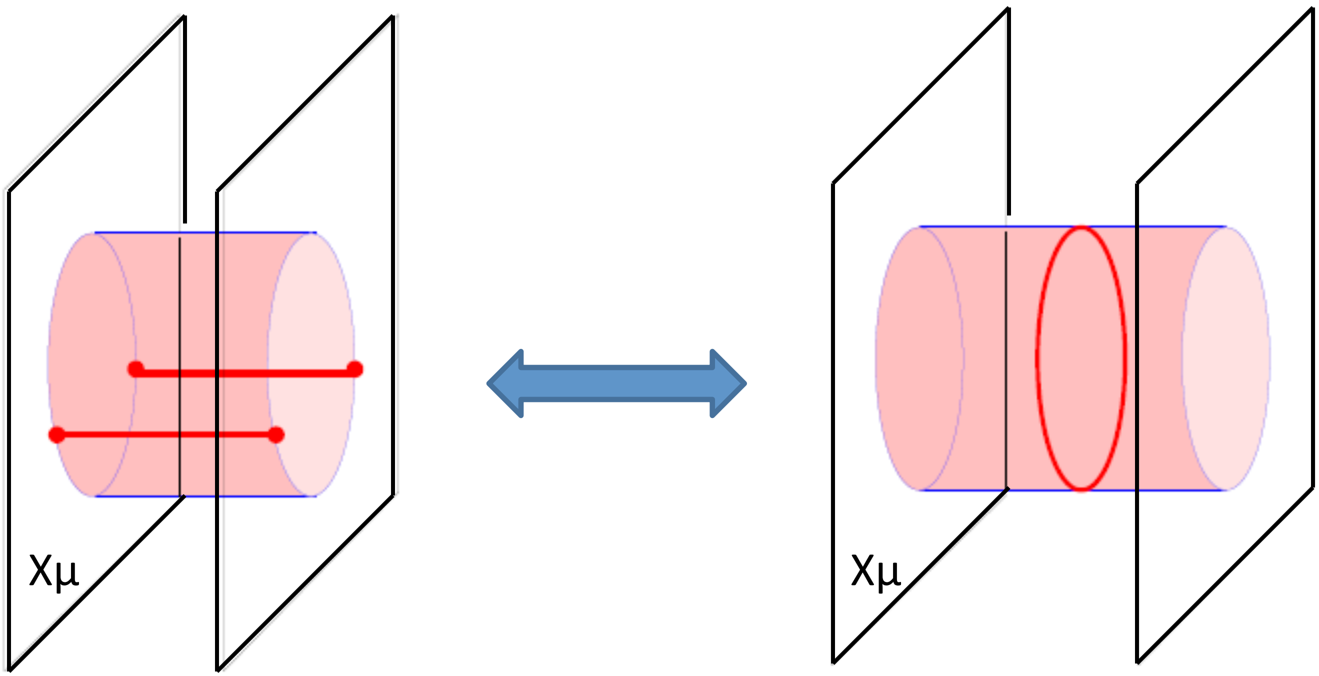
Degrees of freedom of QG in D+1 dim. spacetime volume	=	Degrees of freedom of QFT in D dim. boundary

Heuristic Hint 3: String theory

- Assumption: fundamental constituents are string-like
- Point particles are different modes of a vibrating string

String		Particle
Open		photon (gluon) +…
Closed		graviton +…..

- Open/closed string duality (or: 2 ways of drawing a cylinder)



Open string loop (quantum) ⟺ Closed string propagation (classical)
Quantum Field Theory ⟺ Theory of gravity
Xμ, u (RG scale) ⟺ Xμ, r (radial extra dimension)

- String theory provided the first explicit realization of the holographic correspondence [Maldacena 1997] a.k.a. AdS/CFT

3+1 dim $\mathcal{N}$=4 SU(N) Yang-Mills = IIB string on AdS_5 x S_5

(Conformal Field Theory) (Quantum gravity on Anti de Sitter)

- …and a very detailed map between observables of the corresponding theories [Witten; Gubser, Klebanov, Polyakov, 1998]

- This has allowed to provide both extensions and an enormous amount of quantitative validity checks of the correspondence… e.g.
  - central charges of CFT = volumes of X_5 [e.g. Bertolini, Bigazzi, Cotrone 2005]
  - Wilson loops in QFT = minimal area [e.g. Bigazzi, Cotrone, Griguolo, Seminara 2014]

- … and to use it concretely

Plan

- What is it ?
- How does it work ?
- What is it for ?

“ It works in a very subtle way,
as a strong/weak coupling duality.”

“ For instance: certain regimes where QFT is strongly interacting, mapped into classical (i.e. weakly interacting) gravity! ”

(and the other way around)

- For the master example [Maldacena 1997]:

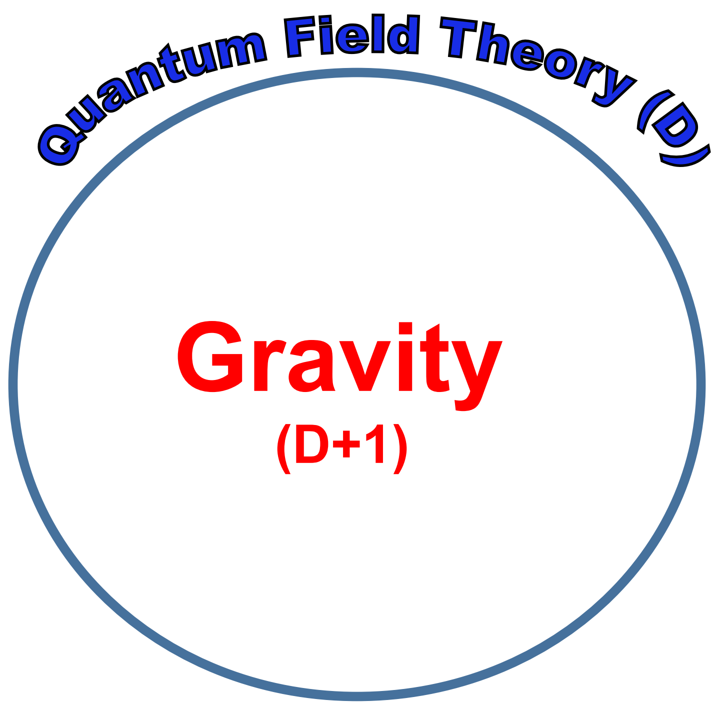
$$\mathcal{N} = 4\ SU(N_c)\ \text{SYM in}\ D = 4\ \text{dual to gravity on}\ AdS_5 \times S^5.$$
$$\text{Classical gravity regime:}\ \ N_c \gg 1, \quad \lambda = g^2_{YM} N_c \gg 1.$$

- Large number of d.o.f. (large N)
- Strong coupling

- Non-perturbative QFT problems can be solved by classical gravity!
- Quantum gravity from a dual perturbative QFT!

How to compute?

QFT vacuum ➔ Gravity background

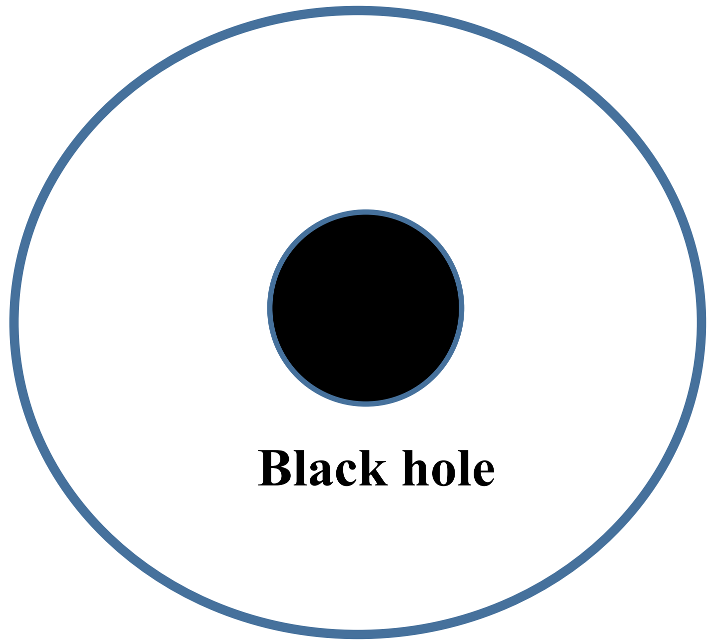


$$Z_{QFT} = Z_{QG(String)} \approx e^{-S_{\mathrm{gravity}}(\mathrm{on-shell})}$$

[Gubser, Klebanov, Polyakov; Witten, 1998]

QFT vacuum → Gravity background

QFT at finite temperature $Z = Tre^{-\frac{H}{T}}$

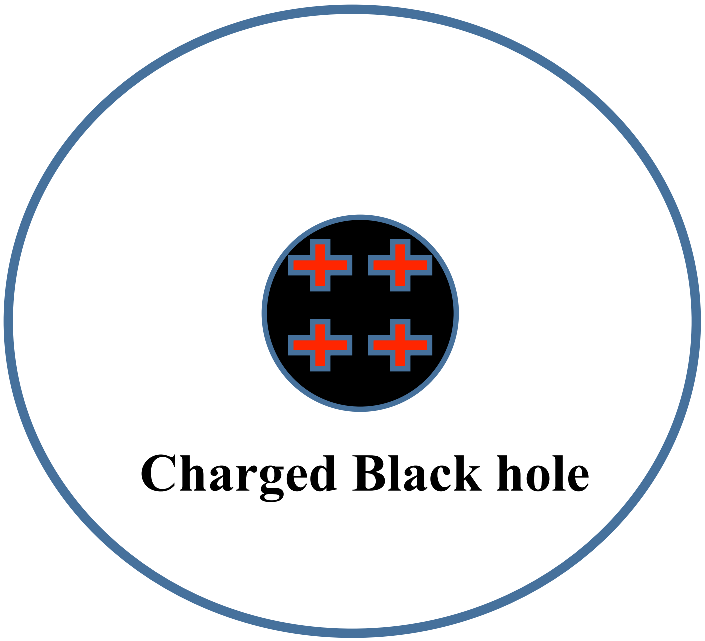


[Witten 98]

$\text{Log } Z \approx -S[\text{gravity on shell}]$

QFT vacuum ➔ Gravity background

QFT at finite temperature and density $Z = Tr e^{-\frac{H-\mu N}{T}}$



QFT density = electric flux on the boundary

Log Z ≈ - S[gravity on shell]

CFT d

???

$$x_\mu \to \lambda x_\mu$$
$$E \to \lambda^{-1} E$$

$$\Longleftrightarrow$$

$$ds^2 = \quad dx_\mu dx^\mu + \quad dr^2$$

$E \approx r$

CFT d

$$x_\mu \to \lambda x_\mu$$
$$E \to \lambda^{-1} E$$

⟺

E ≈ r

AdS d+1

$$ds^2 = \frac{r^2}{R^2} dx_\mu dx^\mu + \frac{R^2}{r^2} dr^2$$

(R=AdS radius)

CFT d

$$x_\mu \to \lambda x_\mu$$
$$E \to \lambda^{-1} E$$

$\Longleftrightarrow$

E ≈ r

AdS d+1

$$ds^2 = \frac{r^2}{R^2} dx_\mu dx^\mu + \frac{R^2}{r^2} dr^2$$

(R=AdS radius)

CFT at finite T

$$Z = Tr e^{-\frac{H}{T}}$$

$\Longleftrightarrow$

AdS black hole

$$ds^2 = \frac{r^2}{R^2}\left[-b[r]dt^2 + dx_i dx_i\right] + \frac{R^2}{r^2}\frac{dr^2}{b[r]}$$

$$b[r] = 1 - \frac{r_h^d}{r^d}$$

$$T_{CFT} = T_{BH} = \frac{r_h\, d}{4\pi R^2}; \qquad S_{CFT} = S_{BH} = \frac{A_h}{4G_N} \sim V_{d-1} T^{d-1}$$

CFT at finite T and μ

$$Z = Tr e^{-\frac{H - \mu N}{T}}$$

$\Longleftrightarrow$

Charged RN-AdS black hole

$$A_t \sim \mu - \frac{\rho}{r^{d-2}}$$

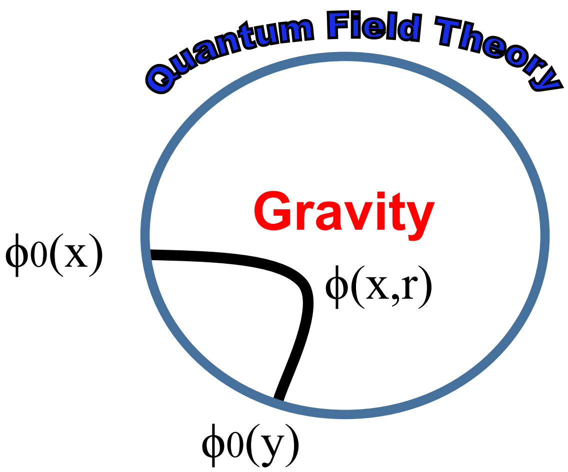
Correlators

- In a D dim. QFT we compute the generating functional

$$Z_{QFT}[\phi_0] = \int D\Psi \exp\left(i[S_{QFT} + \int d^D x \, \phi_0(x) \mathcal{O}[\Psi](x)] \right)$$

- N-point correlators of O[x]: from n-th derivarives of Log Z w.r.t. external source ϕ_0(x).

- Holography: treat external source ϕ_0(x) as boundary value of a gravity field ϕ(x,r) which is "dual" to the operator O[x]



Correlators

- Then: [Gubser, Klebanov, Polyakov; Witten; 1998]

$$Z_{QFT}[\phi_0] = Z_{QG/String} \approx e^{iS_{gravity}[\phi_0]}|_{\text{“}\phi(x,r)\to\phi_0(x)\text{”}}$$

- So that for example:

$$\langle \mathcal{O}(x)\mathcal{O}(y)\rangle \sim \frac{\delta^2 S_{grav}[\phi_0]}{\delta\phi_0(x)\delta\phi_0(y)}|_{\phi_0=0}$$

- Can compute correlators, just solving equation of motion for ϕ(x,r) !

Operator O(x) ➔ Gravity field ϕ(x,r)

- Example 1(stress tensor): $T^{\mu\nu}(x) \to g_{\mu\nu}(x,r)$

- Example 2 (conserved current): $J^{\mu}(x) \to A_{\mu}(x,r)$

- Example 3 (scalar operator): $TrF^2(x) \to \Phi(x,r)$

Plan

- What is it ?
- How does it work ?
- What is it for ?

Strongly correlated QFTs arise in many places:

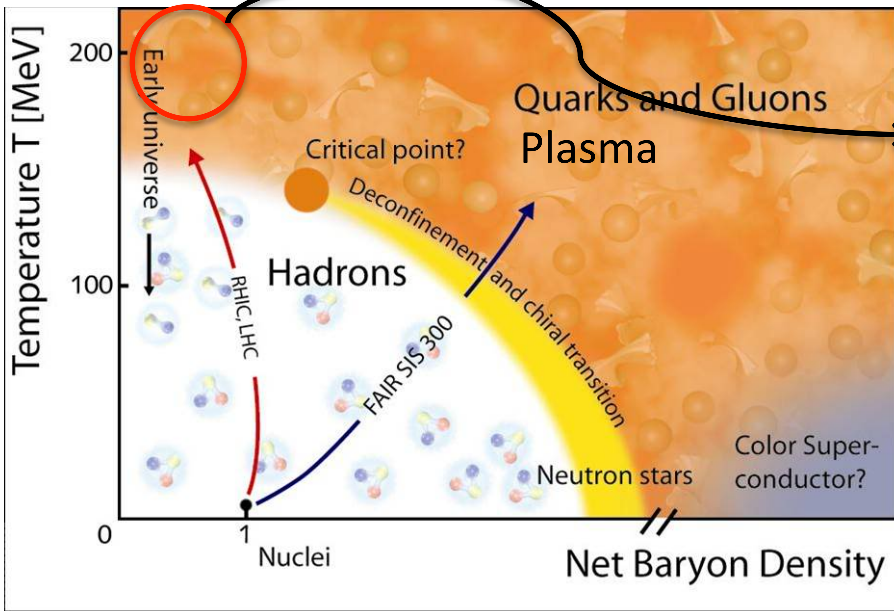
- QCD: confinement, mass gap, quark-gluon plasma phase …
- Quantum critical regions in condensed matter: high Tc superconductors, strange metals …

- Often non-equilibrium, finite density challenging: need novel tools .
- Holography is emerging as a promising one.
- Strongly coupled D-dim QFT solved by classical gravity in D+1

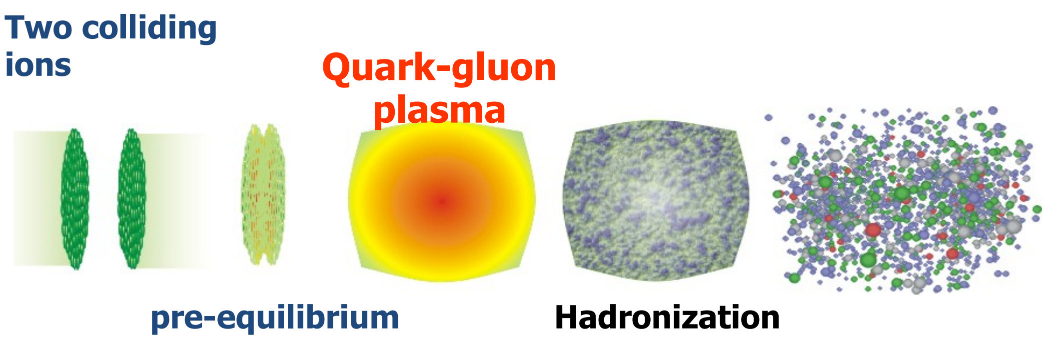
- Often analytic control on the models. Novel intuitions.
- Can deal with both static and real-time dynamical properties.
- Still limited to toy models.
- Provide benchmarks and info on universal behavior at strong coupling.

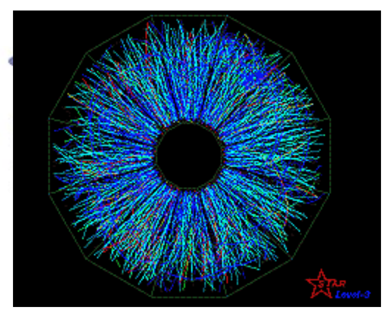
1. Holography and QCD: part I

The Quark-Gluon-Plasma



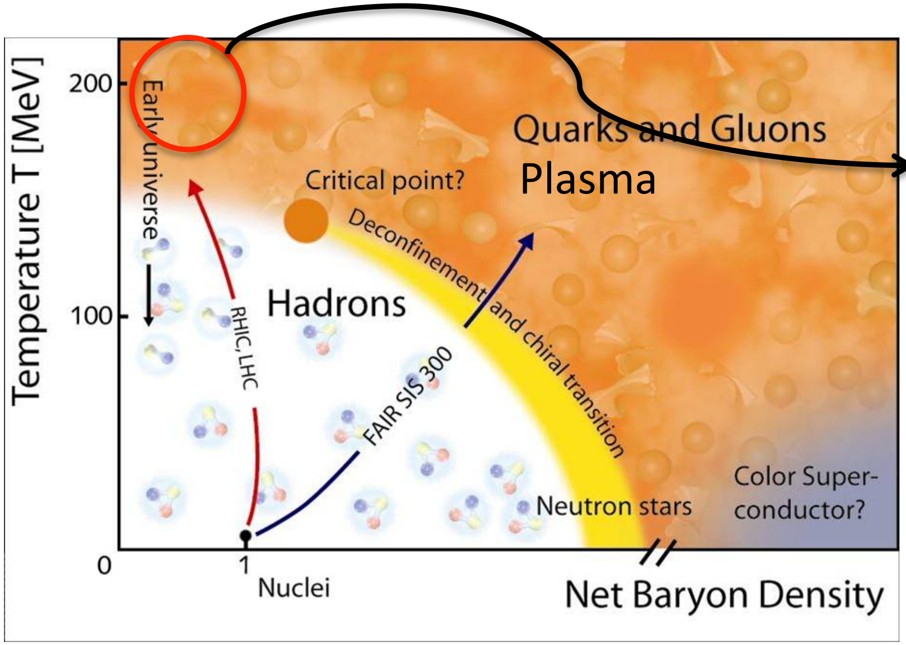
Heavy ion collisions at RHIC …





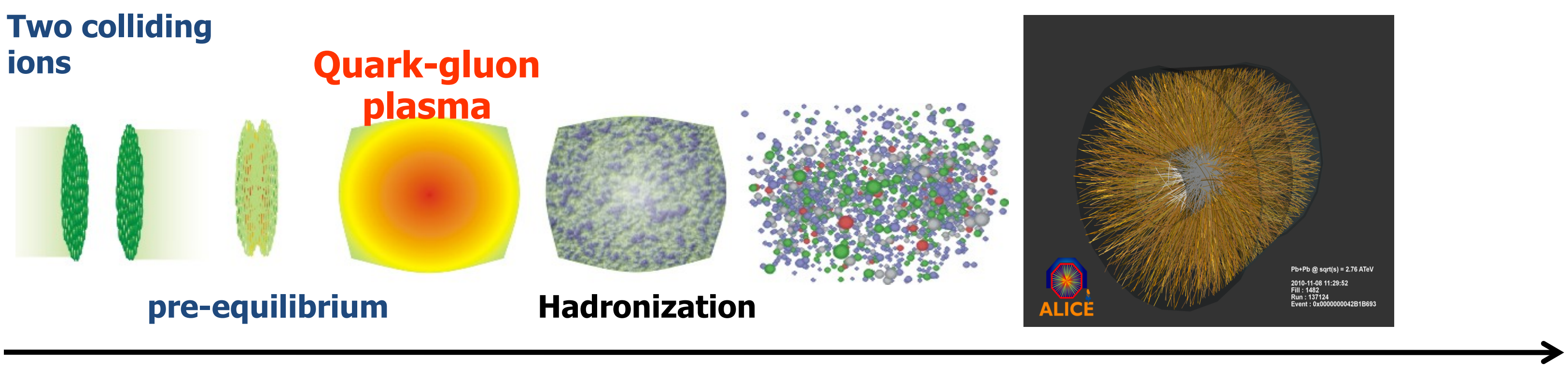
Au+Au collisions at STAR (RHIC). 200 GeV/nucleon pair. Running since 2000.

The Quark-Gluon-Plasma



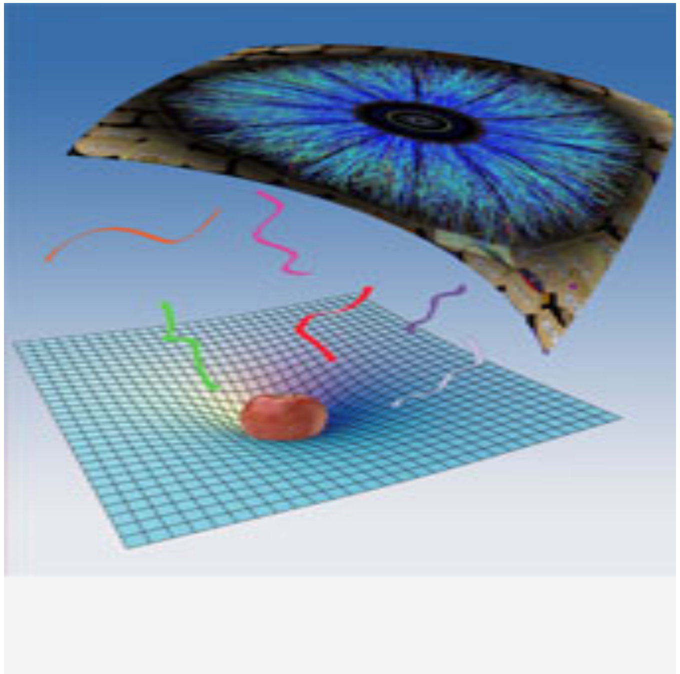
Heavy ion collisions
at RHIC and LHC indicate
formation of QGP:

- behaves like liquid
- Very small (shear viscosity)/s
- Strongly coupled
- Nearly scale invariant
- Very opaque (large jet quenching)



Pb+Pb collisions at ALICE (LHC). 3 TeV/nucleon pair. Running since 2010.

Holography and the Quark Gluon Plasma



- Strongly coupled thermal QFT ➔ Black Hole in higher dim.
- QFT Thermodynamics ➔Black Hole thermodynamics.
- Hydrodynamics ➔ Fluctuations around black hole background

Hydrodynamics

- Effective theory for small frequency, long wavelength fluctuations around local thermal equilibrium
- Characterized by transport coefficients to be extracted from microscopic theory.
- Defined by equations for conservation of stress energy and conserved global currents

$$T^{\mu\nu} = (\epsilon + P)u^\mu u^\nu + P g^{\mu\nu} - \sigma^{\mu\nu}$$

Ideal hydro + Dissipative terms

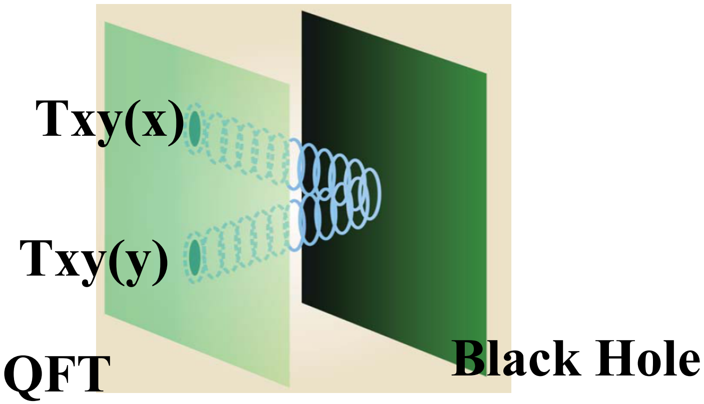
- Dissipative terms given in expansion in derivatives of the 4-velocity
- To first order: shear and bulk viscosities
- In principle can be computed from underlying theory.
- Example: Kubo formula for shear viscosity

$$\eta = -\mathrm{Lim}_{\omega \to 0} \frac{1}{\omega} \mathrm{Im} G^R_{xy,xy}(\omega, \mathbf{0})$$

$$G^R_{xy,xy}(\omega, \mathbf{0}) = \int dt\, d\mathbf{x}\, e^{i\omega t} \theta(t) \langle [T_{xy}(t, \mathbf{x}), T_{xy}(0, \mathbf{0})] \rangle$$

Shear viscosity from holography

- Compute using holographic correspondence!
- Remember: $T_{xy}(x)$ dual to $g_{xy}(x,r)$



QFT correlator = classical scattering of gravitons from black hole

$$\frac{\eta}{s} = \frac{1}{4\pi}\frac{\hbar}{K_B}$$

[Policastro, Son, Starinets, 2001; Kovtun, Son, Starinets 2004]

- Universal: for any isotropic fluid with classical gravity dual
- Surprisingly close to the measured value for
  - The Quark-Gluon-Plasma in QCD
  - Ultracold Fermi atoms at unitarity

Second order transport coefficients

[Romatschke 2009; F.B., Cotrone, Tarrio; F.B. , Cotrone, 2010]

Model: conformality broken by marginally relevant operator

$$\Delta \equiv (1 - 3c_s^2)$$

$$\frac{\eta}{s} = \frac{1}{4\pi} \qquad T\tau_\pi = \frac{2-\log 2}{2\pi} + \frac{3(16-\pi^2)}{64\pi}\Delta \qquad \frac{T\kappa}{s} = \frac{1}{4\pi^2}\left(1 - \frac{3}{4}\Delta\right)$$

$$\frac{T\lambda_1}{s} = \frac{1}{8\pi^2}\left(1 + \frac{3}{4}\Delta\right) \qquad \frac{T\lambda_2}{s} = -\frac{1}{4\pi^2}\left(\log 2 + \frac{3\pi^2}{32}\Delta\right) \qquad \frac{T\lambda_3}{s} = 0$$

$$\frac{T\lambda_4}{s} = 0 \qquad \frac{T\kappa^*}{s} = -\frac{3}{8\pi^2}\Delta \qquad T\tau_\pi^* = -\frac{2-\log 2}{2\pi}\Delta$$

$$\frac{\zeta}{\eta} = \frac{2}{3}\Delta \qquad T\tau_\Pi = \frac{2-\log 2}{2\pi} \qquad \frac{T\xi_1}{s} = \frac{1}{24\pi^2}\Delta$$

$$\frac{T\xi_2}{s} = \frac{2-\log 2}{36\pi^2}\Delta \qquad \frac{T\xi_3}{s} = \frac{T\xi_4}{s} = 0 \qquad \frac{T\xi_5}{s} = \frac{1}{12\pi^2}\Delta \qquad \frac{T\xi_6}{s} = \frac{1}{4\pi^2}\Delta$$

Note: QCD fireball nearly conformal and strongly coupled for $1.5T_c \lesssim T \lesssim 4T_c$ (RHIC, LHC).

Jet quenching parameter

- Transport coefficient characterizing probe parton energy loss
- Evaluated holographically in *N=4 SYM* [Liu, Rajagopal, Wiedemann 06]
- Adding Nf dynamical flavors [Bigazzi, Cotrone, Mas, Paredes, Ramallo, Tarrio 2009]

$$\hat{q} = \frac{\pi^{3/2}\Gamma(\frac{3}{4})}{\Gamma(\frac{5}{4})}\sqrt{\lambda}\, T^3 \left[1 + \frac{2+\pi}{64\pi^2}\lambda\frac{N_f}{N_c} + ...\right]$$

Quarks enhance jet quenching

Extrapolating to QGP: Nc=Nf=3, λ=6π, T=300 MeV, get

q≈ 4÷5 GeV^2/fm
right in the ballpark of data

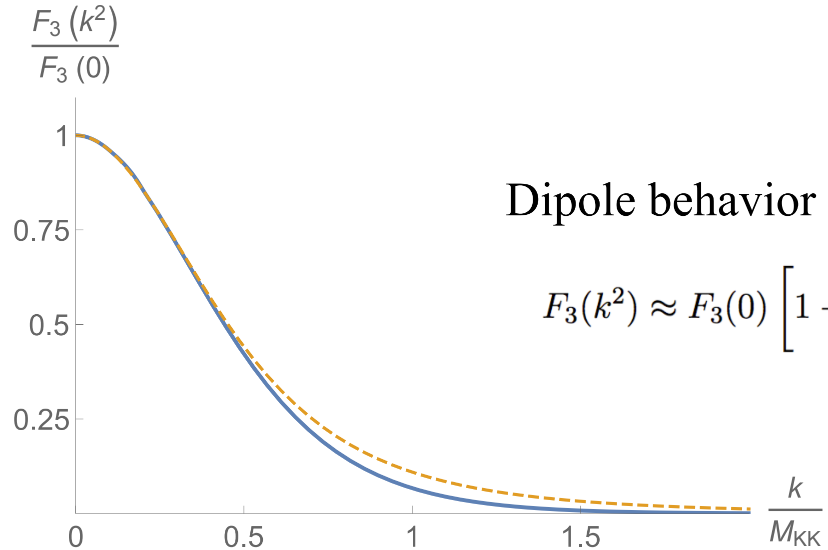
1. Holography and QCD: part II

CP violation in strong interactions: the QCD θ term

$$\mathcal{L}_\theta = \frac{1}{2g_{YM}^2}\mathrm{Tr}F_{\mu\nu}F^{\mu\nu} - \frac{i}{32\pi^2}\theta\,\epsilon^{\mu\nu\rho\sigma}\,\mathrm{Tr}F_{\mu\nu}F_{\rho\sigma}$$

- A non zero θ would induce a non zero neutron electric dipole moment
- Challenging on the Lattice (sign problem: imaginary term)

- Experiments: $|d_n| < 2.9 \times 10^{-26} e\,\mathrm{cm}$
- In a holographic model for QCD:
- Compute $d_n = 1.8\ 10^{-16}\ \theta\ e\ \mathrm{cm}$ [Bartolini, Bigazzi, Bolognesi,Cotrone, Manenti, 2016]
- Find complete electric dipole form factor



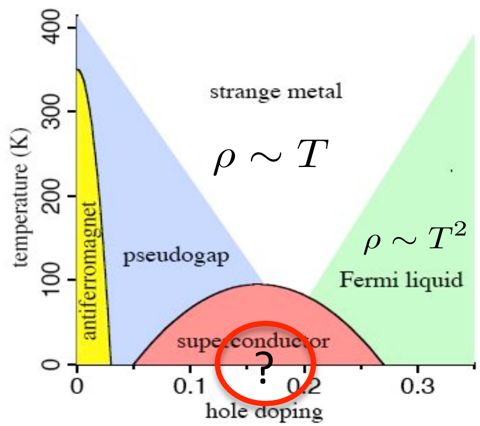
Dipole behavior at small k:

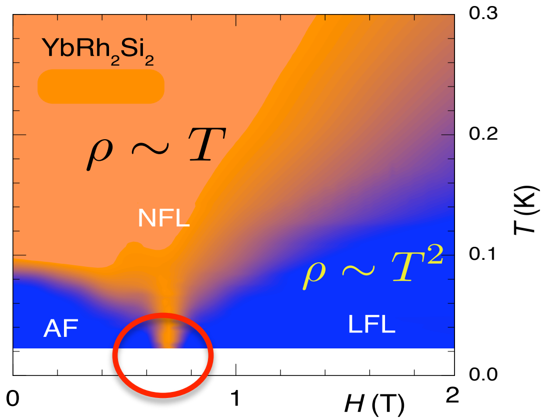
$$F_3(k^2) \approx F_3(0)\left[1 - 2\frac{k^2}{M_{KK}^2}\right]^{-2}$$

2. Holography and condensed matter

2. Strongly correlated electrons (Condensed Matter) (often layered, 2+1)

"High-T_c" Cuprates		Heavy Fermions
$YBa_2Cu_3O_{7-x}$	(YBCO)	$CeCu_{6-x}Au_x$
$La_{2-x}Sr_xCuO_4$	(LSCO)	$CeCoIn_5$
$Bi_2Sr_2CaCu_2O_{8+x}$	(BSCCO)	$YbRh_2Si_2$

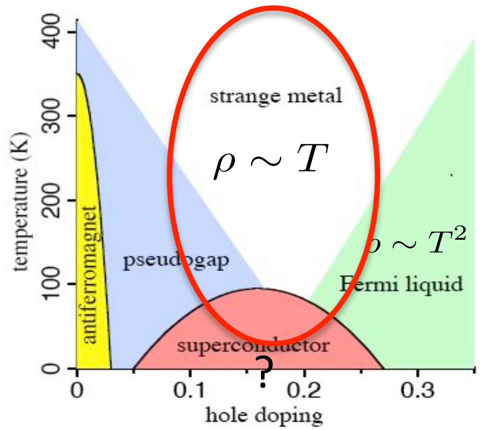


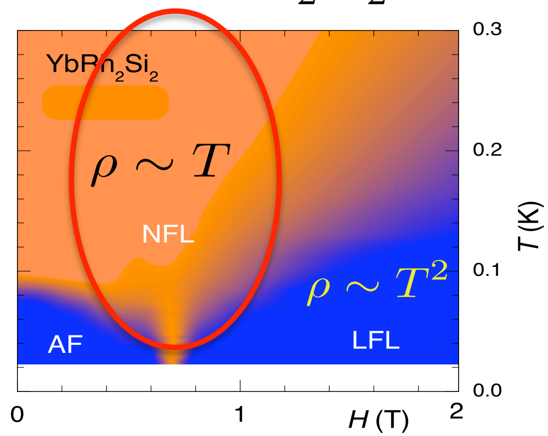


- Quantum phase transitions (T=0). Scale invariant QFT. Very large correlations

2. Strongly correlated electrons (Condensed Matter) (often layered, 2+1)

"High-T_c" Cuprates		Heavy Fermions
$YBa_2Cu_3O_{7-x}$	(YBCO)	$CeCu_{6-x}Au_x$
$La_{2-x}Sr_xCuO_4$	(LSCO)	$CeCoIn_5$
$Bi_2Sr_2CaCu_2O_{8+x}$	(BSCCO)	$YbRh_2Si_2$

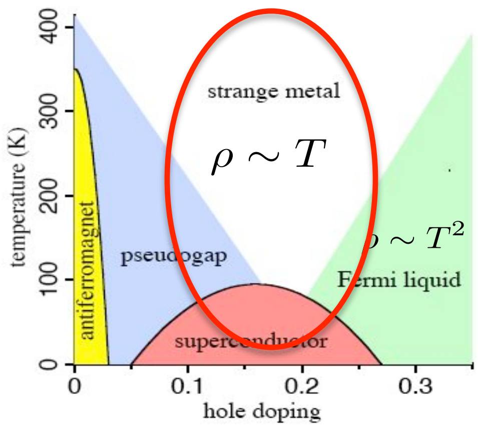


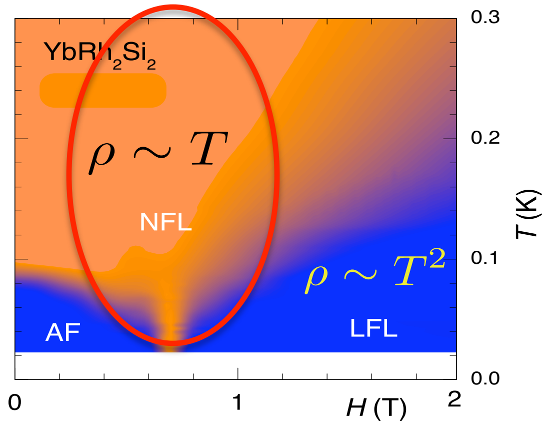


- Quantum phase transitions (T=0). Scale invariant QFT. Very large correlations
- Quantum critical region. Affected by quantum critical point at T=0 [Sachdev]
- Exhibit phases which escape standard paradigms based on quasi-particle description
- Not Landau-Fermi liquids. Not BCS superconductors
- E.g. strange metallic phase with linear (in T) resistivity
- No satisfactory theoretical understanding

- 2. Strongly correlated electrons (Condensed Matter)

"High-T_c" Cuprates		Heavy Fermions
$YBa_2Cu_3O_{7-x}$	(YBCO)	$CeCu_{6-x}Au_x$
$La_{2-x}Sr_xCuO_4$	(LSCO)	$CeCoIn_5$
$Bi_2Sr_2CaCu_2O_{8+x}$	(BSCCO)	$YbRh_2Si_2$





- Example of challenging observable at strong coupling: (optical) conductivity
- Ohm's law: J = σ E.   σ: retarded correlator of U(1) current J;   (ρ=1/Re[σ(0)])

$$\sigma(\omega) = \mathrm{Lim}_{k\to 0}\frac{G^R_{JJ}(\omega,k)}{i\omega}$$

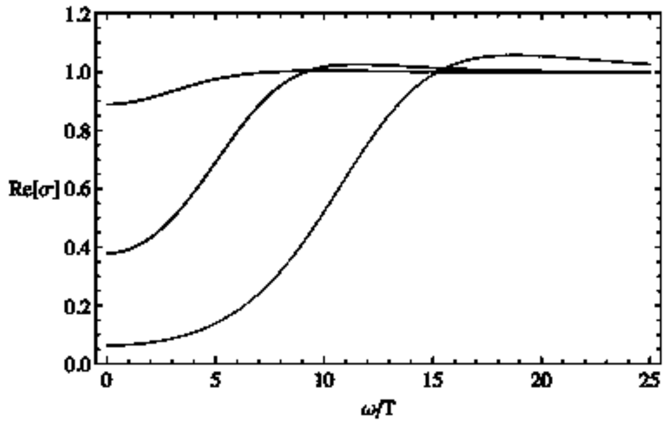
$$G^R_{JJ}(\omega,k) = -i\int d^{d-1}x\,dt\,e^{i\omega t - ikx}\theta(t)\langle[J(t,x),J(0,0)]\rangle$$

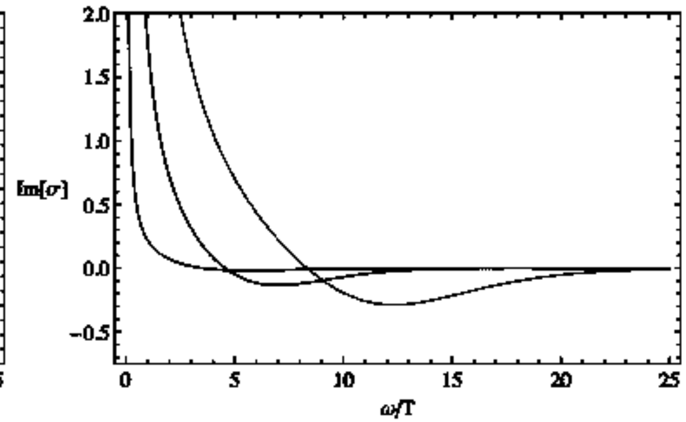
Optical conductivity in d=2+1 from holography

From fluctuations around dual charged black hole

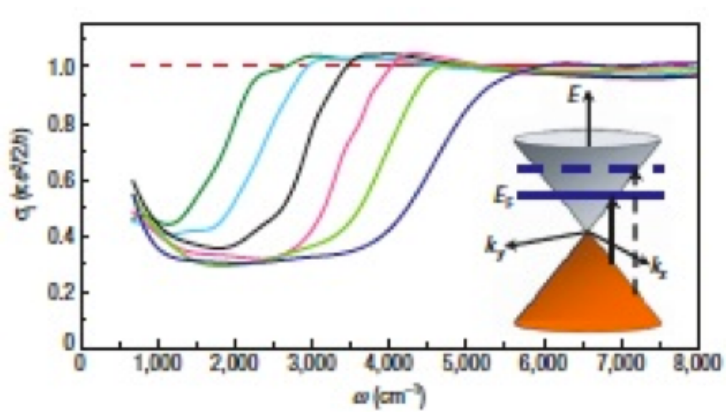
- Jx = σ Ex = - iω σ Ax

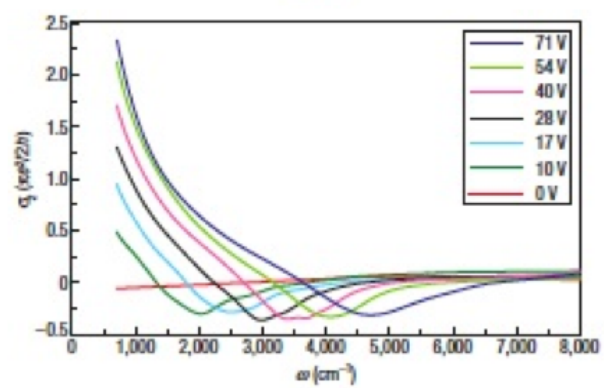
$$\sigma(\omega) = \frac{-iG^R_{J_xJ_x}(\omega)}{\omega}$$





Cfr with graphene (at low energy a relativistic theory in 2+1)



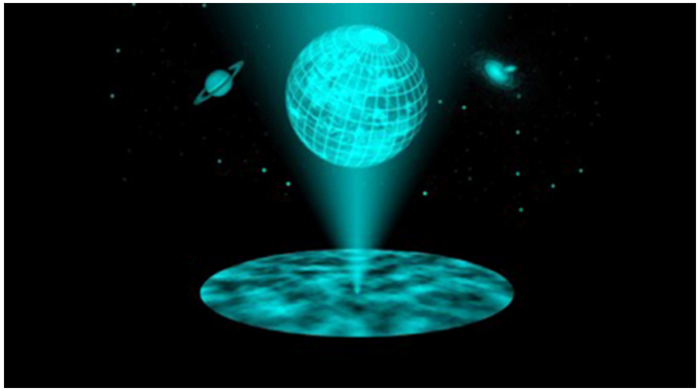


Many other directions

- S-wave, p-wave and d-wave "high Tc" superconductivity [Gubser; Hartnoll, Herzog, Horowits; Erdmenger…]: charged "hairy" black hole solutions.
- Holographic Fermi Surfaces [McGreevy, Liu, Iqbal…]
- Entanglement entropy [Ryu, Takayanagi] from minimization of a surface in gravity having as boundary the subspace A.
- Thermalization and quantum quenches. E.g. black hole formation from colliding shock waves [Gubser et al] or time dependent gravity solutions [Craps et al].
- QCP with generic dynamical critical exponent z: from more general gravity solutions coupled to scalars [Kiritsis, Sachdev et al]
- Spintronics: charge and spin currents-> U(1)xU(1) gravity. Non trivial conductivity matrix (spin-charge conductivity) universally [F.B., Cotrone, Seminara, Musso, Fokeeva, 12]

3. And more…

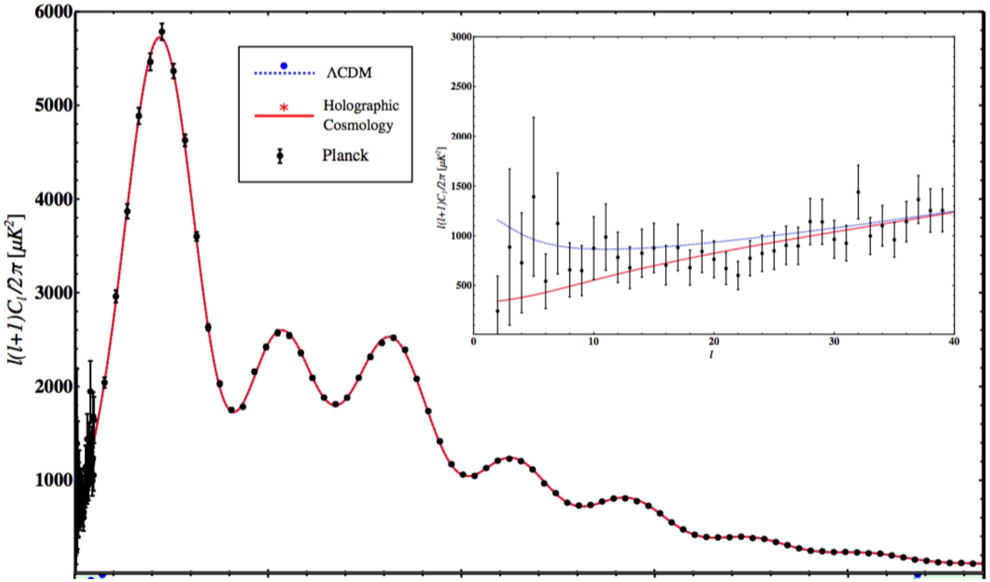
Holographic Cosmology



- Early times Universe has large curvature: need quantum gravity in 4d
- Holography: 4d quantum gravity (on de Sitter) ➔ weakly coupled QFT in 3d
  [McFadden, Skenderis, 2010]

- HC competitive to ΛCDM
- 7 parameters
- Perturbative 3d QFT ok for relatively large multipoles ($l \geq 30$)

[Skenderis et al. 2017 (PRL)]



Holography in Florence

- Staff

Francesco Bigazzi

Aldo L. Cotrone

Domenico Seminara

- Postdocs

Flavio Porri

Gabriele Martelloni

- Ph.D. Students

Sara Bonansea

Thank you