

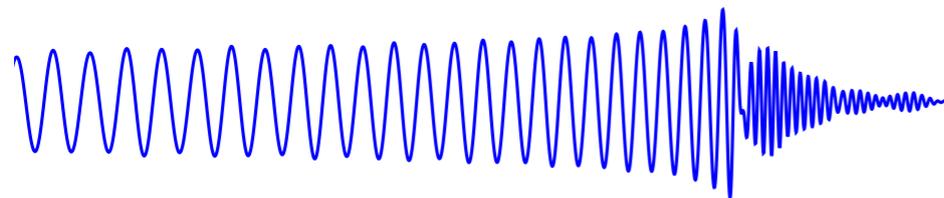
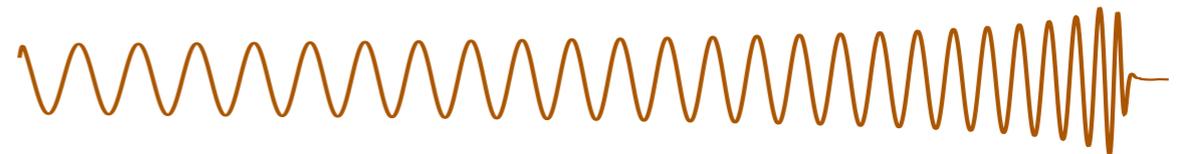
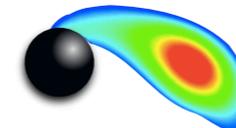
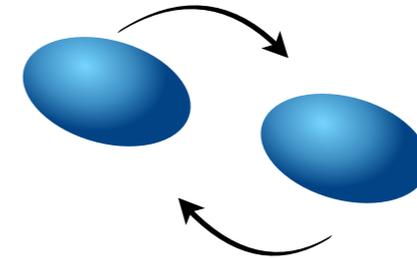
# Tidal effects in the GWs from binary inspirals

Tanja Hinderer  
(AEI Potsdam)

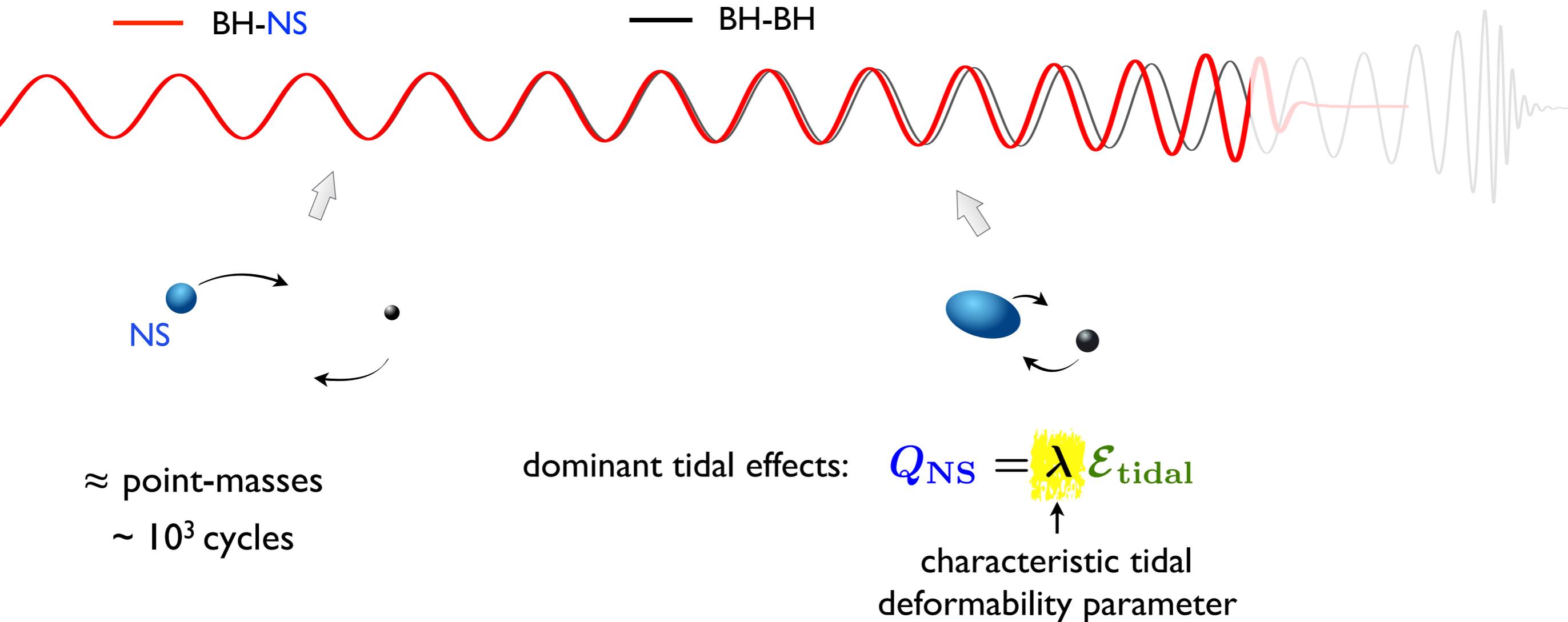
# Overview

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- Main influence of neutron star (NS) **matter** on the **GW signals** during the inspiral: adiabatic tidal effects
- Characteristic tidal deformability parameters
- Dynamical f-mode tides
- Status of models, tests for NS-BH and NS-NS
- Outlook

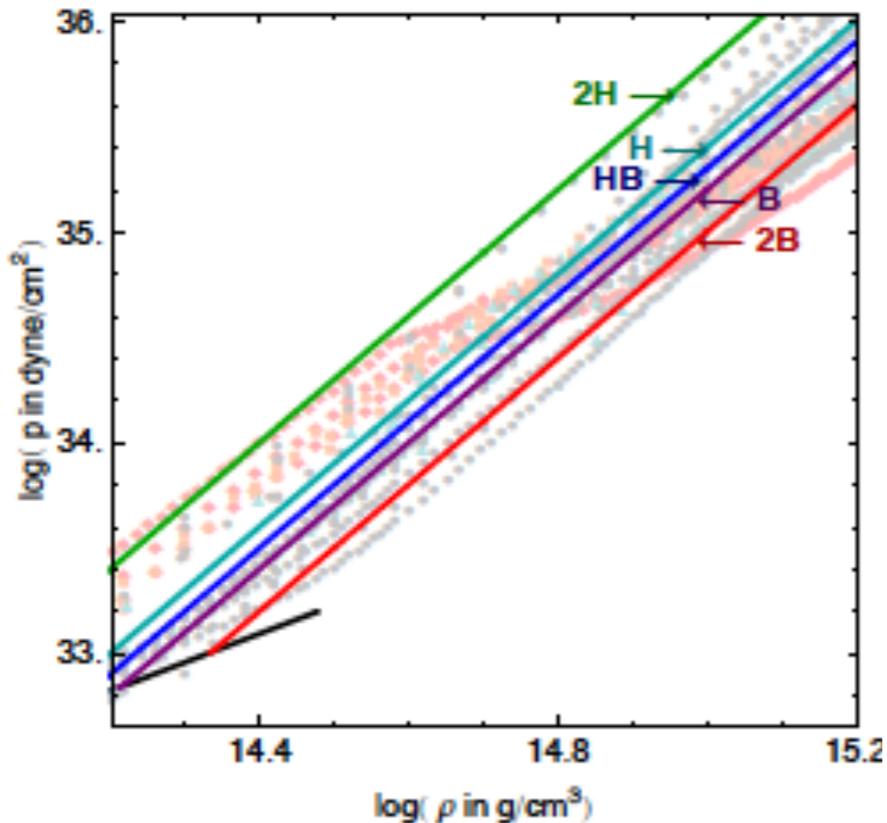


# Influence of NS matter on GWs (inspiral)



# Tidal deformability / Love number parameter

log (pressure - density)

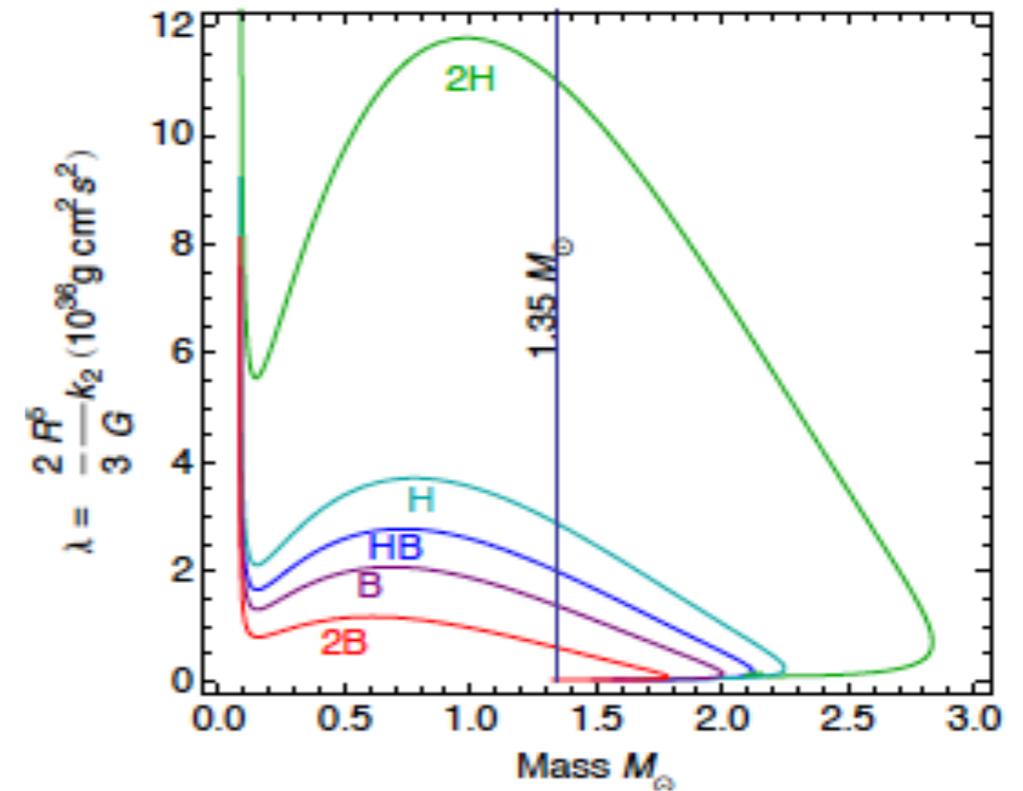


Einstein's Eqs:  
linear, static perturbations  
to equilibrium sol.



$$\lambda_\ell = \frac{2}{(2\ell - 1)!!} k_\ell R^{2\ell+1}$$

$\lambda$ - mass



credit: B. Lackey

- $\lambda=0$  for a BH [Binnington+09, Kol+11]
- also characterizes exotic objects [Pani 15, Uchikata+16, Mendes+16, Cardoso+17, Sennett+17]
- similar treatment for higher multipoles and gravitomagnetic interactions

[Damour+09, Binnington+09, Landry+14]

# Influence on the GWs

- **Energy** goes into deforming the NS

$$E \sim E_{\text{orbit}} - \frac{1}{4} Q_{\text{NS}} \mathcal{E}_{\text{tidal}}$$

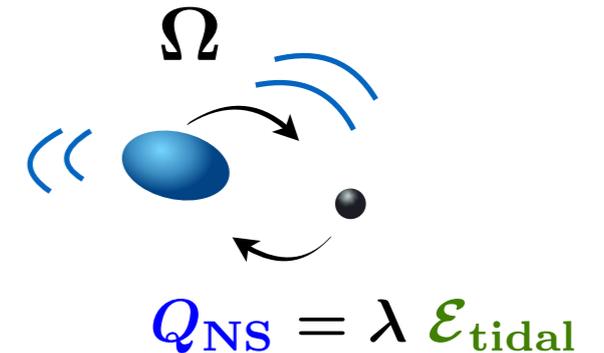
- moving tidal bulges contribute to **gravitational radiation**

$$\dot{E}_{\text{GW}} \sim \left[ \frac{d^3}{dt^3} (Q_{\text{orbit}} + Q_{\text{NS}}) \right]^2$$

- approx. **GW phase**:  $\frac{d\phi_{\text{GW}}}{dt} = 2\Omega$ ,  $\frac{d\Omega}{dt} = \frac{\dot{E}_{\text{GW}}}{dE/d\Omega}$

- for NS-NS: most sensitive to the **weighted average**:

$$\tilde{\Lambda} = \frac{1}{26} \left[ \left( 1 + 12 \frac{m_2}{m_1} \right) \lambda_1 + \left( 1 + 12 \frac{m_1}{m_2} \right) \lambda_2 \right]$$



$$M = m_1 + m_2$$

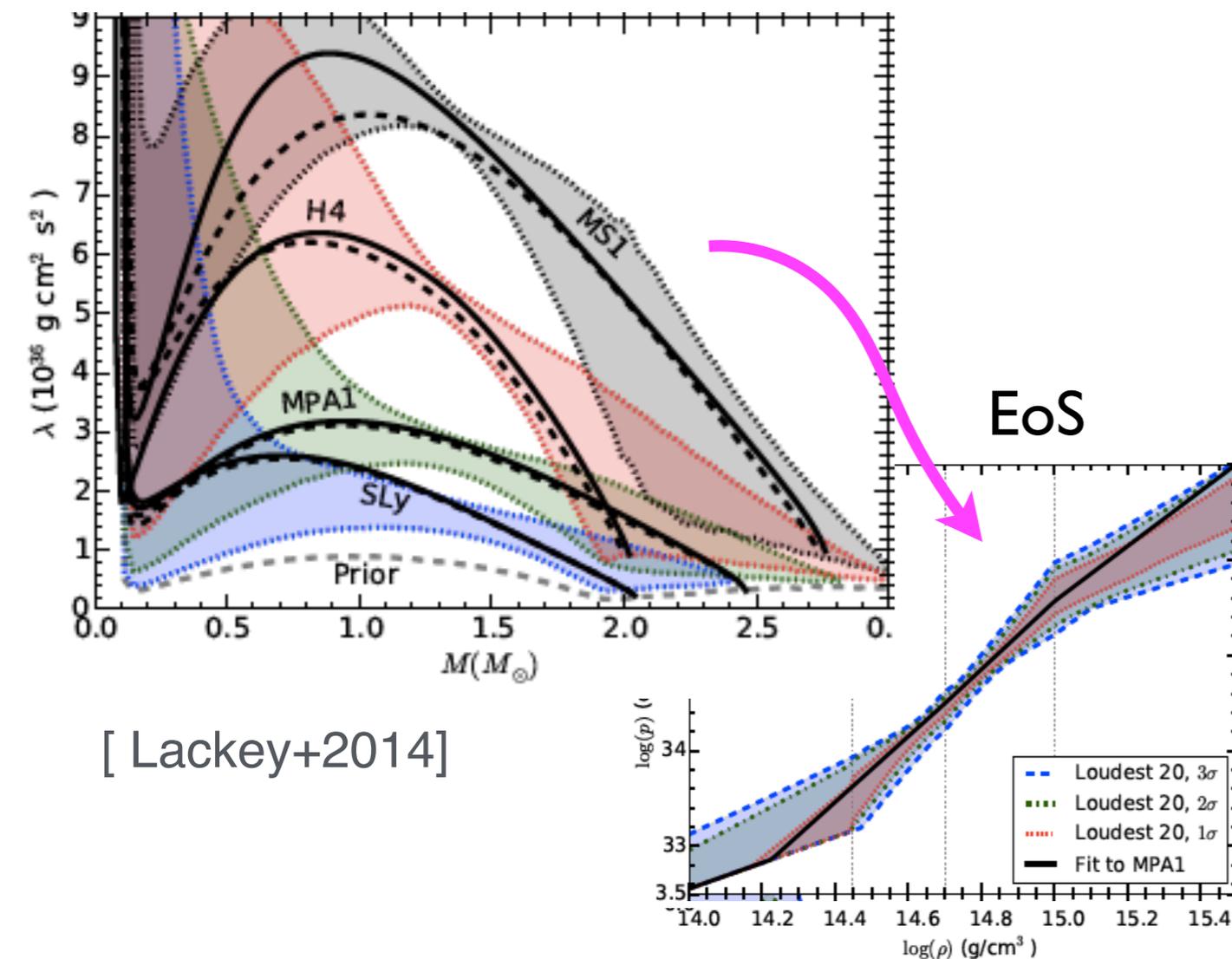
$$\Delta\phi_{\text{GW}}^{\text{tidal}} \sim \lambda \frac{(M\Omega)^{10/3}}{M^5}$$

# Preliminary expectations with aLIGO+Virgo

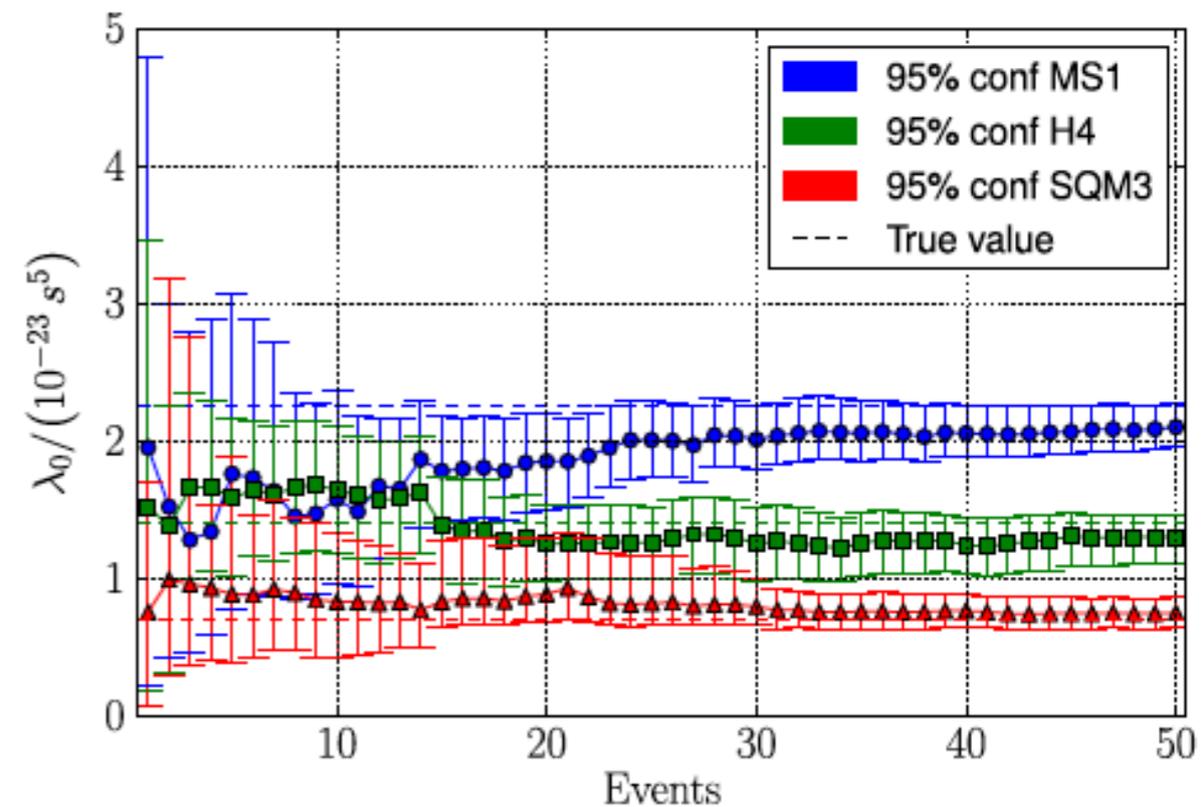
- tens of **NS-NS** events could potentially give: [del Pozzo+2013, Lackey+2014, Agathos+2015]
  - $\lambda$  to  $\sim 10-50\%$ , radius to  $\sim 1-2$  km, pressure to  $\sim$  factor of 2
  - similar conclusions with **hybrid NR** waveforms [Shibata+2016]
- NS-BH** systems:  $\lambda/m^5$  to  $\sim 10-100\%$  [Lackey+ 13, Kumar+16 (full waveforms)]

*caveats*

example results:  $\lambda$

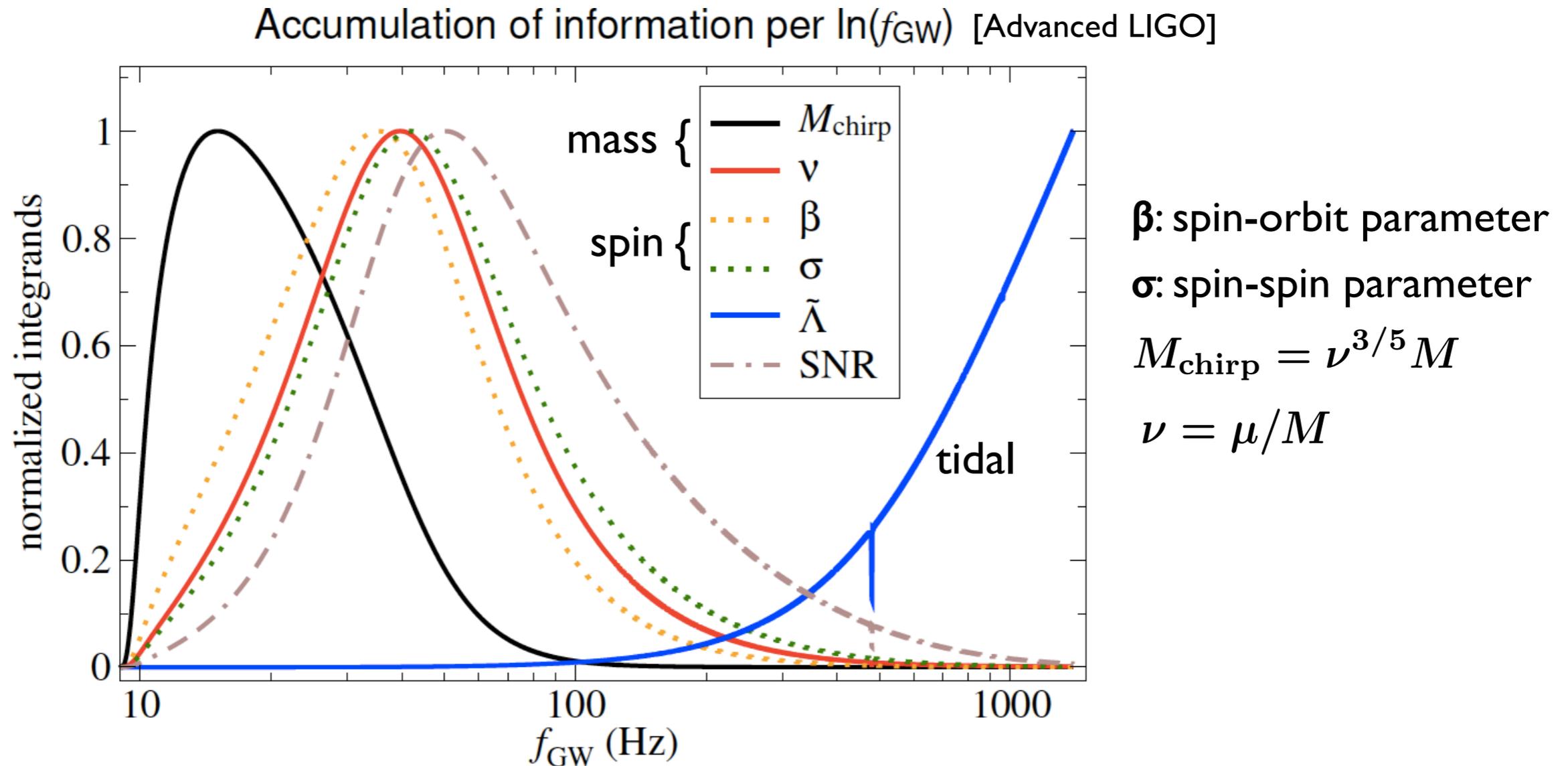


example results:  $\lambda(1.4M_{\text{sun}})$



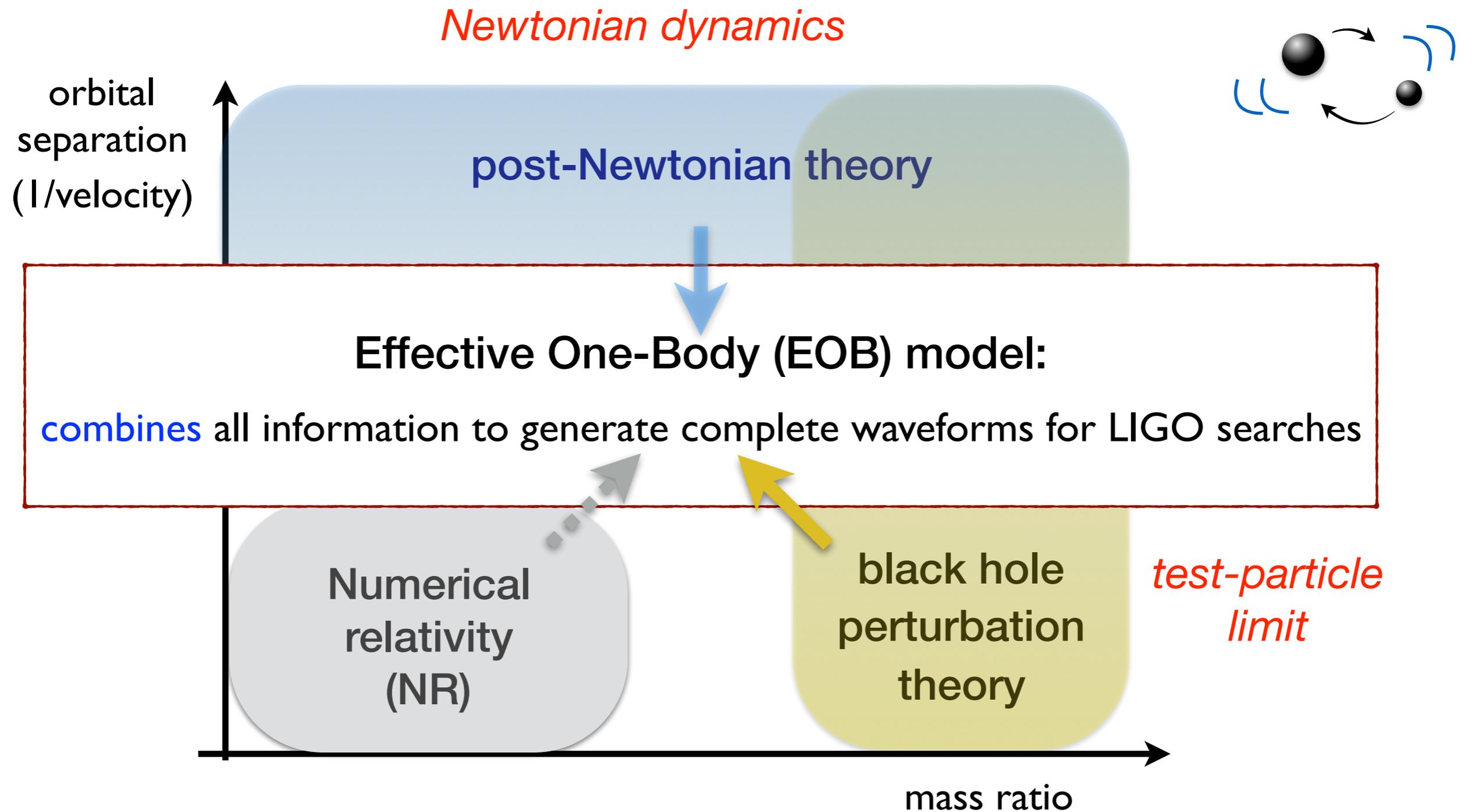
# Information on parameters in the GW signal

- Information about parameters is contained in different frequency ranges

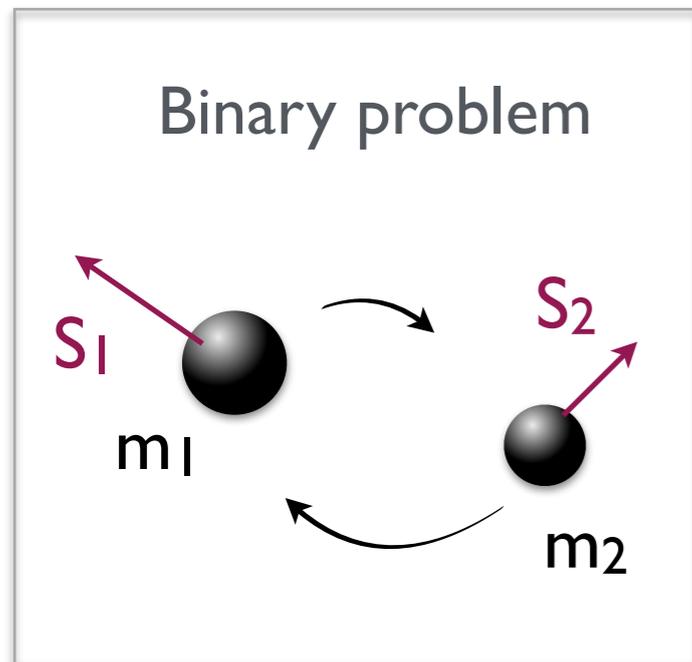


- to avoid large systematics in tidal parameter: need a good point-particle description

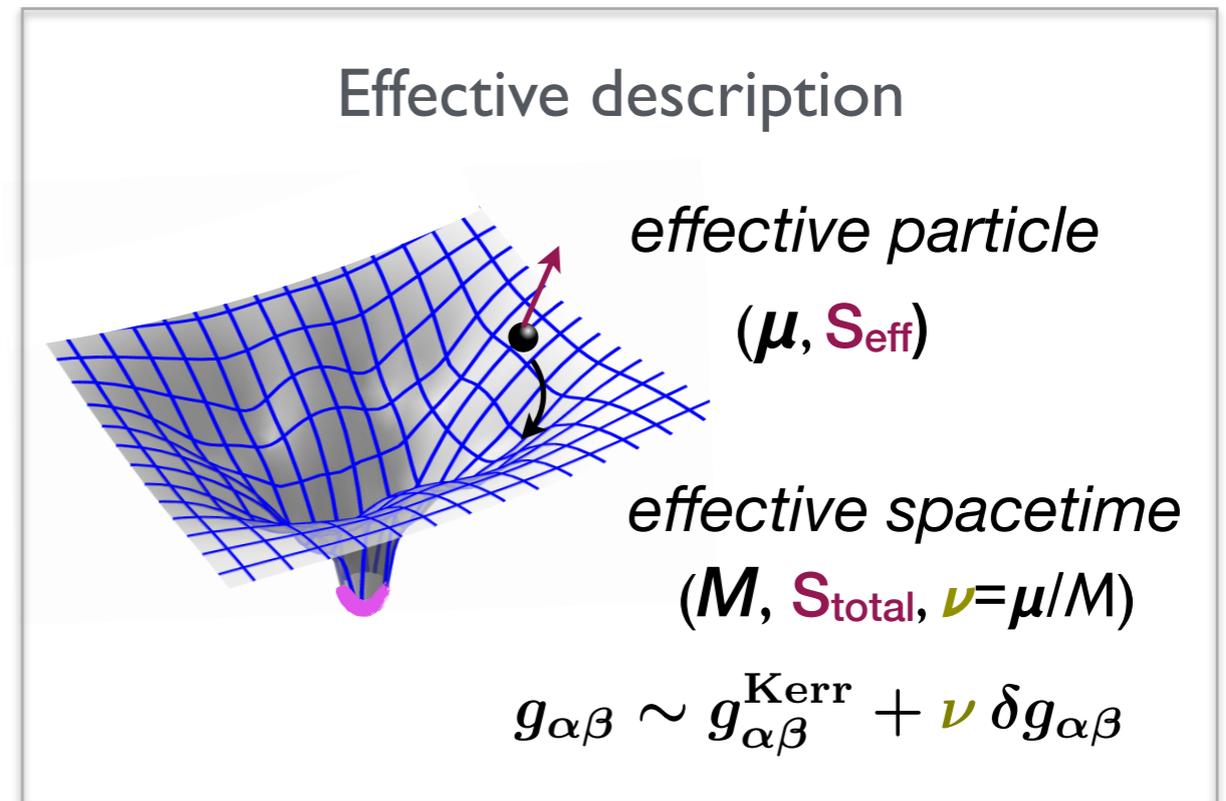
# Approaches to the two-body problem



# Effective One-Body (EOB) model



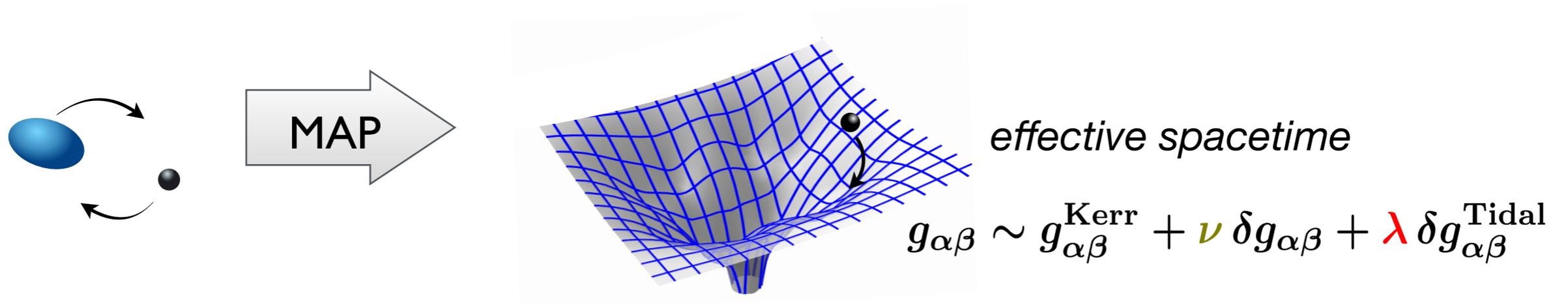
[Buonanno, Damour  
1999, 2000; Barausse+]



$$M = m_1 + m_2 \quad \mu = m_1 m_2 / M$$

EOB **Hamiltonian** + GW dissipation + wave generation + merger-ringdown

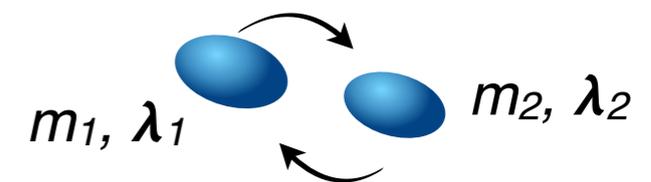
# Adiabatic tidal effects in the EOB model



[Damour, Nagar, Bini, Faye, Bernuzzi+2009-2014]

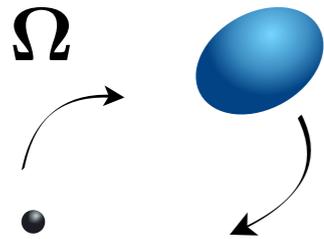
- for NS-NS: dominant tidal contribution in EOB Hamiltonian characterized by

$$\kappa_2^{\text{T}} = 3 \left( \frac{m_2}{m_1} \lambda_1 + \frac{m_1}{m_2} \lambda_2 \right)$$



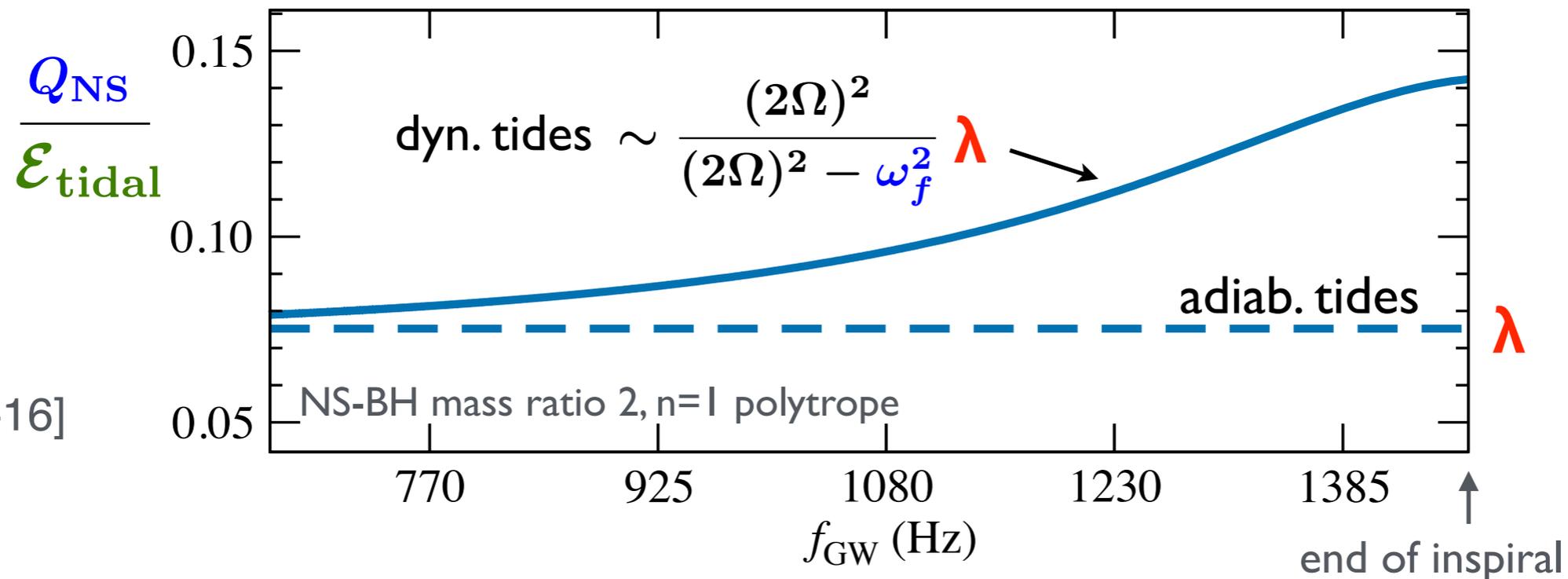
+ tidal correction to GW amplitude [Damour+2012]

# Dynamic $f$ -mode tides



- $Q_{NS}$  corresponds (mainly) to the NS's  $f$  fundamental oscillation modes
- $f$ -mode frequency:  $\omega_f \sim \sqrt{m_{NS}/R^3}$  (internal structure - dependent)
- **tidal forcing** frequency:  $\sim 2\Omega \sim 2\sqrt{M/r^3}$

NS's tidal response during the inspiral

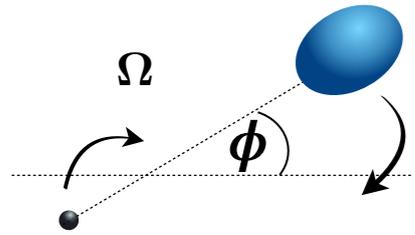


[Hinderer+16]

- separation-dependent tidal **enhancement** also found in **affine** approach

[Ferrari, Gualtieri, Maselli, Pannarale 11, 12]

# Description of dynamic tides



- harmonic-oscillator-like Lagrangian:

$$L = L_{\text{orbit}} + \sum_{\ell \geq 2} \sum_{m=-\ell}^{\ell} \left[ -\frac{z}{2} Q_{\ell m} \underbrace{\mathcal{E}_{\ell m} e^{-im\phi}}_{\text{BH's tidal field (from Weyl tensor)}} + \frac{1}{4\lambda_{\ell} z^2 \omega_{0\ell}^2} \left( \dot{Q}_{\ell m}^2 - z^2 \omega_{0\ell}^2 Q_{\ell m}^2 \right) + L_{\text{frame}} \right]$$

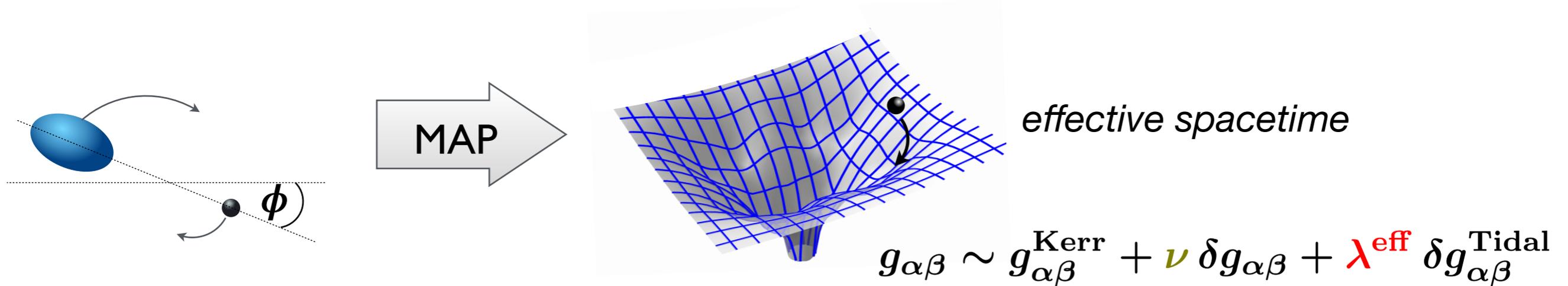
↑ redshift
f-mode frequency
↑ coupling of orbital and  $Q_{\ell m}$ 's angular momentum

- adiabatic limit:

$$L^{\text{AT}} = L_{\text{orbit}} + \sum_{\ell \geq 2} \sum_{m=-\ell}^{\ell} \frac{z}{4} \lambda_{\ell} \mathcal{E}_{\ell m}^2$$

↑ tidal invariants computed from post-Newtonian or gravitational self-force

# EOB with *approximate* dynamic tides



- effective tidal deformability encodes dynamical tides:

$$\frac{\lambda_l^{\text{eff}}}{\lambda_l} \sim \frac{\omega_f^2}{\omega_f^2 - (m\Omega)^2} \& \frac{\omega_f^2}{(\phi - \phi_f)} \& \cos [(\phi - \phi_f)^2] \text{FresnelS}(\phi - \phi_f)$$

↑  
*before  
resonance*

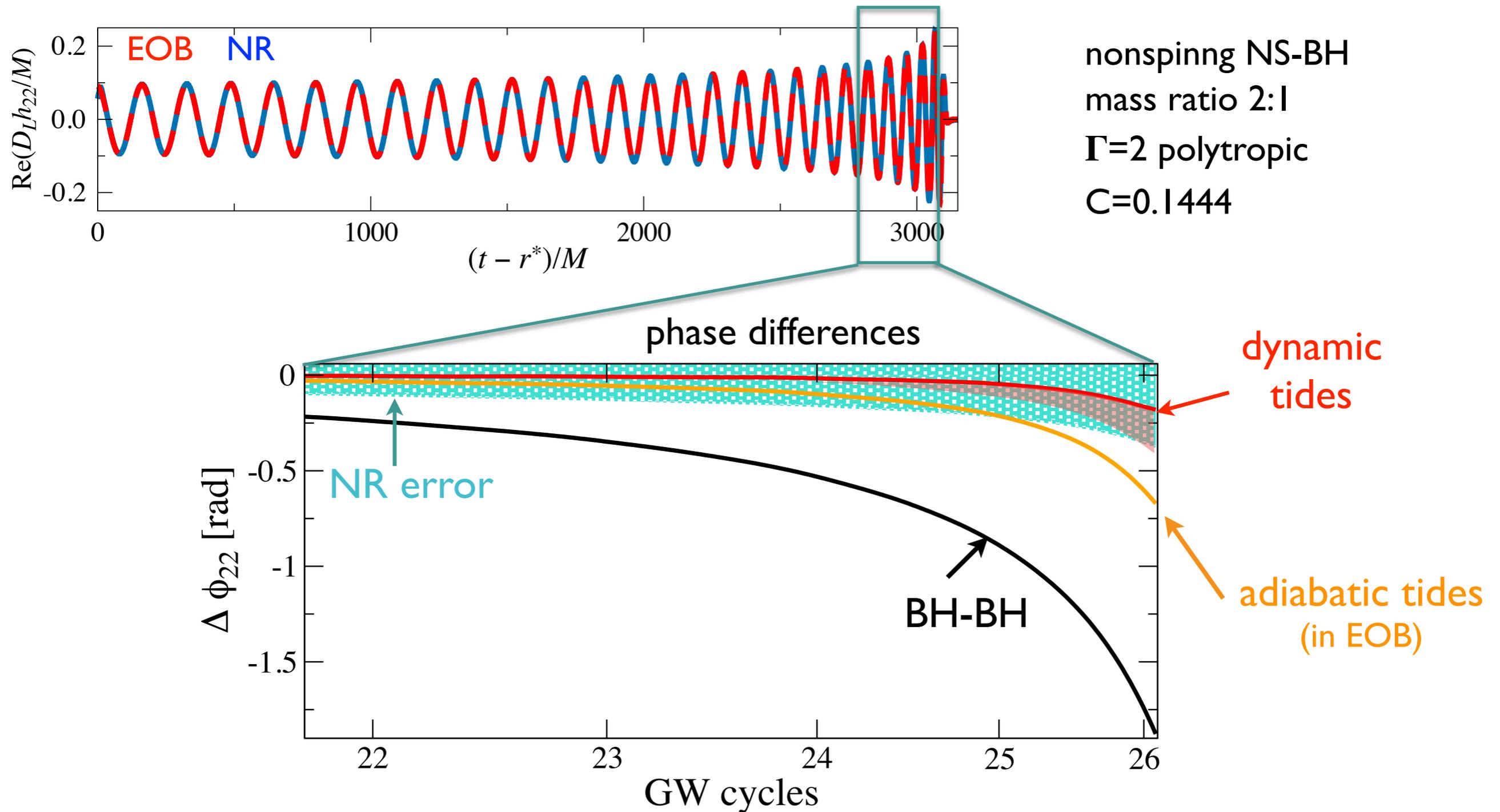
↑  
— *common  
term*

↑  
*near resonance  
where  $\phi \sim \phi_f$*

all fns. of  $\{M, \nu, \omega_f, r\}$

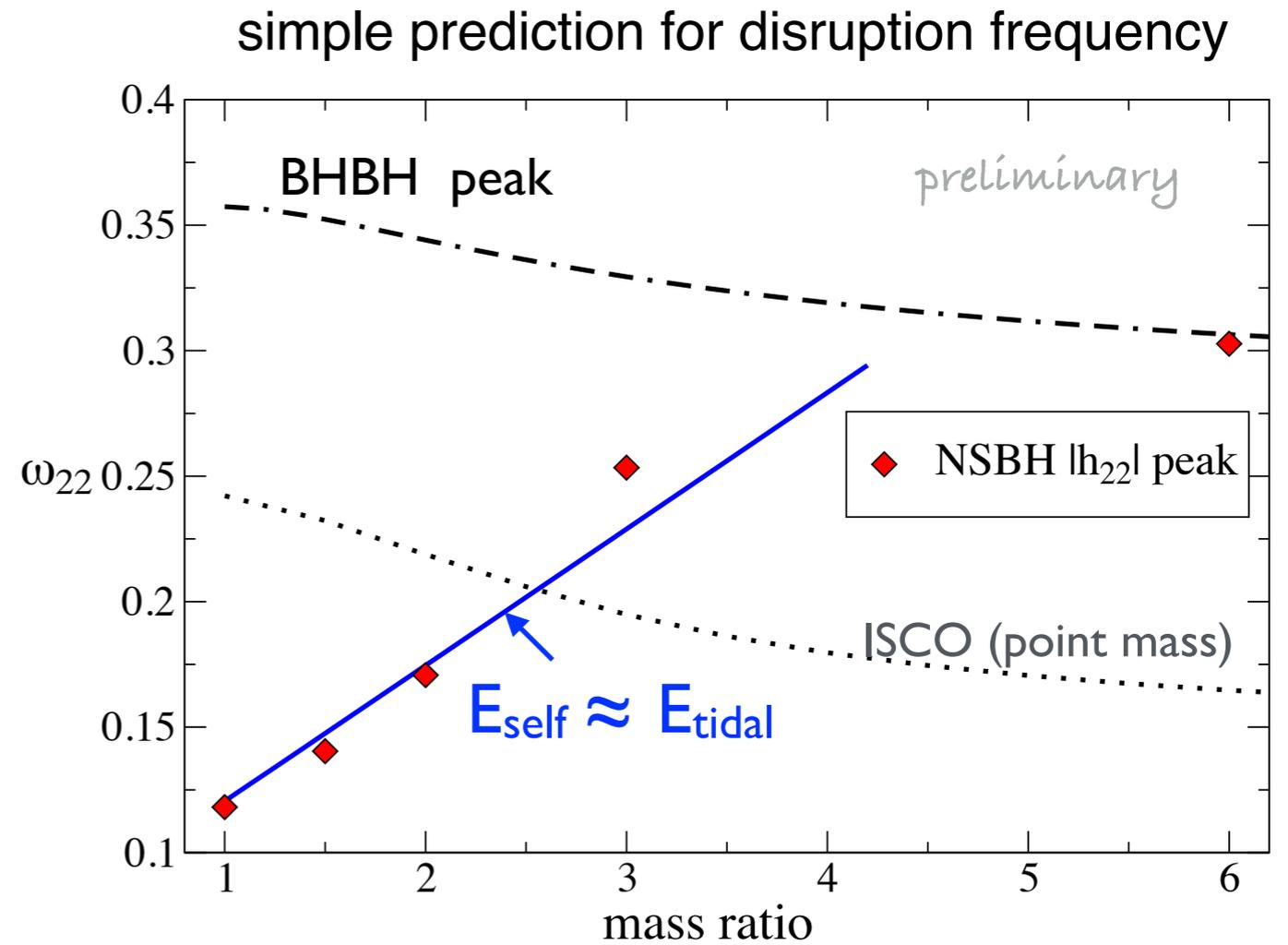
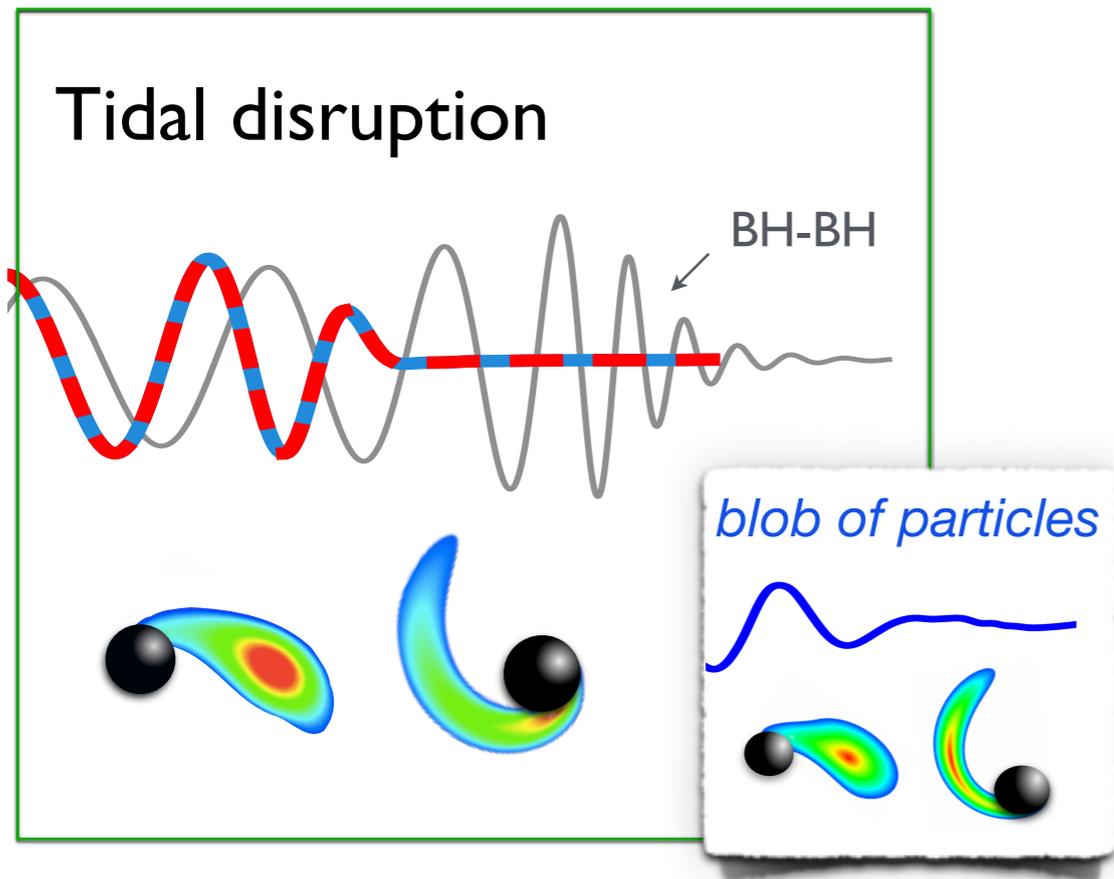
- analogous effective quantity for  $h_{lm}$  mode amplitudes

# Comparison of EOB and NR for NS-BH



NR simulations by F. Foucart, SXS

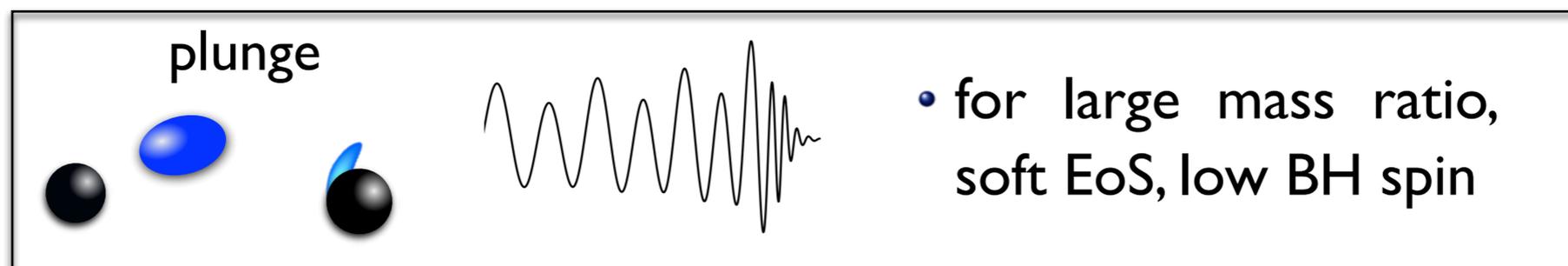
# Final fate of NS-BH binaries



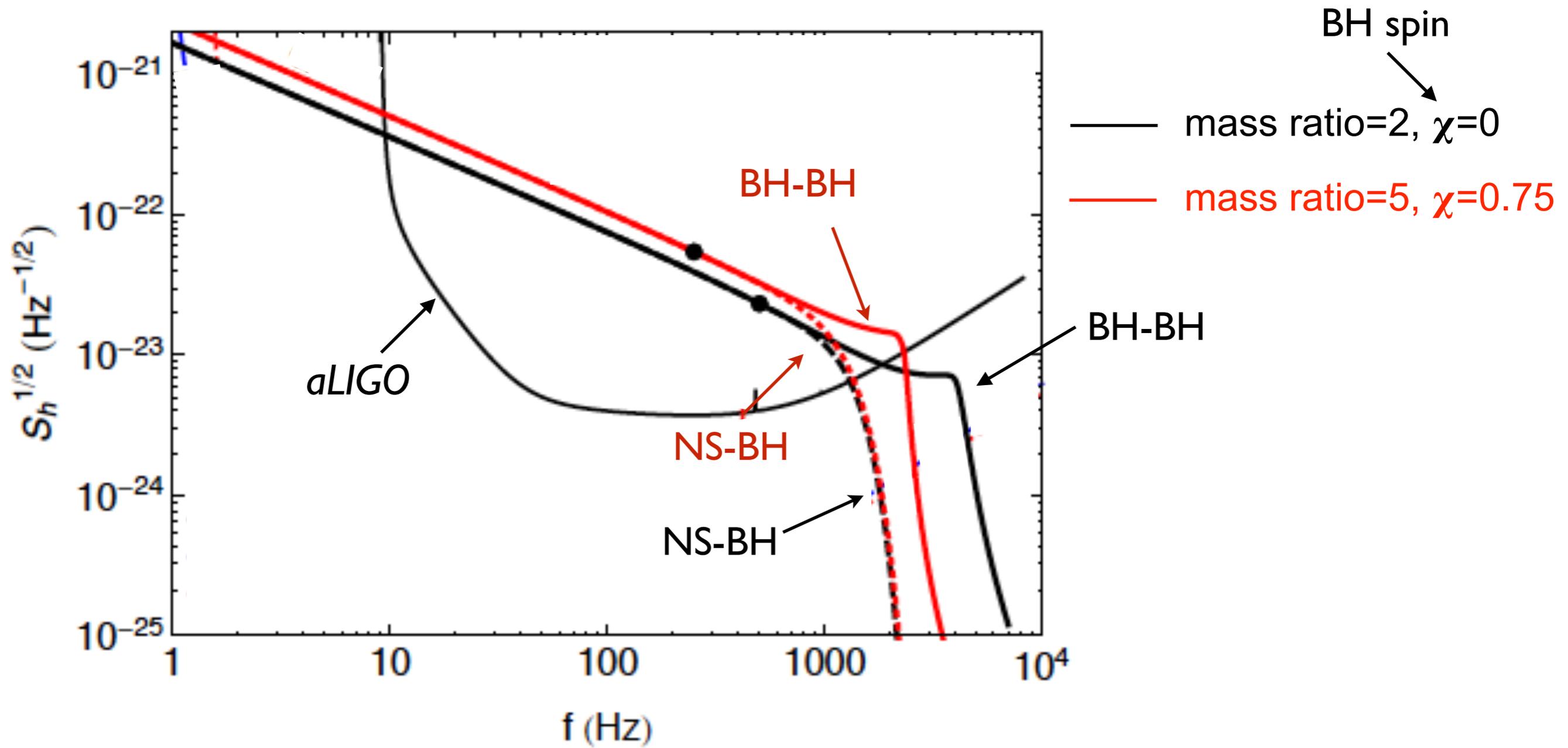
- **GW shutoff frequency:** depends also on  $\Lambda$

$$\Lambda = \frac{\lambda}{m^5}$$

[Lackey+2013, Gualtieri+2014, Kyuotoku+2015]



# Frequency domain NS-BH signal

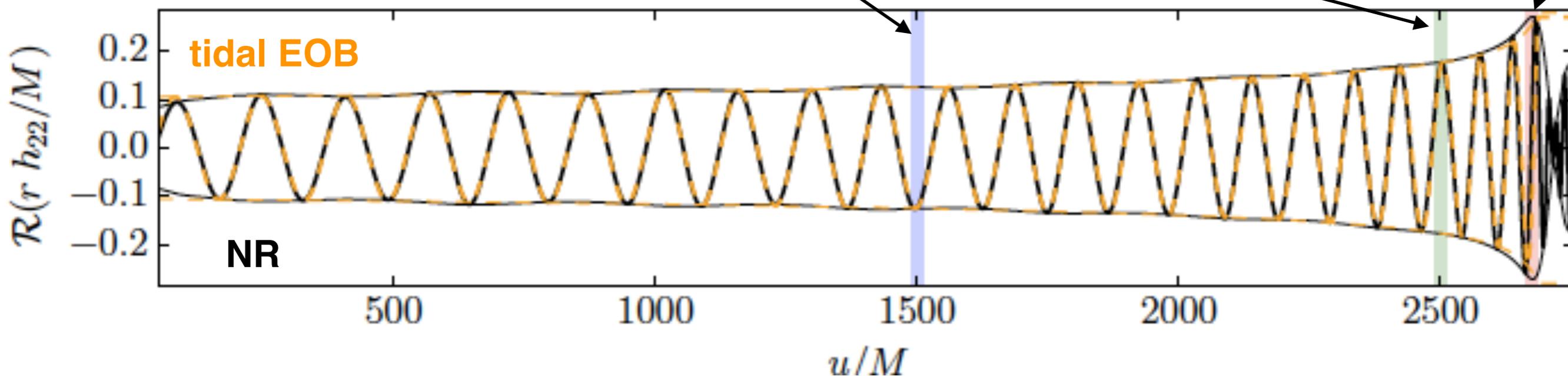
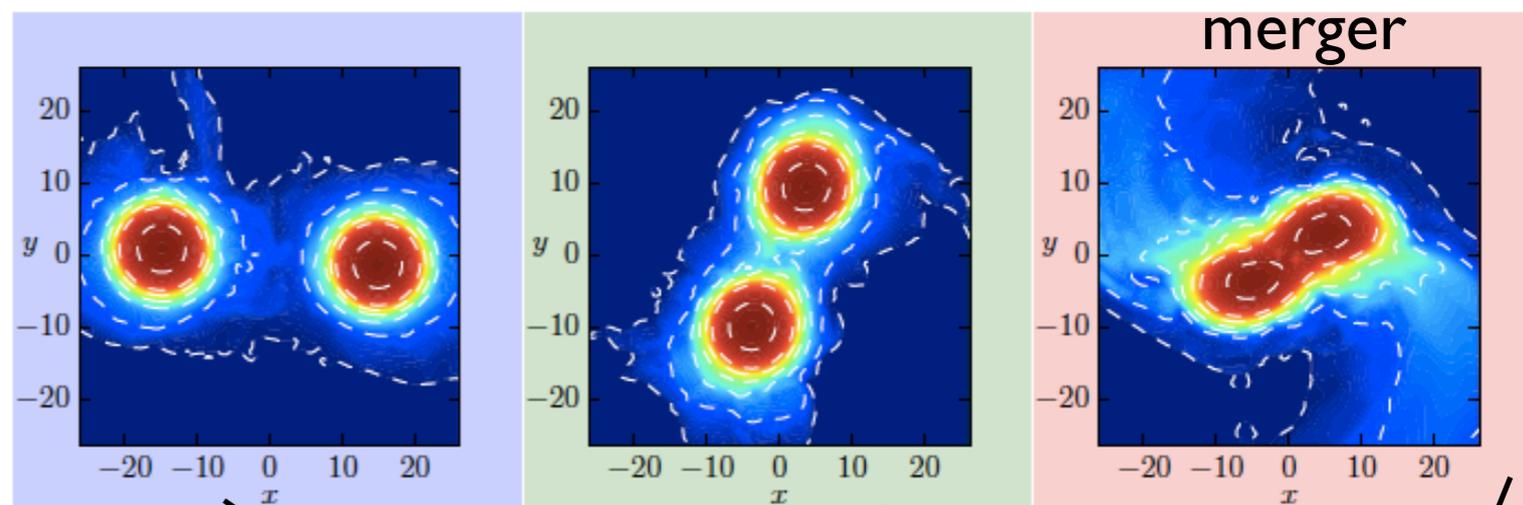


[Lackey+14]

# NS-NS binary inspiral: soft EoS

equal masses  
SLy matter model

NR simulations by  
T. Dietrich (AEI)

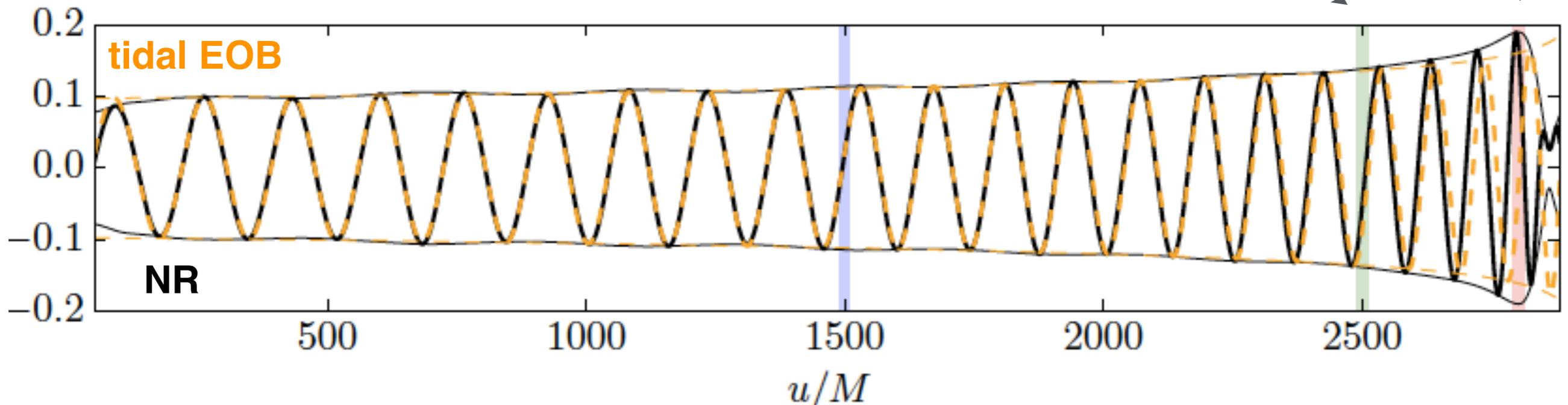
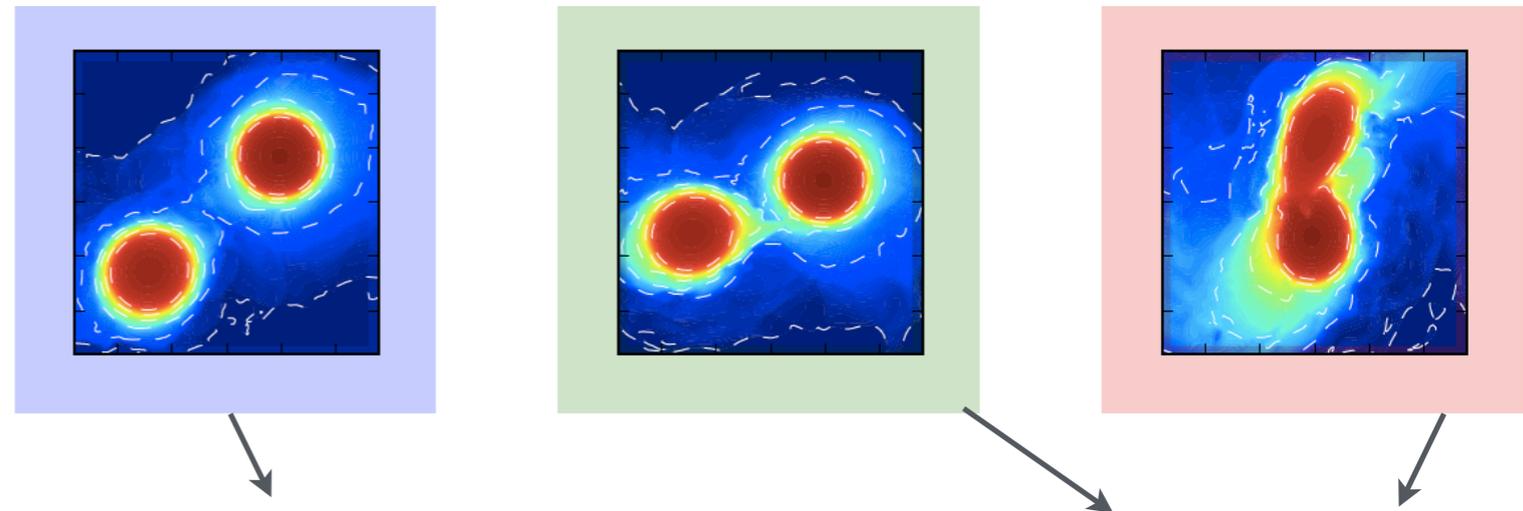


[Dietrich+17]

# Comparison for NS-NS binary inspiral: stiff EoS

mass ratio 1.65:1.1  
MS1b matter model

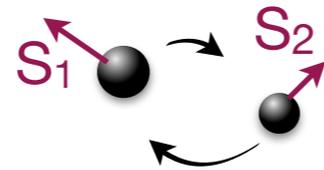
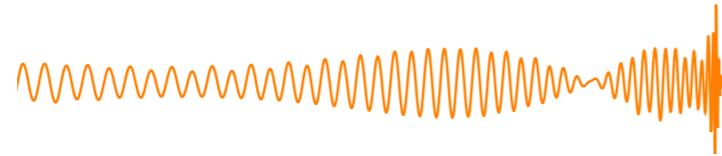
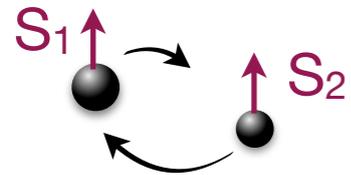
NR simulations by T. Dietrich (AEI)



[Dietrich+ 17]

# Physics included in current GW template models

## spin effects (BHs)



most tests for mass ratio  $< 8$

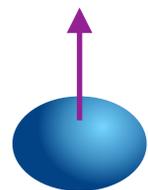
- EOB

[Bohe+16, Nagar+17], [Babak+16]

- PhenomX

[Ajith+ 08, Hannam+ 14, Khan+16]

## matter effects



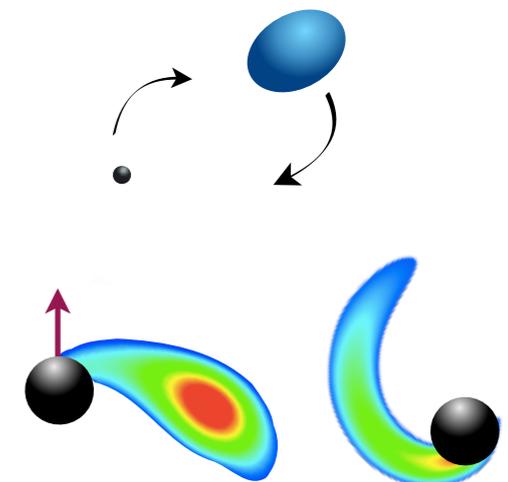
- tidally induced multipoles ( $\ell=2,3,4$ ) [post-Newtonian & EOB]

- spin-induced deformation [post-Newtonian]

- dynamical  $f$ -mode tides [EOB]

- NSBH tidal disruption for moderate aligned BH spin [LEA+]

- tidal heating [Maselli+17]

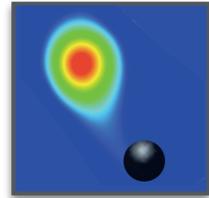
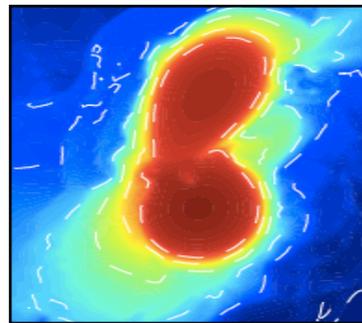


approx. **universal relations** used to reduce EoS parameters in surrogate GW templates to  $\Lambda_1, \Lambda_2$

# Remaining challenges

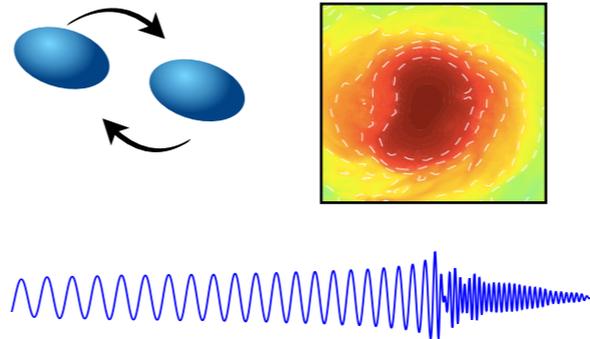
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late inspiral

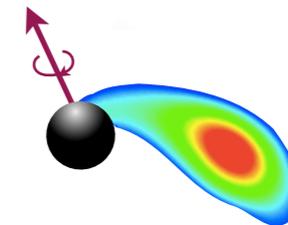


near disruption

complete model



tidal disruption for  
arbitrary spins

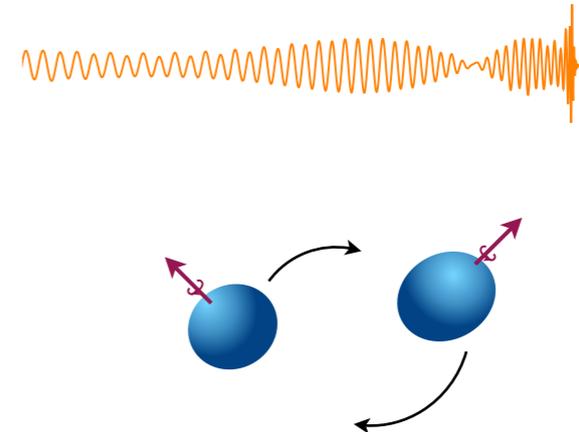


- continue to **test, improve** models — requires increasingly **accurate NR** simulations
- quantify **systematic** uncertainties
- optimize for **data analysis**
- choice of priors? best way to link constraints on  $\Lambda(m, \text{EoS})$  to EoS?

# Remaining challenges

## spins: new phenomena

- spin-induced deformations [Poisson 1998]
- shifts of NS **mode resonances** [Ho & Lai 2000]
- **new tidal interactions** and Love numbers [Pani+ 2016, Landry 2017]
- **gravitomagnetic effects**: zero-frequency modes [Poisson, Landry 2015, 16]



tidally induced fluid  
velocity perturbation

$$\delta v \sim \frac{m_2}{M} R^2 \omega \Omega_{\text{orb}}^{4/3}$$

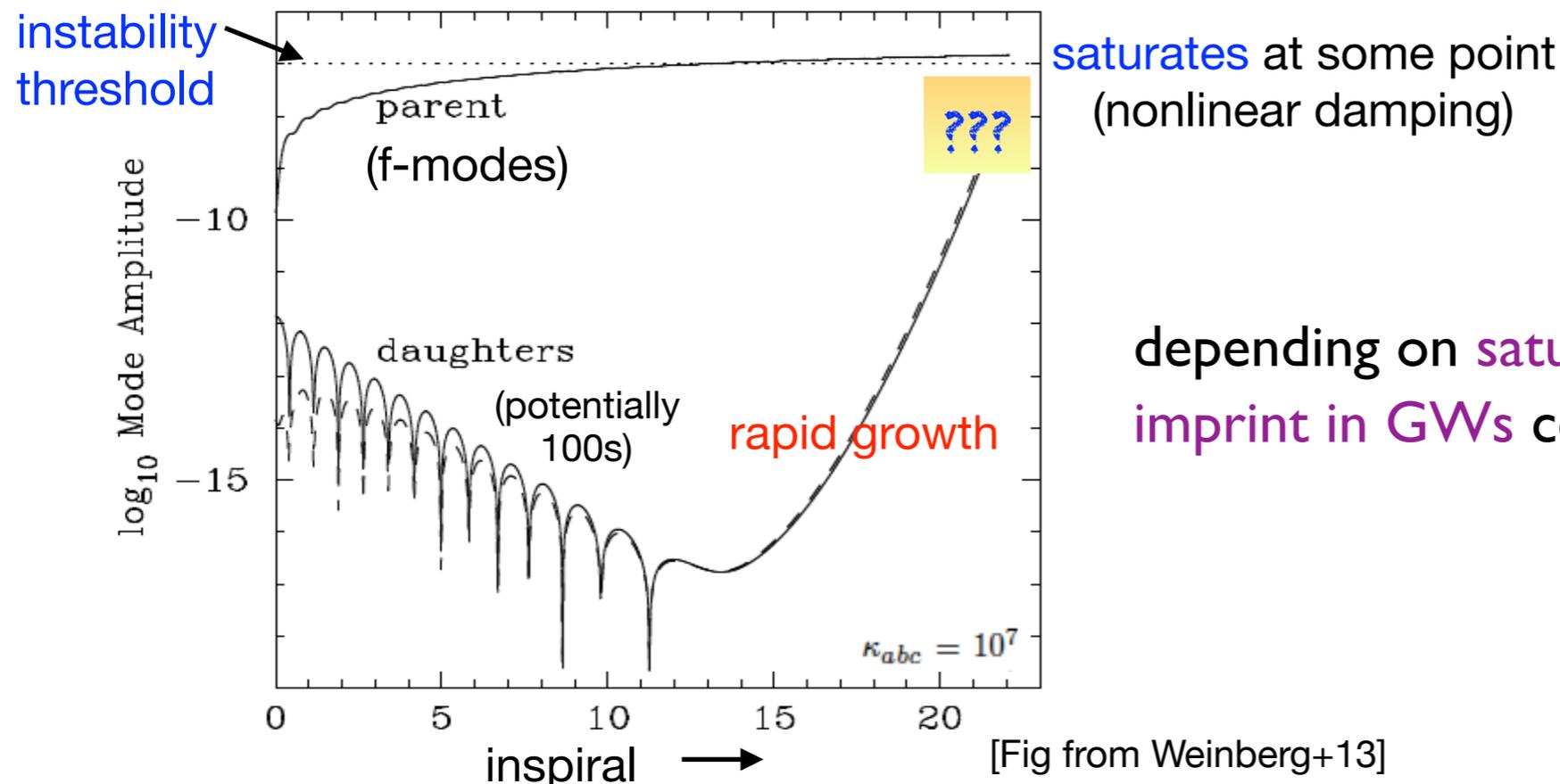
NS spin frequency

- **gravitomagnetic effects** in general (expected to be small)  
[different sign of Love numbers: Poisson, Landry 2014]

# Remaining challenges

- quantify importance of **nonlinear effects** [Weinberg, Essick, Vitale+]

3-mode coupling f, p, g modes



depending on **saturation amplitude & frequency:**  
imprint in **GWs** could be **huge** or **negligible**

- excitation of other **oscillation modes** for more realistic NSs during relativistic inspirals
- orbital **eccentricity**

# Outlook

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- GW detections of NS binaries expected in the coming years.
- Will yield a wealth of new insights into:
  - the properties of **matter** at extremes of density, **astrophysics** of NSs in binaries, the nature of **gravity**, and the central engines of powerful **electromagnetic phenomena**
  - **most readily measurable** information about **NS matter: tidal parameters**
- To take full advantage of this **science** potential requires **increasingly accurate models**
  - recent progress on including more realistic physics
  - tests against NR simulations
  - remaining work
- Further key goal: **envision the science case for future instruments.**

