



UNIVERSITY OF
BIRMINGHAM



Parameter estimation of compact binaries from gravitational-wave signals

Christopher Berry

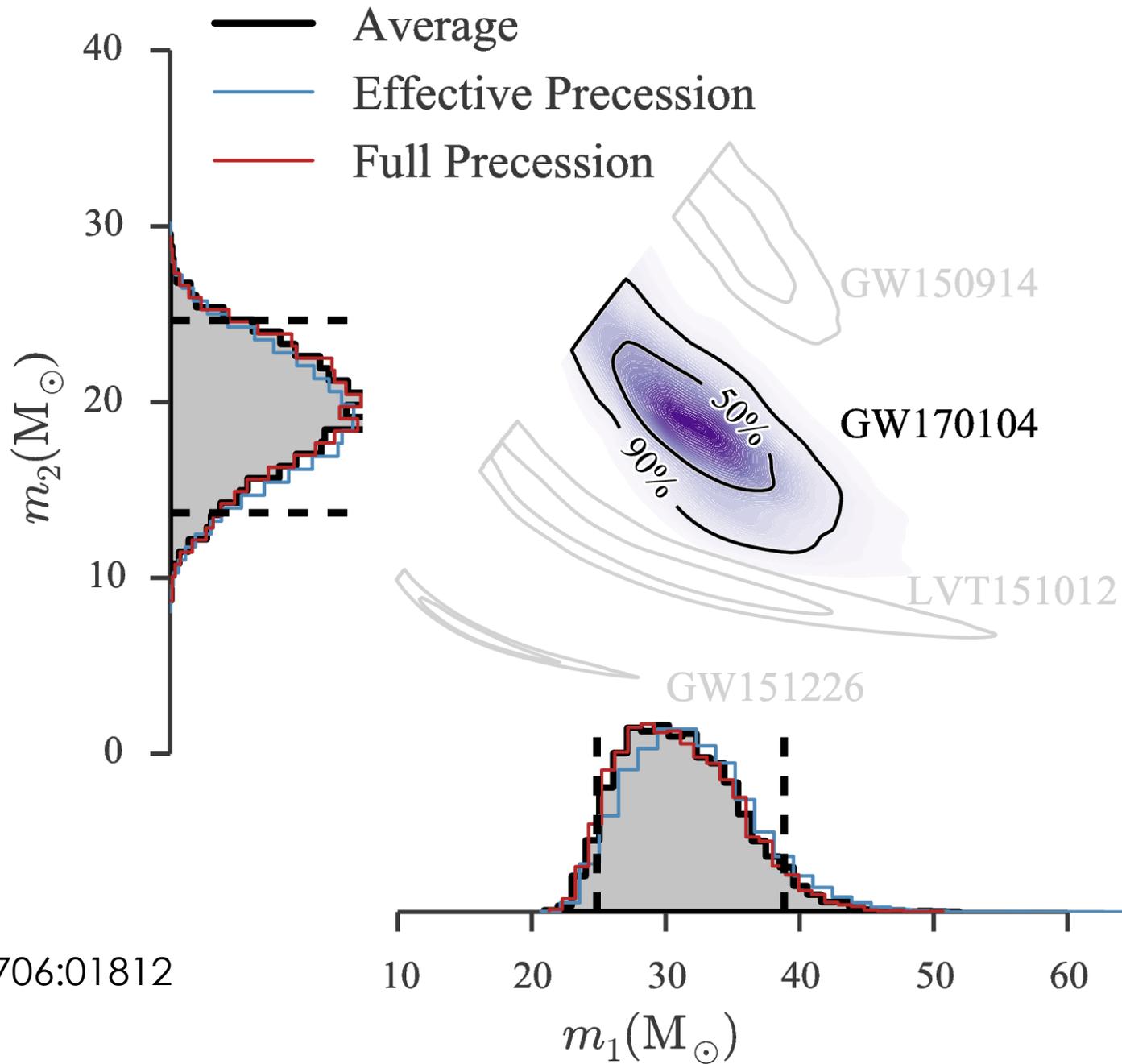
University of Birmingham

@cplberry

On behalf of the
LIGO Scientific & Virgo Collaborations

DCC G1701119

New Frontiers in Gravitational-wave Astrophysics



How to infer source properties

Our measurements so far

Gravitational-wave observations
and formation processes

How to infer source properties

Our measurements so far

Gravitational-wave observations
and formation processes

Bayes' theorem

$$p(\theta|d) = \frac{p(d|\theta) p(\theta)}{p(d)}$$

Bayes' theorem

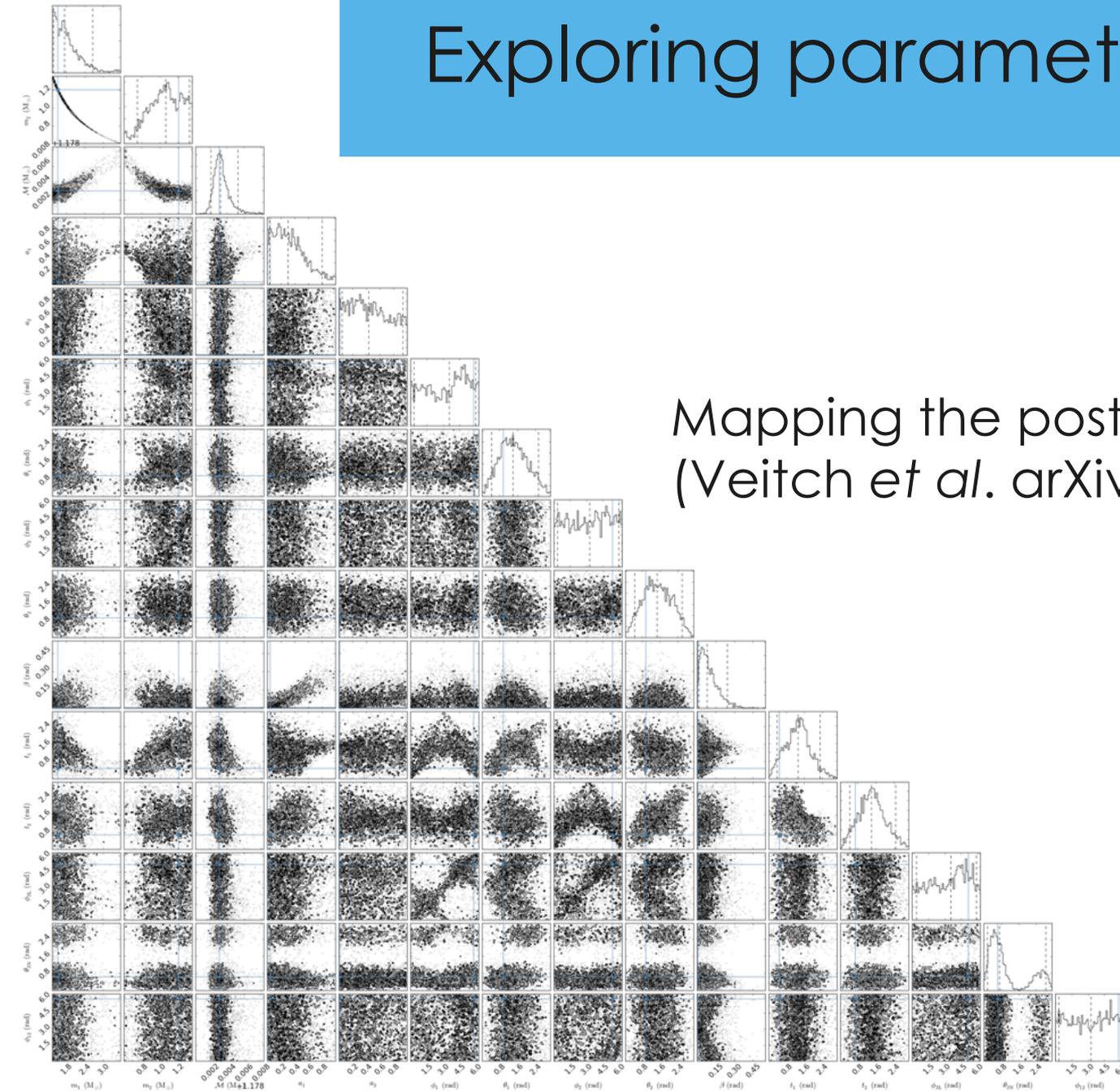
The diagram illustrates Bayes' theorem with the following components:

- Posterior:** $p(\theta|d)$ (blue box)
- Likelihood:** $p(d|\theta)$ (pink box)
- Prior:** $p(\theta)$ (orange box)
- Evidence:** $p(d)$ (green box)

$$p(\theta|d) = \frac{p(d|\theta)p(\theta)}{p(d)}$$

Exploring parameter space

Mapping the posterior is difficult
(Veitch *et al.* arXiv:1409.7215)



Likelihood

$$p(d|\theta) \propto \exp \left[-\frac{1}{2} \sum_k \langle h_k(\theta) - d_k | h_k(\theta) - d_k \rangle \right]$$

Likelihood

$$p(d|\theta) \propto \exp \left[-\frac{1}{2} \sum_k \langle h_k(\theta) - d_k | h_k(\theta) - d_k \rangle \right]$$

Noise-weighting

Likelihood

$$p(d|\theta) \propto \exp \left[-\frac{1}{2} \sum_k \langle h_k(\theta) - d_k | h_k(\theta) - d_k \rangle \right]$$

Noise-weighting

$$h_k(\theta) \rightarrow h_k(\theta) [1 + \delta A_k] \exp [i\delta\phi_k]$$

Likelihood

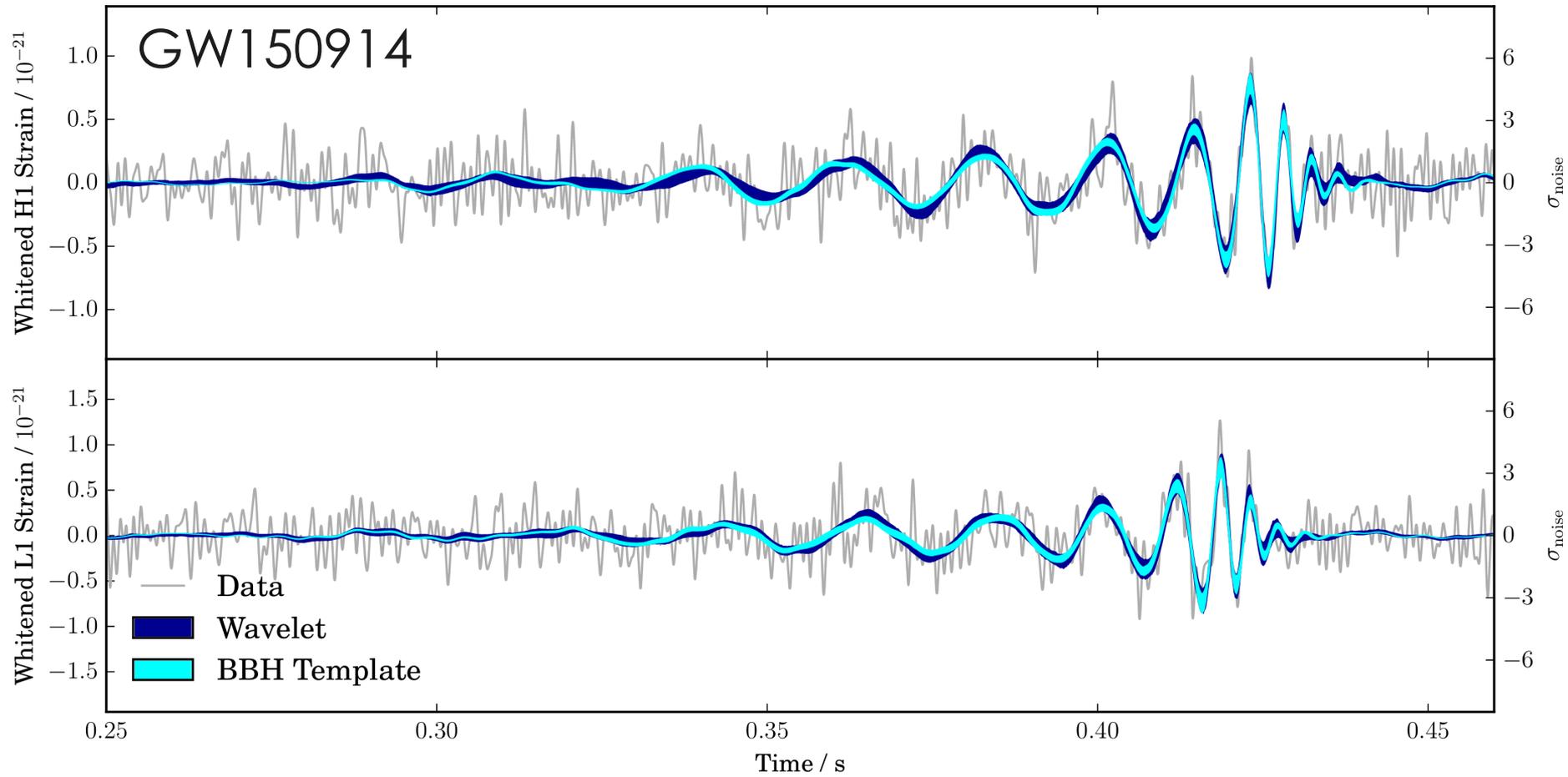
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Noise-weighting

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Waveform

Waveform

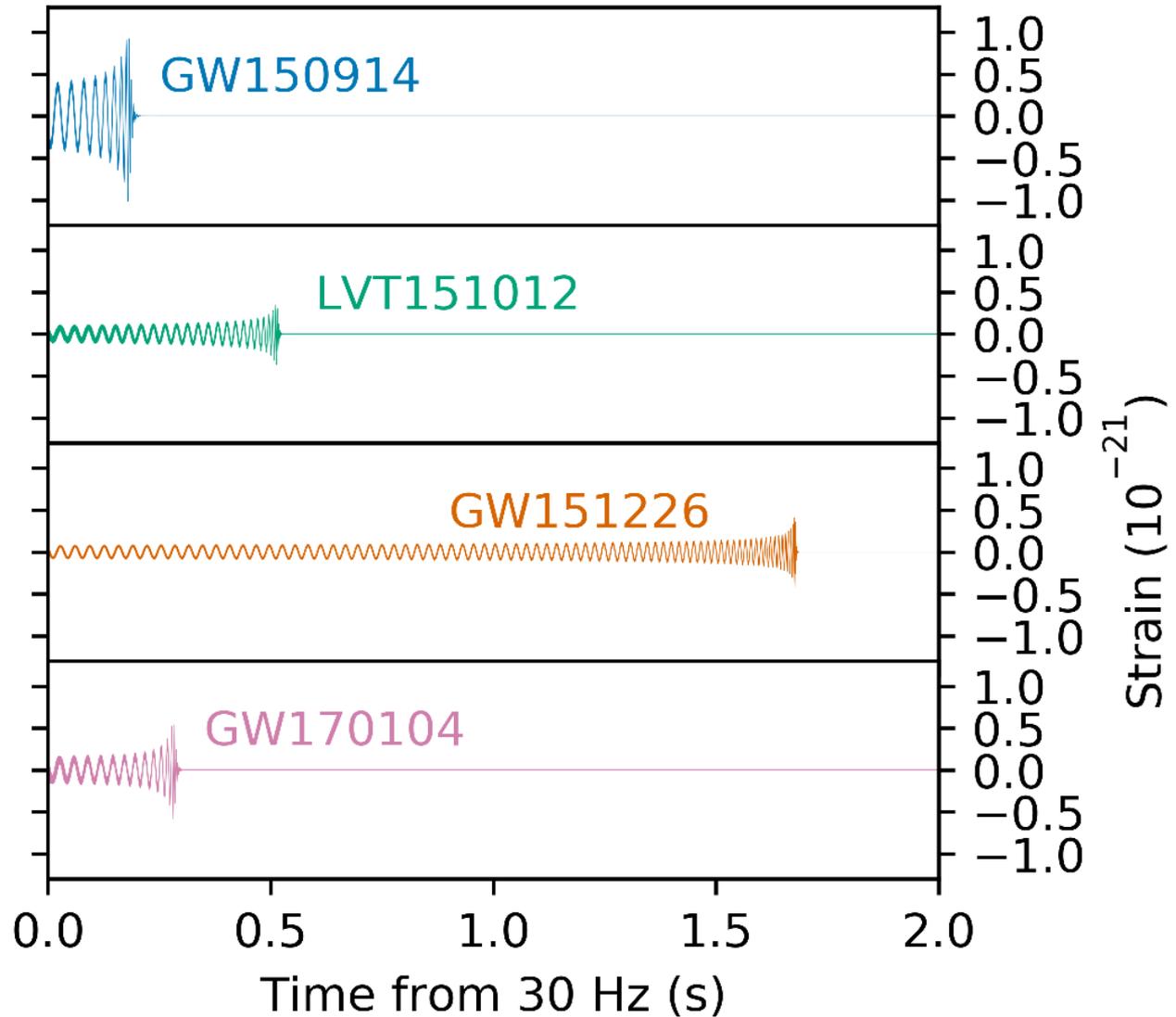


How to infer source properties

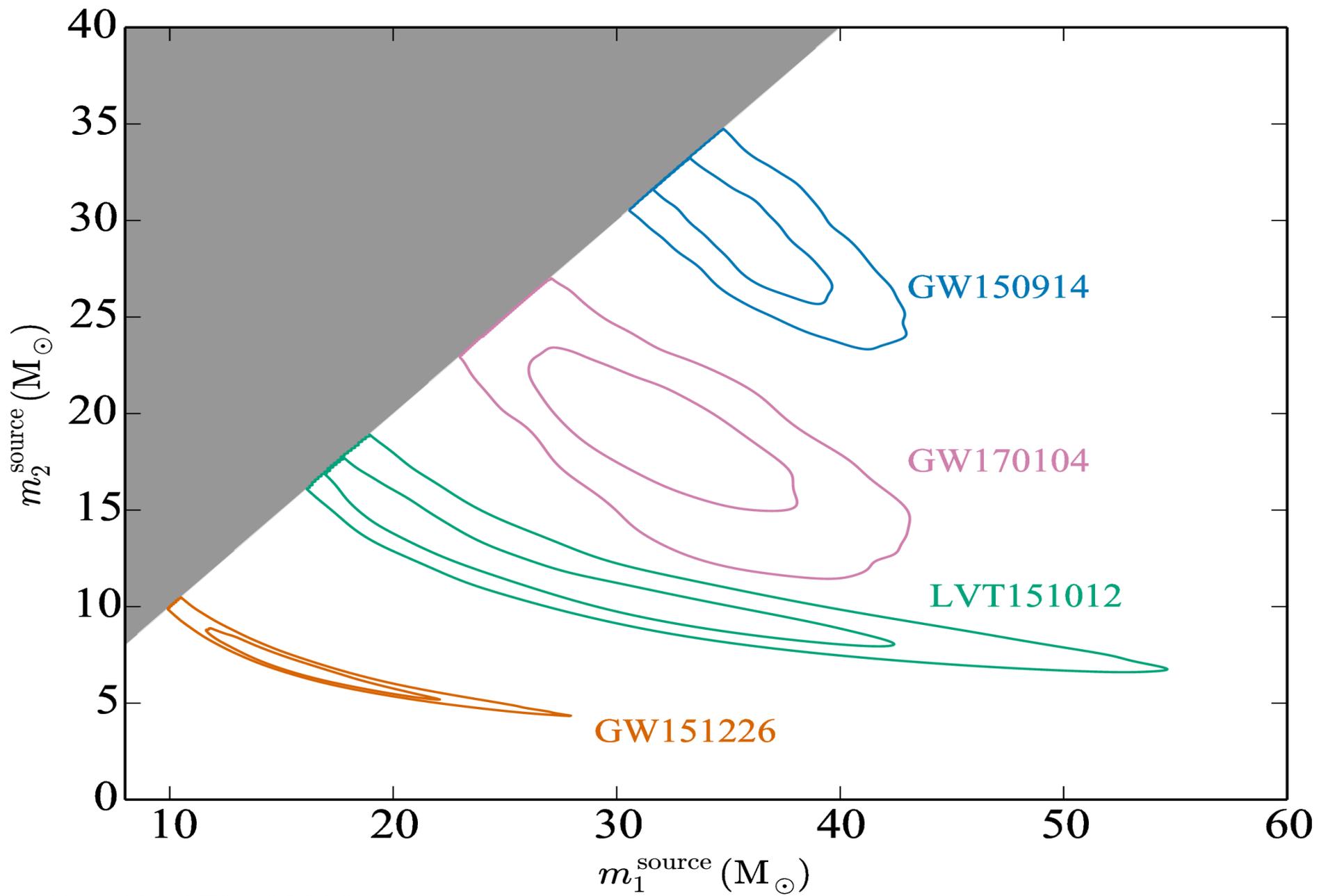
Our measurements so far

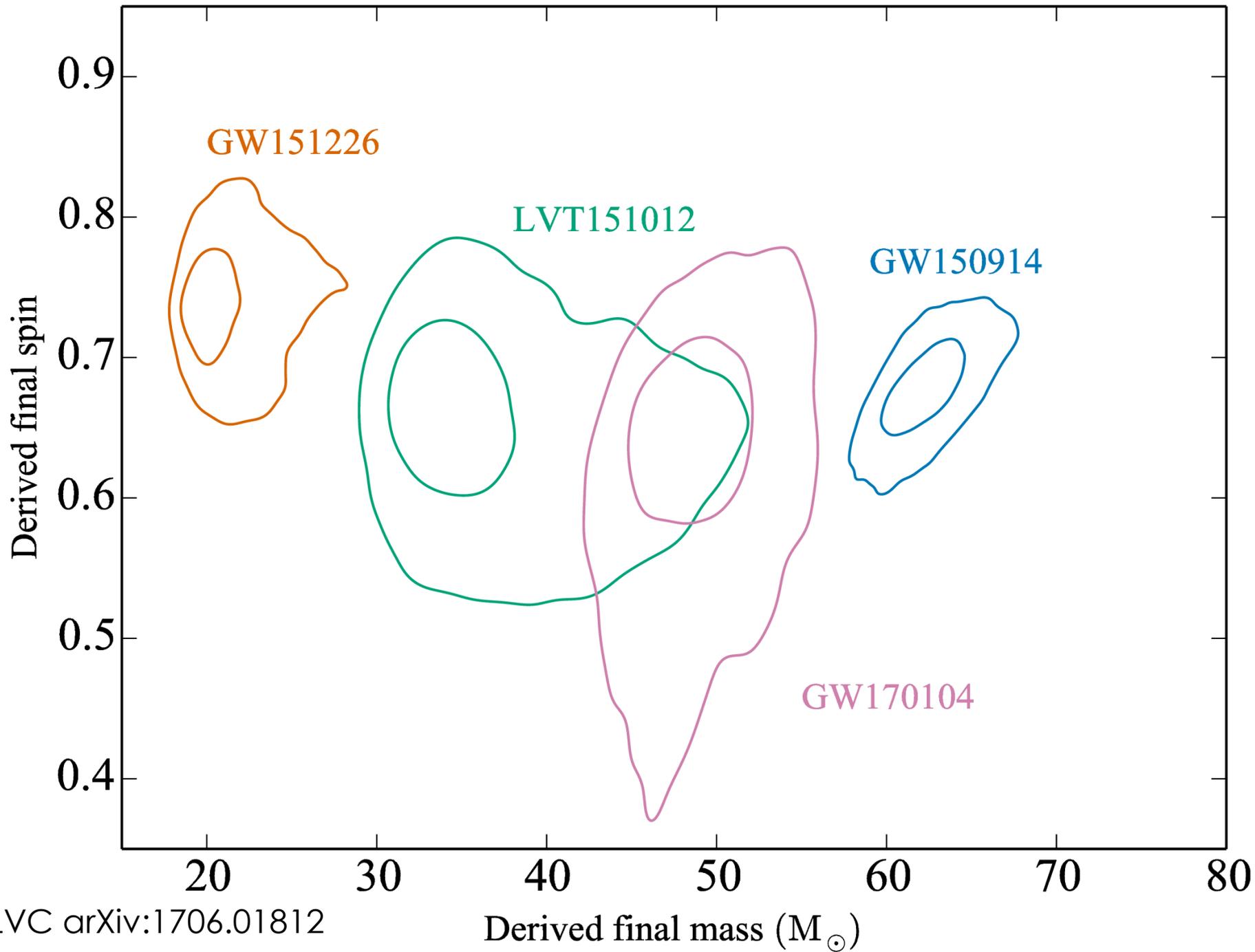
Gravitational-wave observations
and formation processes

Binary black hole signals



LVC
arXiv:1606.04856,
arXiv:1706.01812



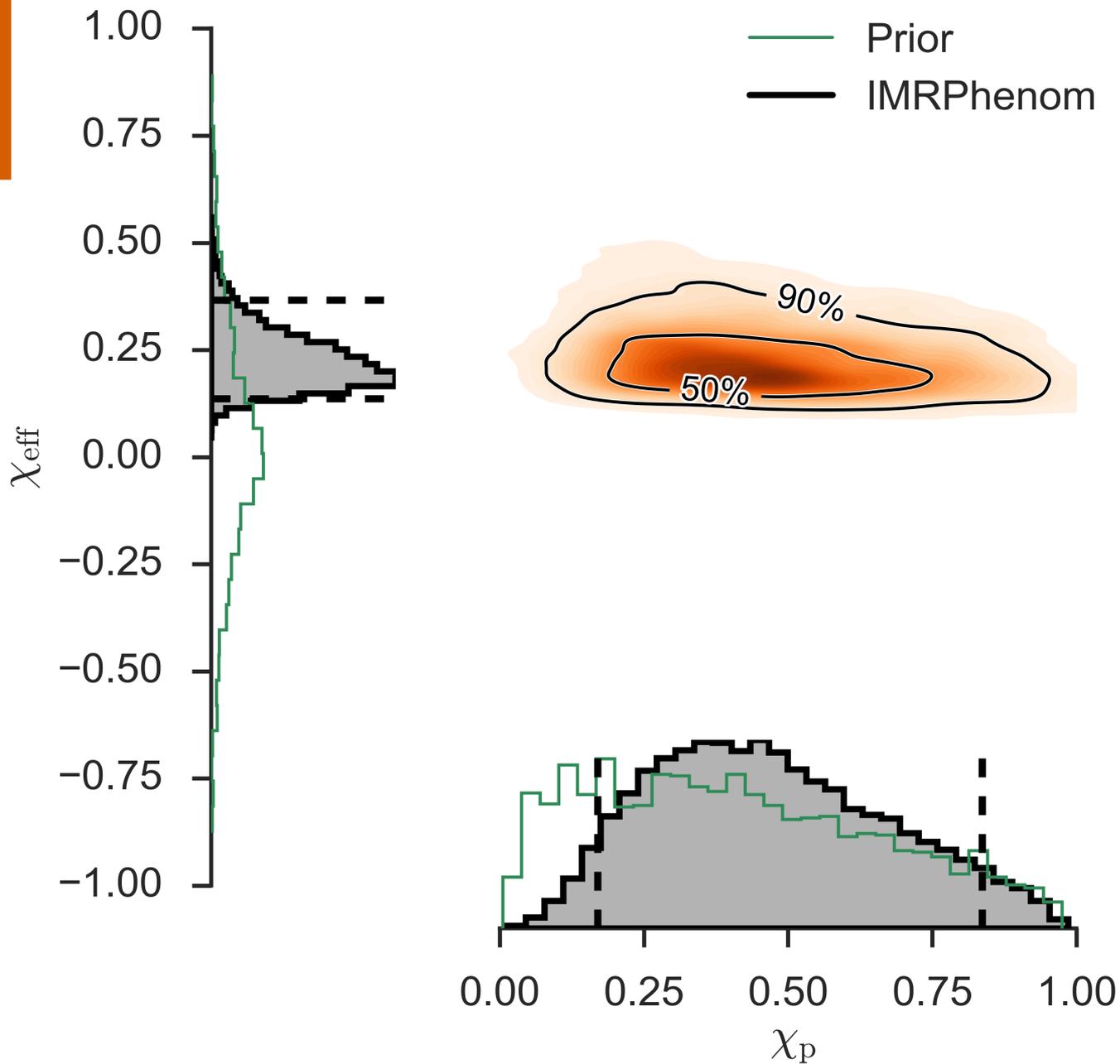


Effective inspiral spin

$$\chi_{\text{eff}} = \frac{c}{GM} \left(\frac{\mathbf{S}_1}{m_1} + \frac{\mathbf{S}_2}{m_2} \right) \cdot \hat{\mathbf{L}}$$

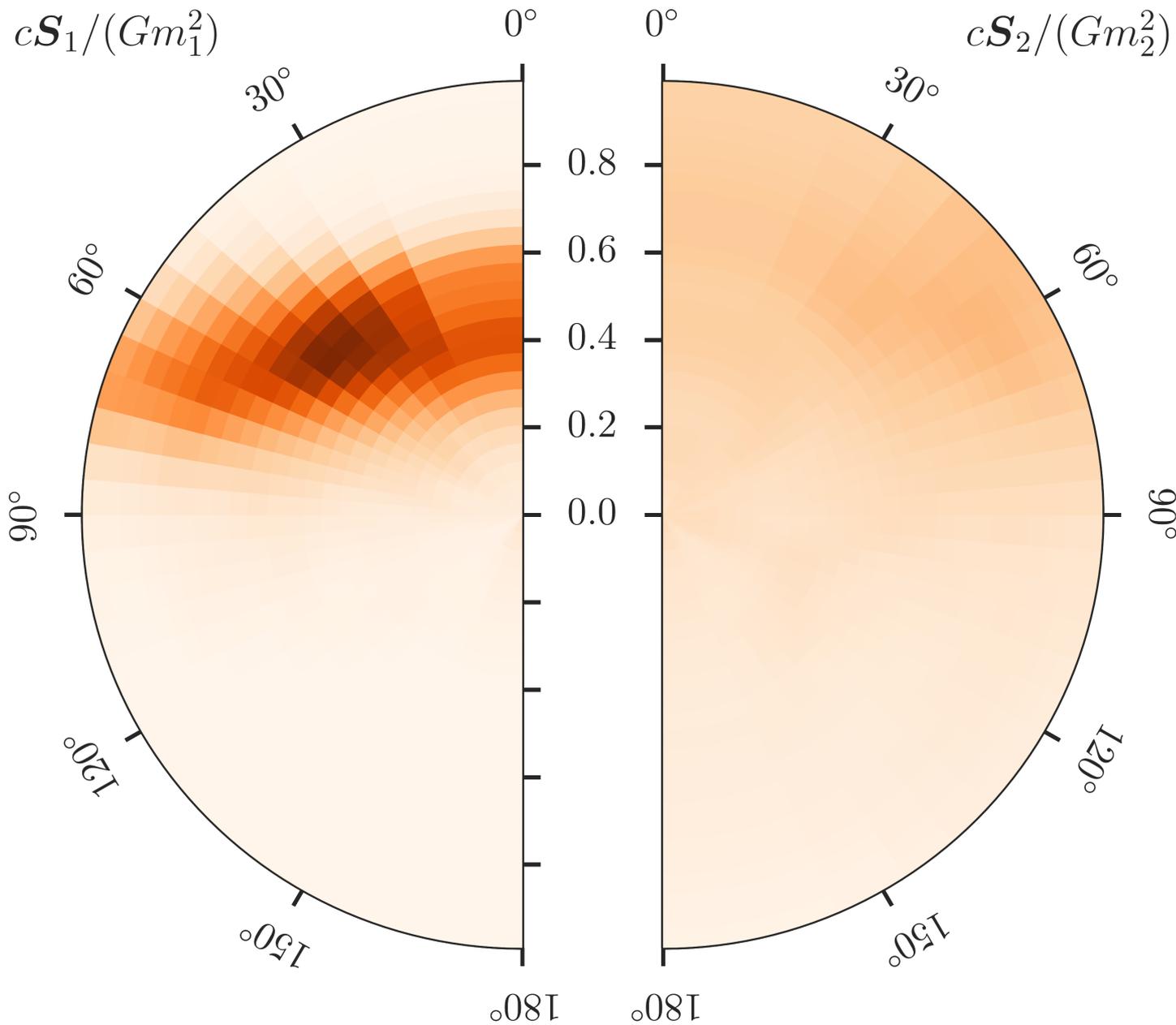
Most important combination of spins for evolution of inspiral (arXiv:0909.2867, 1005.3306)

Spin

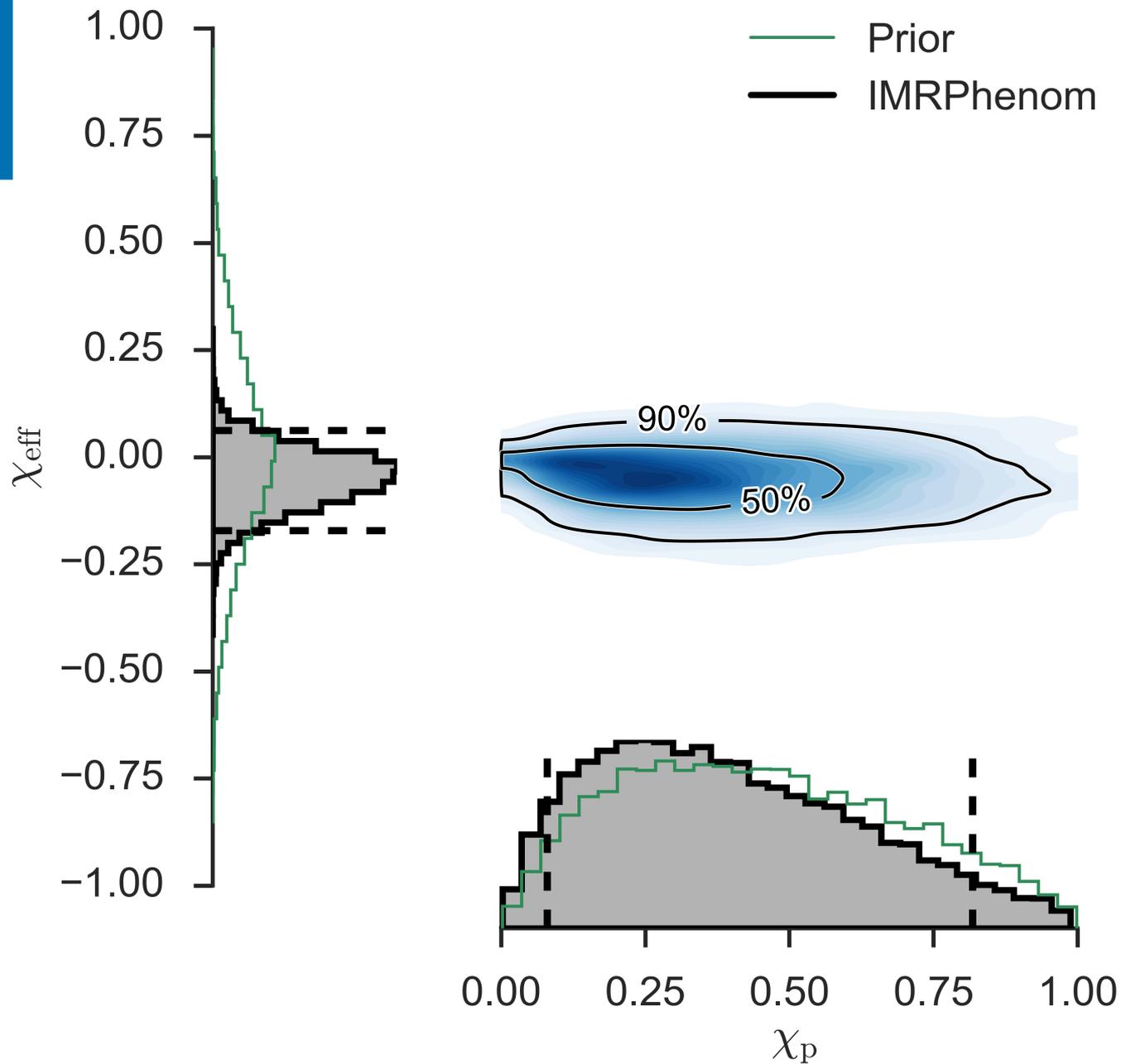


LVC
arXiv:1606.04856

Spin

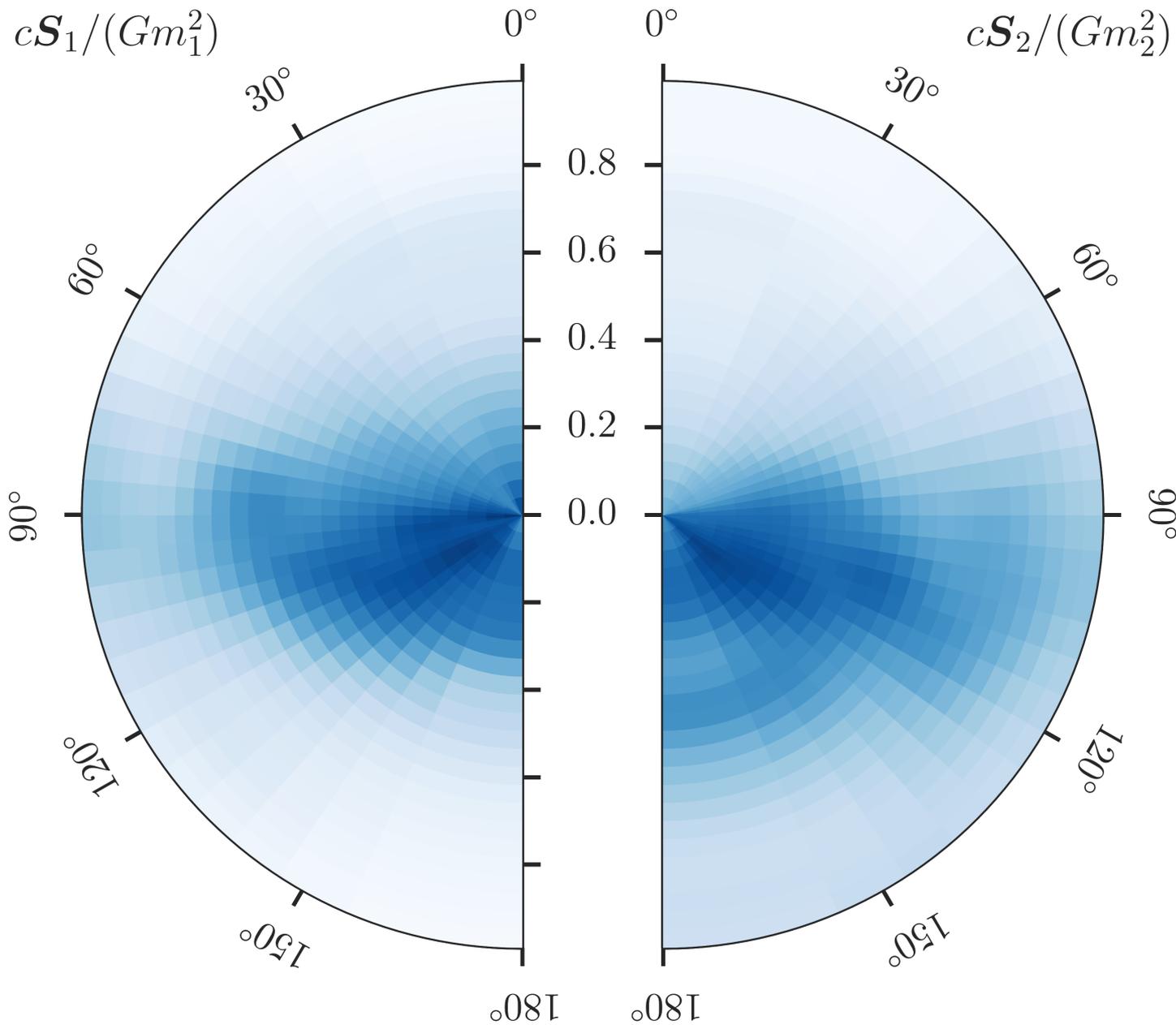


Spin



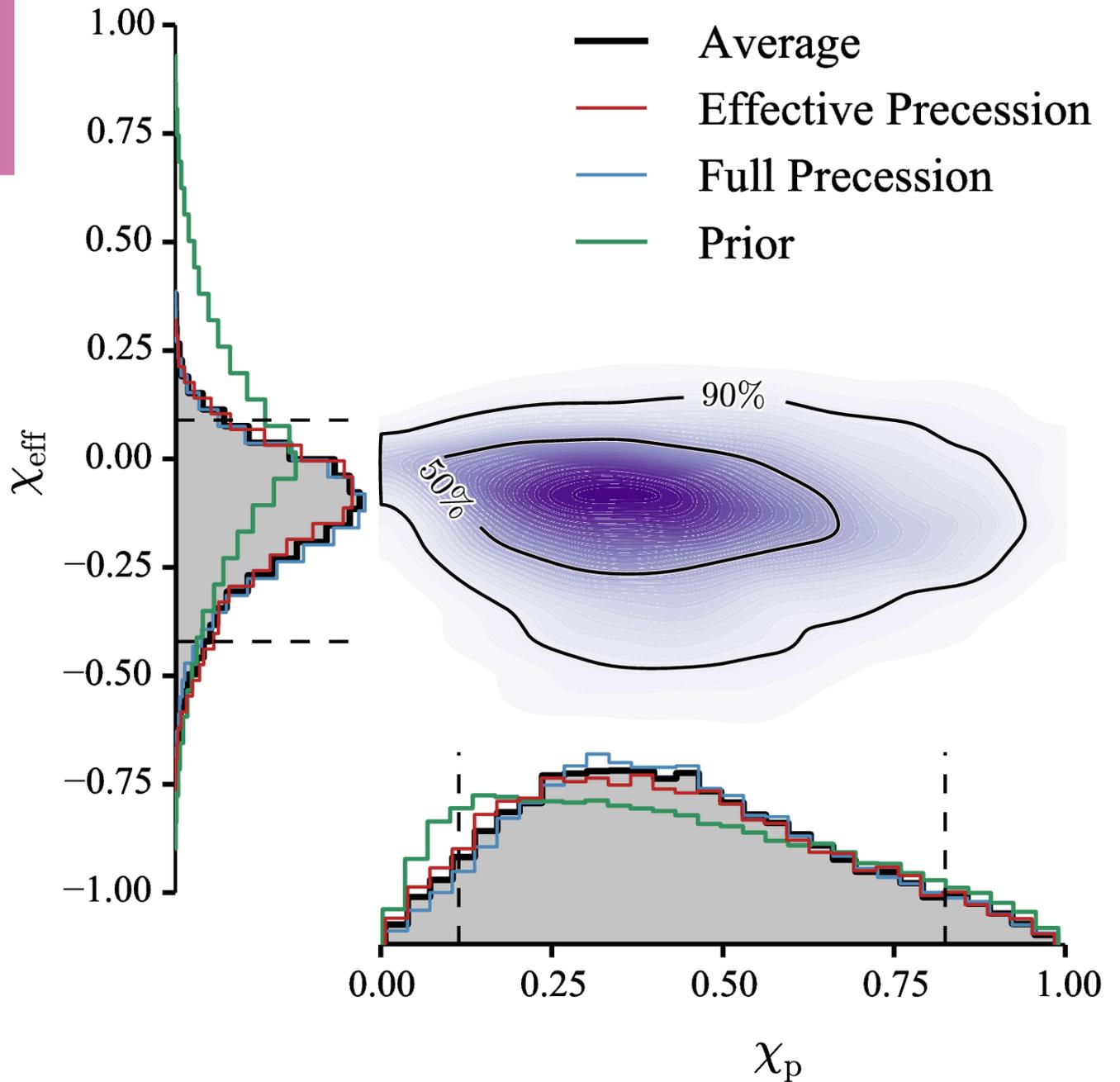
LVC
arXiv:1606.04856
arXiv:1602.03840

Spin

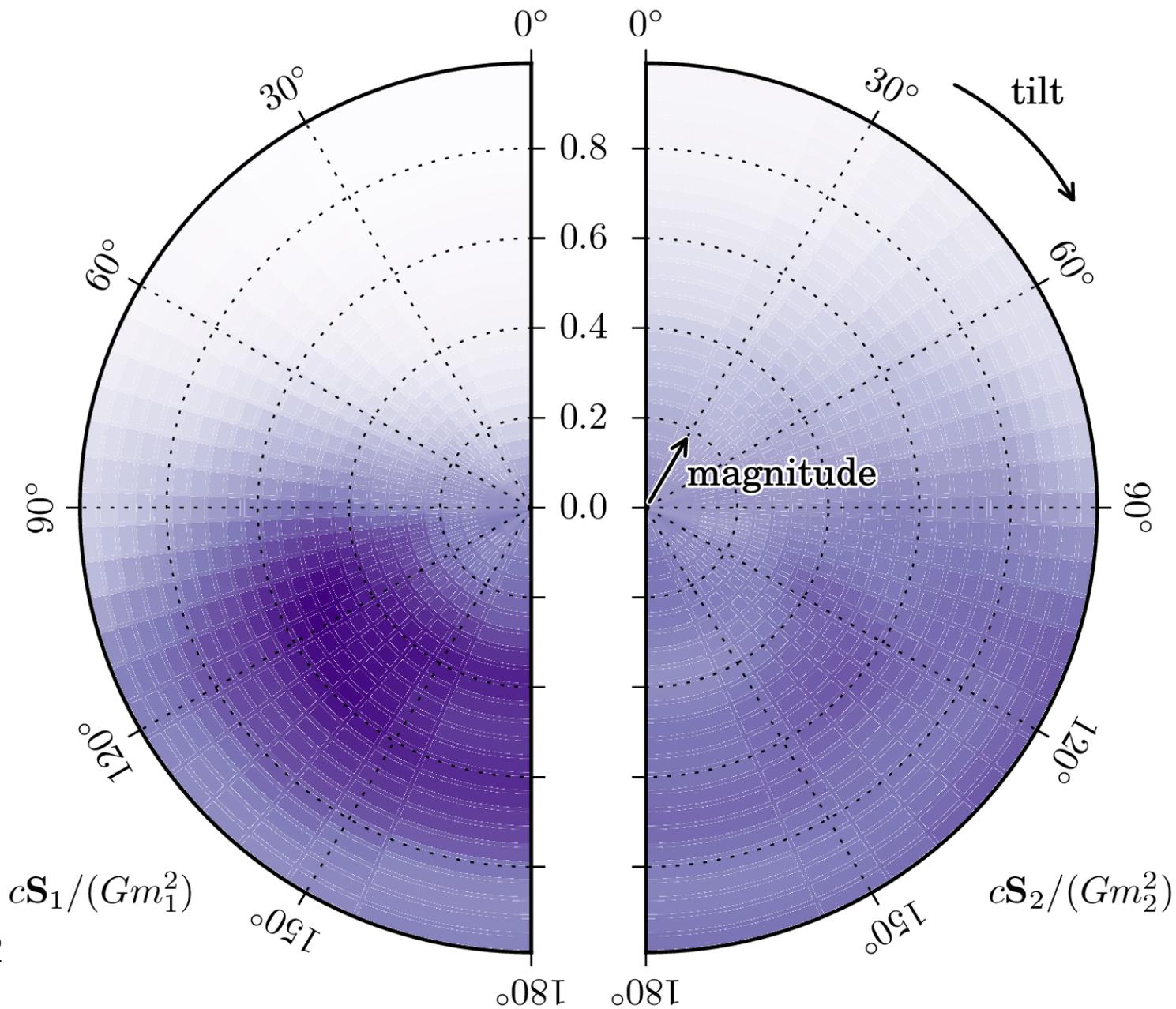


LVC
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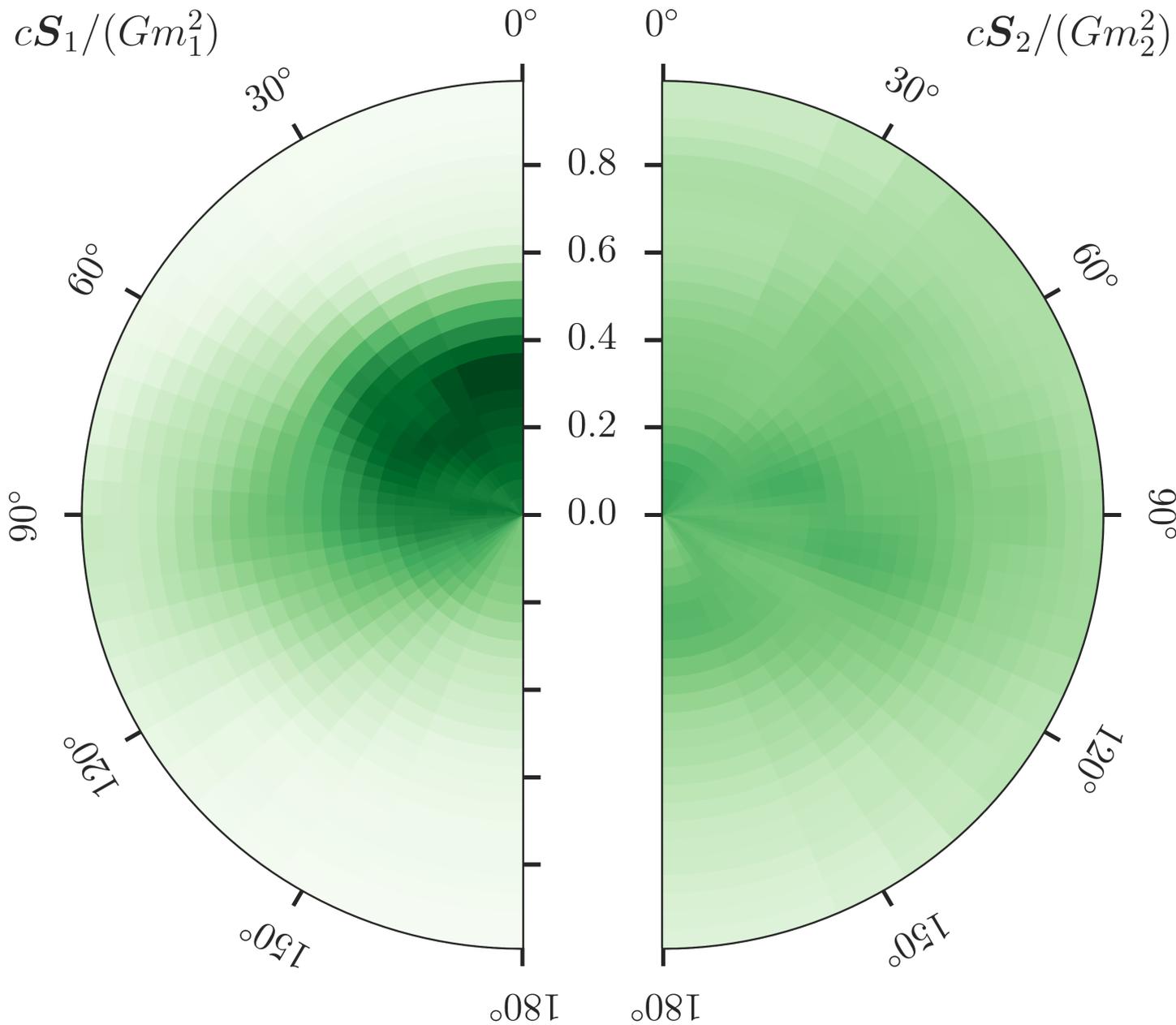
Spin



Spin



Spin

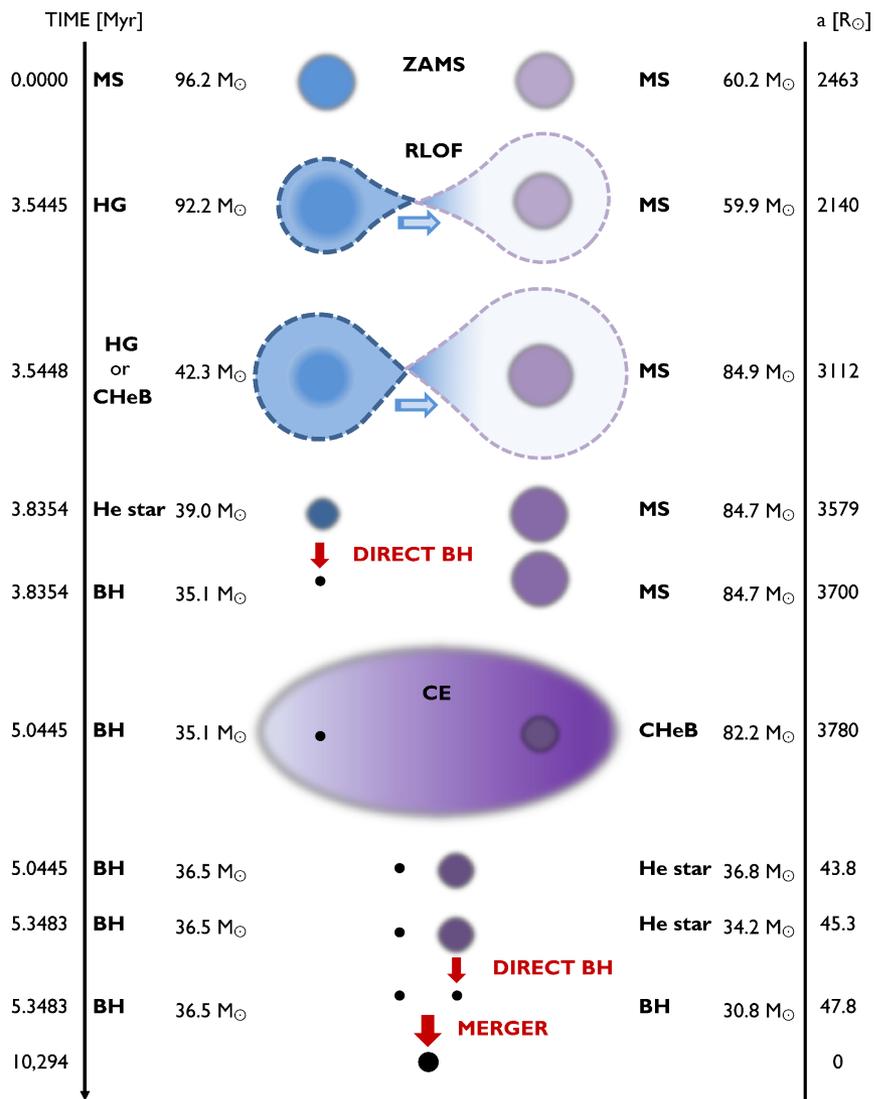


How to infer source properties

Our measurements so far

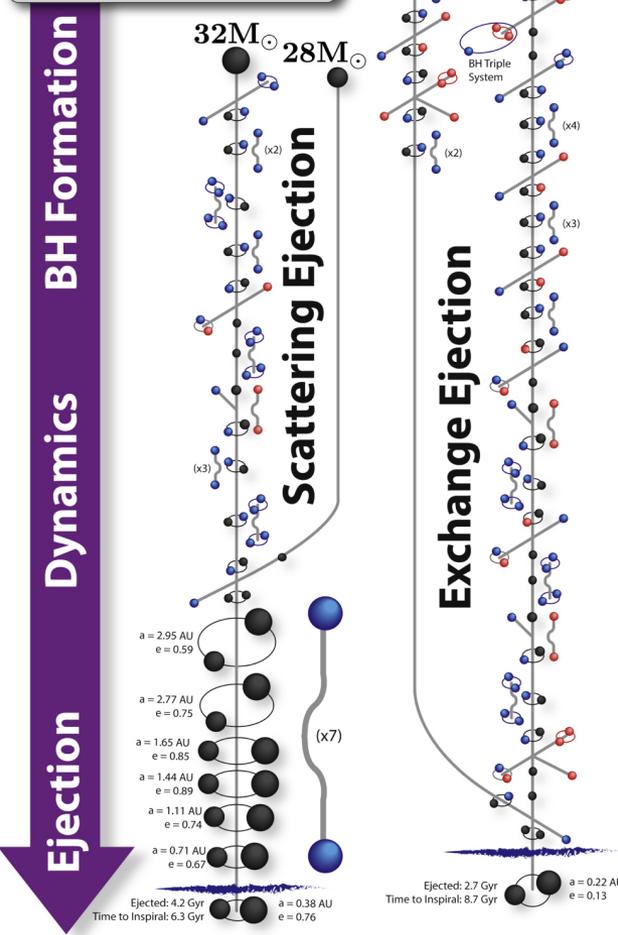
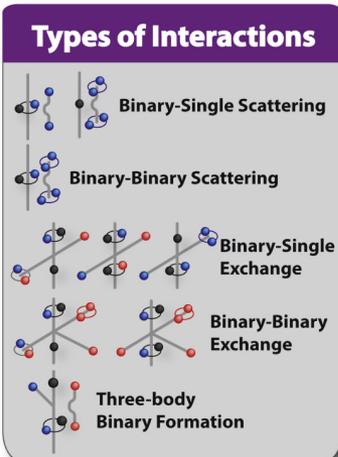
Gravitational-wave observations
and formation processes

Binary formation

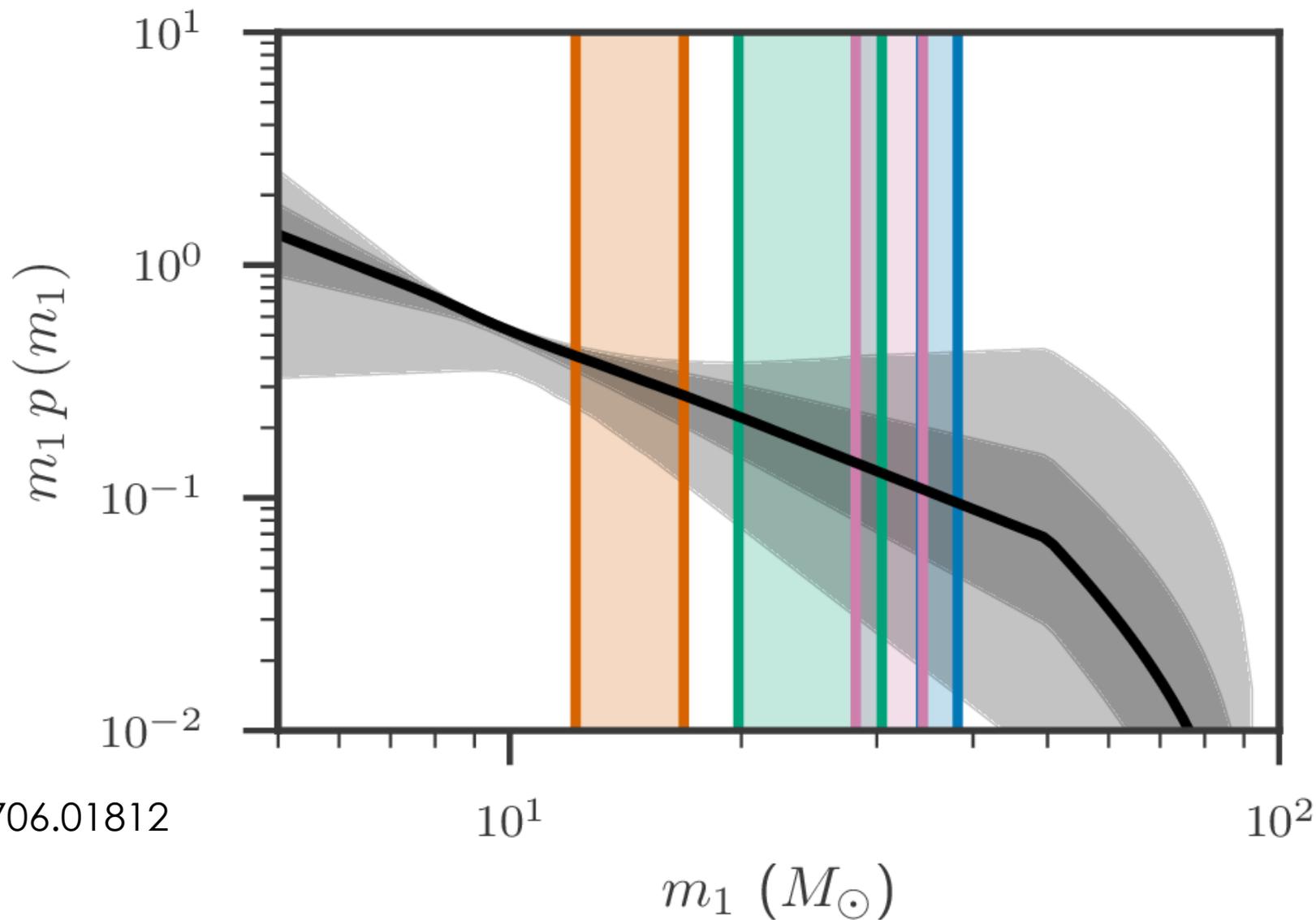


Rodriguez *et al.*
arXiv:1604.04254

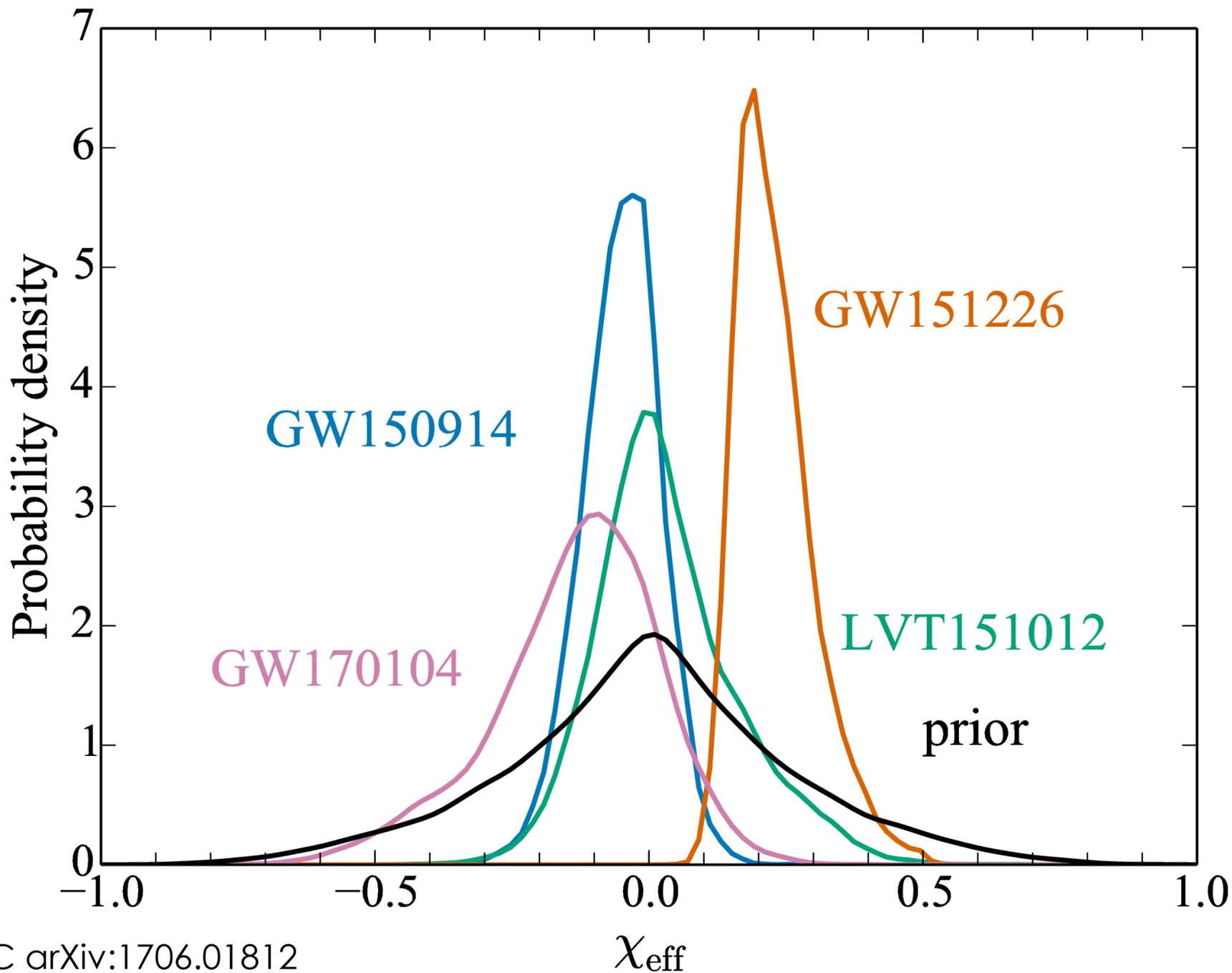
Belczynski *et al.*
arXiv:1602.04531



Mass distribution



LVC
arXiv:1706.01812



Distinguishing Spin-Aligned and Isotropic Black Hole Populations With Gravitational Waves

Will M. Farr, Simon Stevenson, M. Coleman Miller, Ilya Mandel, Ben Farr, Alberto Vecchio

(Submitted on 5 Jun 2017 (v1), last revised 6 Jun 2017 (this version, v2))

The first direct detections of gravitational waves from merging binary black holes open a unique window into the binary black hole formation environment. One promising environmental signature is the angular distribution of the black hole spins; systems formed through dynamical interactions among already-compact objects are expected to have isotropic spin orientations whereas binaries formed from pairs of stars born together are more likely to have spins preferentially aligned with the binary orbital angular momentum. We consider existing gravitational wave measurements of the binary effective spin, the best-measured combination of spin parameters, in the four likely binary black hole detections GW150914, LVT151012, GW151226, and GW170104. If binary black hole spin magnitudes extend to high values we show that the data exhibit a 2.4σ (0.015 odds ratio) preference for an isotropic angular distribution over an aligned one. By considering the effect of 10 additional detections, we show that such an augmented data set would enable in most cases a preference stronger than 5σ (2.9×10^{-7} odds ratio). The existing preference for either an isotropic spin distribution or low spin magnitudes for the observed systems will be confirmed (or overturned) confidently in the near future.

Comments: 32 pages, 9 figures, code and documents
Subjects: **High Energy Astrophysical Phenomena**
Report number: LIGO-P1700067
Cite as: [arXiv:1706.01385](https://arxiv.org/abs/1706.01385) [astro-ph.HE]
(or [arXiv:1706.01385v2](https://arxiv.org/abs/1706.01385v2) [astro-ph.HE])

Vitale *et al.* arXiv: 1503.04307

Gerosa & Berti arXiv: 1703.06223

Fishbach, Holz & Farr arXiv:1703.06869

Stevenson, **CPLB** & Mandel: 1703.06873

Talbot & Thrane arXiv:1704.08370

Inference uses: waveform templates, noise spectrum, calibration uncertainty & a prior

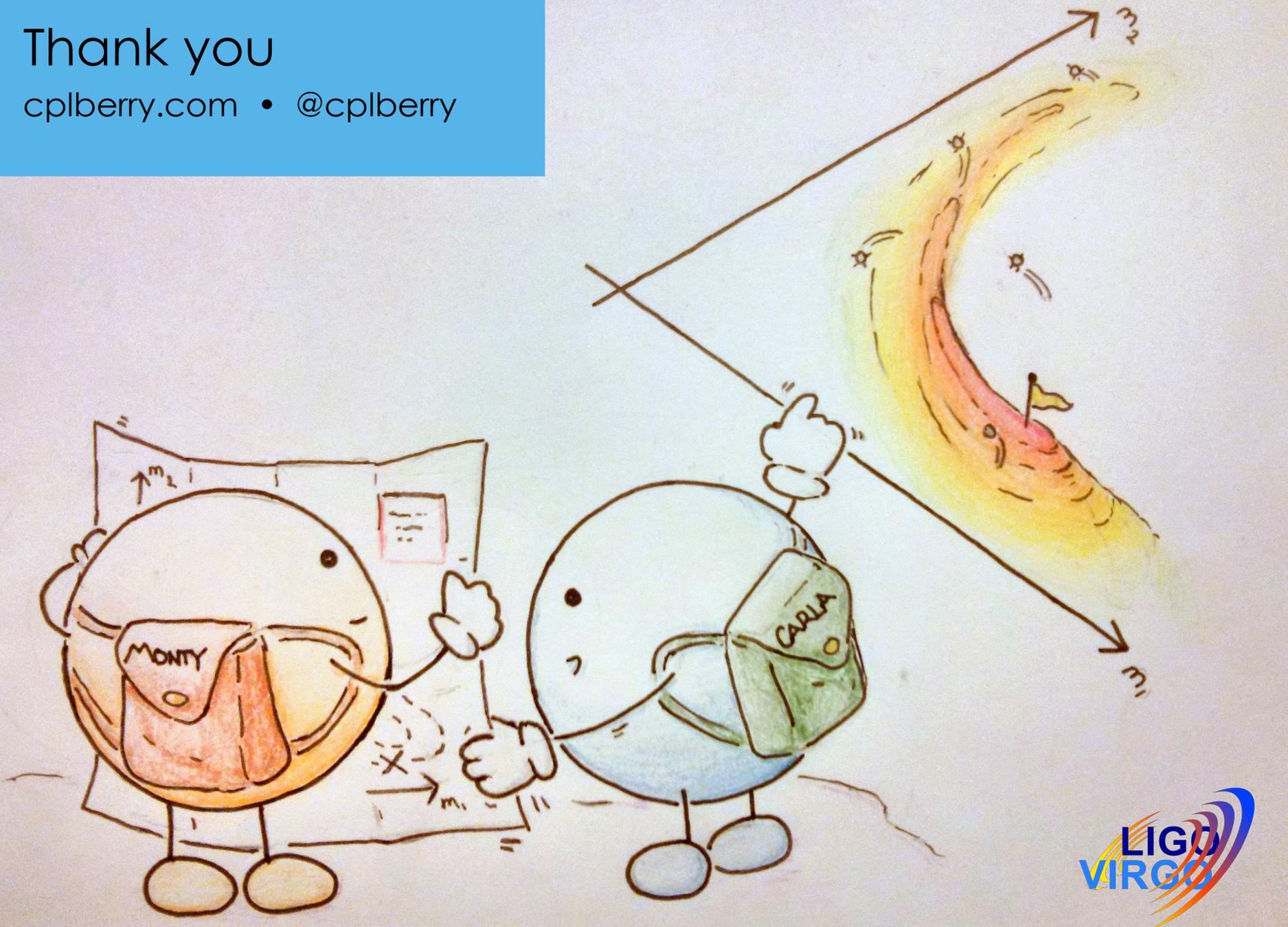
We have observed a family of binary black holes

Masses and spins give hints to formation mechanisms, but need many detections

- Infer source properties by comparing waveform templates to data
- Some parameter (like chirp mass) measured well, others (like in-plane spins) remain uncertain
- Parameters (masses, spins, redshifts) could give insight into evolution
- Need a hierarchical analysis of a population of ~ 100 detections

Thank you

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Bayes' theorem

The diagram illustrates Bayes' theorem with the following components:

- Posterior:** $p(\theta|d)$ (enclosed in a blue box)
- Likelihood:** $p(d|\theta)$ (enclosed in a pink box)
- Prior:** $p(\theta)$ (enclosed in a yellow box)
- Evidence:** $p(d)$ (enclosed in a green box)

The equation is presented as:

$$p(\theta|d) = \frac{p(d|\theta)p(\theta)}{p(d)}$$

Bayes' theorem

$$p(\theta|d, \lambda) = \frac{p(d|\theta, \lambda) p(\theta|\lambda)}{p(d|\lambda)}$$

Posterior

Likelihood

Prior

Evidence

The diagram illustrates Bayes' theorem with the following components:

- Posterior:** $p(\theta|d, \lambda)$ (blue box)
- Likelihood:** $p(d|\theta, \lambda)$ (pink box)
- Prior:** $p(\theta|\lambda)$ (orange box)
- Evidence:** $p(d|\lambda)$ (green box)

Bayes' theorem

$$p(\lambda|\{d\}) = \frac{p(\{d\}|\lambda) p(\lambda)}{p(\{d\})}$$

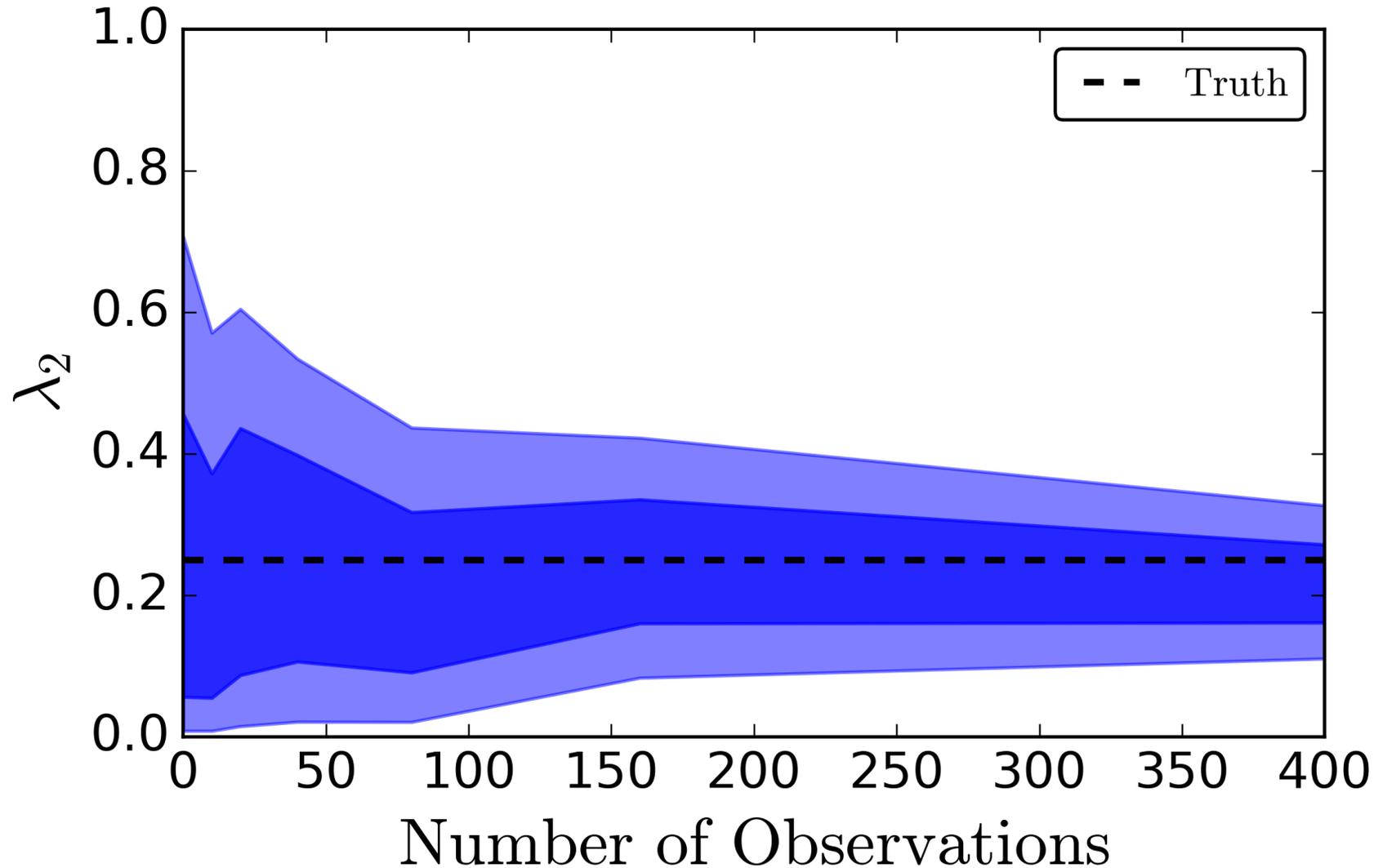
Evidence

Model prior

Model posterior

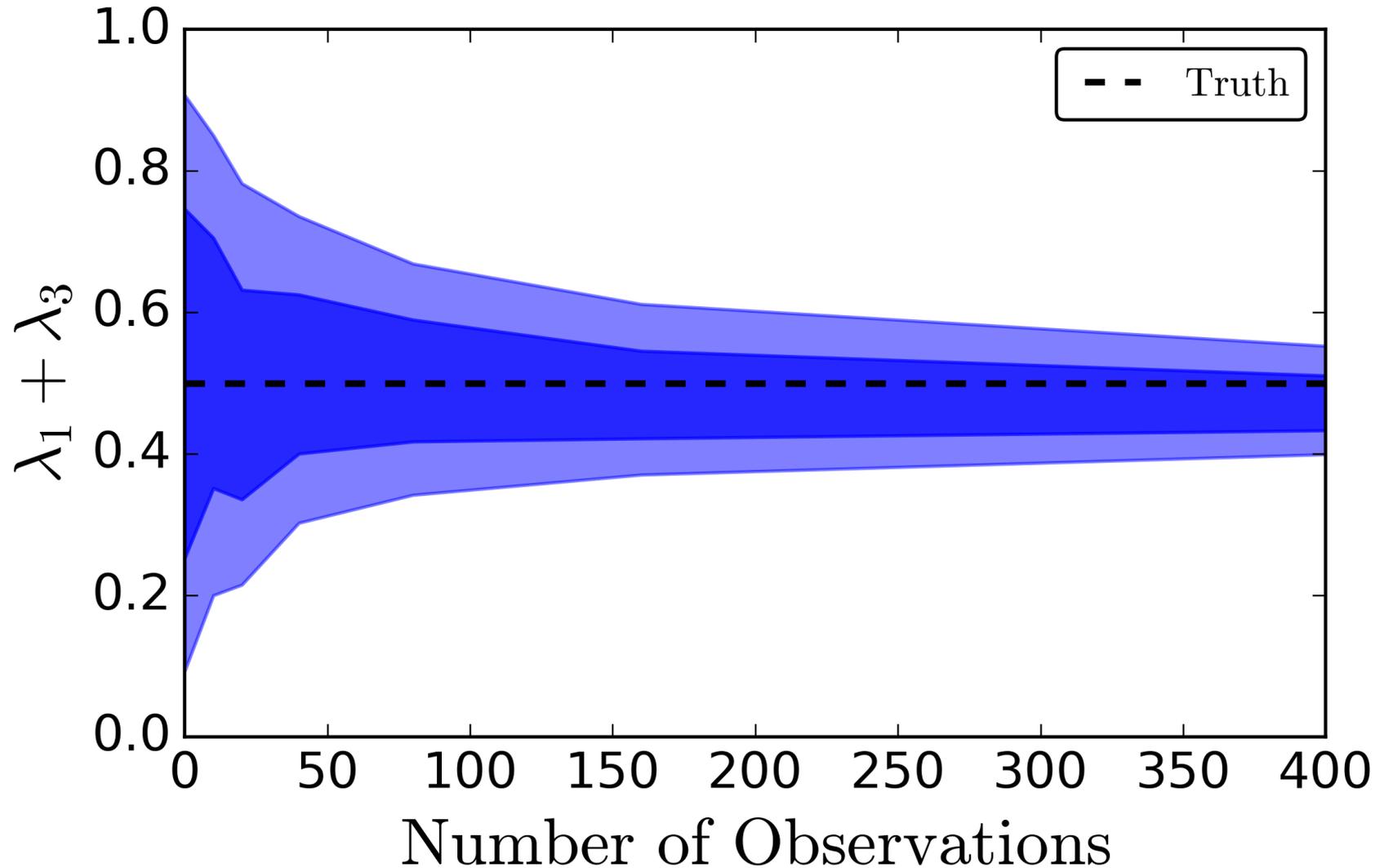
Model inference

Stevenson, CPLB & Mandel
arXiv:1703.06873



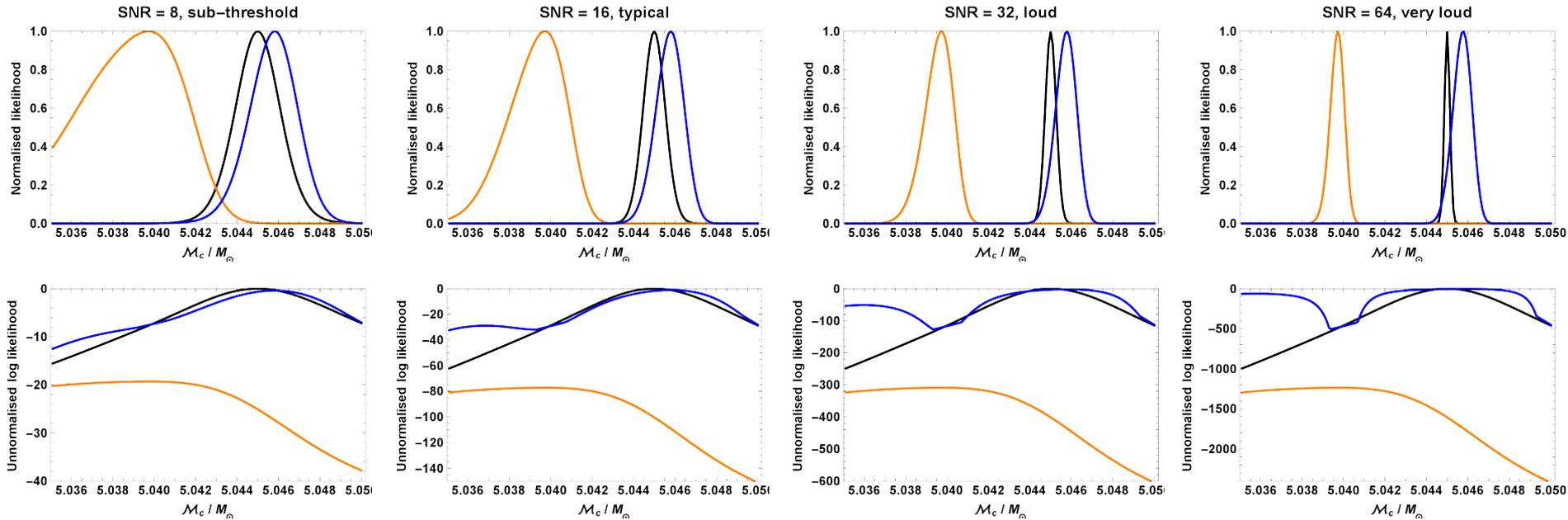
Model inference

Stevenson, CPLB & Mandel
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Waveform error

Waveforms introduce theoretical error (arXiv:0707.2982).
Mitigated using Gaussian processes (arXiv:1509.04066).



Moore *et al.* arXiv:1509.04066



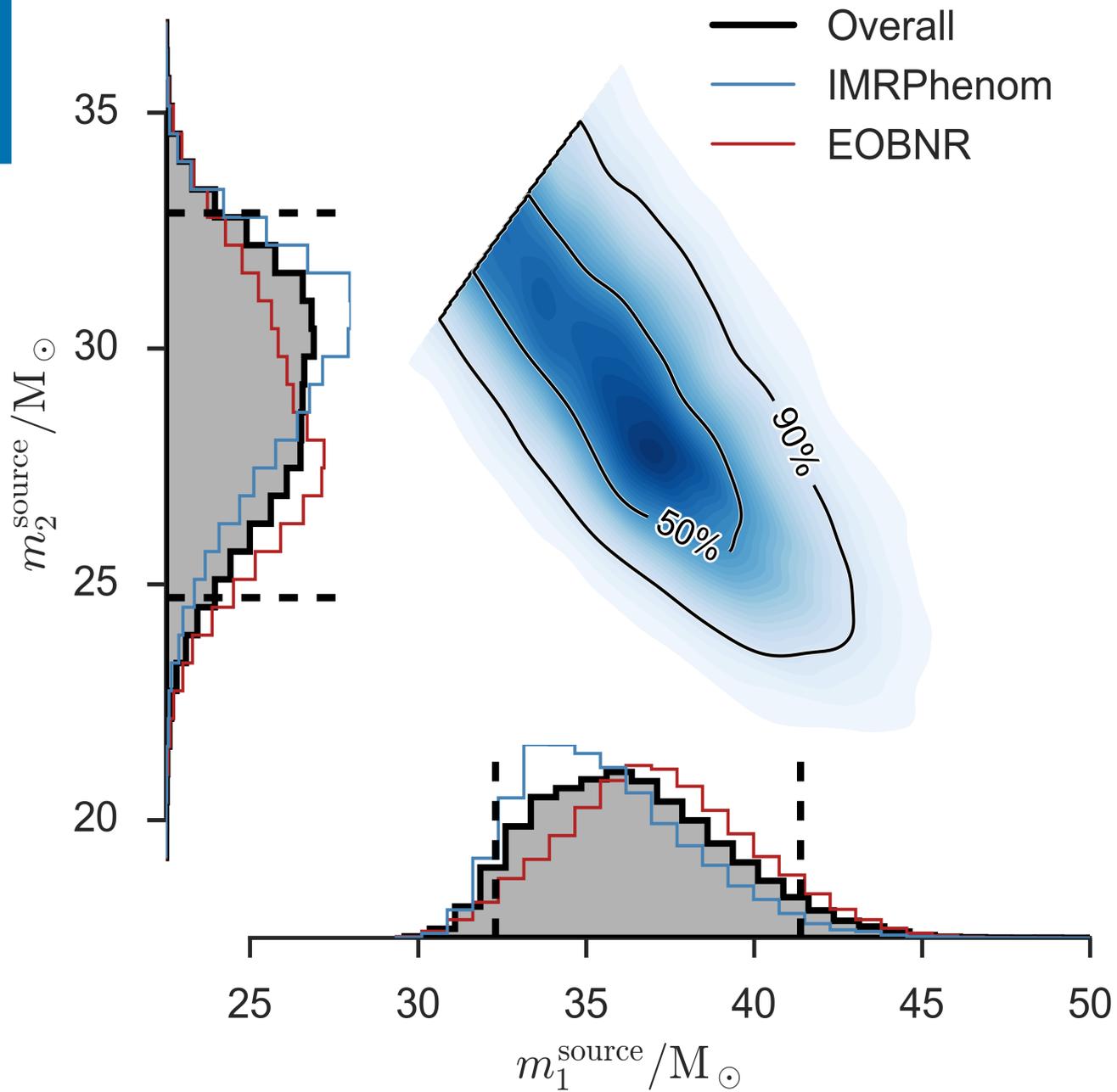
Credit: ButterflyLove1

Chirp mass

$$\mathcal{M}_c = \frac{(m_1 m_2)^{3/5}}{(m_1 + m_2)^{1/5}}$$

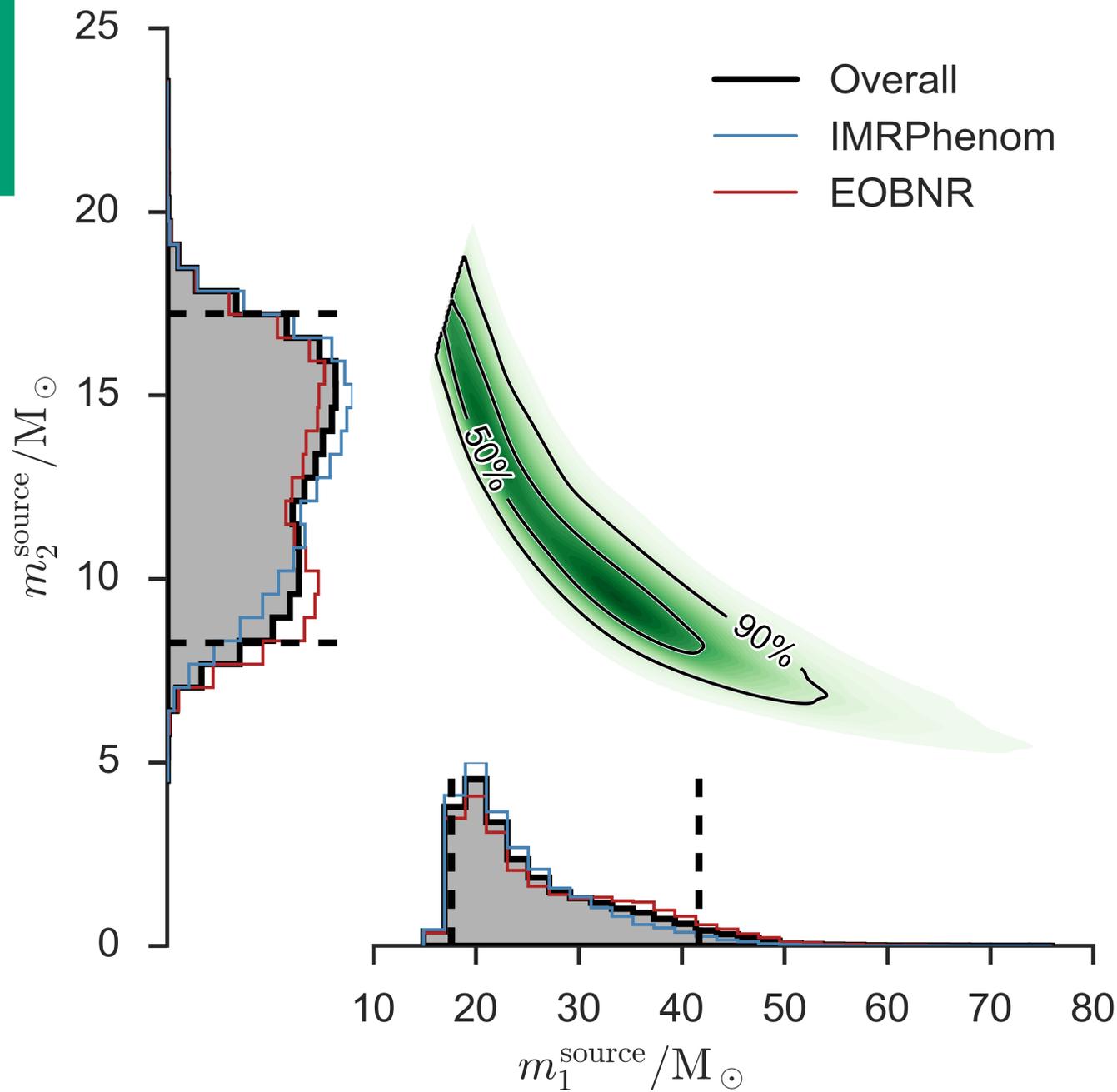
Chirp mass gives leading-order amplitude and phase evolution (Sathyaprakash & Schutz arXiv:0903.0338)

Masses



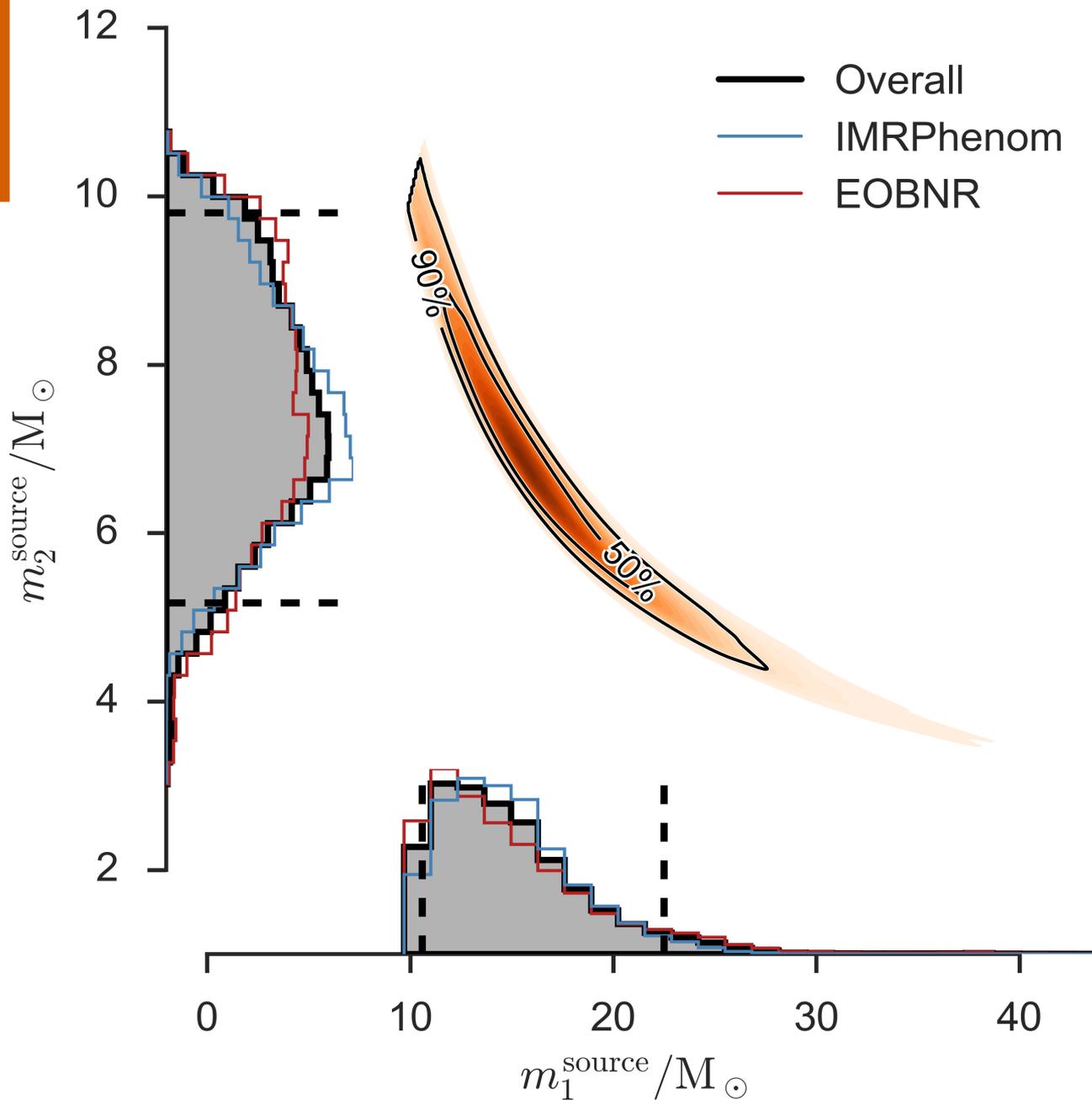
LVC
arXiv:1606.04856
arXiv:1602.03840

Masses



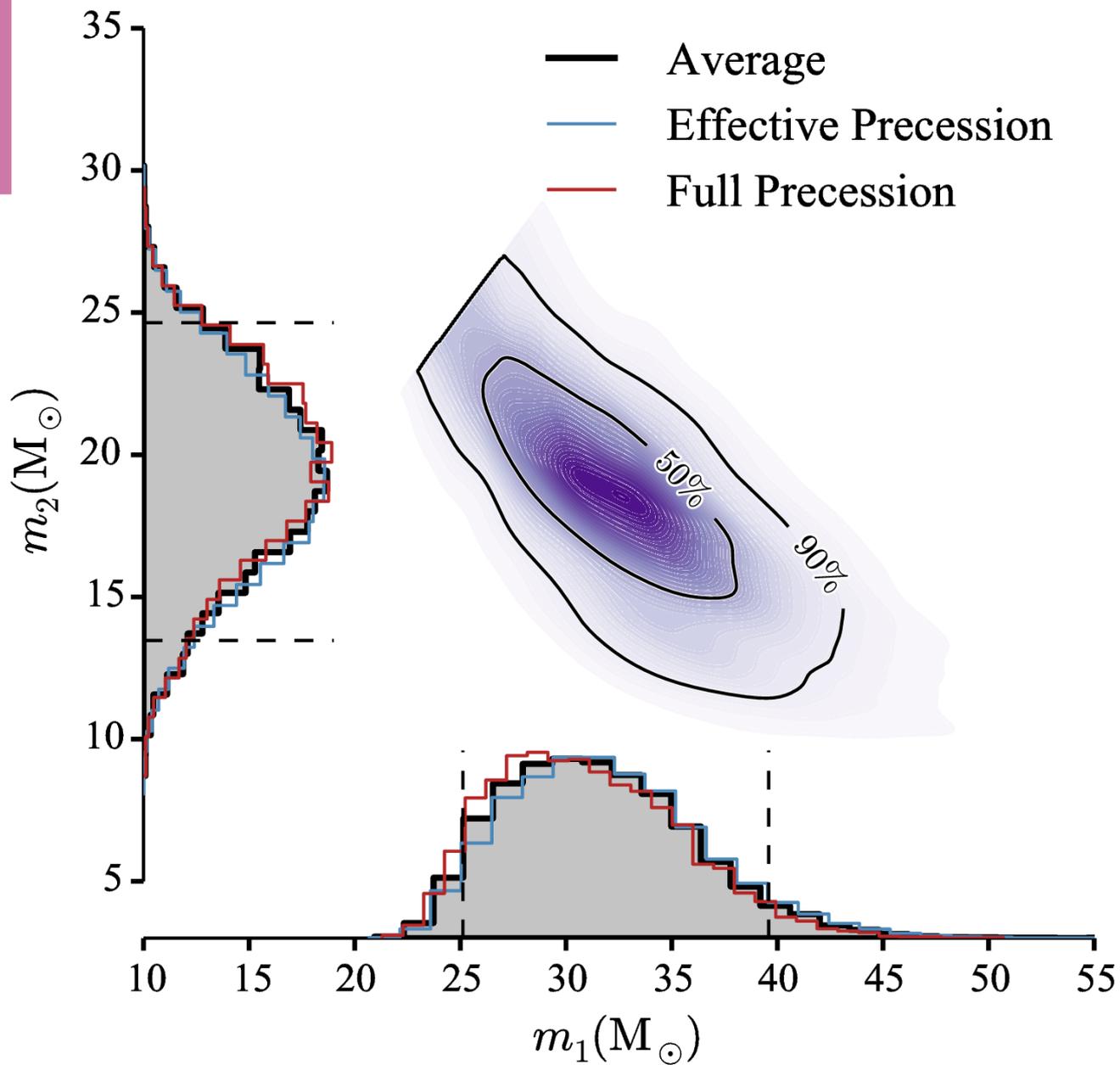
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Masses

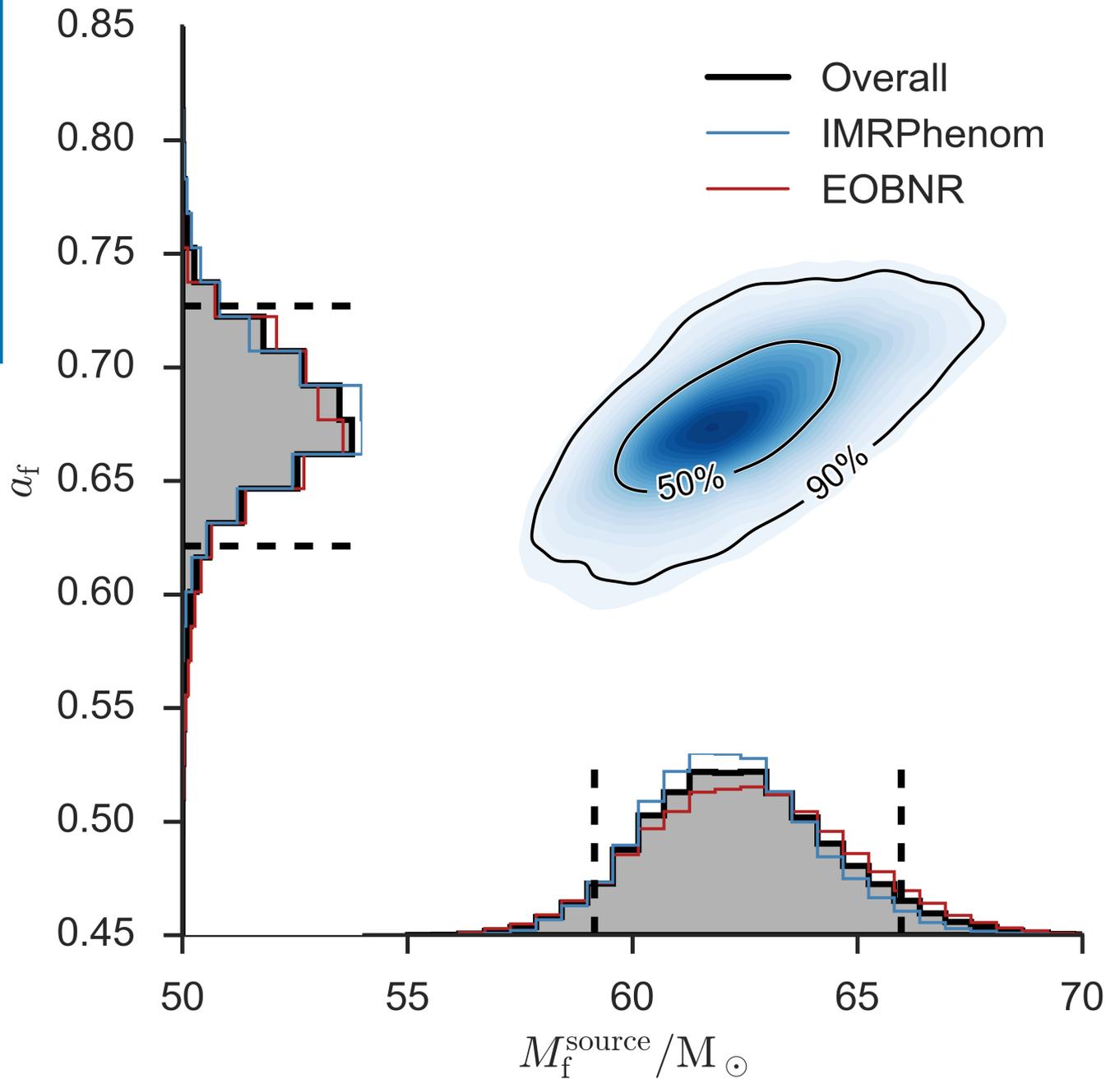


LVC
arXiv:1606.04855

Masses

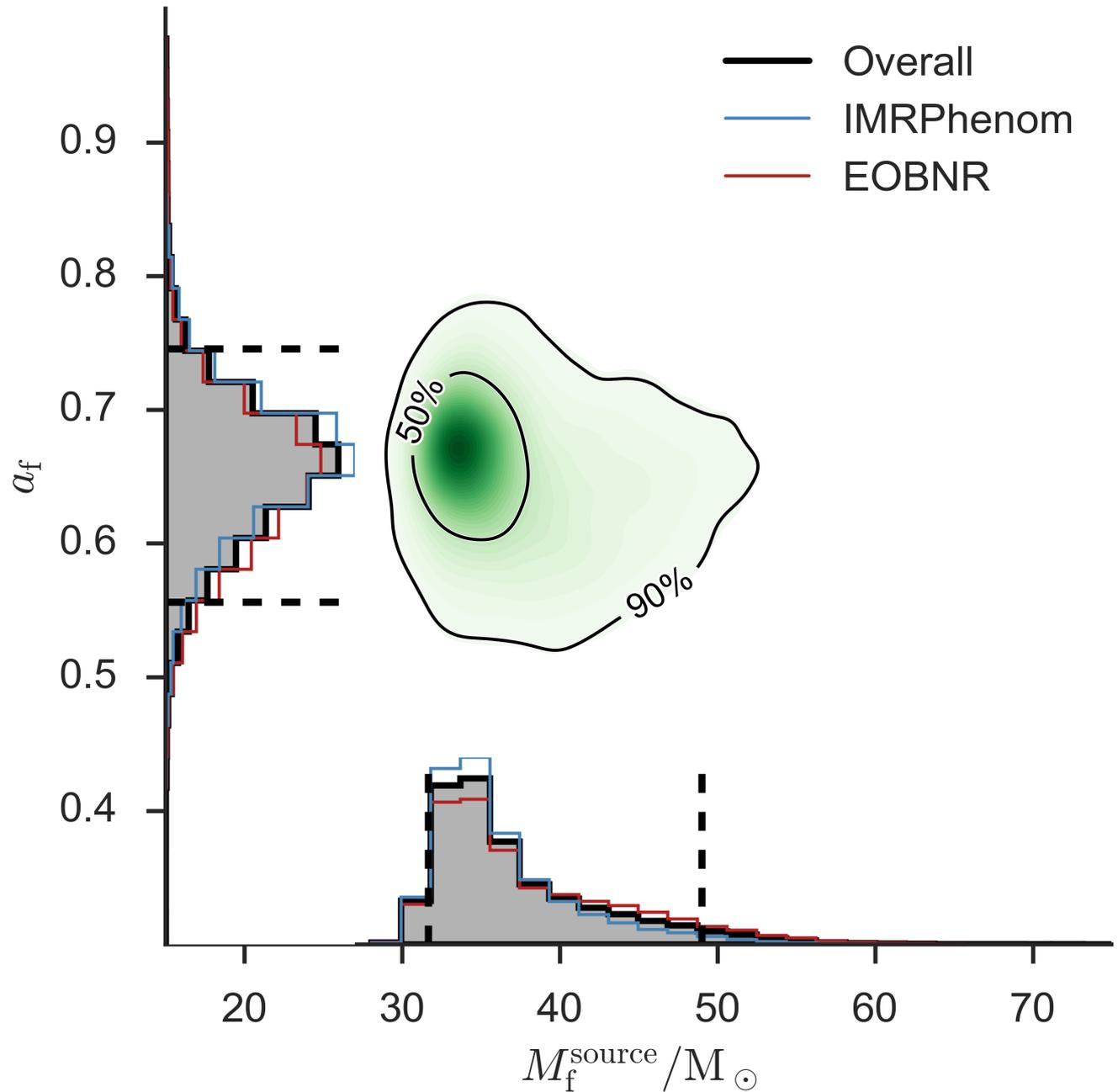


Final mass & spin



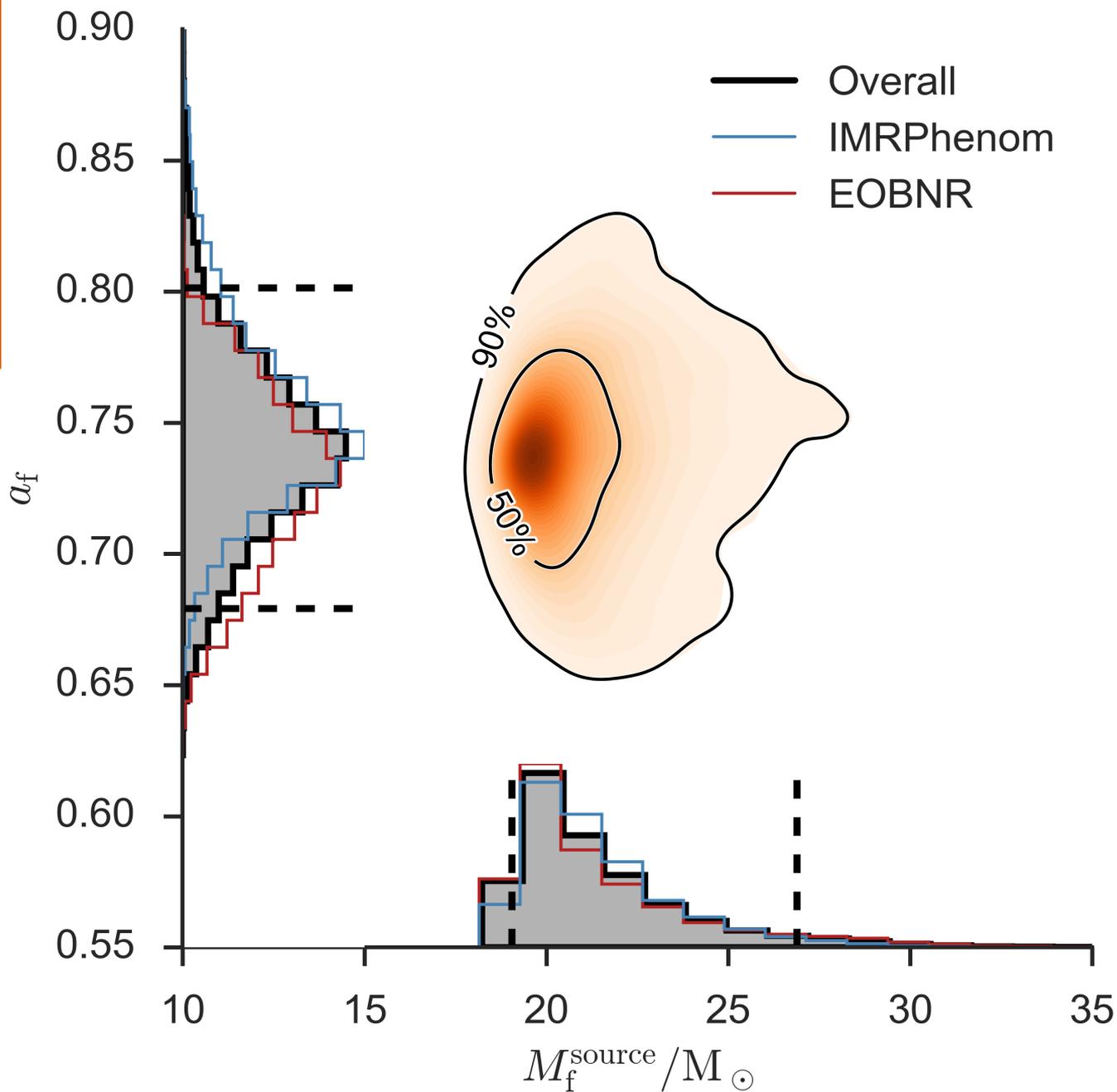
LVC
arXiv:1606.04856
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Final mass & spin



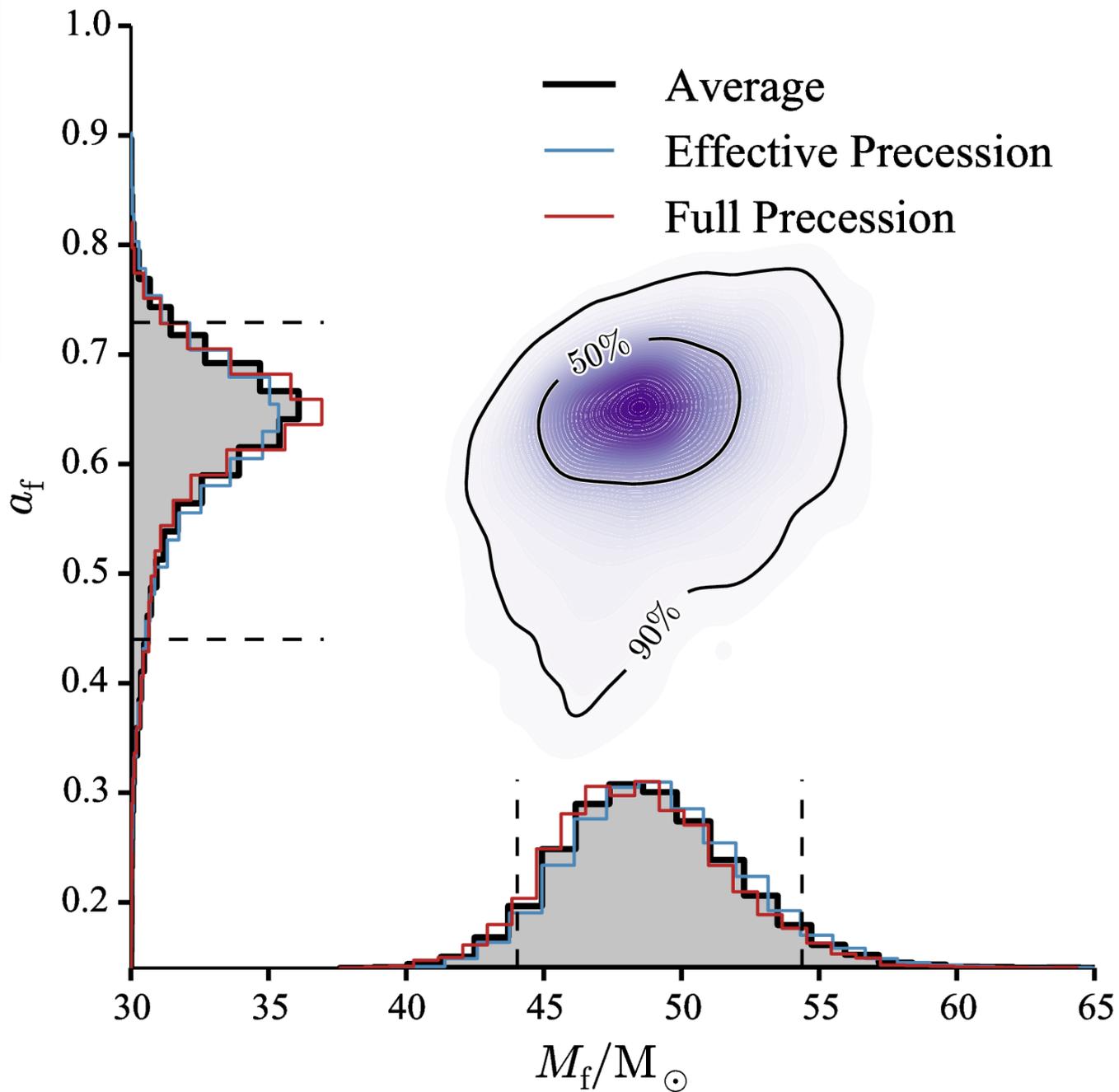
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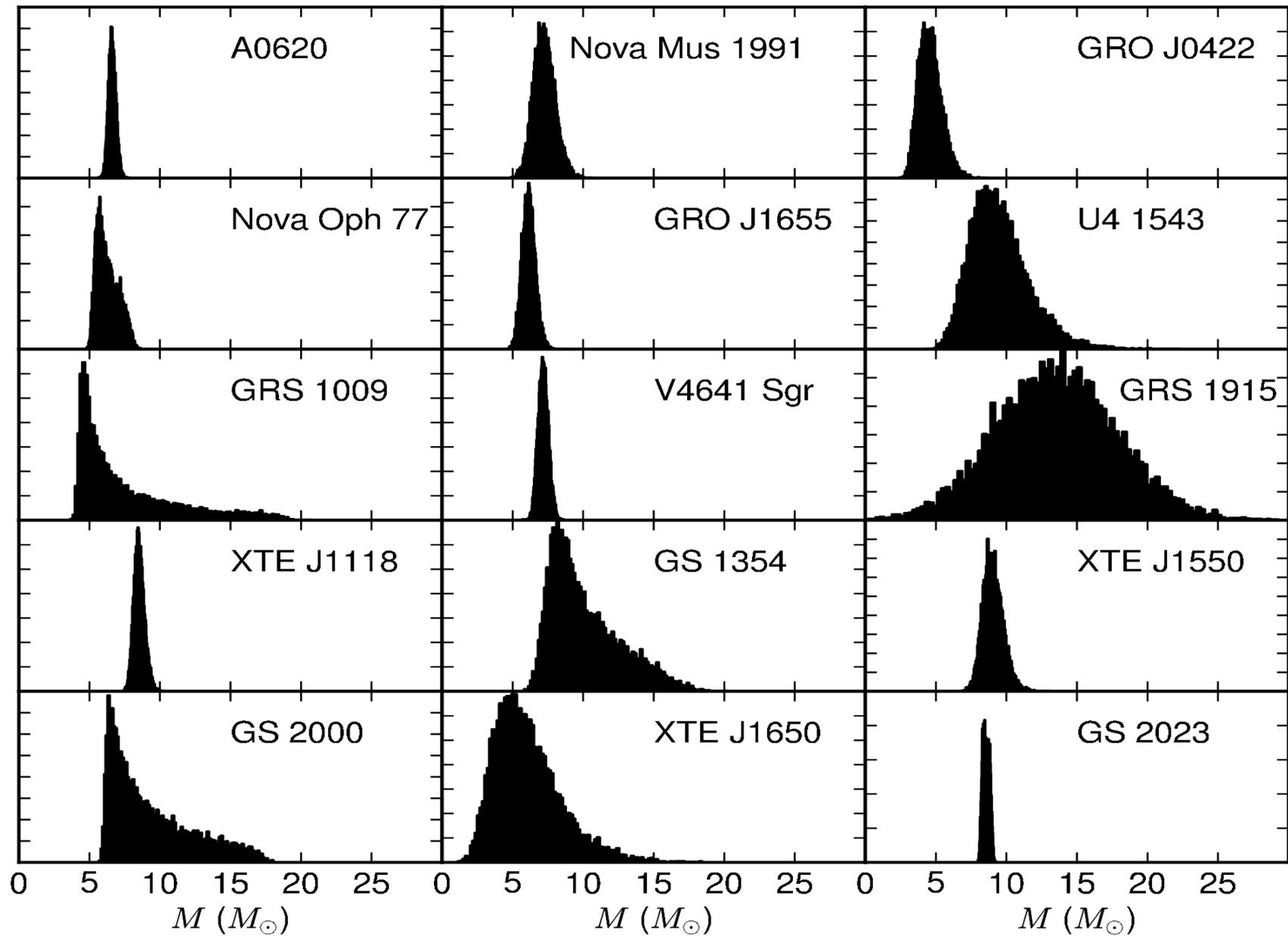
Final mass & spin



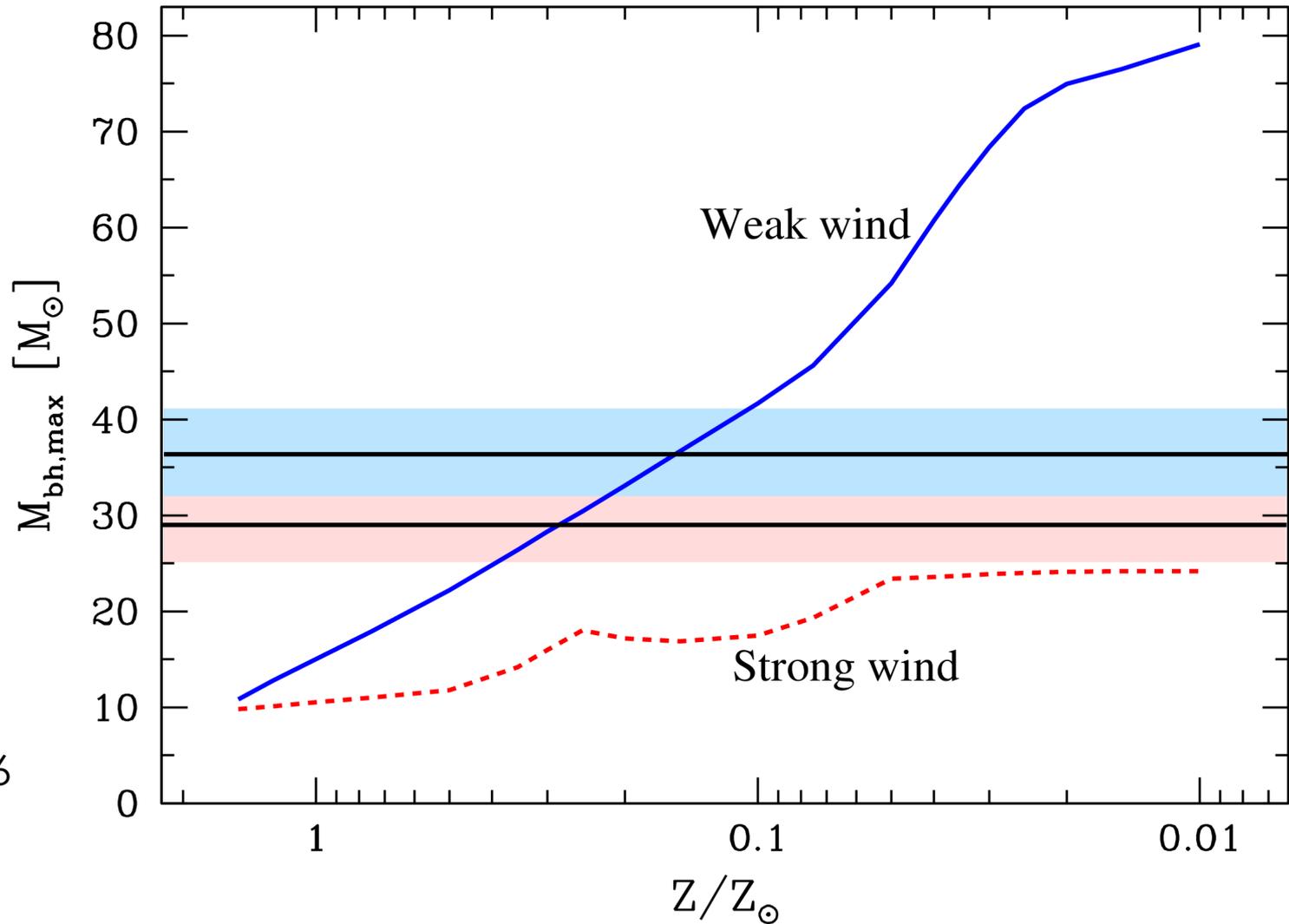
LVC
arXiv:1606.04856

Final mass & spin





Metallicity



LVC
arXiv:1602.03846
Belczynski *et al.*
arXiv:0904.2784

No hair theorem

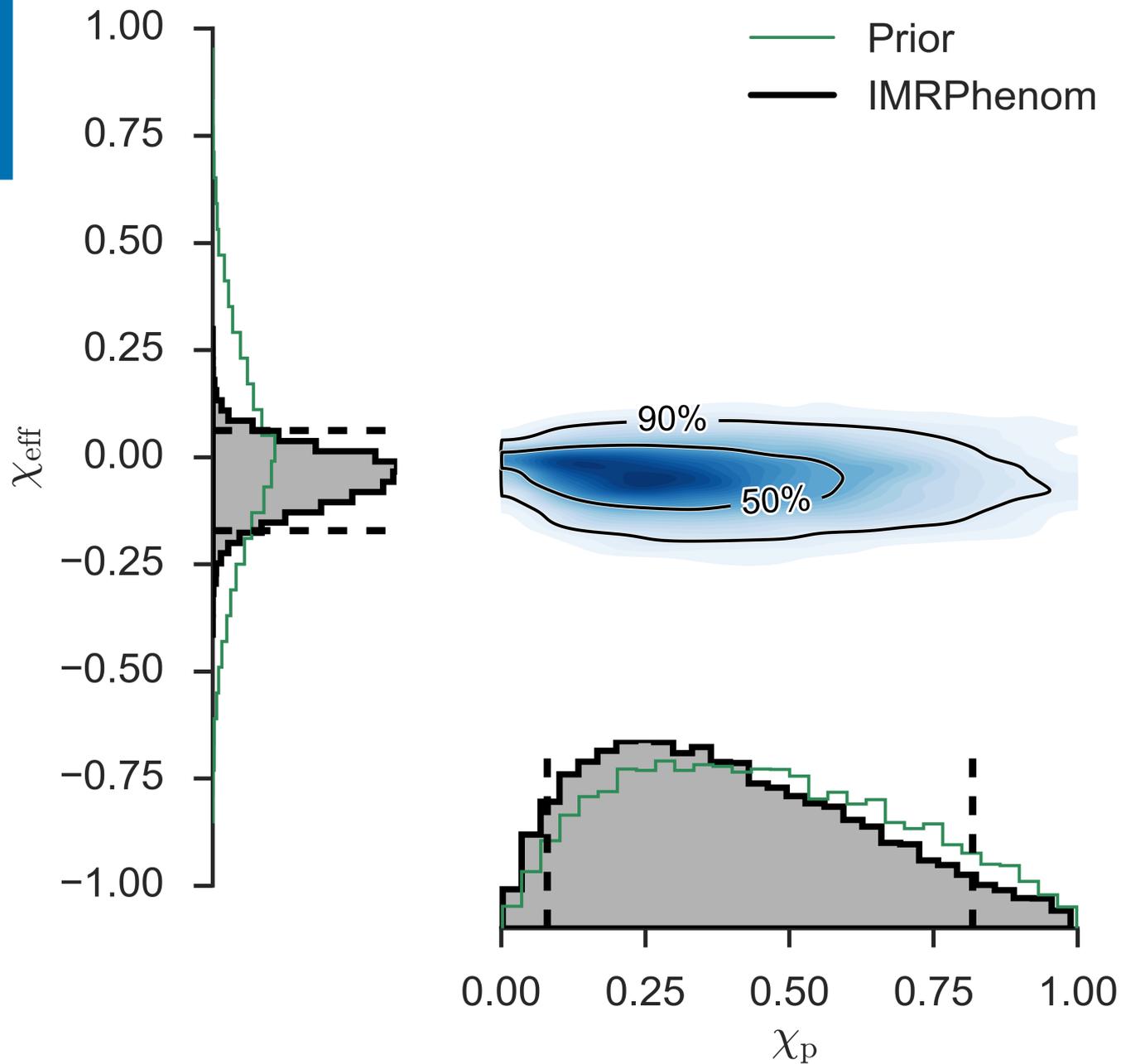


Black holes have:

1. Mass
2. Spin
3. Electric charge

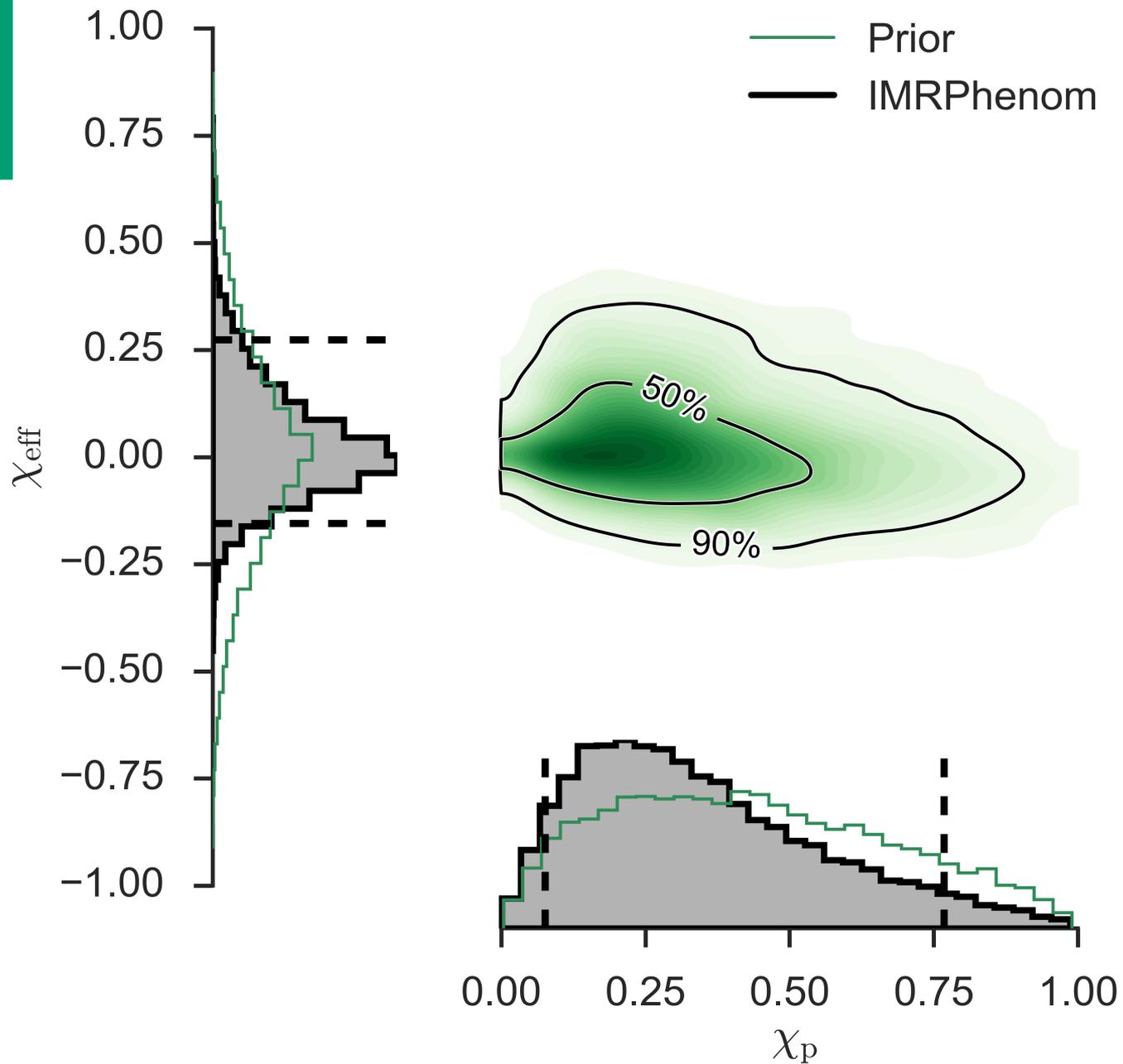
Image: Matt Groening

Spin



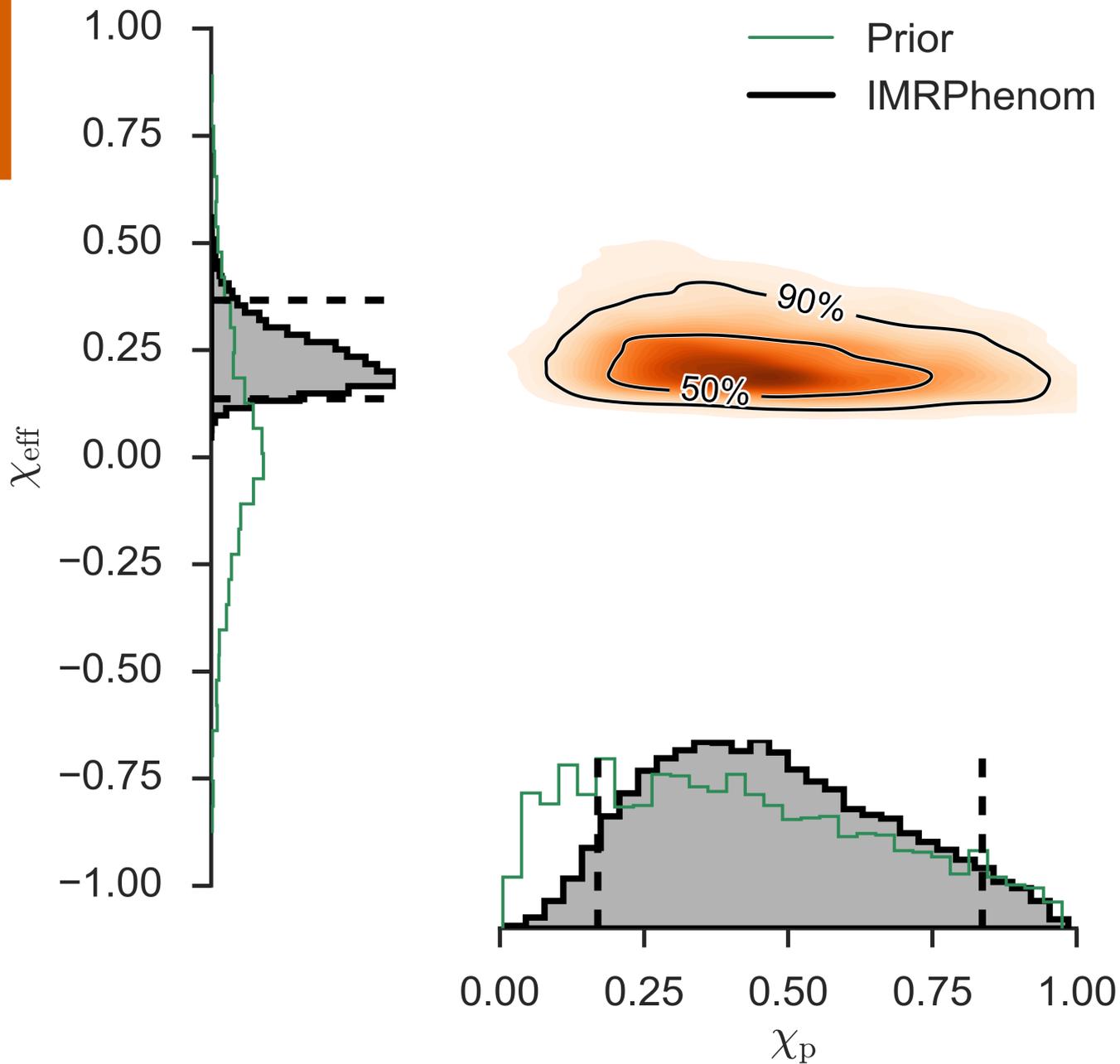
LVC
arXiv:1606.04856
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Spin



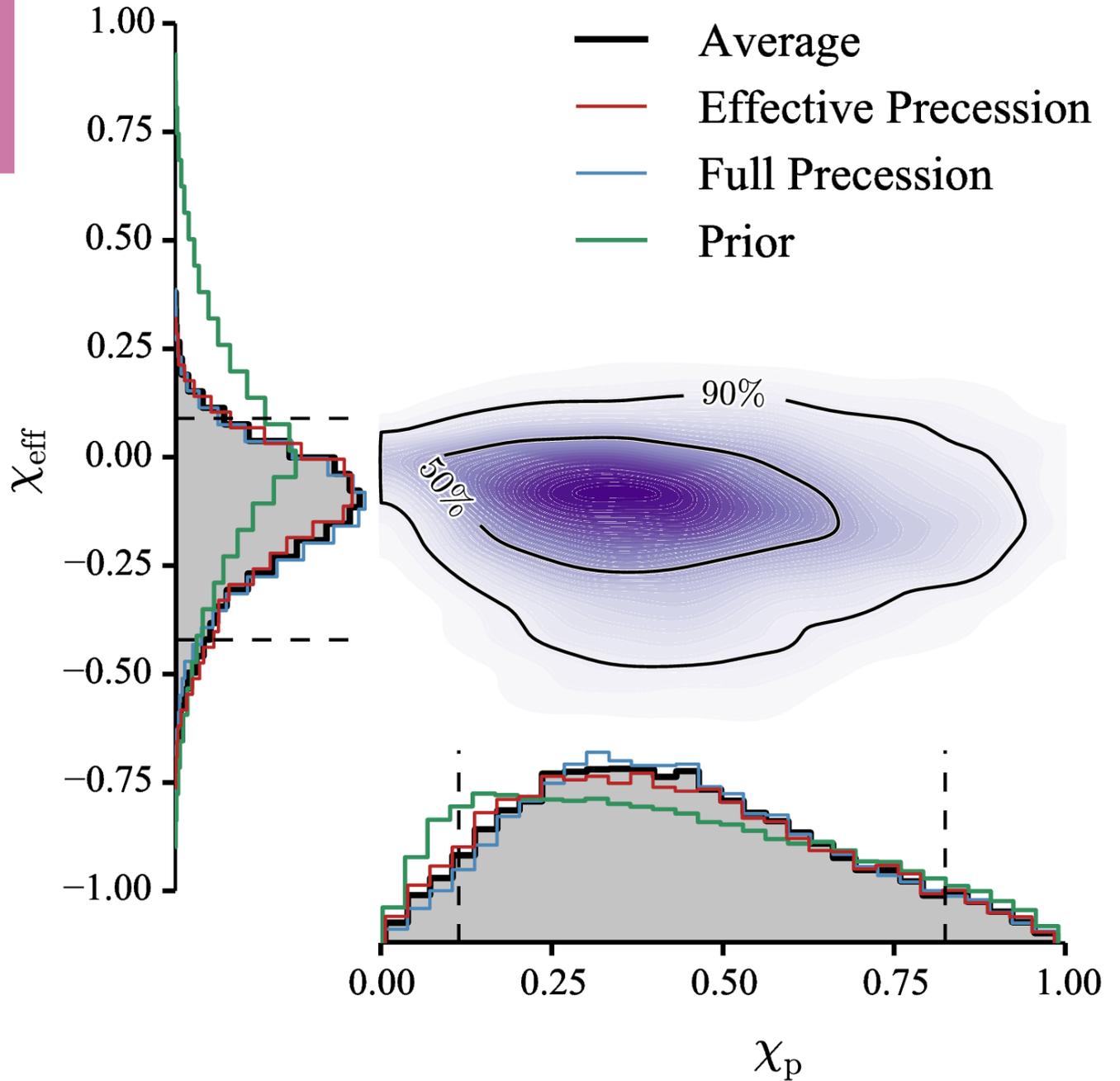
LVC
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Spin

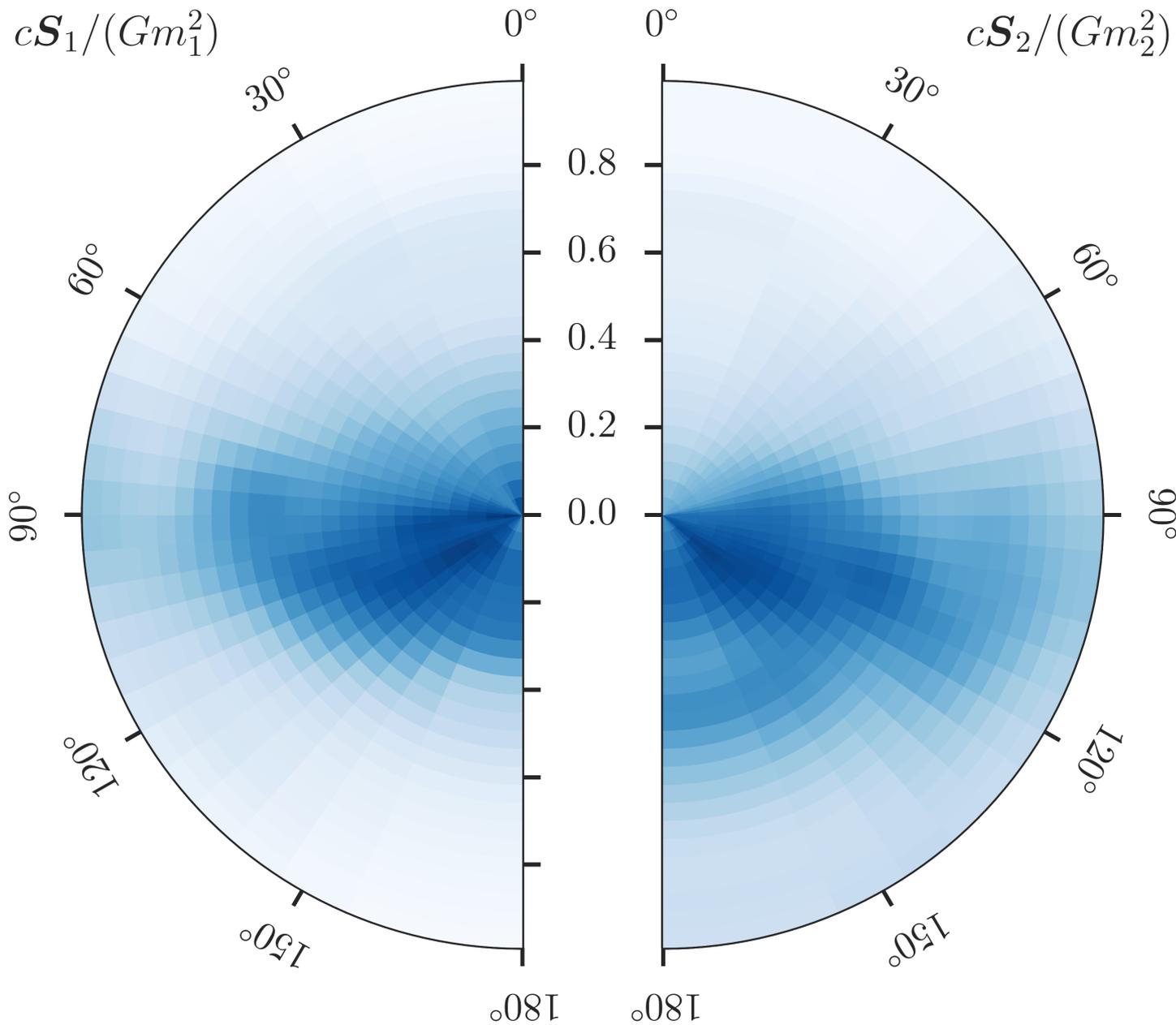


LVC
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Spin

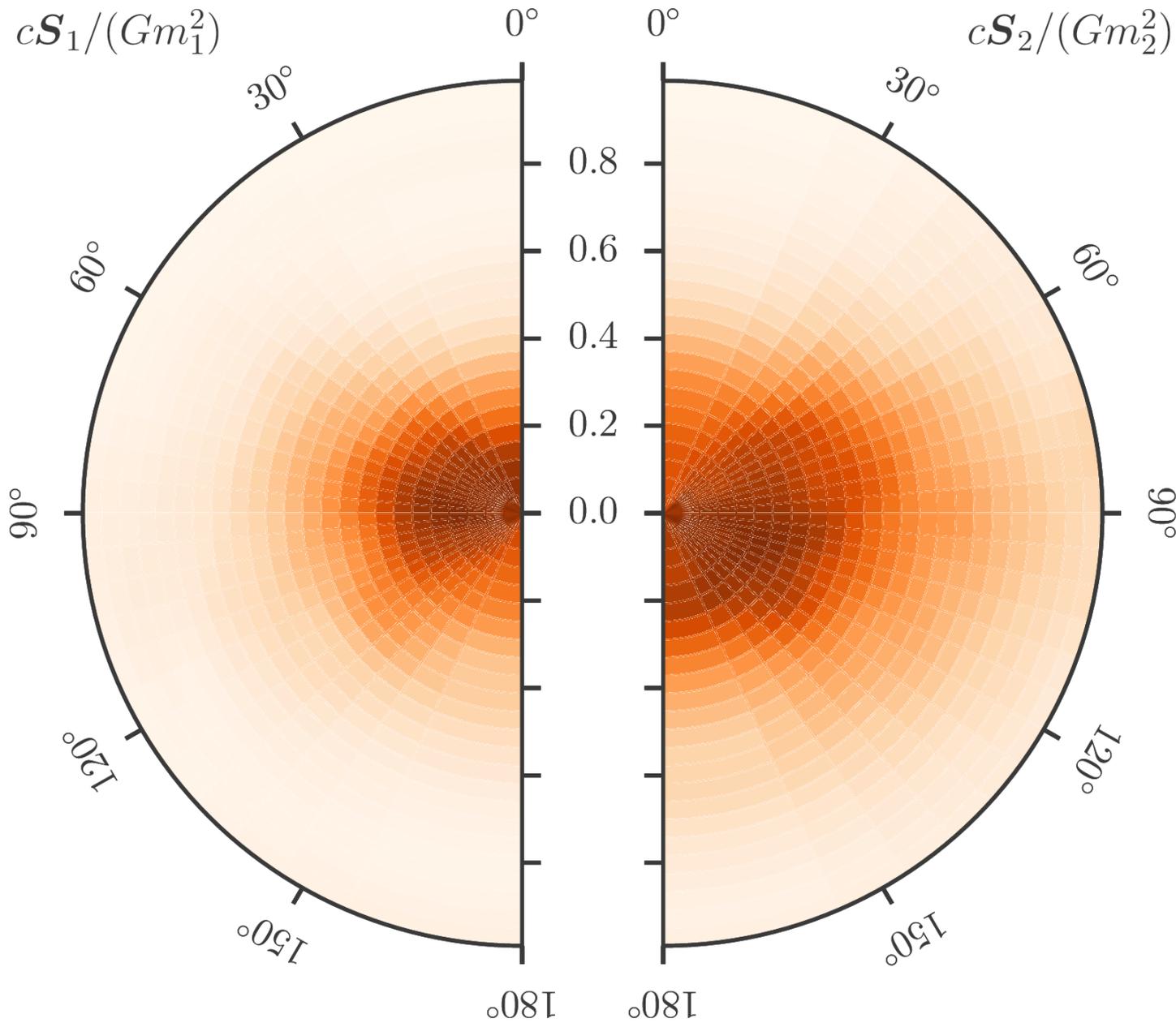


Spin

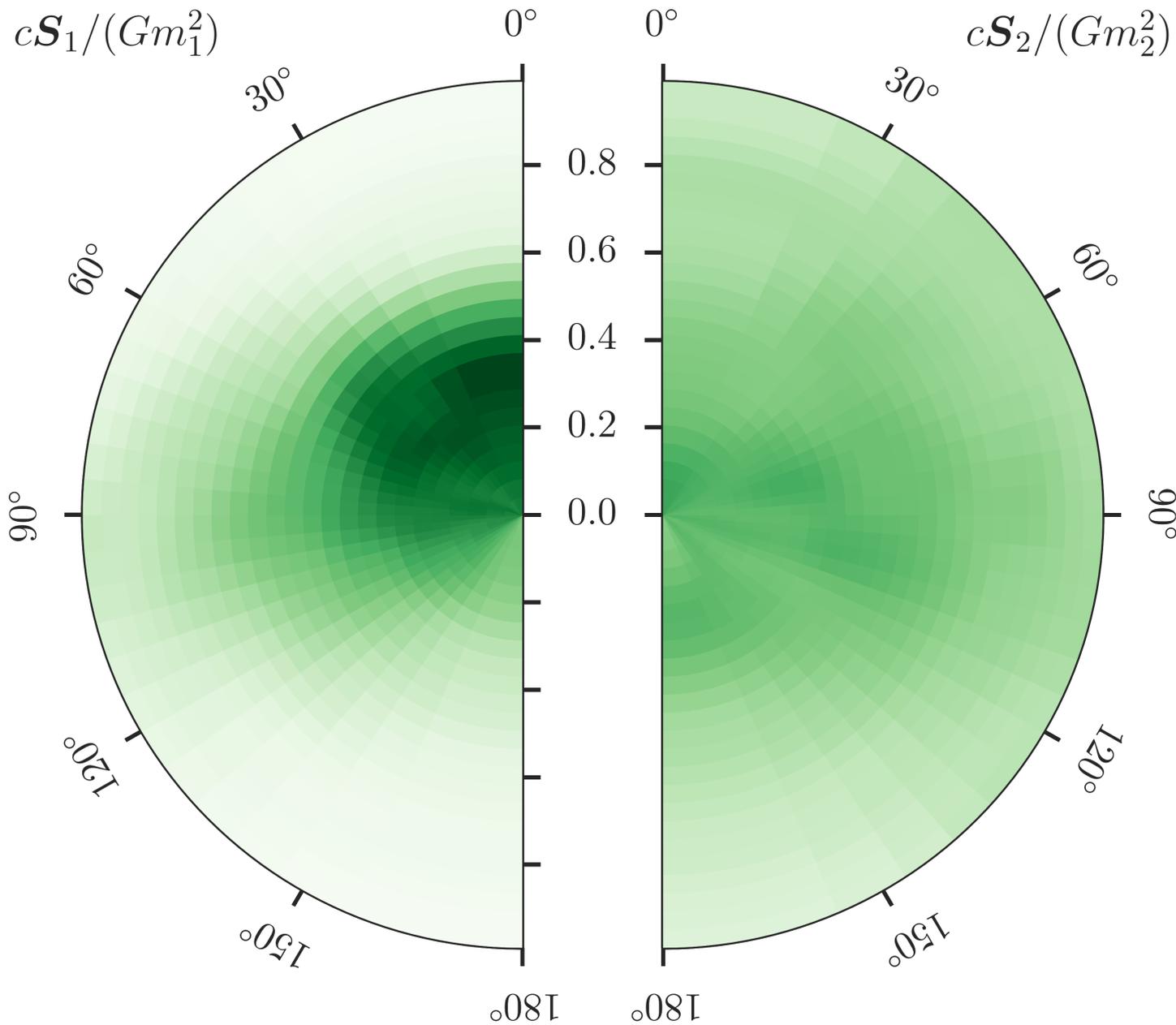


LVC
arXiv:1606.04856
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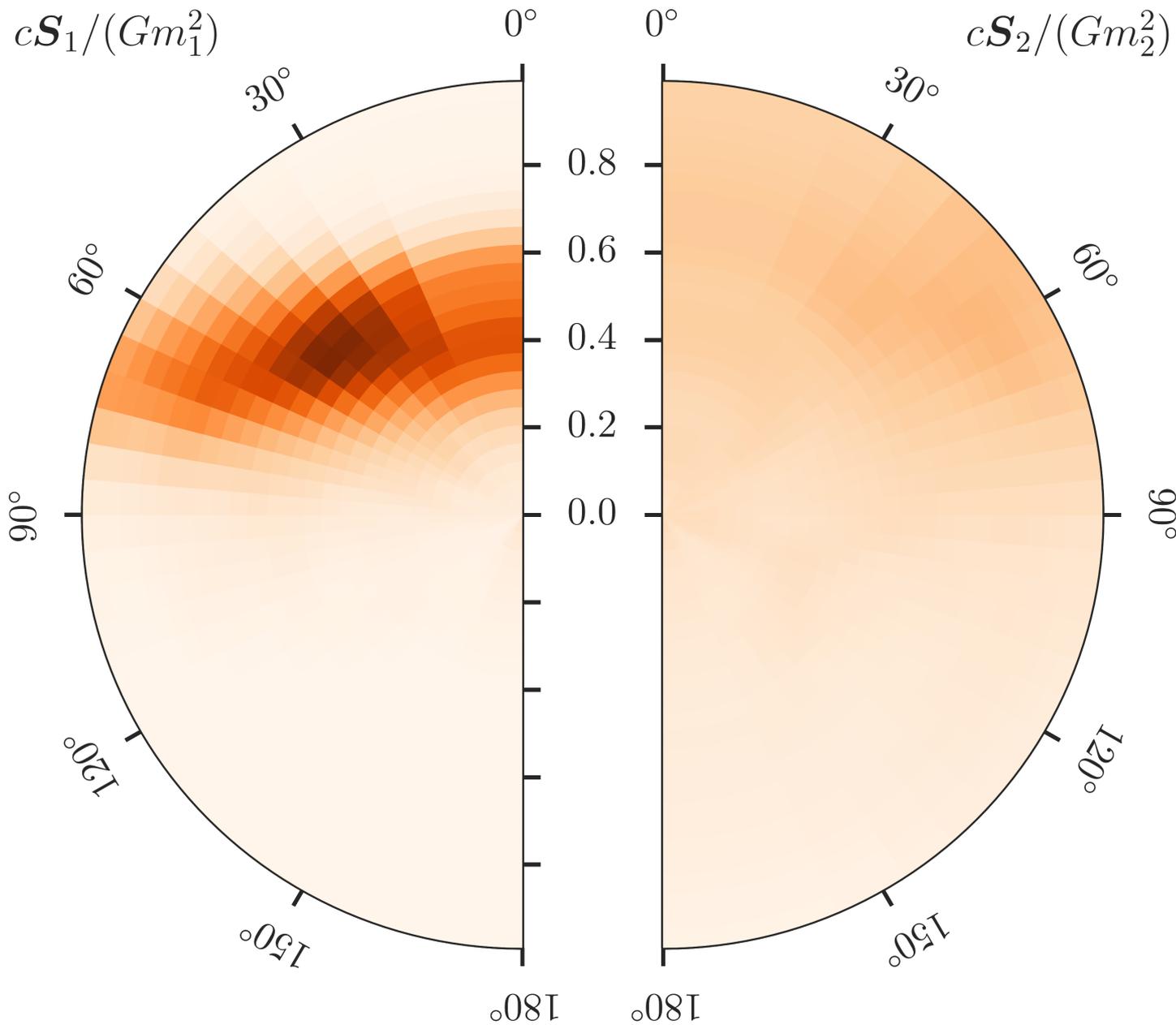
Spin



Spin

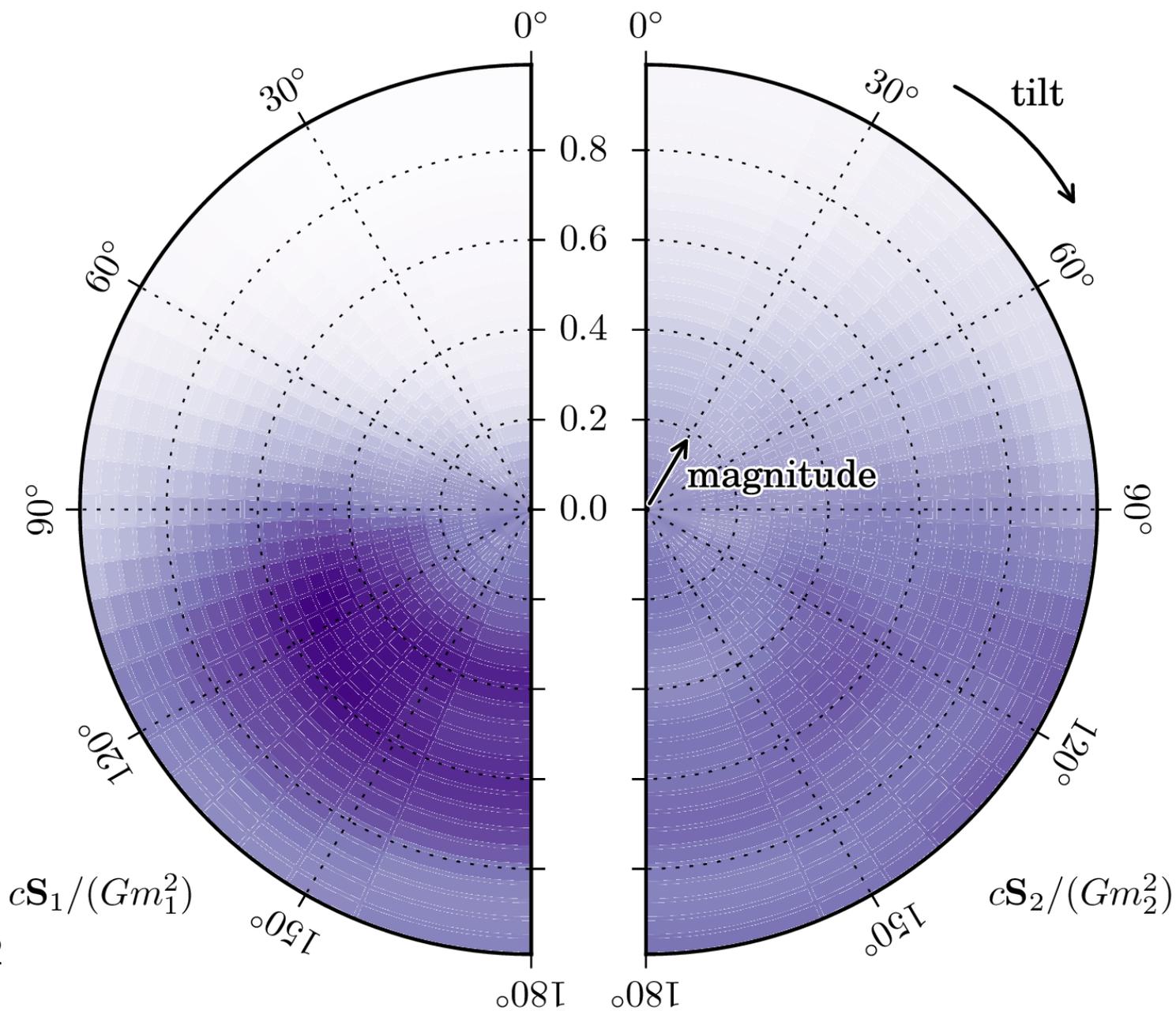


Spin

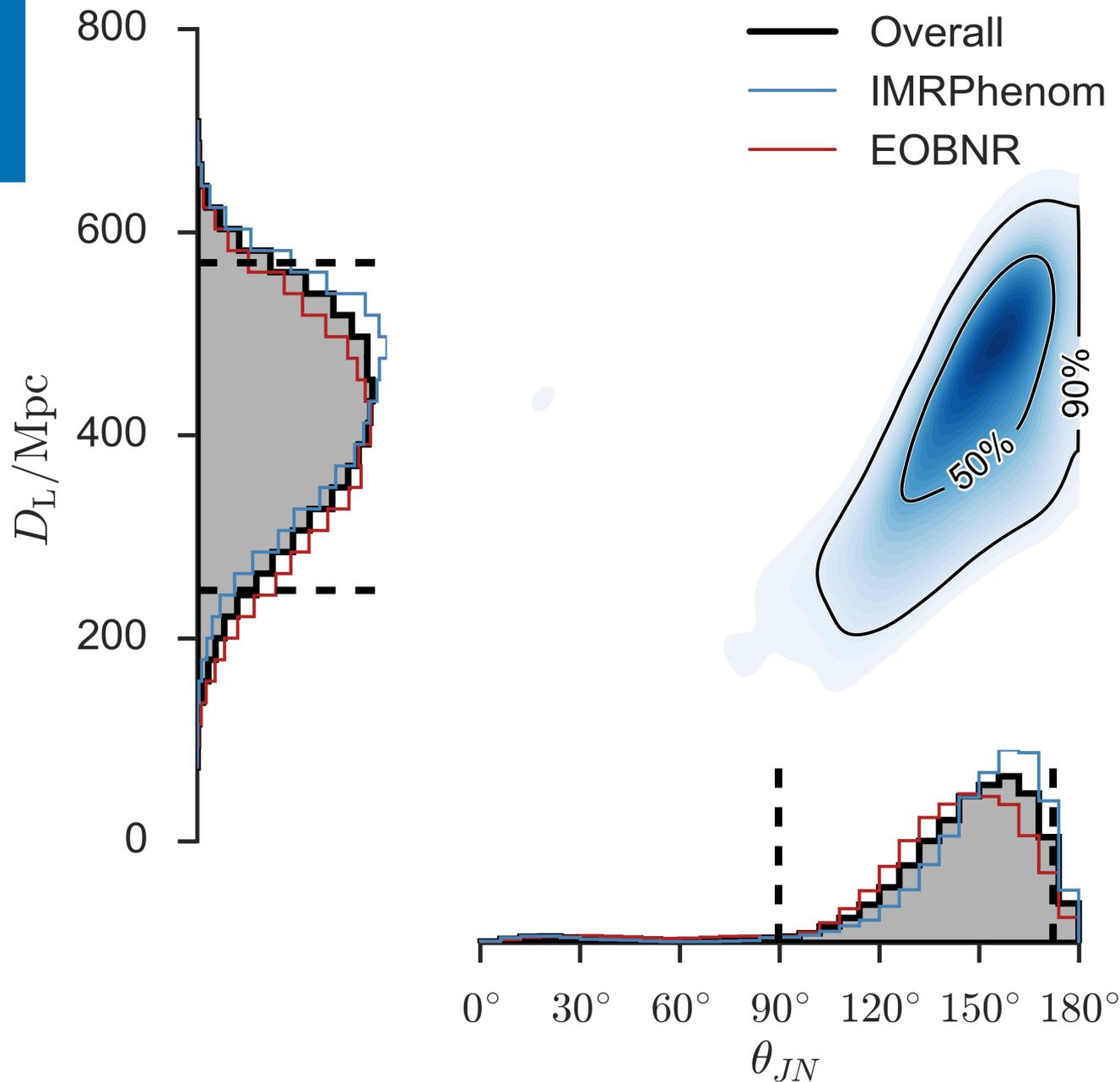


LVC
arXiv:1606.04855

Spin



Distance

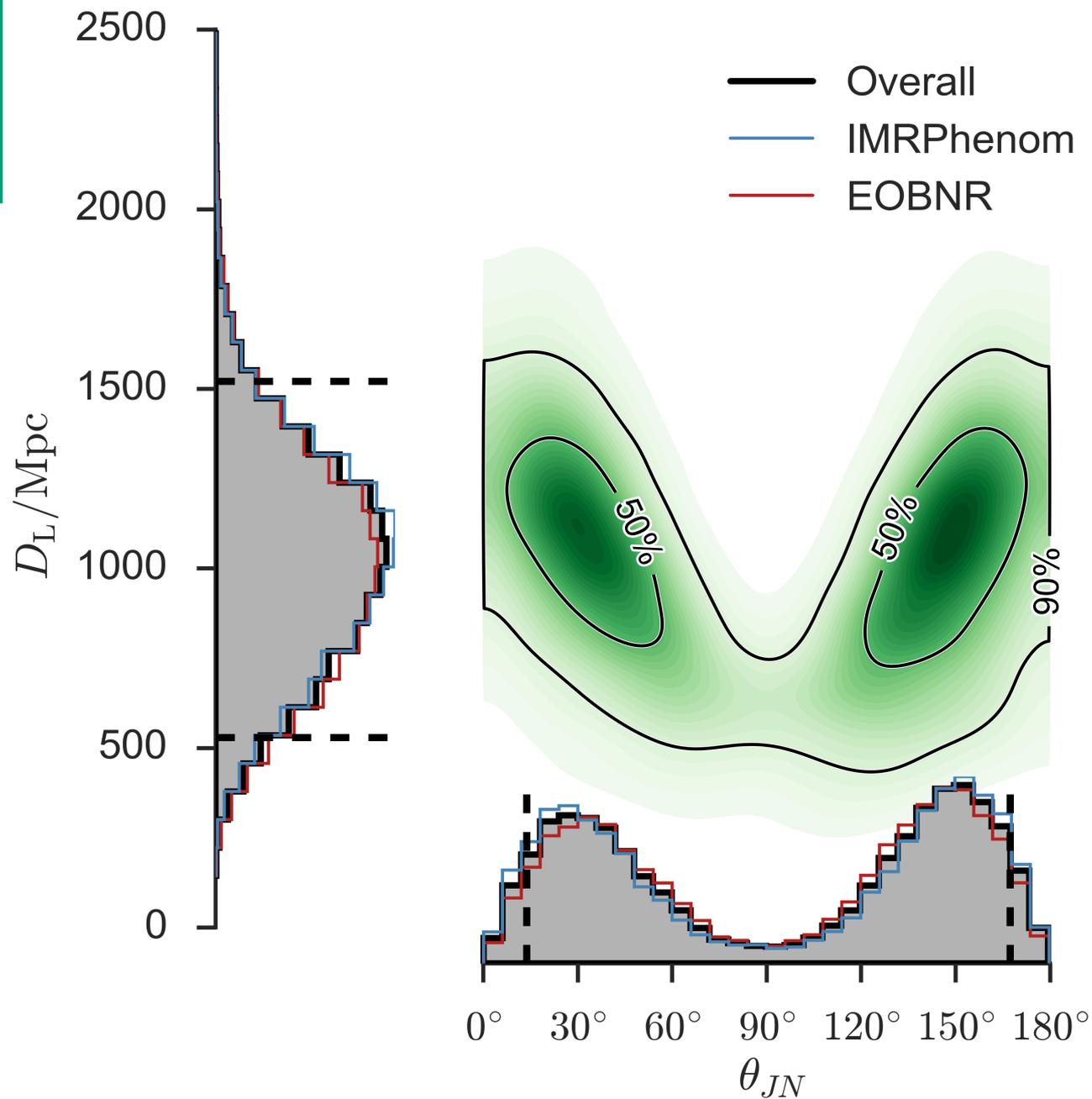


LVC

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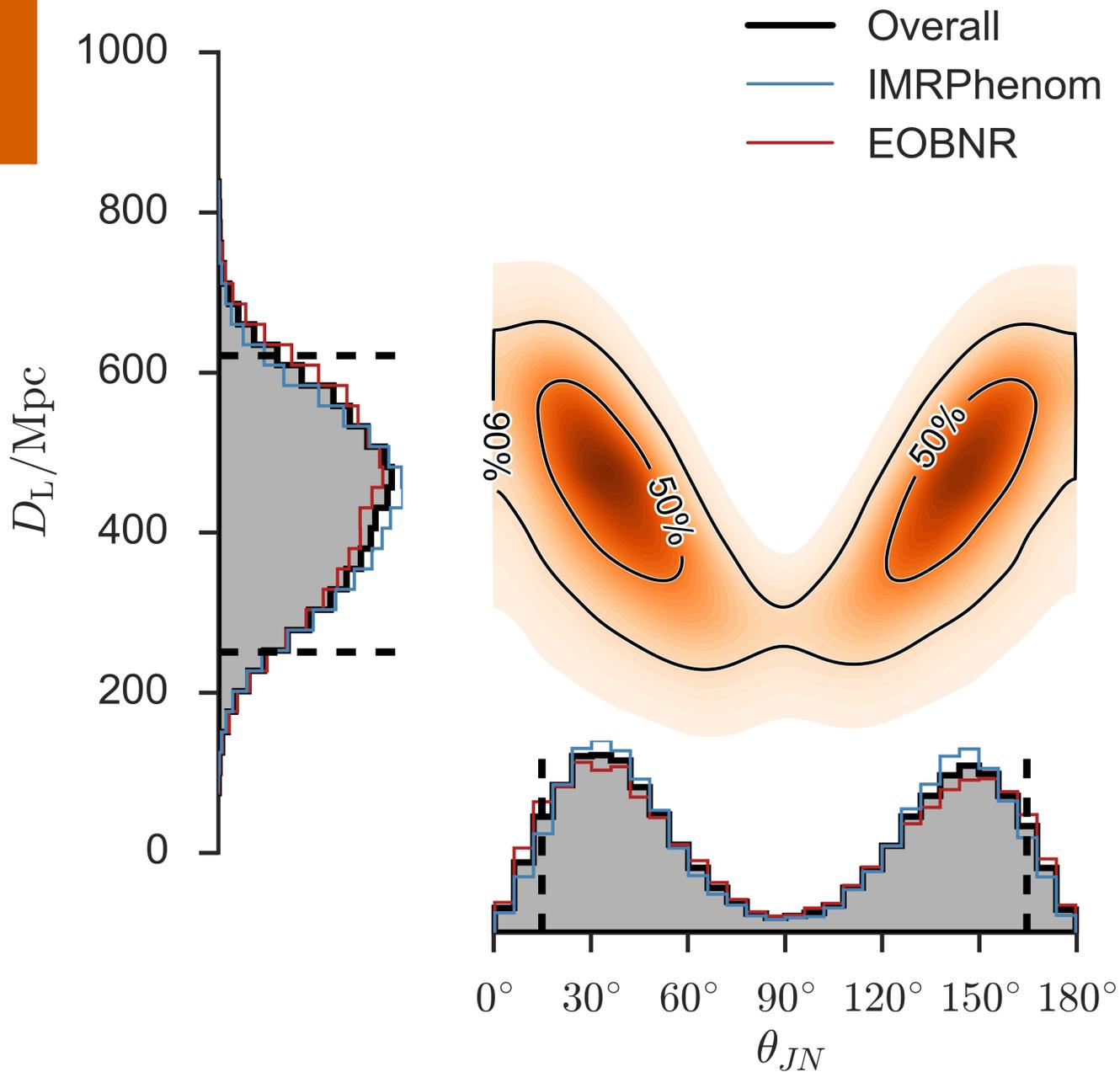
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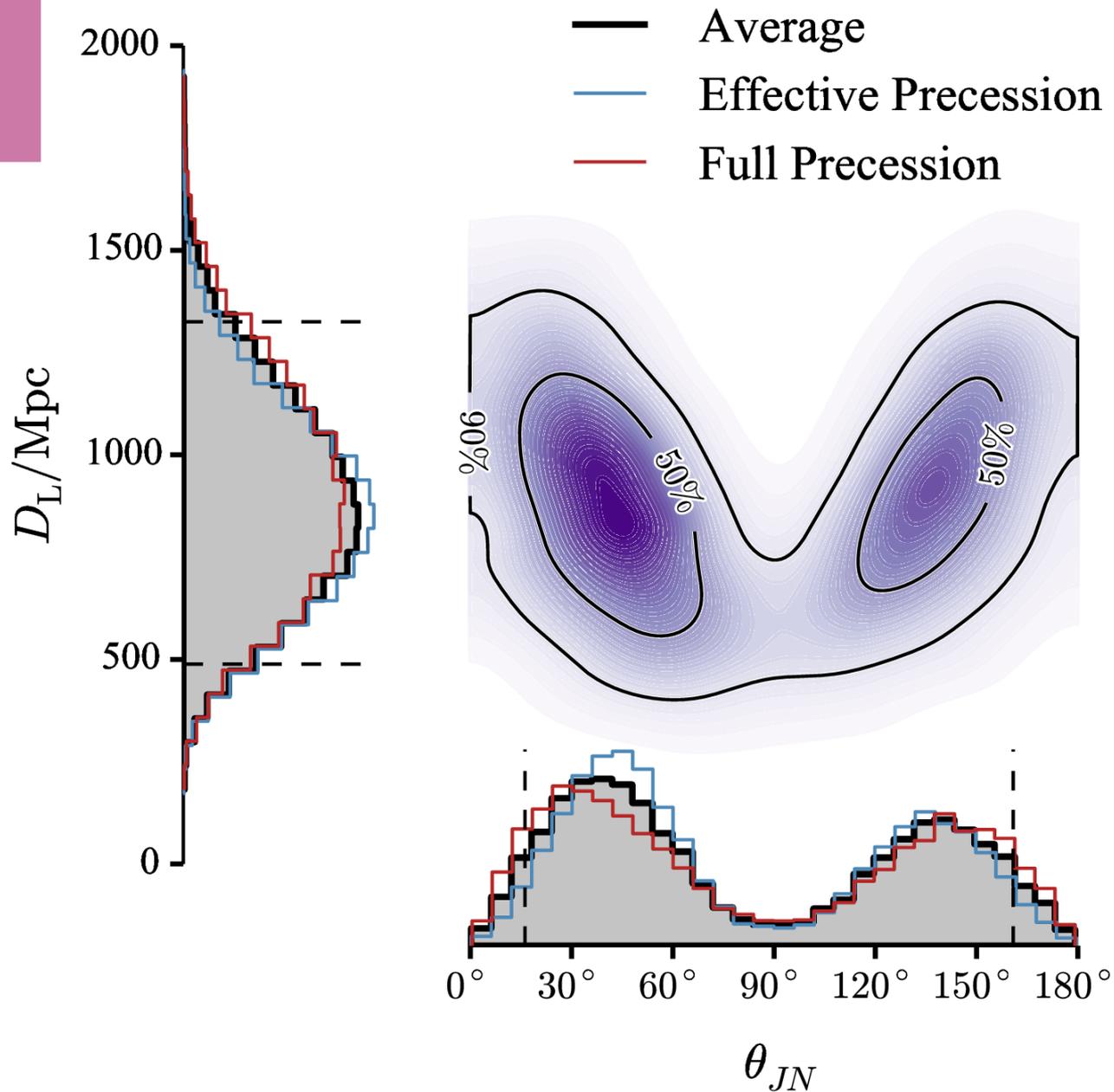
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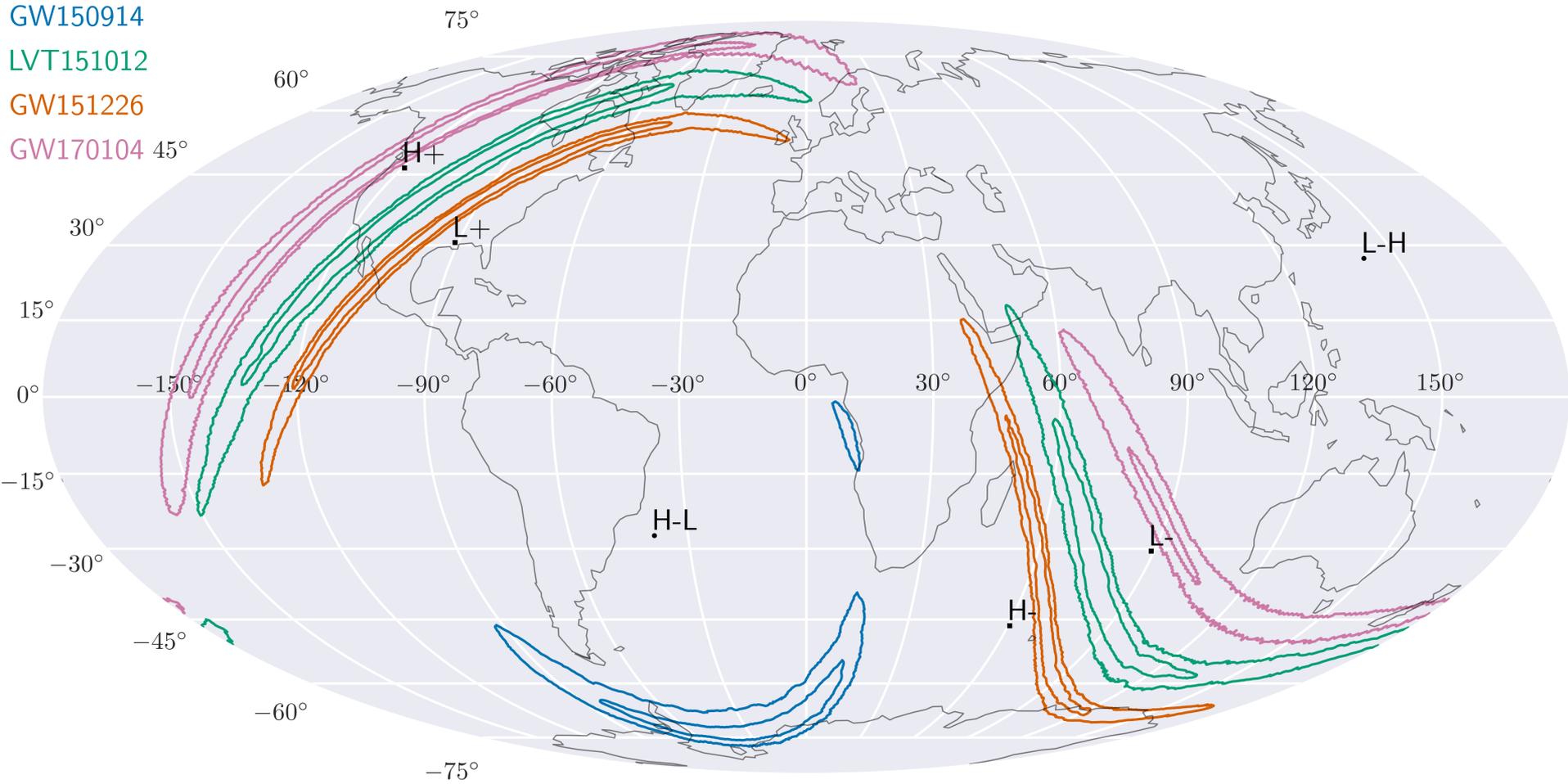
Distance



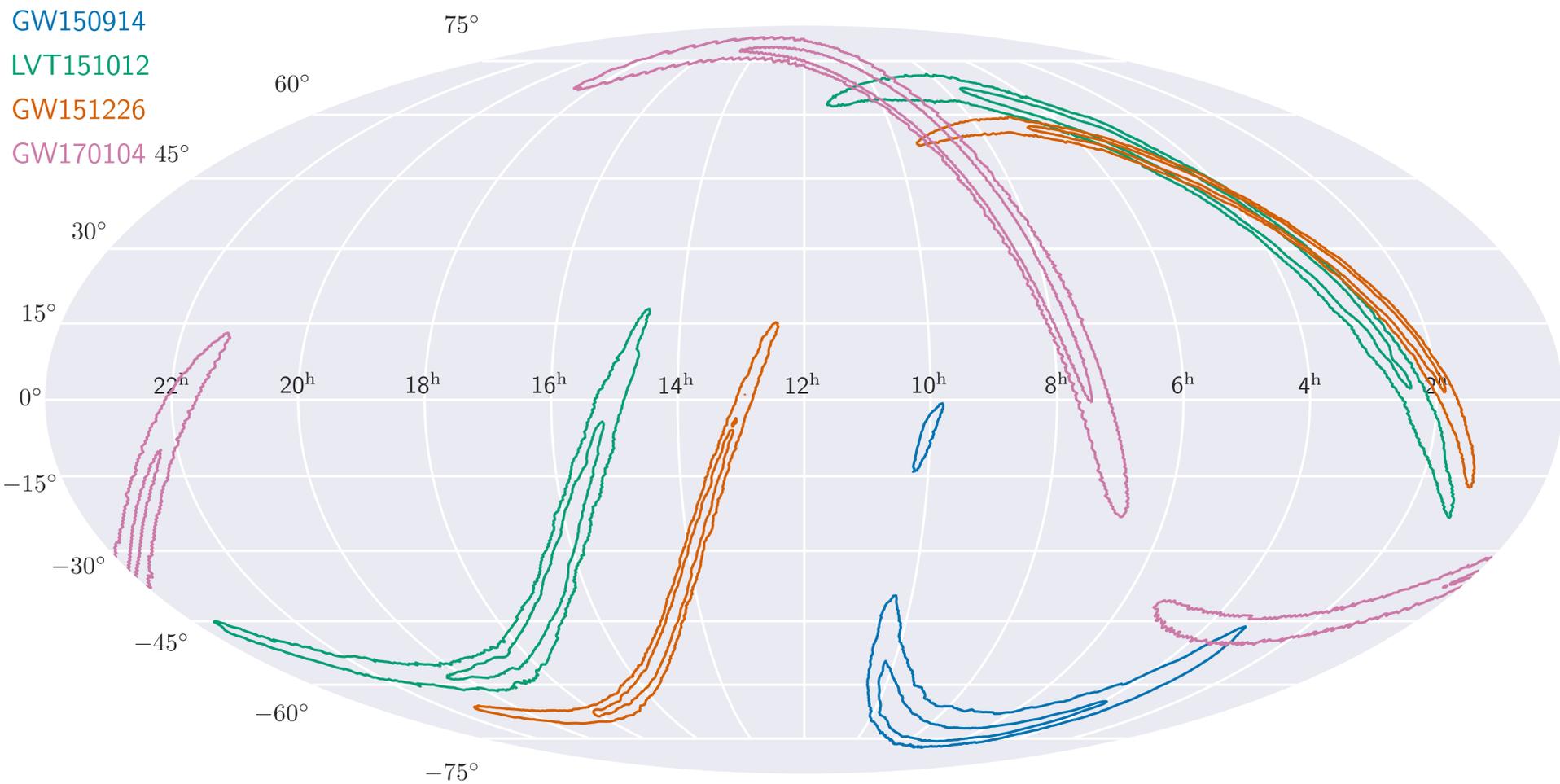
LVC
arXiv:1606.04856

Distance





LVC arXiv:1606.04856
 arXiv:1706.01812



LVC arXiv:1606.04856
arXiv:1706.01812

The First Two Years of Electromagnetic Follow-Up with Advanced LIGO and Virgo

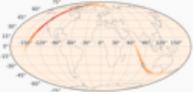
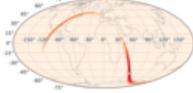
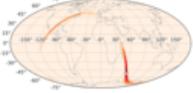
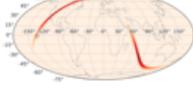
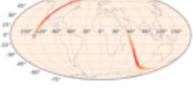
 [Singer et al. 2014](#)
arXiv:1404.5623

 [Berry et al. 2015](#)
arXiv:1411.6934

www.ligo.org/scientists/first2years/
asd.gsfc.nasa.gov/Leo.Singer/going-the-distance/

Catalog of simulated events and sky maps for two-detector, HL, 2015 configuration. This is the same configuration as the 2015 tab, except that the simulated detector noise is data from initial LIGO's  sixth science run, recoloured (filtered) to have the same PSD as the early Advanced LIGO configuration. See also ASCII tables of  simulated signals,  detections, and  parameter-estimation accuracies in [Machine Readable Table](#) format.

This web page provides additional online information related to the paper "Two Years of Electromagnetic Follow-Up with Advanced LIGO and Virgo" and the paper "Parameter Estimation for Binary Neutron Star Coalescences with Advanced LIGO and Virgo".

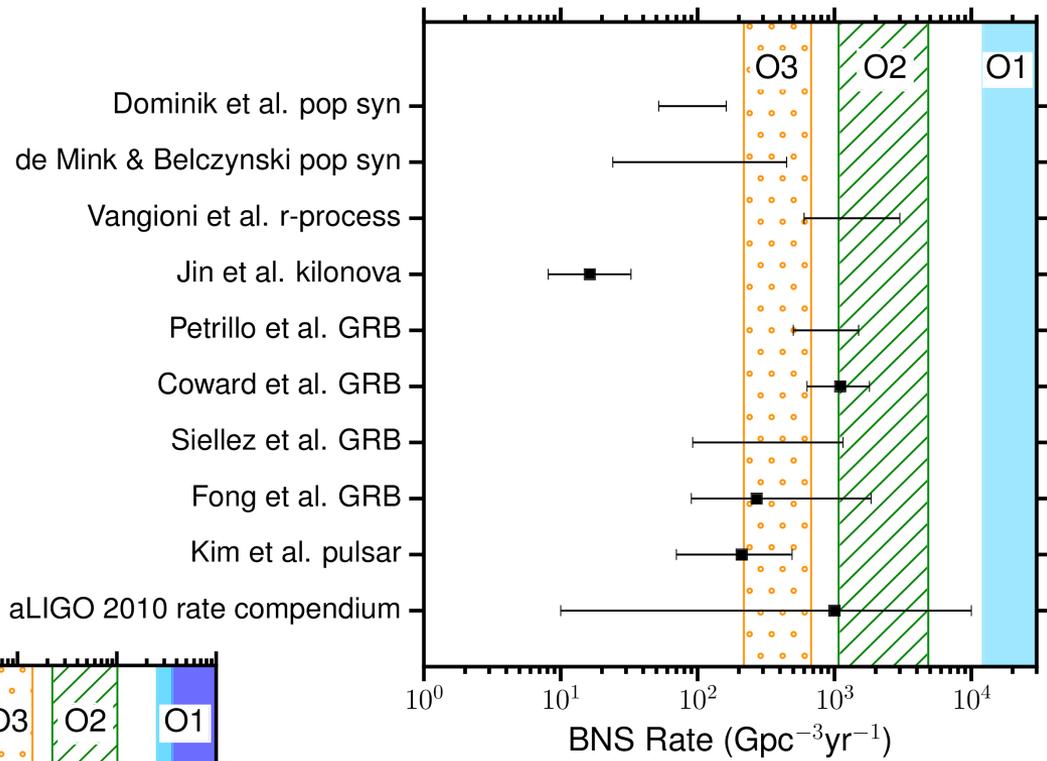
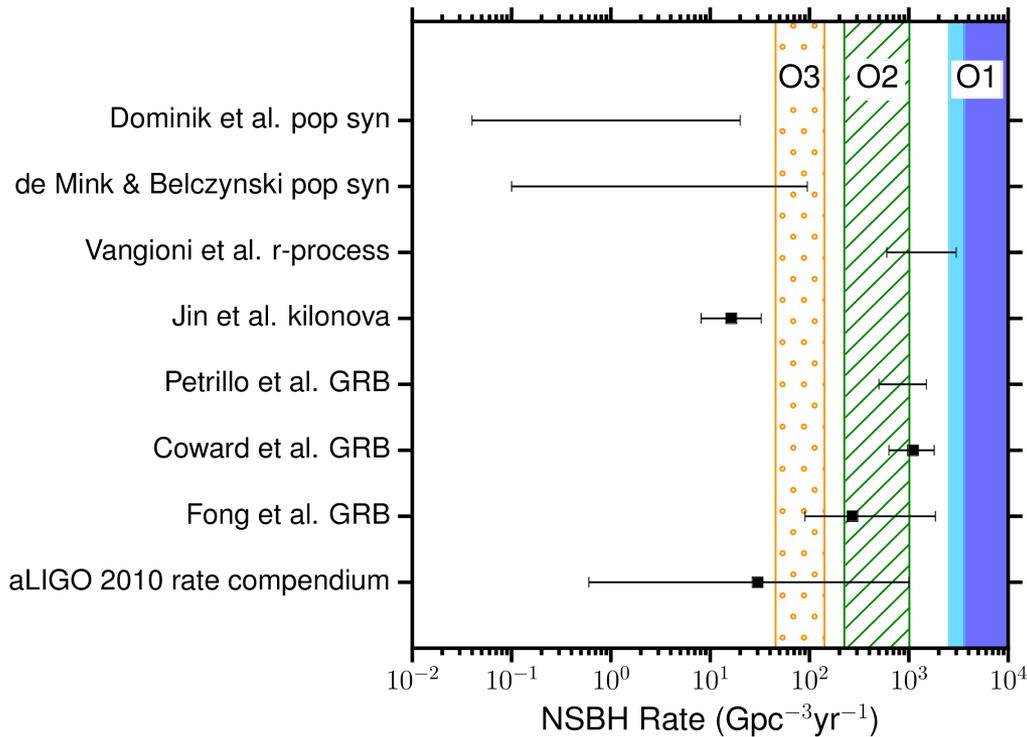
event ID	sim ID	network	SNR			BAYESTAR			LALINFERENCE_NEST			sky maps	
			net	H	L	50%	90%	searched	50%	90%	searched	BAYESTAR	LALINFERENCE_NEST
4532	899	HL	13.9	10.1	9.5	180	750	190	170	790	150		
4572	1243	HL	13.2	10.0	8.7	230	830	45	200	920	33		
4618	1768	HL	10.8	8.0	7.3	160	540	220	130	440	280		
4647	1964	HL	12.4	8.6	9.0	260	890	1200	190	780	780		
4711	2704	HL	10.7	8.0	7.1	370	1200	300	450	1600	520		



LVT151012

**I WANT TO
BELIEVE**

Limits



LVC
arXiv:1607.07456

losc.ligo.org/events/
papers.ligo.org/

