Numerical black holes and mergers beyond general relativity

Leo C. Stein (TAPIR, Caltech)

Okounkova, Stein, Scheel, Hemberger [arXiv:1705.07924]

New frontiers in GW astrophysics — June 21, 2017

Why test GR?

$$G_{ab} = 8\pi \hat{T}_{ab}$$

General relativity successful but incomplete

- Can't have mix of quantum/classical
- GR not renormalizable
- GR+QM=new physics (e.g. BH information paradox)

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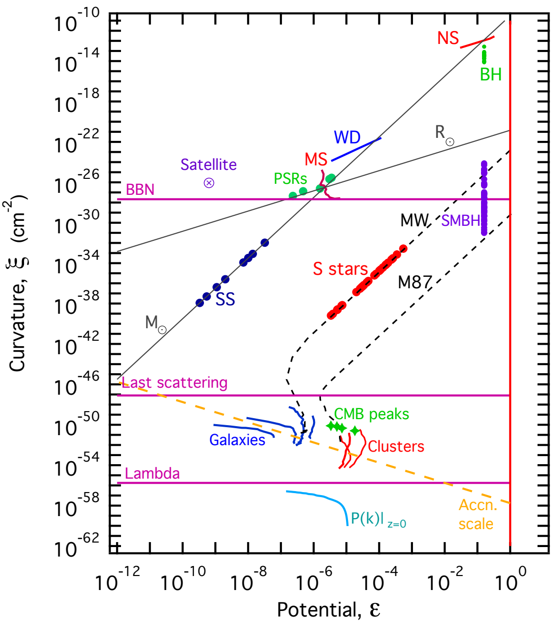
Empiricism

Ultimate test of theory: ask nature

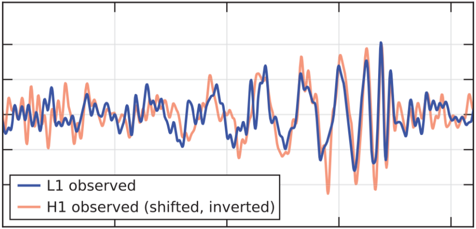
- So far, only weak-field precision tests
- Lots of theories $\approx$ GR
- Need to explore strong-field
  • Strong curvature • non-linear • dynamical

[Baker, Psaltis, Skordis (2015)]

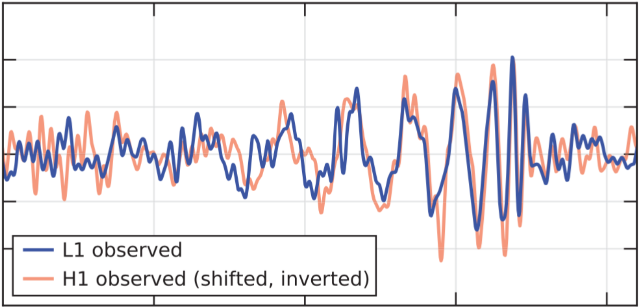


Vision

- Before aLIGO: precision tests of GR in weak field
- Now: first direct measurements of dynamical, strong field regime



LIGO's tests



Selected for a Viewpoint in *Physics*
PRL **116,** 221101 (2016) PHYSICAL REVIEW LETTERS week ending 3 JUNE 2016

Tests of General Relativity with GW150914

B. P. Abbott *et al.*[*]
(LIGO Scientific and Virgo Collaborations)

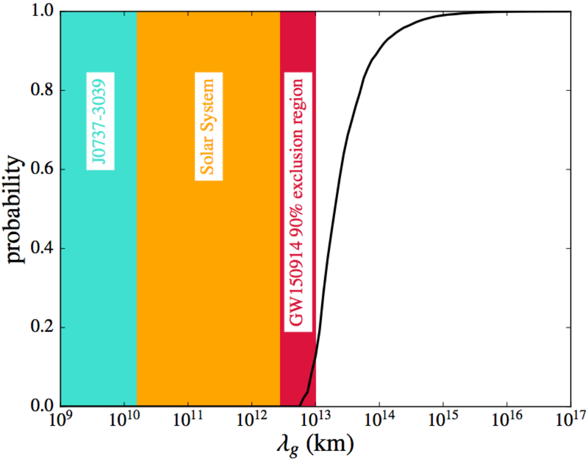
(Received 26 March 2016; revised manuscript received 9 May 2016; published 31 May 2016)

The LIGO detection of GW150914 provides an unprecedented opportunity to study the two-body

LIGO's tests

Two tests I like:

- Any deviation from GR must be below 4% of signal power
- Test of dispersion relation



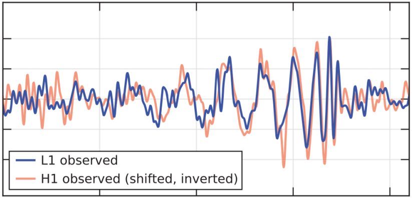
LIGO's tests

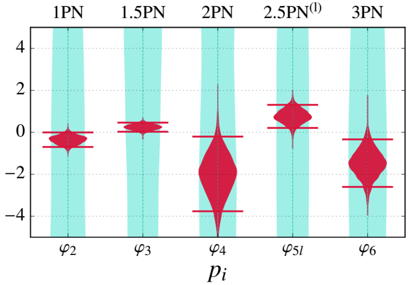
One test I do not particularly like:

- Insert power-law corrections to amplitude and phase $(u^3 \equiv \pi \mathcal{M} f)$

$$\tilde{h}(f) = \tilde{h}_{GR}(f) \times (1 + \alpha u^a) \times \exp[i\beta u^b]$$

- Parameters: (α, a, β, b)
- Inspired by post-Newtonian calculations in beyond-GR theories





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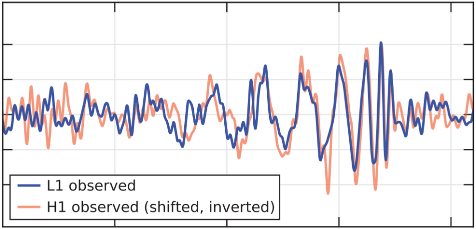

- Future: precision tests of GR in the strong field
  - Changing nuclear EOS is degenerate with changing gravity
  - Need black hole binary merger for precision

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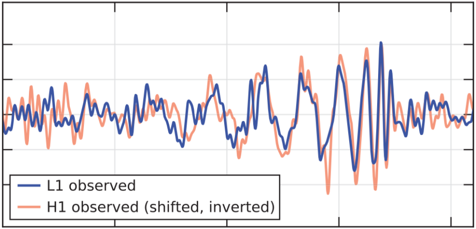

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Question: How to perform precision tests of GR in strong field?

How to perform precision tests

- Two approaches: theory-specific and theory-agnostic
- Agnostic: parameterize, e.g. PPN, PPE

Only 10 numbers in parametrized post-Newtonian [Slide from Wex]

PPN formalism for metric theories of gravity

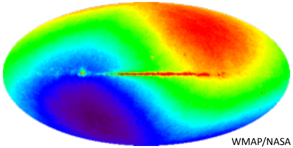
Metric:

$$g_{00} = -1 + 2U - 2\beta U^2 - 2\xi\Phi_W + (2\gamma + 2 + \alpha_3 + \zeta_1 - 2\xi)\Phi_1 + 2(3\gamma - 2\beta + 1 + \zeta_2 + \xi)\Phi_2 \\ +2(1+\zeta_3)\Phi_3 + 2(3\gamma + 3\zeta_4 - 2\xi)\Phi_4 - (\zeta_1 - 2\xi)\mathcal{A} - (\alpha_1 - \alpha_2 - \alpha_3)w^2 U - \alpha_2 w^i w^j U_{ij} \\ +(2\alpha_3 - \alpha_1)w^i V_i + \mathcal{O}(\epsilon^3),$$

$$g_{0i} = -\frac{1}{2}(4\gamma + 3 + \alpha_1 - \alpha_2 + \zeta_1 - 2\xi)V_i - \frac{1}{2}(1 + \alpha_2 - \zeta_1 + 2\xi)W_i - \frac{1}{2}(\alpha_1 - 2\alpha_2)w^i U \\ -\alpha_2 w^j U_{ij} + \mathcal{O}(\epsilon^{5/2}),$$

$$g_{ij} = (1 + 2\gamma U)\delta_{ij} + \mathcal{O}(\epsilon^2).$$

w: motion w.r.t. preferred reference frame



Metric potentials:

$$U = \int \frac{\rho'}{|\mathbf{x} - \mathbf{x}'|} d^3x',$$ (Newtonian potential)

$$U_{ij} = \int \frac{\rho'(x-x')_i(x-x')_j}{|\mathbf{x} - \mathbf{x}'|^3} d^3x',$$

$$\Phi_W = \int \frac{\rho'\rho''(\mathbf{x} - \mathbf{x}')}{|\mathbf{x} - \mathbf{x}'|^3} \cdot \left(\frac{\mathbf{x}' - \mathbf{x}''}{|\mathbf{x} - \mathbf{x}''|} - \frac{\mathbf{x} - \mathbf{x}''}{|\mathbf{x}' - \mathbf{x}''|}\right) d^3x'\, d^3x'',$$

$$\mathcal{A} = \int \frac{\rho'[\mathbf{v}' \cdot (\mathbf{x} - \mathbf{x}')]^2}{|\mathbf{x} - \mathbf{x}'|^3} d^3x',$$

$$\Phi_1 = \int \frac{\rho' v'^2}{|\mathbf{x} - \mathbf{x}'|} d^3x',$$

$$\Phi_2 = \int \frac{\rho' U'}{|\mathbf{x} - \mathbf{x}'|} d^3x',$$

$$\Phi_3 = \int \frac{\rho' \Pi'}{|\mathbf{x} - \mathbf{x}'|} d^3x',$$

$$\Phi_4 = \int \frac{p'}{|\mathbf{x} - \mathbf{x}'|} d^3x',$$

$$V_i = \int \frac{\rho' v'_i}{|\mathbf{x} - \mathbf{x}'|} d^3x',$$

$$W_i = \int \frac{\rho'[\mathbf{v}' \cdot (\mathbf{x} - \mathbf{x}')](x-x')_i}{|\mathbf{x} - \mathbf{x}'|^3} d^3x'.$$

[Will 1993, *Will 2014, Living Reviews in Relativity*]

Parameterized post-Einstein framework

- Insert power-law corrections to amplitude and phase $(u^3 \equiv \pi \mathcal{M} f)$

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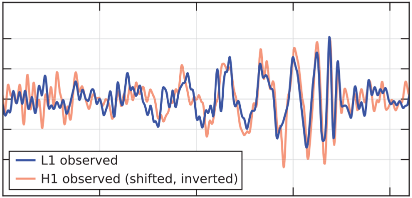

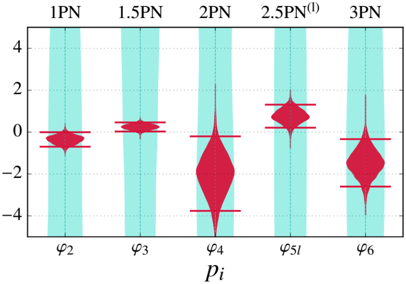

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Problem: Only simulated BBH mergers in GR!*

The problem

From Lehner+Pretorius 2014:

redshifts of $z \simeq 20$ with a SNR ≥ 10. For a recent review see Seoane et al. (2013).] Compounding the problem, despite the large number of proposed alternatives or modifications to general relativity (see, for example, Will 1993, 2006), almost none have yet been presented that (*a*) are consistent with general relativity in the regimes where it is well tested, (*b*) predict observable deviations in the dynamical strong field relevant to vacuum mergers, and (*c*) possess a classically well-posed initial value problem to be amenable to numerical solution in the strong field. The notable exceptions are a subset of scalar tensor theories, though these require a time-varying cosmological scalar field for binary black hole systems (Horbatsch & Burgess 2012) or one or more neutron stars in the merger (see Section 5). Thus there is little guidance on what reasonable strong-field deviations one might expect. Proposed solutions to (at least partially) circumvent these problems include the parameterized post-Einsteinian and related frameworks (Yunes & Pretorius

Don't know if other theories have good initial value problem

Numerical relativity

Numerical relativity

- Nonlinear, quasilinear, 2nd order hyperbolic PDE,
  10 functions, 3+1 coordinates
- Attempts from '60s until 2005.
  Merging BHs for 12 years
- Want to evolve. How do you know if good IBVP?
- Both under- and over-constrained.
  - gauge
  - constraints (not all data free; need constraint damping)

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Every other gravity theory will have at least these difficulties

Some other theories

"Scalar-tensor":

$$G^{\star}_{\mu\nu} = 2\left(\partial_\mu\varphi\partial_\nu\varphi - \frac{1}{2}g^{\star}_{\mu\nu}\partial_\sigma\varphi\partial^\sigma\varphi\right) - \frac{1}{2}g^{\star}_{\mu\nu}V(\varphi) + 8\pi T^{\star}_{\mu\nu}$$

$$\Box_{g^\star}\varphi = -4\pi\alpha(\varphi)T^\star + \frac{1}{4}\frac{dV}{d\varphi}$$

BBH in S-T:

- Massless scalar $\implies \varphi \to 0$, agrees with GR
- Only differ if funny boundary or initial conditions

Helvi+ working on Einstein-dilaton-Gauss-Bonnet

Jai-akson+ paper on Einstein-Maxwell-dilaton on arXiv today

Some other theories

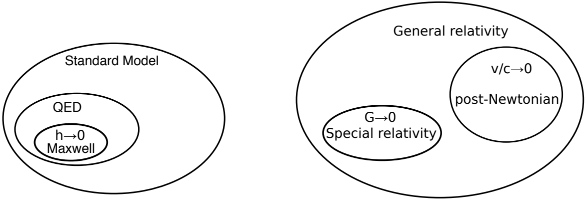
- Higher derivative EOMs
- Ostrogradski instability. H unbounded below
- Some theories try to avoid, e.g. Horndeski, dRGT
- Massive gravity theories. B-D ghost, cured by dRGT.
- Problems even with second-derivative EOMs
- If not quasi-linear, may have $(\partial_t \phi)^2 \simeq$ Source, but ...
- Papallo and Reall paper on Lovelock, Horndeski

Scribble on board

A solution

A solution

- Treat every theory as an effective field theory (EFT)
- Already do this for GR. Valid below some scale
- Theory only needs to be approximate, approximately well-posed



- Example: weak force below EWSB scale (lose unitarity above)

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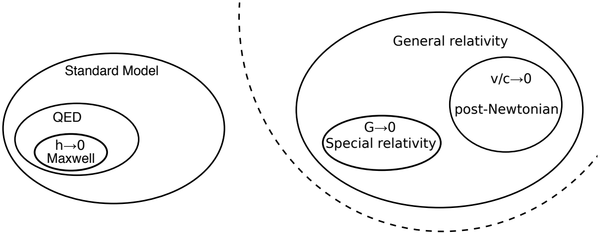

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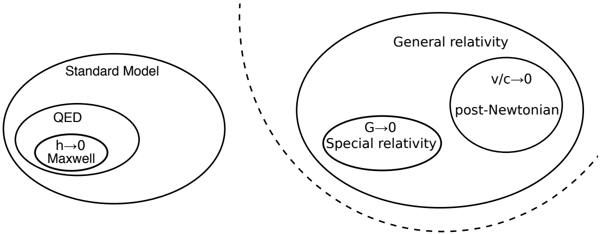

- Same should happen in gravity EFT:
  lose predictivity (bad initial value problem) above some scale
- Theory valid below cutoff $\Lambda \gg E$. Must recover GR for $\Lambda \to \infty$.
- Assume weak coupling, use perturbation theory

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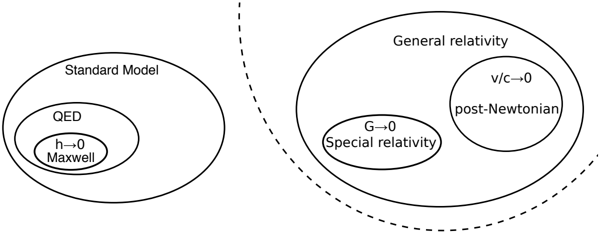

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Example: Dynamical Chern-Simons gravity

What is dynamical Chern-Simons gravity?

- Chern-Simons = GR + pseudo-scalar + interaction

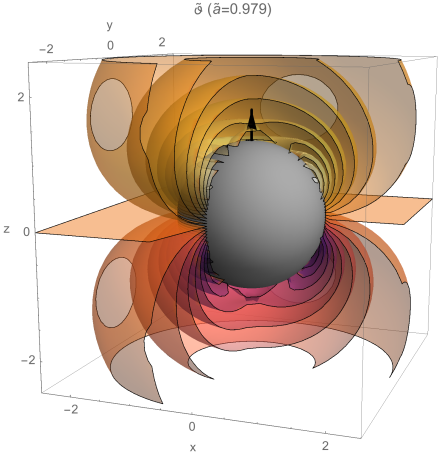
$$S = \int d^4x \sqrt{-g} \left[R - \frac{1}{2}(\partial\vartheta)^2 + \varepsilon\, \vartheta\ {}^*RR \right]$$

$$\Box\vartheta = \varepsilon\, {}^*RR\,, \qquad\qquad G_{ab} + \varepsilon\, C_{ab}[\partial\vartheta\partial^3 g] = T_{ab}$$

- Anomaly cancellation, low-E string theory, LQG. . .
  (see Nico's review Phys. Rept. **480** (2009) 1-55)
- Lowest-order EFT with parity-odd ϑ, shift symmetry (long range)
- Phenomenology unique from other R^2
  (e.g. Einstein-dilaton-Gauss-Bonnet)

Black holes in dCS

- $a = 0$ (Schwarzschild) is exact solution with $\vartheta = 0$
- Rotating BHs have dipole+ scalar hair



LCS, PRD **90** 044061 (2014) [arXiv:1407.2350]

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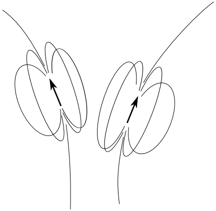
  LCS, PRD **90** 044061 (2014) [arXiv:1407.2350]
- Post-Newtonian of BBH inspiral in
  PRD **85** 064022 (2012) [arXiv:1110.5950]



- More updated phenomenology in
  CQG **32** 243001 (2015) [arXiv:1501.07274]

Back to problem and solution

- DCS had principal part $\partial^3 g$ coming from C_{ab} tensor. *Probably* not well-posed, Delsate+ PRD **91**, 024027.
- Theory is GR + ε $\times$ deformation. Expand everything in ε
- Derivation (on board)
- At every order in ε, principal part is Princ$[G_{ab}]$

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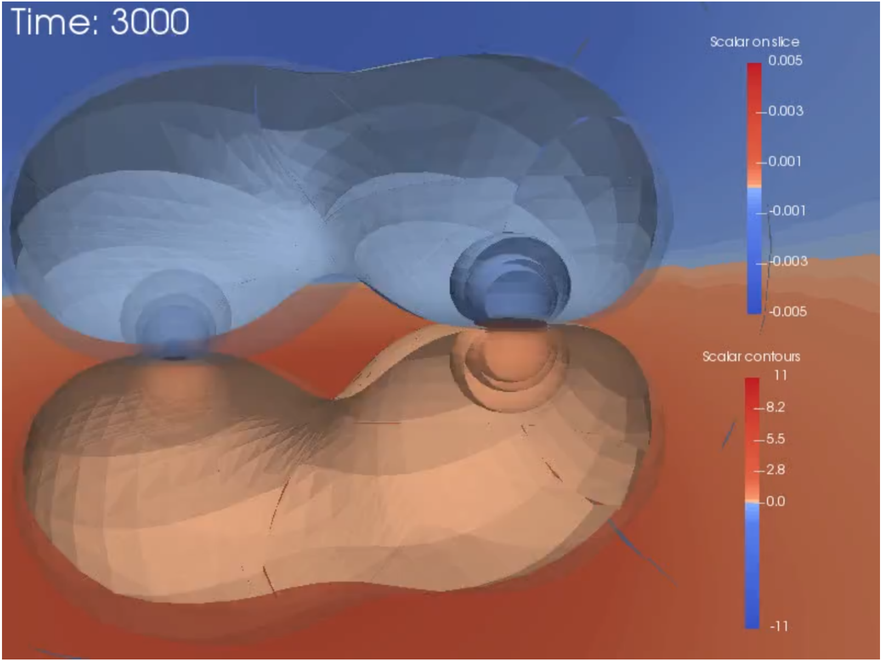
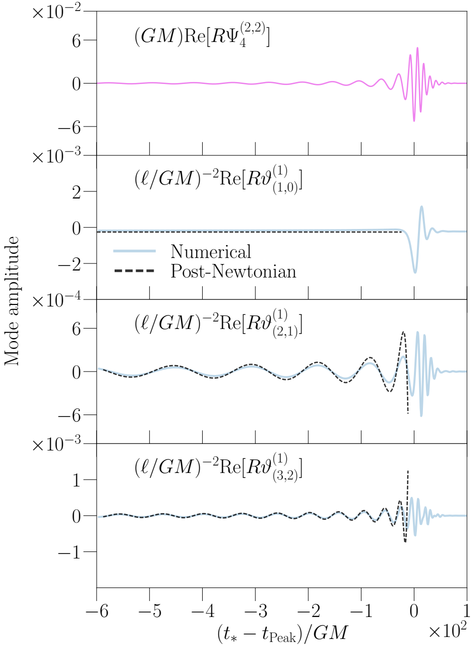
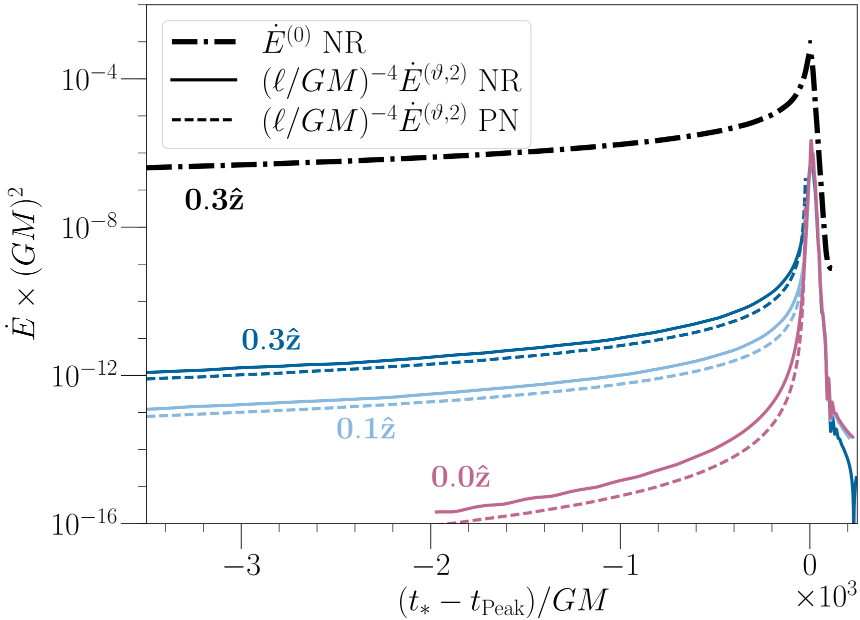
Background dynamics are well-posed $\implies$ perturbations well-posed


Time: 3000
Scalar on slice
0.005
0.003
0.001
-0.001
-0.003
-0.005
Scalar contours
11
8.2
5.5
2.8
0.0
-11


$(GM)\mathrm{Re}[R\Psi_4^{(2,2)}]$
$(\ell/GM)^{-2}\mathrm{Re}[R\vartheta^{(1)}_{(1,0)}]$
Numerical
Post-Newtonian
$(\ell/GM)^{-2}\mathrm{Re}[R\vartheta^{(1)}_{(2,1)}]$
$(\ell/GM)^{-2}\mathrm{Re}[R\vartheta^{(1)}_{(3,2)}]$
Mode amplitude
$(t_* - t_{\mathrm{Peak}})/GM$
$\times 10^2$


$\dot{E}^{(0)}$ NR
$(\ell/GM)^{-4}\dot{E}^{(\vartheta,2)}$ NR
$(\ell/GM)^{-4}\dot{E}^{(\vartheta,2)}$ PN
$\mathbf{0.3\hat{z}}$
$\mathbf{0.3\hat{z}}$
$\mathbf{0.1\hat{z}}$
$\mathbf{0.0\hat{z}}$
10^{-4}
10^{-8}
10^{-12}
10^{-16}
$\dot{E} \times (GM)^2$
-3
-2
-1
0
$\times 10^3$
$(t_* - t_{\mathrm{Peak}})/GM$

Next issues

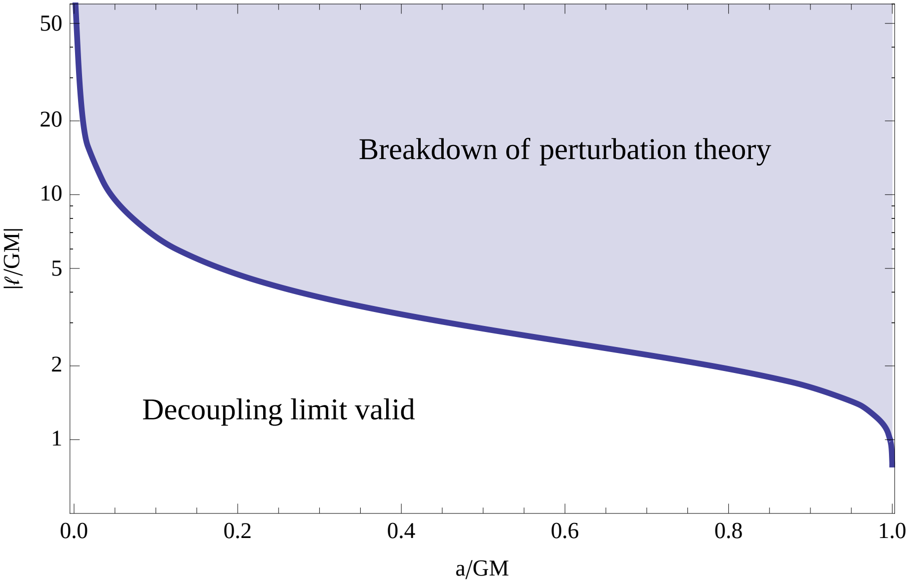
At $\mathcal{O}(\varepsilon^2)$:

- *Two* sets of gauges, constraints
- Find stable gauge
  - Worry from Kerr PT
  - Linearization of damped harmonic
- Regime of validity of perturbation scheme

$$\left\| \varepsilon^2 h^{(2)} \right\| \ll \left\| g^{(0)} \right\|$$

- Renormalization? See e.g. Galley and Rothstein [1609.08268]
- Ways around perturbation theory?

Instantaneous regime of validity



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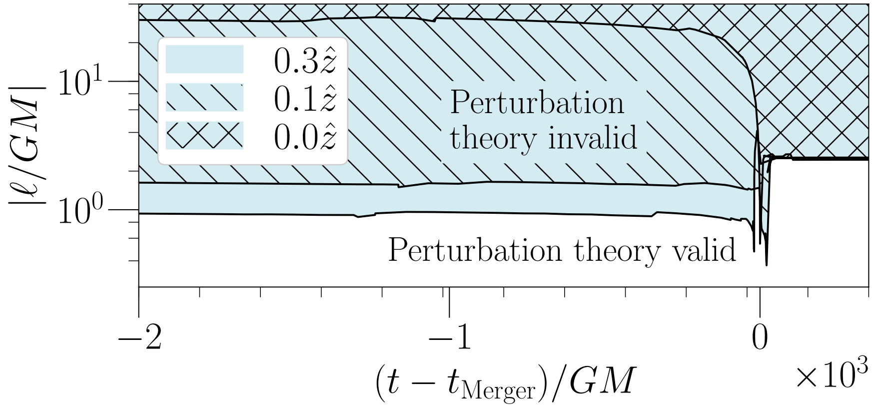


Secular regime of validity — dephasing

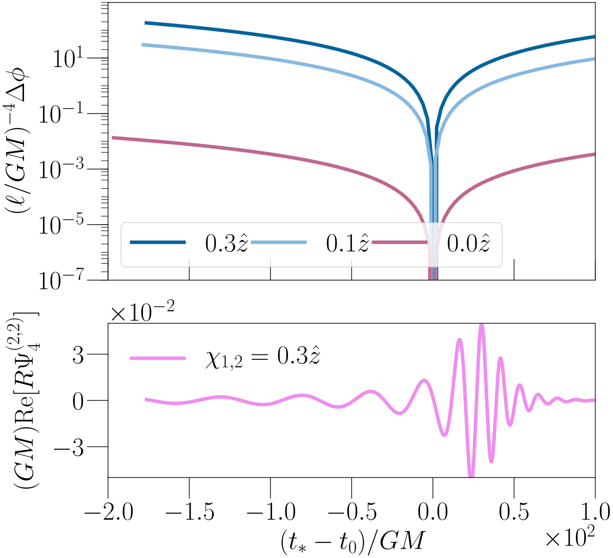
- Expand phase in ε around time t_0

$$\phi = \phi^{(0)} + \varepsilon^2 \Delta\phi + \mathcal{O}(\varepsilon^3)\,,$$

$$\Delta\phi(t) = \Delta\phi(t_0) + (t - t_0)\frac{d\Delta\phi}{dt}\Big|_{t=t_0} + \frac{1}{2}(t-t_0)^2 \frac{d^2\Delta\phi}{dt^2}\Big|_{t=t_0} + \mathcal{O}(t-t_0)^3$$

- Pretend orbits quasicircular, adiabatic $\implies E = E(\omega(t))$
- Use chain rule, relate $d\Delta\omega/dt$ to energy, flux

Secular regime of validity — dephasing



Bounds

$$\Delta\phi_{\mathrm{gw}} = 2\Delta\phi \lesssim \sigma_\phi$$

Spin	M bound	$M \approx 60 M_\odot$
0.3	$\left(\frac{\ell}{GM}\right) \lesssim 0.13 \left(\frac{\sigma_\phi}{0.1}\right)^{1/4}$	$\ell \lesssim 11\ \mathrm{km} \left(\frac{\sigma_\phi}{0.1}\right)^{1/4}$
0.1	$\left(\frac{\ell}{GM}\right) \lesssim 0.2 \left(\frac{\sigma_\phi}{0.1}\right)^{1/4}$	$\ell \lesssim 18\ \mathrm{km} \left(\frac{\sigma_\phi}{0.1}\right)^{1/4}$
0.0	$\left(\frac{\ell}{GM}\right) \lesssim 1.4 \left(\frac{\sigma_\phi}{0.1}\right)^{1/4}$ ✗	—

The TAKEAWAY™

- First binary black hole mergers in dCS
- Inspiral: qualitative agreement with analytics
- Merger: discovered new phenomenology, dipole burst
- Estimated $\Delta\phi$, bound on $\ell \lesssim \mathcal{O}(10)\ \mathrm{km}$
- For better bounds:
  - Higher SNR
  - Longer waveform/lower mass
  - Higher BH spins
- Working on $\mathcal{O}(\varepsilon^2)$

For details, see [arXiv:1705.07924]

The TAKEAWAY™

- General relativity must be incomplete
- New opportunity to test GR in strong-field
- Present tests' shortcomings
    - Almost no theory-specific tests
    - Theory-independent tests need more guidance
- Challenge: Find spacetime solutions in theories beyond GR
    - Our contribution: First binary black hole mergers in dynamical Chern-Simons gravity
    - General method appropriate for many deformations of GR

For details, see [arXiv:1705.07924]