On the spontaneous emission in quantum free-electron lasers (WG 4,5,6&7 - ID. 9)

Hesham Fares[1], Nicola Piovella[2], Gordon R. M. Robb[3]
[1]INFN-LNF, Via Enrico Fermi, 40 00044, Frascati (Roma), Italy.
[2]Dipartimento di Fisica, Universita degli Studi di Milano, via Celoria 16, I-20133 Milano, Italy.
[3]Department of Physics & SUPA, University of Strathclyde, Glasgow, G4 0NG Scotland, United Kingdom.

Abstract

In this work, we investigate the spontaneous emission process and its detrimental effects on coherent free-electron laser (FEL) emission. In our model, the electron dynamics are described by a discrete Wigner distribution coupled to Maxwell equations. For an FEL operating in the quantum regime of single photon recoil, insights on the variation of momentum distribution, bunching factor, and radiation power are presented. We also show a simple differential equation that describes the evolution of the radiated power in the linear regime. It is shown that the essential results of this work agree with those predicted by a density matrix approach.

1. The classical "conventional" FEL

- **High-gain FEL mechanism involves two processes:**

Radiation fields $(\vec{E},\vec{B})$ bunch electrons by Lorentz Force ⟺ Bunched electrons drive radiation (Maxwell's wave Eq.)

$$\vec{F} = -|e|\left[\vec{E} + \vec{v}\times\vec{B}\right] \quad \Longleftrightarrow \quad \nabla^2 E - \frac{1}{c^2}\frac{\partial E^2}{\partial t^2} = \mu_0 \frac{\partial J}{\partial t}$$

- **High-gain FEL is described by [1]:**

$$\frac{d\theta_j}{d\bar{z}} = \bar{p}_j$$

$$\frac{d\bar{p}_j}{d\bar{z}} = -\left(Ae^{i\theta_j} + c.c.\right)$$

$$\frac{dA}{d\bar{z}} = \frac{1}{N}\sum_{j=1}^{N} e^{-i\theta_j} \equiv \langle e^{-i\theta_j}\rangle \quad \text{Bunching factor}$$

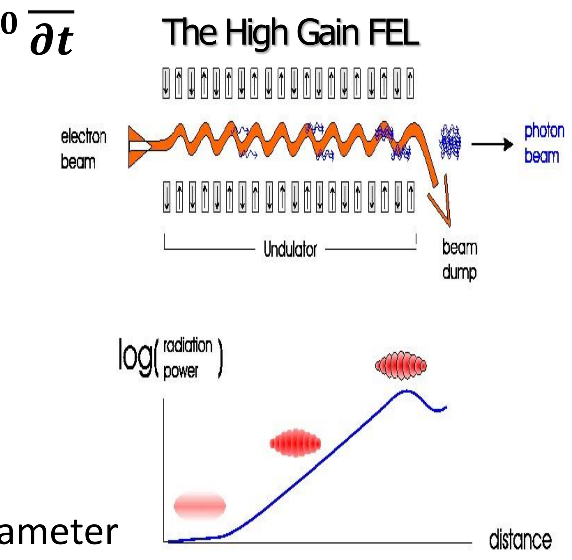
$\theta_j = (k_w + k)z - \omega t_j$ Ponderomotive phase

$\bar{p}_j = (\gamma_j - \gamma_r)/\rho\gamma_r$ Scaled energy change

$|A|^2 = \varepsilon_0 |E|^2/\rho n_e \gamma m c^2$ Scaled EM field intensity

$\bar{z} = z/L_g = 4\pi\rho z/\lambda_w$ Scaled position in wiggler

$\rho = \frac{1}{2\gamma_r}(I/17\,kA)^{1/3}(\lambda_w a_w/2\pi\sigma_b)^{2/3}$ Classical FEL parameter



2. The quantum regime of FEL: What and how to reach?

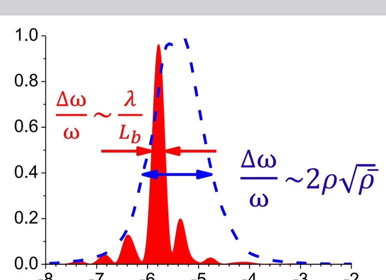
- **Condition of the quantum regime**
  - **Quantum FEL parameter [2]:** $\bar{\rho} = \rho\left(\frac{\gamma mc}{\hbar k}\right)$, $\frac{\Delta\gamma}{\gamma} \sim \rho$

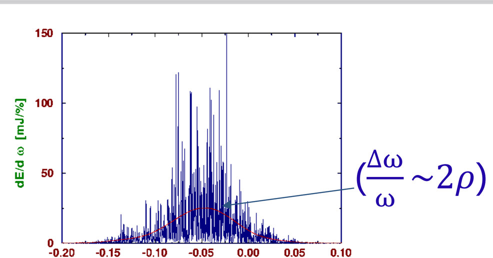
$$\bar{\rho} = \frac{mc\Delta\gamma}{\hbar k} = \frac{\Delta p}{\hbar k}$$

Δp: Electron recoil (momentum Spread); $\hbar k$: Photon recoil

Q-regime $\Delta p < \hbar k$ ($\bar{\rho} < 1$)
- Classical description will break down
- Momentum exchange is discrete, single photon emission is allowed.
- A single line spectrum.

Cl- regime $\Delta p > \hbar k$ ($\bar{\rho} >> 1$)
- Classical description holds.
- Momentum exchange is continues, multi photon emission is allowed.
- A broad-spiky spectrum.





- **Physical interpretation**

E-beam (longitudinal) coherence length is defined as : $L_c = \frac{\lambda_e^2}{\Delta\lambda_e}$ where $\lambda_e = \frac{h}{p}$ is the de Broglie wavelength of the electrons.

In terms of FEL, wave-like nature of electrons should be significant if

$$L_c > \lambda$$

Rewriting L_c in terms of electron momentum, p : $L_c = \frac{h^2}{p^2}\frac{p^2}{h\Delta p} = \frac{h}{\Delta p}$

so $L_c > \lambda$ implies $\frac{h}{\Delta p} > \lambda$ i.e. $\hbar k > \Delta p$

This is the same condition as derived previously for observation of quantum effects.

This suggests that, in this regime, a wavefunction description (or equivalent) of the FEL interaction is required.

- **Possible scheme**
- **The laser undulator FEL can be quantum FEL [3]:**
  (Collective Compton backscattering)

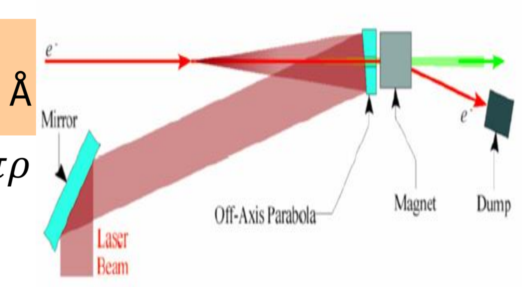
$$\lambda_r = (\lambda_L/4\gamma^2)(1+a_L^2),\quad a_L = \frac{2.4\lambda_L}{r_0}\sqrt{P(TW)}$$

Laser power; Laser radius

λ_L is the laser wavelength.

Intuitive example:
To generate sub-Angstrom QFEL, λ_r=0.1 Å

- The gain length $L_g = \lambda_w(\text{or } \lambda_L/2)/4\pi\rho$
- The QFEL is operated if $\rho \leq \left(\frac{\hbar k}{mc\gamma}\right)$



Undulator FEL:	**Laser undulator FEL**
$\lambda_w = 1$ cm @ $E = 10$ GeV	$\lambda_L = 1\,\mu$m @ $E = 100$ MeV
$\rho \sim 10^{-6}$ & $L_{int} > L_g \sim 1$ km !!	$\rho \sim 10^{-4}$ & $L_{int} > L_g \sim 1$ mm

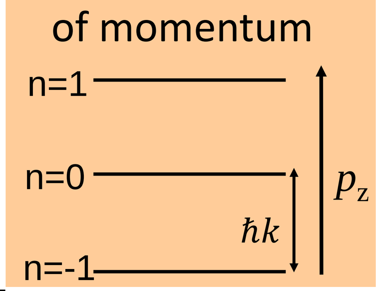
3. Discrete Wigner model for quantum FEL

θ is assumed to be a periodic variable in $(0,2\pi]$. This hypothesis assures that the conjugate momentum variable p is discrete [4].

- **The electron wavefunction :** $\Psi(\bar{z},\theta) = \frac{1}{\sqrt{2\pi}}\sum_{n=-\infty} c_n(\bar{z})\,|n\rangle,$

The momentum state: $|n\rangle = \exp(in\theta)$, Momentum operator: $p = -i\frac{\partial}{\partial\theta}, [\theta,p] = i$ Eigen-value equation: $p|n\rangle = n|n\rangle,$

Discrete changes of momentum



- **The electron dynamic is described by a Schrodinger-like equation:**

$$i\frac{\partial|\Psi(\bar{z},\theta)\rangle}{\partial\bar{z}} = H|\Psi(\bar{z},\theta)\rangle \qquad H = \frac{p^2}{2\bar{\rho}} - i\bar{\rho}\left(Ae^{i\theta} + c.c.\right)$$

- **The coupled equations that describes the FEL including the spontaneous emission:**

$$\frac{\partial w_s(\bar{z},\theta)}{\partial\bar{z}} + \frac{s}{\bar{\rho}}\frac{\partial w_s(\bar{z},\theta)}{\partial\theta} = \bar{\rho}\left(Ae^{i\theta} + c.c.\right)\{w_{s+1/2}(\bar{z},\theta) - w_{s-1/2}(\bar{z},\theta)\} + \sqrt{\bar{\rho}}D\{w_{s+1}(\bar{z},\theta) - w_s(\bar{z},\theta)\}$$

$$\frac{dA}{d\bar{z}} = \sum_{m=-\infty}^{\infty}\int_{-\pi}^{\pi} w_{m+1/2}(\bar{z},\theta)e^{-i\theta}d\theta + i\delta A$$

Spontaneous emission rate $D = \left[\frac{(\alpha a_w^2 mc\gamma_r/6\hbar k)}{\bar{\rho}^{3/2}}\right]$

Detuning $\delta = (\gamma_r - \gamma_0)/\rho\gamma_0$

- **Wigner function is periodic in θ and is expressed by:**

$$w_s(\bar{z},\theta) = \frac{1}{2\pi}\sum_{k=-\infty}^{\infty} w_s^k(\bar{z})e^{ik\theta},\quad w_m^{2k} = c_{m+k}^* c_{m-k},\quad w_{m+1/2}^{2k+1} = c_{m+k+1}^* c_{m-k}.$$

- **From the above equations, we get the coupled equations in the form of:**

Initial population $P_0 = w_0^0$,

$$\frac{dP_0}{d\acute{z}} = -(\acute{A}\acute{B}^* + c.c.) - DP_0,$$

Final population $P_{-1} = w_{-1}^0$,

$$\frac{dP_{-1}}{d\acute{z}} = (\acute{A}\acute{B}^* + c.c.) + D(P_0 - P_{-1})$$

Bunching $\acute{B} = w_{-1/2}^1$,

$$\frac{d\acute{B}}{d\acute{z}} = -(i\acute{\delta} + D)\acute{B} + \acute{A}(P_0 - P_{-1}),$$

$|\acute{A}|^2$=no. of photons per electron

$$\frac{d\acute{A}}{d\acute{z}} = \acute{B}.$$

4.1. Results 1 (Field dynamic)

- **Detrimental effect of spontaneous emission [5]**

- **At resonance** $\delta = 1/2\bar{\rho}$,
  $\acute{B}(0) = 0.01,\quad \acute{A}(0) = 0,$
  $P_0(0) = 1,\quad P_{-1}(0) = 0.$

(i) For D = 0, the system shows a periodic behavior, with maximum of $|\acute{A}|^2$=1.

(ii) The detrimental effect of spontaneous emission is neglected when D <0.05.

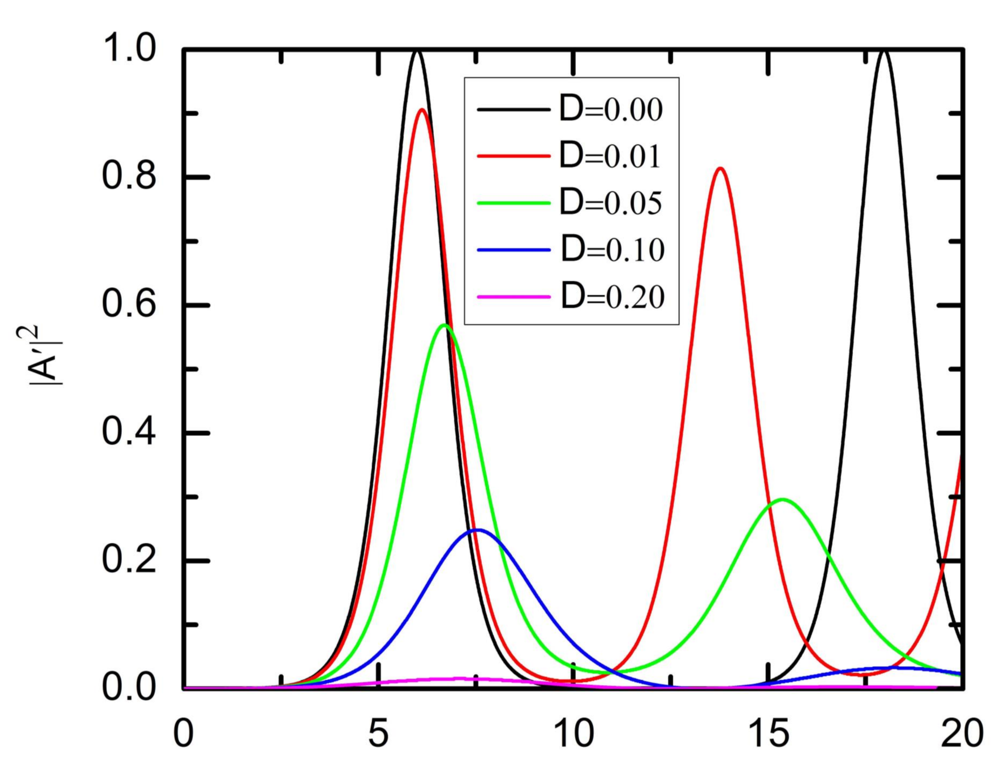


FIG. 1. $|\acute{A}|^2$ **vs.** $\acute{z}$

- **Linear and non-linear regimes**

- **Solving the coupled equations in the linear limit:**
  $\acute{B}(0) = 0.0,\quad \acute{A}(0) = 0,$
  $P_0(0) = 1,\quad P_{-1}(0) = 0,$
  we get,

$$\frac{d^2\acute{A}(\acute{z})}{d\acute{z}^2} + (i\acute{\delta} + D)\frac{d\acute{A}(\acute{z})}{d\acute{z}} - [1 - D\acute{z}]e^{-D\acute{z}}\acute{A} = 0$$

$$\acute{z} = \sqrt{\bar{\rho}}\bar{z},\quad \acute{\delta} = [\delta - (1/2\bar{\rho})]/\sqrt{\bar{\rho}}.$$

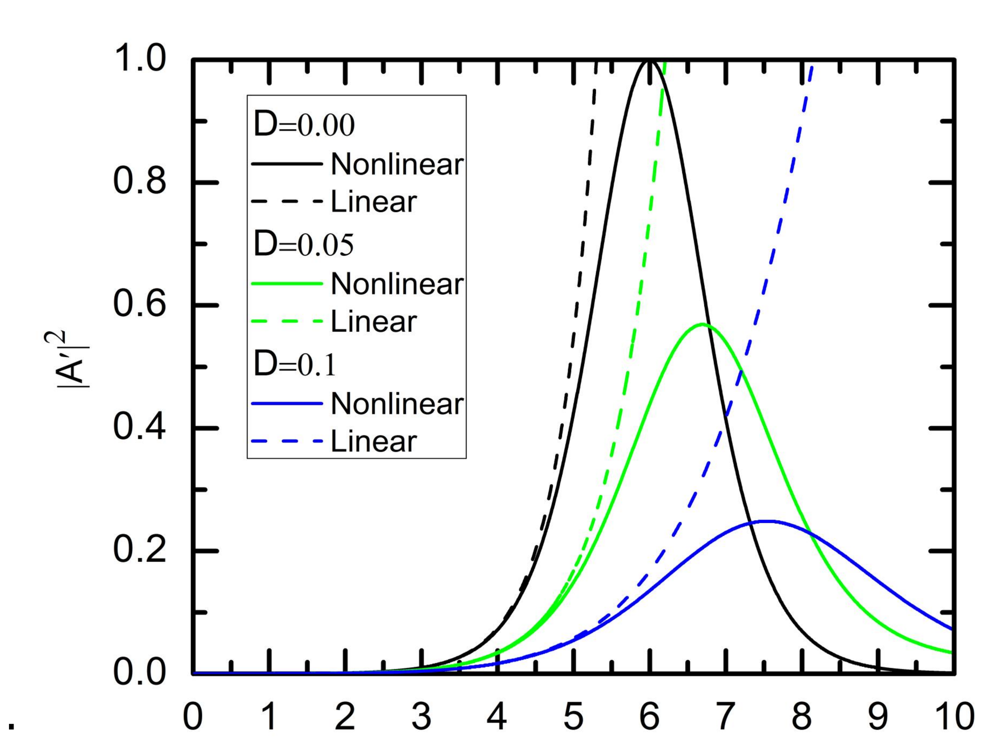
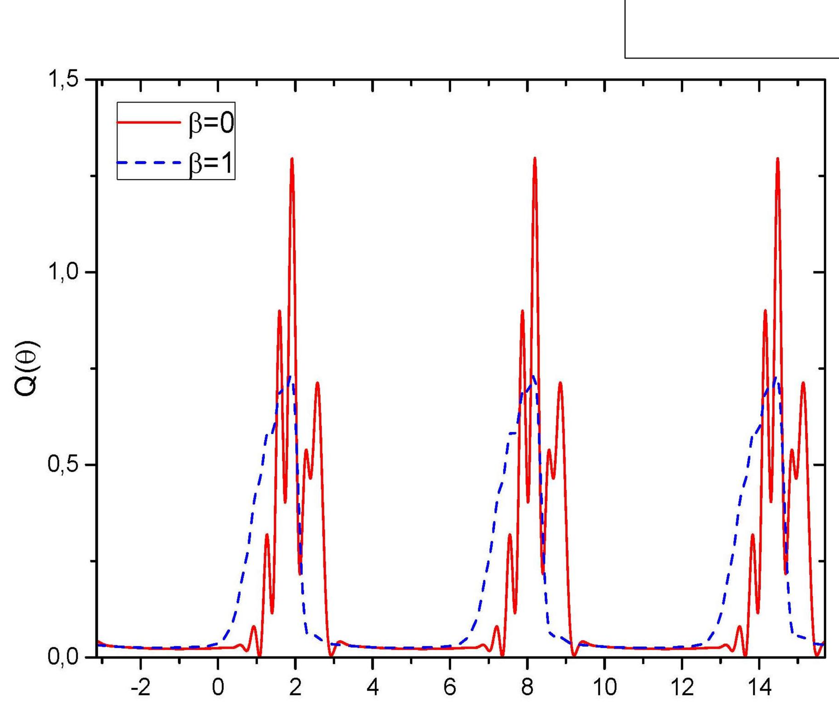


FIG. 2. $|\acute{A}|^2$ **vs.** $\acute{z}$

4.2. Results 2 (Electrons dynamic)

- **A distribution function Q is defined as [5]:**

$$Q(\bar{z},\theta) = \sum_{k=-\infty}^{\infty}\left[w_m(\bar{z},\theta) + w_{m+\frac{1}{2}}(\bar{z},\theta)\right]$$



In FIG. 3, the energy spread induced by spontaneous emission smears the electron spatial distribution.

FIG. 3. The electron distribution function Q vs. θ.

5. Conclusion

We have presented a discrete Wigner model for the quantum FEL, including spontaneous emission. This model describes the momentum as a discrete variable, as it should be assuming spatial periodic boundary conditions. We have shown that, in the quantum regime, the equations reduce to these for two-momentum states coupled to the coherent radiation field. Spontaneous emission is there interpreted as responsible for the loss of coherence (i.e. bunching) and the transfer of electrons into and out of the two momentum states via rate equation terms.

6. References

[1] R. Bonifacio et al, "Physics of the High-Gain Free Electron Laser & Superradiance", Rivista del Nuovo Cimento, Vol. **13**, no. 9, p. 1-69 (1990).
[2] R. Bonifacio, N. Piovella, G.R.M. Robb, A. Sciavi, PRST-AB **9**, 090701 (2006).
[3] R. Bonifacio, H. Fares, M. Ferrario, B. W. J. McNeil, G. R. M. Robb, Optics Communications, **382**, 58 (2017).
[4] N. Piovella, M. M Col, L. Volpe, R. Gaiba, A. Schiavi, R. Bonifacio, Opt. Comm. **274**, 347 (2007).
[5] H. Fares, N. Piovella, G. R. M. Robb, to be published.