

SUMMARY THEORY AND SIMULATIONS

topics - 4 sessions

- 1. Analytical models
- 2. Numerical Results
- 3. Numerical Methods
- 4. Scientific Computing

topics - 4 sessions

- 1. Analytical models
 - #
 transverse bunch matching
- 2. Numerical Results
 - Code development
 - Code update
- 3. Numerical Methods

 - Spectral methods PSATD
 - Adaptive Mesh Refinement (AMR)
 - #Envelope

4. Scientific Computing
Dynamic load balancing
Data Reduction
In situ visalization
use of new libraries

150 years of Maxwell's (other) equations and application to plasma acceleration

Robert Robson¹, Timon Mehrling² and Jens Osterhoff²

¹James Cook University and Griffith University, Australia ²Deutsches Elektronen-Synchrotron DESY, Hamburg, Germany

An approach invented by Maxwell exactly 150 years ago is until today important in science

• Maxwell's original "equations of matter" (velocity moment equation)



• This moment approach can be used to model the channel centroid in hosing in PWFAs

Moment equation for sheath electrons in the quasi-static approximation:

$$\partial_{\xi} \left\langle \Phi \right\rangle = \left\langle \frac{p_r}{1+\psi} \partial_r \Phi \right\rangle + \left\langle \frac{\gamma \dot{\theta}}{1+\psi} \partial_{\theta} \Phi \right\rangle + \left\langle \frac{\gamma F_r}{1+\psi} \partial_{p_r} \Phi \right\rangle + \left\langle \frac{\gamma F_{\psi}}{1+\psi} \partial_{\psi} \Phi \right\rangle \overset{15}{\overset{\text{New}}{\longrightarrow}} \overset{15}{\overset{15}{\longrightarrow}} \overset{15}{\overset{15}{\overset{15}{\longrightarrow}} \overset{15}{\overset{15}{\longrightarrow}} \overset{15}{\overset{15}{\overset{15}{\longrightarrow}} \overset{15}{\overset{15}{\overset{15}{\longrightarrow}} \overset{15}{\overset{15}{\overset{15}{\longrightarrow}} \overset{15}{\overset{15}$$

Setting. $\Phi = x$, etc. \Rightarrow new channel centroid equation:

$$\frac{\partial^2 X_c}{\partial \xi^2} + \frac{k_p^2}{2} \left[c_c(\xi) X_c - c_b(\xi) X_b \right] = 0$$



Saturation of the beam-hosing instability with a betatron chirp in the quasi-linear regime

A varying betatron frequency across the bunch ("betatron chirp") can lead to a saturation of the beam-hosing instability.



Focusing strength is constant across the bunch. Betatron chirp requires an energy spread.

Quasi-linear regime



Focusing strength naturally varies across bunch. No energy spread required.

Confirmation in PIC simulations: Monoenergetic beam in quasi-linear regime

z (mm)

-12 -10

Betatron chirp is observed in the simulation:

Standard hosing scalings

(which neglect betatron

chirp) predict high instability:

_β(ξ) (mm⁻¹) 1.0 0.5 -10Head-to-tail distance ξ (μm) 30 20 10

1.5

(Colormap=instability level)

But actual hosing level saturates at a much lower level:



-8

PIC results

-4

-6

Head-to-tail distance ξ (μm)

Transverse bunch evolution





Development of an analytical model for emittance calculation in external injection scenarios

• Using a single particle DGL and applying a momentum approach, e.g.

 $\langle x^2 \rangle(t) = \int_{-\infty}^{\infty} (x^2(t)) f_0 \mathrm{d}x_0 \mathrm{d}p_{x,0} \mathrm{d}\delta\gamma, f_0 = f_{\perp}(x_0, p_{x,0}) f_{\gamma}(\gamma_0), \int f_0 \mathrm{d}x_0 \mathrm{d}p_{x,0} \mathrm{d}\gamma_0 = 1$

- Two scenarios were analyzed
 - Single witness beam slice without energy gain (E_z zero crossing)
 - Single witness beam slice with energy gain (positioned at defined E_z)
- Formulas for emittance evolution of a single beam slice were derived, e.g. for a scenario without energy gain:

$$\frac{\epsilon_{n,rms}^{2}(\tilde{t})}{\epsilon_{0}^{2}} = \frac{1}{4} \left(\left(\frac{\left\langle x_{0}^{2} \right\rangle}{\left\langle x^{2} \right\rangle_{m}} \right)^{2} + \left(\frac{\left\langle u_{x,0}^{2} \right\rangle}{\left\langle u_{x}^{2} \right\rangle_{m}} \right)^{2} \right) \left(1 - e^{-b\tilde{t}^{2}} \right) + \frac{1}{2} \frac{\left\langle x_{0}^{2} \right\rangle}{\left\langle x^{2} \right\rangle_{m}} \frac{\left\langle u_{x,0}^{2} \right\rangle}{\left\langle u_{x}^{2} \right\rangle_{m}} \left(1 + e^{-b\tilde{t}^{2}} \right) - \frac{\left\langle xu_{x,0} \right\rangle^{2}}{\epsilon_{0}^{2}} e^{-b\tilde{t}^{2}} - \frac{\left\langle xu_{x,0} \right\rangle^{2}}{\epsilon_{0}^{2}} e^{-b\tilde{t}^{2}} = \frac{1}{2} \left(\frac{\left\langle x_{0}^{2} \right\rangle}{\left\langle x^{2} \right\rangle_{m}} \left(1 + e^{-b\tilde{t}^{2}} \right) - \frac{\left\langle xu_{x,0} \right\rangle^{2}}{\epsilon_{0}^{2}} e^{-b\tilde{t}^{2}} - \frac{\left\langle xu_{x,0} \right\rangle^{2}}{\epsilon_{0}^{2}} e^{-b\tilde{t}^{2}} = \frac{1}{2} \left(\frac{\left\langle x_{0}^{2} \right\rangle}{\left\langle x^{2} \right\rangle_{m}} \left(\frac{\left\langle x_{0}^{2} \right\rangle}{\left\langle x^{2} \right\rangle_{m}} \right)^{2} + \frac{1}{2} \left(\frac{\left\langle x_{0}^{2} \right\rangle}{\left\langle x^{2} \right\rangle_{m}} \left(\frac{\left\langle x_{0}^{2} \right\rangle}{\left\langle x^{2} \right\rangle_{m}} \right)^{2} + \frac{1}{2} \left(\frac{\left\langle x_{0}^{2} \right\rangle}{\left\langle x^{2} \right\rangle_{m}} \left(\frac{\left\langle x_{0}^{2} \right\rangle}{\left\langle x^{2} \right\rangle_{m}} \right)^{2} + \frac{1}{2} \left(\frac{\left\langle x_{0}^{2} \right\rangle}{\left\langle x^{2} \right\rangle_{m}} \left(\frac{\left\langle x_{0}^{2} \right\rangle}{\left\langle x^{2} \right\rangle_{m}} \right)^{2} + \frac{1}{2} \left(\frac{\left\langle x_{0}^{2} \right\rangle}{\left\langle x^{2} \right\rangle_{m}} \left(\frac{\left\langle x_{0}^{2} \right\rangle}{\left\langle x^{2} \right\rangle_{m}} \right)^{2} + \frac{1}{2} \left(\frac{\left\langle x_{0}^{2} \right\rangle}{\left\langle x^{2} \right\rangle_{m}} \left(\frac{\left\langle x_{0}^{2} \right\rangle}{\left\langle x^{2} \right\rangle_{m}} \right)^{2} + \frac{1}{2} \left(\frac{\left\langle x_{0}^{2} \right\rangle}{\left\langle x^{2} \right\rangle_{m}} \left(\frac{\left\langle x_{0}^{2} \right\rangle}{\left\langle x^{2} \right\rangle_{m}} \right)^{2} + \frac{1}{2} \left(\frac{\left\langle x_{0}^{2} \right\rangle}{\left\langle x^{2} \right\rangle_{m}} \left(\frac{\left\langle x_{0}^{2} \right\rangle}{\left\langle x^{2} \right\rangle_{m}} \right)^{2} + \frac{1}{2} \left(\frac{\left\langle x_{0}^{2} \right\rangle}{\left\langle x^{2} \right\rangle_{m}} \left(\frac{\left\langle x_{0}^{2} \right\rangle}{\left\langle x^{2} \right\rangle_{m}} \right)^{2} + \frac{1}{2} \left(\frac{\left\langle x_{0}^{2} \right\rangle}{\left\langle x^{2} \right\rangle_{m}} \right)^{2} + \frac{1}{2} \left(\frac{\left\langle x^{2} \right\rangle}{\left\langle x^{2} \right\rangle_{m}} \left(\frac{\left\langle x^{2} \right\rangle}{\left\langle x^{2} \right\rangle_{m}} \right)^{2} + \frac{1}{2} \left(\frac{\left\langle x^{2} \right\rangle}{\left\langle x^{2} \right\rangle_{m}} \left(\frac{\left\langle x^{2} \right\rangle}{\left\langle x^{2} \right\rangle_{m}} \right)^{2} + \frac{1}{2} \left(\frac{\left\langle x^{2} \right\rangle}{\left\langle x^{2} \right\rangle_{m}} \left(\frac{\left\langle x^{2} \right\rangle}{\left\langle x^{2} \right\rangle_{m}} \right)^{2} + \frac{1}{2} \left(\frac{\left\langle x^{2} \right\rangle}{\left\langle x^{2} \right\rangle_{m}} \left(\frac{\left\langle x^{2} \right\rangle}{\left\langle x^{2} \right\rangle_{m}} \right)^{2} + \frac{1}{2} \left(\frac{\left\langle x^{2} \right\rangle}{\left\langle x^{2} \right\rangle_{m}} \left(\frac{\left\langle x^{2} \right\rangle}{\left\langle x^{2} \right\rangle_{m}} \right)^{2} + \frac{1}{2} \left(\frac{\left\langle x^{2} \right\rangle}{\left\langle x^{2} \right\rangle_{m}} \left(\frac{\left\langle x^{2} \right\rangle}{\left\langle x^{2} \right\rangle_{m}} \right)^{2} + \frac{1}{2} \left(\frac{\left\langle x^{2} \right\rangle}{\left\langle x^{2} \right\rangle_{m}} \left(\frac$$

• With (index 'm' denoting beam moments under matching conditions)

$$b = \Delta \gamma^2 / 2\gamma_0, \Delta \gamma = \sigma_\gamma / \gamma, \langle x u_x \rangle_m = 0, \langle x^2 \rangle_m = \epsilon_0 \sqrt{2/\overline{\gamma_0}}, \langle u_x^2 \rangle_m = \epsilon_0 \sqrt{\overline{\gamma_0}/2}$$

• Also providing the final emittance as

 $\lim_{t \to \infty} \epsilon_{n,rms}^2(\tilde{t}) = \frac{\overline{\gamma_0}}{8} \left\langle x_0^2 \right\rangle^2 + \frac{1}{2\overline{\gamma_0}} \left\langle u_{x,0}^2 \right\rangle^2 + \frac{1}{2} \left\langle x_0^2 \right\rangle \left\langle u_{x,0}^2 \right\rangle$



Comparison between analytical, a semi-analytic numerical (SANA) and Particle-in-Cell approaches

Symplectic approach

$$D(z,t) = D(\mathscr{M}_{-t} \circ z_0, t=0)$$

$$\underbrace{\mathbb{M}^t \ J \ \mathbb{M}}_{2D \times 2D} = J \to \dim_{\mathbb{M}} = 2D^2 + D$$





Optically controlled laser-plasma electron accelerators for compact γ - ray sources

- Bi-color stack of sub-Joule pulses is resilient to degradation of the dense plasma ($n_0 \sim 10^{19} \text{ cm}^{-3}$)
- Electron beam quality is preserved
- Electron energy is doubled vs. predictions of scaling

Production of ultra-bright ($B_n > 10^{17}$ A/m²), GeV-scale

Stack-driven LPA produces perfect e-beams for a Thomson scattering-based γ-ray source:

- Photon signal to background ratio exceeds 4:1
- Photon yield in a µsr observation solid angle $\Omega_d = (\pi/2) \langle \gamma_e \rangle^{-2}$: up to 5 × 10⁶ in full bandwidth
- Photon energy is tunable from 4 to 16 MeV, while preserving the yield



Plasma lens simulations

active and passive lens sims



- start-to-end simulations
- GPT + Architect
- Hydro simulations

ionisation injection for PWFA



600 μm pagation	Q[p C]	σ_γ/γ[%]	ε_(n,rms) [μm]
N ¹⁺	22.2	2.9	0.96
Ar ¹⁺	26.7	3.3	0.87
Ne	22.5	3.9	1.2

ionisation injection for LWFA

2.0

107

93

29.1





4.6

5.6

2.5

3.8

optimal values

TNSA max-Energy estimation laws

$$\begin{cases} E_{\max}^{(2D)}(ct) = 0 & \text{for } t < t^{*(2D)} \\ E_{\max}^{(2D)}(ct) = E_{\infty}^{(2D)} \log \frac{ct}{ct^*} & \text{for } t > t^{*(2D)} \end{cases}$$

$$\begin{cases} E_{\max}^{(3D)}(ct) = 0 & \text{for } t < t^{*(3D)} \\ E_{\max}^{(3D)}(ct) = E_{\infty}^{(3D)} \left(1 - \frac{ct^{*(3D)}}{ct}\right)^2 & \text{for } t > t^{*(3D)} \end{cases}$$



PSATD Cherenkov Reduction

Pseudo-Spectral-Analytic-Time-Domain (PSATD)



Intrinsic elimination of NCI in Lorentz-boosted frame simulations

Galilean-PSATD

- Solves PIC in co-propagating Galilean frame
- ▶ No artificial numerical corrections required
- Independent of geometry

Intrinsically free of NCI for drifting plasma

Comparison PSATD / Galilean-PSATD in the boosted frame



Concept & applications: Physics of Plasmas 23, 100704 (2016) Math & stability analysis: Phys. Rev. E 94, 053305 (2016) or checkout **lux.cfel.de/publications/**





Exascale computing

present and future for WarpX - Mesh Refinement

ECP Project WarpX: Exascale Modeling of Advanced Particle Accelerators

Goal (4 years): Convergence study in 3-D of 10 consecutive multi-GeV stages in linear and bubble regime, for laser-& beam-driven plasma accelerators.

How: → Combination of most advanced algorithms

→ Coupling of Warp+BoxLib/AMReX+PICSAR



→ Port to emerging architectures (Xeon Phi, GPU)



Team: LBNL ATAP (accelerators) + LBNL CRD (computing science) + SLAC + LLNL

Ultimate goal: enable modeling of 100 stages by 2025 for 1 TeV collider design!

Noise control



PICon CPU on all Platforms & XFEL-Plasma Modeling



- Got no GPUs? Now runs also on:
 CPU, KNL, ARM, Power, ...!
- open software stack towards exascale 3D3V PIC simulations
- single-source, performance portable C++ (27k LOC)





Photon Scattering in Solid-Density Plasmas

Massive Monte-Carlo X-ray photon scattering

arbitrarily complex 3D density distributions from PIC simulationsmultiple scattering, arbitrary atomic physics









How can one observe the plasma dynamics? Using radiation as synthetic radiation diagnostics



The first 3D LWFA radiation simulation linking spectral signatures to laser and plasma dynamics.

PICon GPU

Radiation signatures all to determine:

- 1. wave breaking
- 2. laser intensity
- 3. laser-bunch interaction
- 4. self-phase modulation



Rise of the open source

Codes

- Epoch
- PiconGPU
- Aladyn / Architect
- SMILEI
- ► FBPIC
- WarpX (2018)

Libraries

- PICSAR (soon)
- PMacc
- Alpaka
- ADIOS
- Standards
 - openPMD
 - Where are the other standards?

Exascale approaches : future of PIC codes

Numerical methods

- Improvements of (pseudo) spectral solvers implementation
- Fighting Cherenkov radiation and instability
- Adaptative Mesh Refinement (AMR)
- Oynamic Load Balancing
- Reduced models (envelope, symplectic, hybrid ...)
- Continuous integration, robustness tests

Diagnostics and data

- In situ visualization
- Radiation diagnostics
- Data reduction

Exascale approaches : future of PIC codes

Numerical methods

- Improvements of (pseudo) spectral solvers implementation
- Fighting Cherenkov radiation and instability
- Adaptative Mesh Refinement (AMR)
- Oynamic Load Balancing
- Reduced models (envelope, symplectic, hybrid ...)
- Continuous integration, robustness tests

Diagnostics and data

- In situ visualization
- Radiation diagnostics
- Data reduction