Numerical studies on capillary discharges as focusing elements for electron beams

Emanuele Brentegani[1], Maria Pia Anania[1], Stefano Atzeni[2], Angelo Biagioni[1], Enrica Chiadroni[1], Michele Croia[1], Massimo Ferrario[1], Francesco Filippi[1], Alberto Marocchino[1], Andrea Mostacci[2], Riccardo Pompili[1], Stefano Romeo[1] , Angelo Schiavi[2], Arie Zigler[1]

1.Laboratori Nazionali di Frascati, Istituto Nazionale di Fisica Nucleare, Via Enrico Fermi 40 - Frascati (Roma)
2.Dipartimento SBAI, Università di Roma 'La Sapienza'

Introduction

The azimuthal magnetic field generated by the discharge current induced by applying a voltage to the extremities of a gas-filled capillary can be used to focus an electron beam passing through the device. The generated magnetic field gradient can reach values higher than those achieved in electromagnet and even permanent magnet quadrupoles.

In principle, optimal focusing condition is reached when the current density is perfectly parallel to the capillary axis and transversely uniform, as in this case the magnetic field intensity has a linear dependence on the distance from the axis.

In a number of experimental situations, this is often not the case; thus, the aim of the present work is to numerically investigate capillary discharges in order to study the causes of the transverse non-linearity of the magnetic field and to allow future studies on mitigation strategies. In fact, the non satisfactory degree of uniformity of the current density distribution could be due to a number of reasons, including but not limited to: the dependence of the plasma resistivity on the local temperature, the shape of the electrodes and the capillary geometry.

Active plasma lens

Working principle

The functionality of an active plasma lens (APL) is based on the magnetic field generated by a current density flowing in a plasma in the same direction as the current of a charged particle beam. The lines of this magnetic field lay on the transverse plane and wrap around the axis of the capillary. The beam particles passing through the device experience a Lorentz force which focuses the beam. This mechanism is exemplified in Figures 1 and 2.

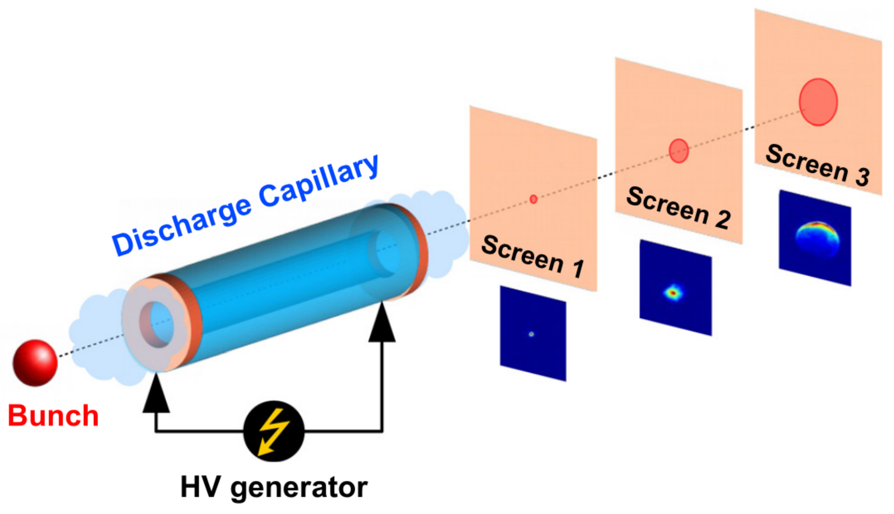

Figure 1: Scheme of principle of an APL [1]

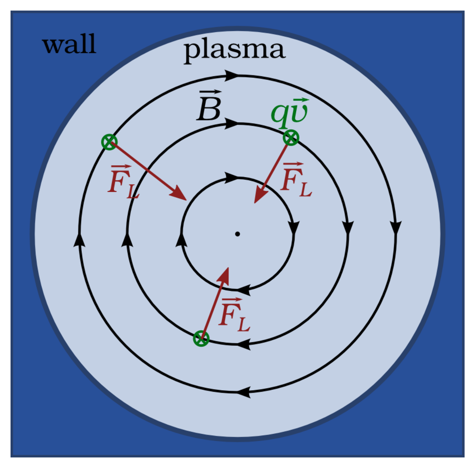

Figure 2: Schematic transverse view of an APL, with a representation of the Lorentz force (F_L) acting on the beam particles.

APLs can reach **magnetic field gradients even higher** than those achievable in permanent magnet quadrupoles, are capable of **focusing a beam in both transverse planes** at the same time, are **compact** objects (few centimeters) and their **focusing force scales like γ^{-1}**, which makes them suitable also at high energy.

Recently, an active plasma lensing scheme has been characterized for the operation with electron beams [1], showing that transverse non linearity of the azimuthal magnetic field can be responsible for emittance growth and increase in minimum spot size attainable. Thus, a full optimization of the operation of the entire device is necessary, in order to exploit all the capabilities of this technology.

Experimental case:

To trigger a gas discharge, a high voltage is applied at the electrodes placed at the extremities of a capillary filled with hydrogen [2]. Figure 3 shows an example of circuit used to power the device. A typical current profile obtained in a 1cm long capillary with a circular aperture of diameter 1 mm is presented in Fig. 4.

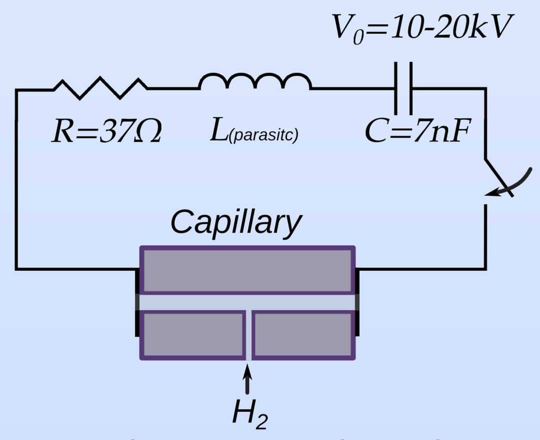

Figure 3: Scheme of the electric circuit powering the gas discharge.

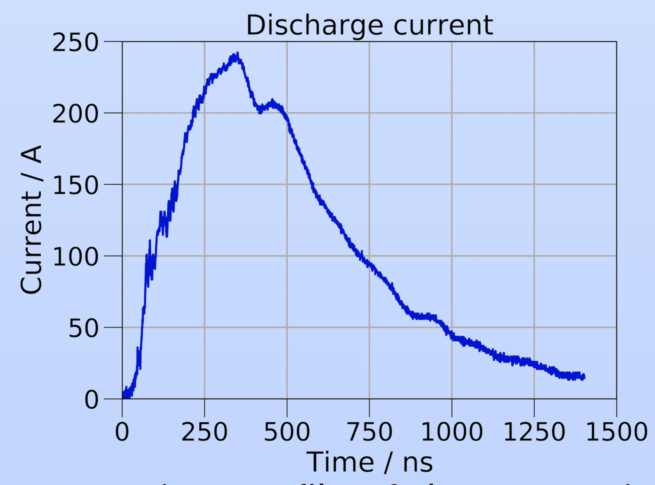

Figure 4: Time profile of the current inside the capillary

Discharge simulation in hydrodynamic approximation

The gas discharge process has to be simulated with a fluid model, as a *particle in cell approach would be computationally impractical*, due to the time scales involved and to the spatial resolution required.

Furthermore, some simplifying assumptions may be made:

- At first order, Lorentz force is negligible with respect to thermal pressure force; skin effect has also very little influence for the cases under study ➜ instead of using a full magneto-hydrodynamic model, a **hydrodynamic** approximation can be employed, the current density is computed using the static current flow approximation and the **ohmic heating term** is added to the energy equation:

$$\begin{cases} \frac{\partial}{\partial t}\rho + \nabla\cdot(\rho\vec{v}) = 0 \\ \frac{\partial}{\partial t}(\rho\vec{v}) + \nabla\cdot(\rho\vec{v}\vec{v}) = -\nabla p \\ \frac{\partial}{\partial t}E + \nabla\cdot(E\vec{v}) = -\nabla\cdot(p\vec{v}) + \nabla\cdot\kappa\nabla T + \eta\|\vec{J}\|^2 \\ \nabla\cdot\vec{J} = 0,\ \vec{J} = -\frac{\nabla V}{\eta} \end{cases}$$

ρ: mass density
$\vec{v}$: fluid velocity
p: thermal pressure
E: total energy density
κ: thermal conductivity
T: plasma temperature
$\vec{J}$: current density
V: electric potential
η: electrical resistivity

- The capillary has a circular cross section ➜ **2D axial symmetry** is assumed
- **Lagrangian approach** (the mesh moves following the matter):
  - **advantage**: no need to mesh the whole domain, as it would be in eulerian approach
  - **disadvantage**: grid pathologies have to be controlled
- An ionization model exploiting a "**local thermodynamic equilibrium**" approximation is suitable

A suitably modified, reduced version of the code **DUED** [3], that implements the above model, has been used to simulate the discharge in a capillary of length 2cm, with a circular aperture of diameter 2mm. The capillary was initially filled by hydrogen with a uniform density of $2\cdot10^{-7}$ g/cm^3 and a temperature of 9000K, in order to provide an initial fictitious ionization. The current is imposed at each time step to mimic the measured profile shown in Fig. 2. The results at 300ns from the start of the discharge are presented in Fig. 5,6,7 .

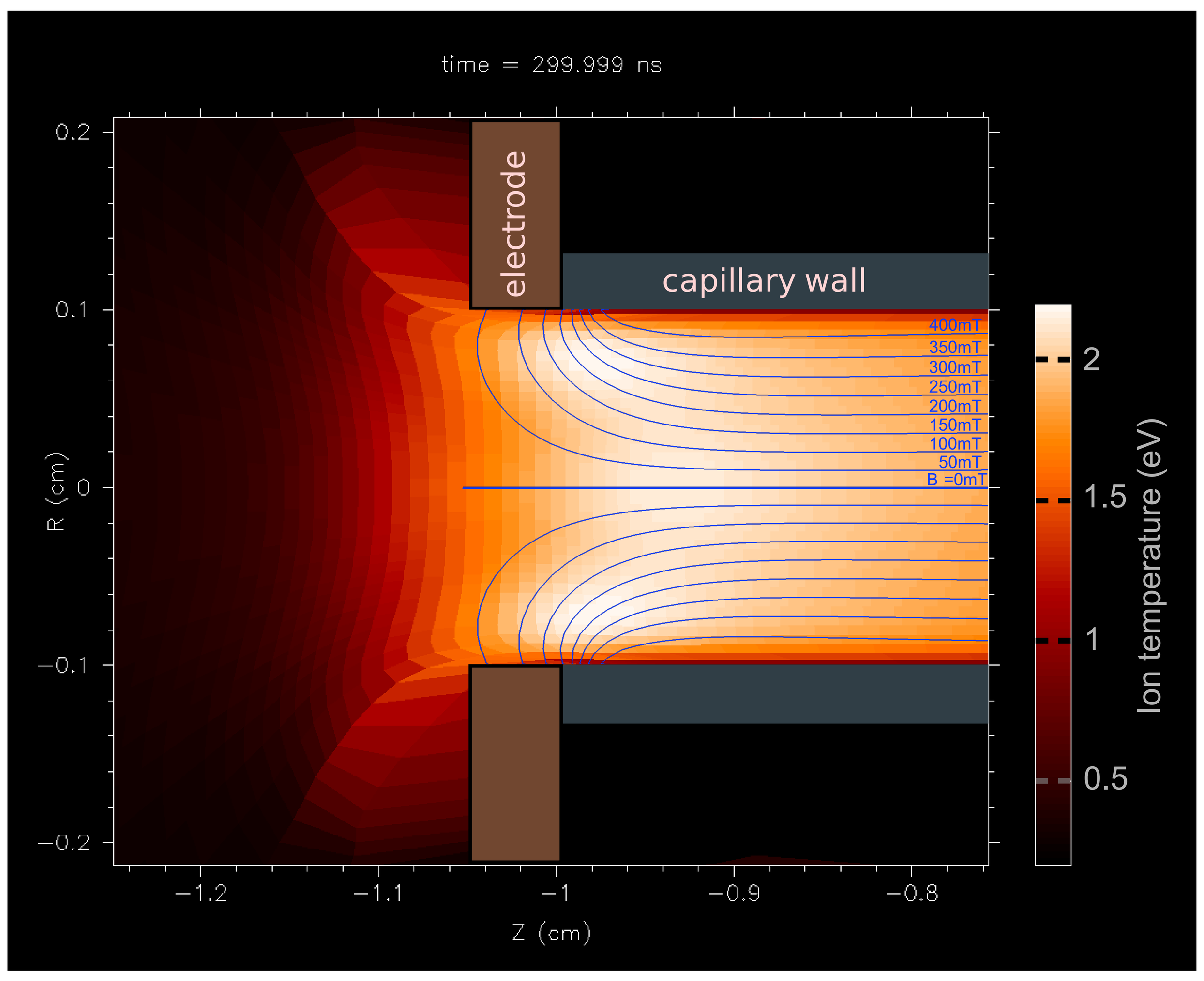

Figure 5: Particular of the plasma temperature (colored map) and azimuthal magnetic field (contour lines) in proximity of the left electrode at 300ns from the start of the discharge.

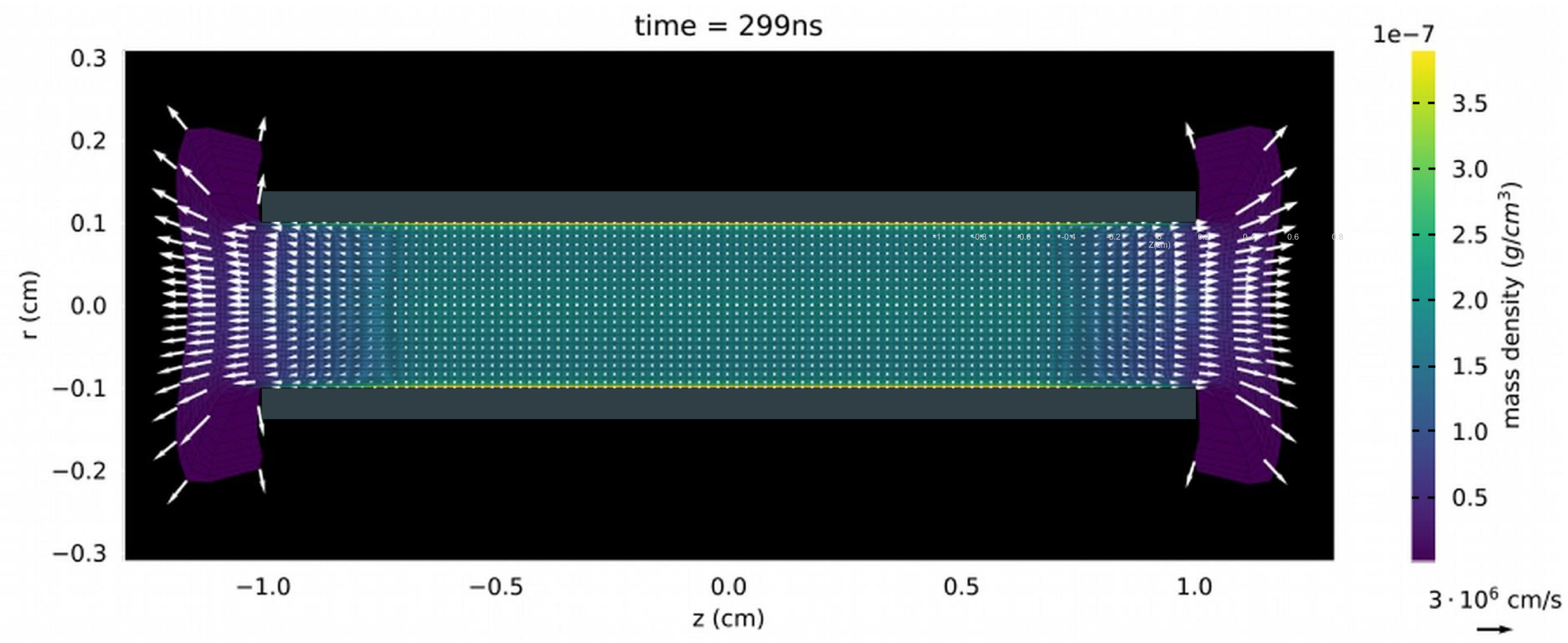

Figure 6: Mass density map and velocity field of the gas flowing from the capillary at 300ns from the start of the discharge.

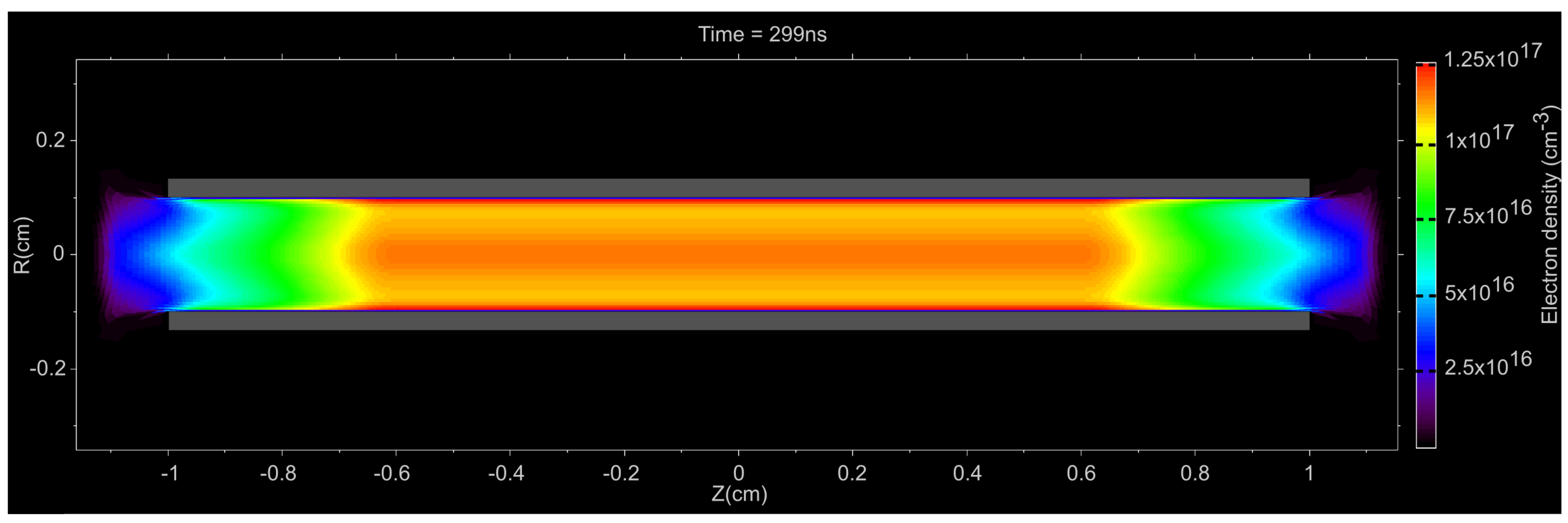

Figure 7: Electron density map at 300ns from the start of the discharge.

Conclusions and perspectives

We have shown the working principle of a gas discharge in a capillary as a focusing element for electron beams. The accurate simulation of the discharge process is crucial as the transverse azimuthal magnetic field distribution depends on the current density, which is in turn heavily dependent on the electric resistivity inside the plasma filling the capillary.

A hydrodynamic model is capable of simulating the plasma heating due to joule losses and the outflow from the open extremities, thus capturing the main phenomena affecting the development of the plasma discharge.

Future steps:

- Thorough comparison of the computed longitudinal electron density profiles with the measured ones;
- Optimization of the design parameters in order to improve the focal properties of the lens.

References

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3. Atzeni et al., Comput. Phys. Commun. 169 (2005) 153