Analytic model for electromagnetic fields in the bubble regime in non-uniform plasma

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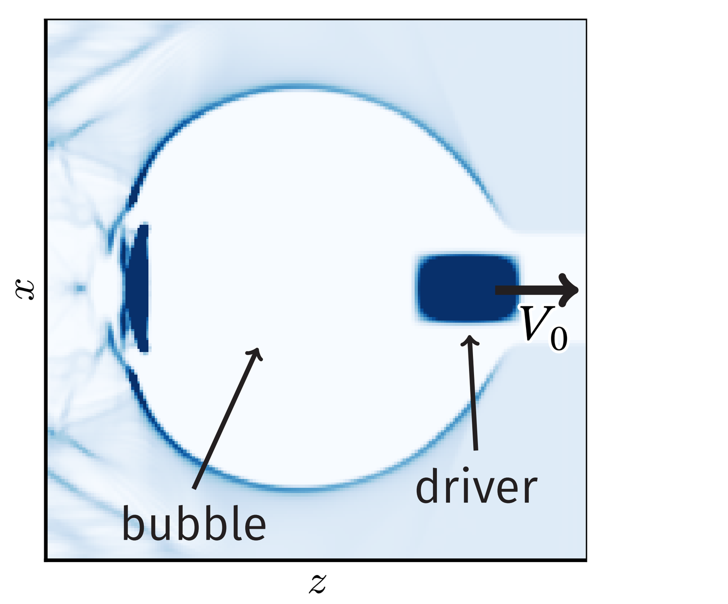
Introduction

We consider the strongly non-linear (bubble) regime of plasma wakefield and build a model describing the electromagnetic field components distributions in the wakefield. The main results are

1. We develop a phenomenological model of the bubble regime, assuming that there are no plasma electrons inside the bubble, while on its boundary there is a thin electron sheath which screens the bubble from plasma.
2. Using the smallness of the electron sheath width, we develop a perturbation theory which allows us to calculate simple explicit expressions for the components of the electromagnetic field both inside and outside the bubble.
3. The theory is verified by particle-in-cell (PIC) simulations.

Basic assumptions



Electron density (**blue**) in a bubble in 3D particle-in-cell (PIC) simulations. The electron bunch pushes plasma electrons, leading to the formation of the bubble.

We consider wakefield in the bubble regime excited either by an electron bunch or a laser pulse. All simulations are done for the electron driver, but the results are applicable for the laser driver.

Assumptions:

- Cylindrical geometry $\mathbf{r} = (r, \phi, z)$.
- Axial symmetry (no dependence on ϕ).
- Radially non-uniform plasma $n(\mathbf{r}) = n(r)$.
- Ions are immobile.

W. Lu et al., *Phys. Rev. Lett.* **96**, 165002 (2006).
J. Thomas et al., *Phys. Plasmas* **23**, 053108 (2016).

Potentials and fields

We use quasi-static approximation, in which fields propagate with the velocity of light, and the structure of the fields does not change

$$f(r, z, t) = f(r, \xi), \quad \xi = t - z,$$

EM fields are described by the vector potential (A_z, A_r) and the wakefield potential $\Psi = \varphi - A_z$.

The solution to the Maxwell's equations is written as

$$E_z = \frac{\partial \Psi}{\partial \xi} \qquad B_\phi = \frac{1}{r}\int_0^r \left(J_z + \frac{\partial^2 \Psi}{\partial \xi^2}\right) r' \, dr'$$

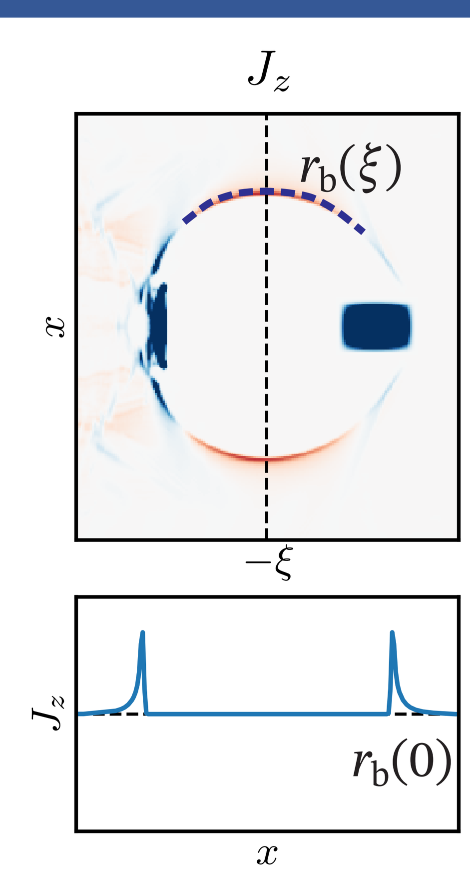
$$E_r = -\frac{\partial \Psi}{\partial r} + B_\phi \qquad \Psi = -\int_r^\infty \frac{dr'}{r'} \int_0^{r'} (J_z - \rho) r'' \, dr''$$

All fields depend on J_z and $J_z - \rho$. Knowing these two sources is sufficient to calculate all field components.

Models for the sources

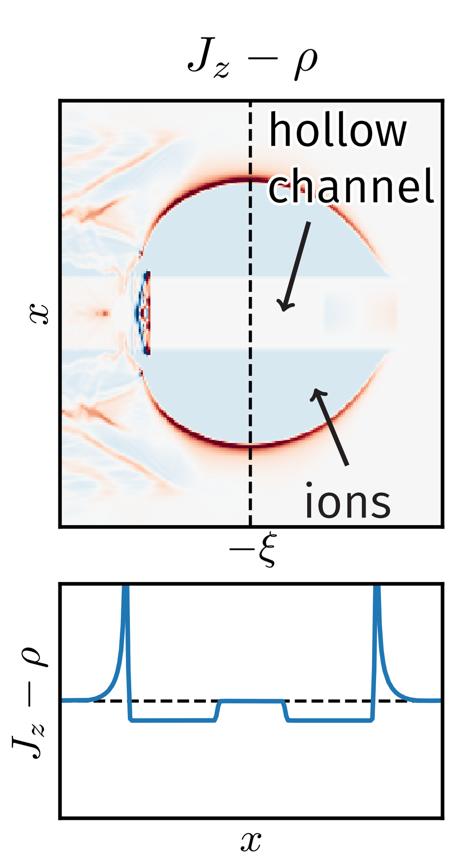
We look at numerical simulations in order to determine proper models for J_z and $J_z - \rho$. Plasma in our simulations has a hollow channel.

Inside the bubble, only electron bunches contribute to J_z. On the boundary, there is a thin electron sheath. Currents in the electron sheath rapidly decay to zero.



$$J_z(\xi, r) = \begin{cases} J_e(\xi, r), & r < r_b(\xi) \\ J_0(\xi) g_J\left(\dfrac{r - r_b(\xi)}{\Delta_J}\right), & r \geq r_b(\xi) \end{cases}$$

The boundary condition for J_0 is $\lim_{r\to\infty} r B_\phi = 0$.



Inside the bubble, $J_z - \rho = -\rho_i$.
Driving and witness bunches do not contribute to $J_z - \rho$.
Outside the bubble, $J_z - \rho$ rapidly decays.

$$J_z - \rho = \begin{cases} -\rho_i(r), & r < r_b(\xi) \\ S_0(\xi) g\left(\dfrac{r - r_b(\xi)}{\Delta}\right), & r \geq r_b(\xi) \end{cases}$$

The boundary condition for S_0 is $\lim_{r\to\infty} \Psi = 0$.

We can calculate the fields as long as we know $r_b(\xi)$, Δ, g, Δ_J, and g_J.

Boundary of the bubble

The width of the electron sheath is small compared to the size of the bubble

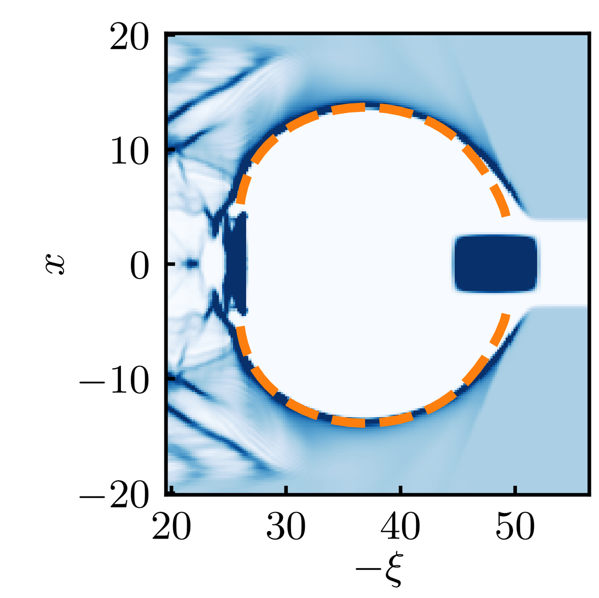
$$\Delta, \Delta_J \ll r_b.$$

Using this assumption, we get an equation for the boundary of the bubble.

$$A(r_b)\frac{d^2 r_b}{d\xi^2} + B(r_b)\left(\frac{dr_b}{d\xi}\right)^2 + C(r_b) = \lambda(\xi)$$

$$\lambda(\xi) = -\int_0^{r_b(\xi)} J_e(\xi, r') r' \, dr'$$

Function $r_b(\xi)$ can be analytically calculated.



Dashed lines correspond to the analytical solution.

A. A. Golovanov et al., *Quantum Electron.* **46**, 295 (2016).

Perturbation theory

At this point, we can analytically calculate all fields distributions. However, the results can be significantly simplified if we use the smallness of the electron sheath thickness $\Delta, \Delta_J \ll r_b(\xi)$.

We begin with the expression for E_z

$$E_z(\xi, r) = -\frac{\partial}{\partial \xi}\left[\int_r^\infty \frac{dr'}{r'} \int_0^{r'} (J_z - \rho) r'' \, dr''\right]$$

In general, this expression requires numerical integration and differentiation.

We develop a perturbation theory with respect to $\epsilon(r_b) = \Delta / r_b$

$$E_z = E_{z,0}(\xi, r) + \epsilon(r_b(\xi)) E_{z,1}(\xi, r) + \dots \approx E_{z,0}$$

The answer is very simple

$$E_z(\xi, r) = \begin{cases} \dfrac{S_i(r_b)}{r_b}\dfrac{dr_b}{d\xi}, & r < r_b(\xi) \\ \dfrac{S_i(r_b)}{r_b}\dfrac{dr_b}{d\xi}\displaystyle\int_{(r-r_b)/\Delta}^\infty g(X)\, dX, & r \geq r_b(\xi) \end{cases}$$

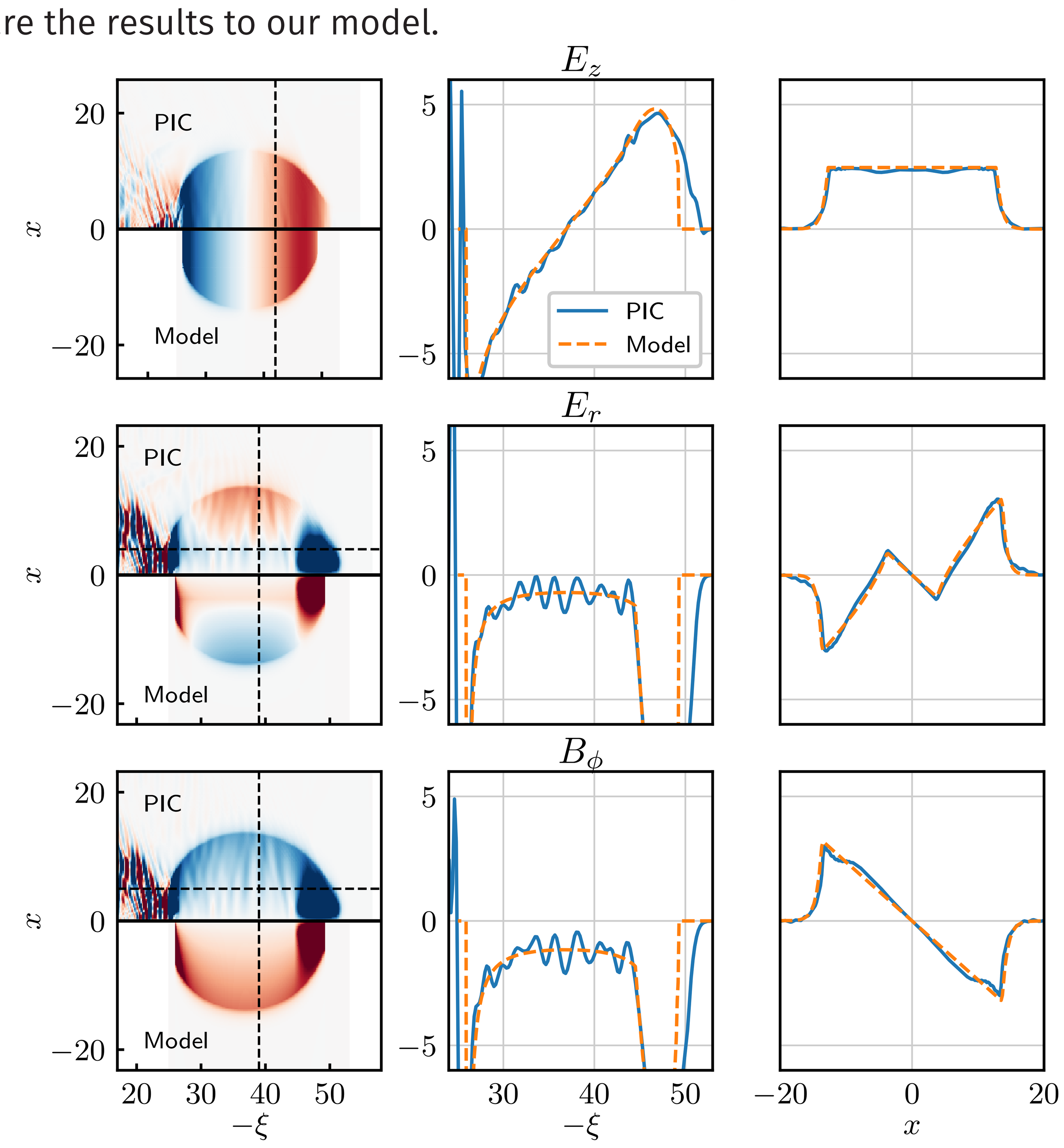
where $S_i(r) = \int_0^r \rho_i(r') r' \, dr'$

A similar procedure is applied to all other field components.

The perturbation theory allows us to find simple explicit expressions for the field components.

Results

We perform simulations with an electron driver using the 3D PIC code Smilei and compare the results to our model.



The comparison shows that the model correctly describes the fields both inside and outside the bubble.

A. A. Golovanov, I. Yu. Kostyukov, J. Thomas, A. Pukhov, *Phys. Plasmas* **24**, 103104 (2017).

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