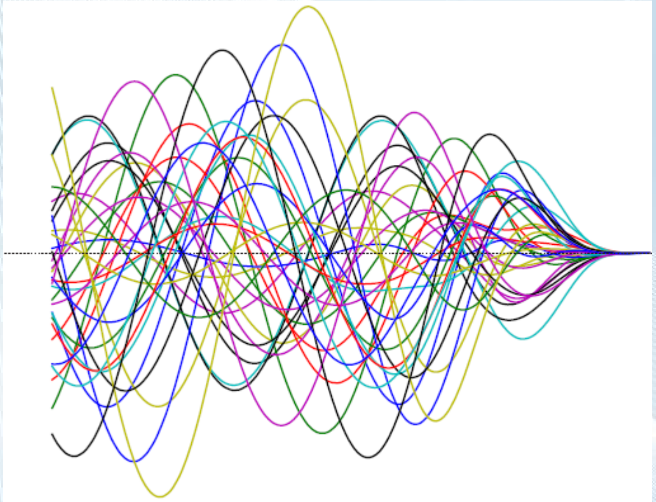
3rd European Advanced Accelerator Concepts Workshop
La Biodola, Isola d'Elba, Italy, 27.09.2017

GENERATION OF CONTROLLABLE PLASMA WAKEFIELD NOISE IN PARTICLE-IN-CELL SIMULATIONS

Roman Spitsyn[a,b], Konstantin Lotov[a,b], Hartmut Ruhl[c], Nils Moschuering[c]



a – Budker Institute of Nuclear Physics
b – Novosibirsk State University
c – Ludwig-Maximilians-Universität

Formulation of the problem

Beam dynamics in plasmas

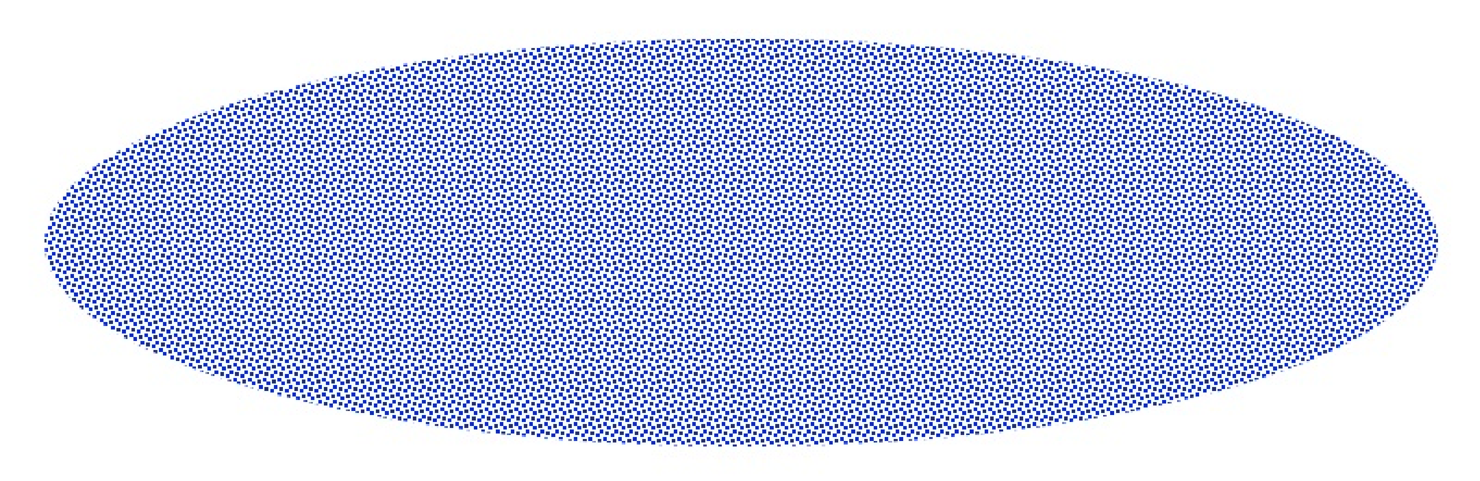
Correct simulations

Overestimated level of wakefield noise

Macroparticles
(Q>>e)

Rapidly increasing instabilities

Formulation of the problem



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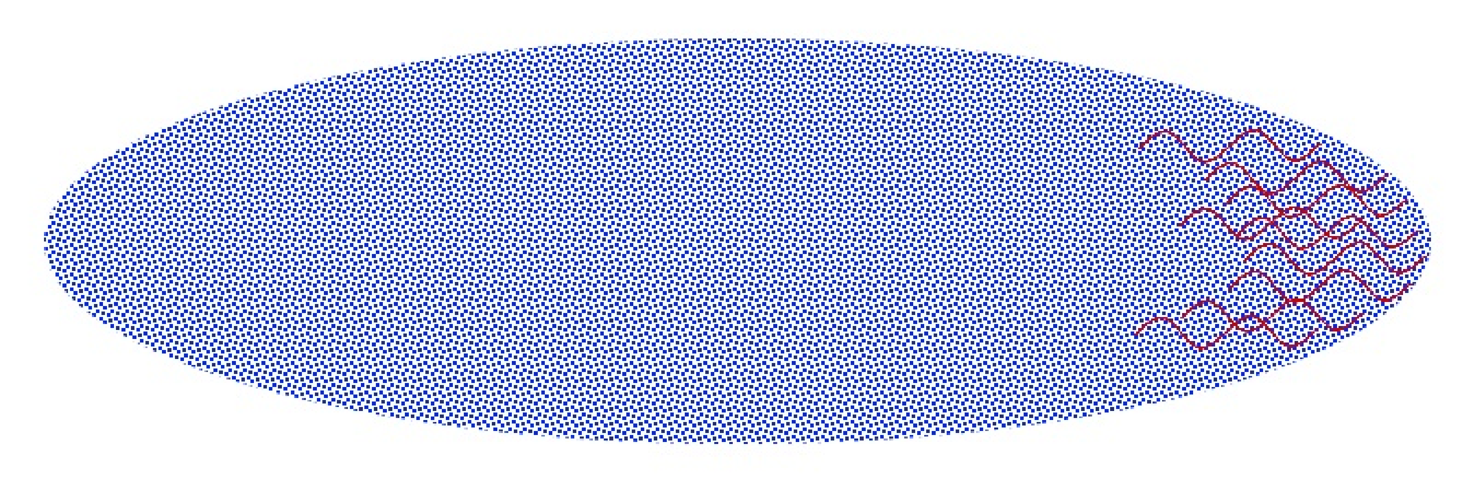

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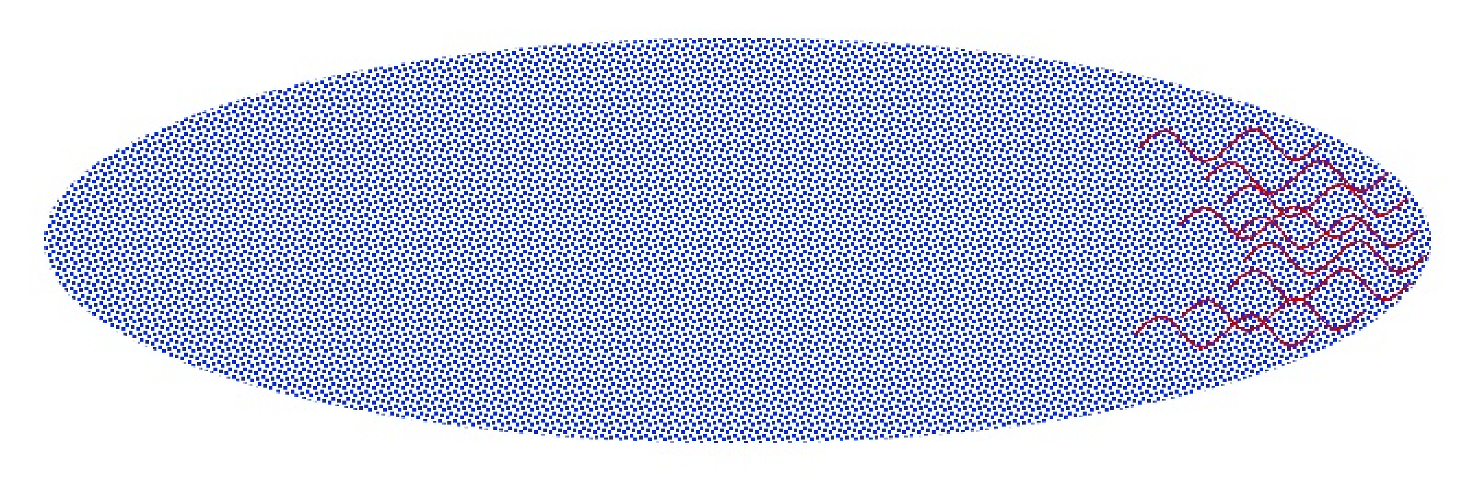

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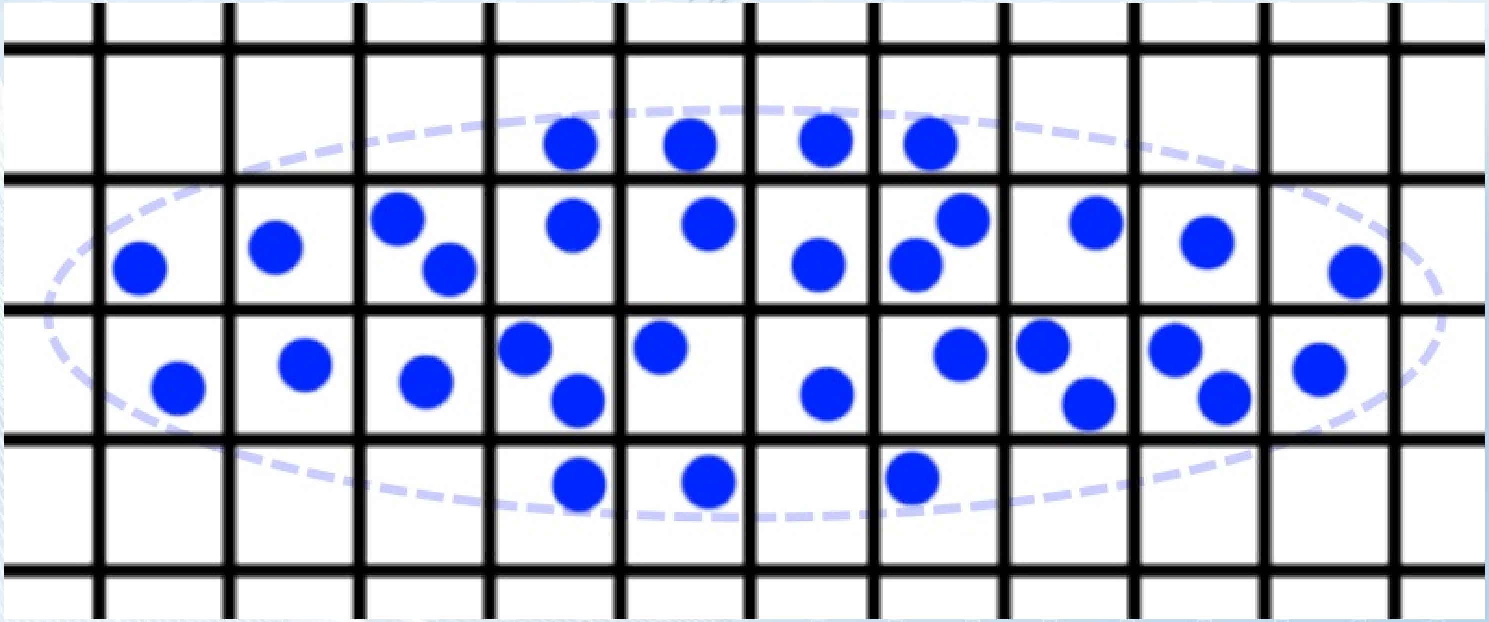

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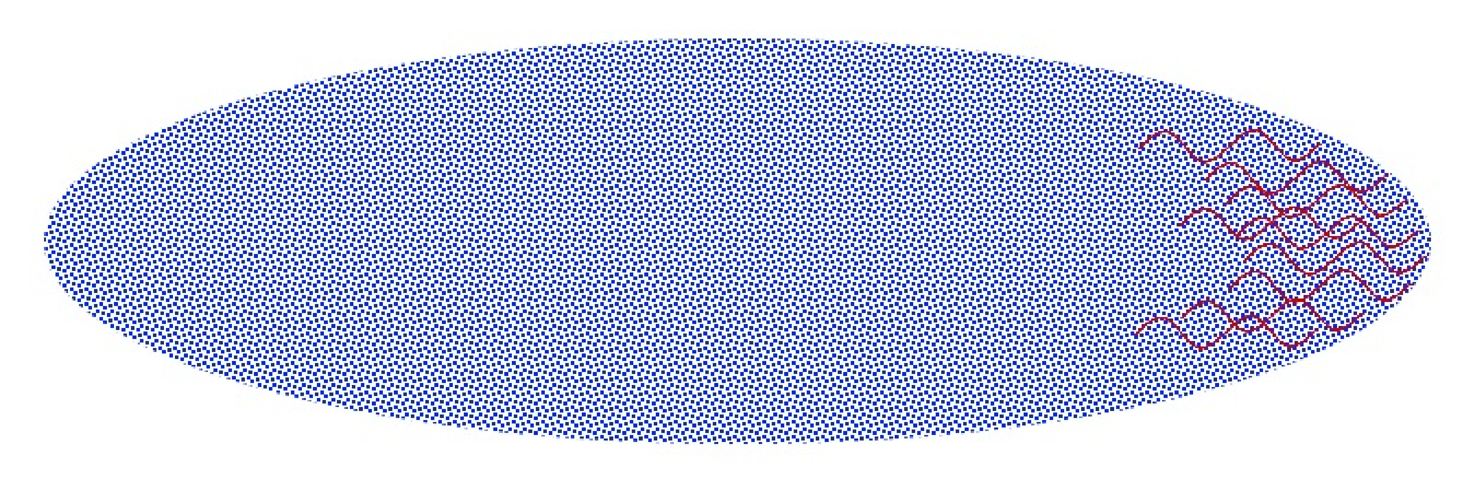

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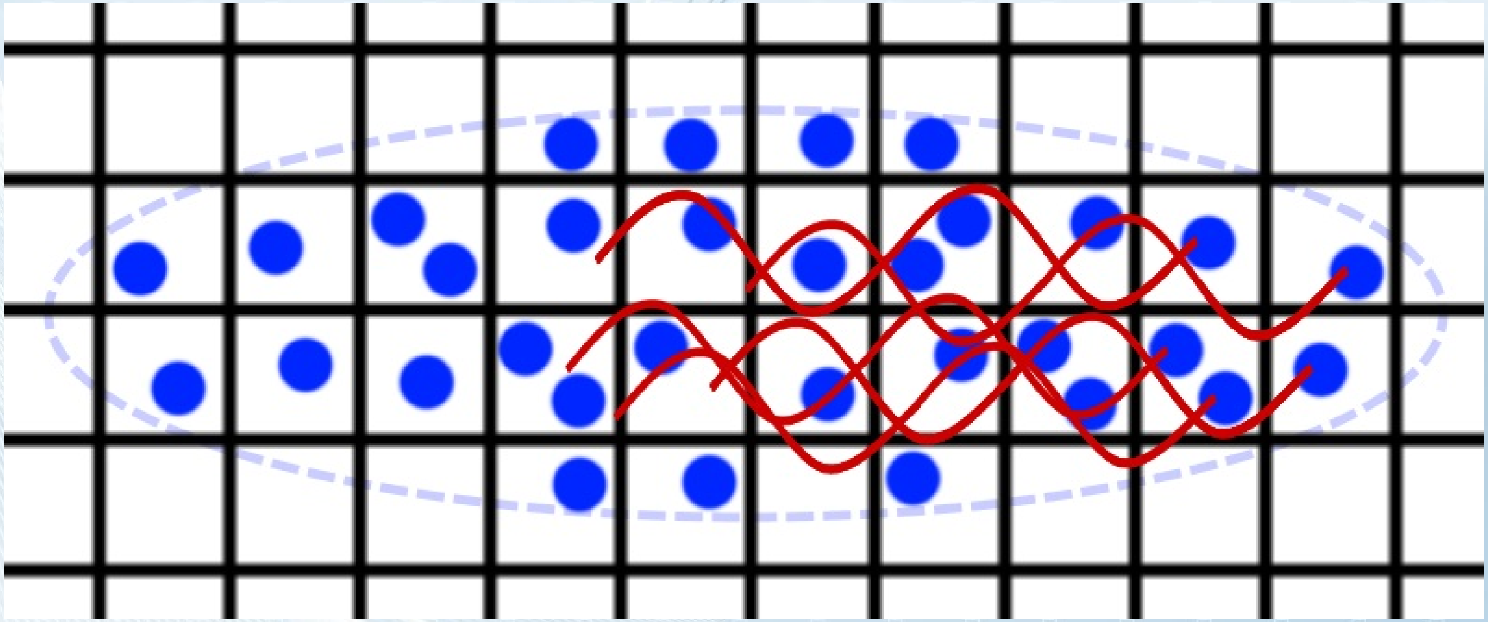

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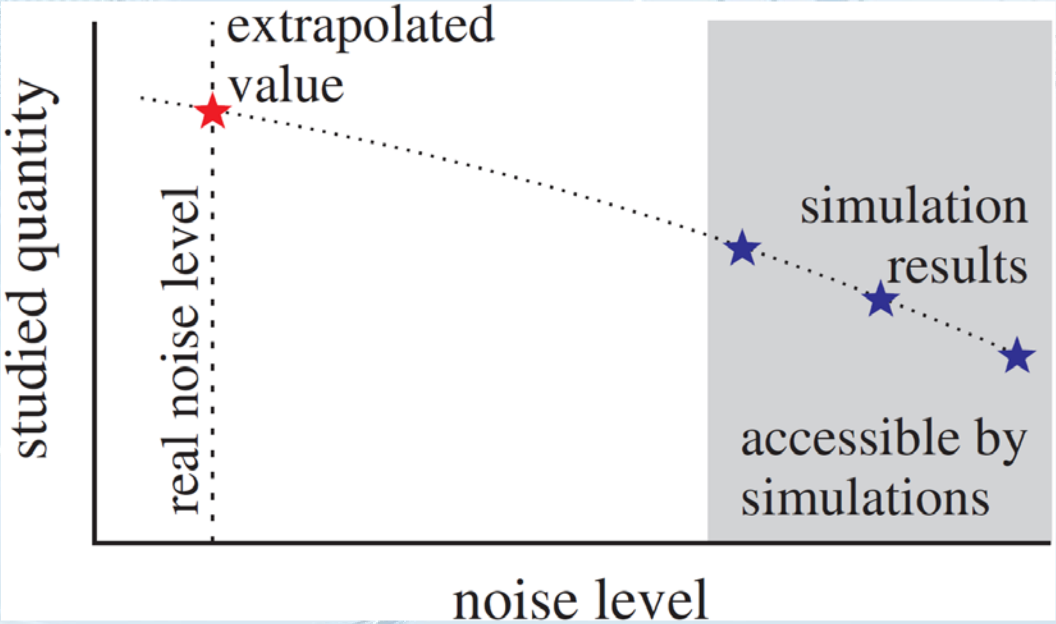

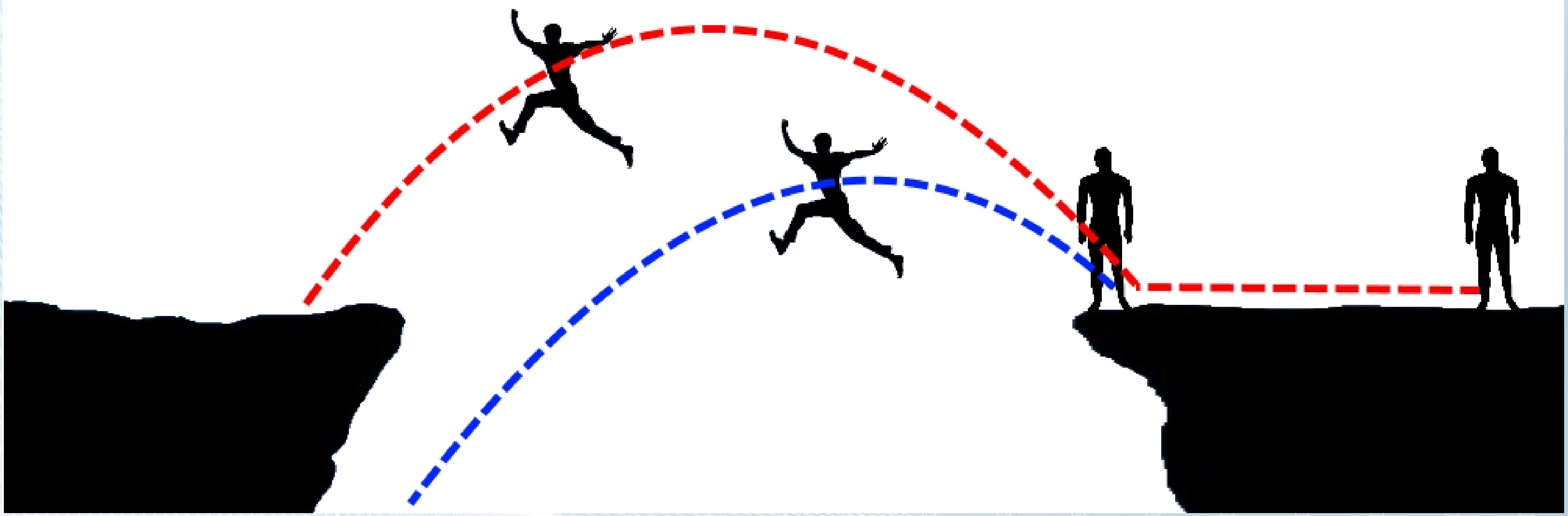

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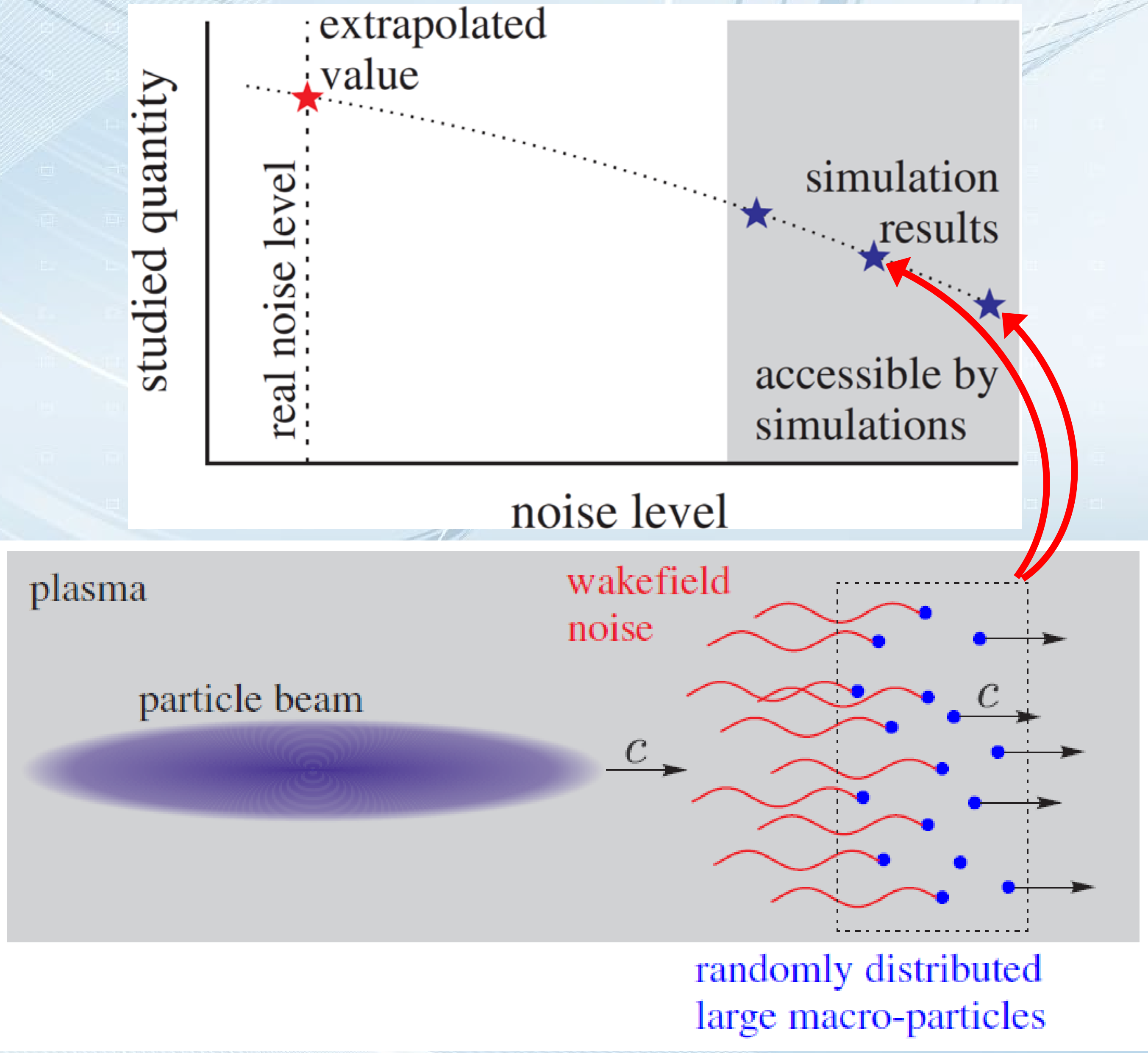

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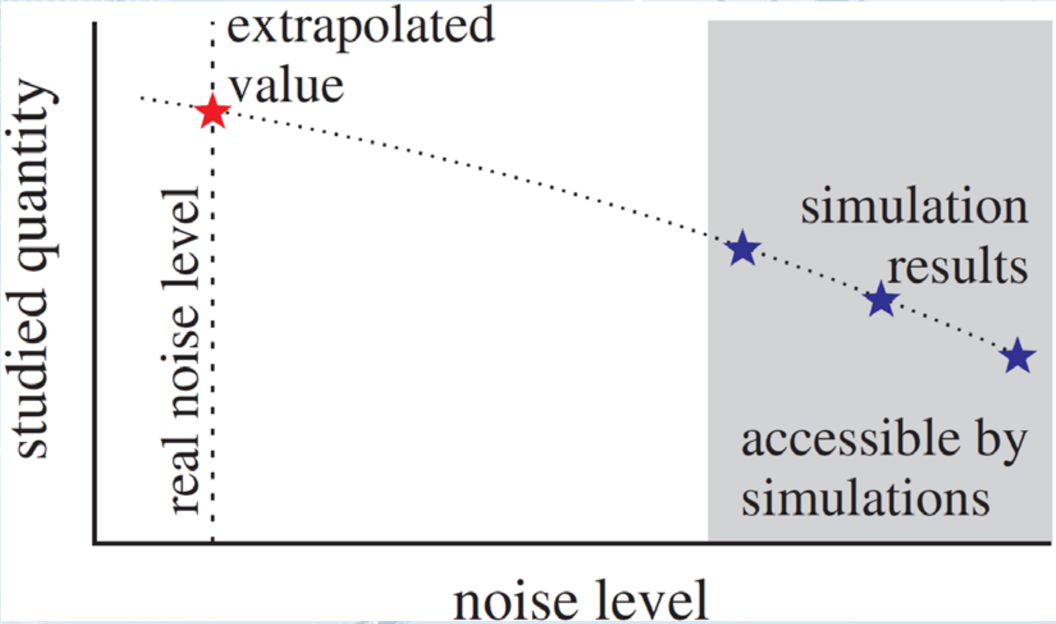
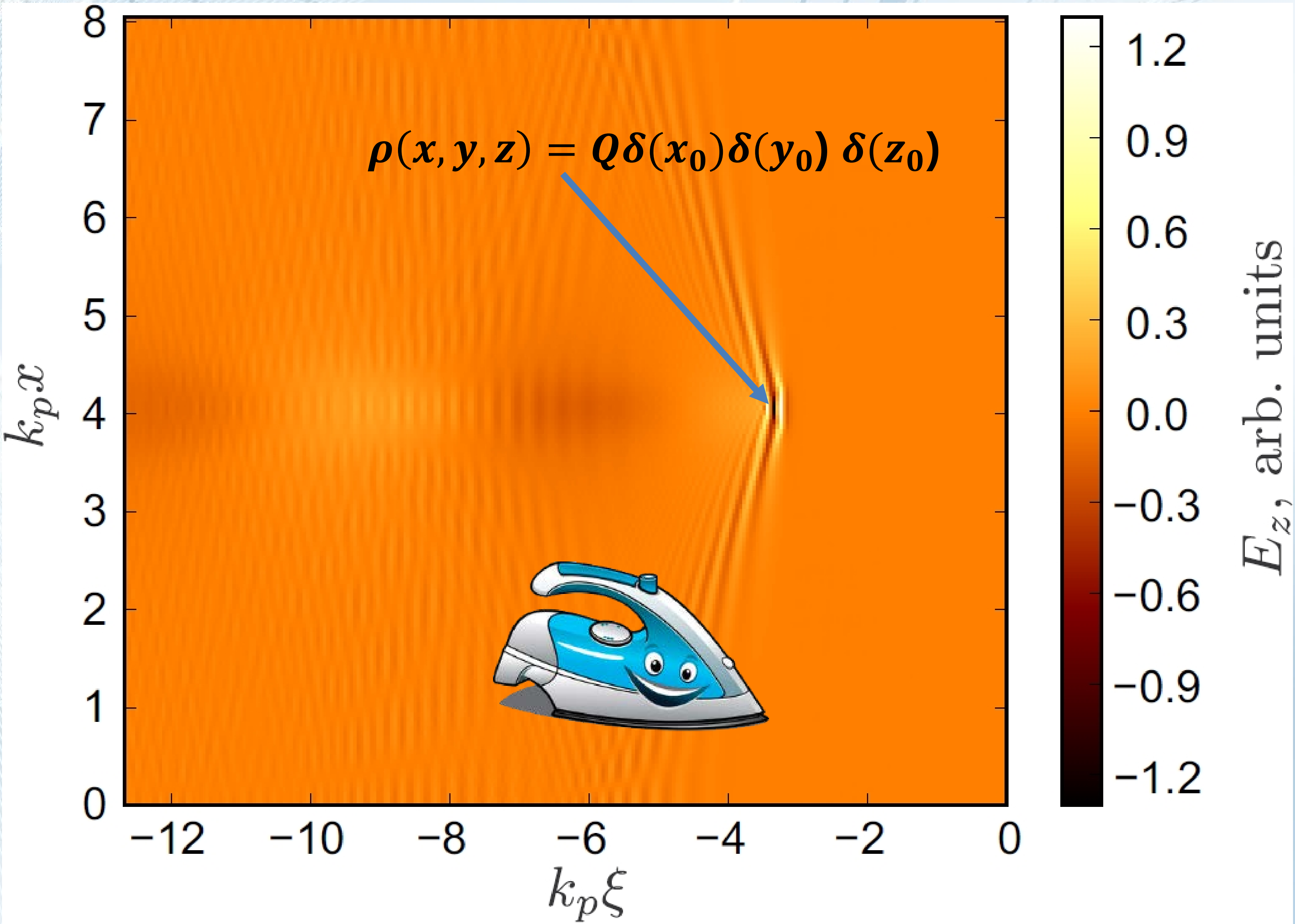


<u>Estimates of the real field level – extrapolation of results</u>

We <u>do not</u> change parameters of simulations (grid, beam, etc.)

<u>BUT!</u> Numerical Cherenkov instability

Point charge



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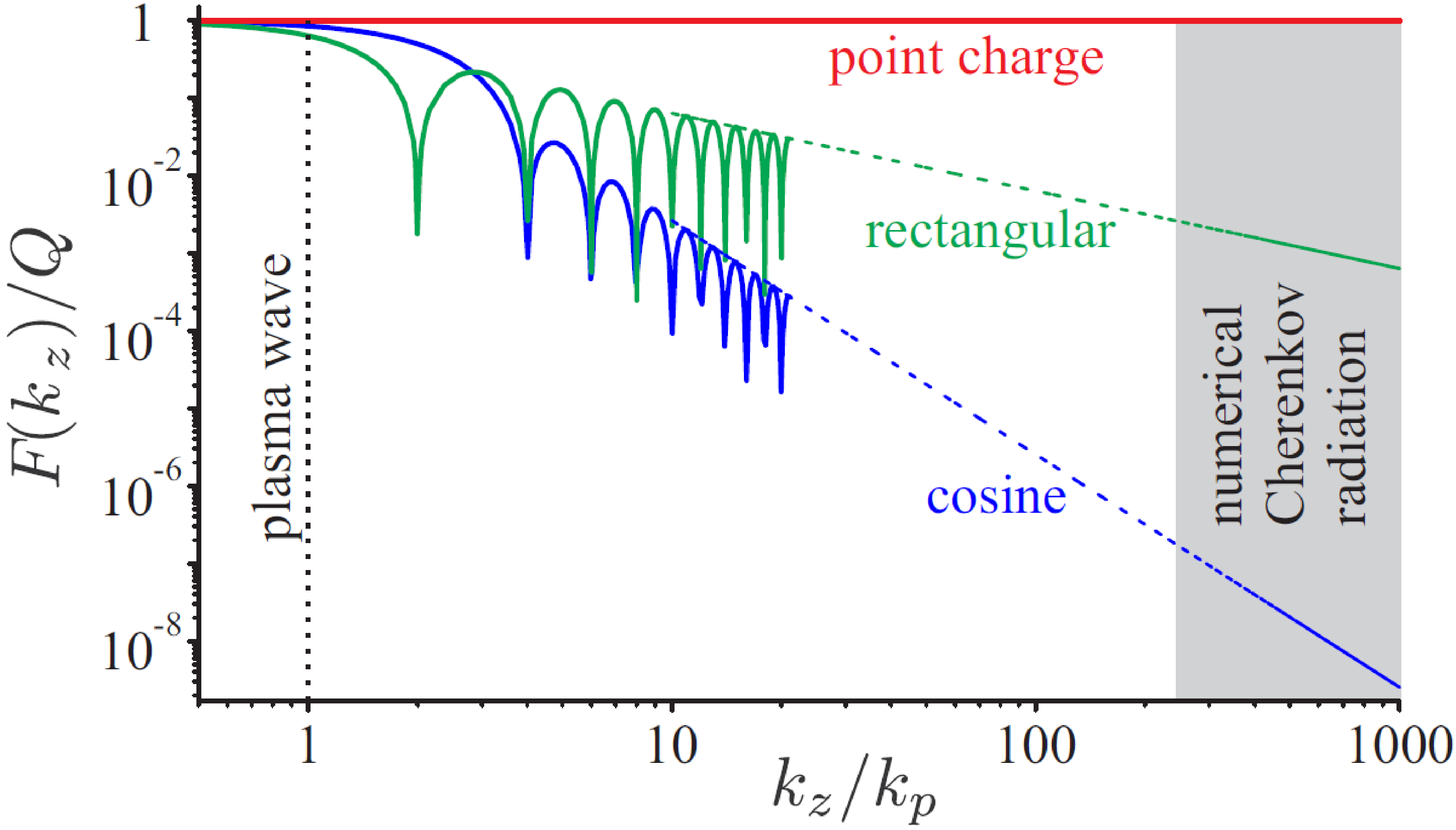
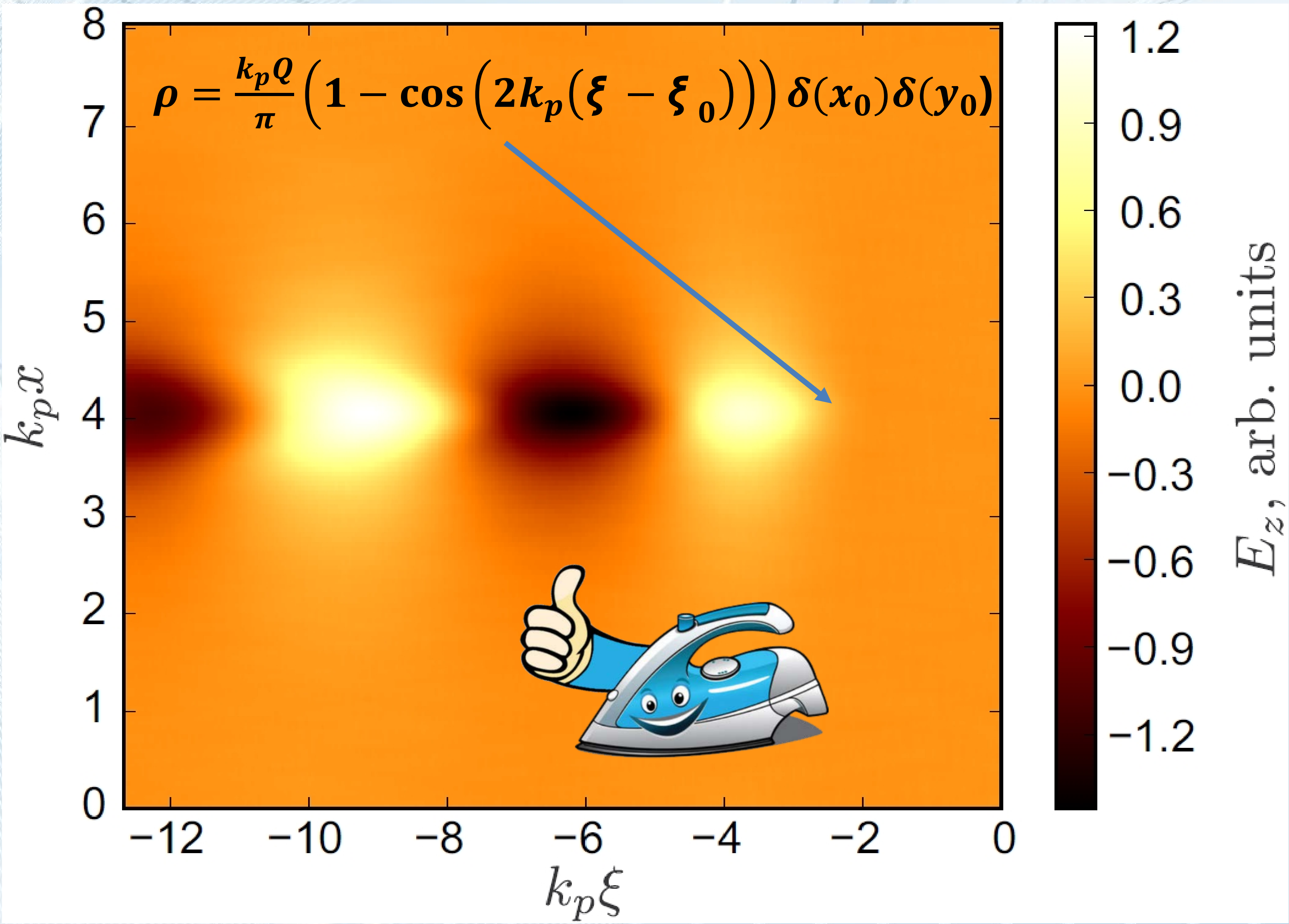
Fourier spectrum

The amplitude of the wave ~ amplitude of Fourier harmonic



Cosine-like rod



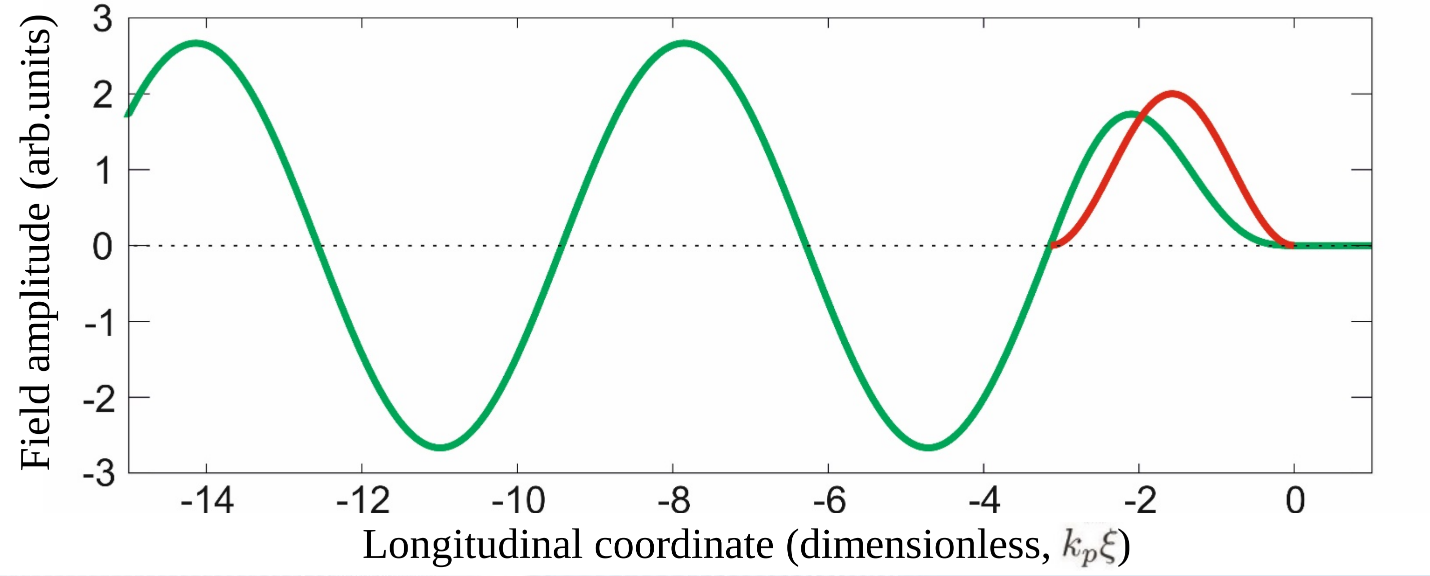
frame moves to the right with the speed of light

The field behind a single rod

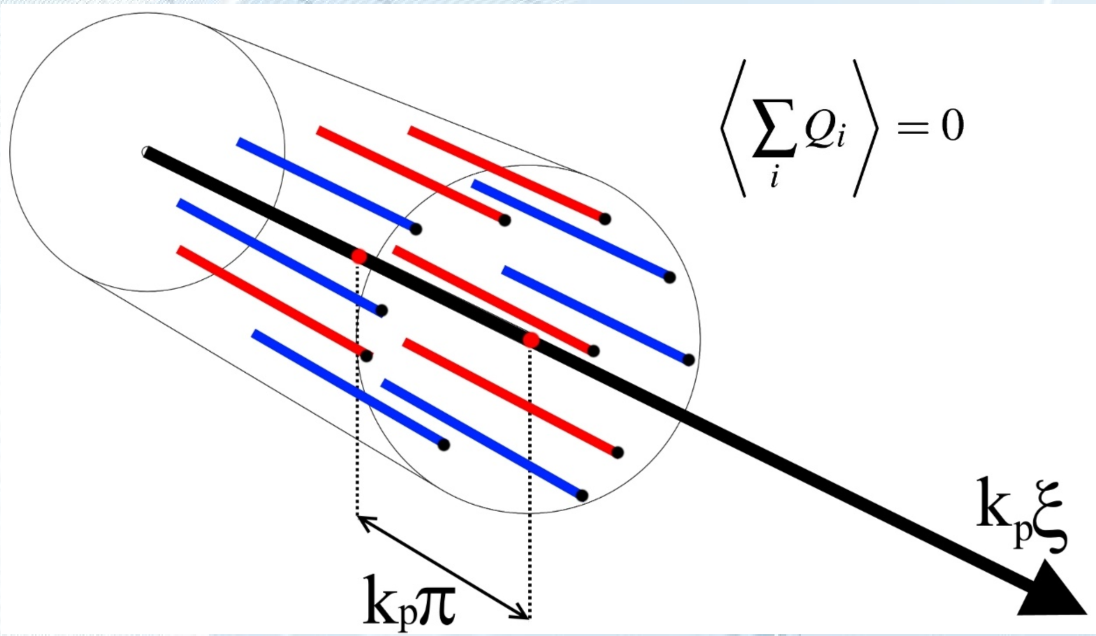
$$E_z(\vec{r}_\perp,\xi) = 2{k_p}^2 \int_\xi^\infty d\xi' \int d\vec{r}'_\perp \rho(\vec{r}'_\perp,\xi') \times$$
$$\times K_0\left(k_p|\vec{r}_\perp - \vec{r}'_\perp|\right) \cos\big(k_p(\xi-\xi')\big) =$$
$$= \boxed{\frac{2k_p^2 Q}{\pi} K_0(k_p r) G(\xi-\xi_0)}$$

T. Katsouleas, et al., Part. Accel. 22, 81 (1987).

$$G(\xi) = \begin{cases} -\frac{8}{3}\sin(\xi), & k_p\xi < -\pi, \\ \frac{2}{3}\big(\sin(2\xi) - 2\sin(\xi)\big), & -\pi < k_p\xi < 0, \\ 0, & \xi > 0. \end{cases}$$



Averaging: N rods



Total charge is neutralized.

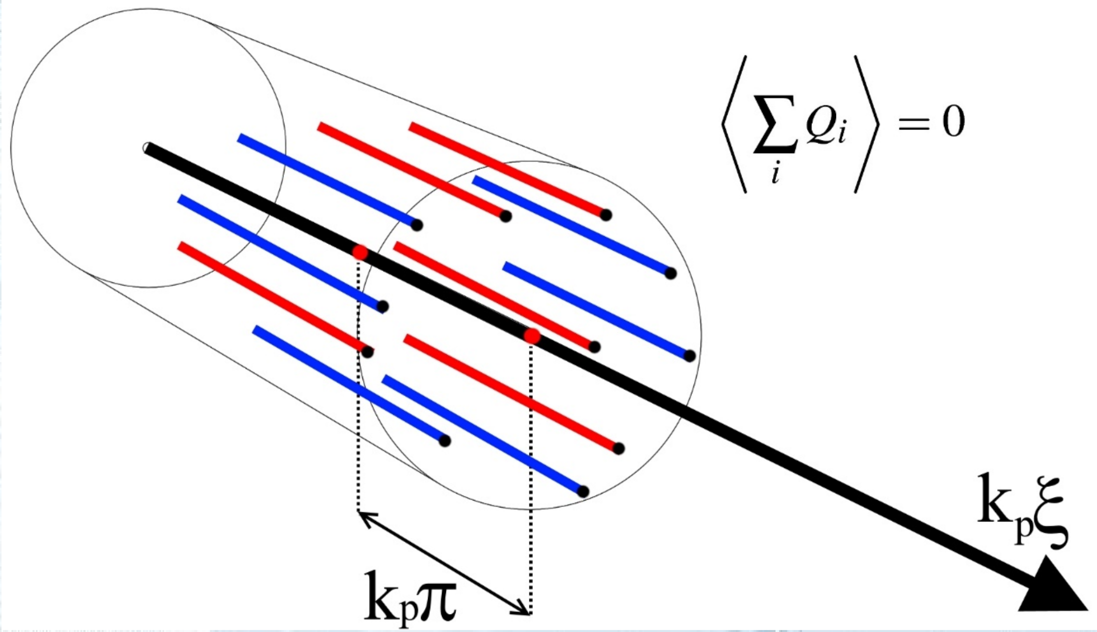
In simulation the sign of the charge of each rod must be chosen **randomly!**

Otherwise:

not all the phases are equiprobable!

1. The averaging of squared field over the uniform distribution in a cylindrical domain (radius R, length $k_p\pi$).
2. Multiplying by N (number of rods).
3. Limit of $R \to \infty$.

Averaging: N rods



Total charge is neutralized.

In simulation the sign of the charge of each rod must be chosen **randomly!**

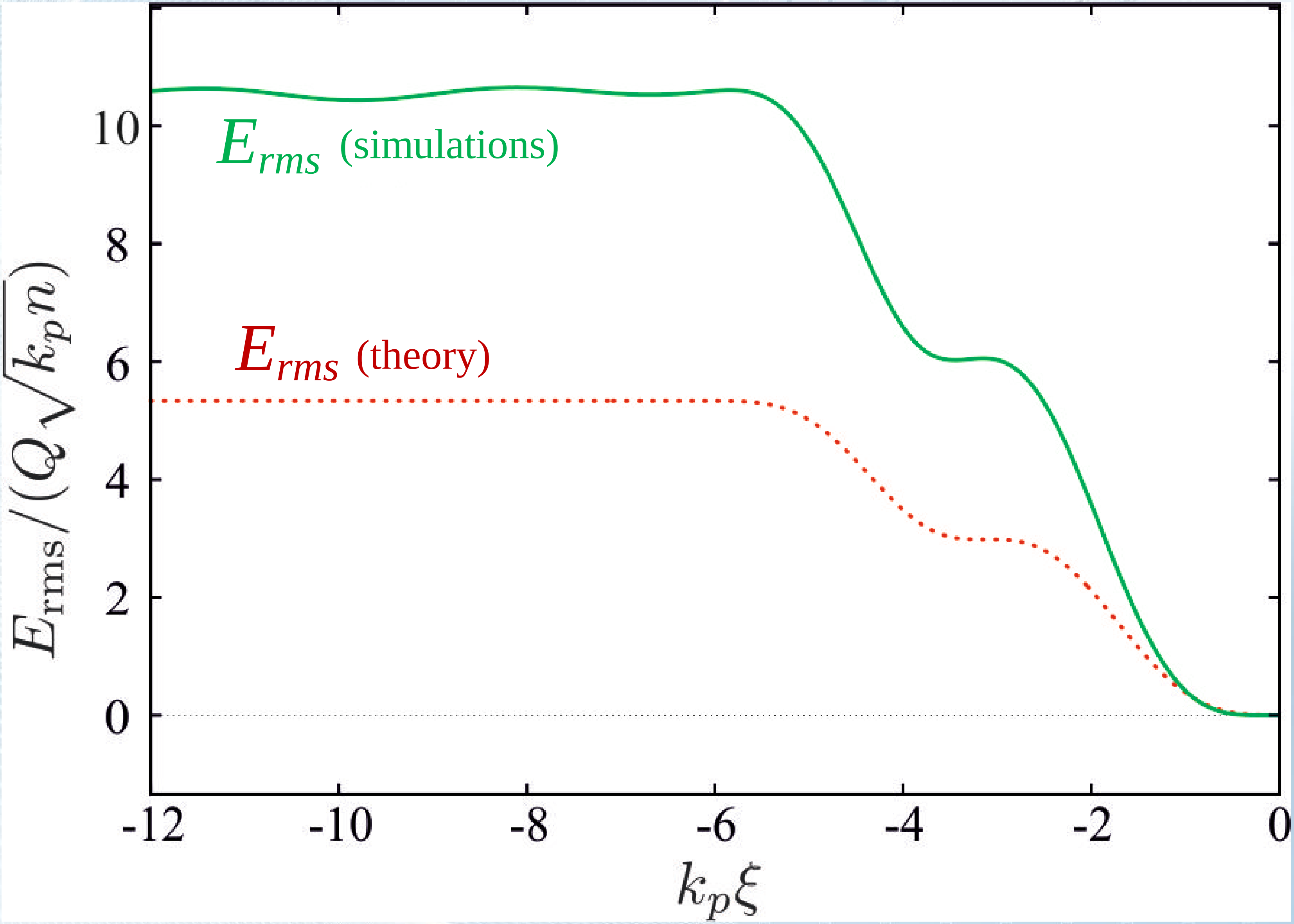
Otherwise:

not all the phases are equiprobable!

$$E_{rms}^2(behind) = N\langle E_z^2\rangle = \frac{256 k_p Q^2 n}{9}$$

2n – the average density of rods in a domain.

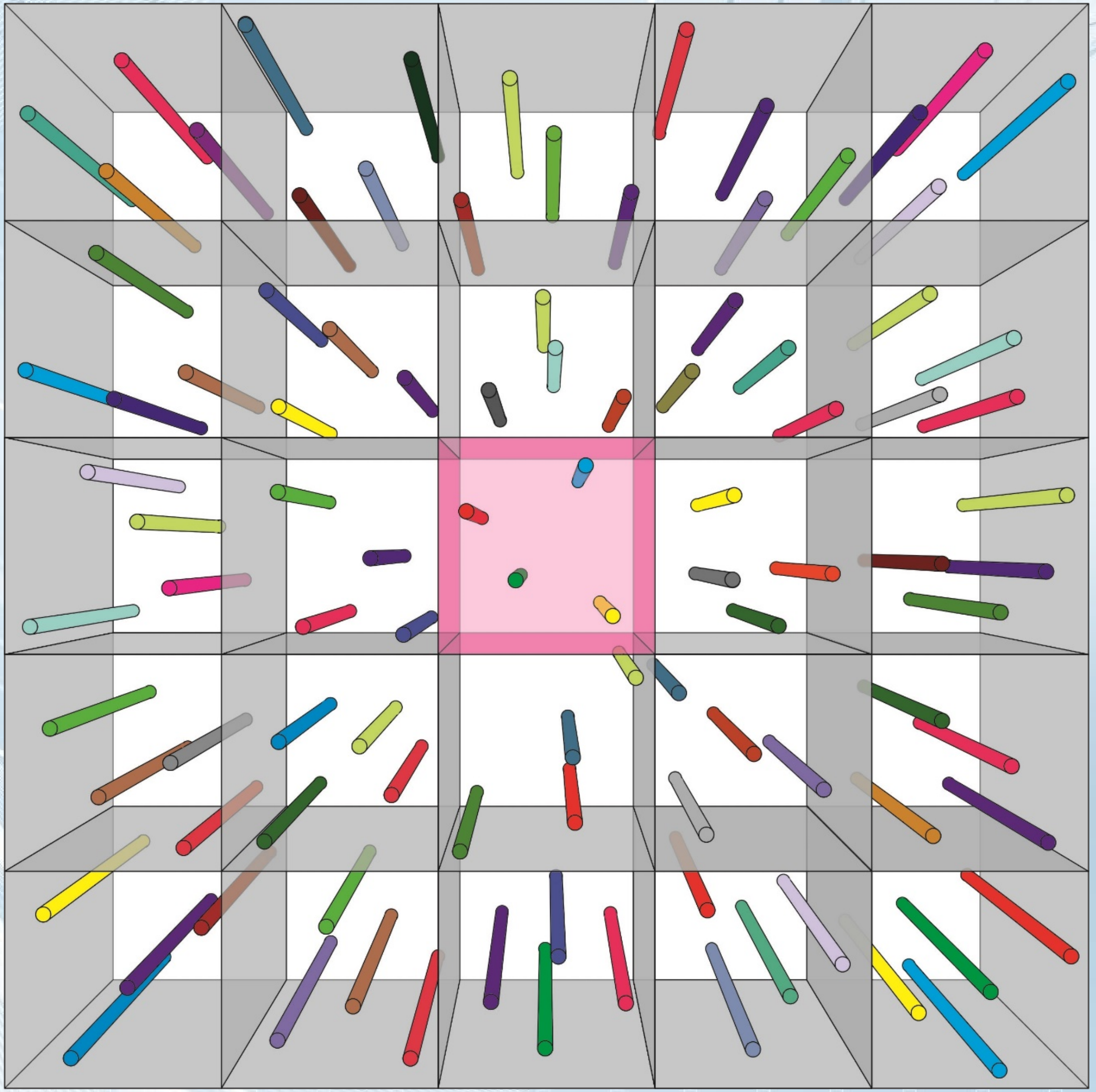
Comparison of theory and simulations



Limitations of the simulation domain

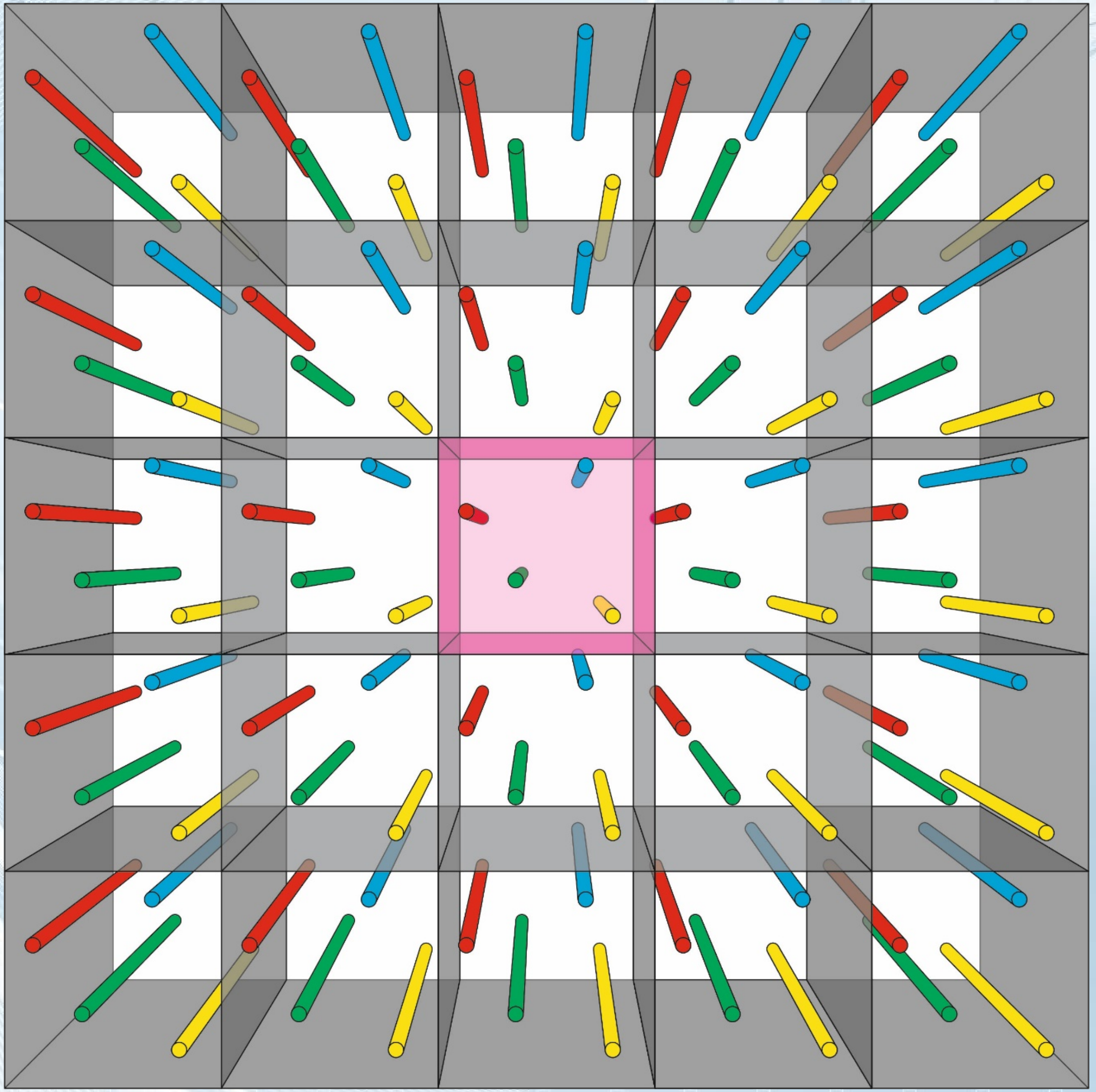
Infinite space – random position **everywhere**

Limitations of the simulation domain



Numerical simulations: a domain + **boundary conditions**

Limitations of the simulation domain

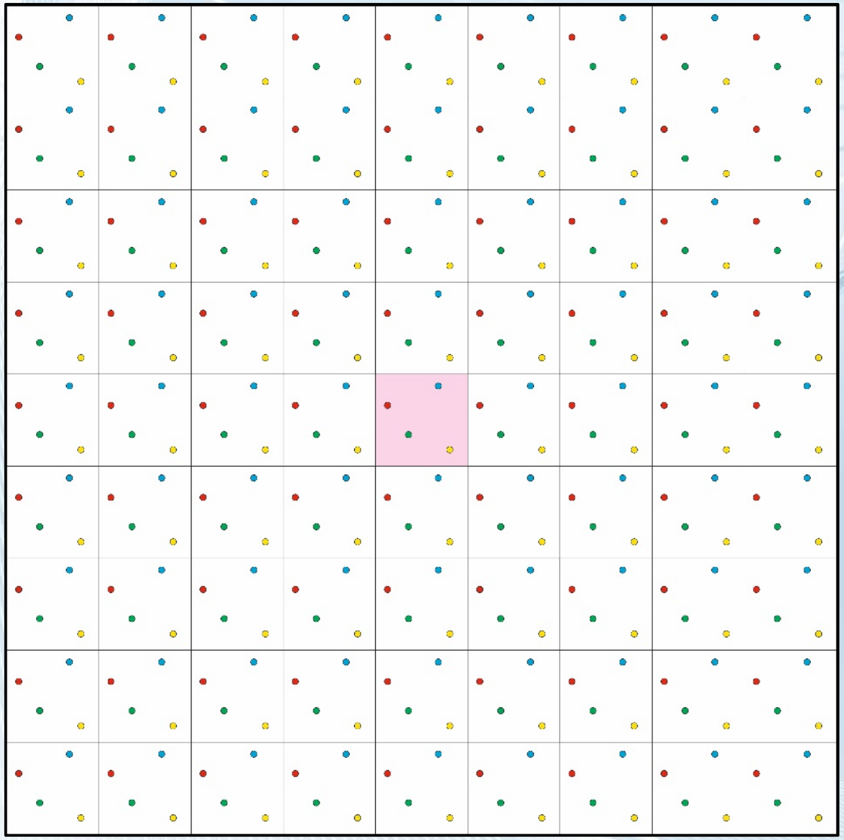


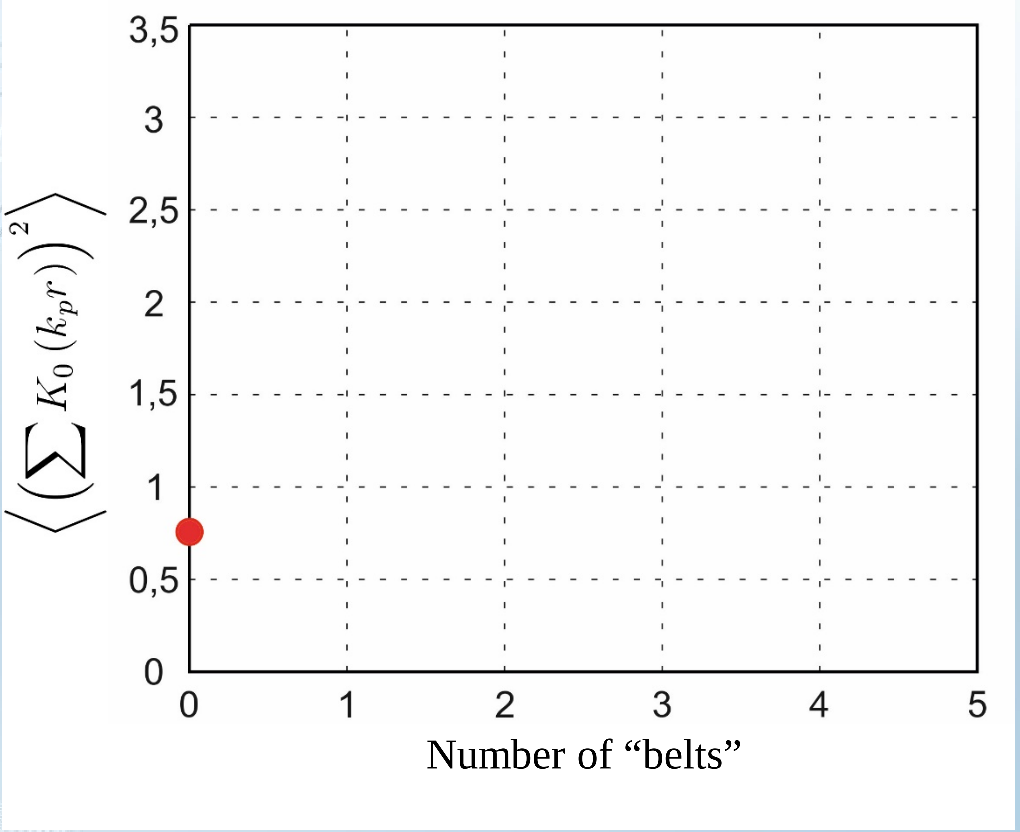
Periodical boundary conditions – random **only inside the frame**

Averaging over N rods, ver.2.0

Firslty: to sum fields of the "same" rods, secondly: to average

$$E_z(\vec{r}_\perp, \xi) = \frac{2k_p^2 Q}{\pi} K_0(k_p r) G(\xi - \xi_0) \quad \Rightarrow \quad \left\langle \left(\sum K_0\left(k_p r\right) \right)^2 \right\rangle$$





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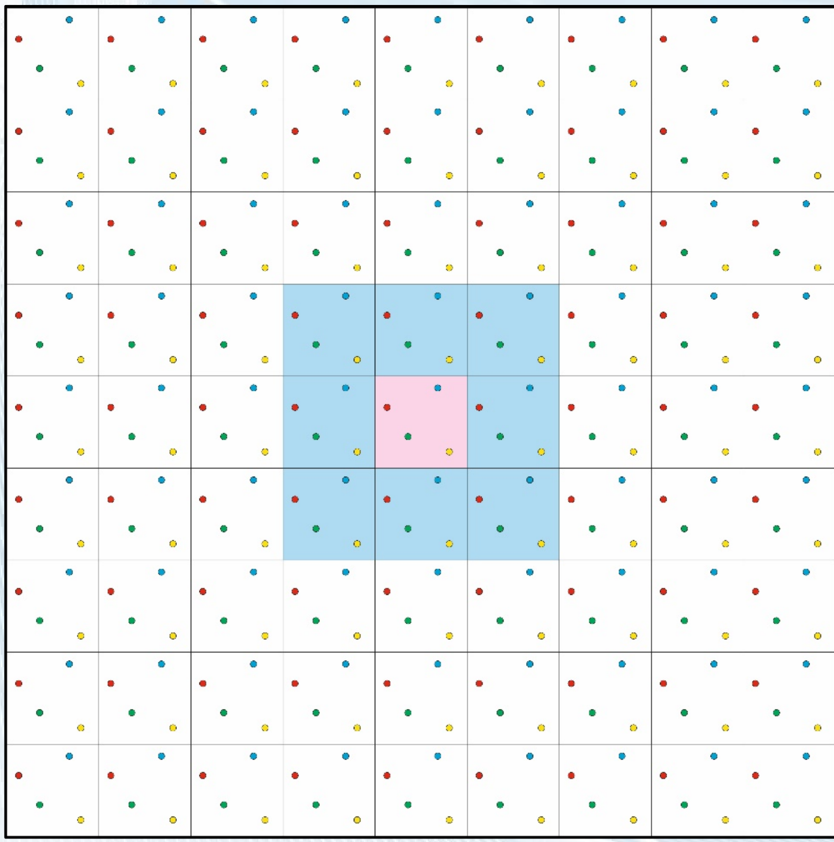
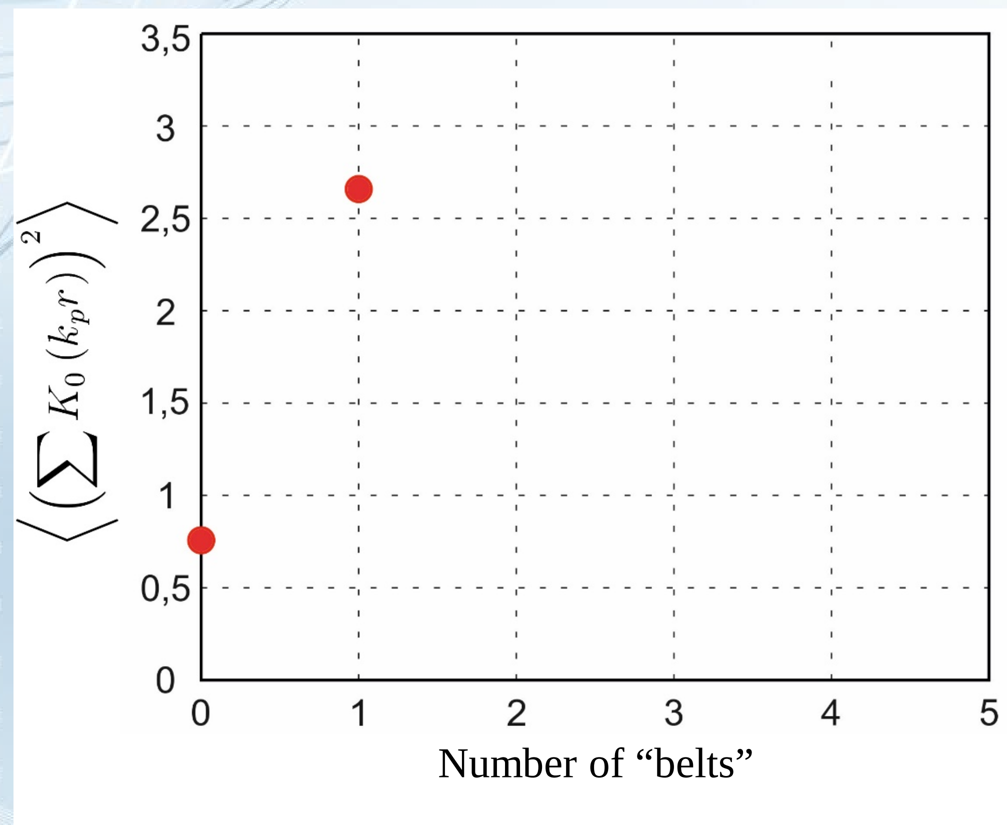
$$E_z(\vec{r}_\perp, \xi) = \frac{2k_p^2 Q}{\pi} K_0(k_p r) G(\xi - \xi_0)$$

=>

$$\left\langle \left(\sum K_0 \left(k_p r \right) \right)^2 \right\rangle$$





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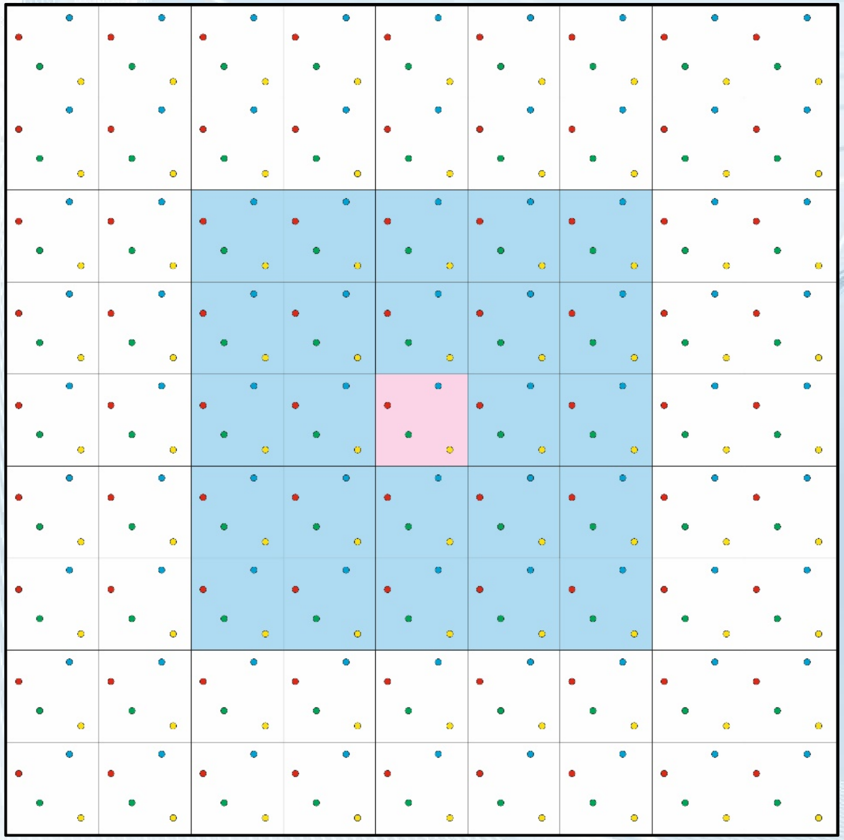

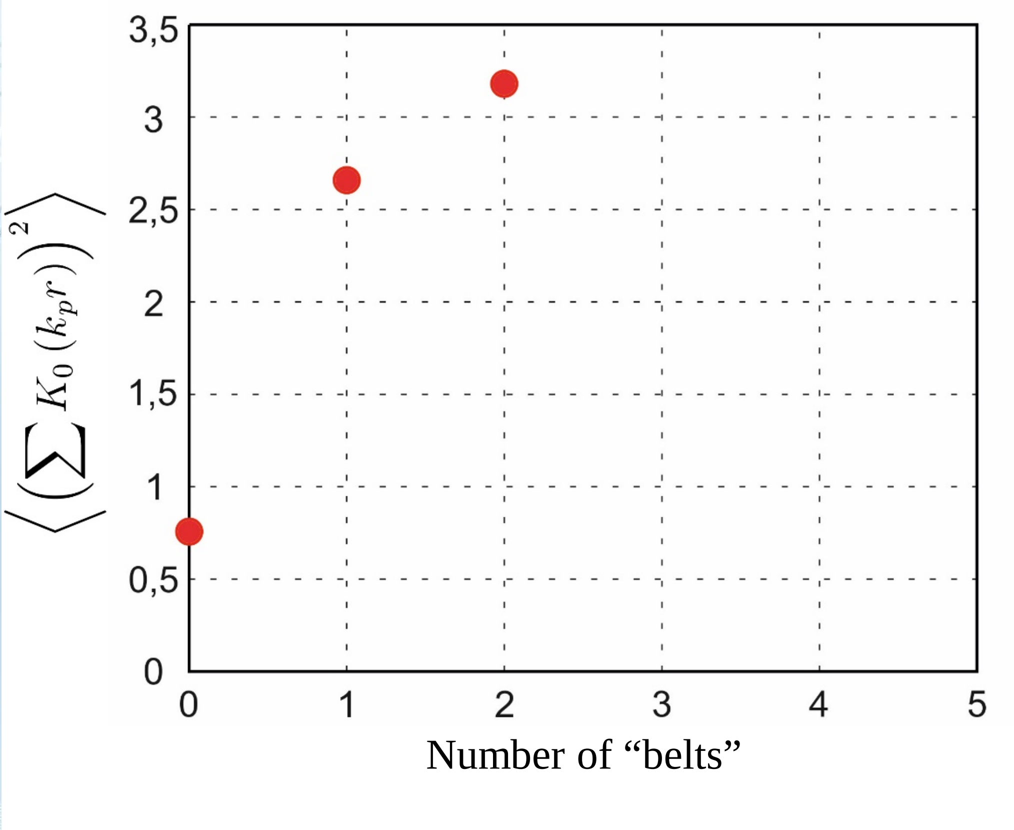

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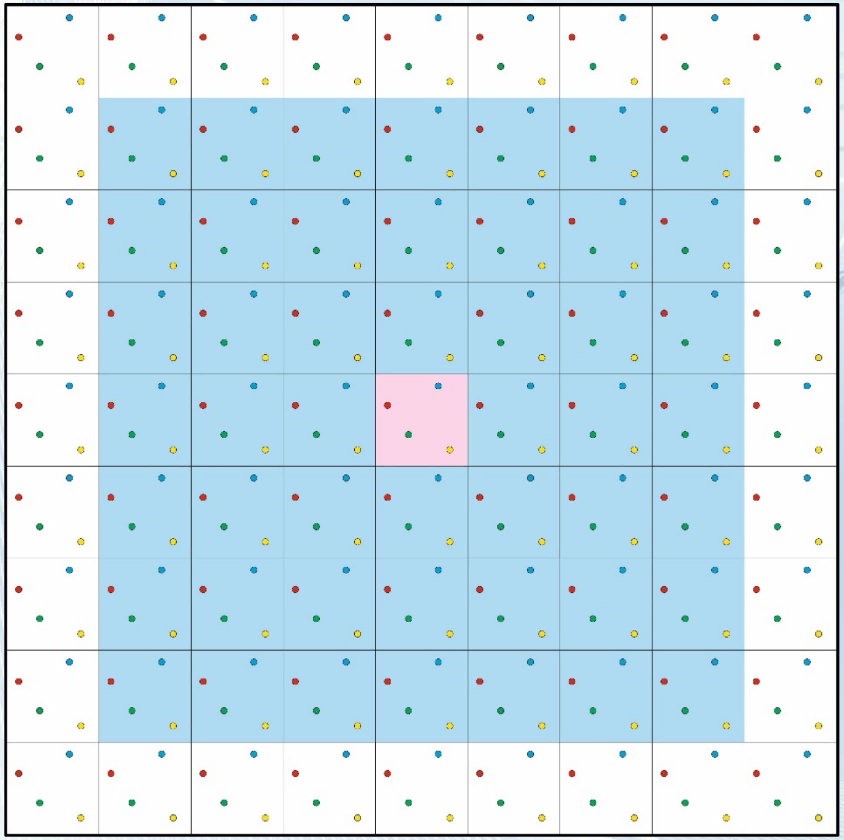

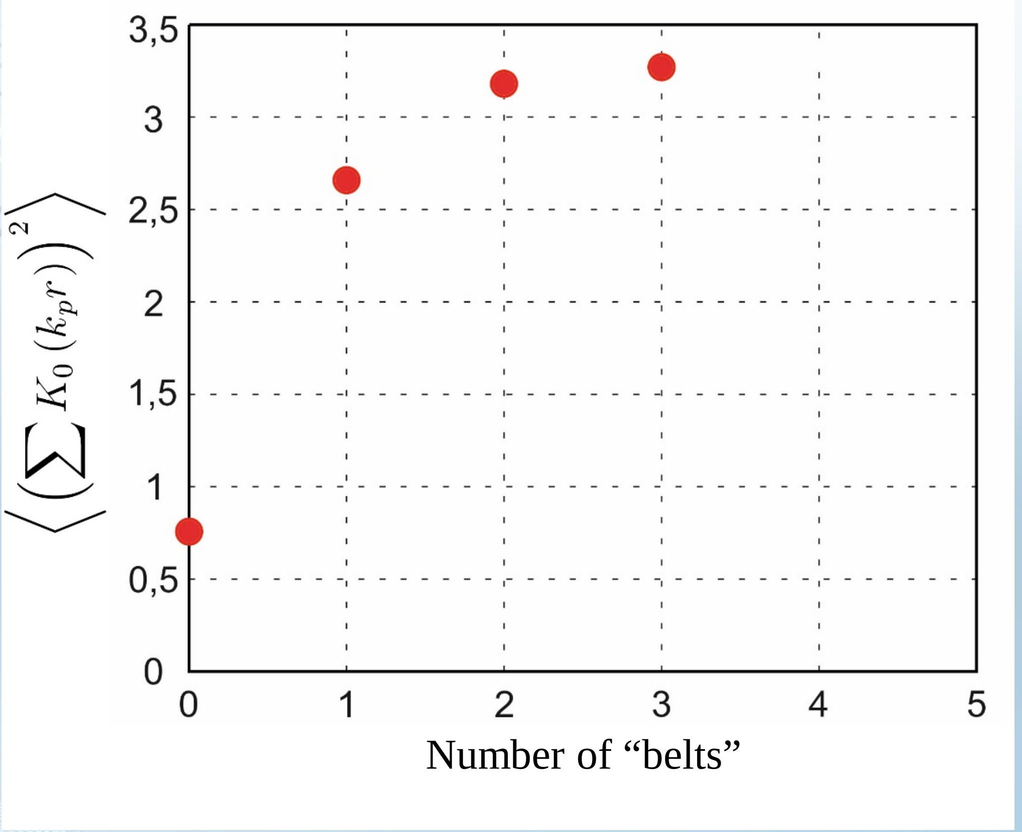

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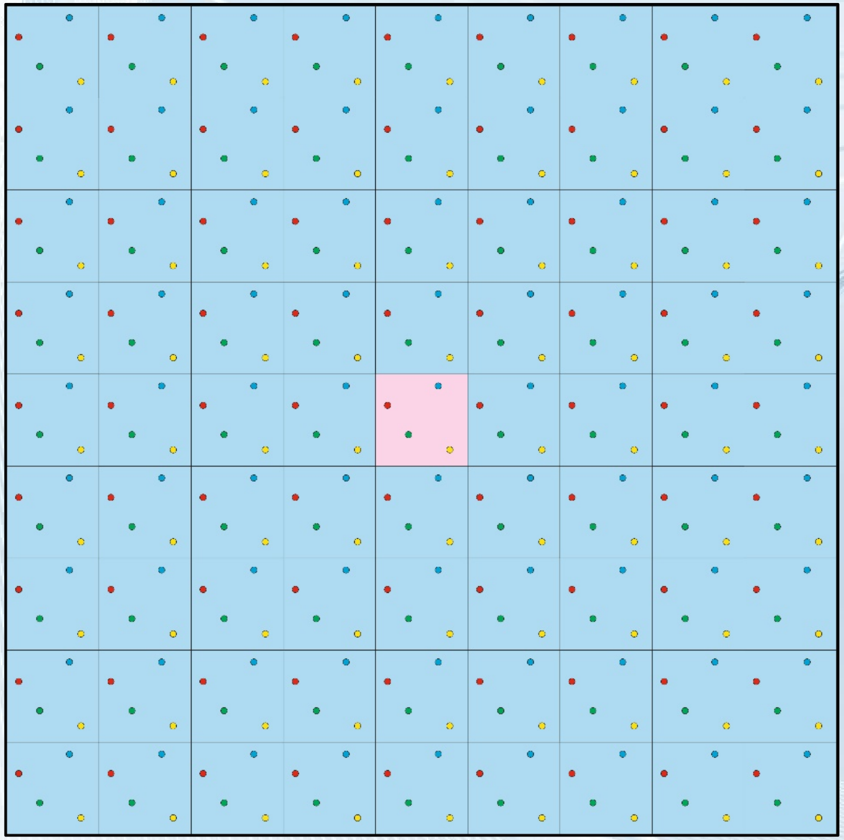

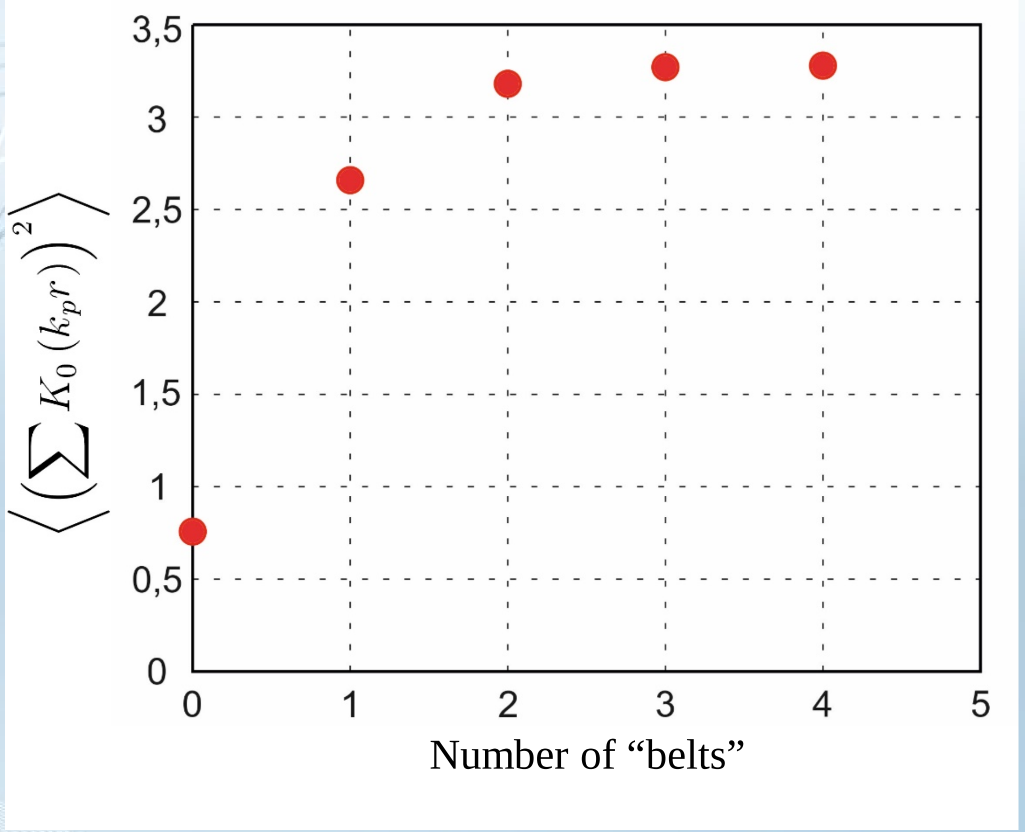

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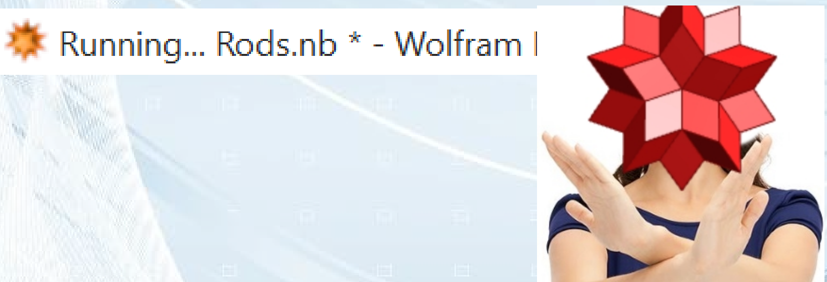
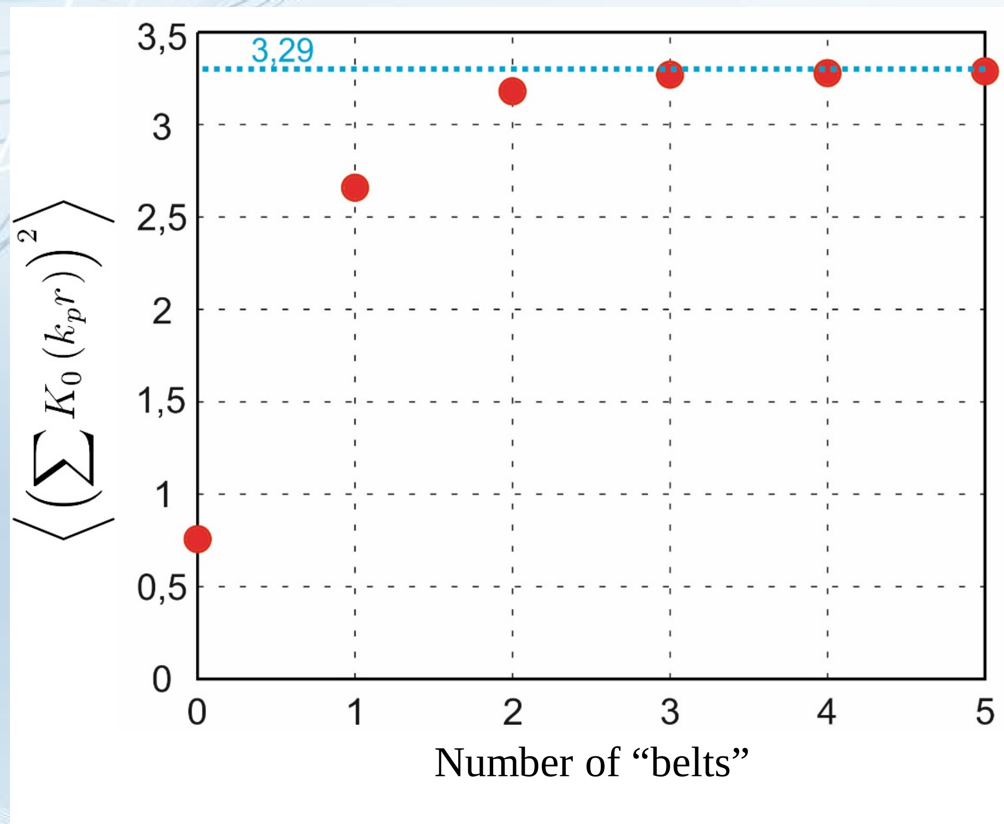


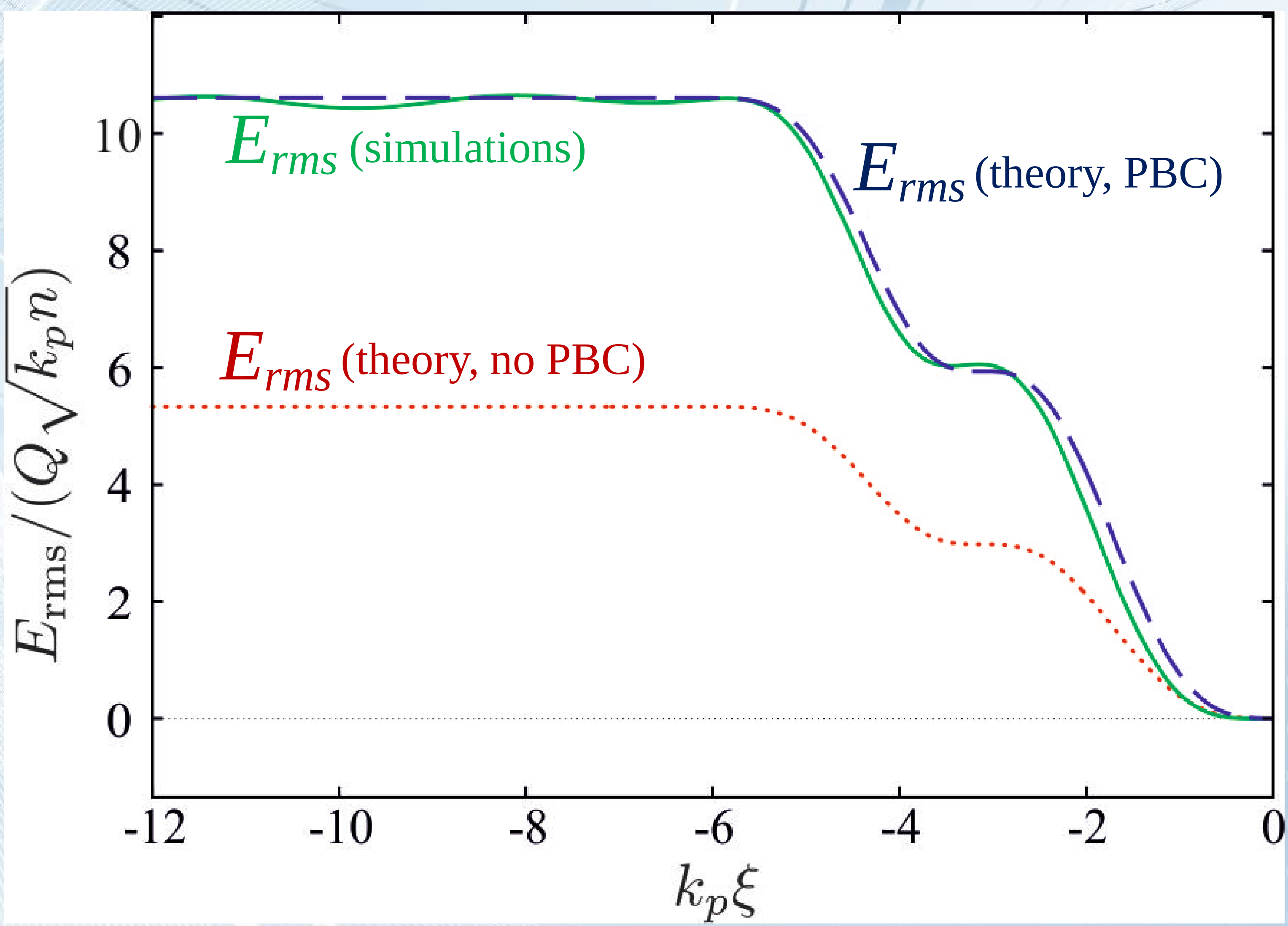
Modified Bessel function of the 2nd kind is rapidly decreasing

⇓

Radial averaging over uniform distribution is substituted by the **coefficient**



Comparison of theory and simulations, ver.2.0



Rods' «heads» are generated at [-π;0], frame moves to the right with the speed of light

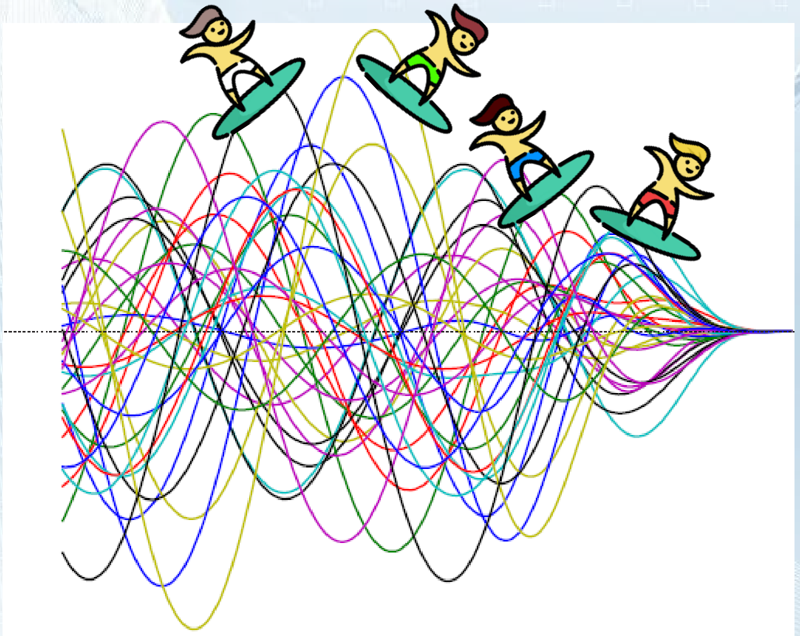
Summary

We have developed a new method of generation the controllable wakefield noise in plasmas.

For more detailed information:

http://arxiv.org/abs/1706.00594

Thank you for your attention!



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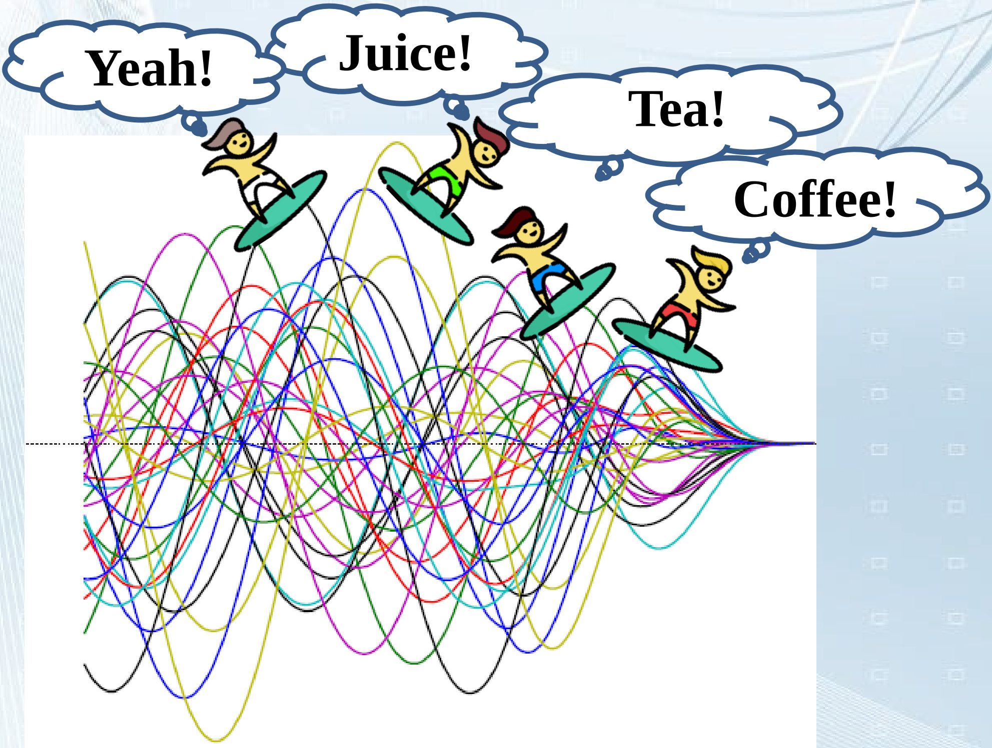

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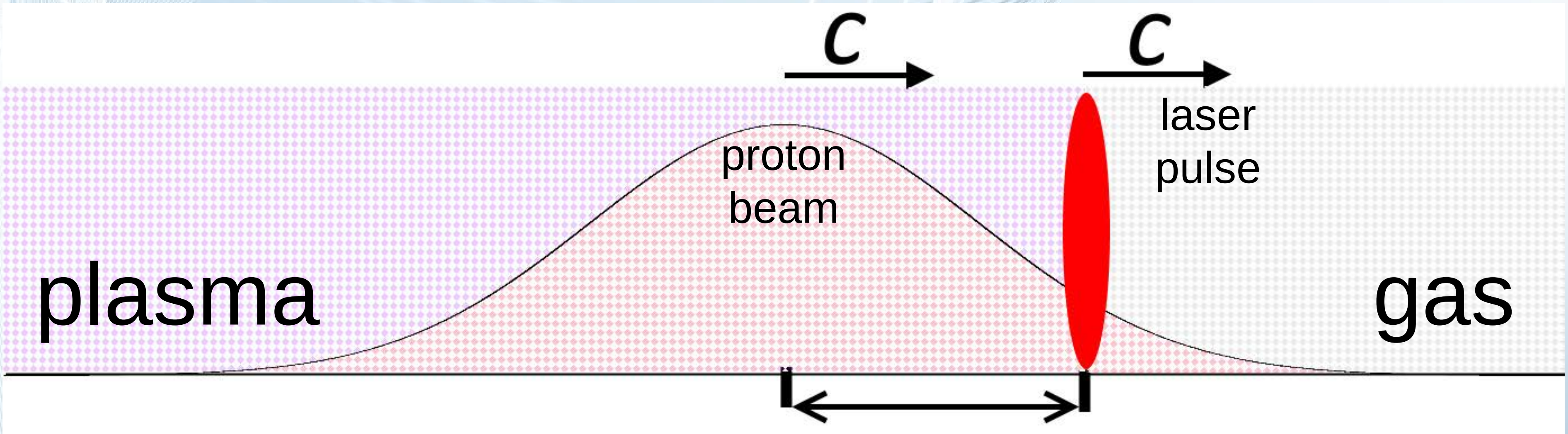
Example 1

1) SMI of the proton beam



Self-modulation instability ⇔ **Hose instability**

2) Filamentation

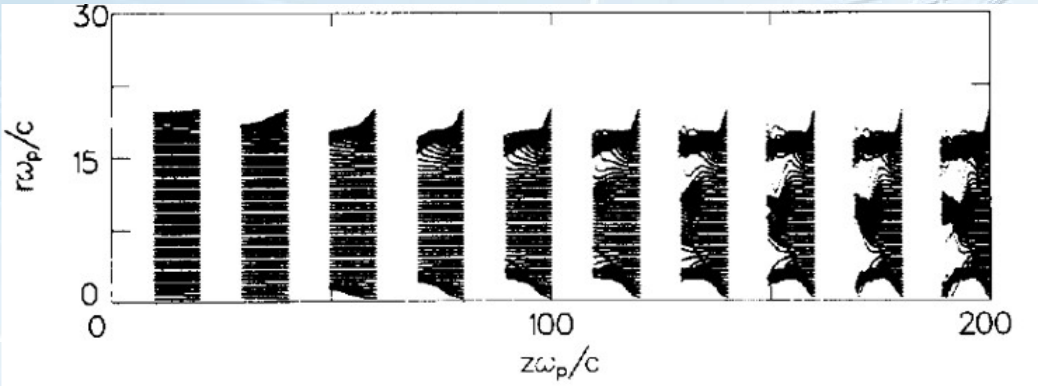


FIG. 4. Snapshots of the beam particles for a simulation with a large radius beam ($\omega_p a/c = 20$). Self-consistent pinching is less pronounced than in the simulation results shown in Fig. 1. A strong filamentation instability is seen in the later snapshots.

Phys. Fluids, Vol. 30, No. 1, January 1987 R. Keinigs and M. E. Jones