Uniwersytet Wrocławski

Transfer matrix and transfer function analysis for grating-type dielectric laser accelerators

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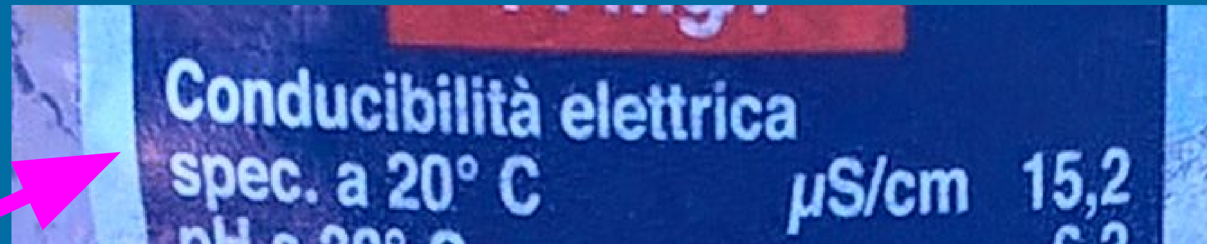
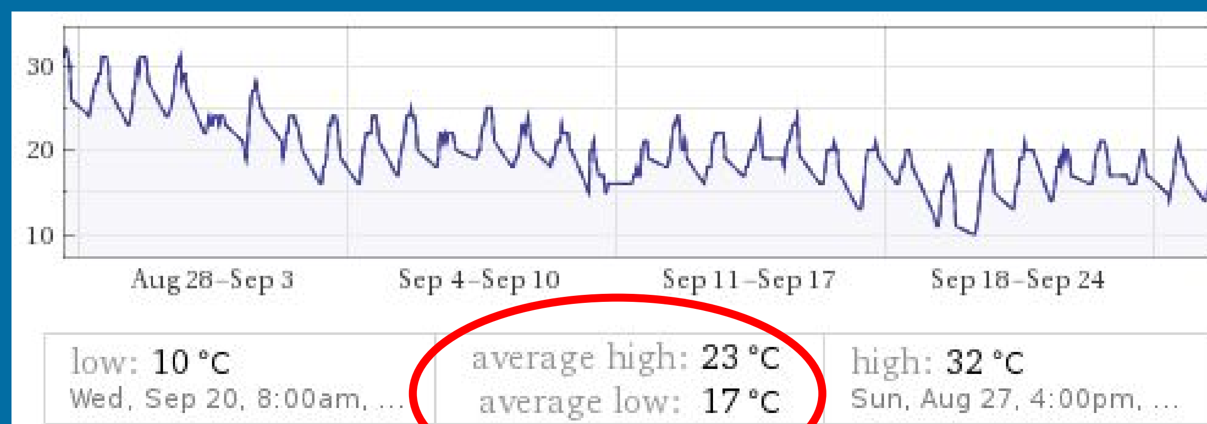
Institute of Experimental Physics
University of Wrocław

EAAC 2017, Elba
Working group 3

Water for physicists

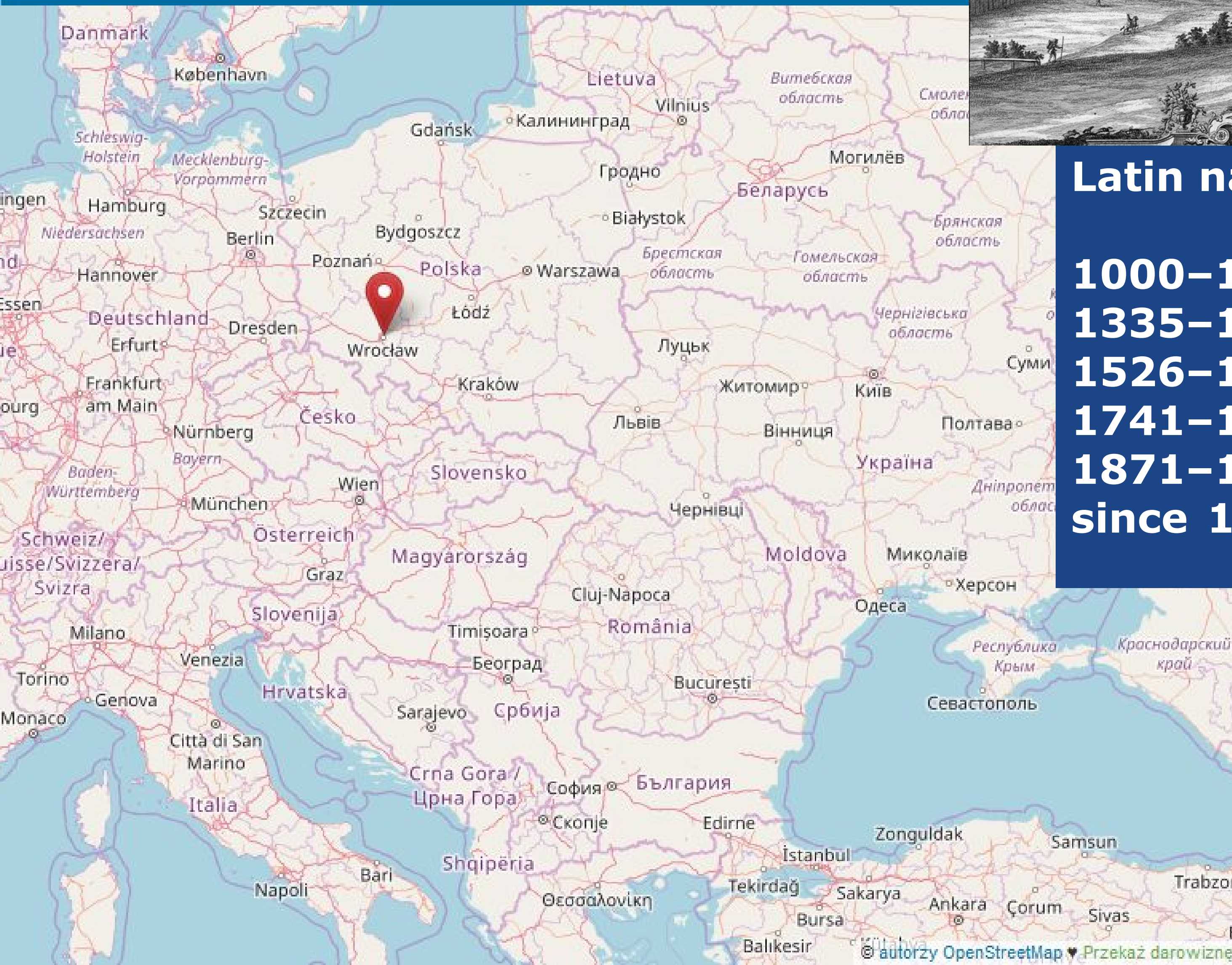


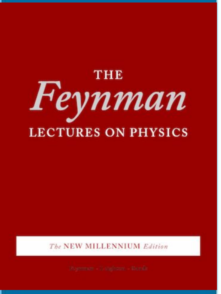


Wroclaw

Latin name: Vratislavia

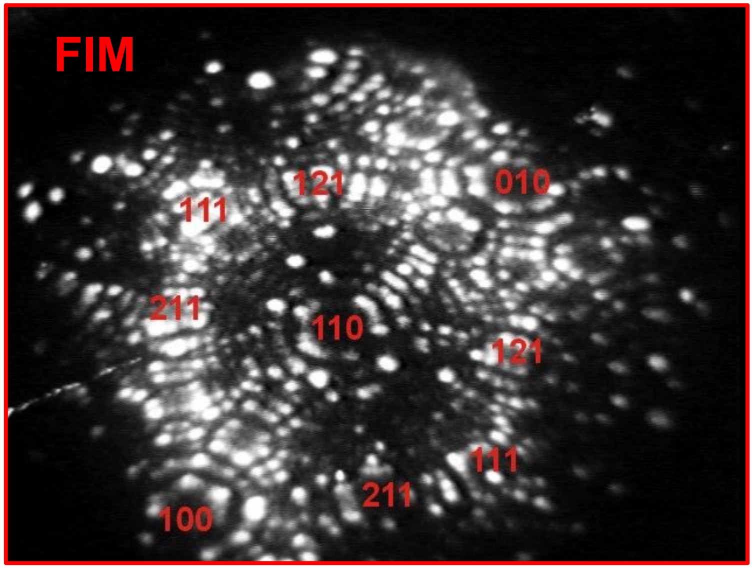
1000–1335 Wrotizla
1335–1526 Wretslaw
1526–1741 Presslaw
1741–1871 Bresslau
1871–1945 Breslau
since 1945 Wrocław

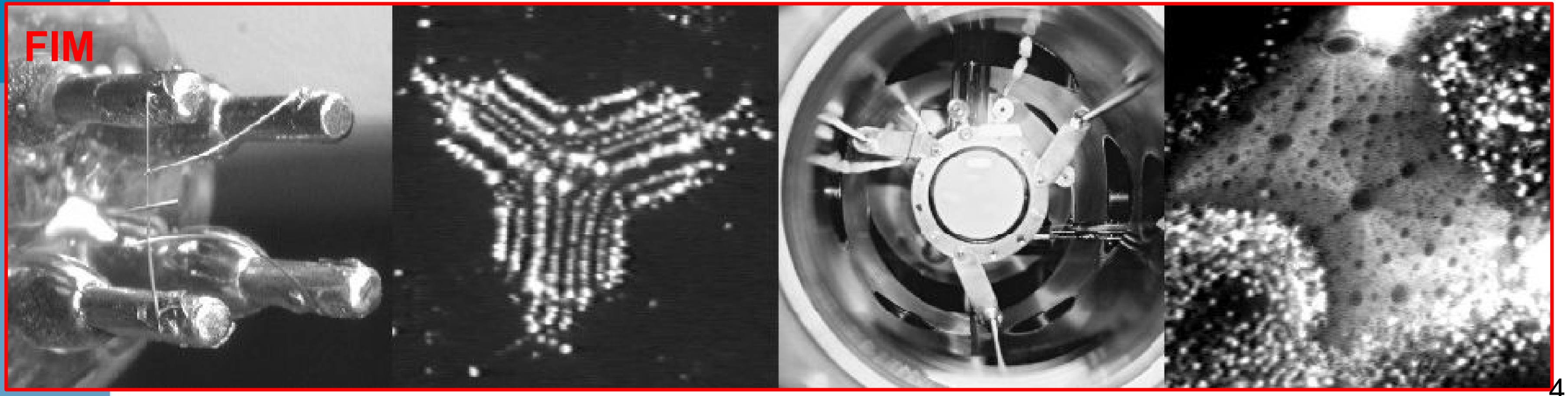




physics and physical chemistry of condensed matter, surface & bulk

Field Ion Microscopy (until 2011)
Scanning Tunneling Microscopy
Atomic Force Microscopy
Low Energy Electron Diffraction
X-ray Photoelectron Spectroscopy
Mössbauer Spectroscopy
…





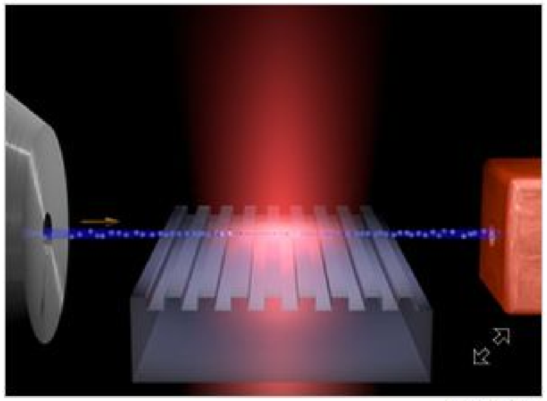
https://physics.aps.org/articles/v6/106

Physics - spotlighting exceptional research - APS Physics
https://physics.aps.org/

Focus: Accelerating Electrons with Light

September 27, 2013 • *Physics* 6, 106

In a new technique, light pulses accelerate electrons more efficiently than traditional accelerators.



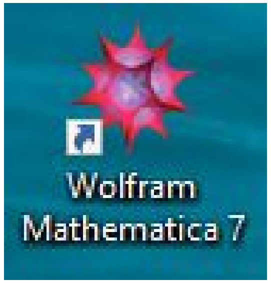
J. Breuer/MPI

Getting a Boost. Shining a laser pulse onto a grating accelerates electrons passing just over it, which could enable lab-sized accelerators and tunable x-ray sources.

Particles in today's accelerators are driven by radiofrequency waves, but using visible light should in principle allow for much smaller machines. In *Physical Review Letters* and *Nature* [1], two teams report the first experimental

My tools :-)

COMSOL



Electron dynamics in grating-type DLA – simple description

- transfer matrix R
- transfer function R

PHYSICAL REVIEW ACCELERATORS AND BEAMS **20,** 081302 (2017)

Application of transfer matrix and transfer function analysis to grating-type dielectric laser accelerators: Ponderomotive focusing of electrons

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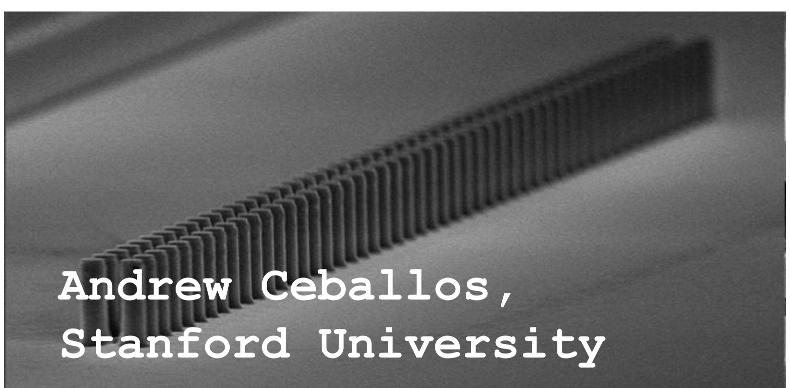
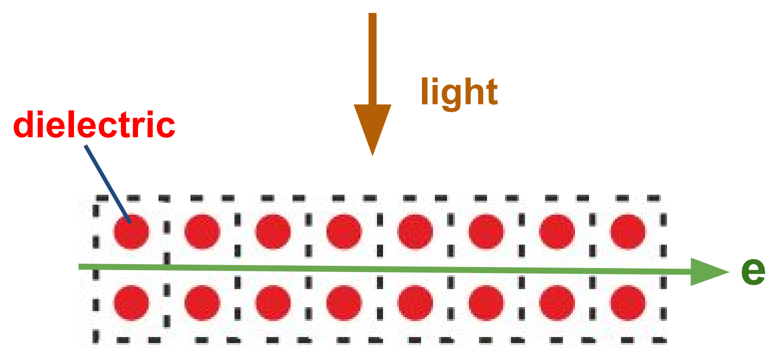
(Received 25 May 2017; published 31 August 2017)

- generalized accelerating/deflecting gradients Gacc/Gdefl

arxiv, proceedings

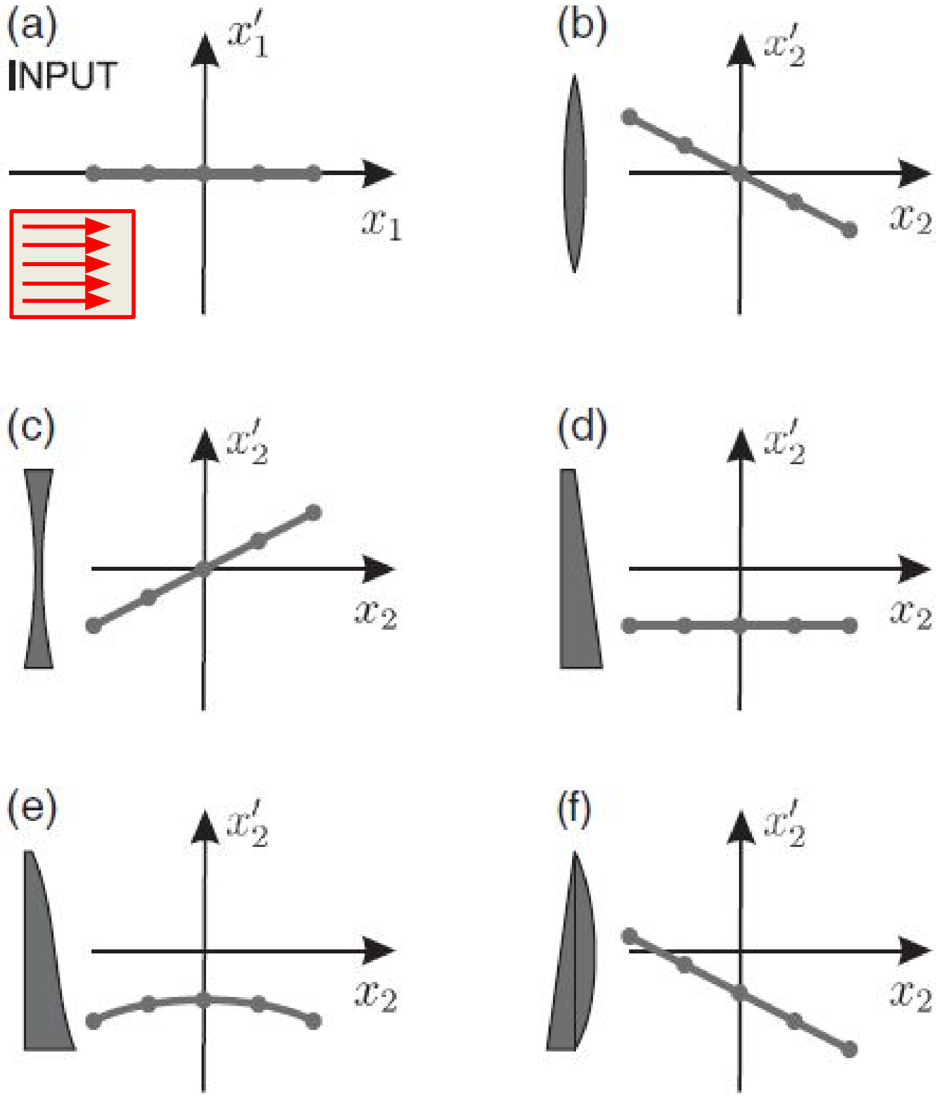
grating-type DLA





Transfer matrix (ABCD matrix) – classical optics

$$\begin{pmatrix} x_2 \\ x_2' \end{pmatrix} = \begin{pmatrix} R_{11} & R_{12} \\ R_{21} & R_{22} \end{pmatrix} \begin{pmatrix} x_1 \\ x_1' \end{pmatrix}$$



Transfer matrix *R* – conventional accelerators

$$\vec{X} = (x, x', y, y', s, \eta)^{\mathrm{T}}$$

[m, –, m, –, m, –]

$$\Delta\mathcal{E}/\mathcal{E}_0$$

$$\vec{X}_2 = \mathcal{R}\vec{X}_1$$

see *Beam by design* by
E. Hemsing, G. Stupakov,
D. Xiang, A. Zholents
Rev. Mod. Phys. 86, 897–941
(2014)

reference trajectory defines a coordinate system which is in general curvilinear, with the distance along the trajectory described by coordinate S (following the notation in Ref. [10]), and with orthogonal coordinates x, y describing the particle position in the transverse plane. The particle on the reference trajectory has *reference energy* $\mathcal{E}_0$ (corresponding to *reference momentum* p_0). The relative position of electrons on the reference trajectory with respect to the beam center is measured by s. The electron location in the six-dimensional phase space comoving with the electron beam is characterized by the vector $\vec{X} = (x, x', y, y', s, \eta)^{\mathrm{T}}$ [10], where $x' = dx/dS$ and $y' = dy/dS$ are the small angles of deflection from the reference trajectory, and $\eta = \Delta\mathcal{E}/\mathcal{E}_0$ is the relative energy deviation (other authors [4,5] use relative momentum deviation $\delta = \Delta p/p_0$ instead of η; in the ultrarelativistic limit $\eta = \delta$). Note that all

Transfer matrix – DLA

Ken Soong's PhD thesis, Stanford University, 2014

B.4 Accelerator Formalism

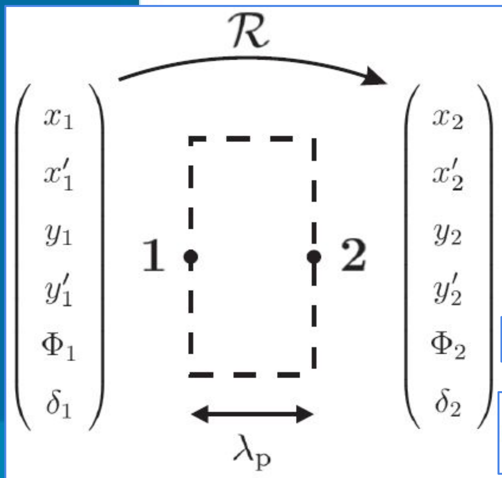
Using the particle tracking simulations described above, we can generate a transfer matrix to summarize the effects of our accelerator structures on the electron beam. This transfer matrix formalism is well established in the accelerator physics community and allows us to leverage other highly developed accelerator codes, which in turn provides us a straight-forward approach to modeling and understanding accelerator beamlines comprised of DLA devices. In accelerator formalism, this transfer matrix is typically denoted as the R matrix and takes the form,

$$\mathbf{X} = R \cdot \mathbf{X}_0 \,, \tag{B.3}$$

where $\mathbf{X}$ and $\mathbf{X}_0$ are the final and initial particle trajectories respectively. $\mathbf{X}$ and $\mathbf{X}_0$ are given explicitly by,

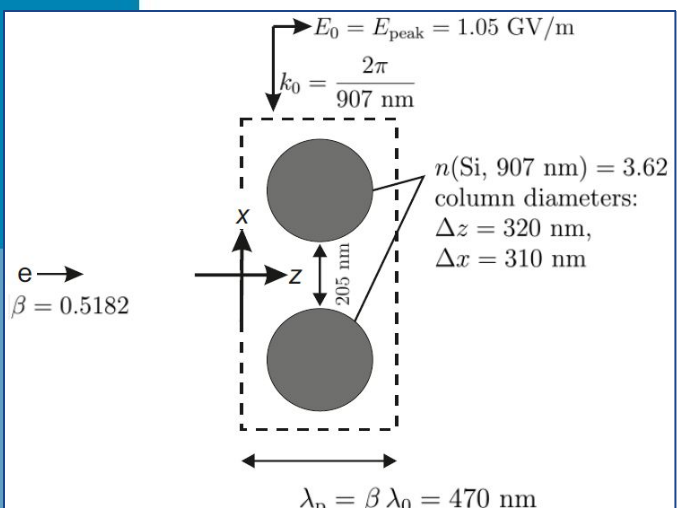
$$\mathbf{x}^{\mathrm{T}} = \left[x,\ x',\ y,\ y',\ z,\ \delta\right] \quad , \quad \mathbf{x}_0^{\mathrm{T}} = \left[x_0,\ x_0',\ y_0,\ y_0',\ z_0,\ \delta_0\right] \quad , \tag{B.4}$$

Transfer function – DLA – unit cell

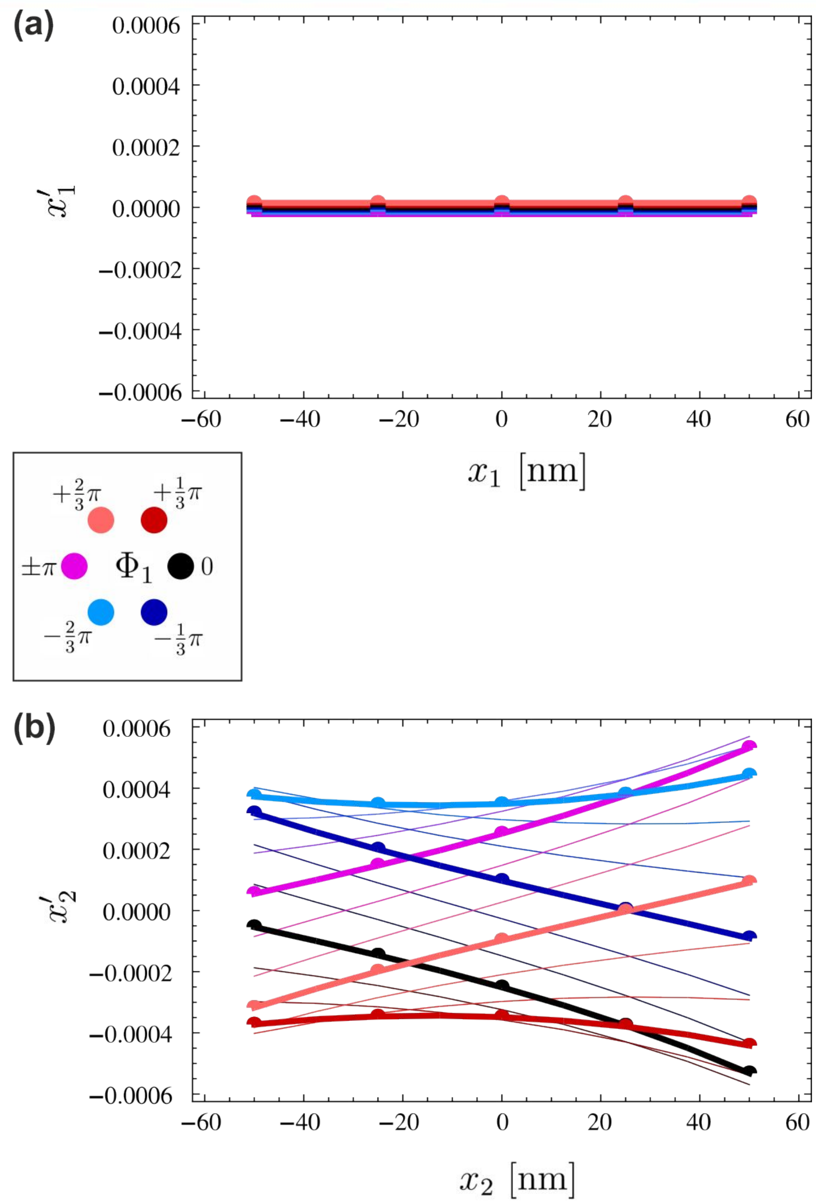


$\Phi_1 = \omega_0 t_1$

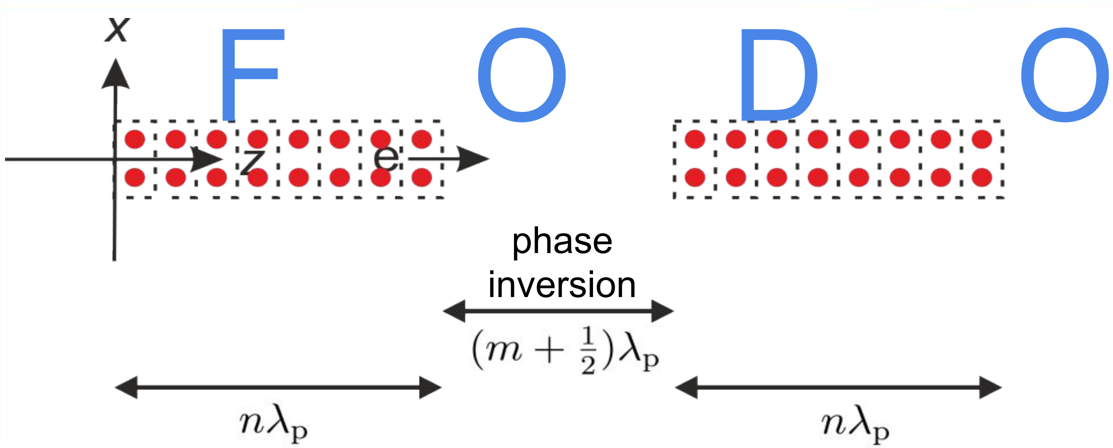
$\delta = \frac{p - p_0}{p_0}$

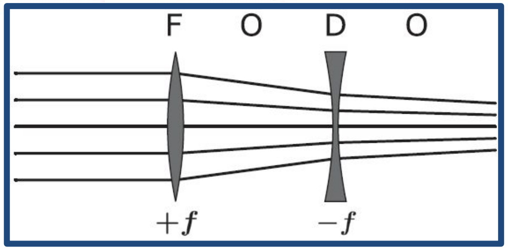


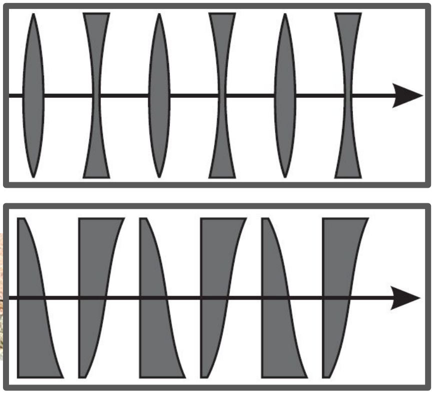
geometry from Leedle et al.,
Opt. Lett. 40, 4344 (2015).



Transfer function of a complex structure







$$\mathcal{R}_{\rm tot} = \mathcal{R}^n \mathcal{O}_{m+1/2} \mathcal{R}^n$$

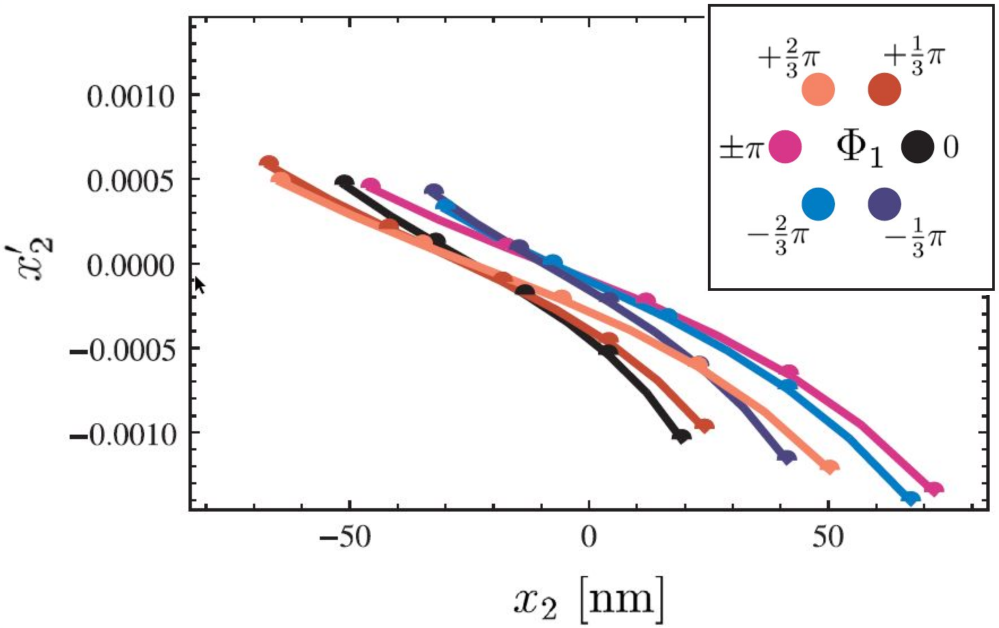


Image 2 Dr Rees demonstrates the production of a pattern on a Chladni plate using a violin bow. Image © the Whipple Museum (Wh.3446).

Ponderomotive focusing – sources, inspirations, circulation of ideas

Landau Lifshitz

Mechanics

A. Szczepkowicz, Application of transfer matrix... Ponderomotive focusing of electrons, PRAB 20, 081302 (2017)

Joel England's talk on Tuesday

J. Breuer, J. McNeur, and P. Hommelhoff, Dielectric laser acceleration of electrons in the vicinity of single and double grating structures—Theory and simulations, J. Phys. B **47**, 234004 (2014).

B. Naranjo, A. Valloni, S. Putterman, and J. B. Rosenzweig, Stable Charged-Particle Acceleration and Focusing in a Laser Accelerator Using Spatial Harmonics, Phys. Rev. Lett. **109**, 164803 (2012).

S. C. Hartman and J. B. Rosenzweig, Ponderomotive focusing in axisymmetric rf linacs, Phys. Rev. E **47**, 2031 (1993).

Thanks are owed to R. K. Cooper for introducing the subject of ponderomotive forces in accelerators to the authors, and to John Smolin, whose initial simulations

E. F. F. Chladni, *Entdeckungen über die Theorie des Klanges* (Weidmanns Erben und Reich, Leipzig, 1787).

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Accelerating and deflecting gradient

$$G_{\mathrm{acc}} = \frac{1}{\Lambda_g} \int_0^{\Lambda_g} E_z(z(t), t)\mathrm{d}z, \quad \textbf{(1)}$$

[V/m]

$$G_{\mathrm{defl}} = \frac{1}{\Lambda_g} \int_0^{\Lambda_g} (E_y(z(t), t) + vB_x(z(t), t))\mathrm{d}z. \quad \textbf{(2)}$$

[V/m]

from Leedle et al, Optica 2, 158 (2015)
see also Plettner, SLAC-PUB-12458 (2007)

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Generalization of accelerating and deflecting gradient

$$G_{\mathrm{acc}} = \frac{1}{\Lambda_g}\int_0^{\Lambda_g} E_z(z(t), t)\,\mathrm{d}z, \quad \textbf{(1)}$$

$$G_{\mathrm{defl}} = \frac{1}{\Lambda_g}\int_0^{\Lambda_g} (E_y(z(t), t) + vB_x(z(t), t))\,\mathrm{d}z. \quad \textbf{(2)}$$

from Leedle et al, Optica 2, 158 (2015)
see also Plettner, SLAC-PUB-12458 (2007)

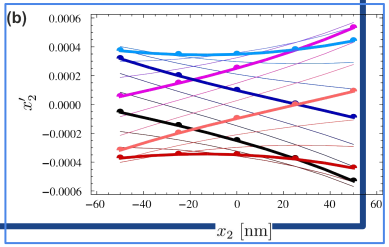
Don't kill the phase information, leave Gacc and Gdefl complex.

almost full description of the DLA unit cell

$$\widetilde{G}_x(x,y) = \int_0^{\lambda_{\mathrm{p}}} \left(\widetilde{E}_x(x,y,z) - c\beta_0 \widetilde{B}_y(x,y,z)\right) e^{ik_{\mathrm{p}}z}\, dz$$

$$\widetilde{G}_y(x,y) = \int_0^{\lambda_{\mathrm{p}}} \left(\widetilde{E}_y(x,y,z) + c\beta_0 \widetilde{B}_x(x,y,z)\right) e^{ik_{\mathrm{p}}z}\, dz$$

$$\widetilde{G}_z(x,y) = \int_0^{\lambda_{\mathrm{p}}} \widetilde{E}_z(x,y,z)\, e^{ik_{\mathrm{p}}z}\, dz$$



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Construction of the transfer function from generalized accelerating and deflecting gradients

$$C = \frac{1}{\sqrt{{x'_1}^2 + {y'_1}^2 + 1}} \approx 1$$

$$x_2 = x_1 + x'_1\,\Delta z$$

$$x'_2 = x'_1 + \frac{\Delta p_x}{p_0}$$

$$y_2 = y_1 + y'_1\,\Delta z$$

$$y'_2 = y'_1 + \frac{\Delta p_y}{p_0}$$

$$\Phi_2 = \Phi_1 + k_0 \frac{\Delta z}{\beta_z}$$

$$\delta_2 = \delta_1 + \frac{\Delta p_z}{p_0}$$

$$\beta_z = \frac{p_0(1+\delta_1)}{\sqrt{p_0^2(1+\delta_1)^2 + m^2c^2}}$$

$$\Delta p_x = (-e)\,\Re(e^{i\Phi_1}\widetilde{G}_x)\,\frac{\Delta z}{\beta_0 c}$$

$$\Delta p_y = (-e)\,\Re(e^{i\Phi_1}\widetilde{G}_y)\,\frac{\Delta z}{\beta_0 c}$$

$$\Delta p_z = (-e)\,\Re(e^{i\Phi_1}\widetilde{G}_z)\,\frac{\Delta z}{\beta_0 c}$$

Particle transfer function expressed by generalized accelerating and deflecting gradients

$$x_2 = x_1 + x_1' \, \Delta z$$

$$x_2' = x_1' + \frac{(-e) \, \Re(e^{i\Phi_1} \widetilde{G}_x)}{\beta_0^2 \gamma_0 m c^2} \, \Delta z$$

$$y_2 = y_1 + y_1' \, \Delta z$$

$$y_2' = y_1' + \frac{(-e) \, \Re(e^{i\Phi_1} \widetilde{G}_y)}{\beta_0^2 \gamma_0 m c^2} \, \Delta z$$

$$\Phi_2 = \Phi_1 + k_0 \, \frac{\sqrt{p_0^2 (1 + \delta_1)^2 + m^2 c^2}}{p_0 (1 + \delta_1)} \Delta z$$

$$\delta_2 = \delta_1 + \frac{(-e) \, \Re(e^{i\Phi_1} \widetilde{G}_y)}{\beta_0^2 \gamma_0 m c^2} \, \Delta z$$

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Conclusions

Simple description of electron dynamics in DLA

1. transfer matrix not applicable
2. use transfer function R
3. generalize accelerating gradients
4. Gacc, Gdefl → R

Thank you:

Funding – University of Wrocław

Inspiration – Erlangen group and Stanford/SLAC groups

Discussions – Martin Kozak, Joshua McNeur, Peter Hommelhoff

andrzej.szczepkowicz@uwr.edu.pl

PHYS. REV. ACCEL. BEAMS **20**, 081302 (2017)

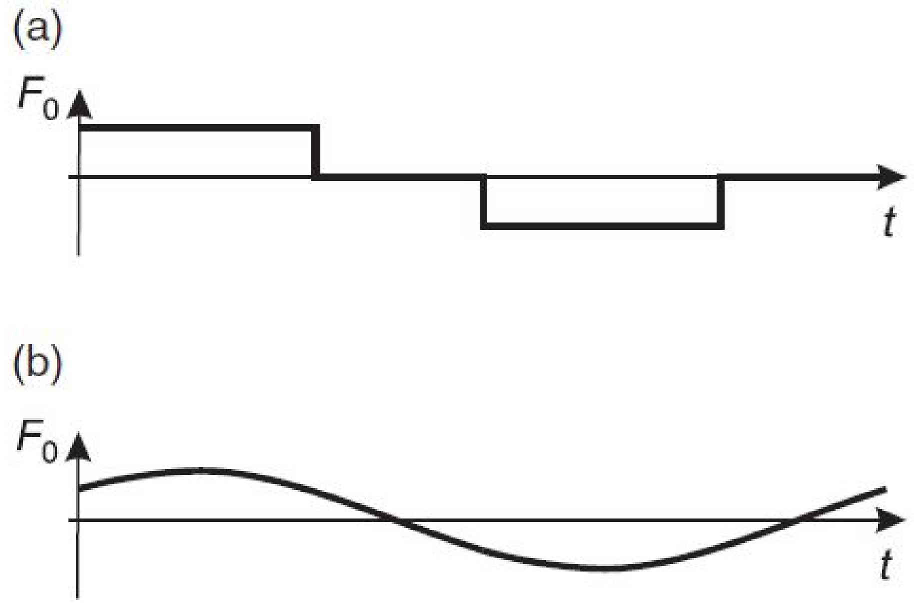

FIG. 11. (a) Time dependence of the focusing force in a FODO-like DLA cell (Fig. 6). (b) Harmonic oscillation leading to classical ponderomotive force.

$$\vec{F} = \vec{F}_0 \cos\omega t \quad \Rightarrow \quad \vec{F}_\mathrm{p} \sim -\frac{1}{\omega^2}\nabla(|\vec{F}_0|^2)$$

APPLICATION OF TRANSFER MATRIX AND …

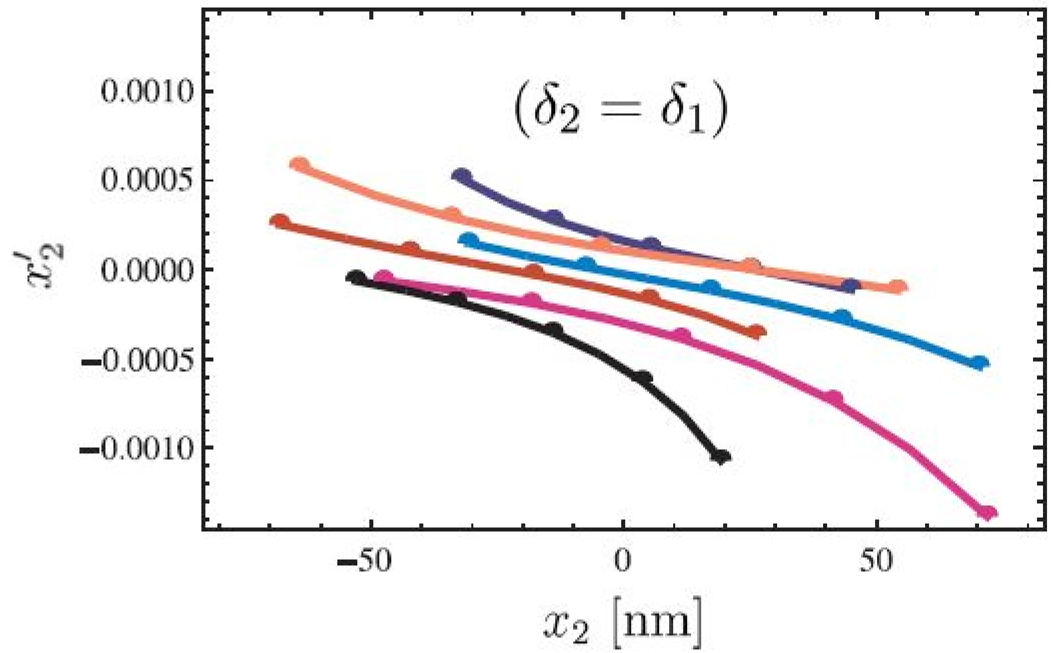

FIG. 10. The focusing properties of the $\mathcal{O}_{5+1/2}\mathcal{R}^8\mathcal{O}_{5+1/2}\mathcal{R}^8$ segment calculated using “spoiled” $(\delta_2 = \delta_1)$ transfer function [compare with the correct result in Fig. 7(c)].

$$x_2 = x_1 + x_1'(z_2 - z_1) \tag{A1a}$$

$$x_2' = \frac{x_1' + \frac{\Delta p_x}{Cp_0(1+\delta_1)}}{1 + \frac{\Delta p_z}{Cp_0(1+\delta_1)}} \tag{A1b}$$

$$y_2 = y_1 + y_1'(z_2 - z_1) \tag{A1c}$$

$$y_2' = \frac{y_1' + \frac{\Delta p_y}{Cp_0(1+\delta_1)}}{1 + \frac{\Delta p_z}{Cp_0(1+\delta_1)}} \tag{A1d}$$

$$\Phi_2 = \Phi_1 + k_0 \frac{z_2 - z_1}{\beta_z} \tag{A1e}$$

$$\delta_2 = (1+\delta_1)C\sqrt{\left(x_1' + \frac{\Delta p_x}{Cp_0(1+\delta_1)}\right)^2 + \left(y_1' + \frac{\Delta p_y}{Cp_0(1+\delta_1)}\right)^2 + \left(1 + \frac{\Delta p_z}{Cp_0(1+\delta_1)}\right)^2} - 1 \tag{A1f}$$

$$C = \frac{1}{\sqrt{x_1'^2 + y_1'^2 + 1}} \tag{A2a}$$

$$\beta_z = C\frac{p_0(1+\delta_1)}{\sqrt{p_0^2(1+\delta_1)^2 + m^2c^2}} \tag{A2b}$$

$$\Delta p_x = \Re\left\{\frac{(-e)}{c}\int_{z_1}^{z_2}\left(\frac{1}{\beta_z}\tilde{E}_x + y_1' c\tilde{B}_z - c\tilde{B}_y\right)\exp\left[i\left(\Phi_1 + k_0\frac{z-z_1}{\beta_z}\right)\right]dz\right\} \tag{A2c}$$

$$\Delta p_y = \Re\left\{\frac{(-e)}{c}\int_{z_1}^{z_2}\left(\frac{1}{\beta_z}\tilde{E}_y + c\tilde{B}_x - x_1' c\tilde{B}_z\right)\exp\left[i\left(\Phi_1 + k_0\frac{z-z_1}{\beta_z}\right)\right]dz\right\} \tag{A2d}$$

$$\Delta p_z = \Re\left\{\frac{(-e)}{c}\int_{z_1}^{z_2}\left(\frac{1}{\beta_z}\tilde{E}_z + x_1' c\tilde{B}_y - y_1' c\tilde{B}_x\right)\exp\left[i\left(\Phi_1 + k_0\frac{z-z_1}{\beta_z}\right)\right]dz\right\}. \tag{A2e}$$

$$E_x(x,y,z,t) = \Re[\tilde{E}_x(x,y,z)e^{i\omega_0 t}], \text{ etc.}$$