TNSA proton maximum energy laws for 2D and 3D PIC simulations

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Our virtual laboratory

piccante
Open Source PIC Code

ALaDyn

jasmine

Propaga

Apart from jasmine, all of our codes are now free (GPL v3)

webpage: https://github.com/ALaDyn

Since ALaDyn became open source[1], almost one year ago, we released these updates:

- Ported ALaDyn to the new Marconi HPC system deployed at CINECA (both the A1 and A2 partition)
- Rewrote the gaseous and solid target specifications and implementations, to ease simulations for recent experiments
- PWFA: new bunch shapes
- Deprecated the previous toolchain, ported everything to CMake
- Rewrote the I/O module to work around machine-level bugs on Marconi
- Add compatibility with old-ALaDyn input files
- Usual bugfixes

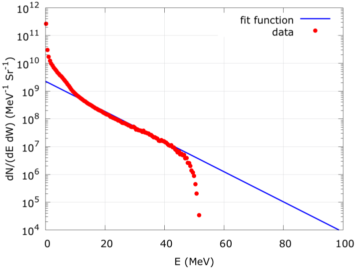
[1] we still have a version in-house a little bit different, we will release all the modules step-by-step

A consolidated regime

TNSA energy spectrum

TNSA has a very well known energy spectrum, based on an exponential distribution with a precise cut-off energy $E_{\max}$ (T is the proton temperature).

$$\begin{cases} dN/dE = (E_{\max}/T)\ e^{-E/T} & \text{for } E < E_{\max} \\ dN/dE = 0 & \text{for } E > E_{\max} \end{cases}$$



- laser pulse duration: $\tau = 40$ fs
- $\lambda = 0.8\,\mu$m, P-polarized
- $I = 2 \cdot 10^{19}$ W/cm^2, $a_0 = 3$
- waist 6.2 μm
- target: uniform Al foil, thickness $0.5\mu\text{m} \leq L \leq 8\mu\text{m}$
- contaminants: layer of H on the rear (non illuminated) side, fixed thickness 0.08μm
- ionization level: fixed, Al^{9+}, H^{+}
- electron densities: $n_e^{Al} = 100\,n_c$, $n_e^{H} = 10\,n_c$.
- neglected preplasma (the temporal contrast is assumed as infinite)

The problem
Numerical simulations

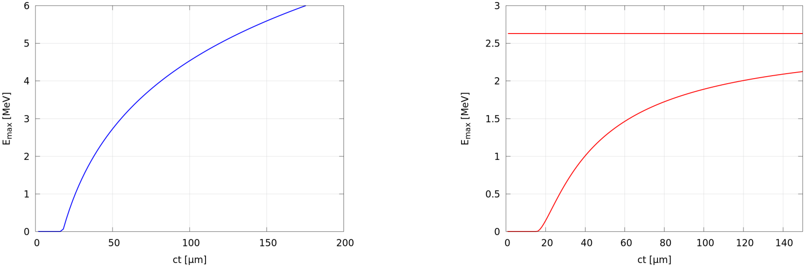


Figure 1: Maximum energy rise in time. Left: the 2D case, right: the 3D case.

In 2D PIC TNSA simulations, a monotonic rise of $E_{\max}$ with time is observed whereas in 3D a slow trend towards a possible saturation to an asymptotic value is usually observed.

Two EMPIRICAL laws for $E_{\max}(t)$

Work originated from Schreiber et al. model[2]. The acceleration of protons (contaminants) is due to the positive surface charge created on the rear target, thanks to the electron escape.

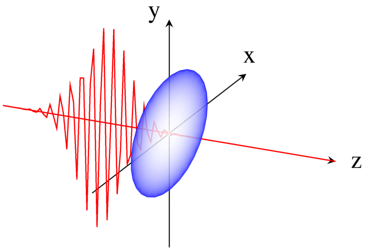


Figure 2: Left: 3D case (charge on a disc of radius R). Right: 2D case (charge on a strip of infinite length).

[2] Phys. Rev. Lett., 97:045005, Jul 2006

Proposed laws

The 3D case

Laser pulse: propagates along the z axis
Hypothesis: electrostatic potential V which vanishes at $z = 0$, where a uniform charge density σ, within a disc of radius R, is located.

$$V(\zeta) = 2\pi R\,\sigma\left(\sqrt{1+\zeta^2} - \zeta - 1\right) \qquad \zeta = \frac{z}{R}$$

A particle initially at rest accelerates and the law of motion is obtained from energy conservation. Since $V(0) = 0$, we have

$$m\frac{v^2}{2} + eV(z) = 0 \qquad v = \dot{z}$$

Proposed laws
The 3D case

so that

$$E_\infty = m\frac{v_\infty^2}{2} = 2\pi e R\sigma \qquad v_\infty = \dot{z}(\infty)$$

The kinetic energy of the particle, after integrating the equation of motion, is

$$E_{\text{kin}}(t) \simeq E_\infty \left(1 - \frac{t^*}{t}\right)^2 \qquad t > t^* = \frac{R}{4v_\infty}$$

Since this is an asymptotic law, we may assume that $E(t) = 0$ for $t < t^*$.

Proposed laws

Fits for 3D simulations

$$
\begin{cases}
E_{\max}^{(3D)}(ct) = 0 & \text{for } t < t^{*(3D)} \\
E_{\max}^{(3D)}(ct) = E_{\infty}^{(3D)} \left(1 - \dfrac{ct^{*(3D)}}{ct}\right)^2 & \text{for } t > t^{*(3D)}
\end{cases}
$$

We can perform a linear fit by defining $y = \sqrt{E}$ and $x = 1/ct$, so that the previous law becomes

$$
y = a + bx \qquad E_{\infty}^{(3D)} = a^2 \qquad ct^{*(3D)} = -\frac{b}{a}
$$

Laser pulse: propagates along the z axis
Hypothesis: electrostatic potential V which vanishes at $z = 0$, where a uniform charge density σ, on an infinite strip along the y axis and with a size R along the x axis, is located.

$$
\begin{aligned}
V(z) &= 4R\sigma \left(-\zeta \arctan \frac{1}{\zeta} + \log \frac{1}{\sqrt{1+\zeta^2}} \right) \\
&\simeq -4R\sigma \, \log(1+\zeta)
\end{aligned}
$$

where we defined $\zeta = z/R$.

The potential in this case diverges logarithmically and consequently the particle accelerates indefinitely. We approximate the potential energy with

$$e\hat{V}(z) = -E_\infty \log(1+\zeta)$$

so that

$$E_\infty \equiv m\frac{v_\infty^2}{2} = 4eR\sigma$$

Integrating the equations of motion we have

$$E_{\text{kin}}(t) = E_\infty \log\left(\frac{t}{t^*}\right) \qquad t \geq t^* = \frac{R}{v_\infty}$$

Since this is an asymptotic law, we may assume that $E(t) = 0$ for $t < t^*$.

Proposed laws

Fits for 2D simulations

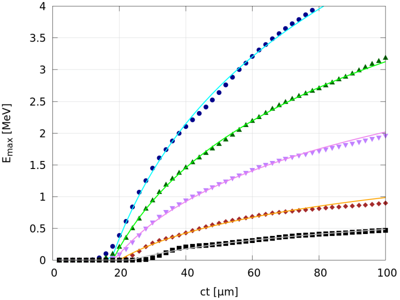
$$\begin{cases} E_{\max}^{(2D)}(ct) = 0 & \text{for } t < t^{*(2D)} \\ E_{\max}^{(2D)}(ct) = E_{\infty}^{(2D)} \log \dfrac{ct}{ct^*} & \text{for } t > t^{*(2D)} \end{cases}$$

We perform a linear fit by defining $y = E$ and $x = \log ct$, so that the previous law becomes

$$y = a + bx \qquad\qquad E_{\infty}^{(2D)} = b \qquad ct^{*(2D)} = e^{-a/b}$$

Results for 2D simulations

Normal incidence, different target thicknesses



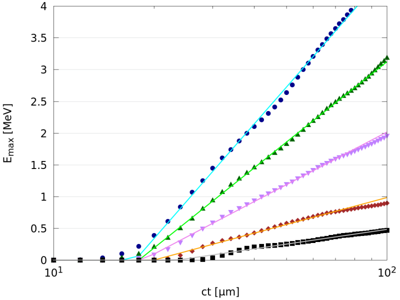


Figure 3: Left: $E_{\max}$ versus ct, PIC simulation (symbols) vs fit (continuous line): blue $L = 0.5\,\mu$m, green $L = 1\,\mu$m, violet $L = 2\,\mu$m, orange $L = 4\,\mu$m, black $L = 8\mu$m. Right: the same as the left panel but in a logarithmic scale for ct which clearly shows the linearity and the accuracy of the fit.

Results for 2D simulations

Non-normal incidence, fixed target thickness

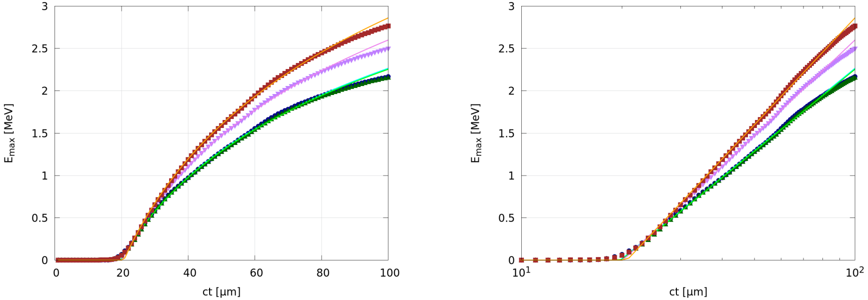


Figure 4: Left: $E_{\max}$ versus ct, PIC simulation (symbols) vs fit (continuous line): $\alpha = 5°$ green, $\alpha = 10°$ violet and $\alpha = 15°$ orange. Right: the same data are plotted as with a logarithmic scale for ct, which shows how the data stay on a line and the accuracy of the linear fit.

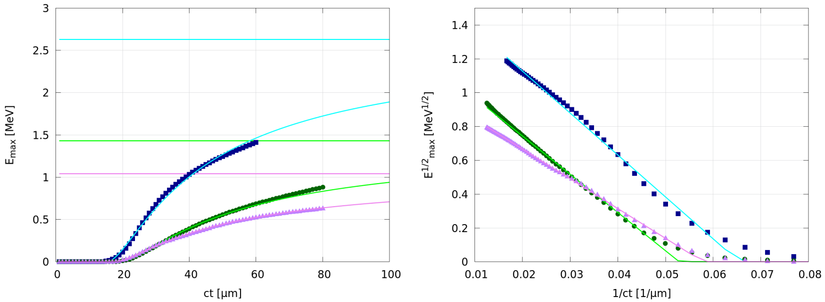


Figure 5: Left: E_{max} versus ct, PIC simulation (symbols) vs fit (continuous line): $L = 0.5\mu$m blue, $L = 1\mu$m green and to $L = 2\mu$m violet. Right: Plot of $\sqrt{E_{\mathrm{max}}}$ versus $1/ct$ which shows their linearity and the accuracy of the fit.

Comparison with experiments

Where are we?

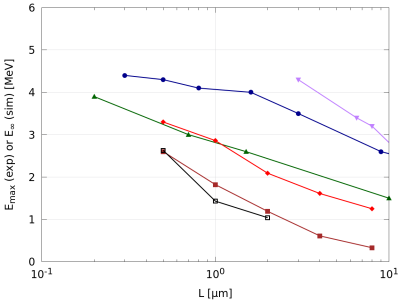


Figure 6: $E_{\max}$ versus L from various experiments ($a_0 \sim 3$ and a metal target): Ceccotti's (45° incidence angle) (blue), Neely's (30°) (green), Flacco's (45°) (violet), fits from our 2D PIC simulation at zero degree incidence (brown), fits from 2D sims at 30° incidence (red) and fits from 3D PIC simulation at zero degree incidence (black).

Conclusions

What we knew

- The asymptotic value of the cut-off energy of protons, which is what is measured in experiments, is difficult to extract from PIC simulations
- The 2D results do not exhibit a saturation
- The 3D results show that a saturation might be reached, despite at a large time ($ct > 200\,\mu$m), which is computationally too expensive

Conclusions

What is new

- We formulated two empirical laws for 2D and 3D simulations, which depend on the asymptotic energy E_∞
- The fits to the 2D and 3D results coming from PIC simulations are quite good and the statistical uncertainties are a few percent
- The extrapolated values E_∞^{2D} and E_∞^{3D} are comparable
- E_∞^{2D} and E_∞^{3D} can be fully calculated fitting the results obtained before $ct \leq 50 \sim 60\,\mu$m, which is a distance reachable also in 3D simulations

Conclusions

What is new

- The fitting appears to be satisfactory also for small incidence angles, even though the model was developed for normal incidence
- The proposed phenomenological model is adequate to avoid the arbitrariness in the choice of the time at which the asymptotic cut-off energy is chosen in numerical simulations
- 2D simulations may have a quantitative value, with an adequate extrapolation, rather than being of purely qualitative nature