Plasma acceleration limitations due to betatron radiation

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Is there a limit to plasma wake-field acceleration?

$$P_{br} \approx r_e m_e c^3 \gamma^2 k_p^2 r_\beta^2 = c^2 m_e \omega_p \approx P_{wf}$$

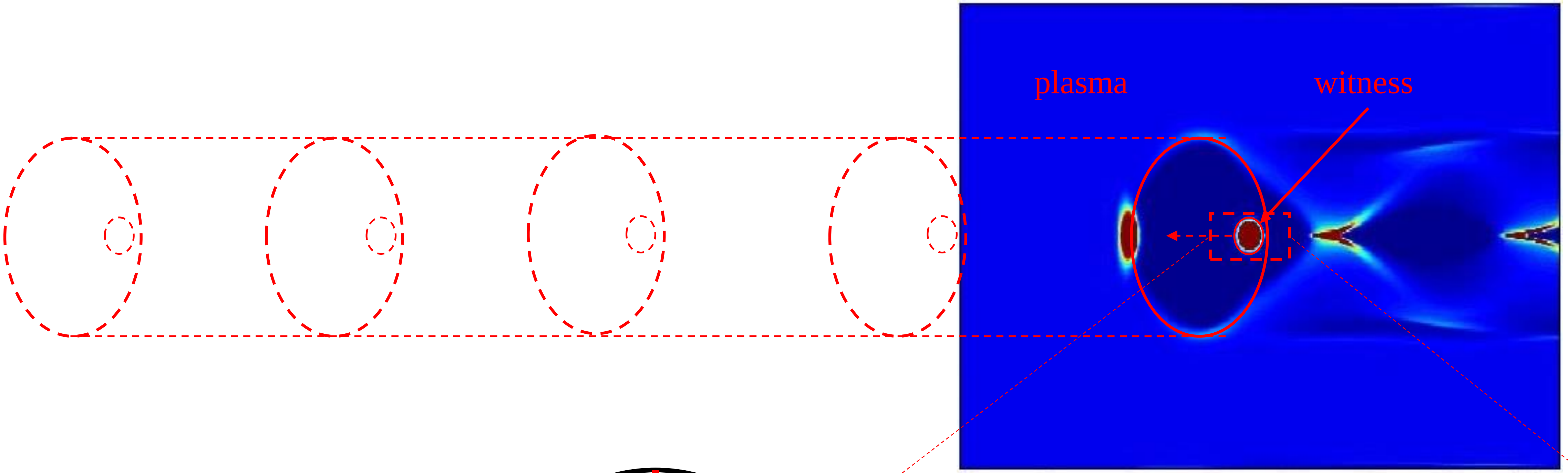
How betatron radiation will affect the electron beam:

- cooling of the beam
- energy spread generation

[1] F.Zimmerman, Possible limits of plasam linear colliders, Journal of Physics: Conf.Series 874 (2017) 012030. doi:10.1088/1742-6596/874/1/012030.

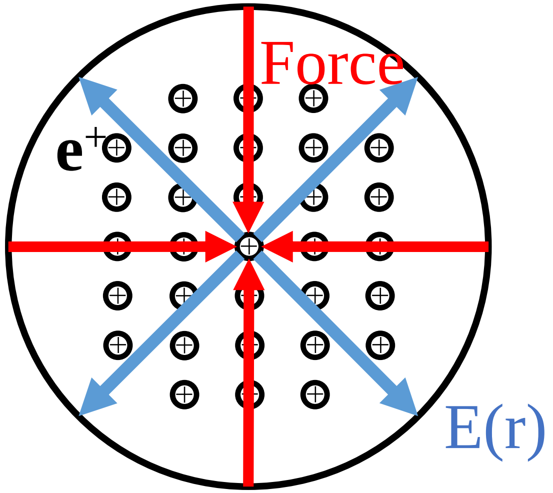
[2] A.Deng et al., Electron beam dynamics and self-cooling up to pev level due to betatron radiation in plasma-based accelerators, Phys.Rev.Accel.andBeams 15 (2012) 081303. doi:10.1103/PhysRevSTAB.15.081303.

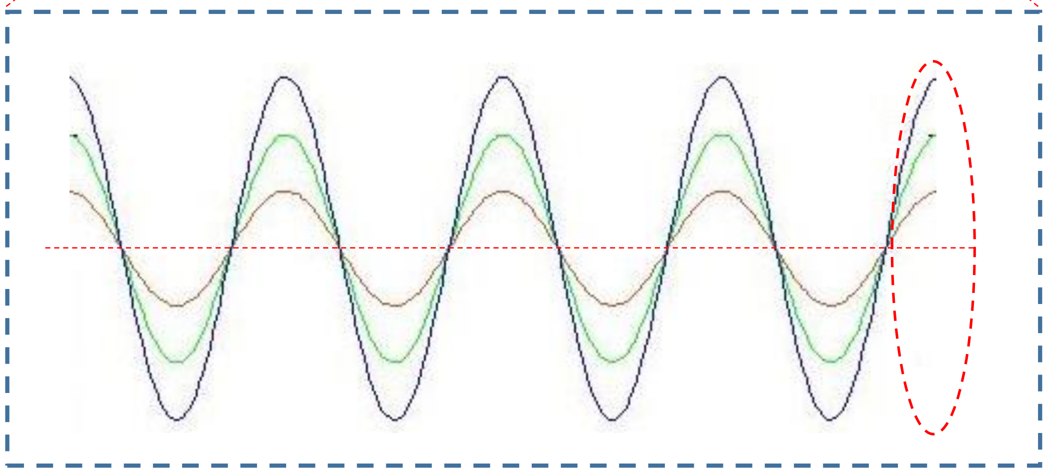
[3] W.A.Barletta, E.P.Lee, R.Bonifacio, L.De Salvo, Limitations on plasma acceleration due to synchrotron losses, Nucl.Instr.andMeth A 423 (1999) 256–259. doi:10.1016/S0168-9002(98)01306-0.



What we need to take into account:

- losses due to BR
- focusing of the beam
- cooling of the beam
- energy spread generation

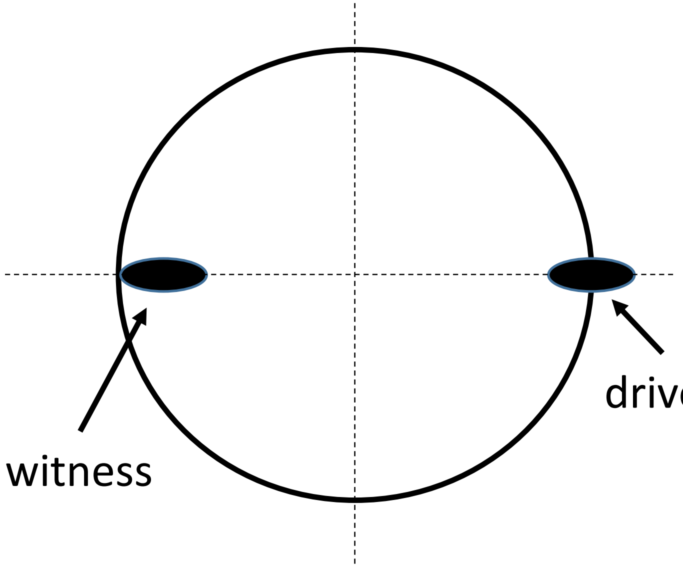




$$\frac{dp_z}{dt} = F_{z,accel} - F_{x,rr}$$

$$\frac{dp_x}{dt} = F_{x,accel} - F_{x,rr}$$

Spherical model

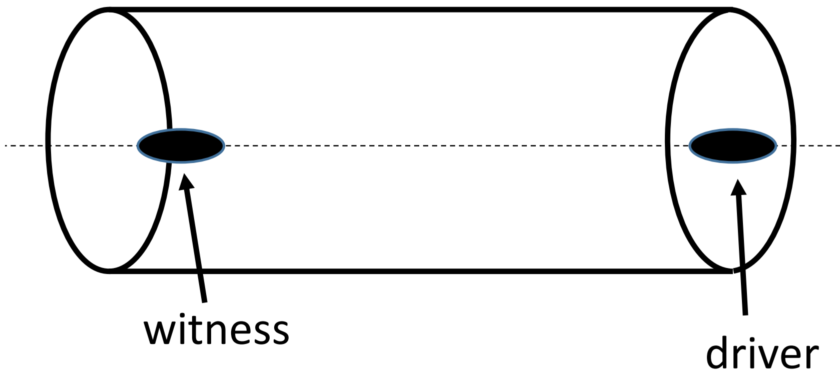


$$\vec{E} = \frac{k_p^2 m_e c^2}{3e} \times \vec{r}$$

Cylindrical model

$$E_{z,max} = \frac{m_e c \omega_p}{e}$$

$$E_x = \frac{k_p^2 m_e c^2}{2e} \times x$$



$$F_z^{rad} \approx F^{rad} = -\frac{e^2}{6\pi\epsilon_0 m_e^2 c^4}\left(\frac{dp_\mu}{d\tau}\right)^2$$

$$F_\mu^{rad} = \frac{e^2}{6\pi\epsilon_0 m_e^2 c^4}\left(\frac{d^2 p_\mu}{d\tau} - \frac{p_\mu}{m_e^2 c^2}\left(\frac{dp_\mu}{d\tau}\right)^2\right)$$

Spherical model

$$\frac{d^2x}{dz^2} + \left(\frac{k_p^2}{3\gamma}z + \frac{e^2}{18\pi\epsilon_0 m_e c^2}k_p^2\right)\frac{dx}{dz} + \frac{k_p^2}{3\gamma}x = 0$$

$$\frac{d\gamma}{dz} = \frac{k_p^2}{3}z - \frac{e^2}{54\pi\epsilon_0 m_e c^2}k_p^4\gamma^4 x^2$$

Cylindrical model

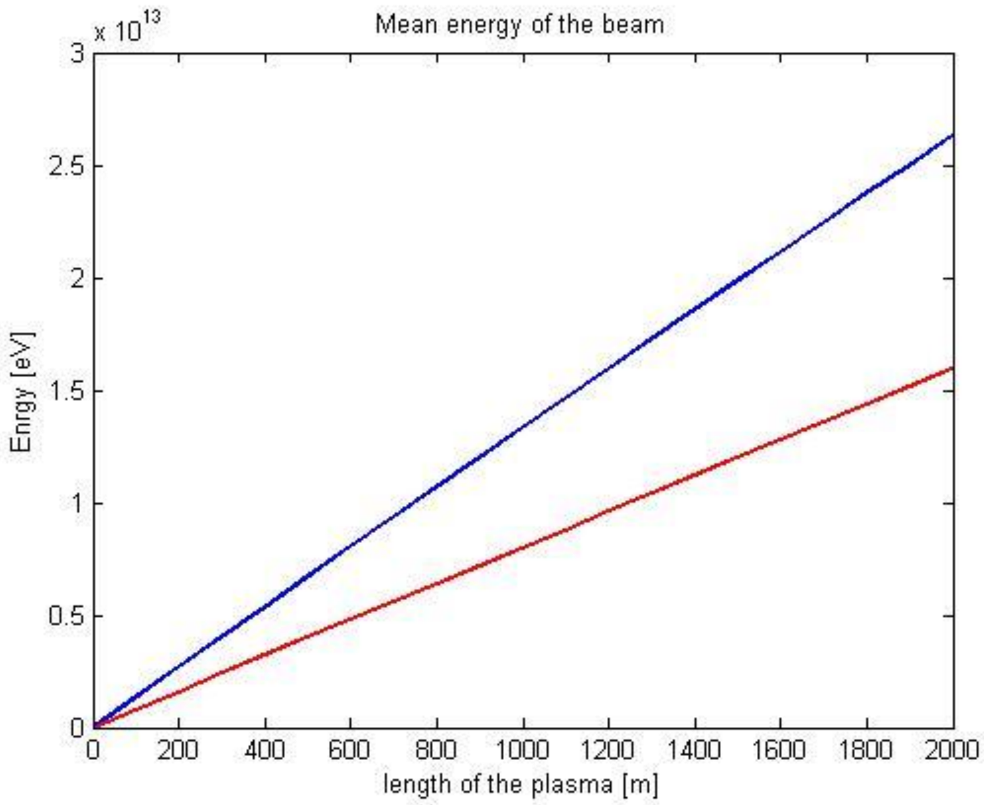
$$\frac{d^2x}{dz^2} + \left(\frac{k_p^2}{\pi\gamma}z + \frac{e^2}{12\pi\epsilon_0 m_e c^2}k_p^2\right)\frac{dx}{dz} + \frac{k_p^2}{2\gamma}x = 0$$

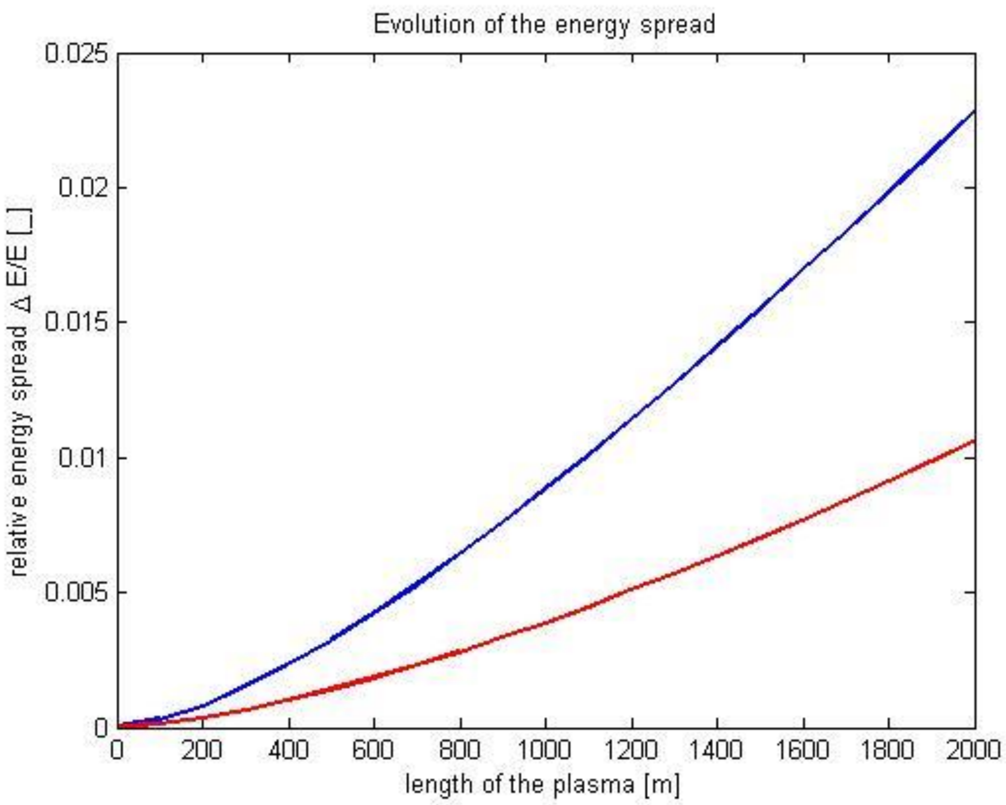
$$\frac{d\gamma}{dz} = \frac{k_p^2}{\pi}z - \frac{e^2}{24\pi\epsilon_0 m_e c^2}k_p^4\gamma^4 x^2$$

$$\frac{d\gamma}{dz} = \frac{k_p^2}{\pi} z - \frac{e^2}{24\pi\epsilon_0 m_e c^2} k_p^4 \gamma^4 x^2$$

Spherical model

Cylindrical model



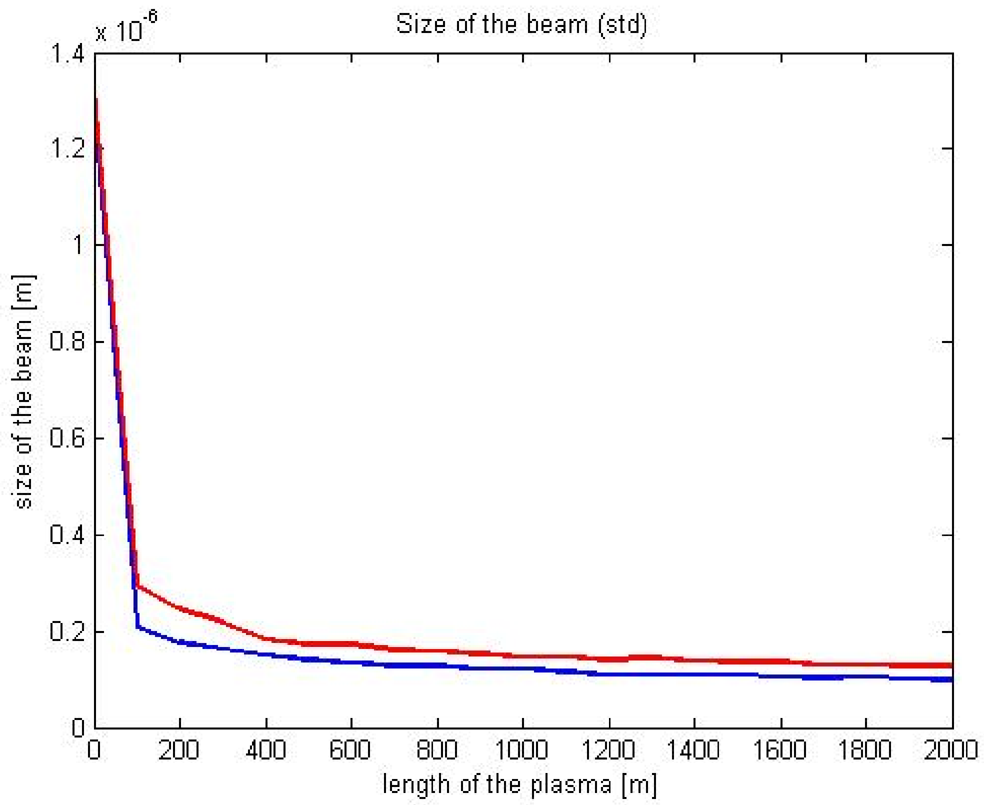


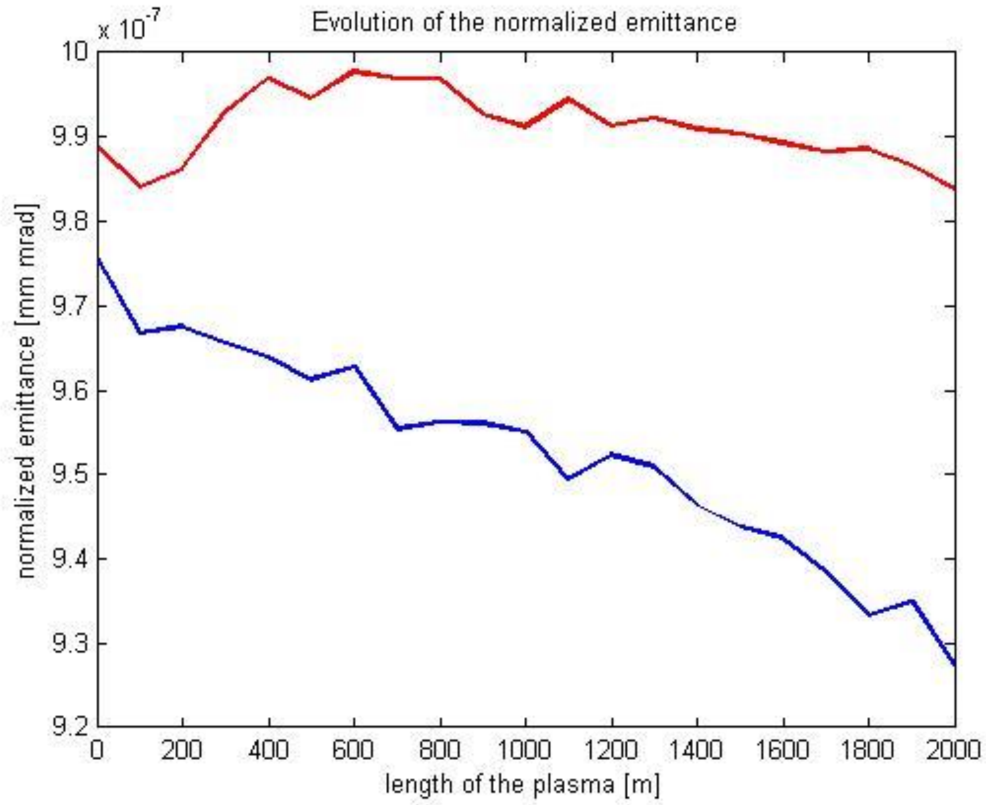
size and emittance of the beam

$$\frac{d^2x}{dz^2} + \left(\frac{k_p^2}{\pi\gamma}z + \frac{e^2}{12\pi\epsilon_0 m_e c^2}k_p^2\right)\frac{dx}{dz} + \frac{k_p^2}{2\gamma}x = 0$$

Spherical model

Cylindrical model

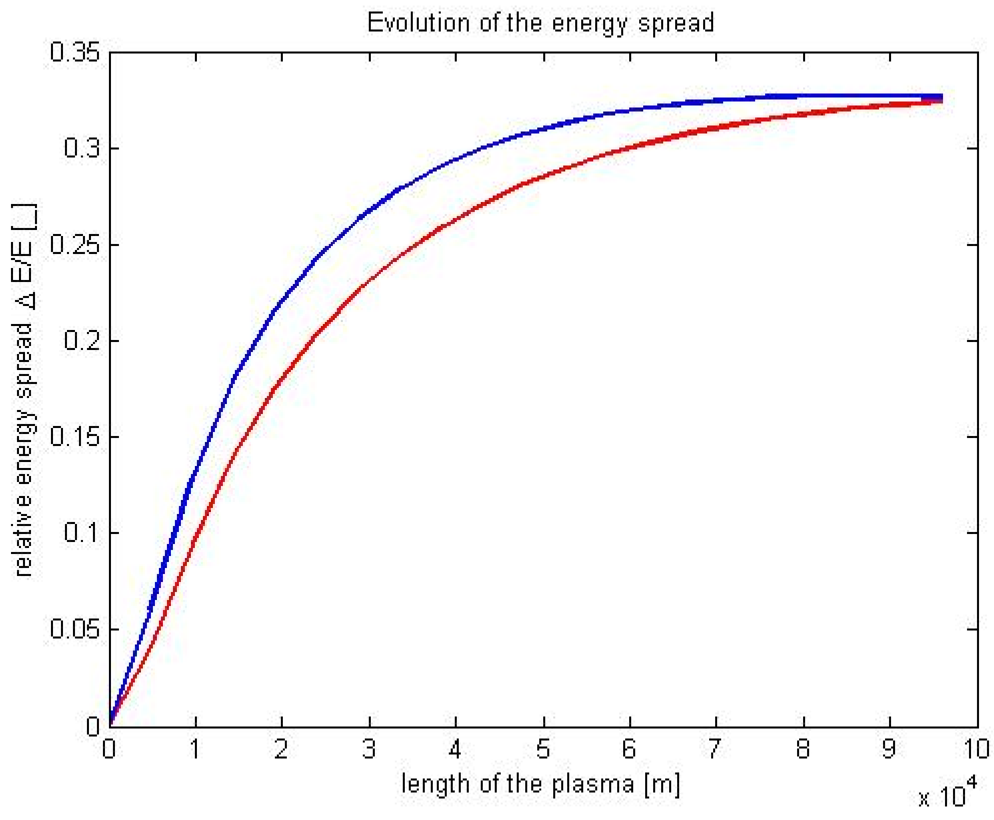


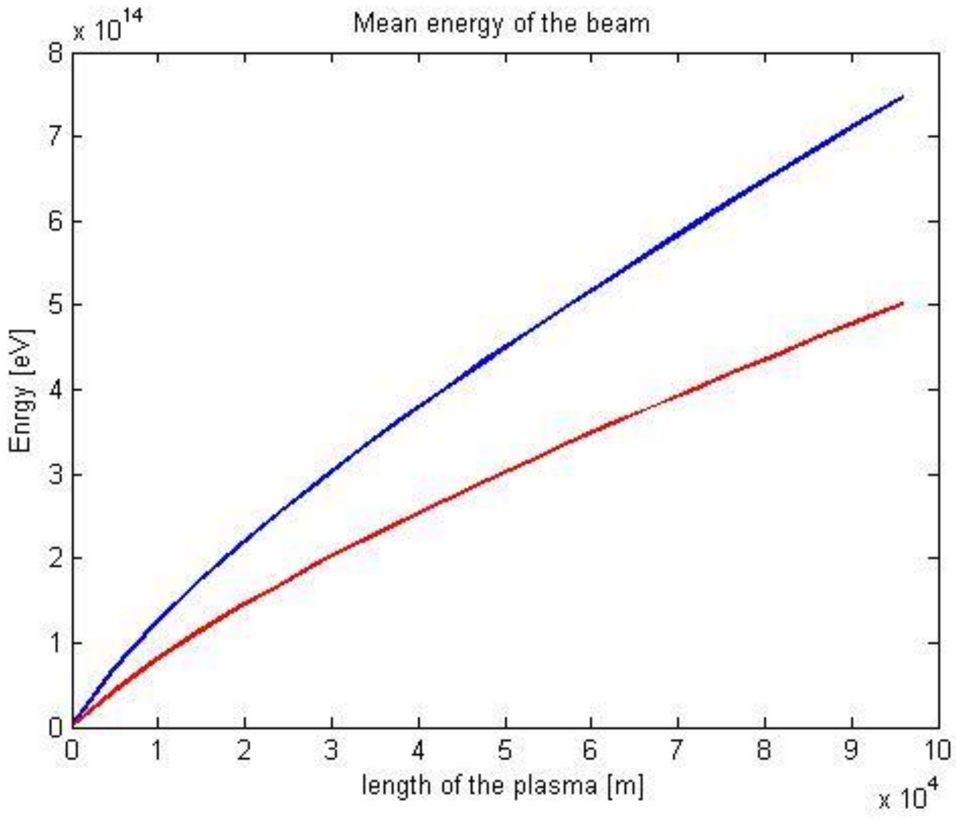


$$\frac{d\gamma}{dz} = \frac{k_p^2}{\pi} z - \frac{e^2}{24\pi\epsilon_0 m_e c^2} k_p^4 \gamma^4 x^2$$

Spherical model

Cylindrical model





- We have found no limit to acceleration …
- but energy spread will prevent it from actually use the beam…
- thus the actual energy will be limited to a couple of tens TeV in ideal conditions
- the cooling of the beam present, but its significance is debatable

Thank You!