Better Measurement of Plasma Wakefields:

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Fast and sensitive measurements of plasma wakefields with TESS

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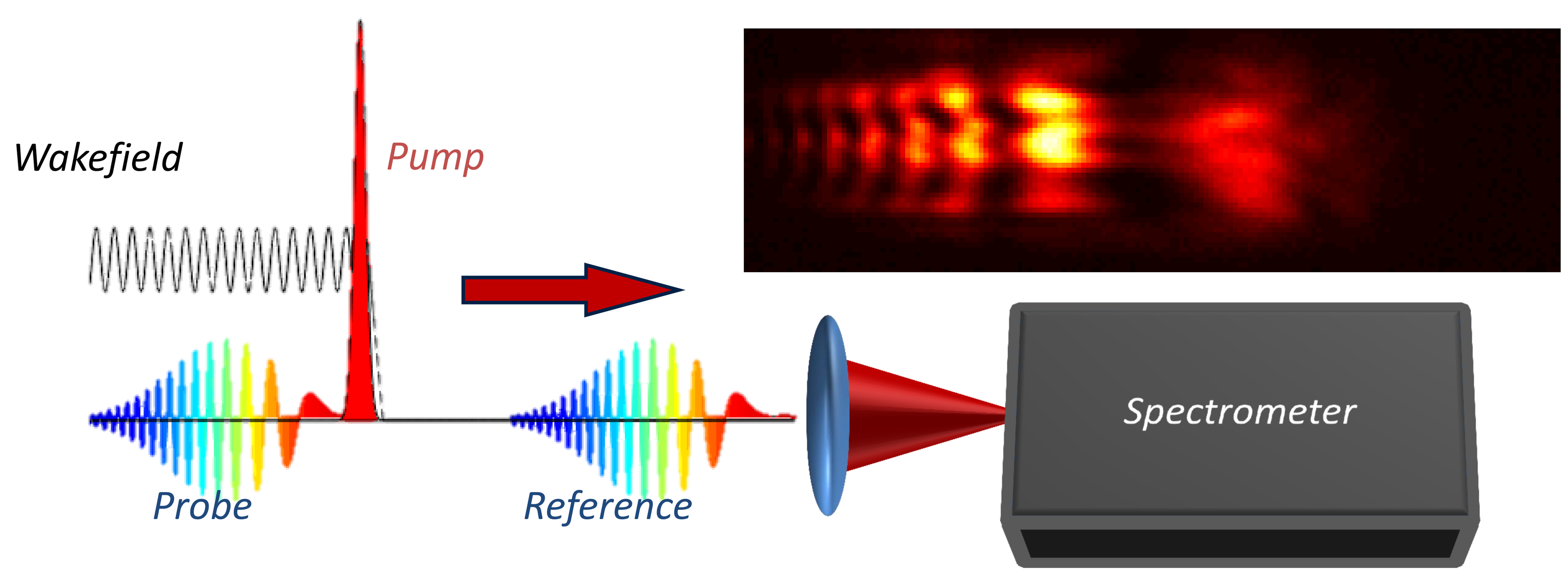
Outline:

- Frequency Domain Holography **(FDH)** has already been shown to measure wakefields with great accuracy
- Temporally Encoded Spectral Shifting **(TESS)** can now process data from the same experimental set up faster and requiring fewer calibrations and less human input
- We have experimentally demonstrated that TESS can measure extremely weak wakefields with sub-percent accuracy, agreeing with simulation and theory
- An improved TESS scheme would allow us to measure non-linear wakefields without assuming identical or Gaussian probe and reference pulses

Experimental Set Up:

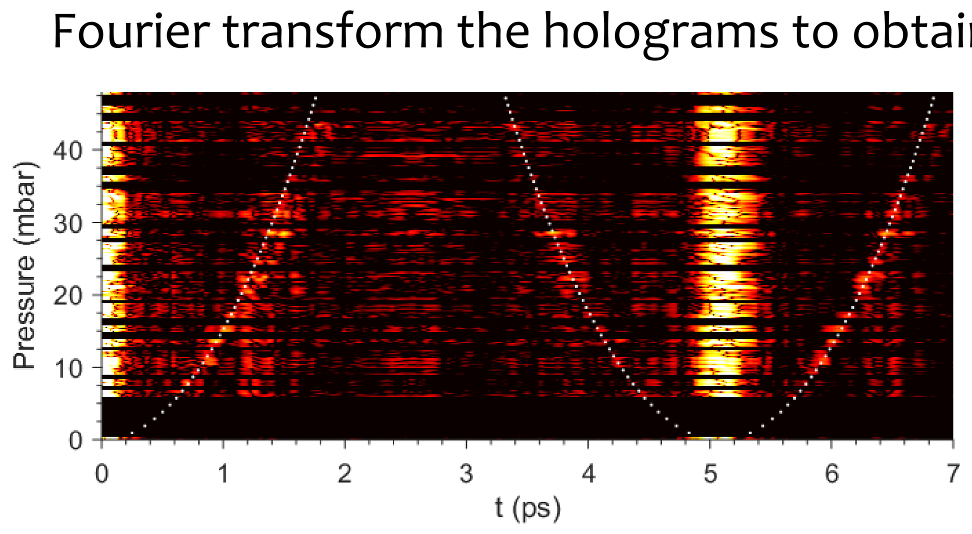
1. Probe the wakefield with a long (ps scale) chirped probe pulse
2. Interfere the probe with a preceding reference pulse inside a spectrometer, creating a **Frequency Domain Hologram** [2]



- Probe, reference and pump pulses are all co-linear through interaction
- The phase difference between the probe and the reference is proportional to the density structure of the wakefield $\varphi(\zeta) \propto L\,\delta n_e(\zeta)$

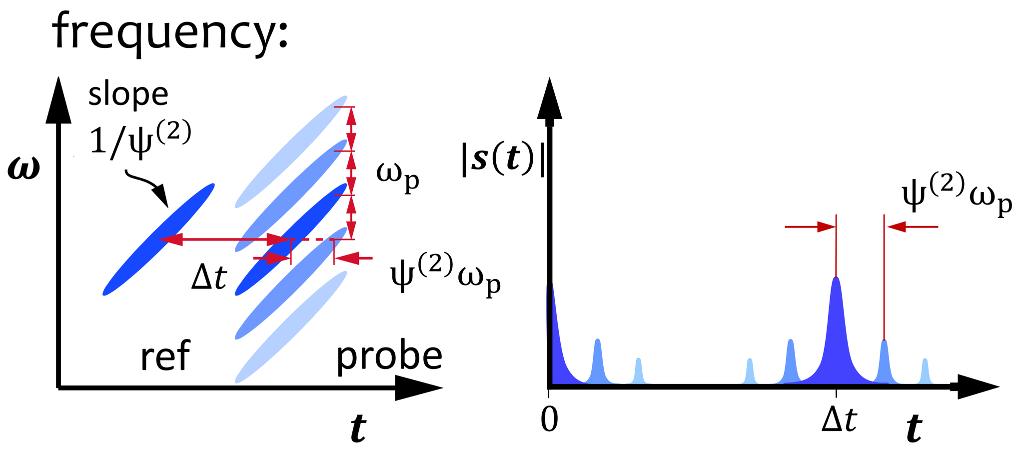
Results:

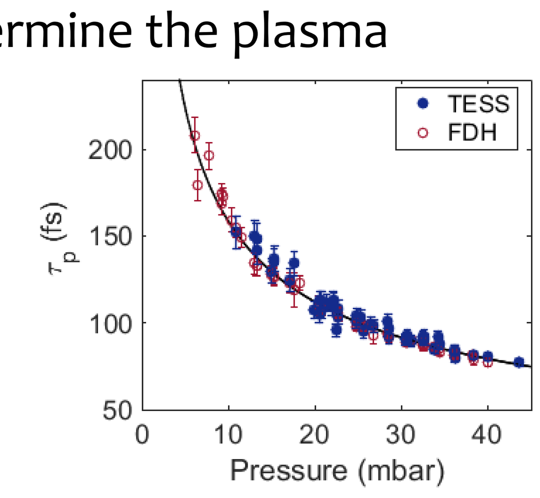
- Fourier transform the holograms to obtain the TESS spectrum[1]:



Waterfall plot of TESS spectra: Each horizontal line is a lineout of a TESS spectrum, with peaks in brighter colours. White lines are shown at expected peak positions: $t = \psi^{(2)}\omega_p$ and $t = \Delta t \pm \psi^{(2)}\omega_p$

- Locations of peaks in the TESS spectrum determine the plasma frequency:

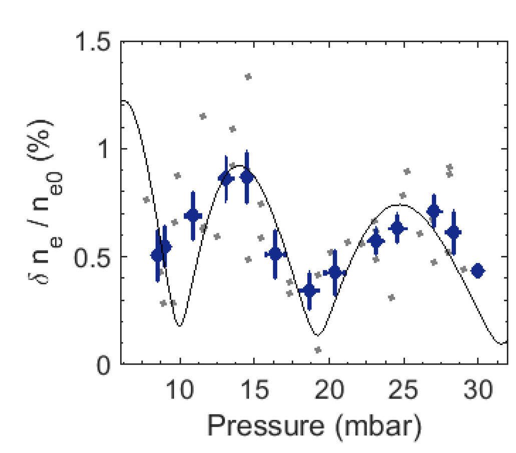




- Results from TESS and FDH agreed well
- Peak heights determine the wake amplitude:

$$\left|\frac{k\text{th peak}}{\text{sideband}}\right| \equiv r_k = \frac{J_k(\phi_1)}{J_0(\phi_1)}\mathcal{F}(k\omega_p)$$

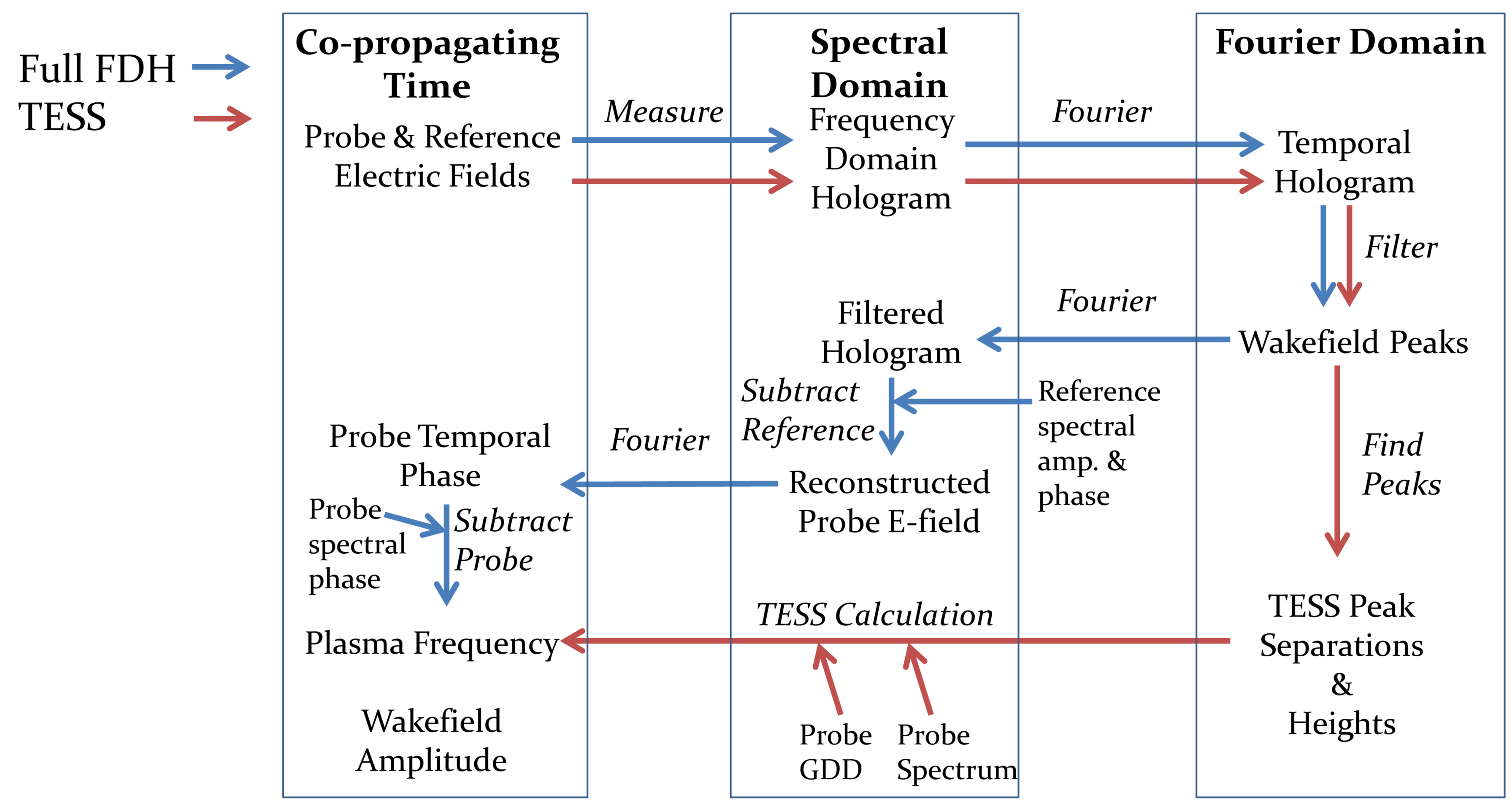
$$\text{where} \quad \phi_1 = \frac{\omega_p{}^2 L}{2\omega_0 c}\frac{\delta n_e}{n_{e0}}$$



- Using TESS, we measured wakefields with $\frac{\delta n_e}{n_{e0}} \approx 1\%$

Advantages of TESS:

- Faster process, with no phase reconstruction required – no staring at phase maps!
- Intrinsic noise filtering makes small features clearer
- Quantitative analysis, suiting automation, on-shot analysis & batch processing



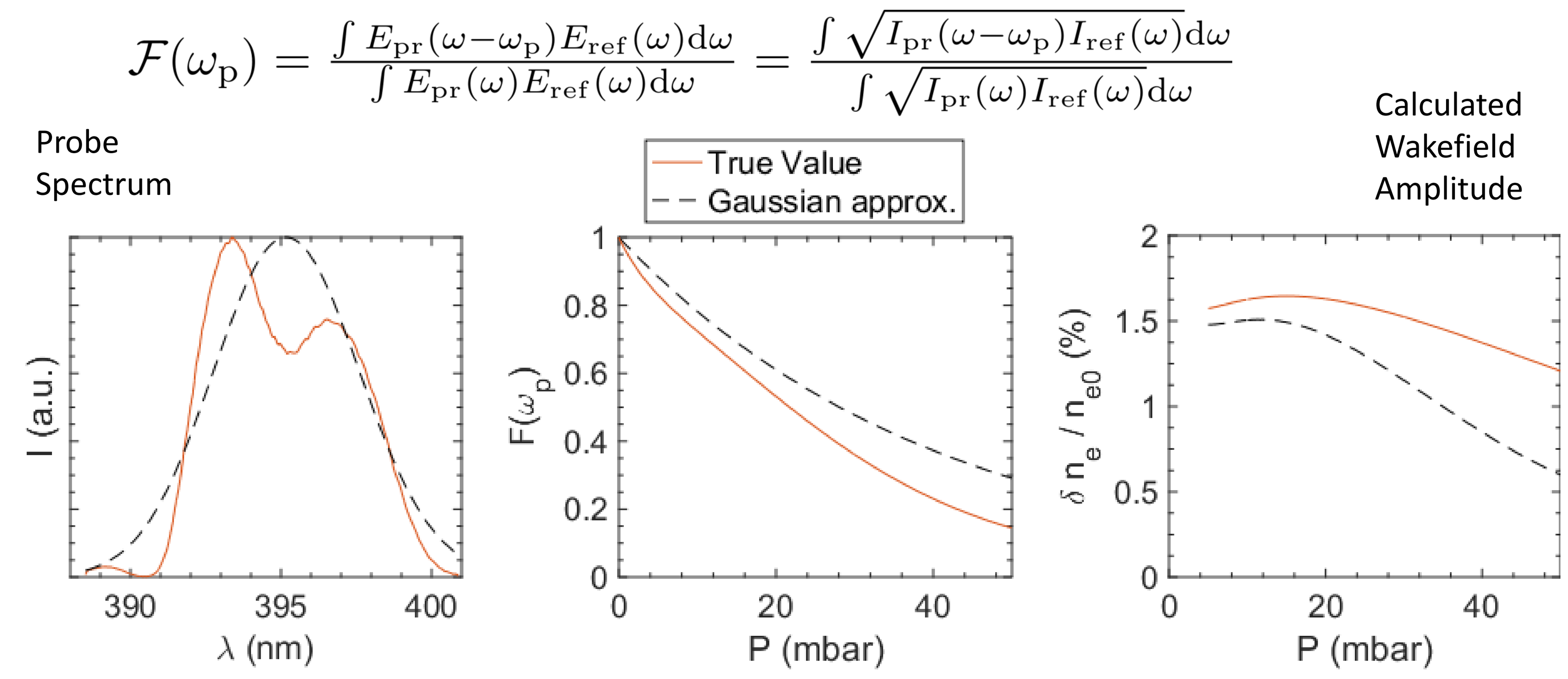
Arbitrary Probe and Reference Spectra:

- Matlis et al. [1] estimated the spectral overlap factor as:

$$\mathcal{F}_{\text{Gauss}}(\omega_p) = \exp\left[-\frac{1}{4}\left(\frac{\omega_p}{\delta\omega}\right)^2\right]$$

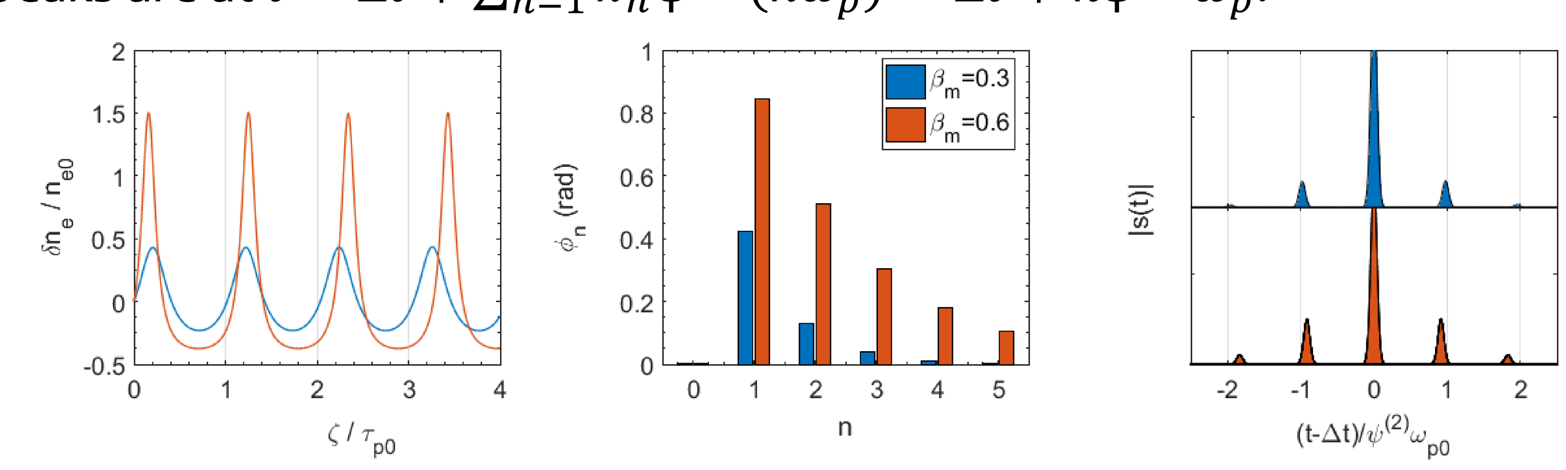
- True expression for arbitrary spectra is:

$$\mathcal{F}(\omega_p) = \frac{\int E_{\text{pr}}(\omega-\omega_p)E_{\text{ref}}(\omega)\text{d}\omega}{\int E_{\text{pr}}(\omega)E_{\text{ref}}(\omega)\text{d}\omega} = \frac{\int\sqrt{I_{\text{pr}}(\omega-\omega_p)I_{\text{ref}}(\omega)}\text{d}\omega}{\int\sqrt{I_{\text{pr}}(\omega)I_{\text{ref}}(\omega)}\text{d}\omega}$$



Non-linear Wakefields

- Periodic non-linear wakefields contain harmonics of the plasma frequency, which we can truncate to N terms.
- Each of these creates carrier waves at frequencies $k_n(n\omega_p)$, so TESS peaks are at $t = \Delta t + \sum_{n=1}^{N} k_n\psi^{(2)}(n\omega_p) = \Delta t + \kappa\psi^{(2)}\omega_p$:

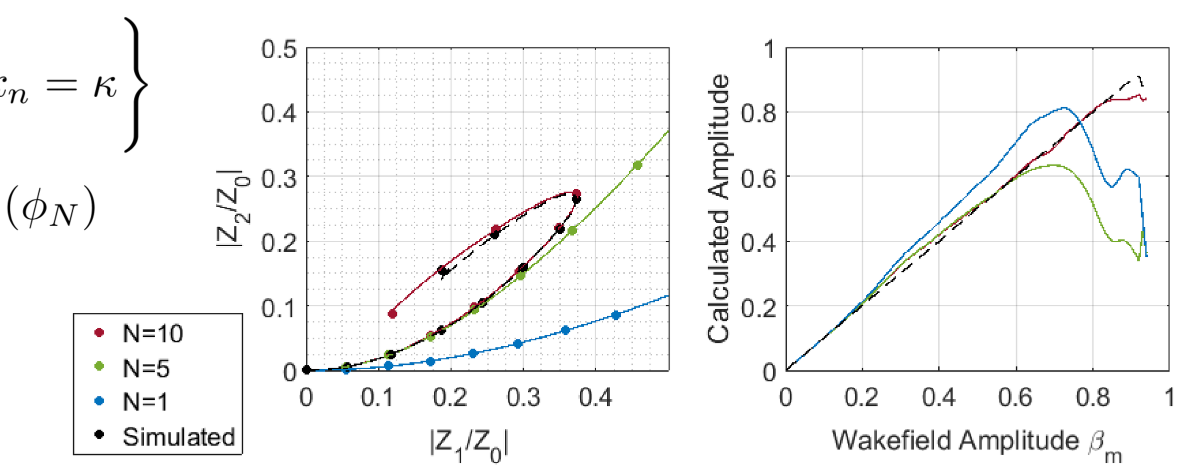


- Inversion is difficult – each peak contains contributions from many carrier waves, which interfere with each other. We must solve a Diophantine equation, and calculate the peak height for each solution:

$$S_\kappa = \left\{\{k_1,\ldots,k_N\} : \sum_{n=1}^{N} nk_n = \kappa\right\}$$

$$Z_\kappa = \sum_{\{k_n\}\in S_\kappa} J_{k_1}(\phi_1)\ldots J_{k_N}(\phi_N) \cdot e^{i\sum_{n=1}^{N}k_n\theta_n}$$

$$r_\kappa = \left|\frac{Z_\kappa(\phi,\theta)}{Z_0(\phi,\theta)}\right|\mathcal{F}(\kappa\omega_p)$$



[1] TESS - N.H.Matlis et al. Optics Letters **41**, 5503 (2016)
[2] FDH - N.H.Matlis et al. Nature Physics **2** 749(2006)
[3] Experiment - J. Cowley et al, PRL **119**, 044802 (2017)

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