International Workshop on Impedances and
Beam Instabilities in Particle Accelerators
Benevento 2017

The circulant matrix formalism and the role of beam-beam effects in coherent instabilities

X. Buffat

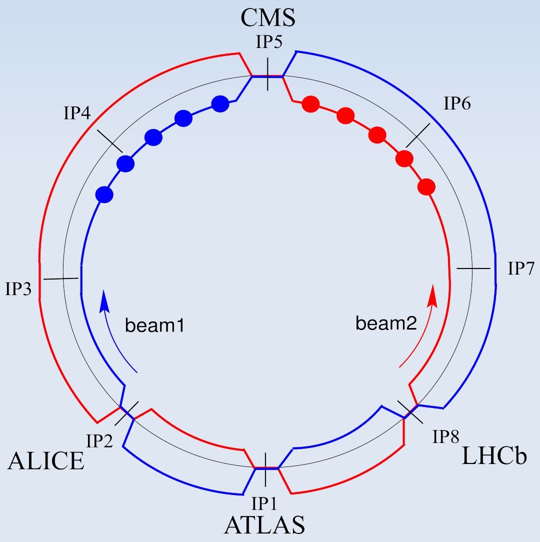
Many thanks to L. Barraud, W. Herr, K. Li, A. Maillard, T. Pieloni, M. Schenk, and S.M. White for their work on this topic as well as for the several fruitful discussions

Content

- Coherent stability of colliding beams
  - The circulant matrix model
- The mode coupling instability of colliding beams
  - Mitigations
  - Observations
- Landau damping of weak head-tail instabilities due to the incoherent spread of beam-beam interactions
- Summary

Stability of colliding beams

- How can we estimate the stability of beams in collision ?



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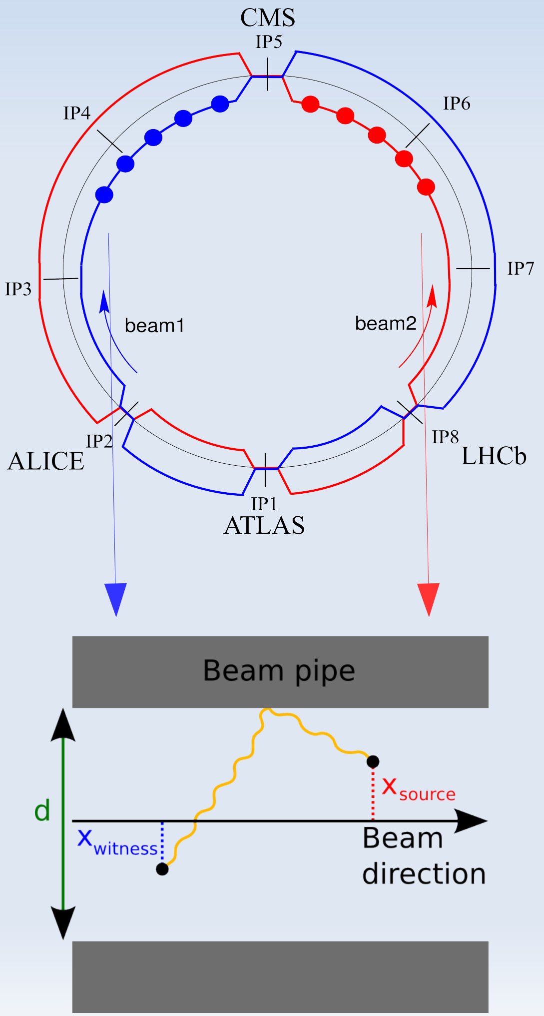

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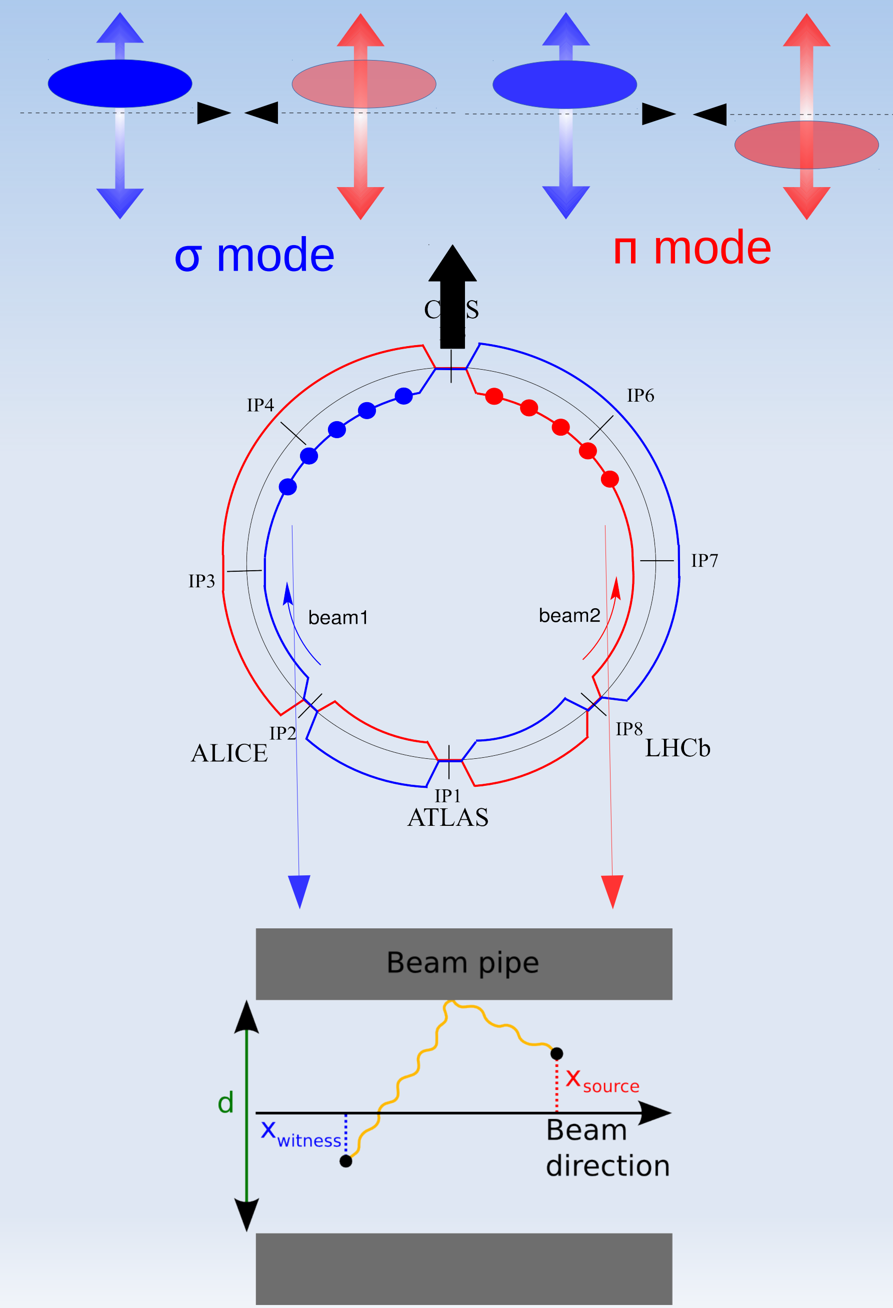
  - In the so-called strong-strong regime, the dynamic of the two beams needs to be treated self-consistently



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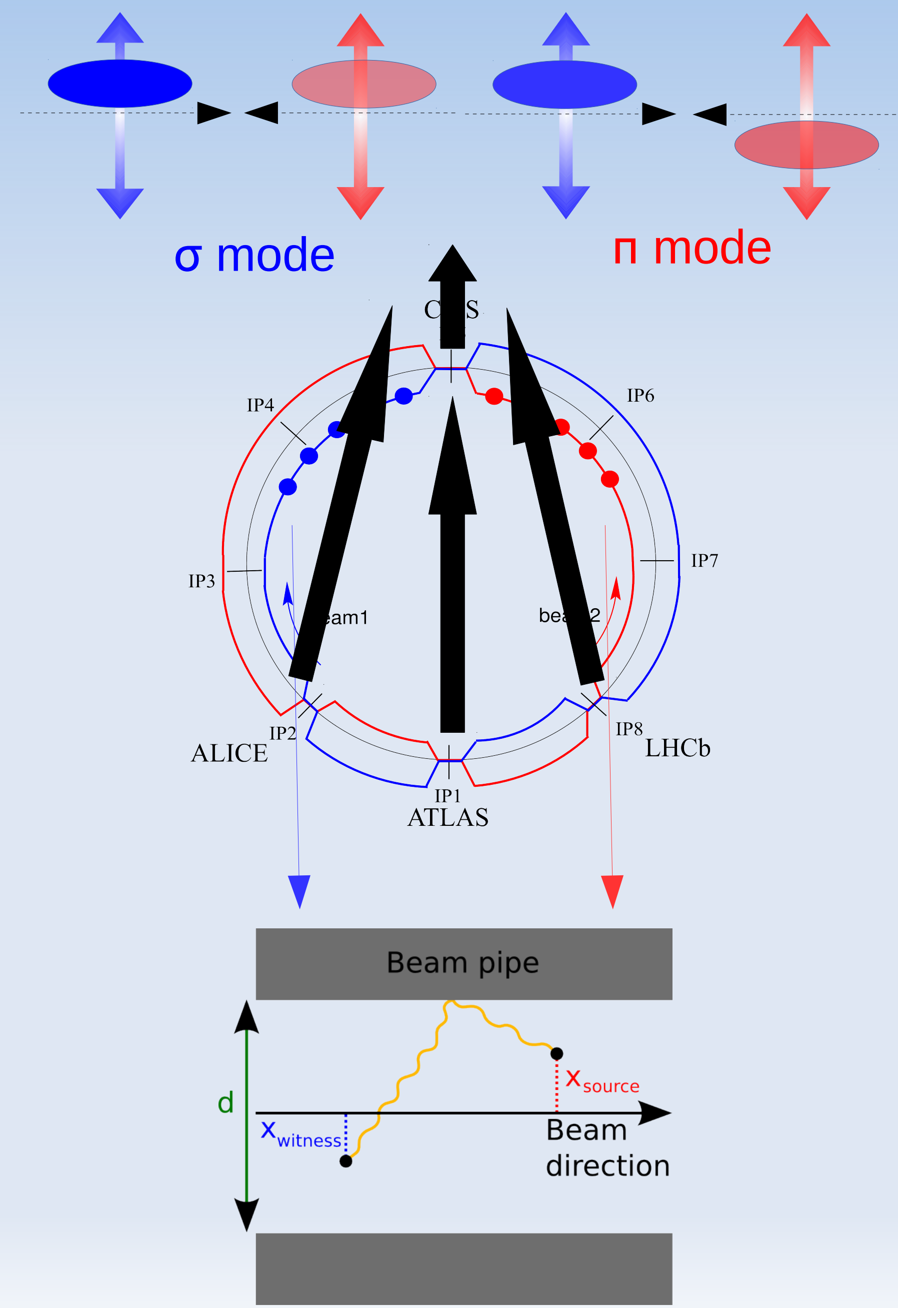
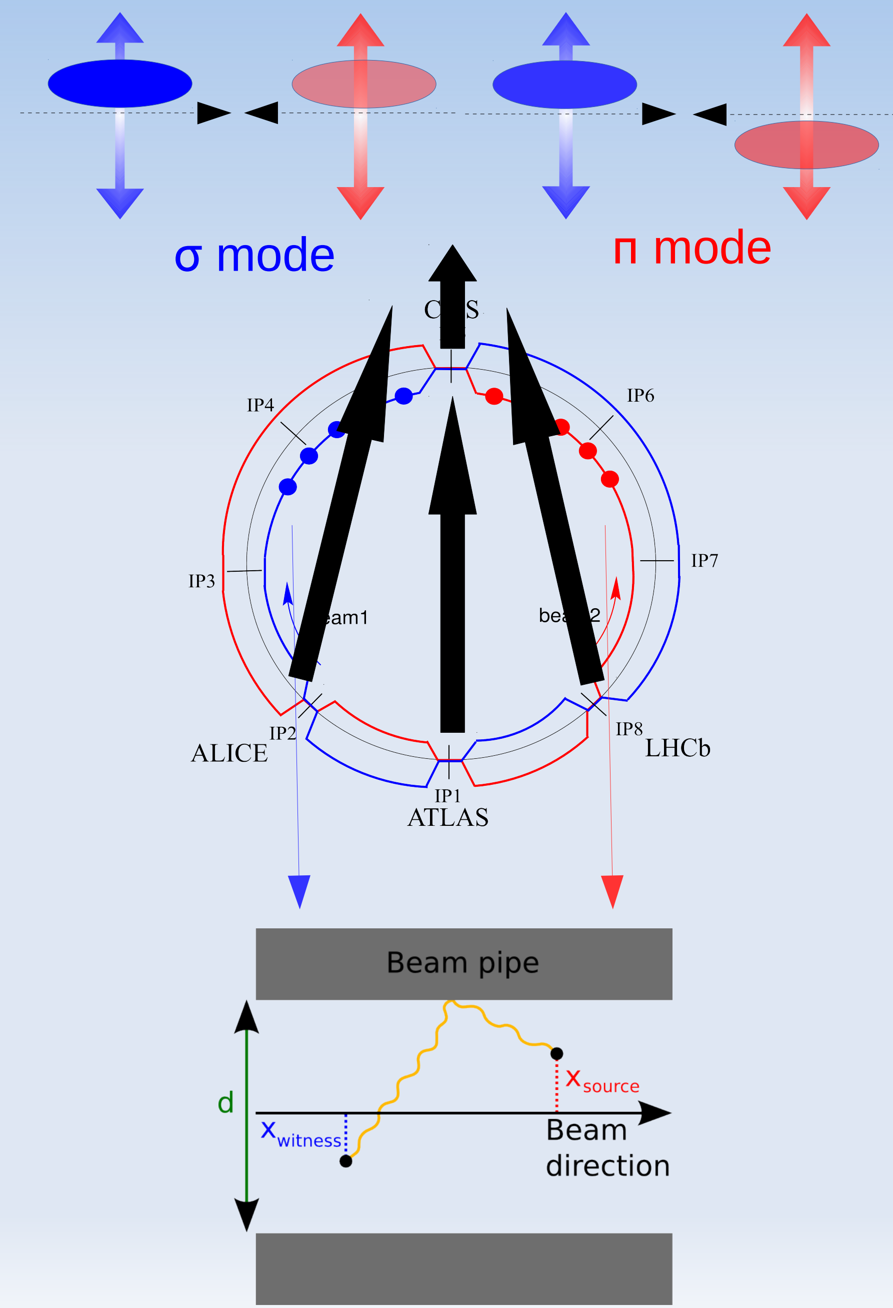
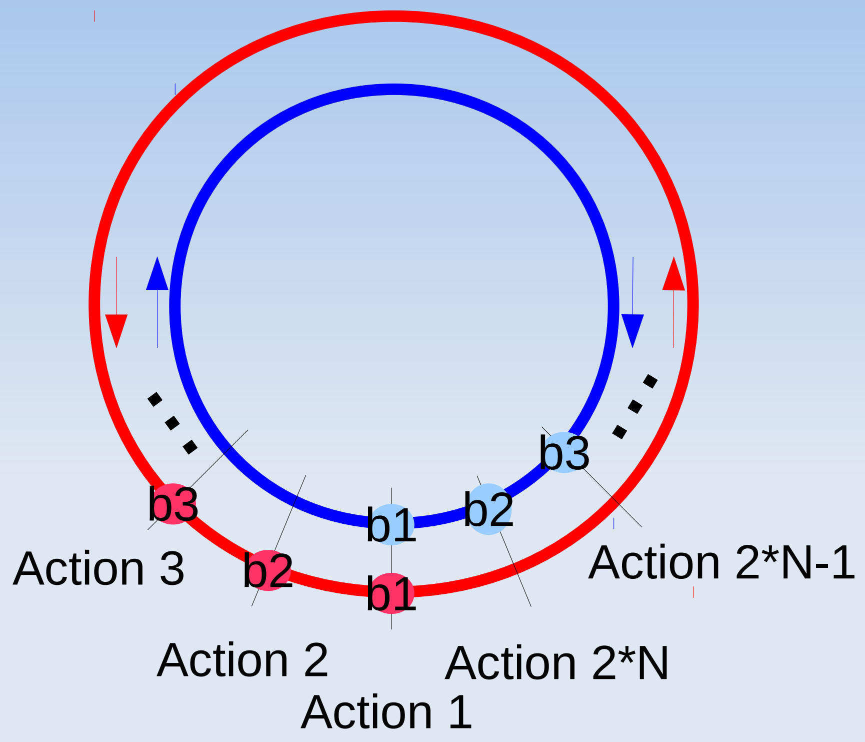


Stability of colliding beams

- How can we estimate the stability of beams in collision ?
  - In the so-called strong-strong regime, the dynamic of the two beams needs to be treated self-consistently
- Two main approaches based on normal mode analysis of :
  - Coupled Vlasov equations [K. Yokoya, Y. Alexahin]
  - Coherent one turn matrix [V.V. Danilov, E.A. Perevedentsev, I.N Nesterenko, S.M. White, X. Buffat]



Coherent one turn matrix



- In a general way, the model can be understood as the development and normal mode analysis of the one turn matrix for all bunches of the two beams
  - Need to derive the linearised coherent forces in a given basis

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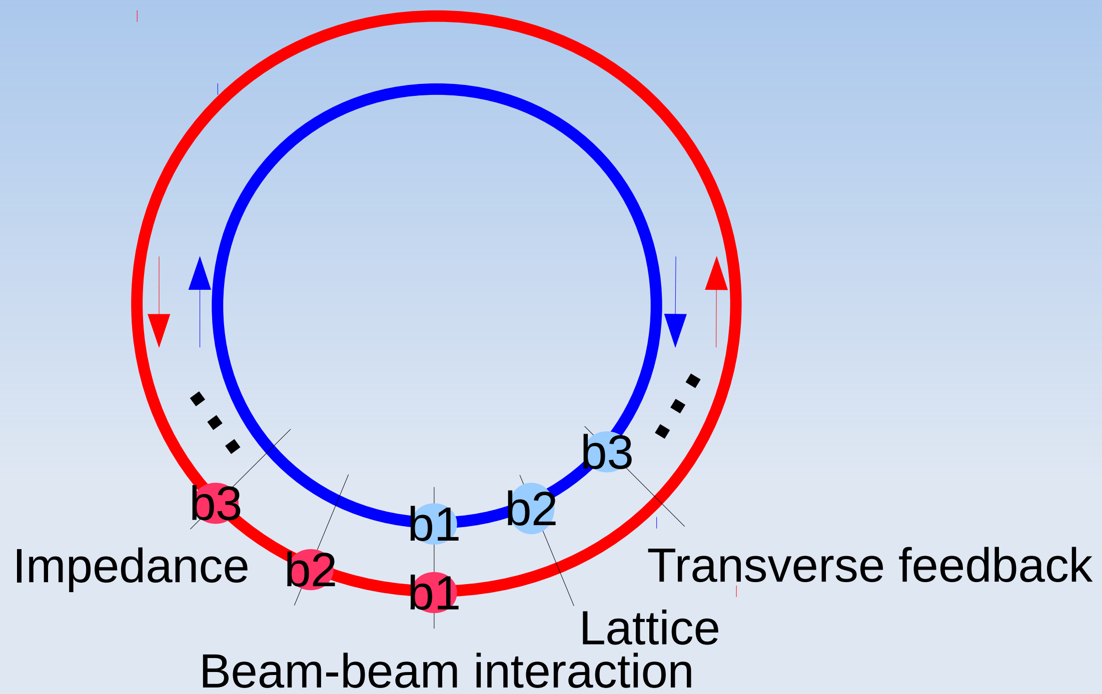

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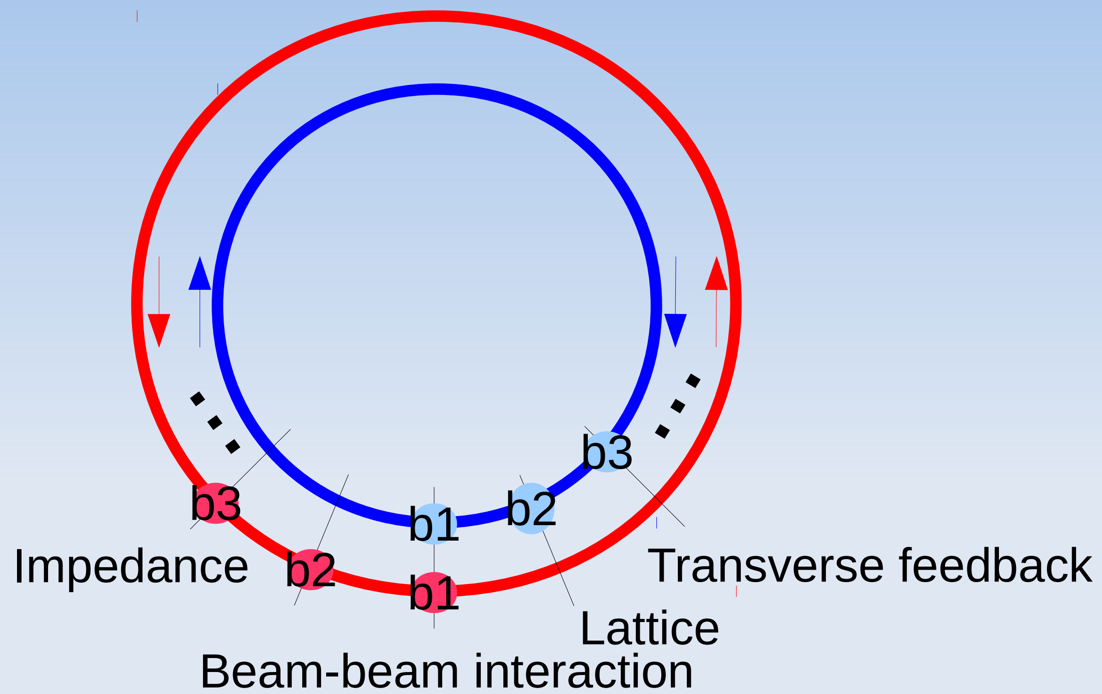

$$\underline{x}(step\,1) = M_{step\,1}\,\underline{x}(0)$$

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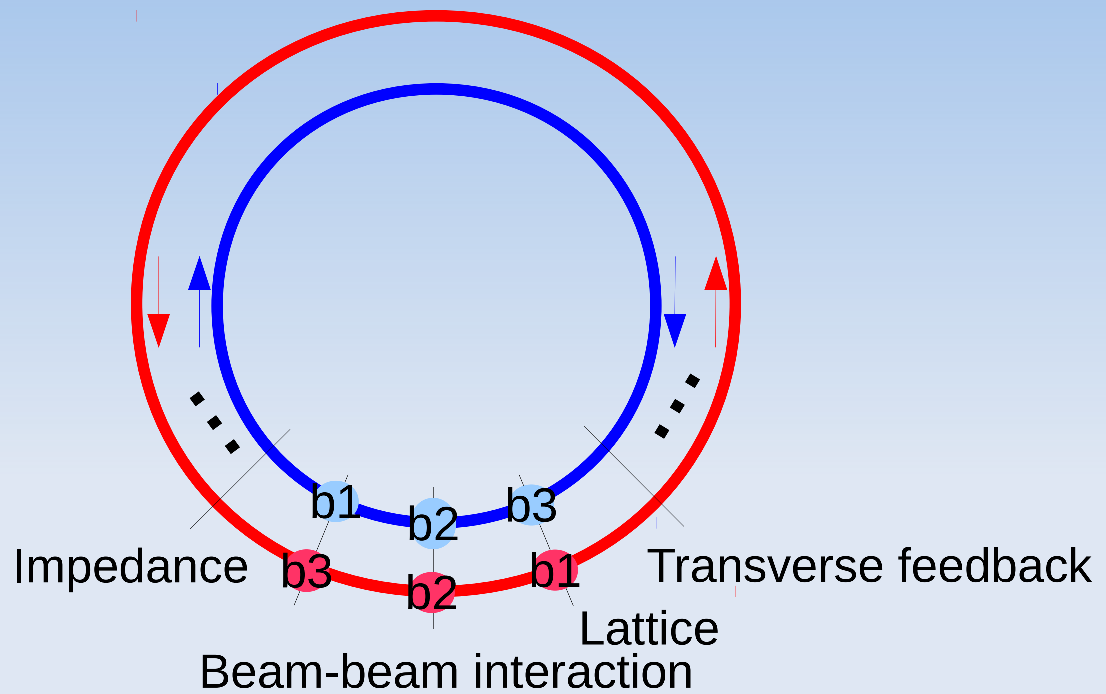

$$\underline{x}(step\,2)=M_{step\,2}\cdot M_{step\,1}\,\underline{x}(0)$$

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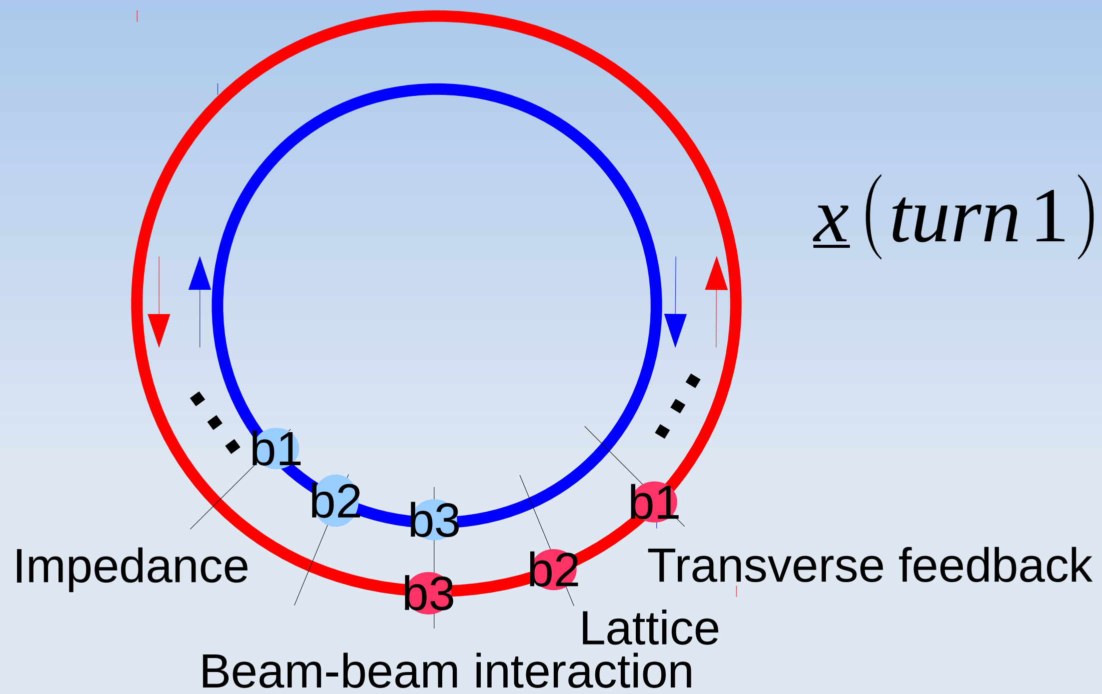

$$\underline{x}(turn\,1) = M_{step\,2N} \cdot \ldots \cdot M_{step\,2} \cdot M_{step\,1}\, \underline{x}(0)$$

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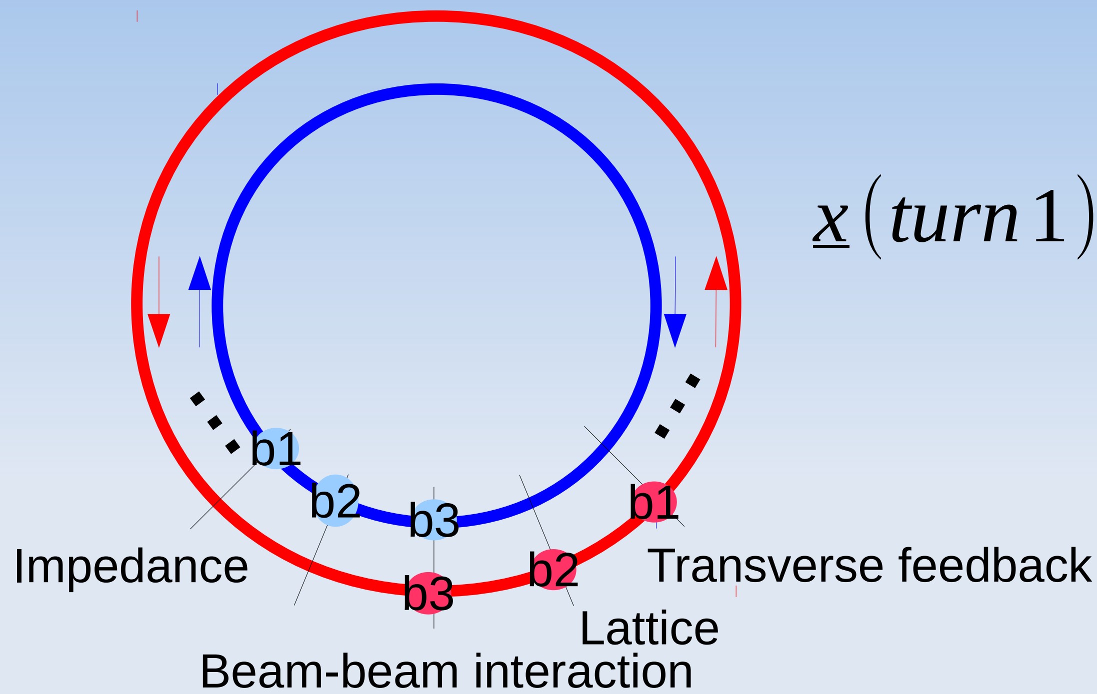

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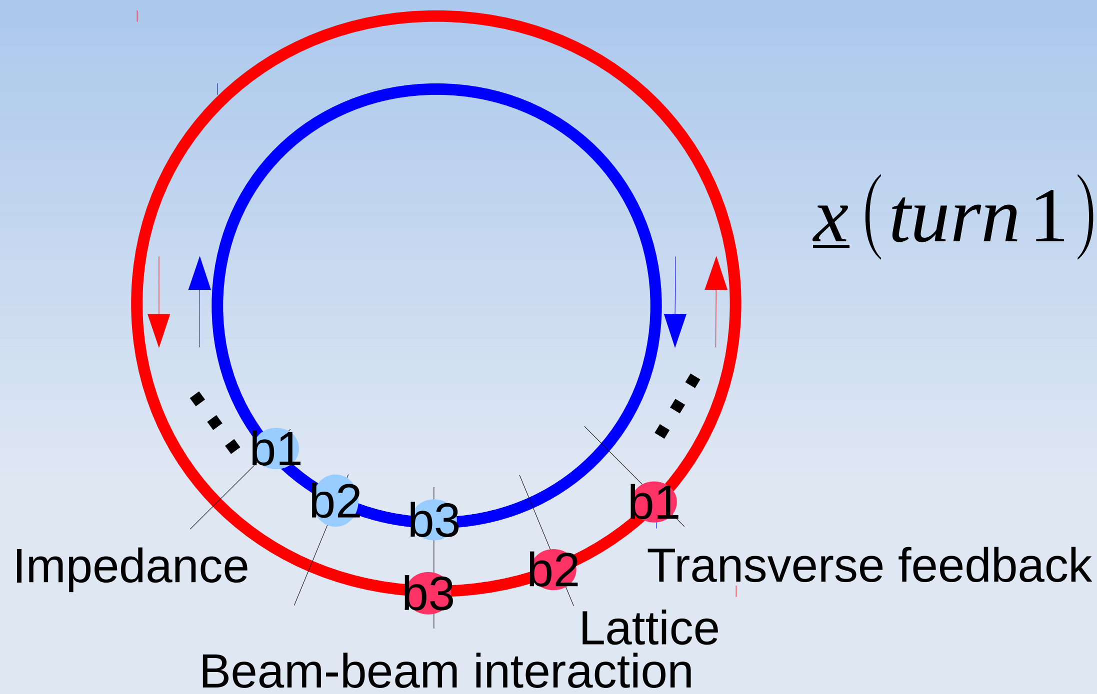

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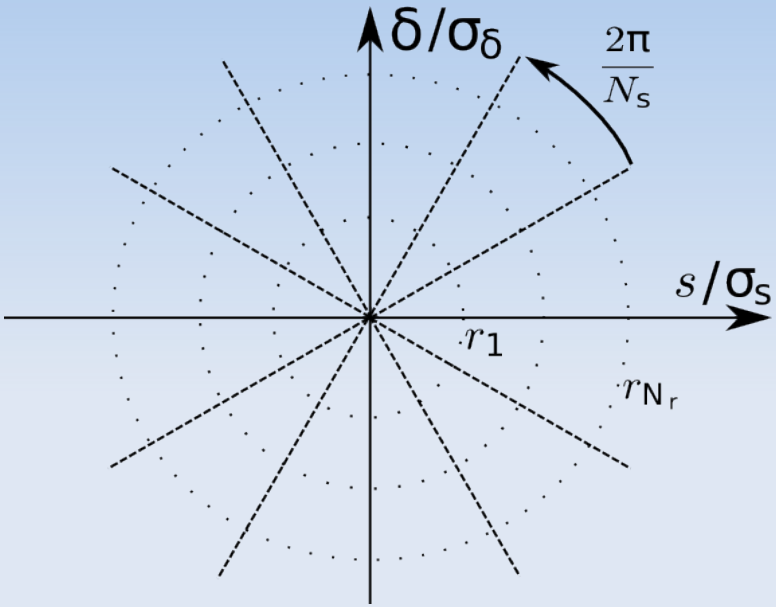
    → The circulant matrix model basis

The circulant matrix model basis

- Polar discretisation of the longitudinal phase space in cells (slices and rings)
  - The dynamical variables are the transverse positions and momentum (1 or 2 planes) of the cells
  - The synchrotron motion corresponds to a rotation of the slices → circulant matrix
  - The basis can be easily extended to describe several bunches per beam
- Initially developed to study the stabilisation of the TMCI with a feedback [V.V. Danilov] and for coherent synchrobetatron beam-beam modes in VEPP-2M [E.A. Perevedentsev]



$$\underline{x}(t) = M^{t}_{One\,turn}\,\underline{x}(0) = \sum_{j} e^{-2\pi i Q_j t}\,\underline{v}_j$$

The unperturbed circulant matrix

$$M_{1\mathrm{b}} = \frac{1}{N_r N_s} \mathbb{I}_{N_r} \otimes P_{N_s}^{N_s Q_s} \otimes B_0(2\pi Q_{y,0})$$

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Unperturbed betatron motion (w/o chromaticity)

$$B_0 = \begin{pmatrix} cos(2\pi Q) & \beta sin(2\pi Q) \\ \frac{-1}{\beta} sin(2\pi Q) & cos(2\pi Q) \end{pmatrix}$$

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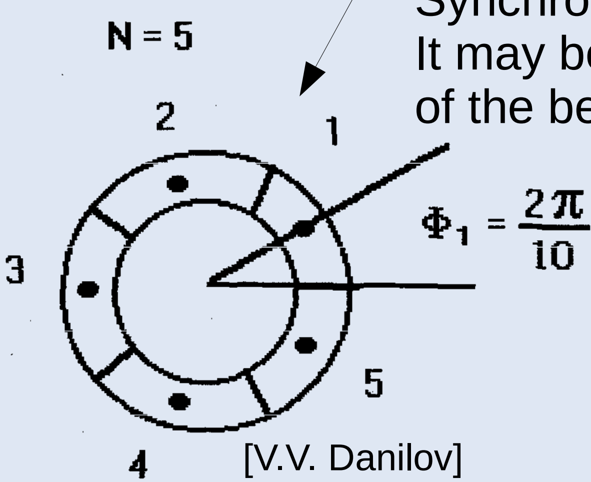
Synchrotron motion within each ring
It may be extended to include the chromatic shift of the betatron phase (see backup)



[V.V. Danilov]

$$P_{N_s} = \begin{pmatrix} 0 & 1 & & & \\ & 0 & 1 & & \\ & & \ddots & \ddots & \\ 1 & & & 0 & 1 \end{pmatrix}$$

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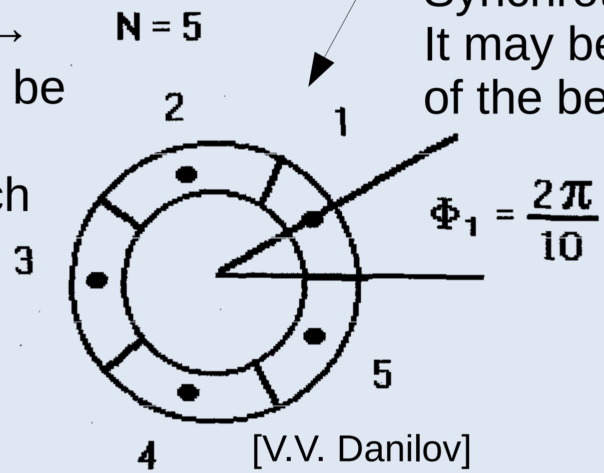
Identical synchrotron tune for each ring → the matrix can also be constructed with a different Qs for each ring



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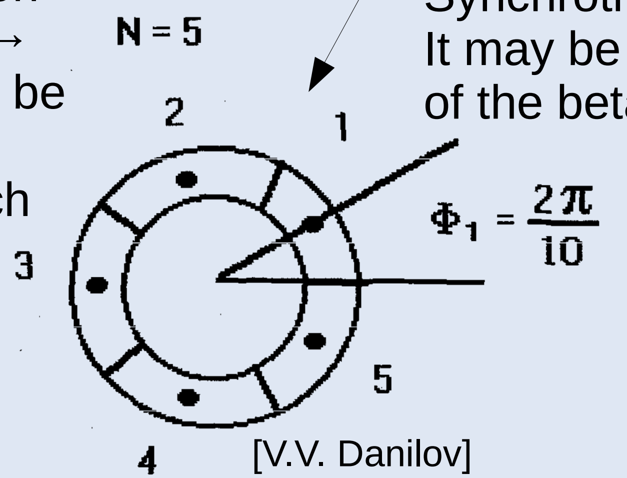

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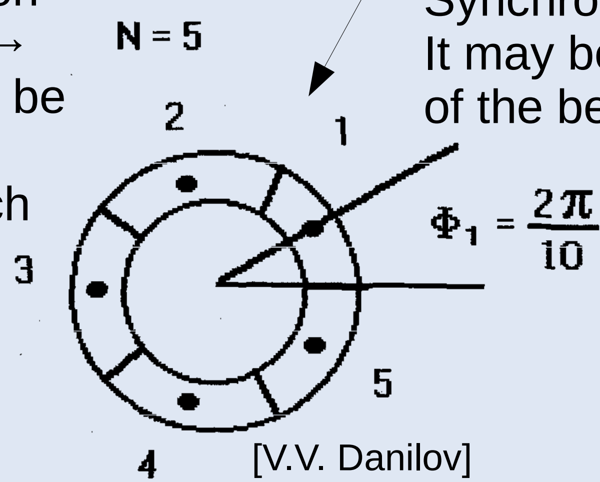

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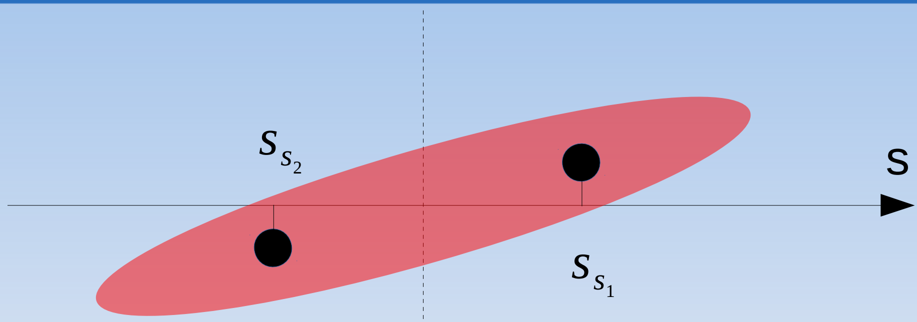
For multiple identical beam / bunches : $M = \mathbb{I}_{N_{\mathrm{beam}}} \otimes \mathbb{I}_{N_{\mathrm{bunch}}} \otimes M_{1\mathrm{b}}$

In practice, the matrix of each beam/bunch can be build based on different parameters

Example : Wake fields



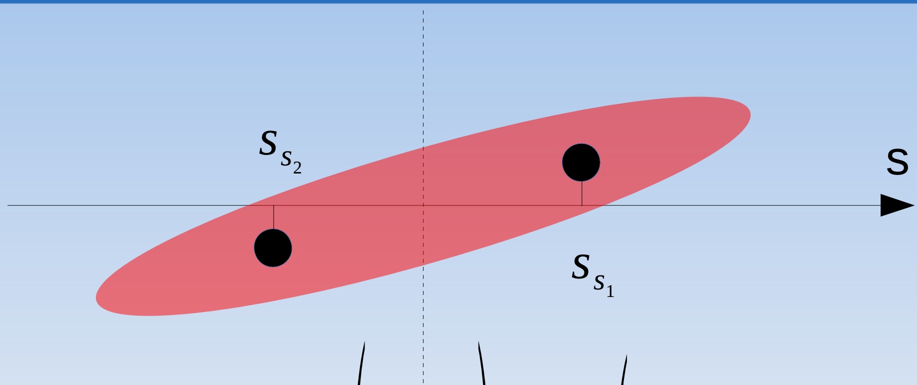
$$\Delta x_{s2}' = W_{dip}(s_{s2} - s_{s1})\, x_{s1}$$

$$\begin{pmatrix} x_{s1} \\ x_{s1}' \\ x_{s2} \\ x_{s2}' \end{pmatrix}_{t+1} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ W_{dip} & 0 & 0 & 1 \end{pmatrix} \cdot M \begin{pmatrix} x_{s1} \\ x_{s1}' \\ x_{s2} \\ x_{s2}' \end{pmatrix}_{t}$$

- The wake field is introduced as binary interaction between particles within the cells of each bunch and of the other bunches of the beam based on the integrated wake function
  - The single beam model including the effect of the impedance (wake fields) is equivalent to Vlasov approach to second order in wake field strength assuming a slow synchrotron motion and a reasonable amount of slices

    ($1 << N_s << 1/Q_s$) [A. Maillard]
- This method is not appropriate in the presence of strong multiturn wake (e.g. resonators)
  - Can be modeled by computing the effect of the multiturn wake assuming a given multibunch mode number → Requires normal mode analysis of all multibunch modes individually

Example : Wake fields



$$\Delta x_{s2}' = W_{dip}(s_{s2} - s_{s1})\, x_{s1} + W_{quad}(s_{s2} - s_{s1})\, x_{s1}$$

$$\begin{pmatrix} x_{s1} \\ x_{s1}' \\ x_{s2} \\ x_{s2}' \end{pmatrix}_{t+1} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ W_{dip} & 0 & W_{quad} & 1 \end{pmatrix} \cdot M \begin{pmatrix} x_{s1} \\ x_{s1}' \\ x_{s2} \\ x_{s2}' \end{pmatrix}_{t}$$

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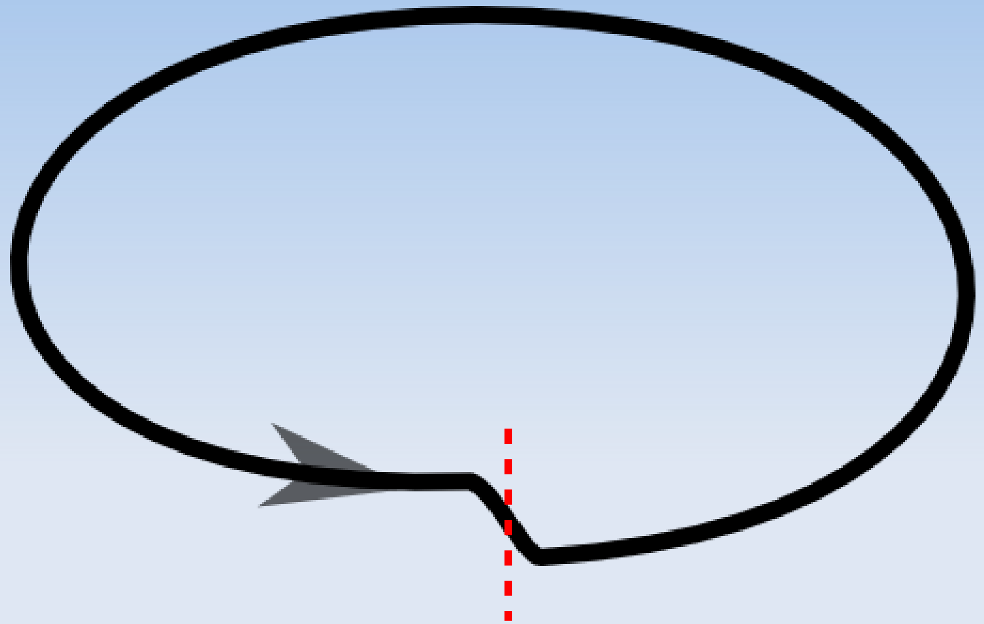
Example : Beam-beam interaction



$$\begin{pmatrix} x_1 \\ x_1' \end{pmatrix}_{t+1} = \begin{pmatrix} \cos(2\pi Q) & \sin(2\pi Q) \\ -\sin(2\pi Q) & \cos(2\pi Q) \end{pmatrix} \begin{pmatrix} x_1 \\ x_1' \end{pmatrix}_t$$

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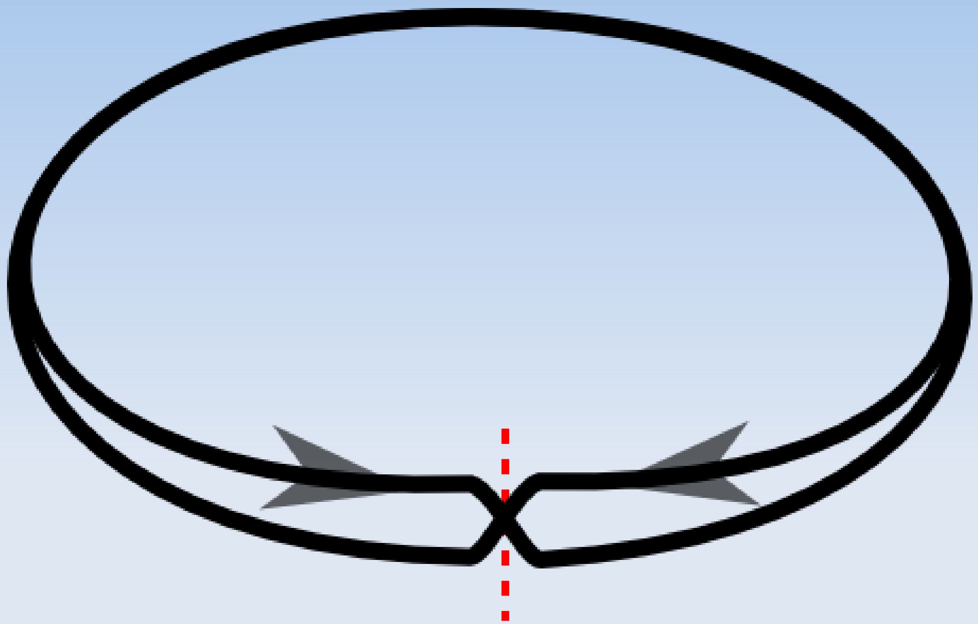

$$\begin{pmatrix} x_{B1} \\ x_{B1}' \\ x_{B2} \\ x_{B2}' \end{pmatrix}_{t+1} = \begin{pmatrix} \cos(2\pi Q) & \sin(2\pi Q) & 0 & 0 \\ -\sin(2\pi Q) & \cos(2\pi Q) & 0 & 0 \\ 0 & 0 & \cos(2\pi Q) & \sin(2\pi Q) \\ 0 & 0 & -\sin(2\pi Q) & \cos(2\pi Q) \end{pmatrix} \begin{pmatrix} x_{B1} \\ x_{B1}' \\ x_{B2} \\ x_{B2}' \end{pmatrix}_{t}$$

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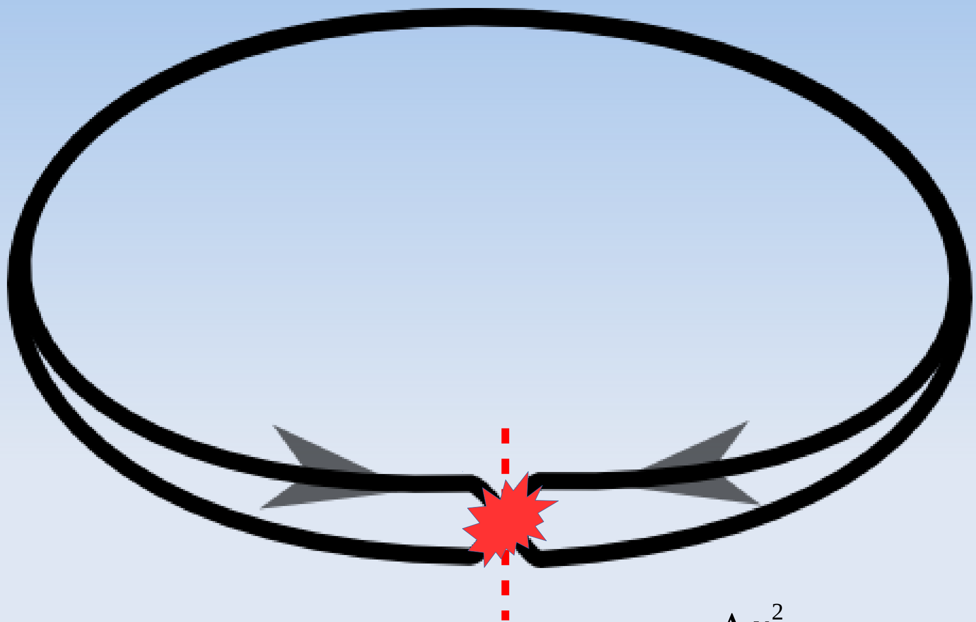

$$\Delta x'_{B1} = \frac{-2 r_0 N}{\gamma_r}\frac{1}{\Delta x}\left(1-e^{\frac{-\Delta x^2}{4\sigma^2}}\right) \approx k\left(x_{B1}-x_{B2}\right)$$

(linearised coherent force)

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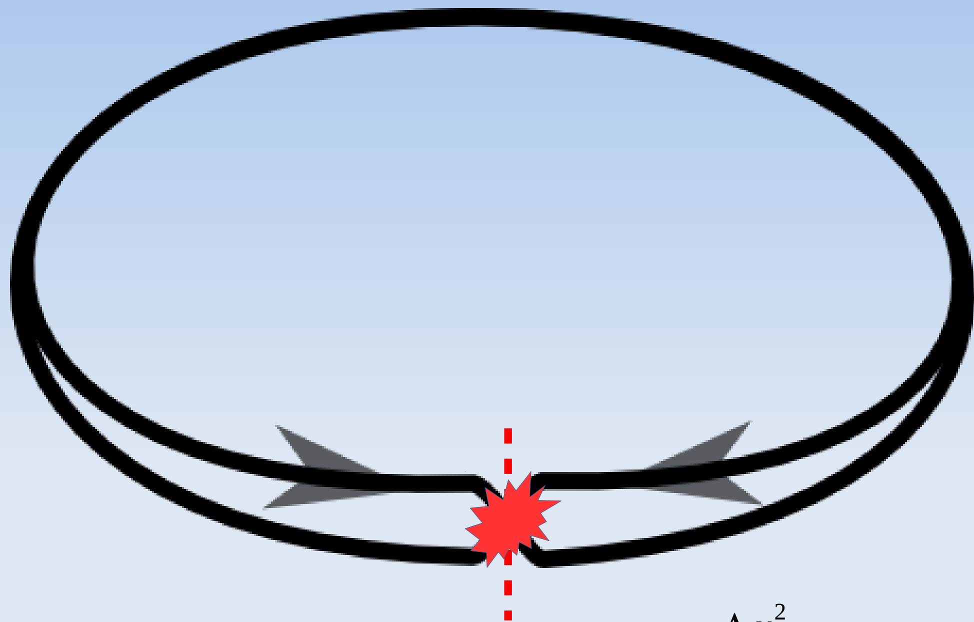

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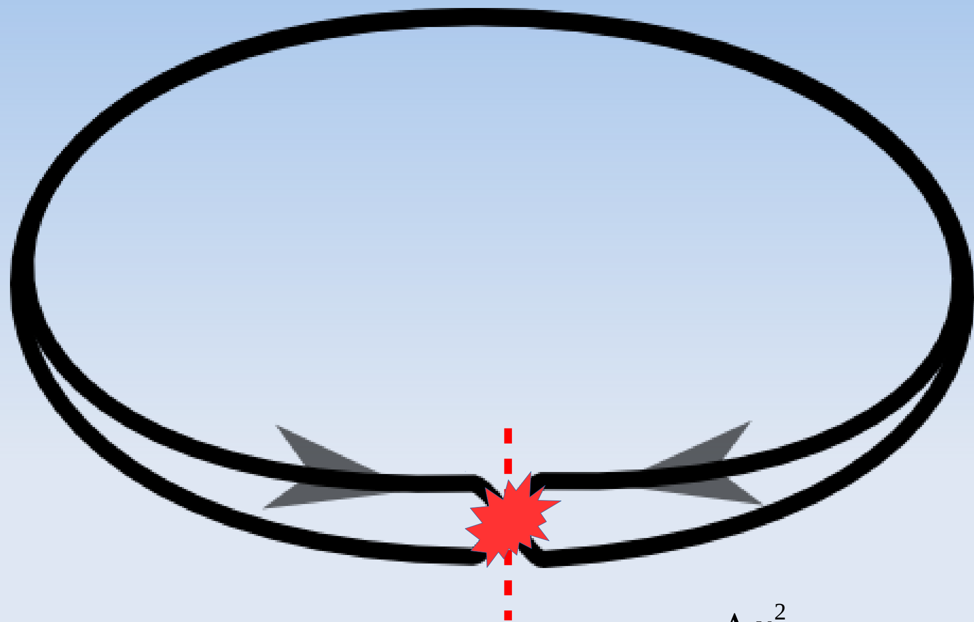

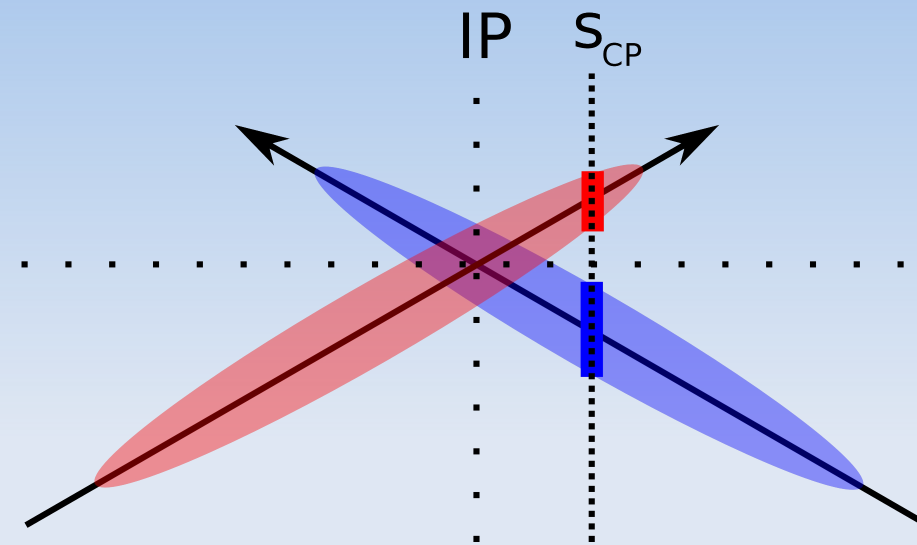

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→ This procedure is extended to binary collision of all the cells (possibly including the crossing angle and the hourglass effects)

Dos and don'ts

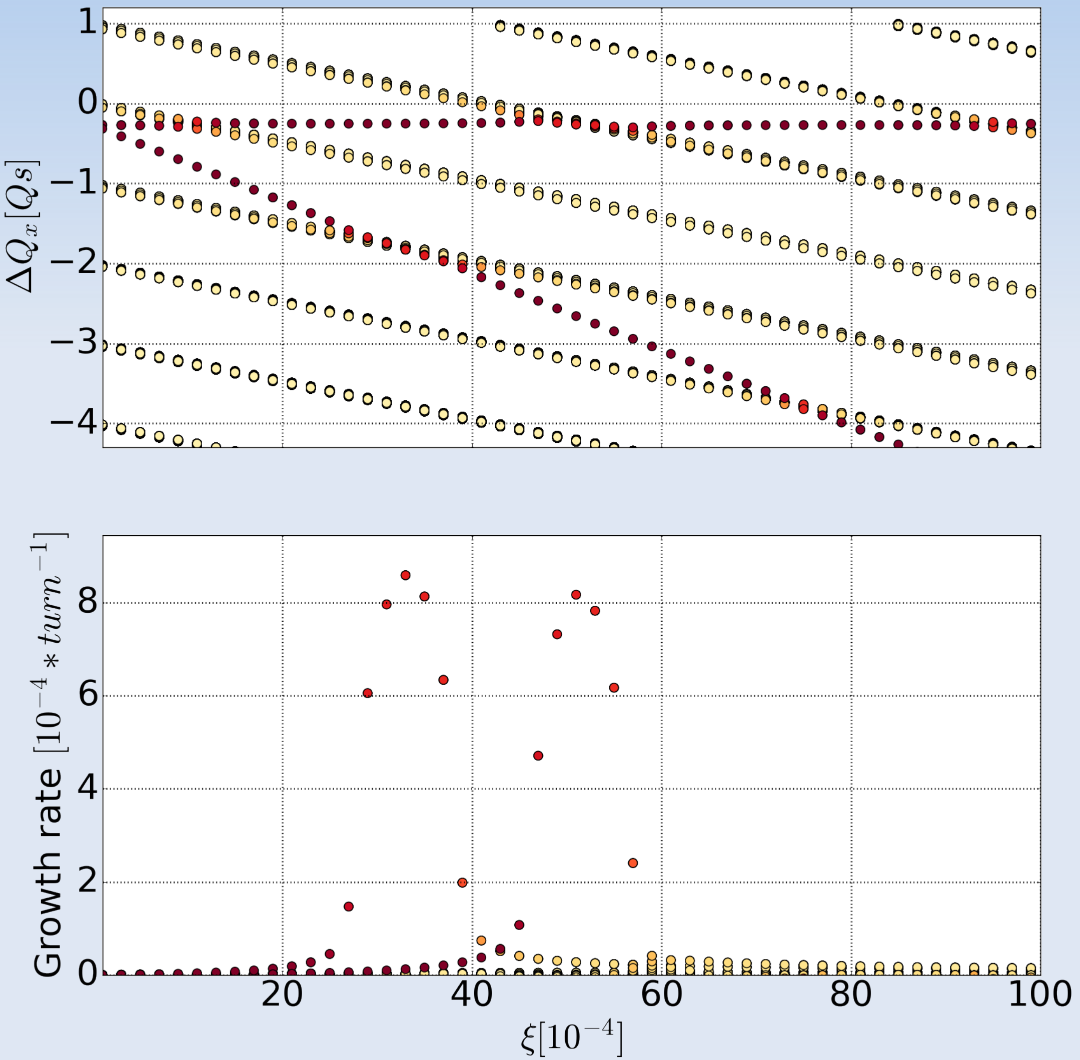
- Linear synchrobetatron motion
  - Non-linear RF
  - (Non-)Linear chromaticity
  - RF-Quadrupole [M. Schenk]
- Single turn dipolar and quadrupolar wake fields
- 5D beam-beam interaction (no energy change)
- Transverse feedback (including bandwidth limitation)
- Electron clouds ?
- Space charge ?

- Non-linear transverse motion
  - Transverse Landau damping
  - Yokoya factor
- Multiturn wake fields (can be done under strong assumptions)
- The limited memory of common computers limits the number of bunches
- Non-normal effects limit the predicting power of the normal mode analysis in some configurations (see backup)

- BimBim benchmark campaigns :
  - Impedance : Multibunch HEADTAIL - PyHEADTAIL – DELPHI – NHTVS
  - Beam-beam and impedance : COMBI – BeamBeam3D

Mode coupling instability of colliding beams

- Considering a configuration with low sychrobetatron coupling :
  - Small Piwinski angle (small crossing angle)
  - Large β* w.r.t the bunch length
- Mode coupling instability occurs when the frequency of coherent beam-beam modes with a centre of mass motion (red dots) reach the frequency of the head-tail modes



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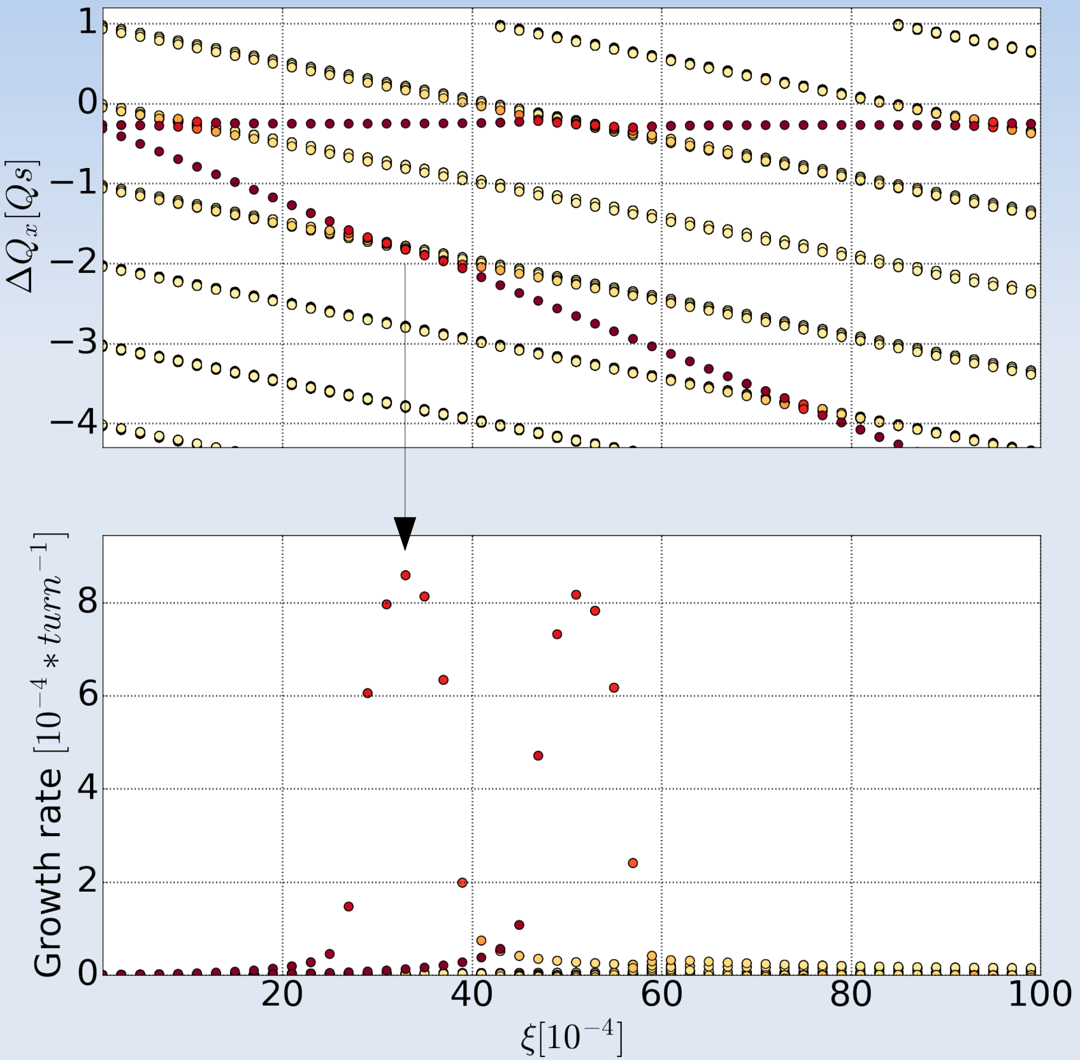

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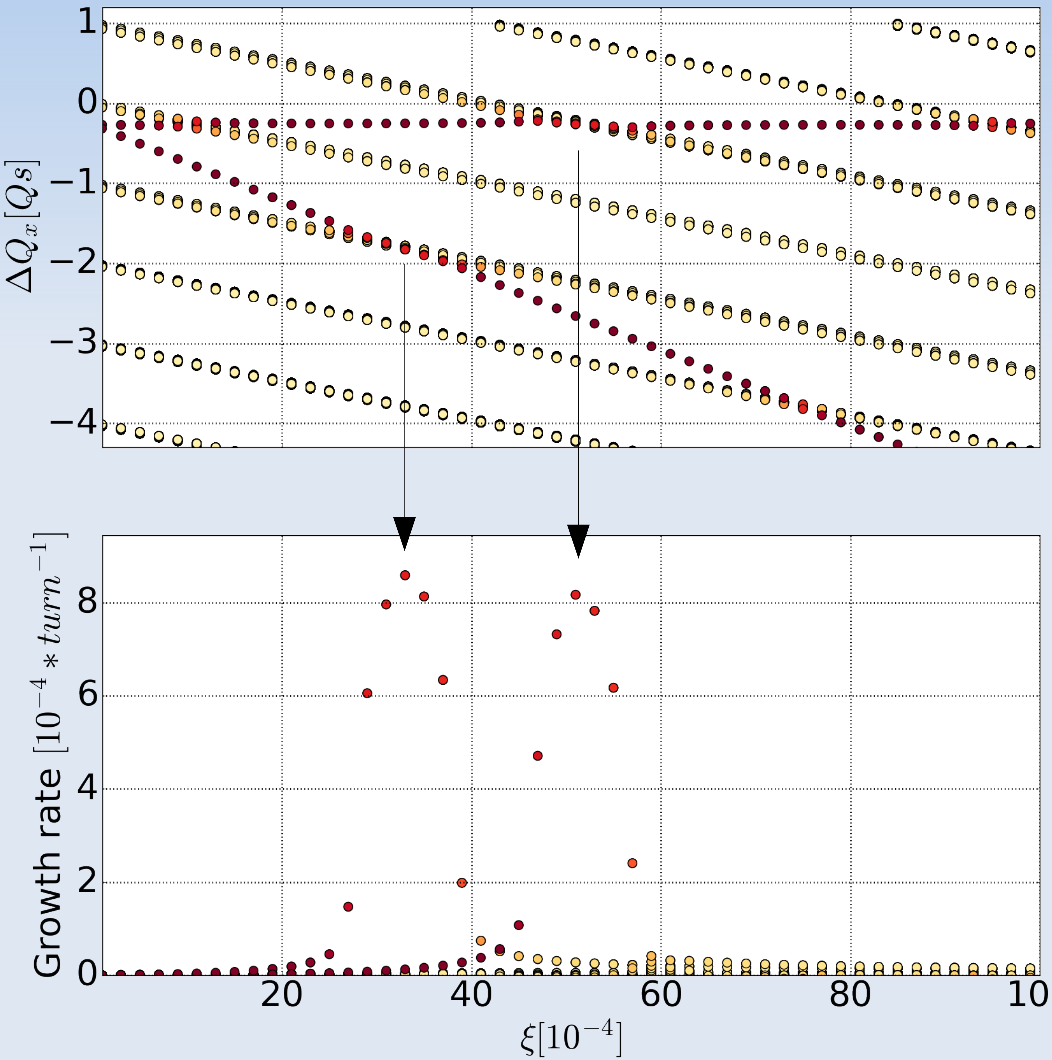
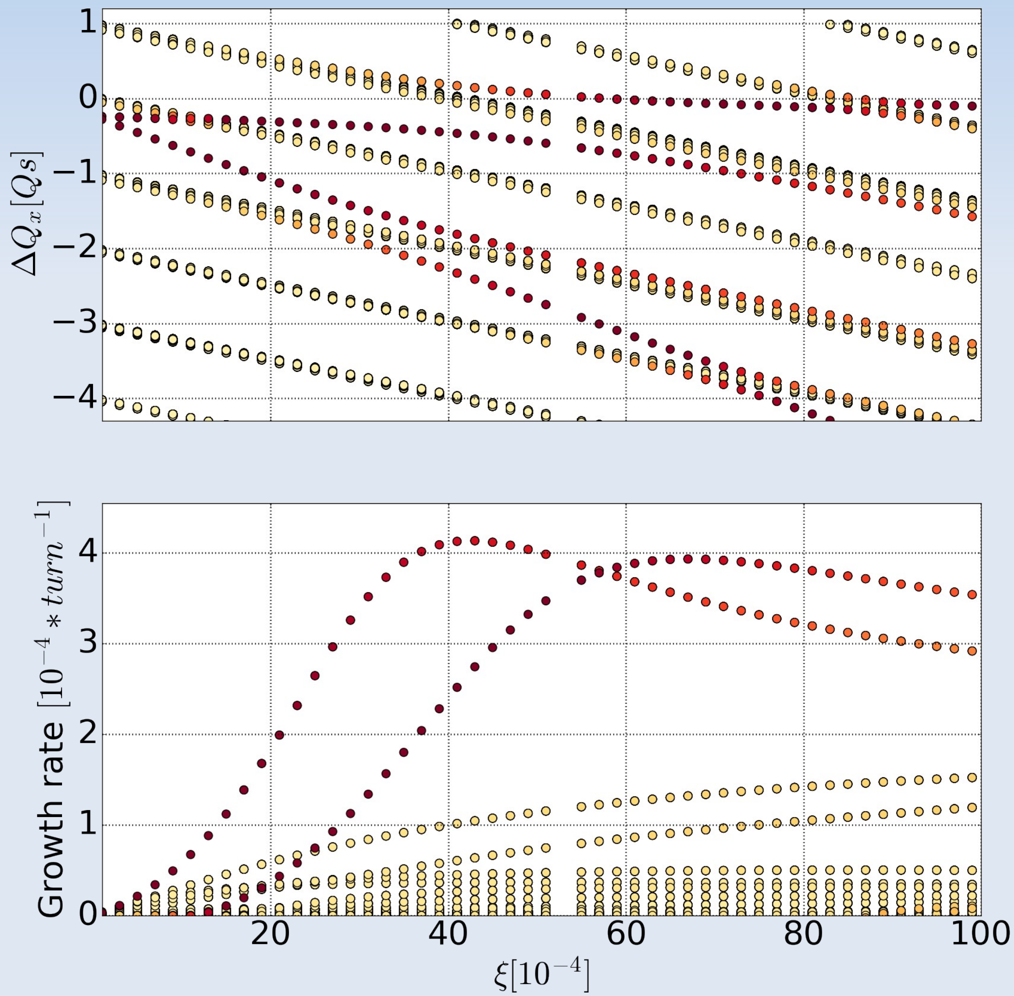


Stability mechanisms
Landau damping

- In configurations with strong synchrobetatron coupling (e.g. HL-LHC), the instability occur at any beam-beam tune shift

L. Barraud, Master Thesis, UPMC, Paris



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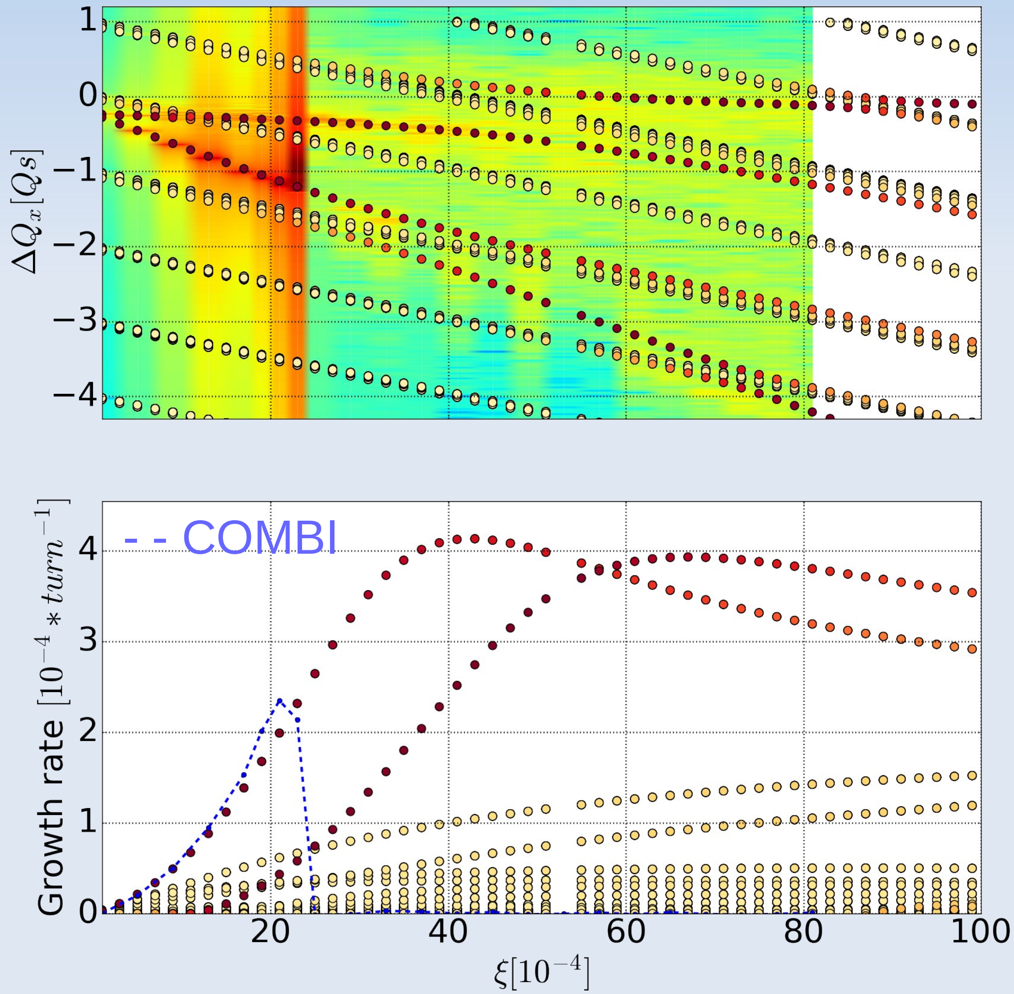
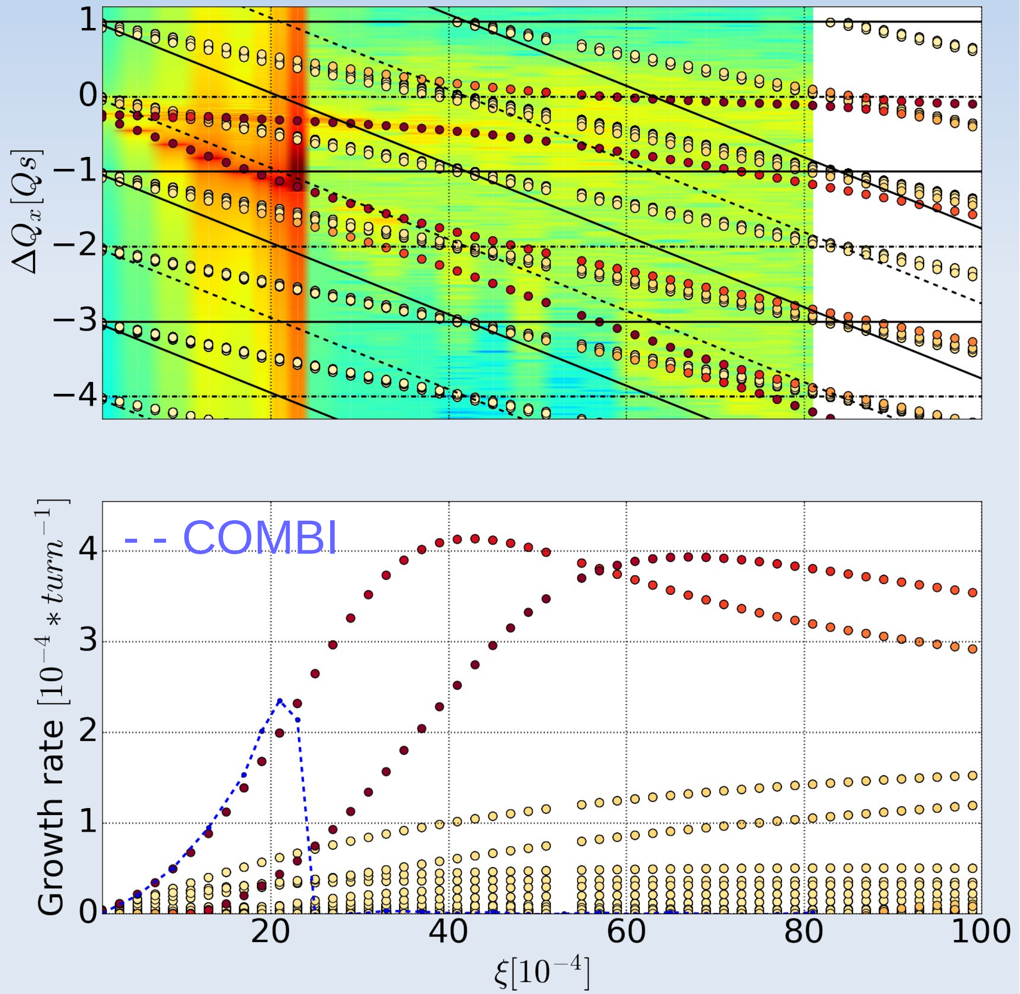


Stability mechanisms
Landau damping

- In configurations with strong synchrobetatron coupling (e.g. HL-LHC), the instability occur at any beam-beam tune shift
- Landau damping from the beam-beam induced tune spread is usually effective if the coherent modes frequency is within the incoherent tune spread and its side bands [Y. Alexahin, W. Herr]

L. Barraud, Master Thesis, UPMC, Paris



Stability mechanisms
Landau damping

- In configurations with strong synchrobetatron coupling (e.g. HL-LHC), the instability occur at any beam-beam tune shift
- Landau damping from the beam-beam induced tune spread is usually effective if the coherent modes frequency is within the incoherent tune spread and its side bands [Y. Alexahin, W. Herr]

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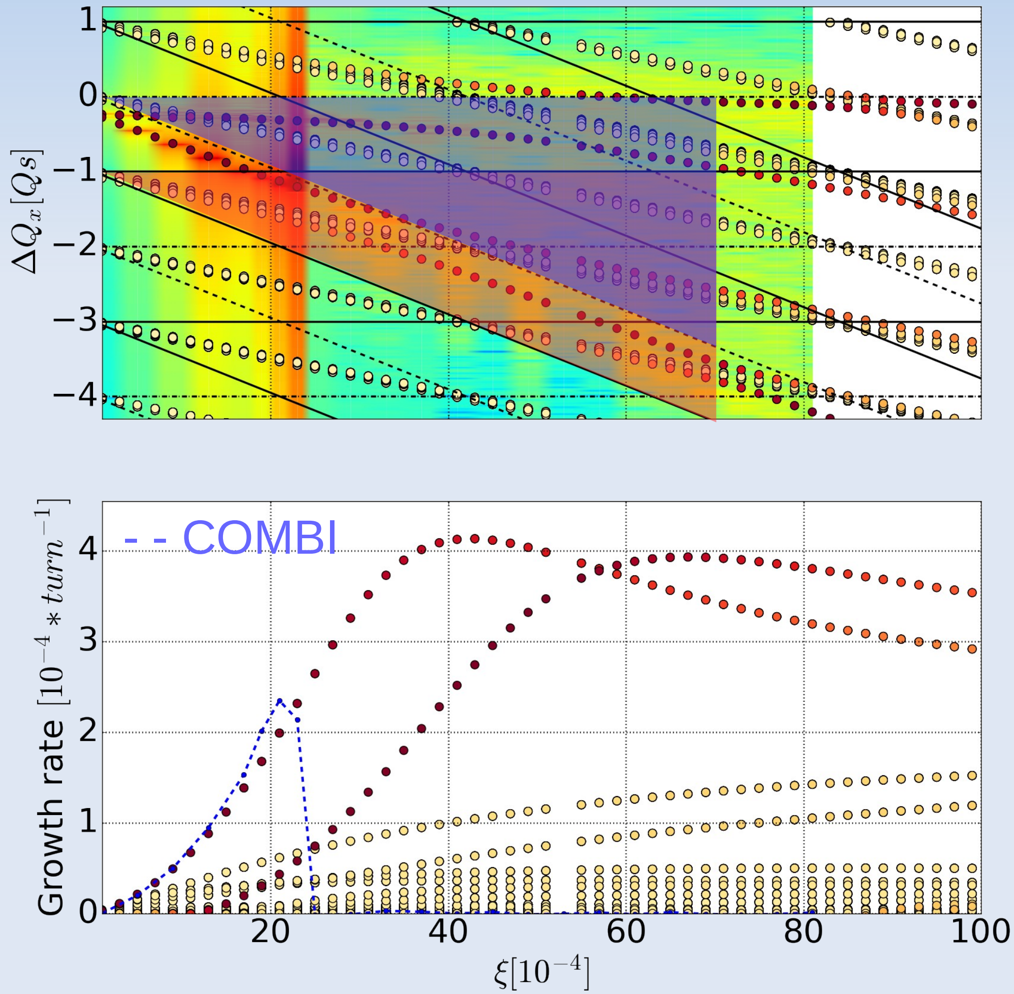

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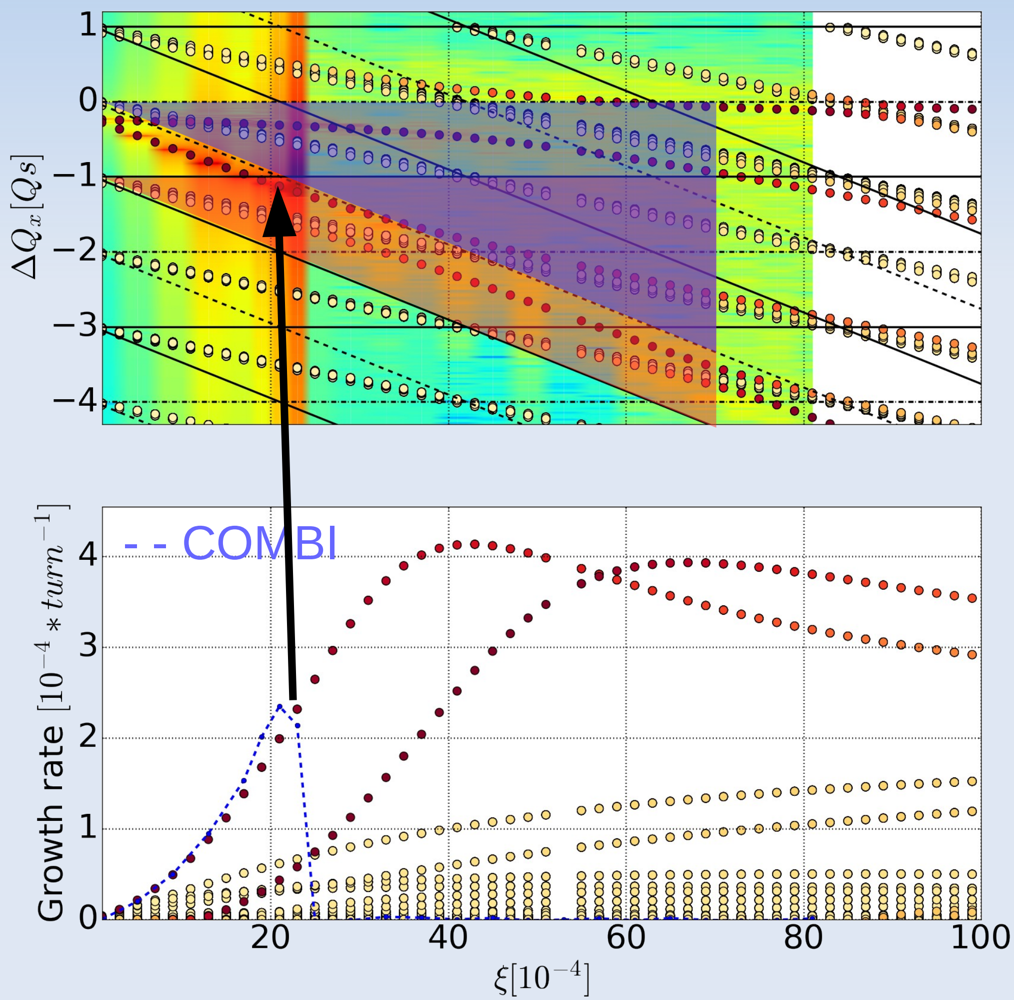

Stability mechanisms
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- Landau damping from the beam-beam induced tune spread is usually effective if the coherent modes frequency is within the incoherent tune spread and its side bands [Y. Alexahin, W. Herr]
  - Can become an issue in the case of head-on tune spread compensation with an e^- lens [L. Jin, S.M White]

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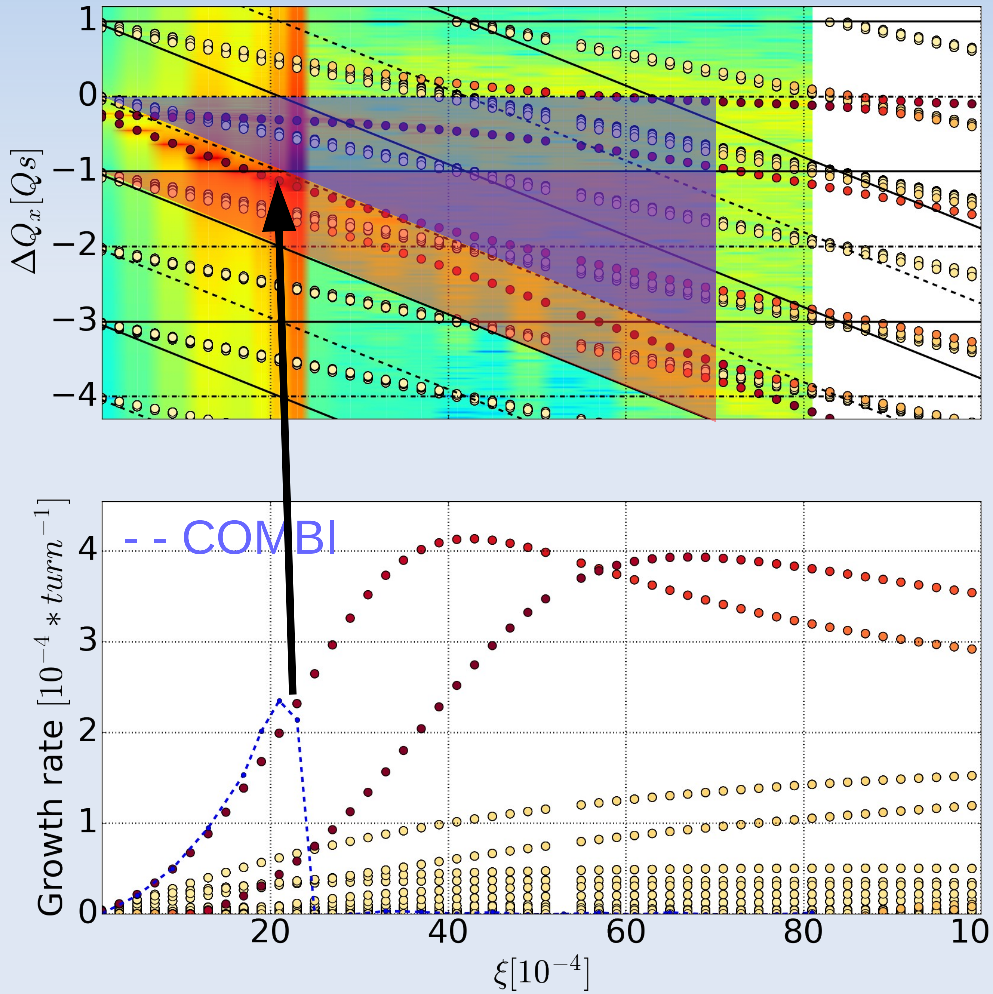

Stabilisation mechanisms
Transverse feedback

- A transverse feedback based on the centre of mass motion of the bunches is effective against the zero-mode coupling instability [S.M. White]
    - Coupling instability of higher order head-tail mode is still observed for beam-beam interaction with sychrobetatron coupling (crossing angle, hourglass effect)

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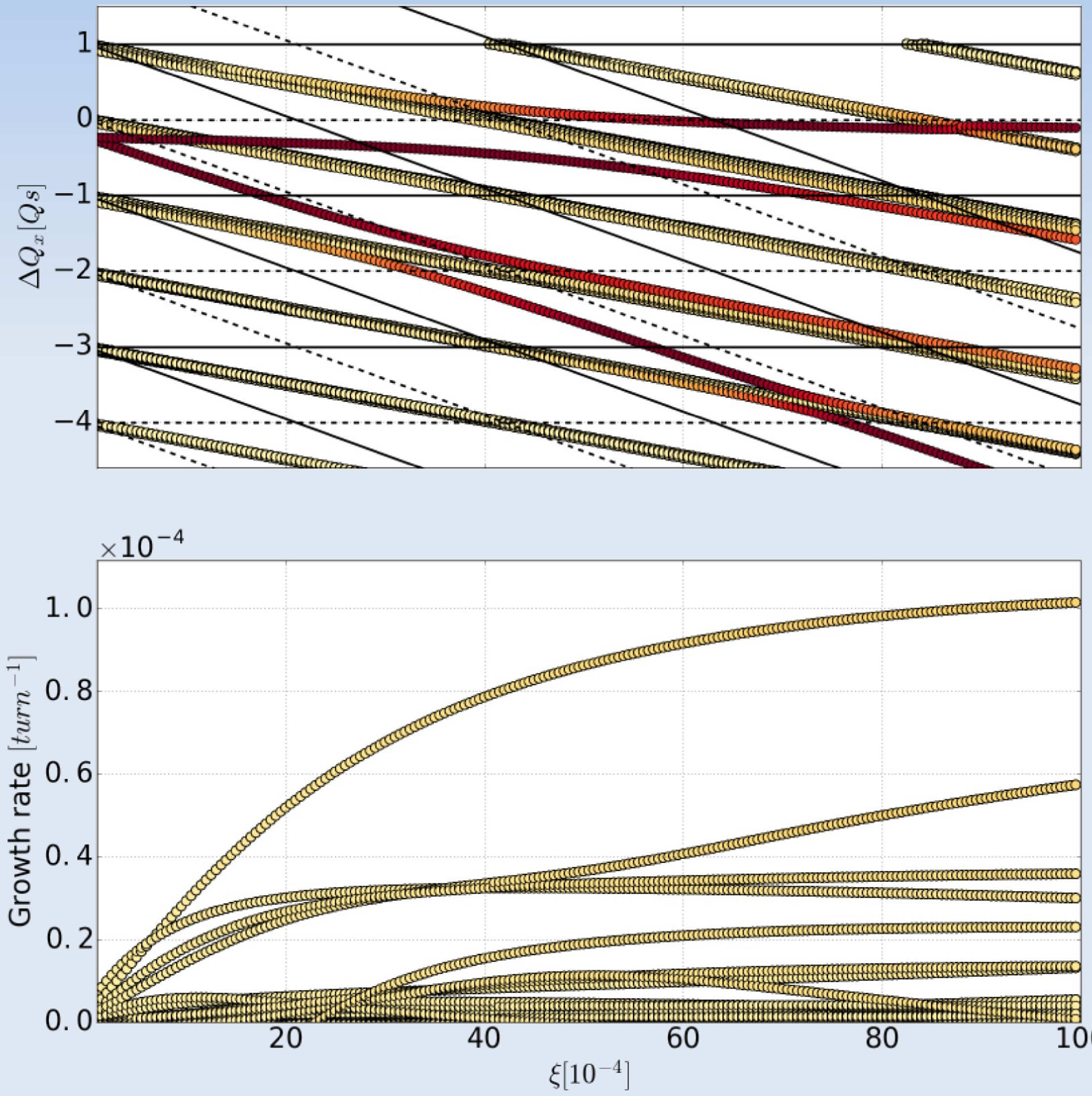
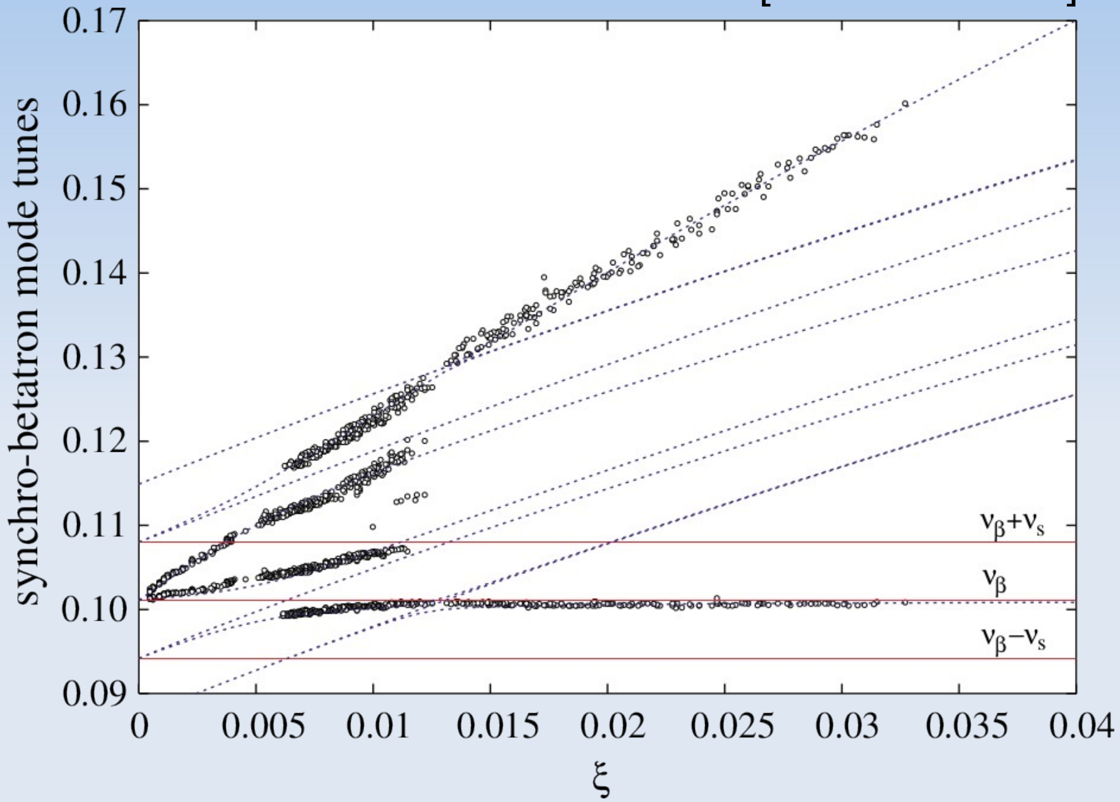


Observations
VEPP - 2M

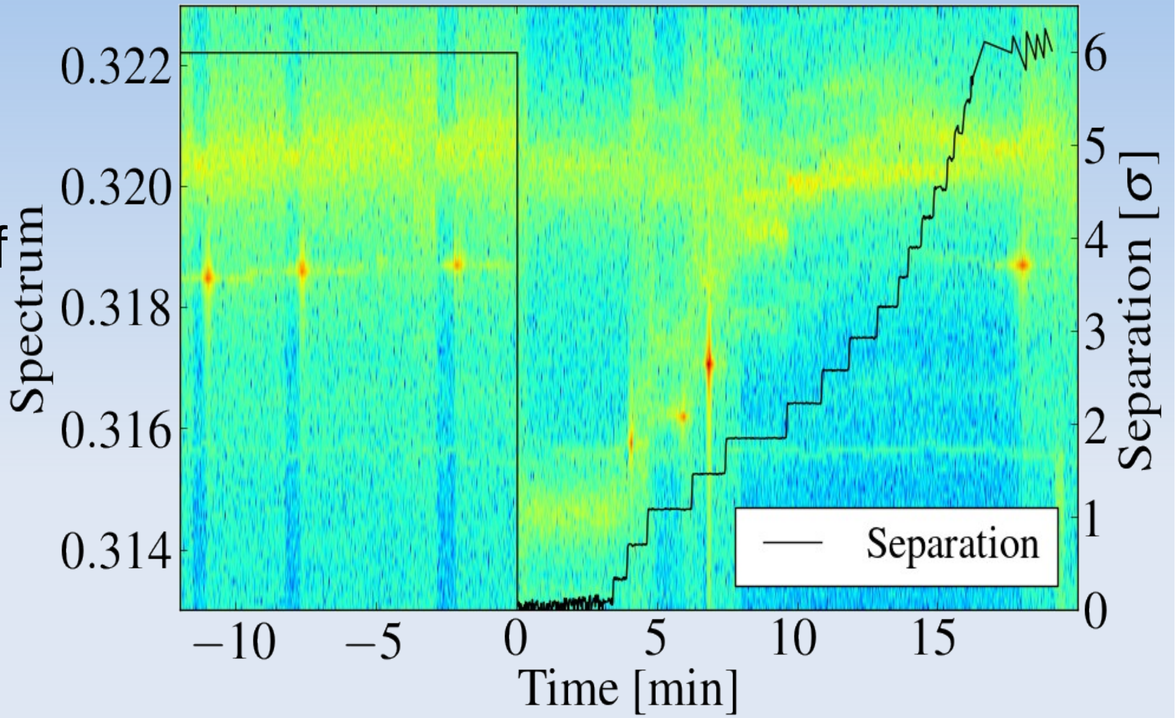
[N.I. Nesterenko]



- Electron-positron collision in two IPs with β* = 6 cm and σ_s = 3.5 cm
  - Significant hourglass effect → synchrobetatron coupling
- Measured mode frequencies matches the prediction of the circulant matrix model
- No instabilities were observed due to the low impedance and strong radiation damping
  - Oscillation measured by kicking the beams in the vertical plane

Observations LHC

- Two-beam Instability observed for intermediate separations between the beams corresponding to the frequency of the mode coupling



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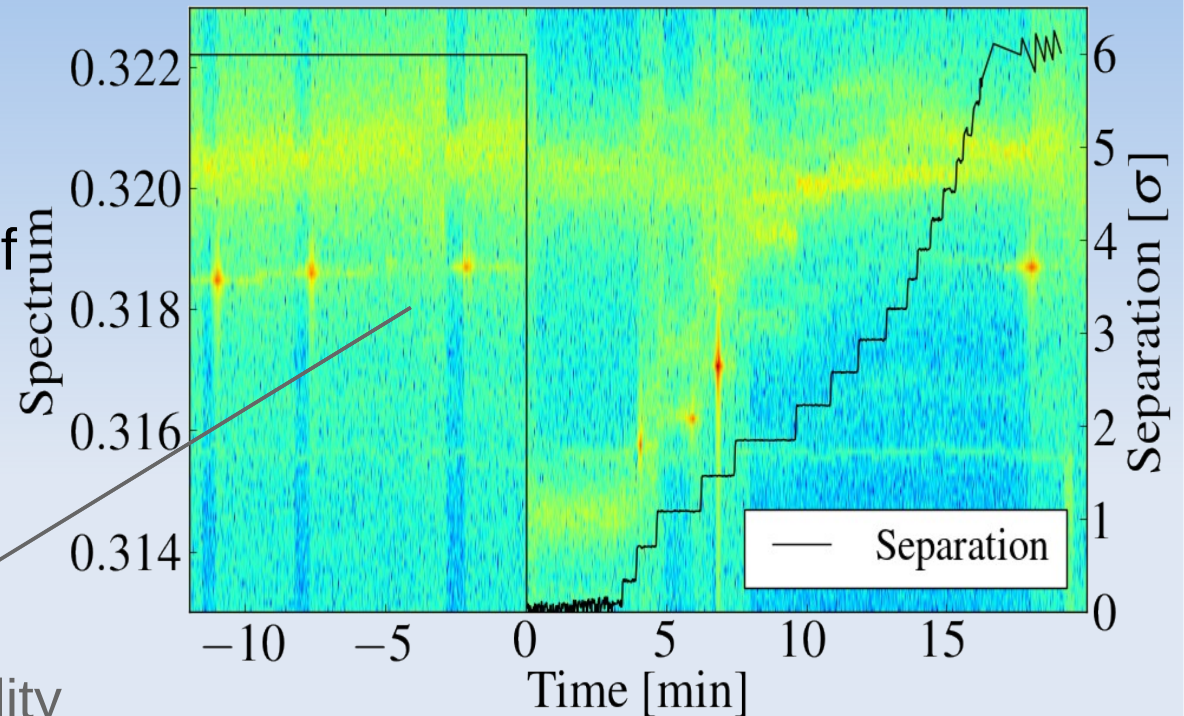
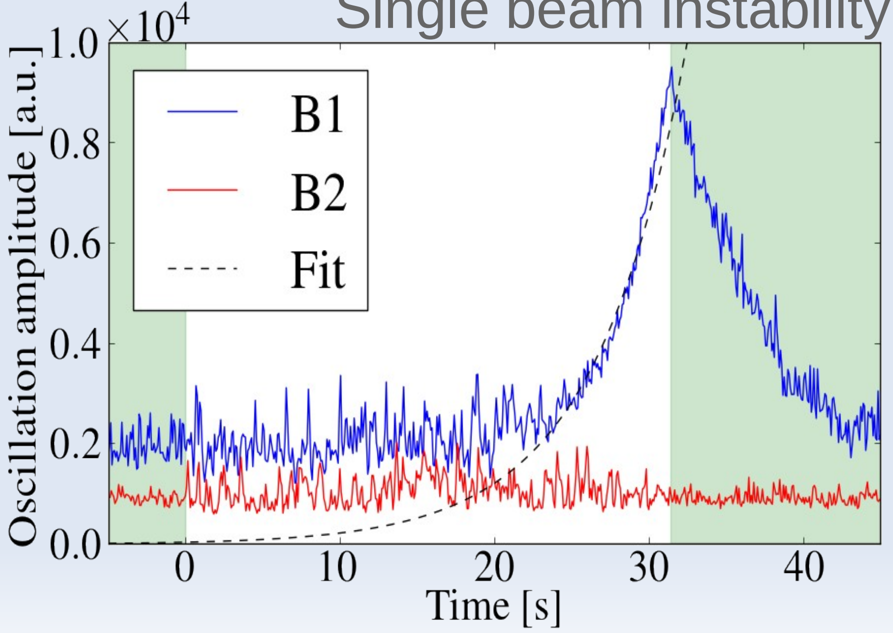


Single beam instability



Observations LHC

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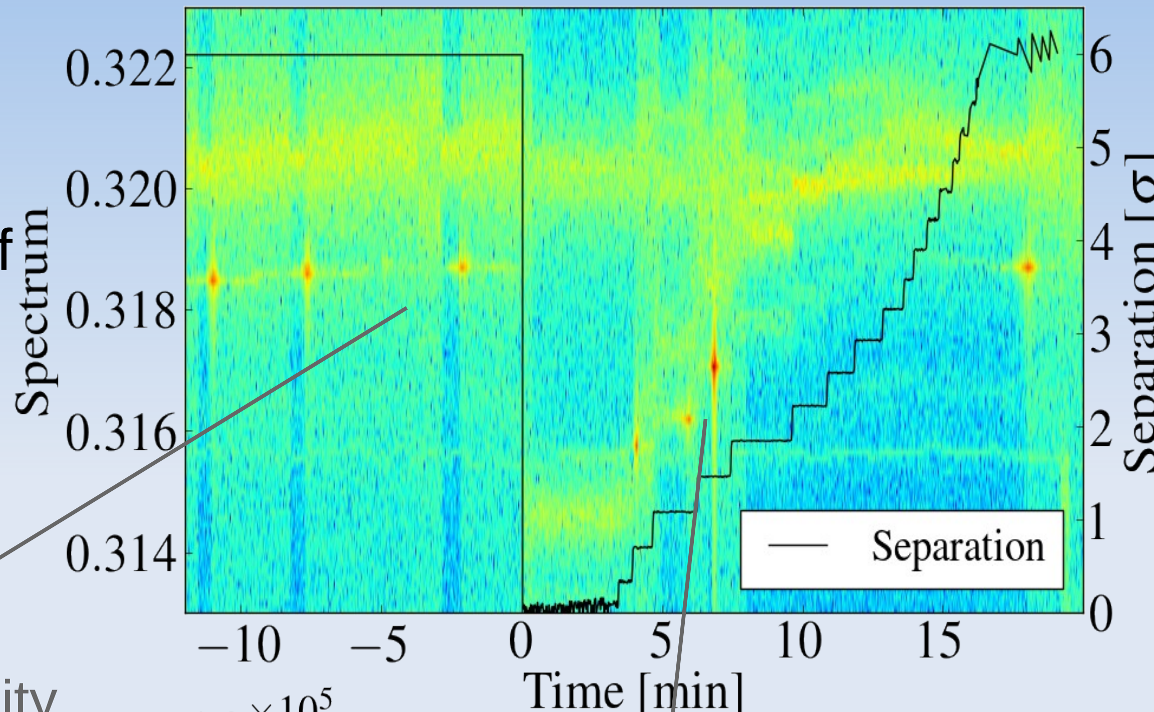

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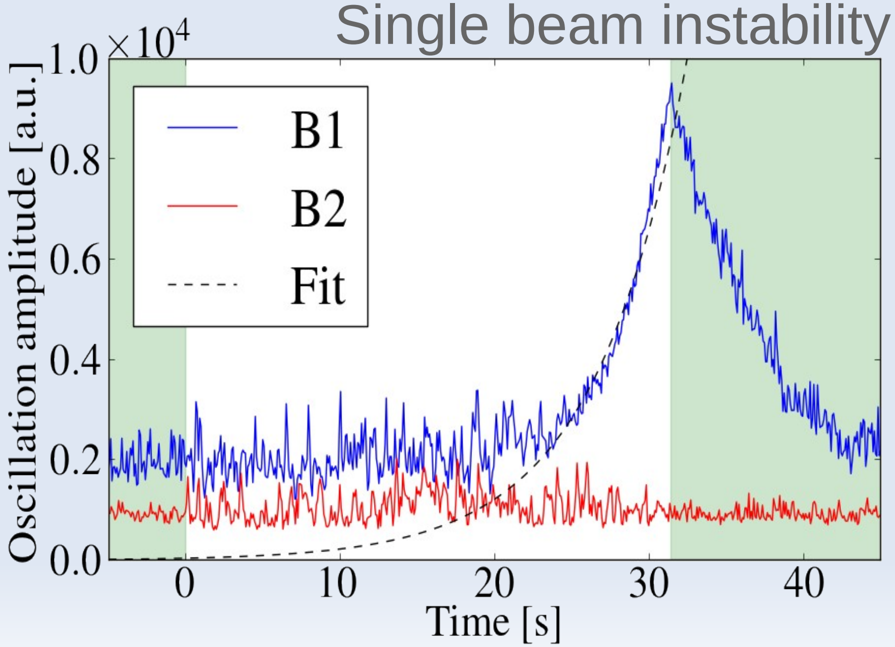
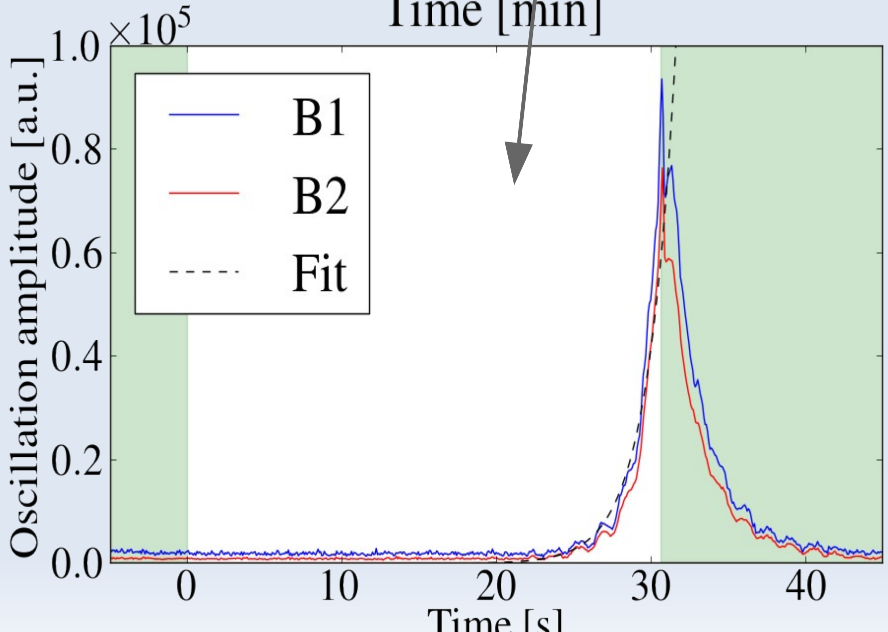
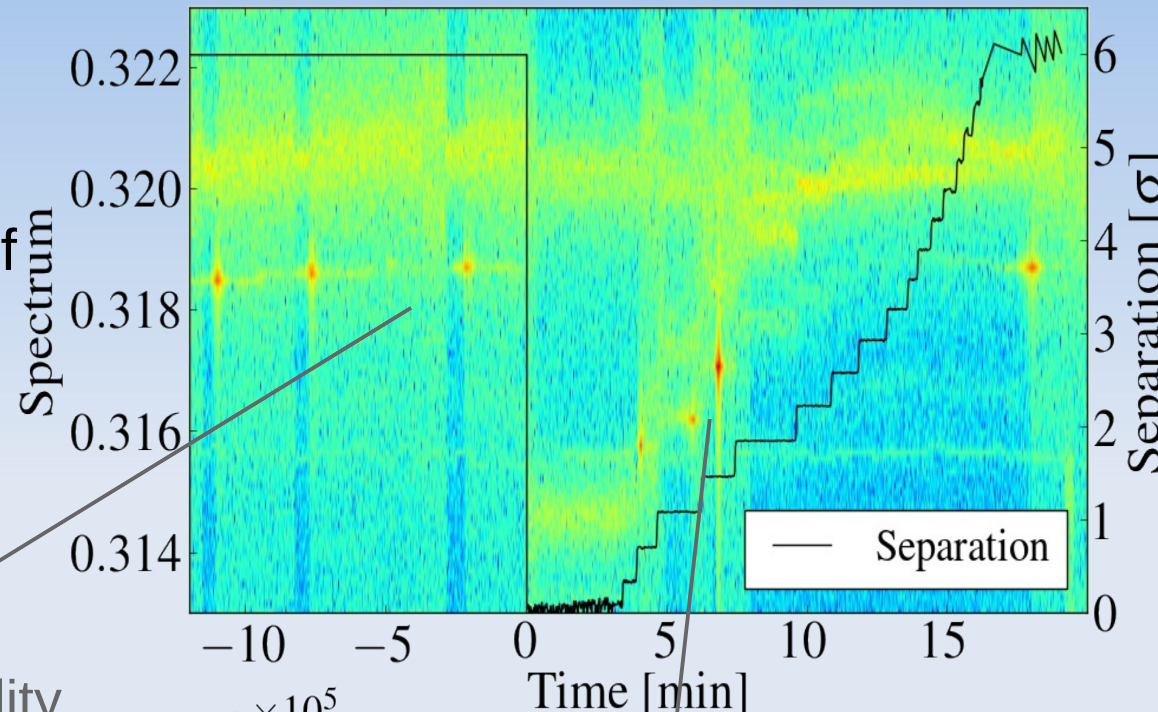


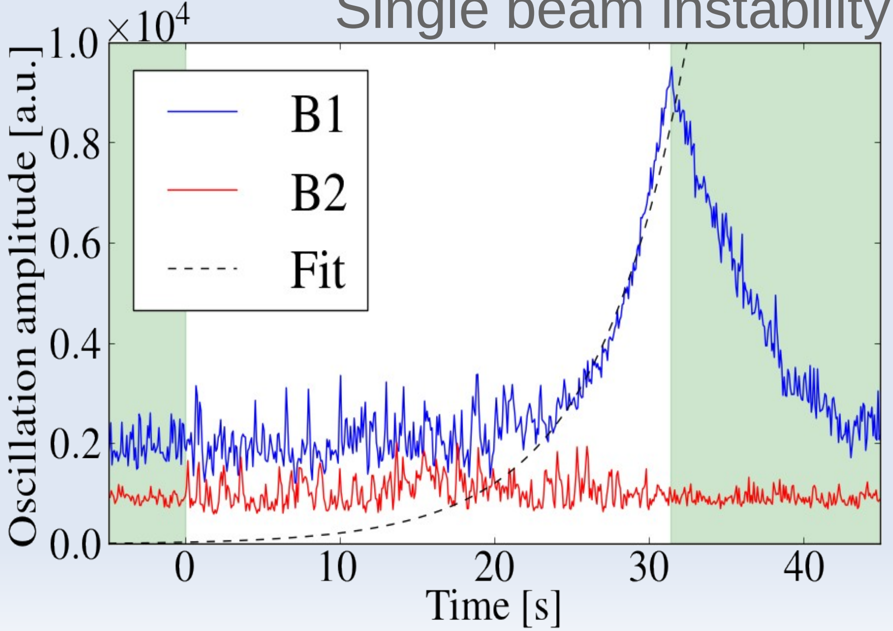


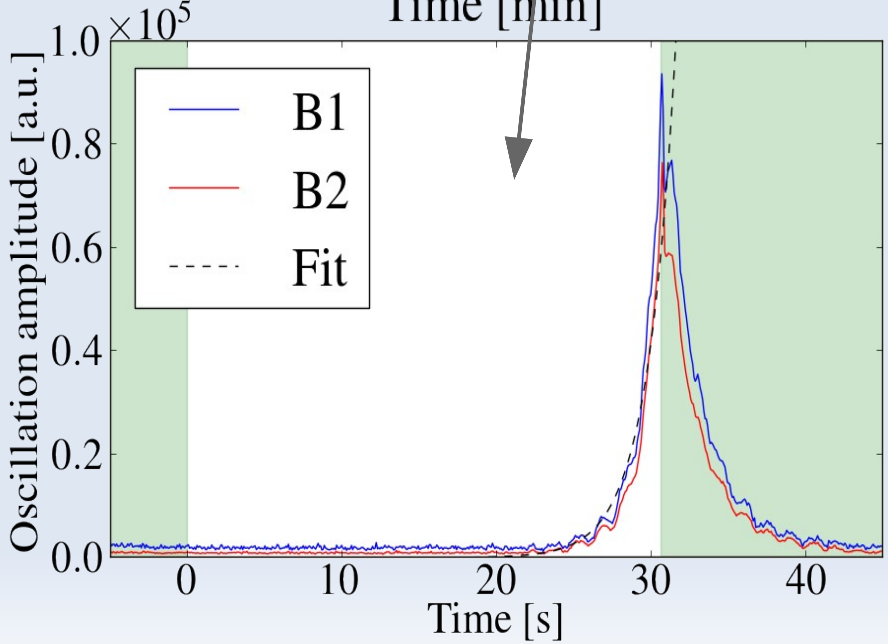
Observations LHC

- Two-beam Instability observed for intermediate separations between the beams corresponding to the frequency of the mode coupling
- The stability was always ensured by the transverse feedback



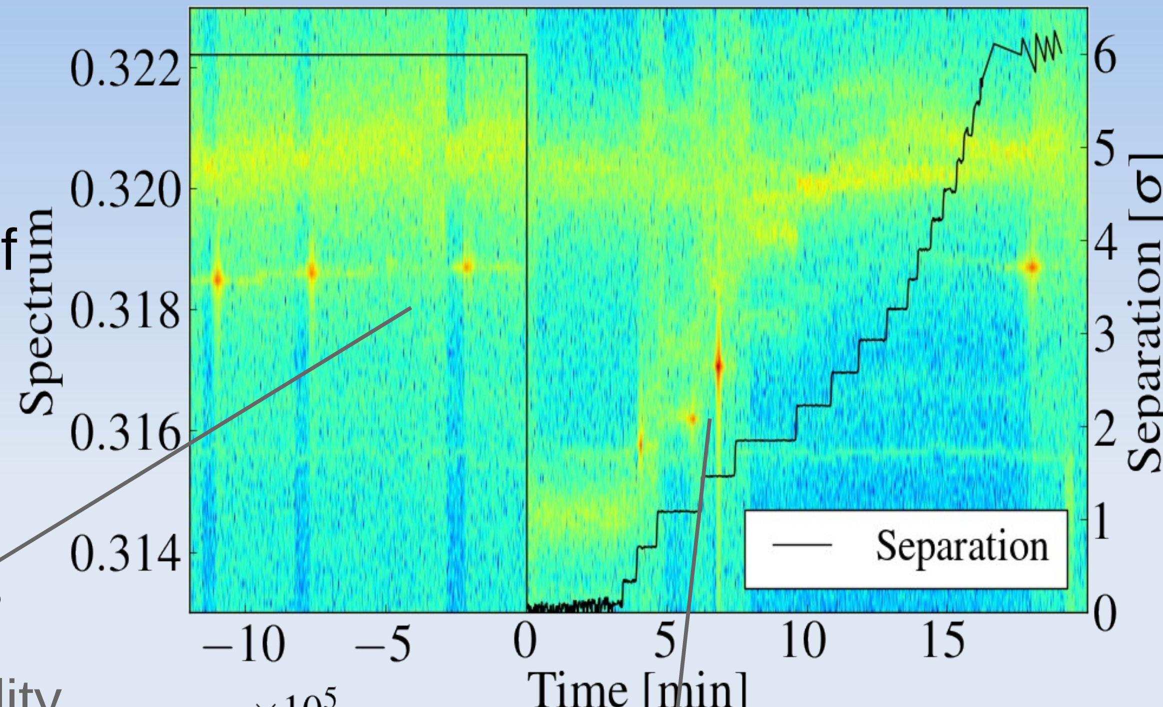
Single beam instability



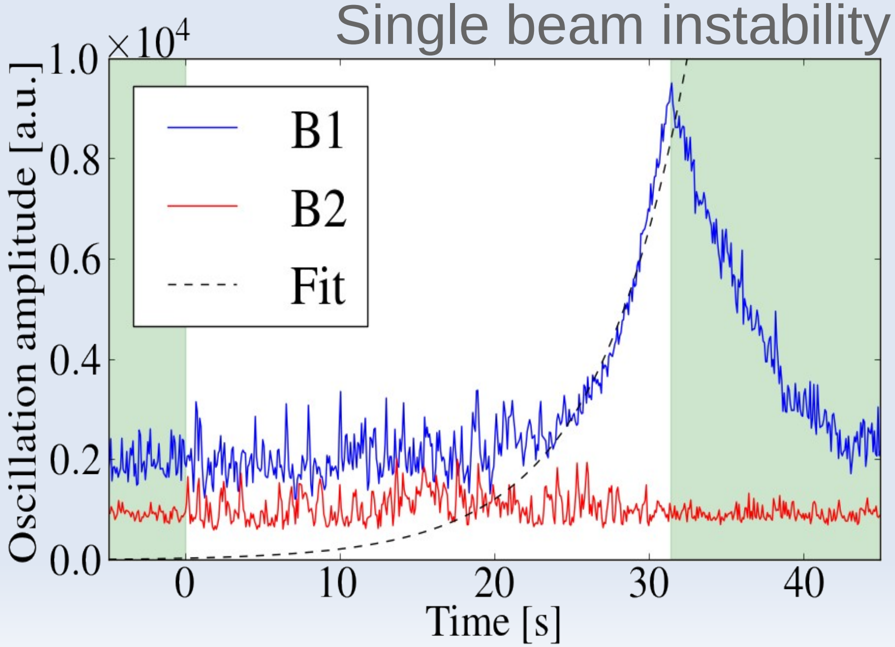


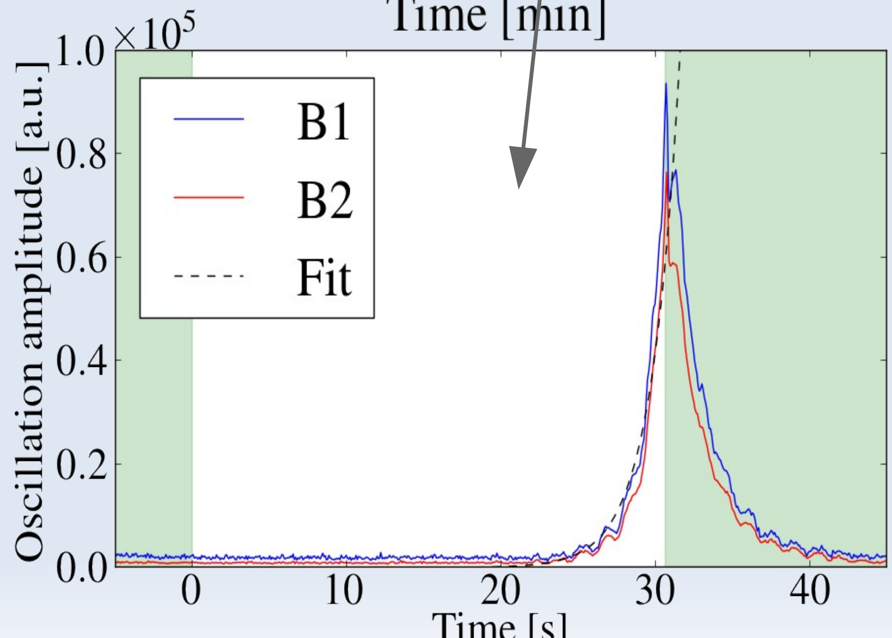
Observations LHC

- Two-beam Instability observed for intermediate separations between the beams corresponding to the frequency of the mode coupling
- The stability was always ensured by the transverse feedback
  - In agreement with the models



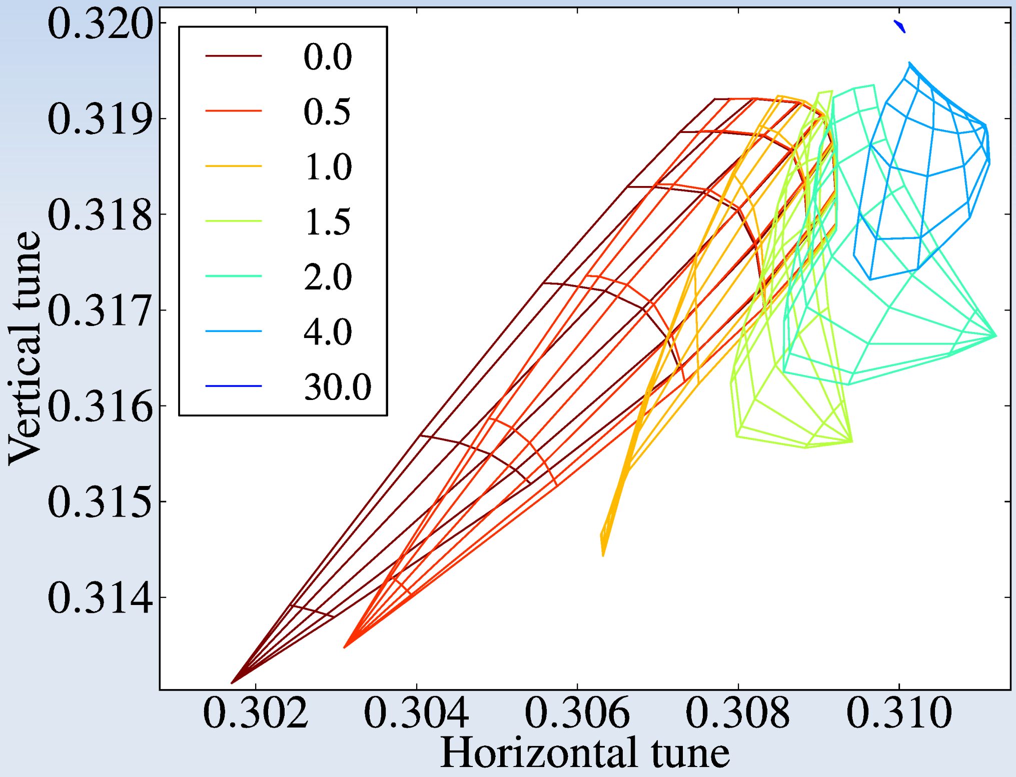
Single beam instability





Landau damping of the weak head-tail instability

- In configurations were the coherent aspects of the beam-beam interactions are well suppressed (e.g. with a feedback), the incoherent effect of beam-beam interaction remain
  - Landau damping of the weak head-tail instability is strongly affected by the presence of beam-beam interactions [S. Meyers, L. Vos, X. Buffat, C. Tambasco]
  - More details tomorrow with C. Tambasco



Summary

- The circulant matrix model is based on the derivation and normal mode analysis of the transverse one turn matrix of a discretised longitudinal distribution
  - The longitudinal motion is modelled through a circulant matrix
- This model was successfully benchmarked with several codes in their own validity domain
- Single and two beam stabilisation mechanisms can be studied (transverse feedback, RFQ, Q', Q'')
  - Non-normal aspects of the coupled bunch instability can be modelled
- It predicted accurately the frequency of synchrobetatron beam-beam mode frequencies at VEPP-2M as well as the mode coupling instability of colliding beams in the LHC
  - It is currently used to evaluate the stability of colliding beams in the HL-LHC
- The non-linear effect of beam-beam interactions (→ Transverse Landau damping) has to be modelled differently
  - Comparison of the linearised model with fully self-consistent multiparticle tracking simulation can be used to quantify the effect of Landau damping

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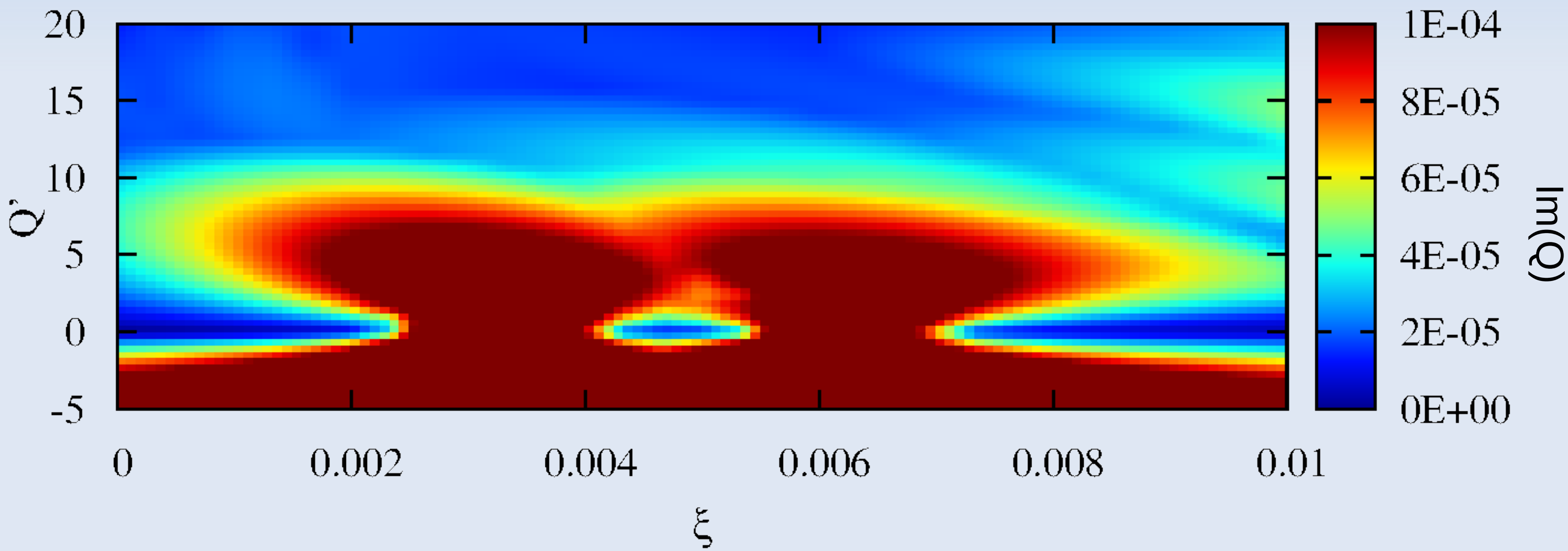
- C. Tambasco, PhD Thesis, EPFL, 2017

Backup
Chromaticity

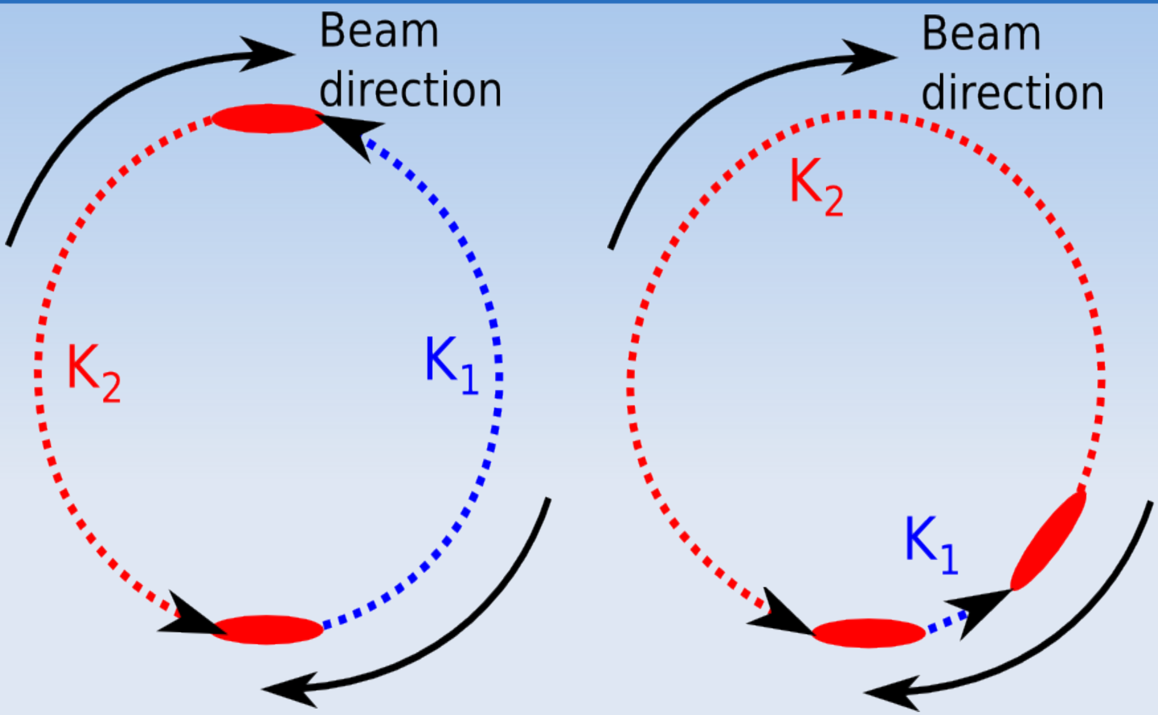
- To achieve exact convergence with the Vlasov perturbation theory, the chromaticity may be introduced by modifying the circulant matrix [A. Maillard] :

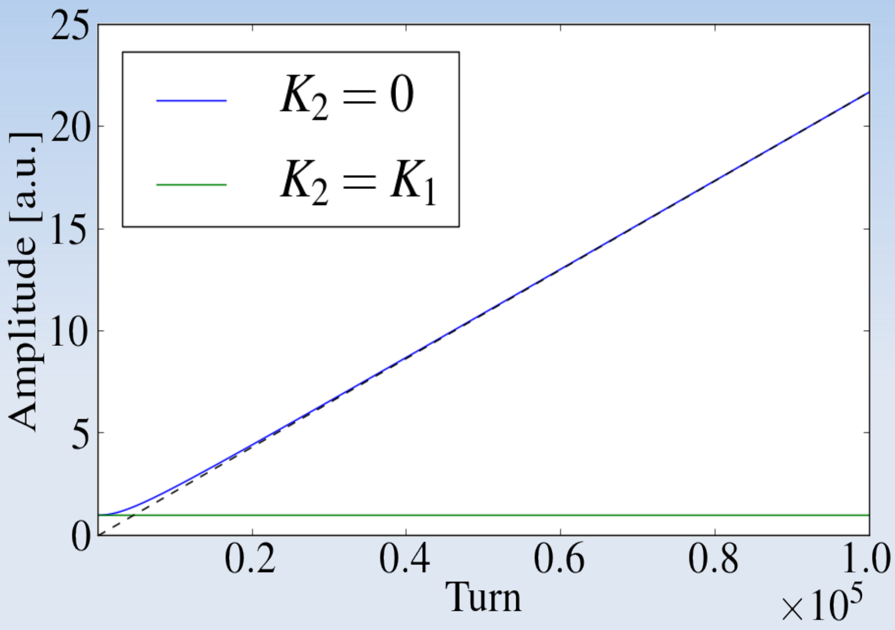
$$\left[P_{N_s}\begin{pmatrix} e^{2\pi j\epsilon\frac{Q'_y}{N_sQ_s}\delta_1} & \cdots & 0\\ \vdots & \ddots & \vdots \\ 0 & \cdots & e^{2\pi j\epsilon\frac{Q'_y}{N_sQ_s}\delta_{N_s}}\end{pmatrix}\right]^{N_sQ_s}$$



- By reducing the effect of the impedance on low order head-tail mode (with a center of mass motion), the chromaticity allows for a mitigation of the mode coupling instability
  - The situation is different when considering beam-beam interaction introducing a strong synchrobetatron coupling

Backup
Non-normal analysis





- In a linear accelerator, the effect of each bunch on the trailing ones creates a non-normal instability, known as the multibunch beam breakup instability
- In a ring with K1 >> K2, the short term behaviour may be dominated by an instability close to the multibunch beam breakup instability

Backup
Beam breakup instability

$$M=\begin{pmatrix} B & 0 \\ 0 & B \end{pmatrix}\cdot\begin{pmatrix} 1 & 0 \\ K_1 & 1 \end{pmatrix}$$

$$M_d=\begin{pmatrix} \lambda_1 & 1 & 0 & 0 \\ 0 & \lambda_1 & 0 & 0 \\ 0 & 0 & \lambda_2 & 1 \\ 0 & 0 & 0 & \lambda_2 \end{pmatrix}$$

Consider a vector in the subspace associated to λ_i:

$$\vec{V}=a_1\vec{e}_1+a_2\vec{e}_2 \quad \text{with} \quad \vec{e}_1=\begin{pmatrix} 1 \\ 0 \end{pmatrix} \quad \vec{e}_2=\begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

$$\longrightarrow \quad \vec{V}_n=\lambda_i^n a_1\vec{e}_1+\lambda_i^n a_2\vec{e}_2+\sum_{k=0}^{n-1}(\lambda_i a_2)^k\vec{e}_1$$

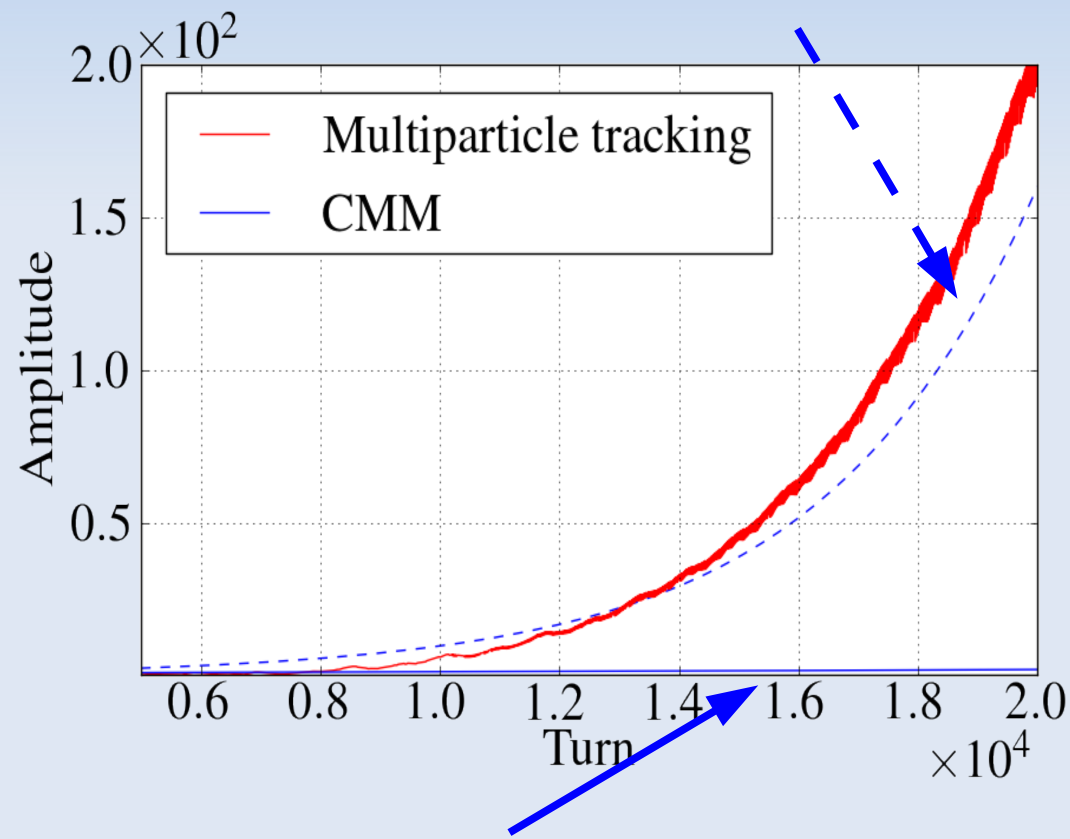
- Linear growth which depends on the initial condition
- The behaviour of the system under a small perturbation is no longer independent of the perturbation
- Non-linear growth expected for more complicated systems

BACKUP

Non-normality of multibunch instabilities

- The dynamics of the system is not well described by normal mode analysis
- Modern analysis tools are required to analyse the matrix
  - In the LHC (mostly short short range wake fields) the non-normal behaviour of the beams is well mitigated by the transverse feedback

$$\underline{x}(t) = M^{t}_{One\ turn}\,\underline{x}(0)$$



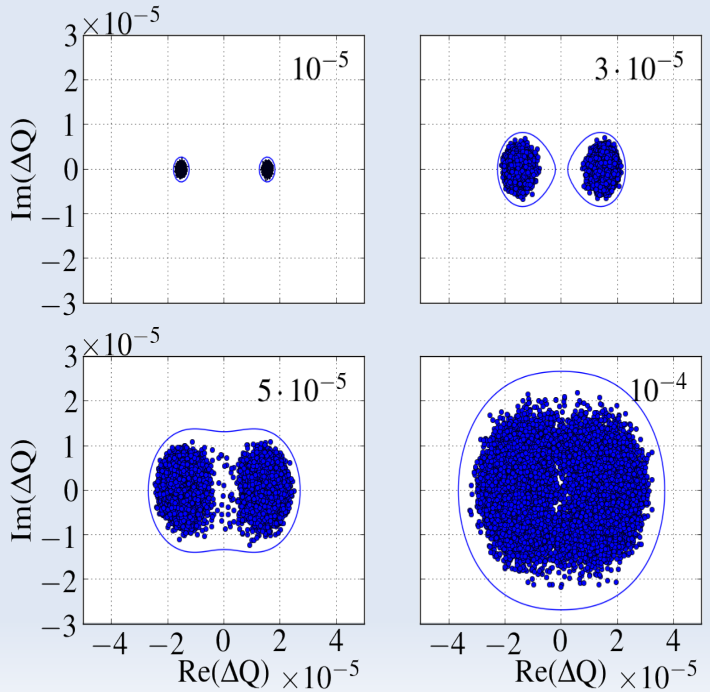
Normal mode analysis

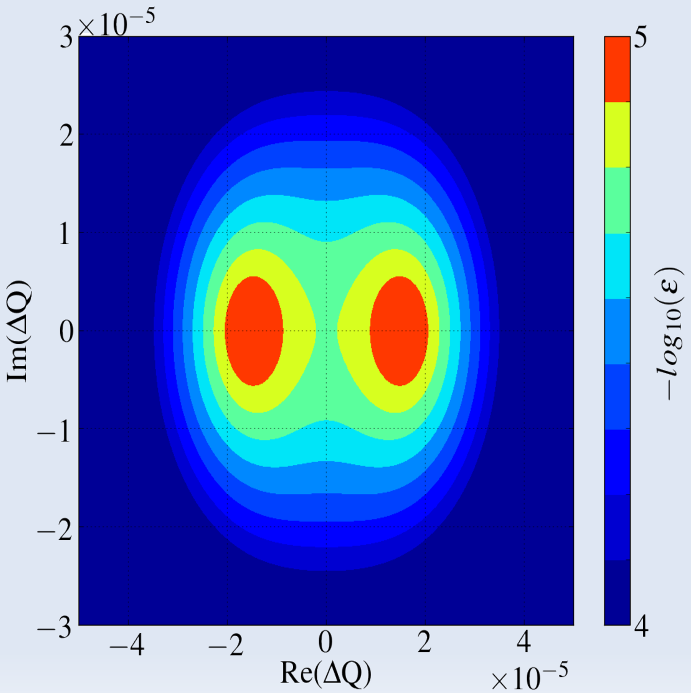
Backup
Pseudospectrum

$$Spectrum(M)=\{\lambda\in\mathbb{C} \mid \exists\vec{v}:(M-\lambda I)\cdot\vec{v}=0\}$$

$$Pseudospectrum(M,\epsilon)=\{\lambda\in\mathbb{C} \mid \exists\vec{v}:\|(M-\lambda I)\cdot\vec{v}\|<\epsilon\}$$

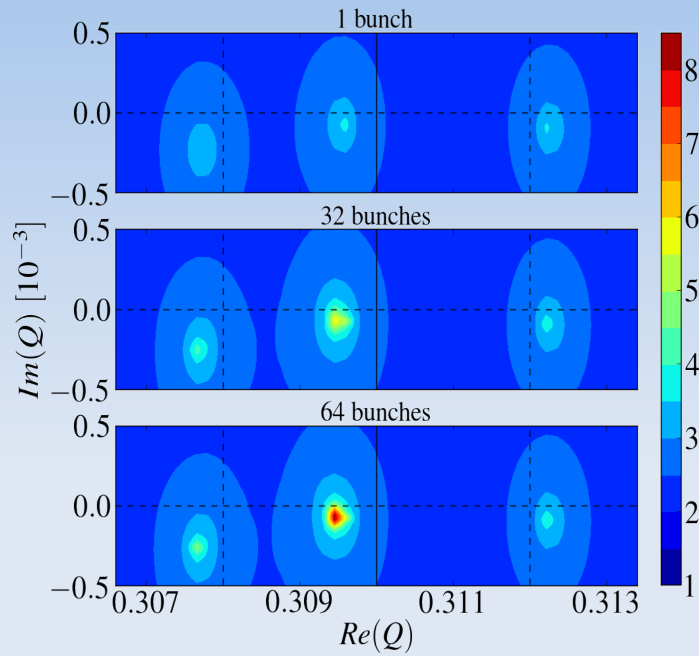
- The pseudospectrum is a tool to describe the non-normality of a system
- Qualitatively, it provides the extend of the spectrum of the system under a perturbation of order ε

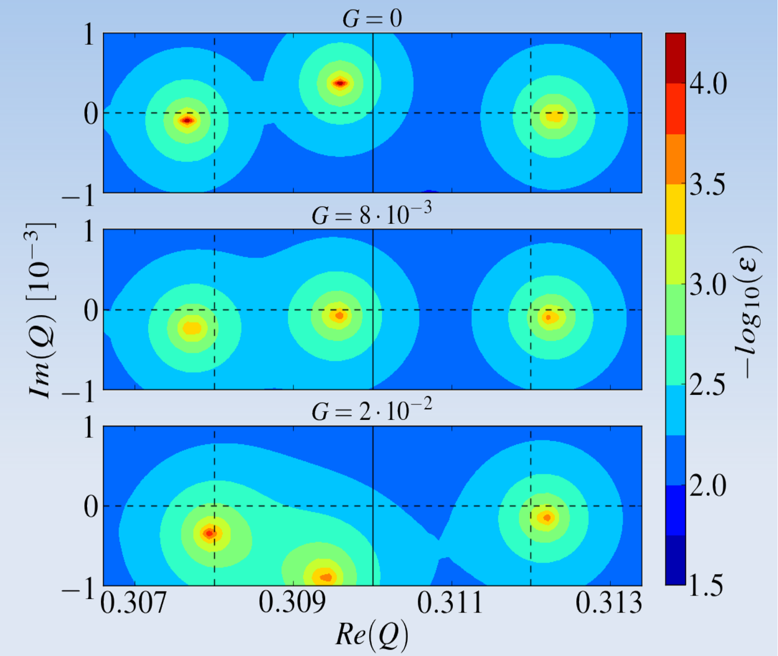


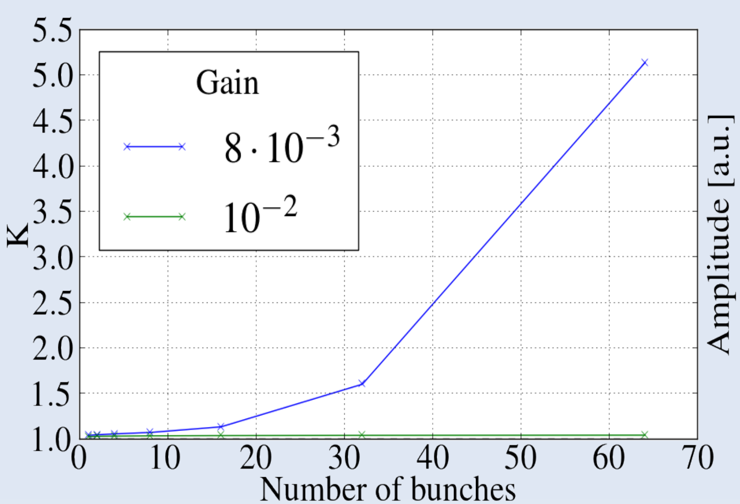


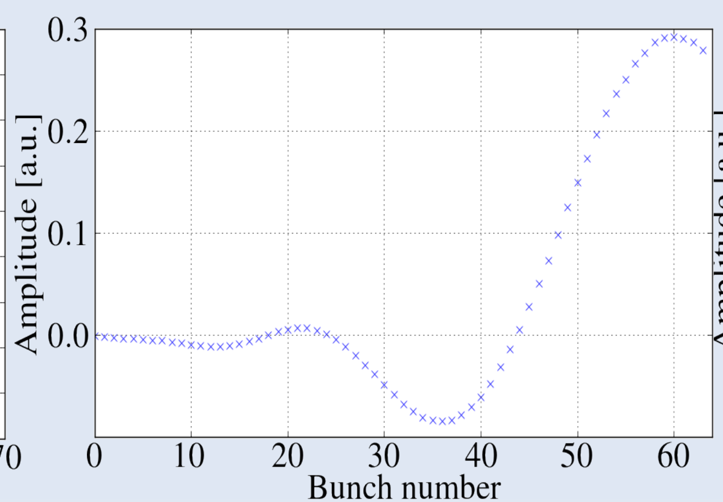
Backup
Non-normal analysis

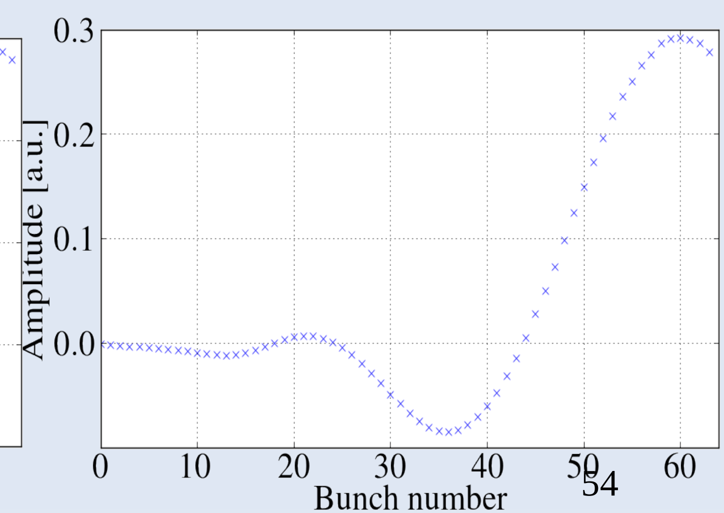
- The pseudospectrum gives insight into the non-normality of the system
- The transverse feedback acts effectively against coupled bunch instability



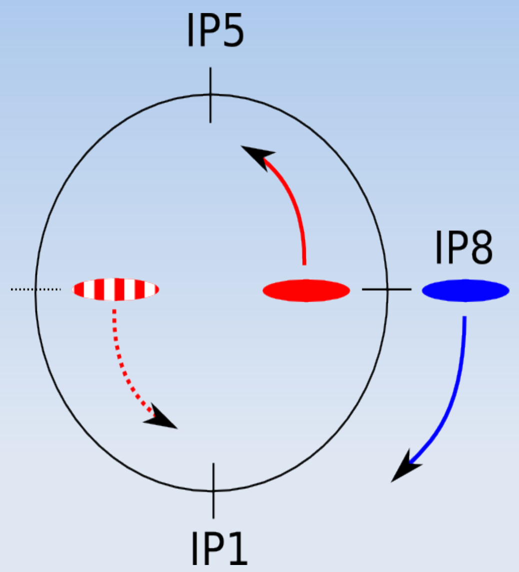


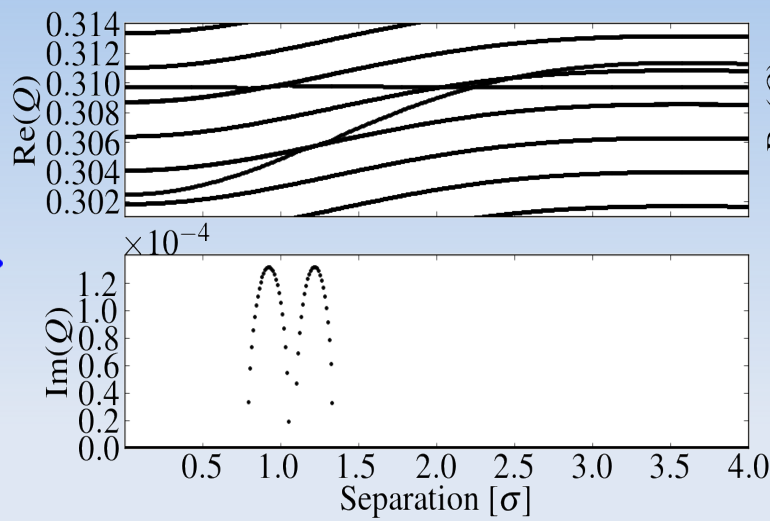


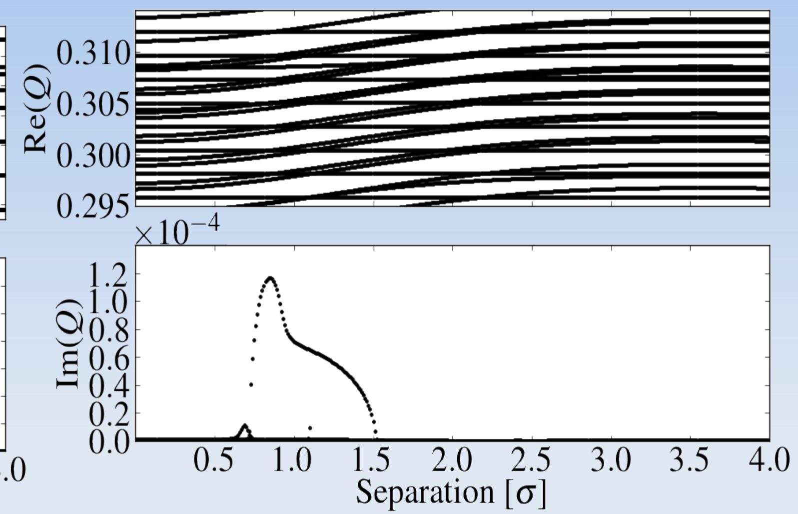




Backup
Asymmetric collision scheme







- The coupled instability due to the offset collision in IP8 is well mitigated by the Landau damping of the head-on interactions in IPs 1 and 5