Role of space charge in coherent instabilities

Michael Blaskiewicz BNL

- A caveat: I will be discussing synchrotrons, not linacs or other short cycle machines. Things like surface waves and turbulent emittance growth will not be discussed.
- Introduction
- Balbekov/Burov Model
- Simulations and checking
- Pitfalls
- Example

Introduction

- Negative mass instability is known since antiquity.
- Low frequency longitudinal space charge is essentially a negative inductance, albeit a very large one.
- For simulation purposes the simplest picture is to consider a macro-particle at (x_j,y_j,z_j) with a charge density $\rho_0(x-x_j,y-y_j,z-z_j)$. The charge densities of other macroparticles are translated in space but not rotated.
- Lorentz transform to the rest frame of the bunch and assume motion there is non-relativistic.
- Everything becomes electrostatic and the field is the gradient of a scalar potential. Forces are averaged spatially over ρ.

- The kicks are of the form

$$\vec{F}_j = \frac{\partial}{\partial \vec{x}_j} \sum_{k=1}^{N} U(\vec{x}_j - \vec{x}_k)$$

$$\approx \vec{z} C_z \frac{\partial}{\partial z_j} \sum_{k=1}^{N} \lambda(z_j - z_k) + \vec{x} C_x \sum_{k=1}^{N} (x_j - x_k) \lambda(z_j - z_k) + \vec{y}...$$

- Where $\lambda(z) = \iiint dxdyds\ \rho_0(x, y, s) \rho_0(x, y, z + s)$
- In the second expression only leading order terms are kept and there is more than one way to get there.
- Some newer codes like Synergia [1] keep higher order terms that lead to nonlinear coupling and enhanced collisionless damping.
- For this talk I stick with the simple expression.

- For coasting beams both transverse and longitudinal space charge can be handled well.
- For smooth bunched beams transverse space charge includes a new term, detuning, that is different from an impedance.
- In the simplest macro-particle model one has

$$
\begin{aligned}
F_{sc,j} &\propto \sum_{k=1}^{N} (x_j - x_k)\lambda(\tau_j - \tau_k) \\
&= x_j \sum_k \lambda(\tau_j - \tau_k) - \sum_k x_k \lambda(\tau_j - \tau_k) \\
&= x_j \rho(\tau_j) - \bar{x}(\tau_j)\rho(\tau_j)
\end{aligned}
$$

- The term proportional to x_j is the detuning. This term is the reason that no direct space charge tune shift can be measured using a centroid beam position monitor.

Consider the following equations of motion

$$\frac{d^2 x_j}{d\theta^2} + Q^2(\varepsilon_j)x_j = \kappa Z_{sc}\sum_{k=1}^{N}(x_j - x_k)\lambda(\tau_j - \tau_k) + \kappa V_x(\theta,\tau_j)$$

$$\frac{d\varepsilon_j}{d\theta} = \frac{qV(\theta,\tau_j)}{2\pi}, \qquad \frac{d\tau_j}{d\theta} = \frac{T_{rev}}{2\pi}\eta\frac{\varepsilon_j}{\beta^2 E_T}$$

V_x is the transverse voltage due to the wall induced wakes.

Assume the single sideband approximation is valid and take 1<Q<2 while modifying κ appropriately.

Update several (≈10) times per turn (betatron oscillation).

Nonlinear V(θ,τ) includes time dependent longitudinal space charge as well as synchrotron tune spread; important for collisionless damping.

Moment equations of corresponding Vlasov equation close in linear order.

Balbekov, Burov model [2,3]

- Starts with the moment equation (new variables)

$$\left(\frac{\partial}{\partial t}+\omega_s u\frac{\partial}{\partial \theta}-\omega_s\theta\frac{\partial}{\partial u}\right)^2 X(t,u,\theta)+\omega_x^2(u)X = 2\omega_x\omega_s\rho(\theta)(X-\overline{X}(t,\theta))+G(t,\theta)$$

- They use the single sideband approximation with

$$X(t,u,\theta)=Y(\theta,u)\exp(-i\Omega_c t+i\chi\theta)+small$$

- To make real progress they take

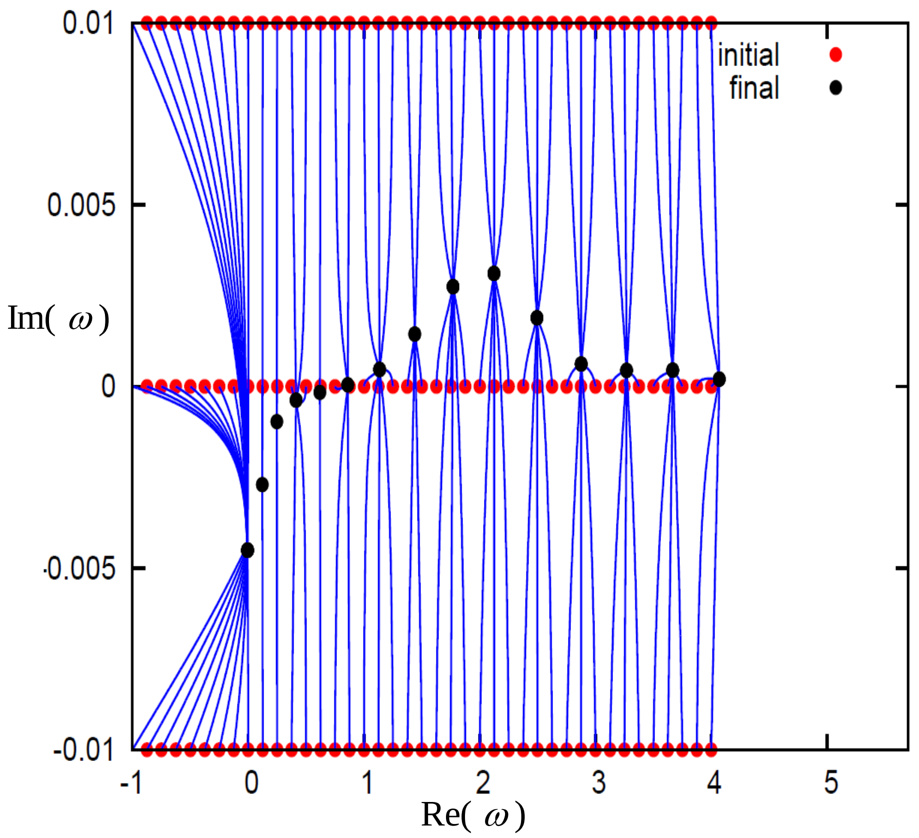
$$Y(\theta,u)=y_0(\theta)+uy_1(\theta)+u^2y_2(\theta)+small$$

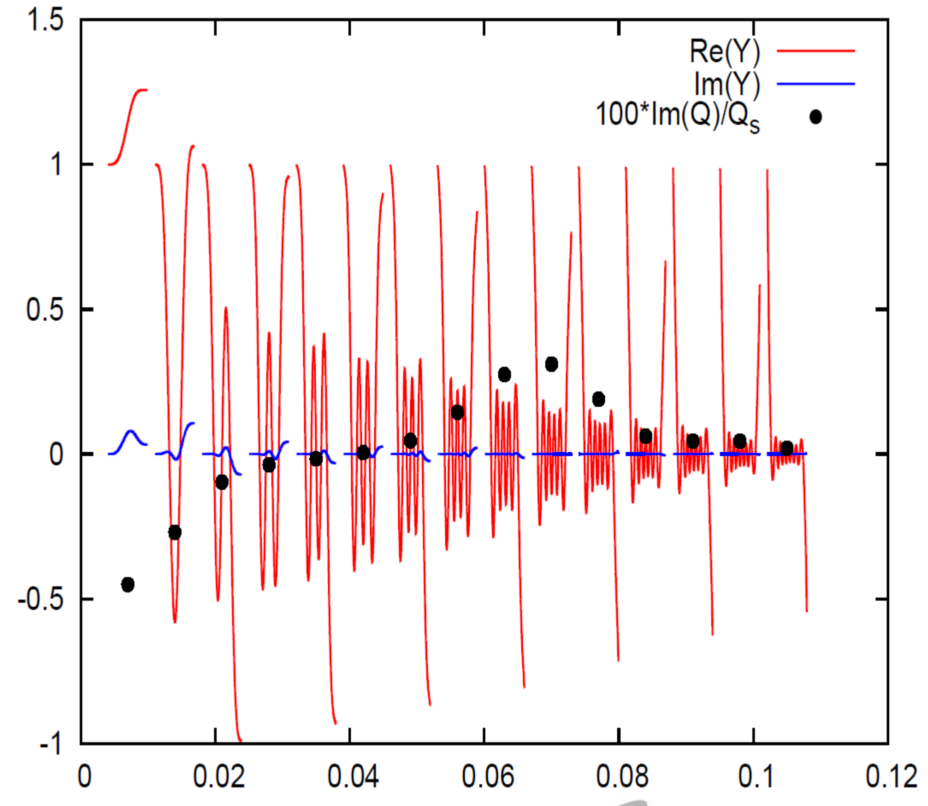
- The equations close approximately, $\overline{Y}(\theta)=y_0(\theta)+U^2(\theta)y_2(\theta)$

$$\omega\overline{Y}(\theta)+U^2\left(\frac{\overline{Y}'}{\rho+\omega}\right)'-\theta\frac{\overline{Y}'}{\rho+\omega}=G(\theta)\propto\int_{-\hat{\theta}}^{\theta}d\theta_1\overline{Y}(\theta_1)\rho(\theta_1)W_x(\theta-\theta_1)e^{i\xi(\theta-\theta_1)/\eta}$$

$$\rho=\frac{\Delta Q_{sc}}{Q_s}\left(1-\frac{\theta^2}{\hat{\theta}^2}\right)^{\alpha},\quad U^2=\frac{\hat{\theta}^2-\theta^2}{2\alpha+2},\qquad \omega=\frac{\Delta Q_c}{Q_s}$$

- I have limited the bunch shape to get a simple analytic form for ρ and U^2, and kept only single bunch wakes so that G vanishes at the head of the bunch.
- Can get accurate numerical solutions [4] .
- α=3, HT phase =-18, resonator wake,$\Delta Q_{sc}/Q_s$=65.





Simulations and checking

- For estimates that are sufficiently realistic to design an accelerator it appears that simulations are necessary.
- How do you know your code is right?
- For SC with linear RF and a parabolic line density we have Neuffer's exact longitudinal solutions [5].
- For boxcar bunches in linear RF with SC we have Sacherer's exact transverse solutions [6].
- For hollow bunches in a square well we have numerically exact transverse solutions with SC and wake potentials that are sums of (complex) exponentials. [7,8]
- A new basis expansion technique generalizing [6] appears to give *convergent solutions with wakefields* [9]

Pitfalls

- For a longitudinal smoothing time $\Delta\tau$ need

$$\Delta \tau \geq \frac{T_{rev}}{N_{update}} \eta \frac{\Delta p}{p}$$

Otherwise particles will jump over each other without interacting

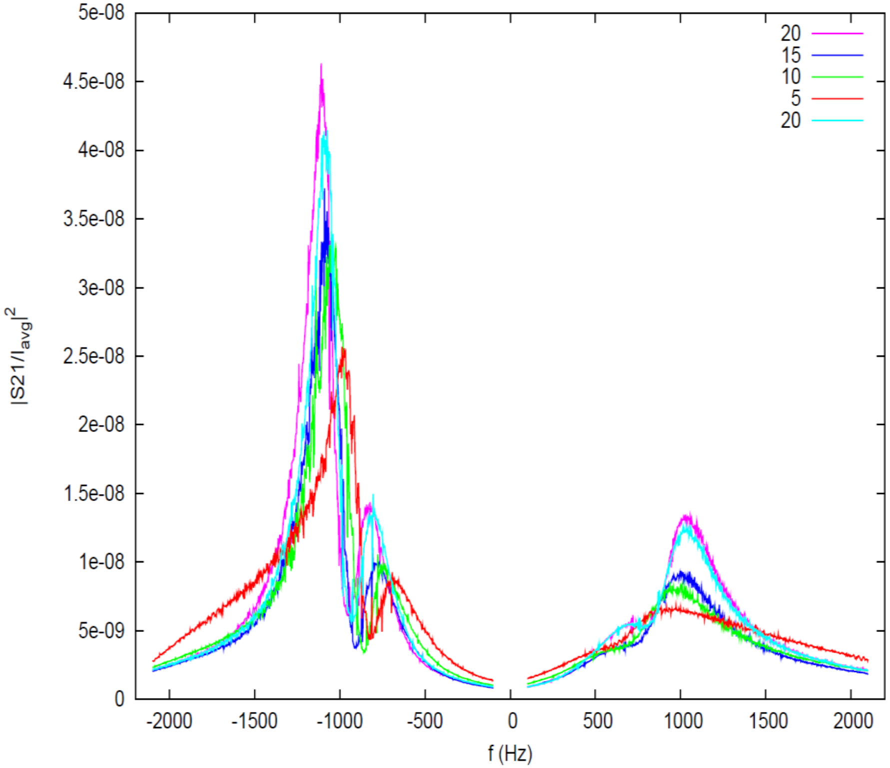
- With linear transverse space charge need $N_{update} \geq 4Q_x$

Otherwise you don't Nyquist sample both sine and cosine components of the betatron oscillation.

- With nonlinear forces you need to update more often depending on what order of nonlinearity you want to include.
- Need enough macro-particles per bunch so that instability and finite N_p effects can be resolved (more later).

RHIC proton injection as an example

- Want to know the impedance and to predict instability thresholds.
- Measured beam transfer functions at 250 MHz with much help from K. Mernick and the operations crew.
- Data are shown.
- The emittance changed with intensity keeping ΔQ_{sc} fixed so Z_{sc} was varied in sims.
- Best fit gave 10 MΩ/m which was 4 or 5 times bigger than Z_{sc}= 0 result.
- Variation of amplitude with intensity is not understood.



What does an instability look like?

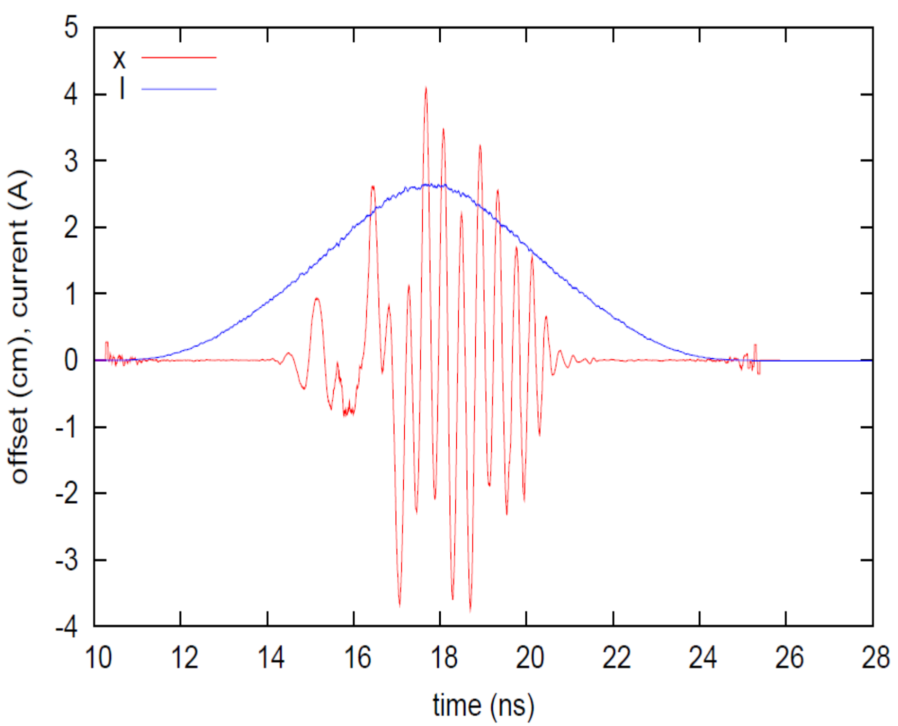
Unstable beam with parameters close to RHIC proton injection

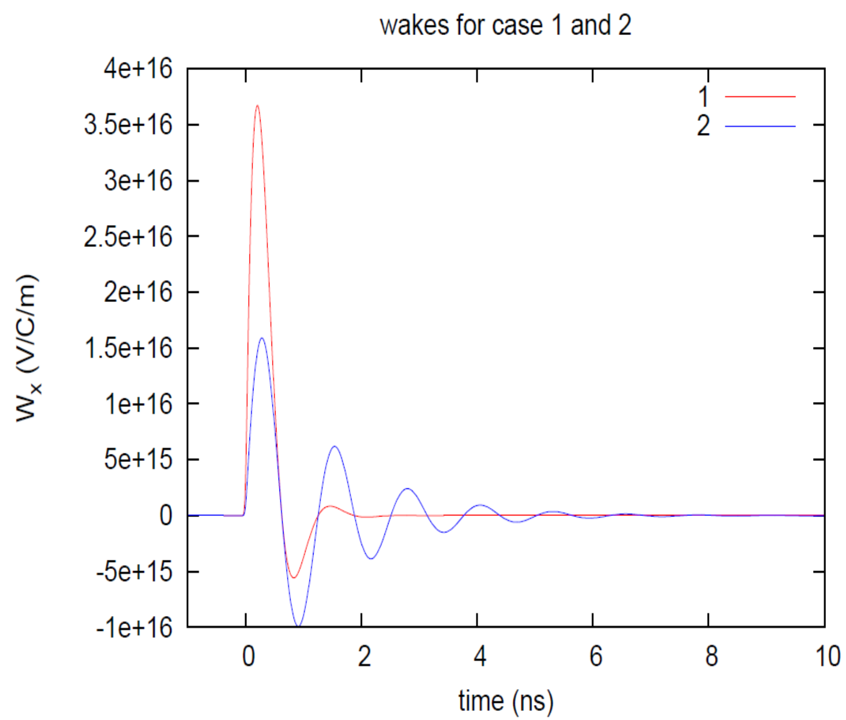
$\approx 10^{11}$ protons/bunch, C=3834m, Υ= 25.5, Υ_T=22.8,

100kV (h=360), 20 kV (h=7*360), Q=29.25,ξ=4 ,Z_{sc}=25MΩ/m.

Results below for 1×10^{11} , wake 1.

Since RHIC is stable the actual wakefield is probably different.

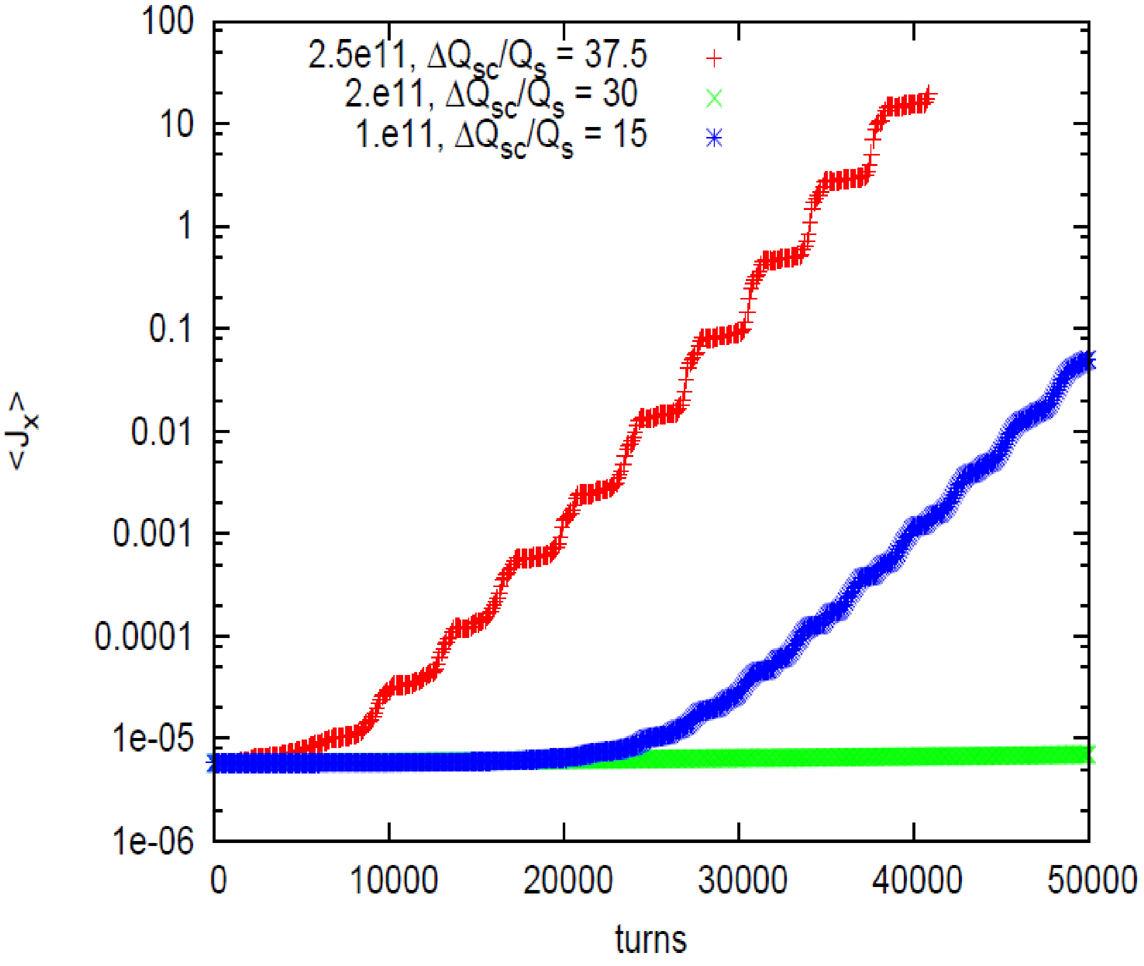




Growth rate is not monotonic in bunch charge.

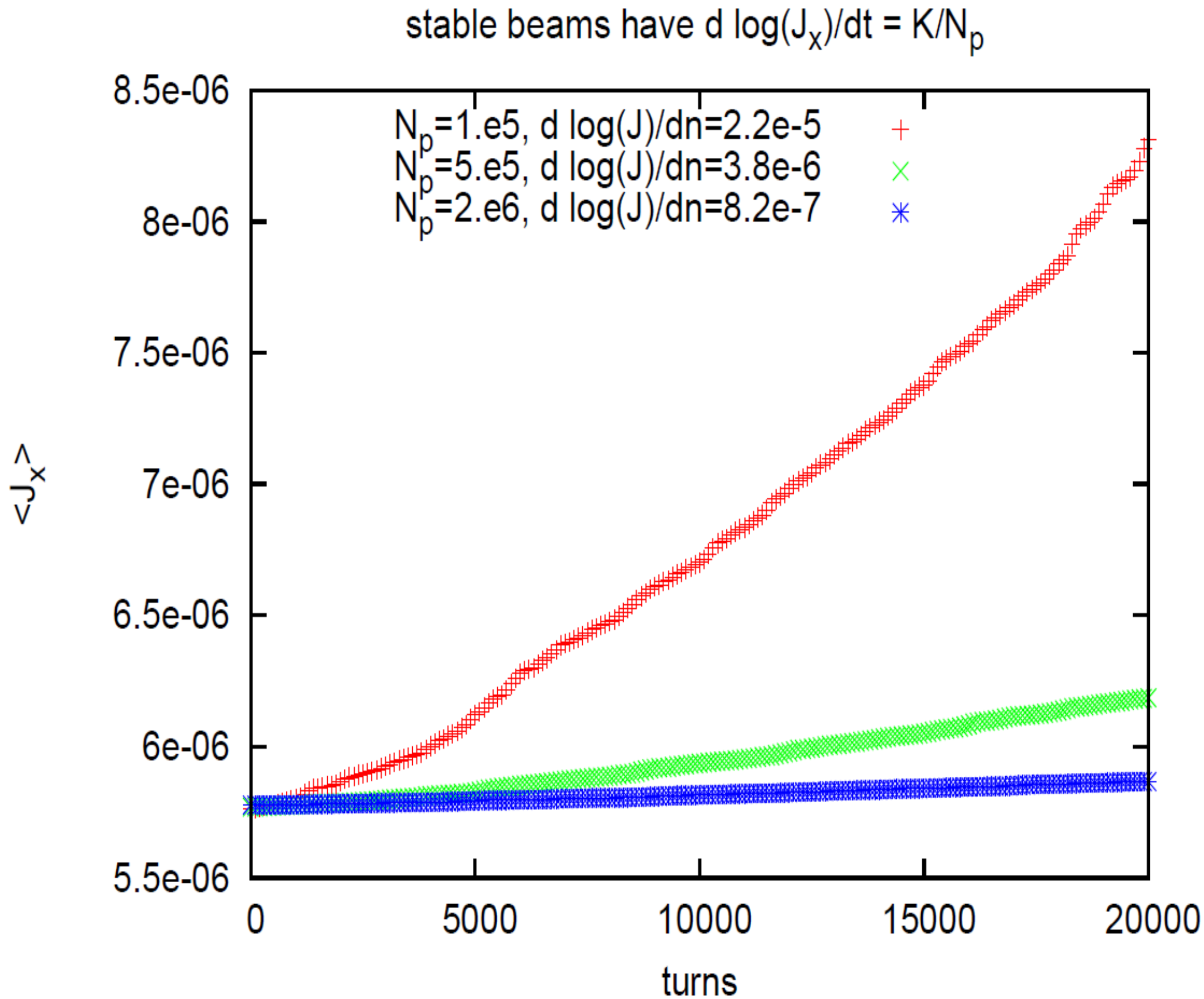
Blue trace corresponds to previous figure.

Only changed charge per macro-particle.



Check stable mode more carefully

- If bunch is stable expect stochastic cooling in reverse.



Stochastic cooling in reverse

$$\ddot{x}_j + (\Omega + j\Delta)^2 x_j = \frac{\alpha}{N}\sum_{k=-N}^{N}\dot{x}_k$$

$$x_m = a_m \exp(\lambda t - i\Omega t)$$

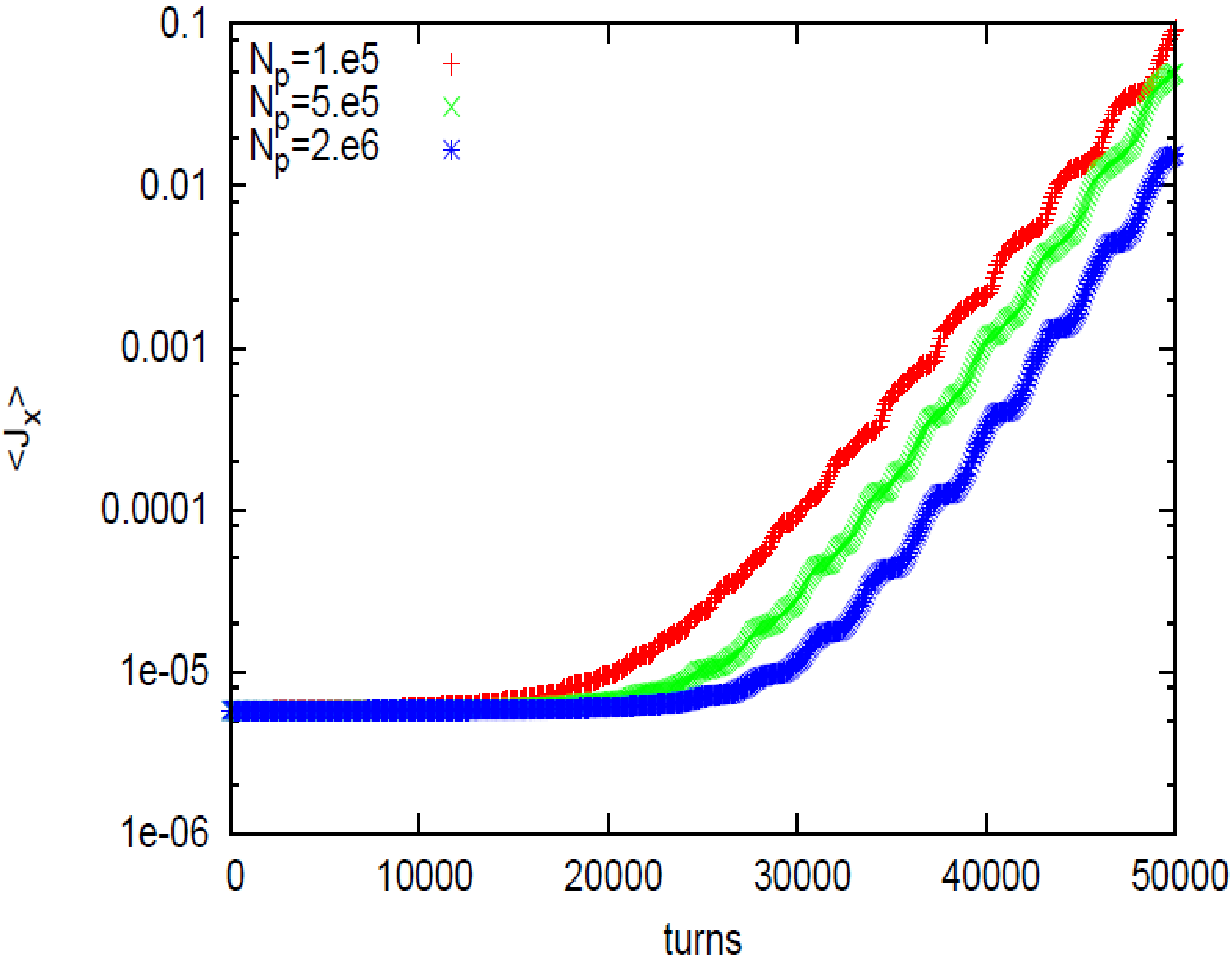
$$(\lambda + im\Delta)a_m \approx \frac{\alpha}{2N}\sum_{k=-N}^{N} a_k, \qquad \lambda = (\delta - in)\Delta$$

$$\frac{2\Delta N}{\alpha} = \sum_{m=-N}^{N}\frac{1}{\delta + i(m-n)} \approx \sum_{m=-Big}^{Big}\frac{1}{\delta + i(m-n)} - \int_{-Big}^{-(N+.5)\Delta}\frac{dm}{i(m-n)} - \int_{(N+.5)\Delta}^{Big}\frac{dm}{i(m-n)}$$

$$= \pi\frac{\exp(2\pi\delta)+1}{\exp(2\pi\delta)-1} + i\log\left|\frac{N+.5+n}{N+.5-n}\right| + O\left(\frac{1}{Big}\right)$$

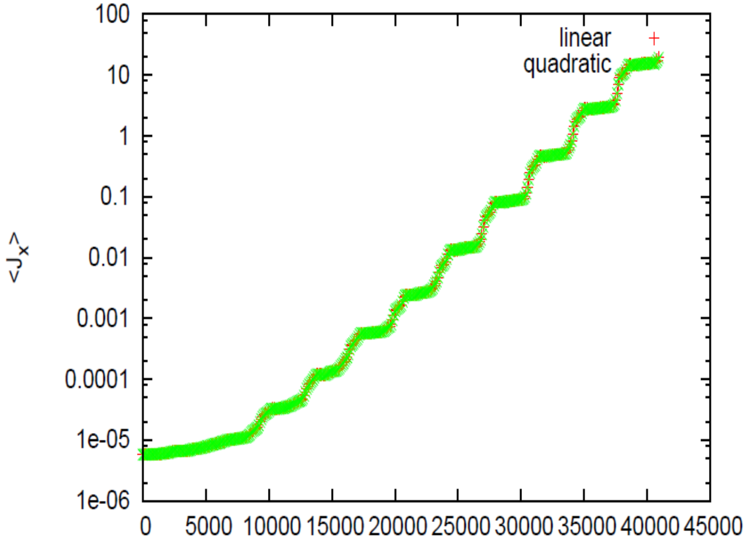
$$\exp(2\pi\delta) = \frac{1+\dfrac{\alpha\pi}{2N\Delta} - \dfrac{i\alpha}{2N\Delta}\log\left(\dfrac{N+.5+n}{N+.5-n}\right)}{1-\dfrac{\alpha\pi}{2N\Delta} - \dfrac{i\alpha}{2N\Delta}\log\left(\dfrac{N+.5+n}{N+.5-n}\right)} = 1 + \frac{\alpha\pi}{N\Delta} + O(\alpha^2), \qquad \mathrm{Re}(\lambda) \approx \frac{\alpha}{2N}$$

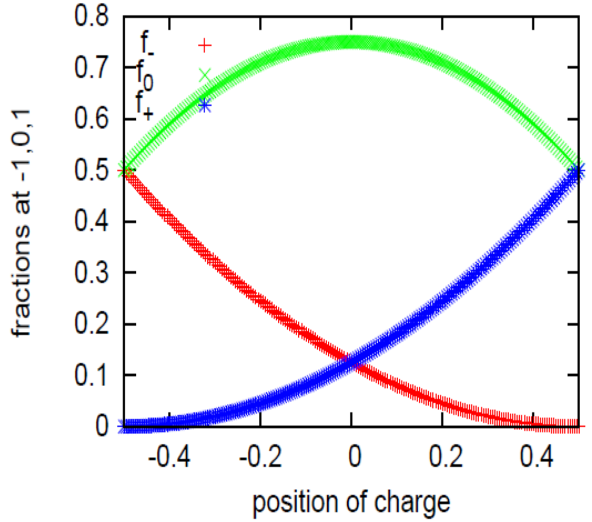
The growth at 10^{11} is real

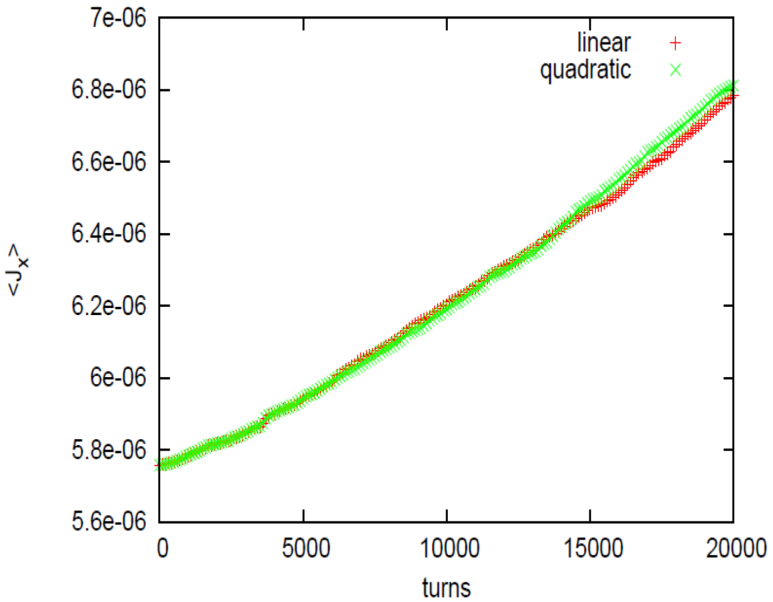


Code uses linear interpolation and smoothing

- Higher order deposition does not appear to be important.



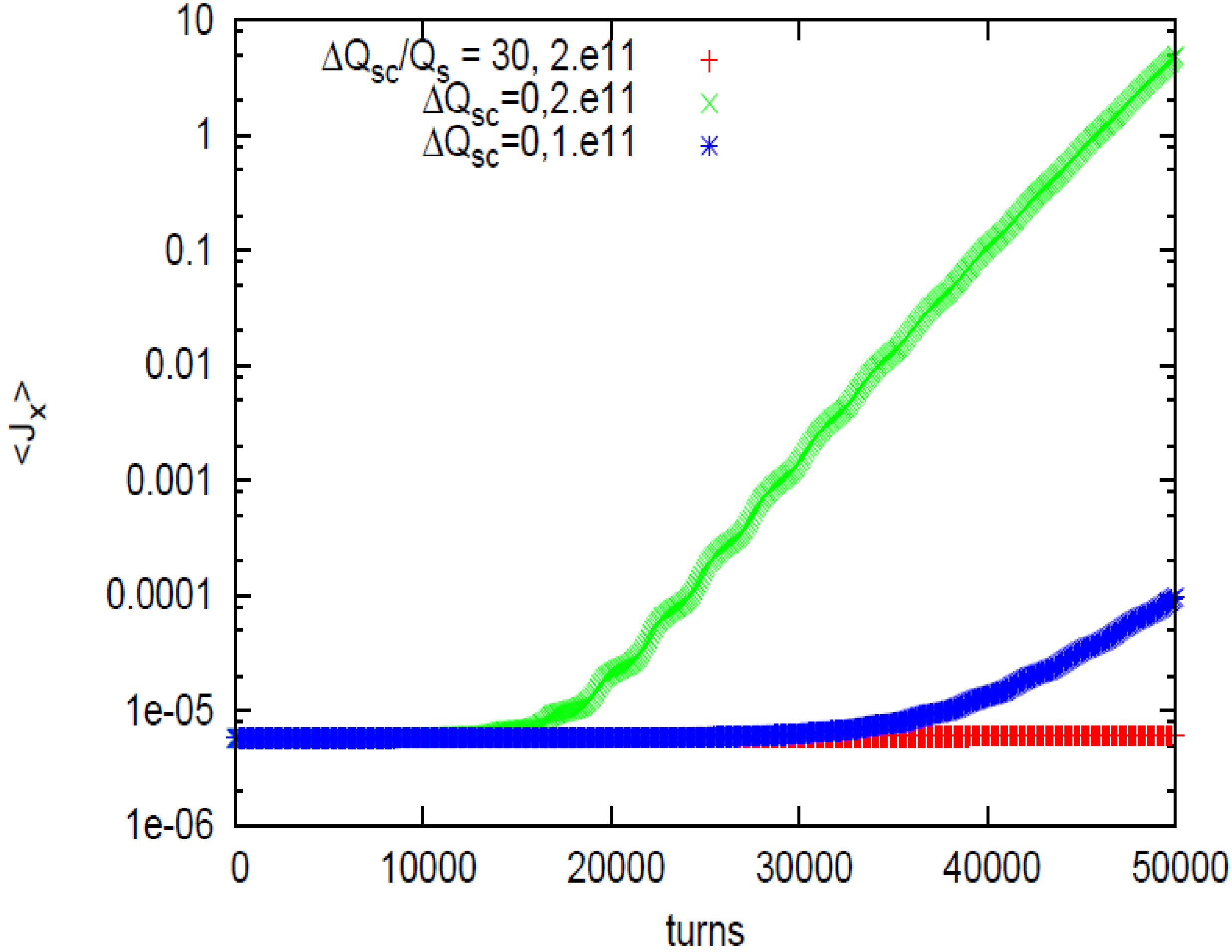




relative weights for nodes at -1,0,1 with 2nd order deposition.

Space charge can damp instabilities.

Simulations using wake for case 2.



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Summary and Conclusions

- Took vertical BTF data at 250 MHz for RHIC at 25.5 GeV
- Space charge reduced tune shift by factor of 4 over no SC.
- Took measured Z_y but guessed at f_{res} and Q.
- "Reasonable" parameters led to marginally stable beams, need to investigate this more.
- Exact solutions with space charge exist and should be used to test code during development.
- Stable simulations exhibit emittance growth scaling as $1/N_p$, which can be useful when testing for the presence of actual instability.
- For fixed impedance the growth rate can be non-monotonic in intensity.
- As seen before, space charge can stabilize things.

References

[1] A. Macridin et.al., PRSTAB, 18, 074401, 2015

[2] V. Balbekov, PRSTAB, 12, 124402, 2009

[3] A. Burov, PRSTAB, 12, 044202, 2009

[4] M. Blaskiewcz, BNL report C-A/AP/#475, 2012

[5] D. Neuffer, Particle Accelerators, 11, p23, 1980

[6] F. Sacherer, CERN report, CERN/SI-BR/72-5, 1972

[7]V.V. Danilov, E.A. Perevedentsev, 15th International Conference on High Energy Accelerators, p 1163, 1992.

[8] M. Blaskiewicz, PRSTAB, 1, 044201, 1998.

[9] V. Balbekov, PRSTAB, 20, 034401, 2017