Analytical Impedance Models for Very Short Bunches

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ICFA Workshop on Impedances and Beam Instabilities

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HELMHOLTZ GEMEINSCHAFT

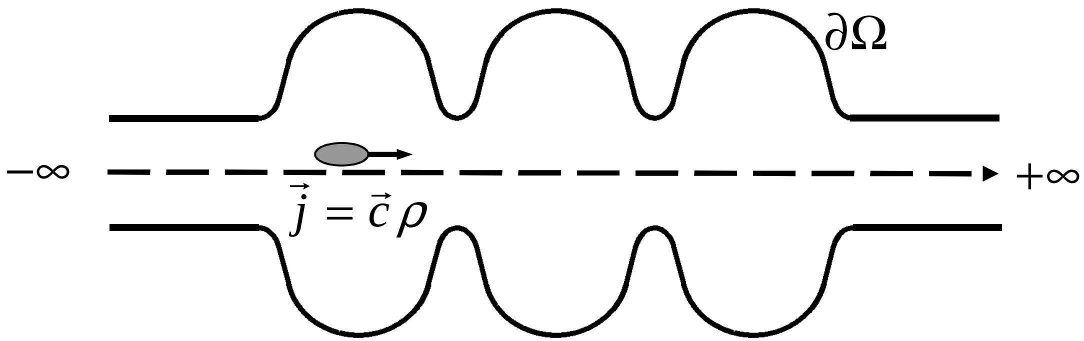
DESY

Overview

- Motivation
- Optical Model
- Diffraction Model
- Surface Impedance
- Limiting Value of the Wake at the Origin
- Combining Computations and Analytics
- Impedance Database Model

Motivation

Wake field calculation – estimation of the effect of the material properties of the chamber on the bunch



Wake potential

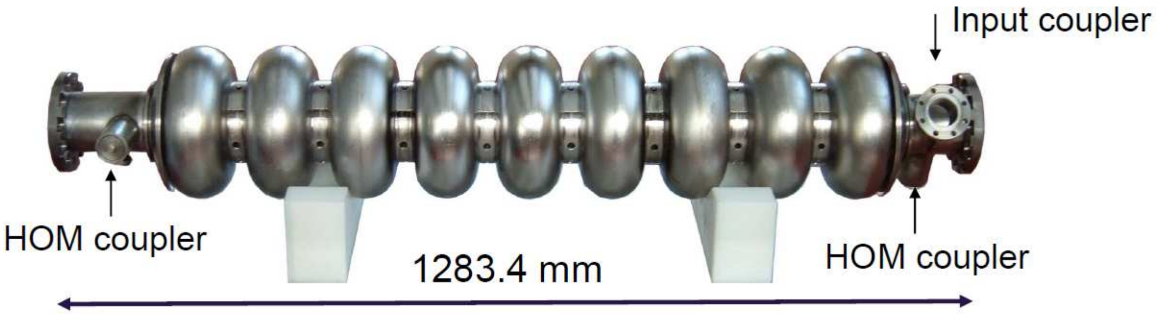
$$W_{\parallel}(\mathbf{r}_0,\mathbf{r},s) = \frac{1}{Q}\int_{-\infty}^{\infty} E_z\left(\mathbf{r}_0,\mathbf{r},z,\frac{z-s}{c}\right)dz$$

$$\frac{\partial}{\partial s}\mathbf{W}_{\perp}(\mathbf{r}_0,\mathbf{r},s) = \nabla_{\perp}W_{\parallel}(\mathbf{r}_0,\mathbf{r},s)$$

Impedance

$$\mathbf{Z}(\mathbf{r}_0,\mathbf{r},\omega) = \frac{1}{c}\int_{-\infty}^{\infty}\mathbf{W}(\mathbf{r}_0,\mathbf{r},s)e^{i\omega\frac{s}{c}}ds$$

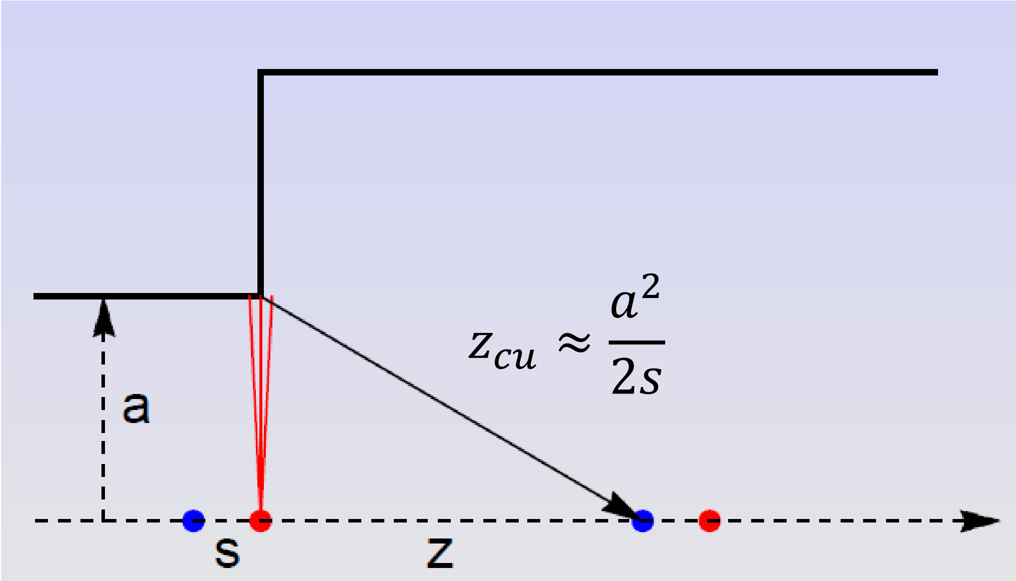
Motivation



	RMS bunch length σ_z, µm
ILC	300
European XFEL	5-25
LCLS-II	25-1000

The difficulty of **numerical** calculation of wakefields can be assotiated with a small parameter σ_z/a, where σ_z is the RMS bunch length and a is the typical size of the structure (f.e. iris radius)..

Motivation



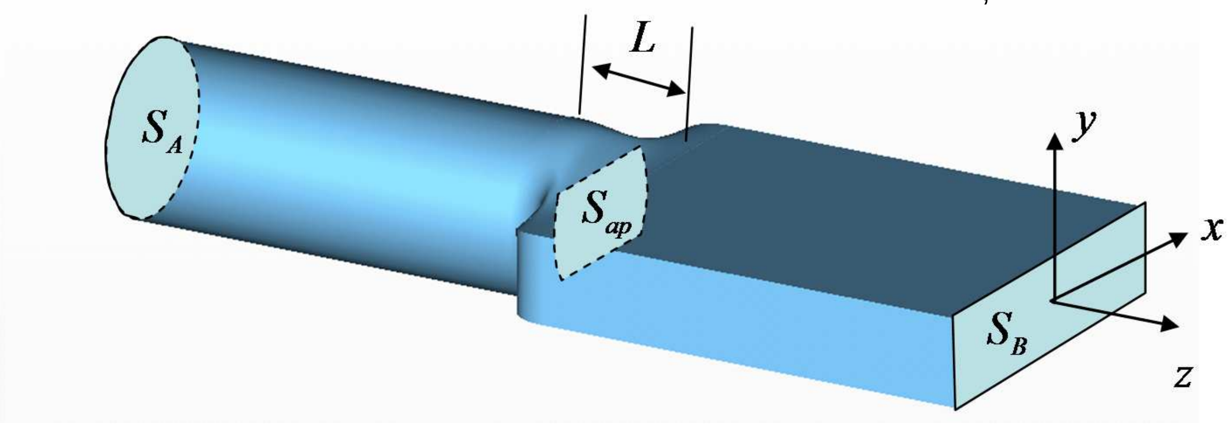
G. Stupakov,
ICFA Workshop on HOM in Superconducting Cavities, Chicago, 2014

For Gaussian bunch with RMS length $\sigma_z = 25$µm and $a = 35$ mm the formation length (~transient region) is ~ 25m .
It there is a long outgoing pipe we can stop a numerical calculation after “geometry variation” und use “indirect integration”*. Otherwise the numerical simulations require huge resources.
On the other hand the small parameter ${}^{\sigma_z}/_{a}$ allows to develop asymptotic **analytical** models.

*I. Zagorodnov, Phys. Rev. STAB **9**, 102002 (2006)

Optical Model

The length of the transition is much smaller than the formation length



$$L << a^2 / \sigma_z$$

$$Z_{\|}(\mathbf{r}_1,\mathbf{r}_2) = \frac{2\varepsilon_0}{c}\left[\int_{S_B} \nabla\varphi_B(\mathbf{r}_1,\mathbf{r})\nabla\varphi_B(\mathbf{r}_2,\mathbf{r})ds - \int_{S_{ap}} \nabla\varphi_A(\mathbf{r}_1,\mathbf{r})\nabla\varphi_B(\mathbf{r}_2,\mathbf{r})ds\right]$$

In the optical regime $Z_{||}$ is real and independent of the frequency.

$$\Delta\varphi_A(\mathbf{r}_i,\mathbf{r}) = -\varepsilon_0^{-1}\delta(\mathbf{r}-\mathbf{r}_i)$$

$$\mathbf{r} \in S_A$$

$$\varphi_A(\mathbf{r}_i,\mathbf{r}) = 0$$

$$\mathbf{r} \in \partial S_A$$

$$\Delta\varphi_B(\mathbf{r}_i,\mathbf{r}) = -\varepsilon_0^{-1}\delta(\mathbf{r}-\mathbf{r}_i) \quad i = 1,2$$

$$\mathbf{r} \in S_B$$

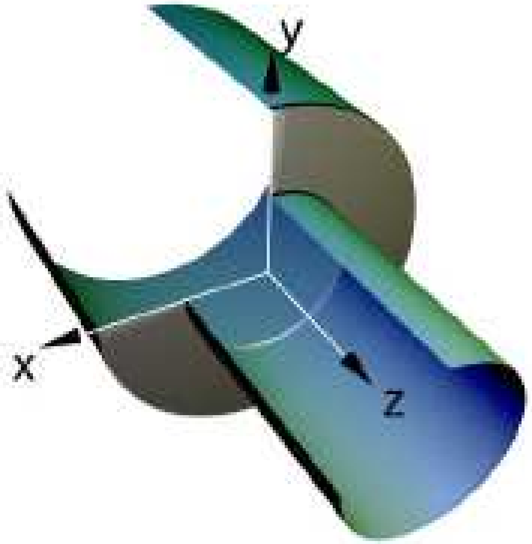
$$\varphi_B(\mathbf{r}_i,\mathbf{r}) = 0$$

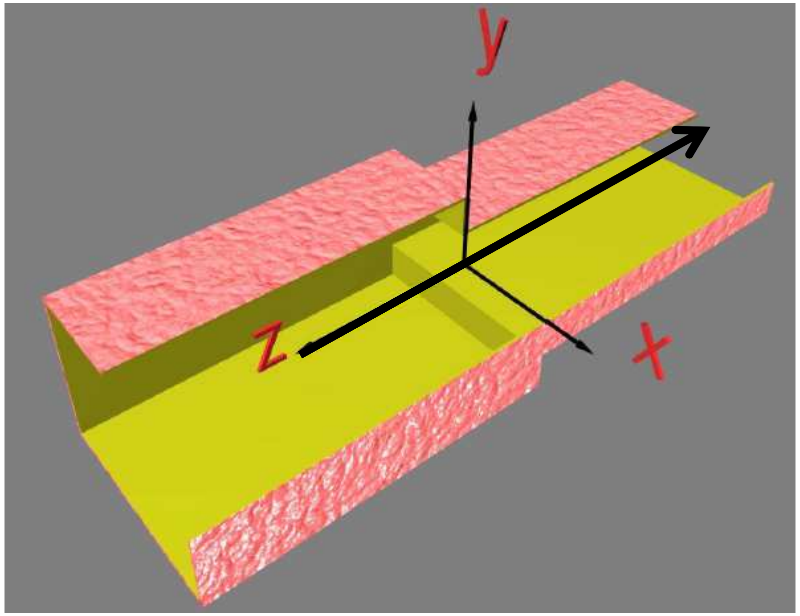
$$\mathbf{r} \in \partial S_A$$

G. Stupakov, K. Bane, I. Zagorodnov, Phys. Rev. STAB **10**, 054401 (2007)
K. Bane, G. Stupakov, I. Zagorodnov, Phys. Rev. STAB **10**, 074401 (2007)

Optical Model

Step-In

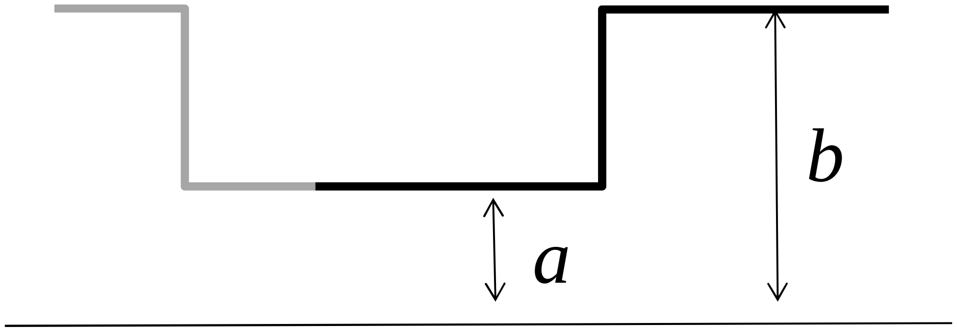


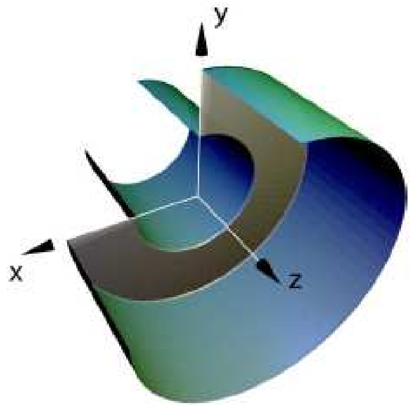


Longitudinal and transverse wakes are 0!

Optical Model

Step-Out, Long Collimator





$$Z_{\parallel} = \frac{Z_0}{\pi} \ln \frac{b}{a}$$

$$Z_{\perp} = \frac{Z_0 c}{\omega \pi} \left(\frac{1}{a^2} - \frac{1}{b^2} \right)$$

$$w_{\parallel}(s) = Z_{\parallel} c \delta(s)$$

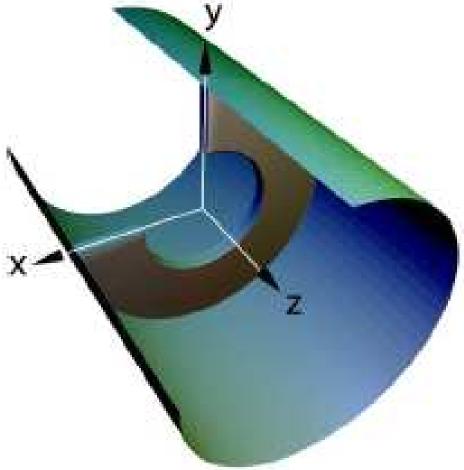
$$w_{\perp}(s) = \omega Z_{\perp} = const$$

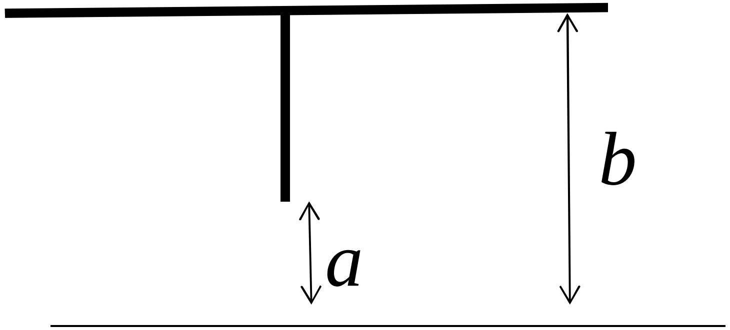
$$k_{\parallel} = \frac{c}{2\sqrt{\pi}\sigma_z} Z_{\parallel}$$

$$k_{\perp} = \frac{\omega}{2} Z_{\perp} = const$$

Optical Model

Iris, Short collimator



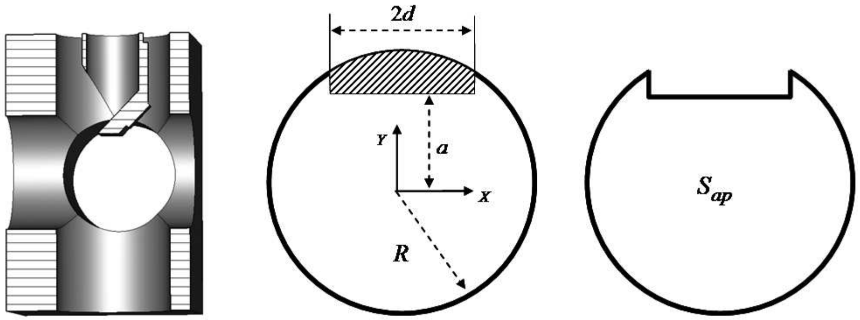


$$Z_{\parallel} = \frac{Z_0}{\pi} \ln \frac{b}{a} \qquad Z_{\perp} = \frac{Z_0 c}{2\omega\pi}\left(\frac{1}{a^2} - \frac{a^2}{b^4}\right)$$

The longitudinal wake is the same.
The transverse wake is ~ two times smaller than for the long collimator.

Optical Model

Transverse Impedance of Laser Mirror of RF Gun



M. Dohlus et al, EPAC 2008

$$Z_y(\omega,\mathbf{y}) \approx Z_y^{(m)}(\omega) + Z_y^{(d)}(\omega)y_1 + Z_y^{(q)}(\omega)y_2$$

$$Z_y^{(m)}(\omega) = \frac{1}{2\varepsilon_0\pi^2\omega a R^2}\left[\left(R^2-2a^2\right)\alpha + ad\left(1+\ln\frac{R^2}{a^2+d^2}\right)\right]$$

$$Z_y^{(d)}(\omega) = A^{-1}\left[aR^4 d - 4a^3 d^3 + R^4 d^2\alpha - a^2\left(2d^3 Q + R^4\beta + R^2 d\left(Q - 4d(\alpha+\beta)\right)\right)\right]$$

$$Z_y^{(q)}(\omega) = \frac{1}{AB}\left[ad\left(R^4(d^2-a^2) + \left(a^2+d^2\right)\left(R^2+6d^2\right)aQ\right) + B\left(a^2\left(R^4-8d^4\right)\beta + \left(R^4-8a^4\right)d^2\alpha\right)\right]$$

$\sigma = 0.5$ mm	$k_y(0,0)$, V/pC	$k_y^{(d)}$, V/pC/m	$k_y^{(q)}$, V/pC/m
Analytical	0.124	13.1	12.1
Numerical	0.120	13.1	11.6

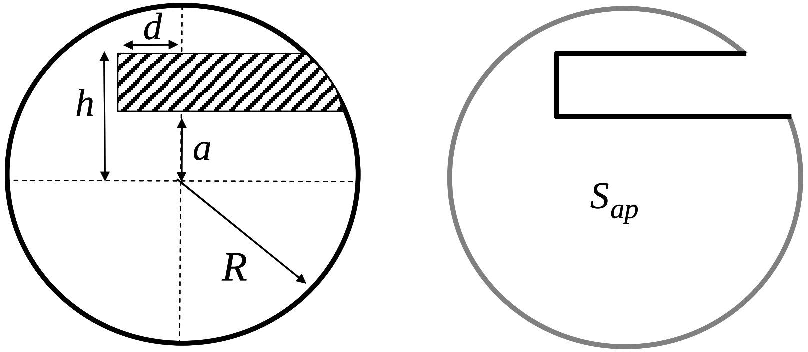
$$\alpha = \tan^{-1}\left(\frac{d}{a}\right) \qquad \beta = \cot^{-1}\left(\frac{d}{Q}\right) - \tan^{-1}\left(\frac{a}{d}\right)$$

$$Q = \sqrt{R^2 - d^2}$$

$$B = a^2 + d^2 \qquad A = 4\pi^2\varepsilon_0\,\omega a^2 R^4 d^2$$

Optical Model

Transverse Impedance of OTR Screens



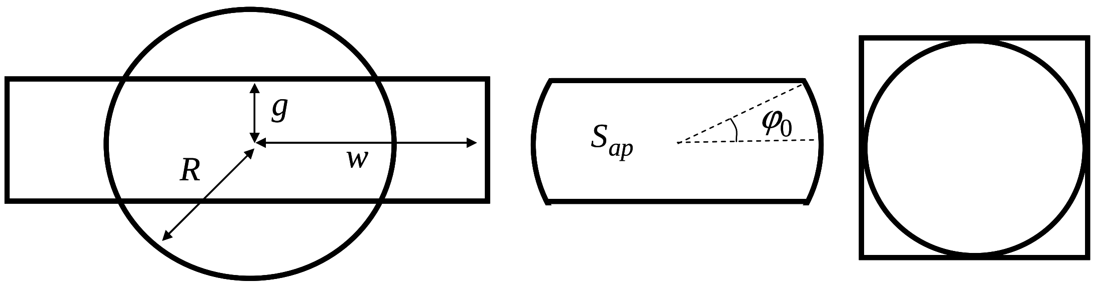
$$Z_y^{(m)}(\omega) = \frac{1}{4\pi^2 \varepsilon_0 \omega a R^2 h}\left[F(a,h) - F(h,a)\right]$$

$$F(x,y) = \left(R^2 - 2x^2\right) y \left(\cot^{-1}\left(\frac{x}{\sqrt{R^2 - x^2}}\right) + \tan^{-1}\left(\frac{d}{x}\right)\right) + ay\left(\sqrt{R^2 - x^2} + d \ln\left(d^2 + y^2\right)\right)$$

M. Dohlus et al, DESY 10-063, 2010

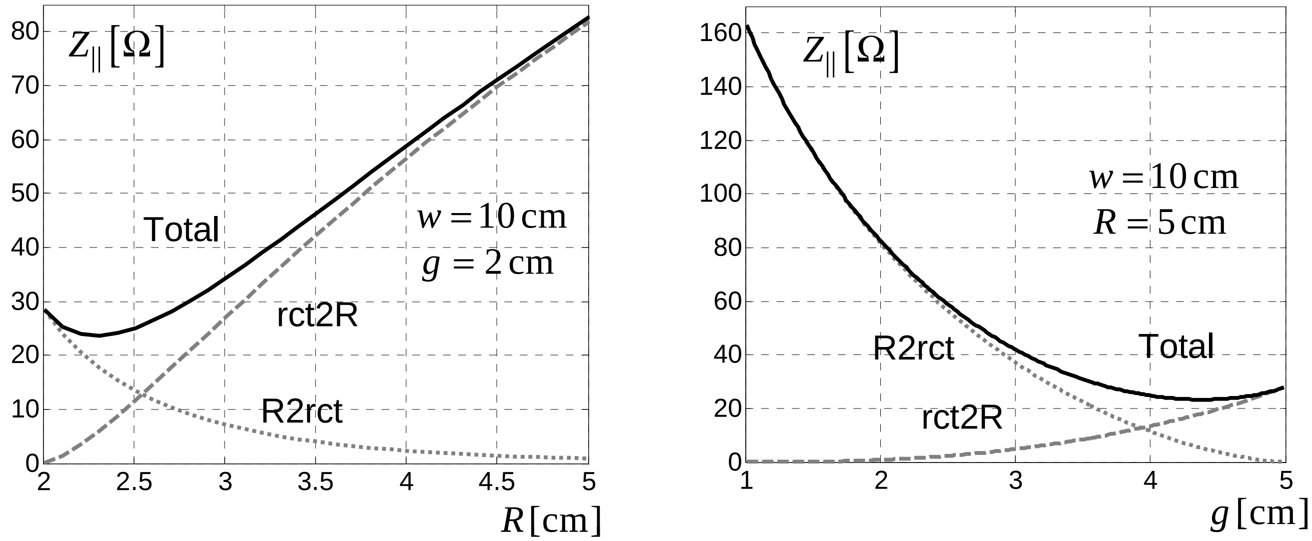
Optical Model

Longitudinal Impedance of Round-to-Rectangular Transitions in Bunch Compressors



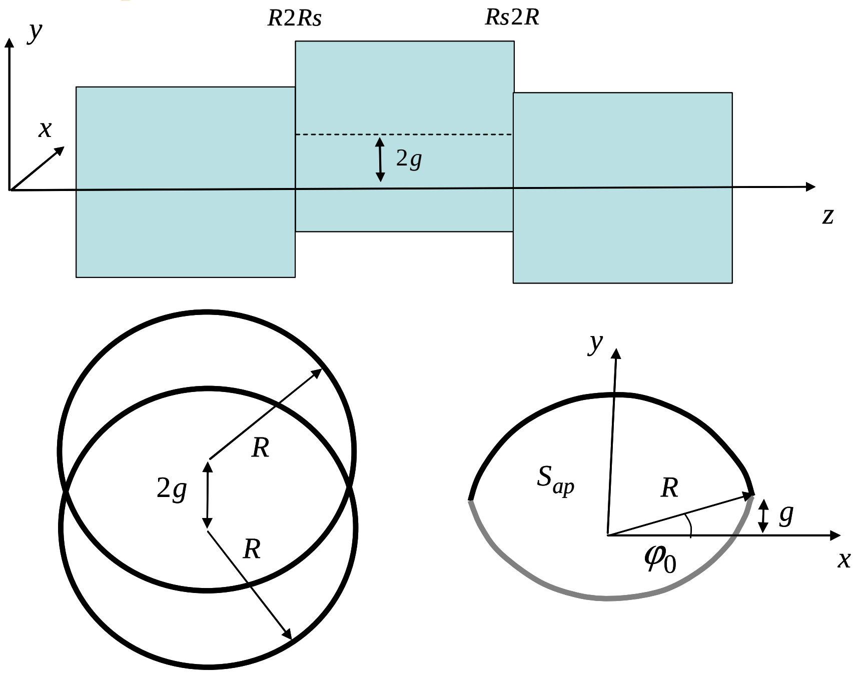
$$Z_{\|}^{(R2S)} = 9\,\Omega$$

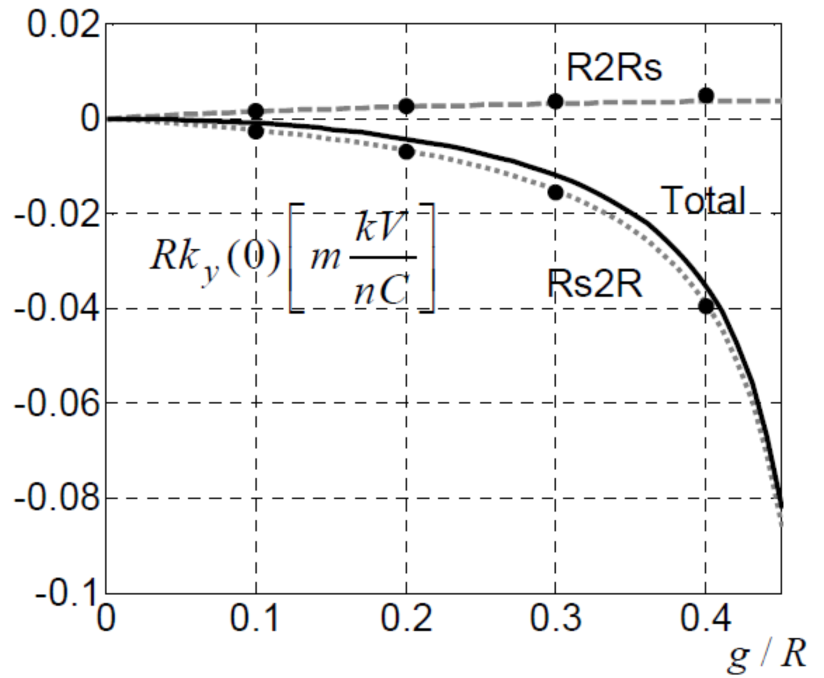
$$Z_{\|}^{(S2R)} = 0$$



Optical Model

Impedances of Round Misaligned Pipe





$$k_y^{(R2Rs)}(0) \equiv k_y^{(R2Rs)}(0,0) = -\frac{Z_0 c}{8\pi^2}\left(F_0\left(\frac{\pi}{2}\right) - F_0(\varphi_0) \right),$$

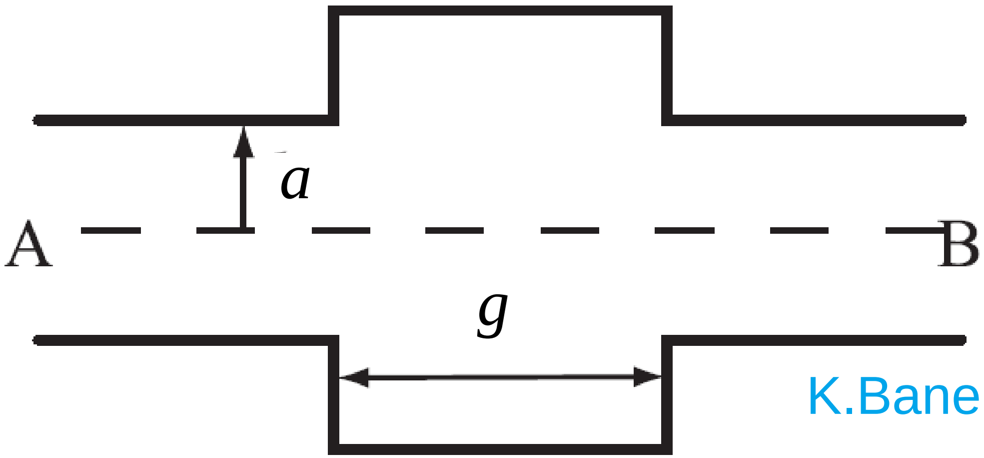
$$F_0(\varphi) = \frac{4\cos\varphi}{R} + \frac{8\varphi\alpha^2 - \varphi + 2\tan^{-1}[x,y]}{R\left(4\alpha^3 - \alpha\right)},$$

$$x = \left(4\alpha^2 - 1\right)\cos\frac{\varphi}{2} - 2\alpha\sin\frac{\varphi}{2}, \quad x = \left(4\alpha^2 - 1\right)\sin\frac{\varphi}{2} - 2\alpha\cos\frac{\varphi}{2}, \qquad \alpha = \frac{g}{R}, \ \varphi_0 = \sin^{-1}\left(\frac{g}{R}\right)$$

DESY

Diffraction Model

Cavity and Gap Wakes



The optical theory ignores diffraction effects. It predicts zero impedance for the pillbox cavity or periodic irises.

K.Bane, M.Sands, SLAC-PUB-4441 (1987)

$$w_{\|}(s) = \frac{Z_0 c}{\sqrt{2}\pi^2 a}\sqrt{\frac{g}{s}} \qquad w_{\perp}(s) = \frac{2}{a^2}\frac{\sqrt{2}Z_0 c}{\pi^2 a}\sqrt{gs}$$

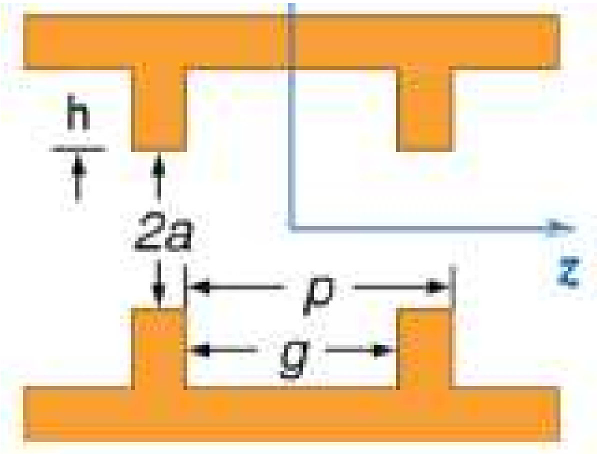
$$k_{\|} = \frac{Z_0 c}{4a\pi^{2.5}}\Gamma\left(\frac{1}{4}\right)\sqrt{\frac{g}{\sigma}} \qquad k_{\perp} = \frac{2}{a^3}\frac{Z_0 c}{\pi^{2.5}}\Gamma\left(\frac{3}{4}\right)\sqrt{g\sigma}$$

This impedance can be derived from parabolic equation (PE) approach.

G.Stupakov, New Journal of Physics 8, 280 (2006)

Diffraction Model

Periodic chain of cavities



K. L. F. Bane, K. Yokoya, in Proceedings of the 1999 PAC99, New York.

$$Z_{\|}(k) = \frac{Z_0}{2\pi a}\left[\eta^{-1} - ik\frac{a}{2}\right]^{-1}$$

$$\eta = \frac{(1+i)}{\sqrt{k}}\left[\alpha p\sqrt{\frac{\pi}{g}}\right]^{-1}$$

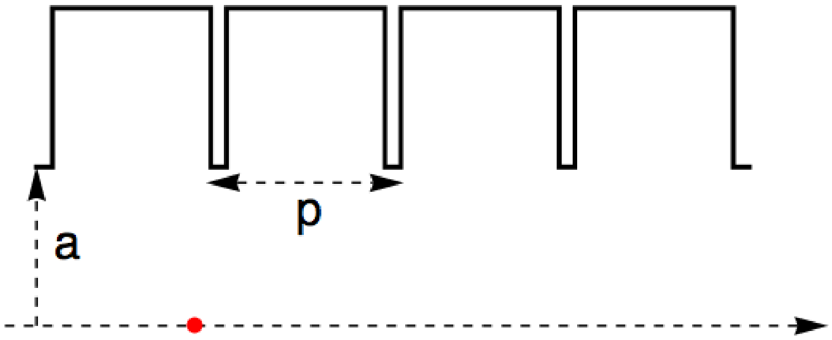
$$\alpha(\gamma) = 1 - 0.4648\sqrt{\gamma} - (1 - 2\cdot 0.4648)\gamma$$

$$w_{\|}(s) = \frac{Z_0 c}{\pi a^2} e^{\frac{s}{s_0}} erfc\left(\sqrt{\frac{s}{s_0}}\right)$$

$$s_0 = \frac{g}{2\pi}\left(\frac{a}{\alpha(g/p)p}\right)^2$$

Diffraction Model

Periodic array of irises



$$Z_{\|}(k) = \frac{Z_0}{2\pi a}\left[\eta^{-1} - ik\frac{a}{2}\right]^{-1}$$

Bane-Yokoya

$$\eta = \left[\frac{(1-i)}{2}\alpha\sqrt{gk\pi}\right]^{-1}$$

Stupakov

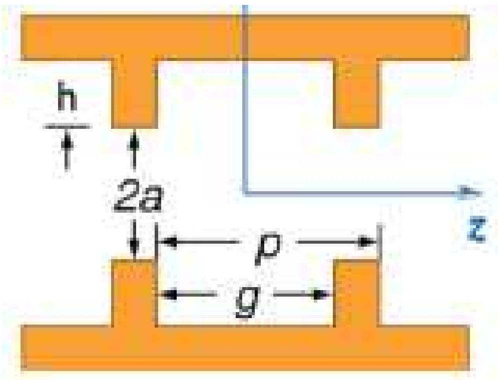
$$\eta = \left[\frac{(1-i)}{2}\alpha\sqrt{gk\pi} + \frac{1}{2}\right]^{-1}$$

$$\alpha = 0.4648$$

G. Stupakov, in Proceedings of the 1995 PAC, Dallas, Texas (IEEE, Piscataway, NJ,1996), p. 3303.

Diffraction Model

Periodic array of irises (G.Stupakov)



$$Z_{\|}(k) = i\frac{Z_0}{\pi k a^2}\left[1+(1+i)\frac{1}{a}\sqrt{\frac{g\pi}{k}}\alpha + i\frac{1}{ka}\right]^{-1}$$

Periodic array of cavities (my guess)

$$Z_{\|}(k) = i\frac{Z_0}{\pi k a^2}\left[1+(1+i)\frac{p}{a}\sqrt{\frac{\pi}{kg}}\alpha\left(\frac{g}{p}\right) + i\frac{p}{a}\frac{1}{kg}\right]^{-1}$$

$$\eta = \left[\frac{(1-i)}{2}\alpha\left(\frac{g}{p}\right)p\sqrt{\frac{k\pi}{g}} + \frac{1}{2}\frac{p}{g}\right]^{-1}$$

DESY

Diffraction Model

$$w_1(s) = \frac{Z_0 c}{\pi a^2} e^{\frac{s}{s_0}} erfc\left(\sqrt{\frac{s}{s_0}}\right) \quad - \text{from exact Fourier transform of BY}$$

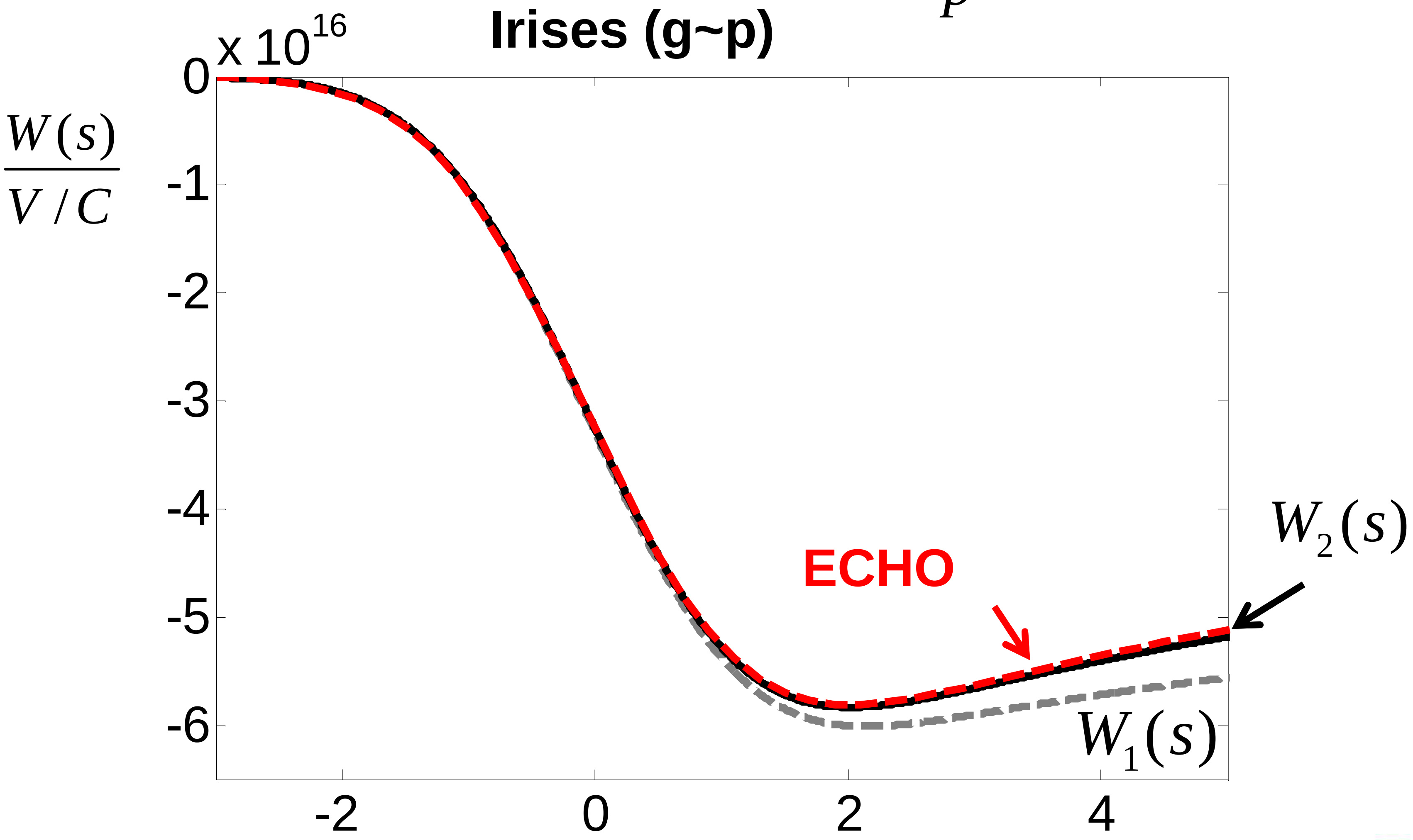
$$s_0 = \frac{g}{2\pi}\left(\frac{a}{\alpha(g/p)p}\right)^2$$

$$w_2(s) = \frac{Z_0 c}{\pi a^2} e^{-\sqrt{\frac{s}{s_1}}-\frac{s}{s_2}} \quad - \text{approximation of the new equation}$$

$$s_1 = s_0\left(\frac{\pi}{4}\right) \qquad s_2 = s_1\left(\frac{1}{2} - \frac{\pi}{4} + \frac{s_1}{a}\frac{p}{g}\right)^{-1}$$

Diffraction Model

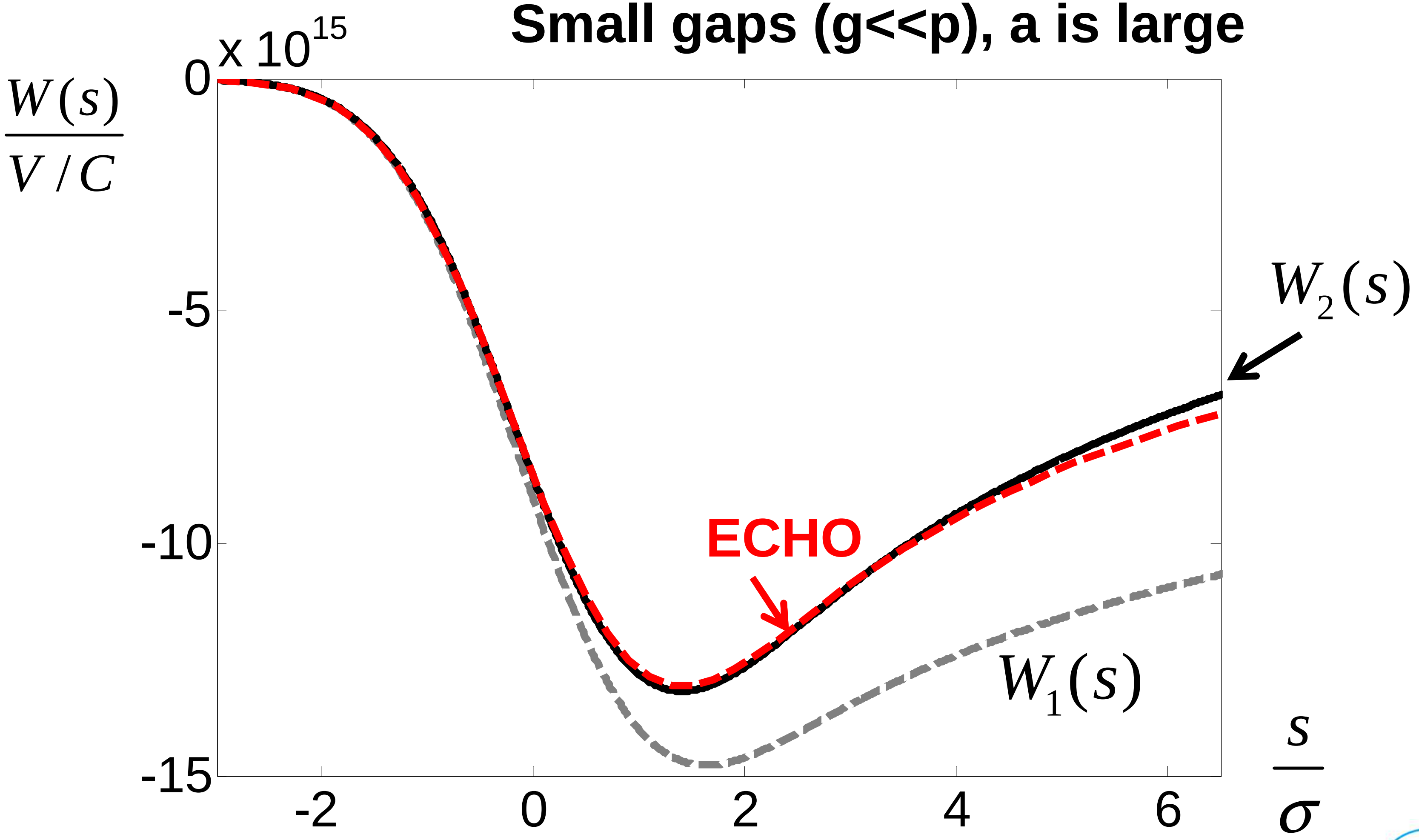
$$\sigma = 10\mu\text{m}, \quad a = 0.7\text{mm}, \frac{g}{p} = 0.98, \; h = 0.5\text{mm}$$



Diffraction Model

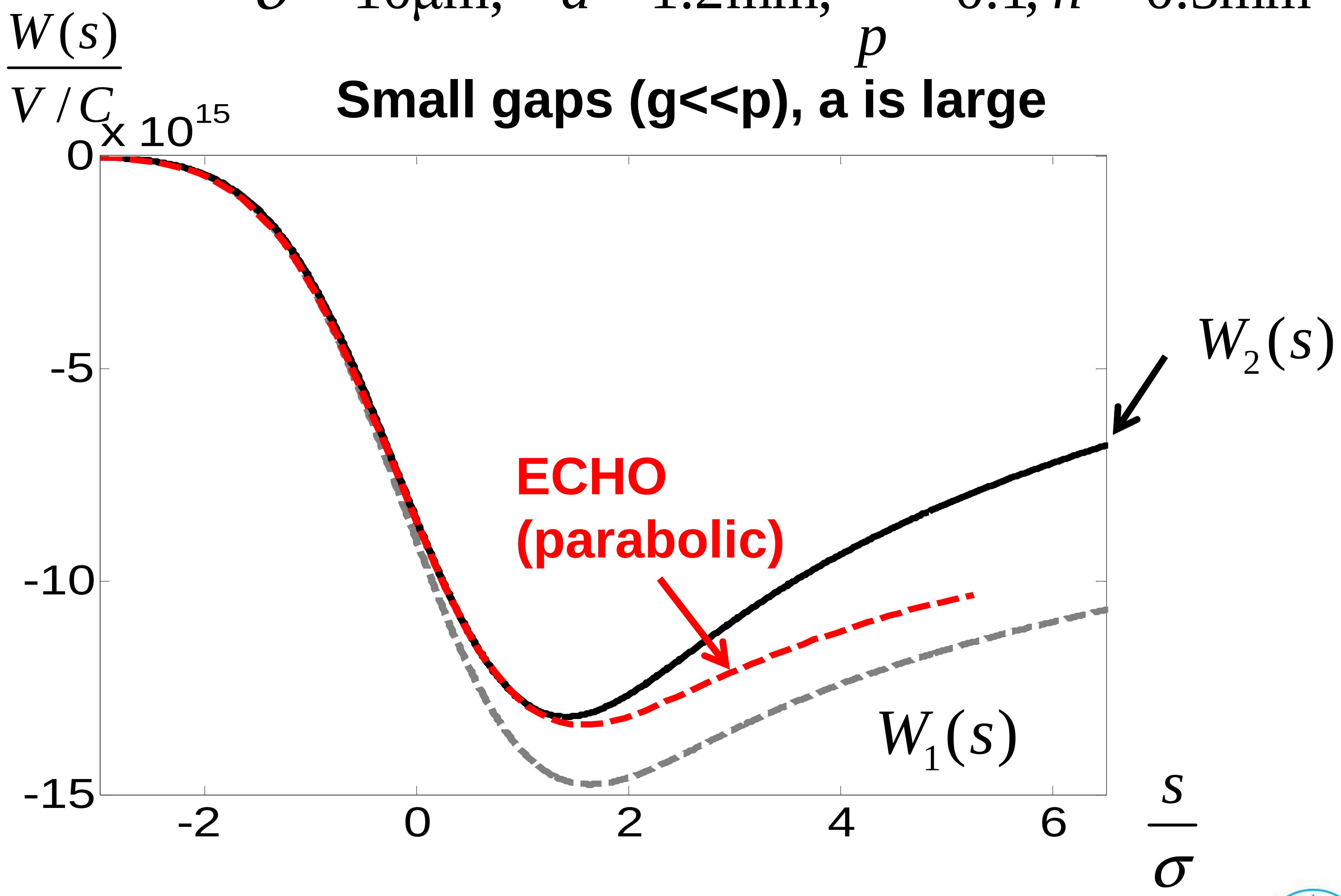
$$\sigma = 10\mu\text{m}, \quad a = 1.2\text{mm}, \frac{g}{p} = 0.1, h = 0.5\text{mm}$$

Small gaps (g<<p), a is large



Diffraction Model

$$\sigma = 10\mu\text{m}, \quad a = 1.2\text{mm}, \frac{g}{p} = 0.1, h = 0.5\text{mm}$$



Surface impedance

Round Resistive Pipe with Roughness and Oxide Layer

$$Z_{\|}(k) = \frac{1}{2\pi a}\left[\eta^{-1} - ik\frac{a}{2}\right]^{-1}$$

M.Dohlus. TESLA 2001-26, 2001
A.Tsakanian et al, TESLA-FEL 2009-05

$$\eta(\omega) = \frac{1}{Z_0}\sqrt{\frac{j\omega\mu_0}{\kappa(\omega)}} \qquad \kappa(\omega) = \frac{\kappa_0}{1 + j\omega\tau}$$

The effect of the oxide layer and the roughness can be taken into account through the inductive surface impedance

$$\overline{\eta}(\omega) \approx \eta(\omega) + i\omega\frac{L}{Z_0} \qquad L = \mu_0\left(\frac{\varepsilon_r - 1}{\varepsilon_r}d_{oxid} + 0.01 d_{rough}\right) \qquad \varepsilon_r \sim 2$$

$$Z_{\perp} = \frac{2}{ka^2}Z_{\|}$$

Surface impedance

Rectangular

K. Bane and G. Stupakov, Phys. Rev. STAB **18**, 034401 (2015)

$$Z_{\parallel}(x_0, y_0, x, y, k) = \frac{1}{w}\sum_{m=1}^{\infty} Z(y_0, y, k_{x,m}, k)\sin(k_{x,m}x_0)\sin(k_{x,m}x), \quad k_x = \frac{\pi m}{2w}$$

$$Z(y_0, y, k_x, k) = Z^{cc}(k_x, k)\cosh(k_x y_0)\cosh(k_x y) + Z^{ss}(k_x, k)\sinh(k_x y_0)\sinh(k_x y)$$

$$Z^{cc}(k_x, k) = \frac{Z_0 c}{2a}\operatorname{sech}^2(X)\left[\eta^{-1} - ika\frac{\tanh(X)}{X}\right]^{-1}, X = ak_x$$

$$Z^{ss}(k_x, k) = \frac{Z_0 c}{2a}\operatorname{csch}^2(X)\left[\eta^{-1} - ika\frac{\coth(X)}{X}\right]^{-1}$$

Corrugated structure

$$\eta = \left[\frac{(1-i)}{2}\alpha\left(\frac{g}{p}\right)p\sqrt{\frac{k\pi}{g}} + \frac{1}{2}\frac{p}{g}\right]^{-1}$$

Conductive layer

$$\eta = \frac{1}{Z_0}\sqrt{\frac{i\omega\mu}{\kappa}}$$

Limiting Value of the Wake at the Origin

The wakefield experienced by a point-like charge (loss factor) in a waveguide of fixed transverse dimensions is independent of the detailed properties of the slowdown layer (dielectric, conductive, corrugations)

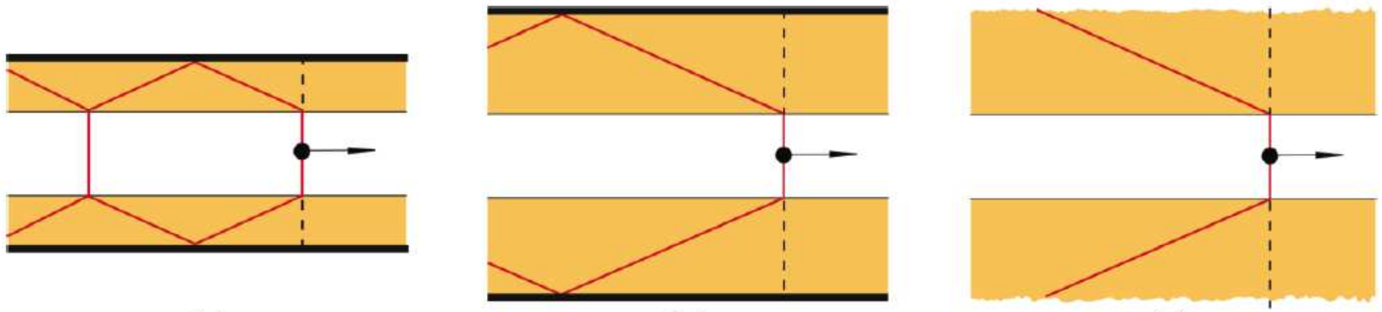


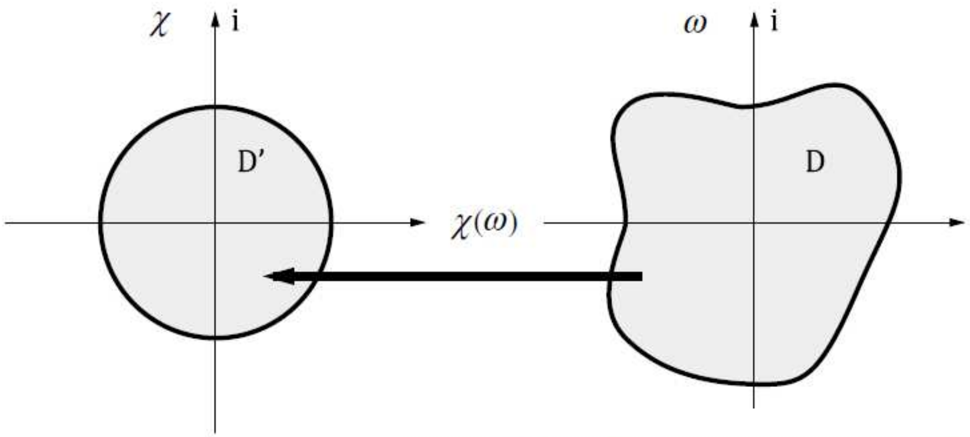
Figure 1. Cherenkov wakefield cones of a point-like charge moving along (a) a waveguide with a thin arbitrary slowdown layer on a metal surface; (b) waveguide with a thick layer; (c) infinite medium.

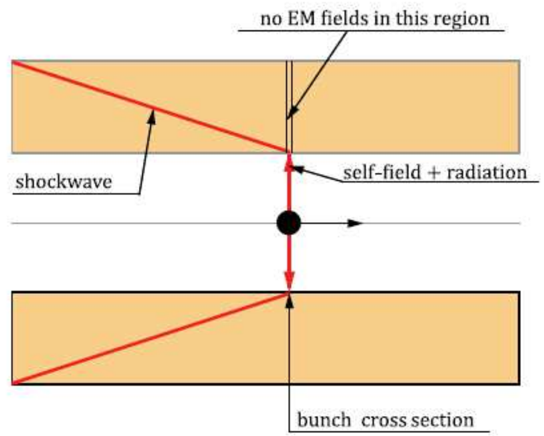
$$\kappa_c = \frac{1}{2\pi a_c^2 \varepsilon_0}, \qquad \kappa_p = \frac{1}{2\pi a_p^2 \varepsilon_0}\frac{\pi^2}{16},$$

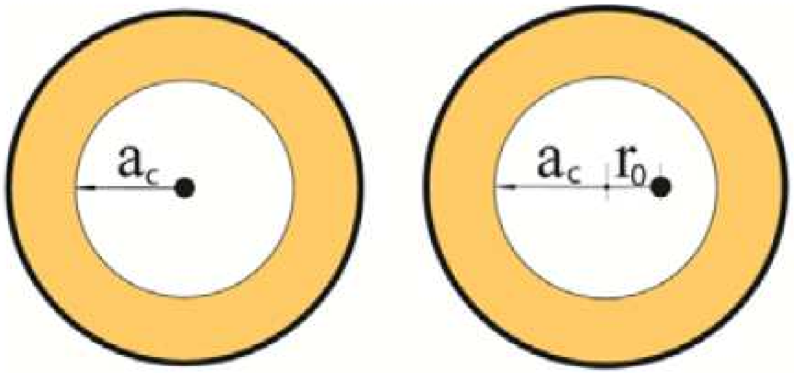
K.L.F. Bane, SLAC-pub-11829, (2006).

K.L.F. Bane and G. Stupakov, Phys. Rev. ST-Accel. Beams 6, 024401 (2003)

Limiting Value of the Wake at the Origin







$$\kappa_{\perp}^{c} \approx \frac{2}{\pi a_c^4 \varepsilon_0}\left(1+3\left(r_0 / a_c\right)^2\right); \qquad r_0 / a_c \ll 1.$$

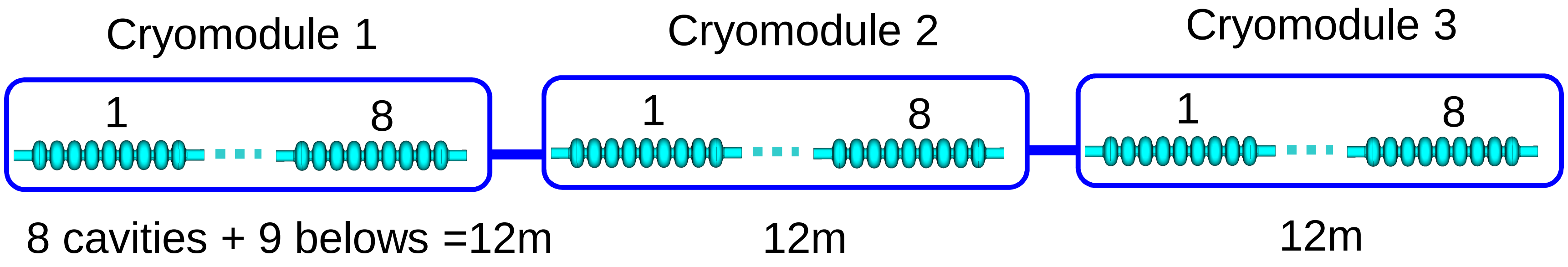
S. S. Baturin and A. D. Kanareykin, Phys. Rev. Lett. 113, 214801 (2014)
S. S. Baturin and A. D. Kanareykin, Phys. Rev. AB, 19, 051001 (2016)

Limiting Value of the Wake at the Origin

Optical model	Diffraction model (cavity)	Diffraction model (cavity chain)	Slow down layer
$w_{\parallel}(s) \sim \delta(s)$	$w_{\parallel}(s) = O\left(\sqrt{\frac{1}{s}}\right)$		$w_{\parallel}(s) = O(1)$
$w_{\perp}(s) = O(1)$	$w_{\perp}(s) = O\left(\sqrt{s}\right)$		$w_{\perp}(s) = O(s)$

Combining Computations and Analytics

Wakefunctions of TESLA Cryomodule



- Wakes for short bunches up to 50um have been studied
- To reach the steady state solution 3 cryomodules are considered
- For longitudinal case the wakes were studied earlier by A. Novokhatski et al*. The transverse results are calculated with ECHO**.

*Novokhatski A et al, DESY, TESLA-1999-16, 1999

**Weiland T., Zagorodnov I, DESY, TESLA-2003-19, 2003

Combining Computations and Analytics

Wakefunctions of TESLA Cryomodule

Periodic structure

$$w_{\|}(s) = A\frac{Z_0 c}{\pi^2 a}\exp(-\sqrt{s/s_0}) \quad \sim O(1)$$

$$w_{\perp}(s) = \frac{2}{a^2} A\frac{Z_0 c}{\pi^2 a} 2s_1 \left(1-\left(1+\sqrt{s/s_1}\right)e^{-\sqrt{s/s_1}}\right) \quad \sim O(s)$$

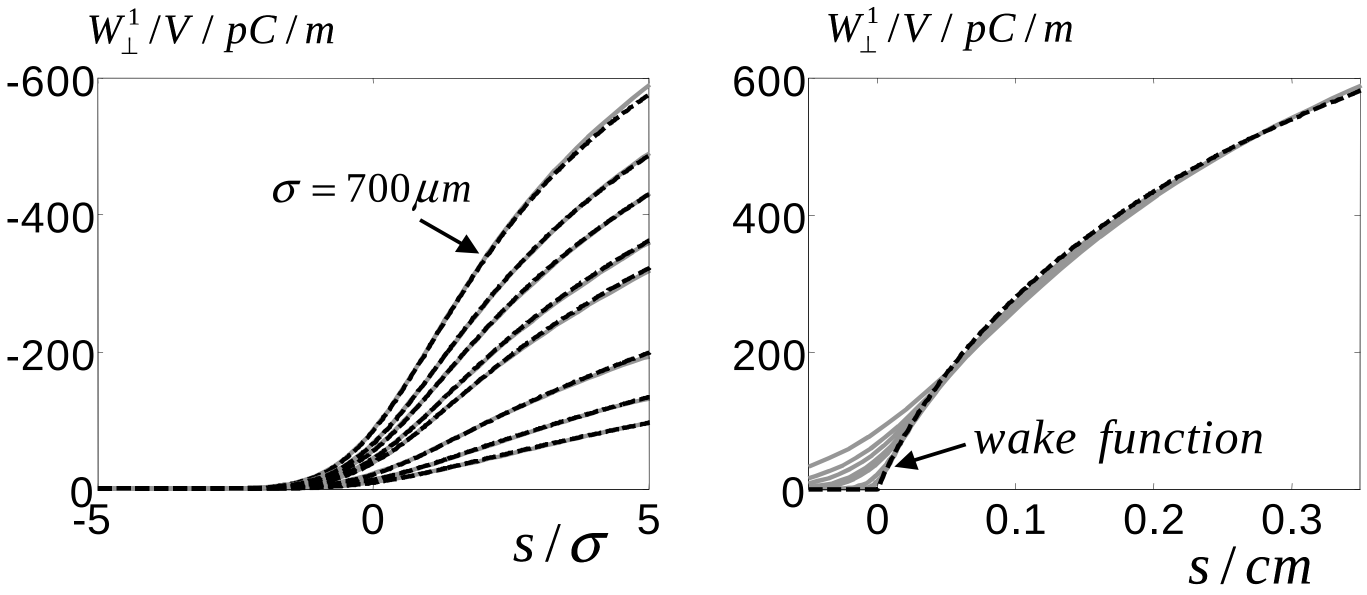
a – iris rtadius, g – cavity gap

A, s_0, s_1 - fit parameters

K.L.F.Bane, SLAC-PUB-9663, LCC-0116, 2003

Combining Computations and Analytics

Wake functions of TESLA Cryomodule



Comparison of numerical (grays) and analytical (dashes) transverse wakes

$$w_{\parallel}(s) = 344 \exp(-\sqrt{s/s_0})[\mathrm{V/pC/module}] \qquad O(1), s \to 0$$

$$w_{\perp}(s) = 10^3 \left(1 - \left(1 + \sqrt{\frac{s}{s_1}}\right) \exp\left(-\sqrt{\frac{s}{s_1}}\right)\right)\left[\frac{\mathrm{V}}{\mathrm{pC} \times \mathrm{m} \times \mathrm{module}}\right] \qquad O(s), s \to 0$$

$$s_0 = 1.74 \cdot 10^{-3} \qquad s_1 = 0.92 \cdot 10^{-3} \qquad A = 1.46 \qquad a = \bar{a} = 35.57 \mathrm{mm}$$

Combining Computations and Analytics

Recently another method was suggested*.

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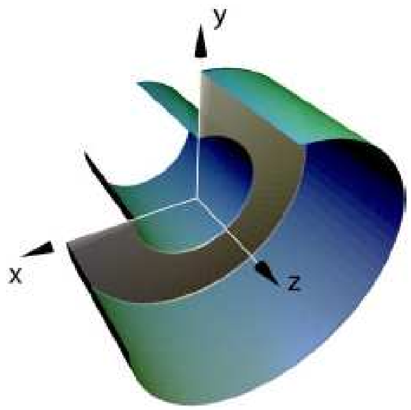
The idea behind the method is to use a combination of computer simulations with an analytical form of the wake function $w_s(s)$ for a given geometry in the high-frequency limit (optical or diffraction model).



$$w(s) = w_s(s) + d(s)$$

$$w_s(s) = -\frac{1}{\pi\varepsilon_0}\ln\left(\frac{b}{a}\right)\delta(s) \qquad d(s) = (\alpha + \beta s)\theta(s)$$

The crucial element of the method is that the smooth function $\mathrm{d}(s)$ can be obtained from simulations with long bunch by fitting to the formula.

*Podobedov B., Stupakov G., PRST-AB 16, 024401 (2013)

Impedance Database

Wake function model

$$w(s) = w^{(0)}(s) + \frac{1}{C} + Rc\delta(s) - c\frac{\partial}{\partial s}\left[Lc\delta(s) + w^{(-1)}(s)\right]$$

regular part

singular part
(cannot be tabulated directly)

$$Z(\omega) = Z^{(0)}(\omega) - \frac{1}{i\omega C} + R + i\omega\left[L + Z^{(-1)}(\omega)\right]$$

capacitive — resistive — inductive

$W \sim \int \lambda(s)ds$ — $W \sim \lambda(s)$ — $W \sim \lambda'(s)$

$$\frac{\partial}{\partial s} w^{(-1)}(s) = o(s^{-1}), \qquad s \to 0,$$ it describes singularities $s^{-\alpha}$, $\alpha<1$

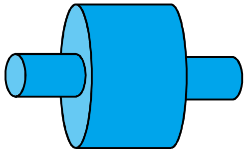
O. Zagorodnova, T. Limberg, in Proceedings of 2009 PAC,(Vancouver, 2009).

Impedance Database

$$w(s) = w^{(0)}(s) + \frac{1}{C} + Rc\delta(s) - c\frac{\partial}{\partial s}\left[Lc\delta(s) + w^{(-1)}(s)\right]$$

Pillbox Cavity

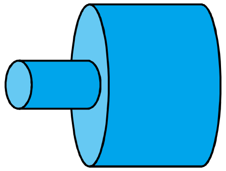
$$w(s) = \frac{Z_0 c}{\sqrt{2}\pi^2 a}\sqrt{\frac{g}{s}}$$



$$w^{(-1)}(s) = -\frac{Z_0}{\sqrt{2}\pi^2 a}\sqrt{sg}$$

Step-out transition

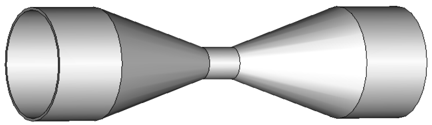
$$w(s) = c\frac{Z_0}{\pi}\ln\left(\frac{b}{a}\right)\delta(s)$$



$$R = \frac{Z_0}{\pi}\ln\left(\frac{b}{a}\right)$$

Tapered collimator

$$w(s) = -c^2\left(\frac{Z_0}{4\pi c}\int r'dr\right)\frac{\partial}{\partial s}\delta(s)$$



$$L = \frac{Z_0}{4\pi c}\int r'dr$$

Impedance Database

Wake potential for arbitrary bunch shape

$$W(s) = -\int_{-\infty}^{s} w^{(0)}(s-s')\lambda(s')ds' - \frac{1}{C}\int_{-\infty}^{s} \lambda(s')ds' - Rc\lambda(s) -$$

$$-c^2 L\lambda'(s) - c\int_{-\infty}^{s} w^{(-1)}(s-s')\lambda'(s)ds'$$

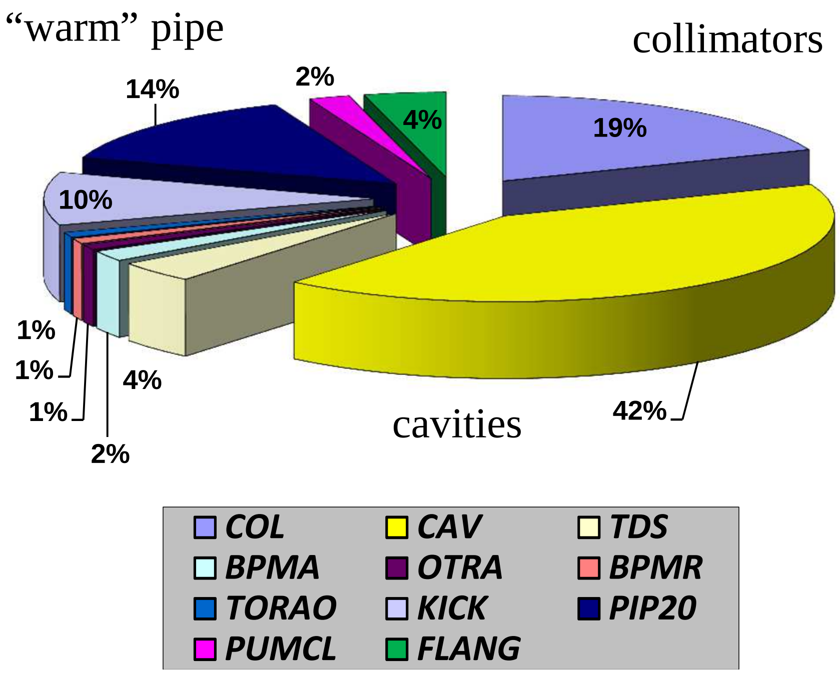
derivative of the bunch shape

Impedance Database

Accelerator wakes. Q=1nC

Impedance Budget (list of elements)

El.type	Num.	Loss (kV/nC)	%	Spread (kV/nC)	%	Peak (kV/nC)	%
BPMF	4	4.075E+01	0	1.858E+01	0	5.804E+01	0
COL	7	6.725E+03	19	3.373E+03	22	1.058E+04	21
KICK	3	3.645E+03	10	1.459E+03	9	5.283E+03	10
PIP20	1	5.116E+03	14	3.661E+03	24	8.959E+03	18
PUMCL	78	5.605E+02	2	2.363E+02	2	7.946E+02	2
CAV	808	1.481E+04	42	8.842E+03	57	2.814E+04	56
CAV3	8	8.084E+01	0	3.010E+01	0	1.117E+02	0
FLANG	500	1.330E+03	4	5.610E+02	4	1.886E+03	4
TDS	8	1.507E+03	4	7.348E+02	5	2.174E+03	4
OTRB	8	1.584E+02	0	7.251E+01	0	2.254E+02	0
STEP1	1	3.010E+00	0	5.969E-01	0	3.441E+00	0
BPMA	107	5.654E+02	2	2.896E+02	2	8.670E+02	2
OTRA	12	3.078E+02	1	1.274E+02	1	4.494E+02	1
BPMC	56	4.431E+01	0	2.138E+01	0	6.805E+01	0
BPMR	26	2.993E+02	1	1.304E+02	1	4.501E+02	1
DCM	4	1.644E+01	0	7.479E+00	0	2.315E+01	0
BPMB	27	5.744E-02	0	1.587E-01	0	6.023E-01	0
BAM	5	3.319E+00	0	1.494E+00	0	4.768E+00	0
TORA	3	3.147E+01	0	1.609E+01	0	4.763E+01	0
TORAO	6	1.856E+02	1	7.684E+01	0	2.700E+02	1
		3.530E+04	100	1.540E+04	100	5.037E+04	100



Impedance Database

Longitudinal+Transverse Wakes 3D

Taylor Expansion of wake function

(Test particle coordinates $-\{x_t, y_t\}$)

$$w_{//}(x, x_t, y, y_t, s) = w_0(s) + \begin{pmatrix} w_1(s) \\ w_2(s) \\ w_3(s) \\ w_4(s) \end{pmatrix}^T \begin{pmatrix} x \\ y \\ x_t \\ y_t \end{pmatrix} + \begin{pmatrix} x \\ y \\ x_t \\ y_t \end{pmatrix}^T \begin{pmatrix} w_{11}(s) & w_{12}(s) & w_{13}(s) & w_{14}(s) \\ w_{12}(s) & -w_{11}(s) & w_{23}(s) & w_{24}(s) \\ w_{13}(s) & w_{23}(s) & w_{33}(s) & w_{34}(s) \\ w_{14}(s) & w_{24}(s) & w_{34}(s) & -w_{33}(s) \end{pmatrix} \begin{pmatrix} x \\ y \\ x_t \\ y_t \end{pmatrix}$$

In the special case (monopole+dipole wake) non-vanishing coefficients are: →

$$w_0(s) = w_{//}^{(monopole)}(s)$$
$$w_{13}(s) = w_{24}(s) = 0.5 \cdot w_{//}^{(dipole)}(s)$$

Wake file is in ASCII format and is a "multi-table" describing up to 13 coefficient functions.
Each function is described by following model:

$$w(s) = w^{(0)}(s) + \frac{1}{C} + Rc\delta(s) - c\frac{\partial}{\partial s}\left[Lc\delta(s) + w^{(-1)}(s)\right]$$

M. Dohlus et al, DESY 12-012, 2012.

I. Zagorodnov et al, NIM A 837 (2016) 69-79.

https://github.com/ocelot-collab/ocelot