Beam measurements of frequency characteristics of (longitudinal) impedance

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ICFA mini-Workshop on Impedance and Beam Instabilities in Particle Accelerators

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Introduction and motivation

- The impedance of an accelerator is often developed with the help of beam based measurements. These allow to probe the effective impedance: product of the beam spectrum and the machine impedance.

- Various methods are applicable to evaluate the frequency characteristics of the impedance. In this presentation, we will focus on the evaluation of the reactive part of the longitudinal impedance.

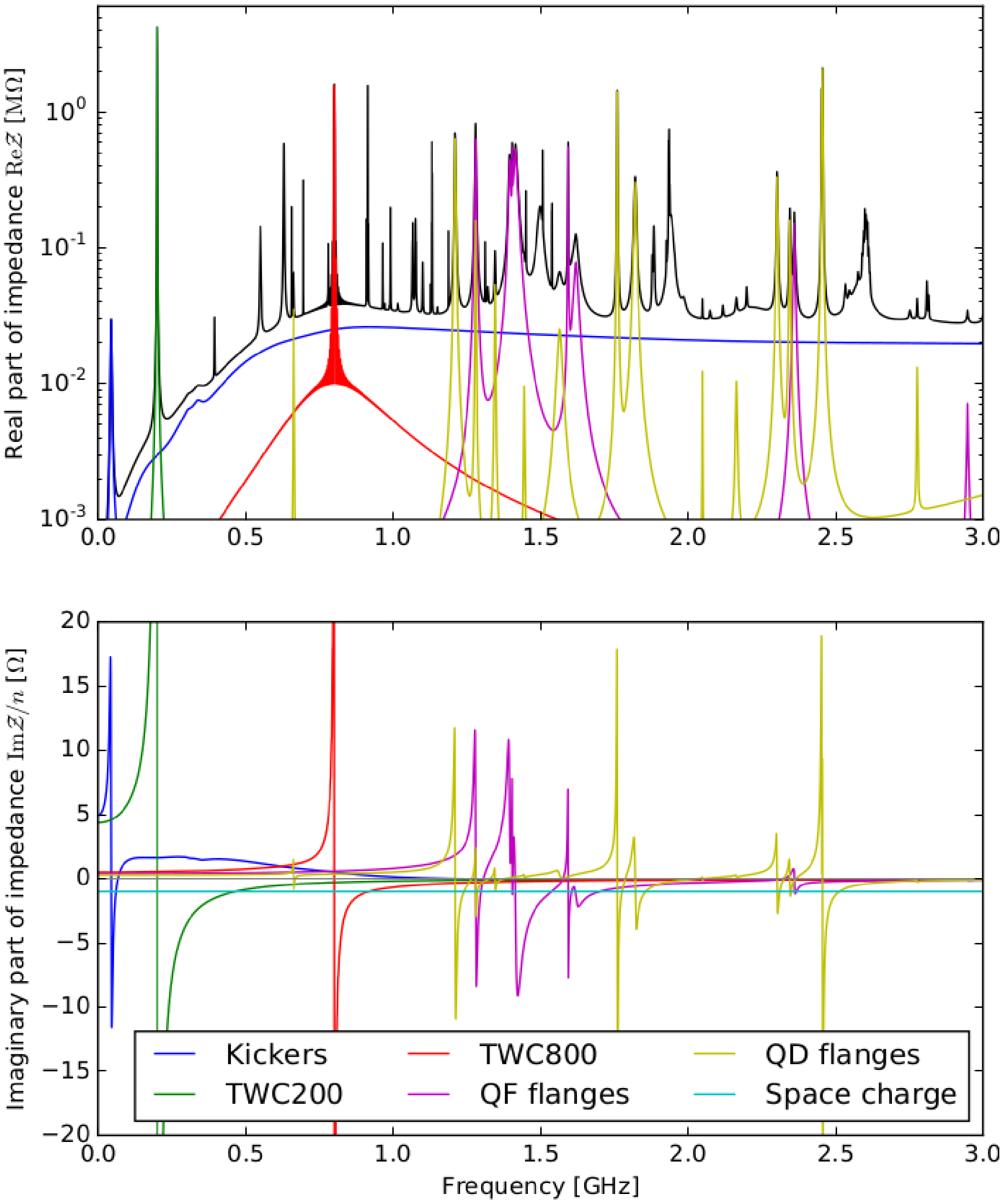
- Studies were done for the SPS at CERN, by measuring the synchrotron frequency shift with intensity.

- Longitudinal instabilities in the SPS are one of the main limitations for the High Luminosity-LHC project. A reliable impedance model is necessary to understand and find means to mitigate these instabilities.

Outline

- The SPS impedance and beam measurements techniques
- Synchrotron frequency shift measured from quadrupole oscillations
    - Theory
    - Measurements
    - Simulations
- Identification of possible missing impedance and its frequency characteristics

The present SPS impedance model



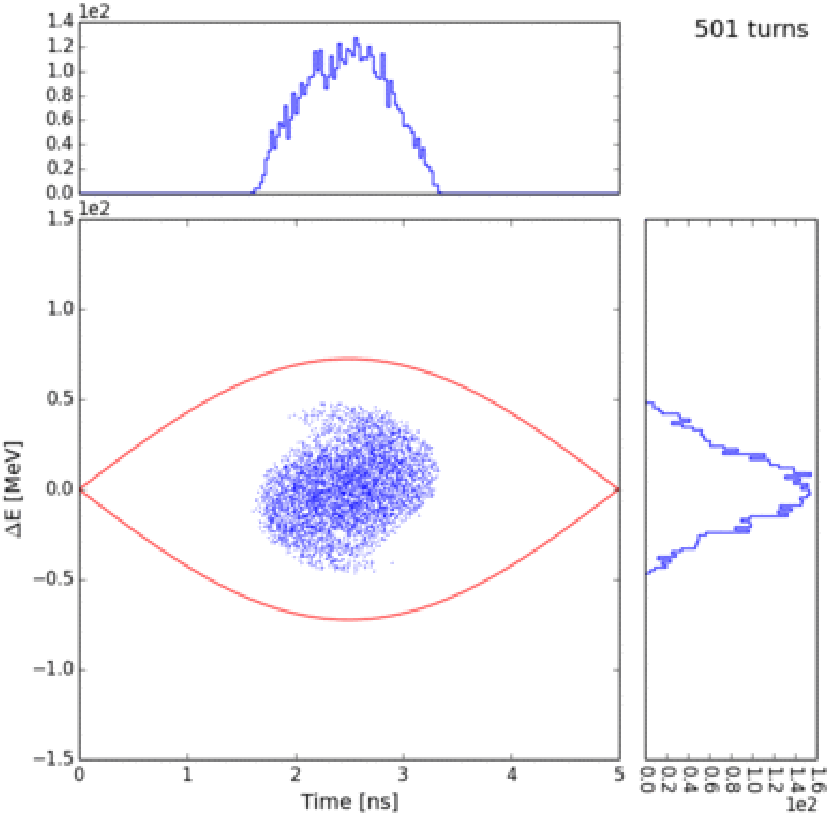
- Includes many contributions obtained from electromagnetic simulations and bench measurements

- Model has been constantly evolving over the past years

- Many sources (RF, flanges…) can be described by a resonator impedance:

$$\mathcal{Z}(f) = \frac{R_s}{1 + jQ\left(\frac{f}{f_r} - \frac{f_r}{f}\right)}$$

Effective impedance



- *Linear synchrotron motion*

$$\ddot{\tau} + \omega_{s0}^2 \tau = \frac{\omega_{s0}^2 q N_b}{2\pi V_{\mathrm{rf}}\, h \cos\phi_s}\left[Z_0 + \tau f_{\mathrm{rev}} Z_1\right]$$

$= 2\pi f_{s0}$, linear synchrotron frequency

- *Effective resistive impedance*

$$Z_0 \underset{\hat{\tau}\to 0}{\approx} \frac{1}{f_{\mathrm{rev}}} \int_{-\infty}^{\infty} \mathcal{S}\left(f\right) \mathrm{Re} \mathcal{Z}\left(f\right) df$$

- *Effective reactive impedance*

$$Z_1 \underset{\hat{\tau}\to 0}{\approx} -\frac{2\pi}{f_{\mathrm{rev}}^2} \int_{-\infty}^{\infty} f\, \mathcal{S}\left(f\right) \mathrm{Im} \mathcal{Z}\left(f\right) df$$

These equations are only valid in the linear regime, for particles with small amplitude of oscillations $\hat{\tau}$

Beam measurements of the impedance

- Various methods are applicable to measure the impedance of a synchrotron and evaluate its frequency characteristics

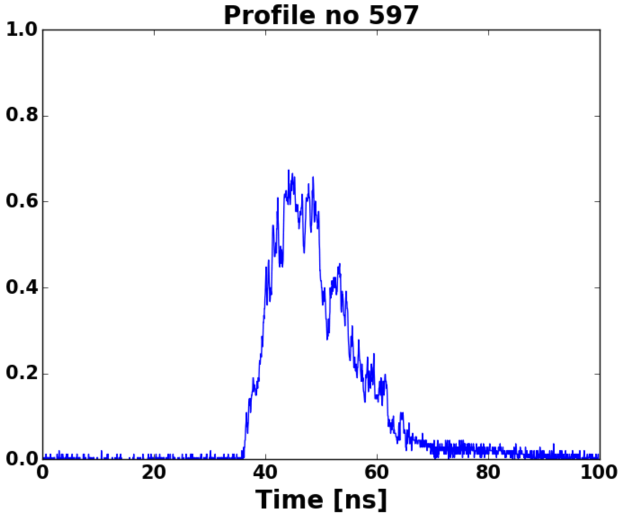
- For stable bunches:
  - Synchrotron frequency shift
  - Debunching time
  - Synchronous phase shift

Bunch length

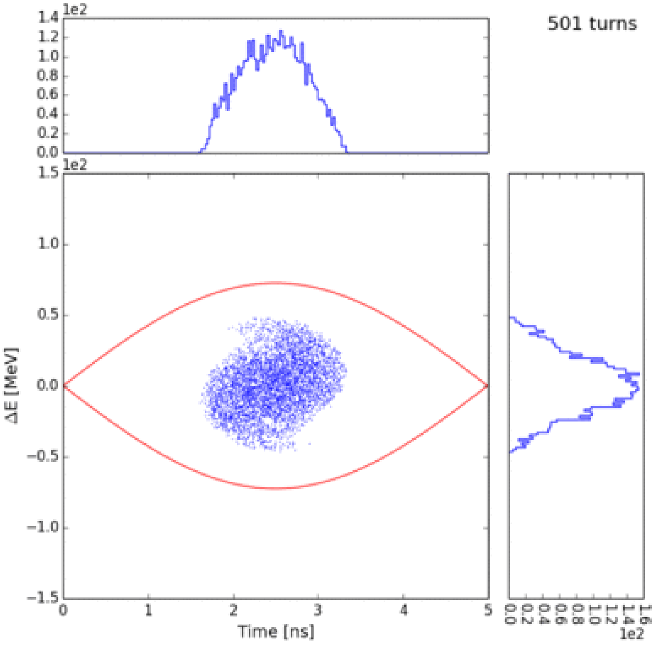
$$Z_1 \underset{\hat{\tau}\to 0}{\approx} -\frac{2\pi}{f_{\mathrm{rev}}^2}\int_{-\infty}^{\infty} f\,\mathcal{S}(f)\,\mathrm{Im}\mathcal{Z}(f)\,df$$

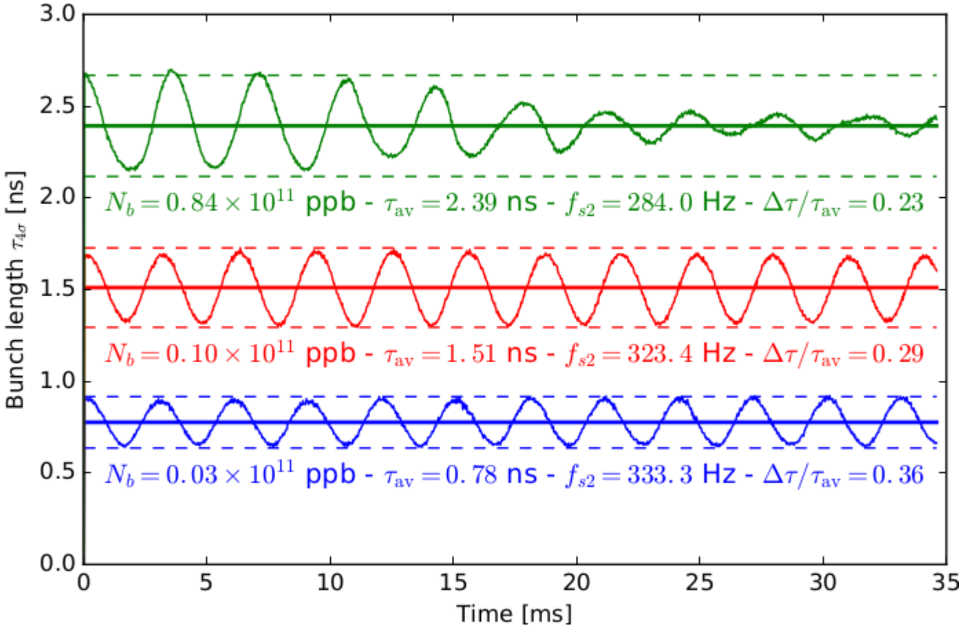
$$Z_0 \underset{\hat{\tau}\to 0}{\approx} \frac{1}{f_{\mathrm{rev}}}\int_{-\infty}^{\infty} \mathcal{S}(f)\,\mathrm{Re}\mathcal{Z}(f)\,df$$

- For unstable bunches
  - Growth rates and thresholds (dependence on longitudinal emittance)
  - Direct measurements from the beam spectrum (identification of the vacuum flanges impedance at 1.4 GHz)



Quadrupole frequency measurements



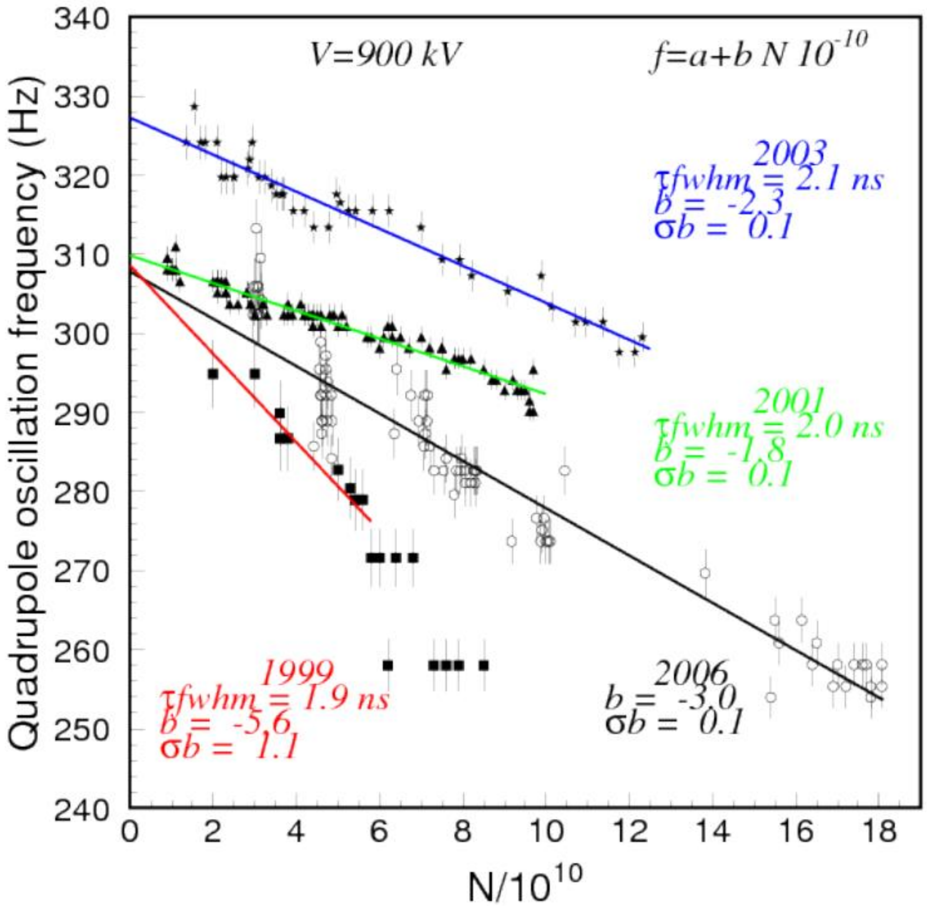


Measurement method:

- Quadrupole oscillations provoked by injection of short bunches into mismatched RF voltage (oscillations are mainly performed by particles with large amplitude of oscillations $\hat{\tau}$)

- Frequency of bunch length (or peak amplitude) oscillations, $\approx 2 f_{s0}$ ($f_{s0} = 172.4$ Hz), is measured for bunches with different intensities keeping same average bunch length

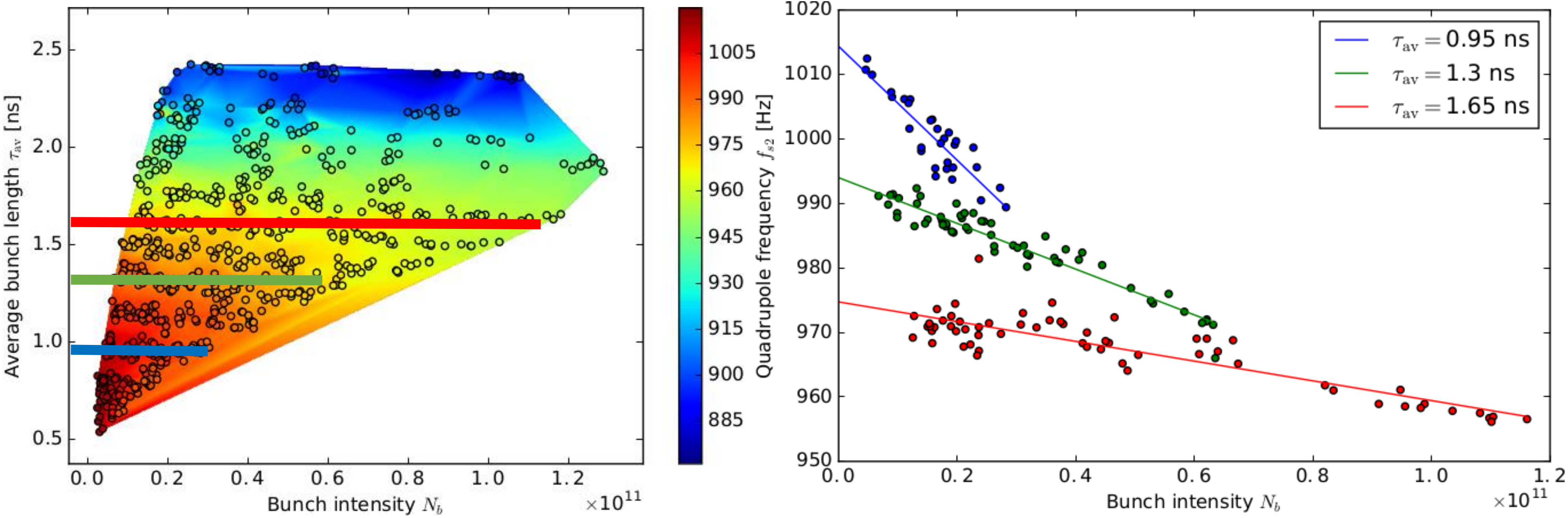
Synchrotron frequency shift in the CERN SPS



- 2000: *SPS impedance reduction*
- *2003&2006: addiction of extraction kickers for LHC*

- Reference measurements performed over many years to follow up the evolution of the impedance
- Measurements performed for bunches with different intensity, keeping the same average bunch length
- The slope b of the shift with intensity depends on the effective reactive impedance Z_1
- Small variations in bunch length can lead to inconsistent results

Quadrupole frequency dependences

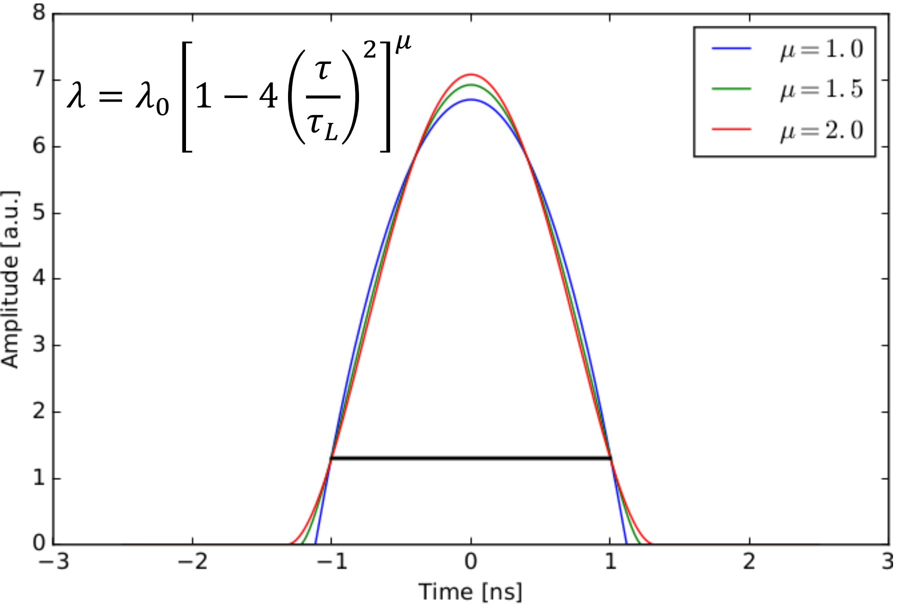


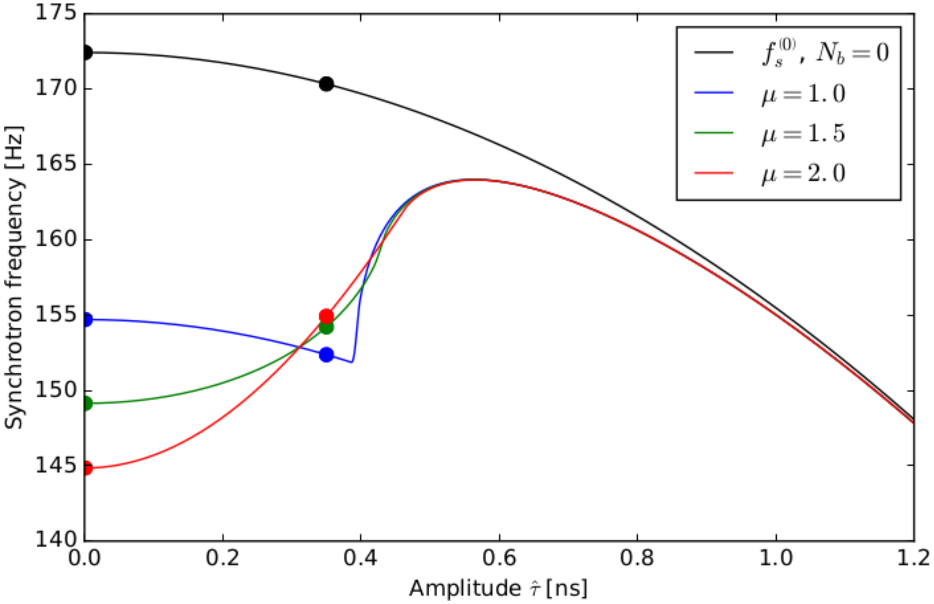
- The quadrupole frequency strongly depends on the average bunch length (emittance)

- The shift with intensity depends on the effective reactive impedance and contains
  - The incoherent part (from stationary bunch distribution)
  - The coherent part (from the perturbation)

$$f_{s,m}(N_b) \approx m f_{s0} + m \Delta f_{\mathrm{inc}}(N_b) + \Delta f_{\mathrm{coh},m}(N_b)$$

Incoherent synchrotron frequency shift



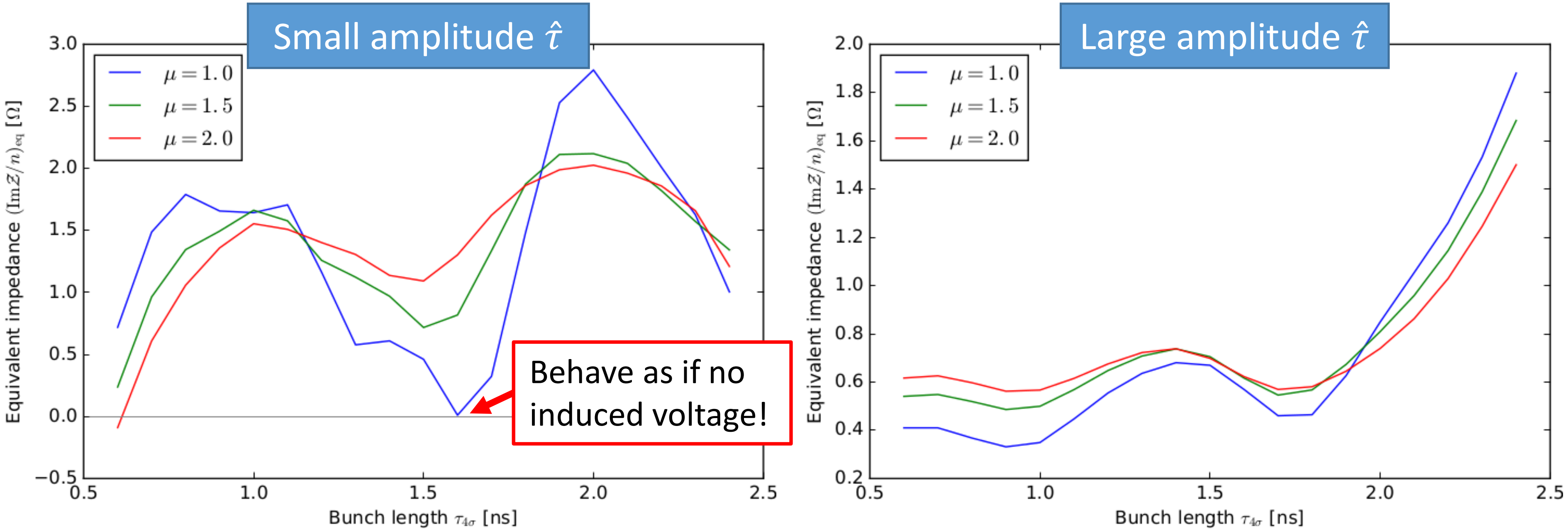


- Defined by the induced voltage of the stationary bunch distribution

- For a parabolic bunch ($\mu = 1$) and a constant reactive impedance ImZ/n ($n = f/f_{\mathrm{rev}}$), the incoherent shift is:

$$f_{2s}(\tau) \approx f_{2s}^{(0)}(\tau_L) - f_{2s}^{(0)}(\tau_L) \frac{6e}{h V_{\mathrm{rf}} \omega_0^2 {\tau_L}^3} \left(\frac{\mathrm{Im} Z}{n}\right) N_b$$

SPS equivalent impedance



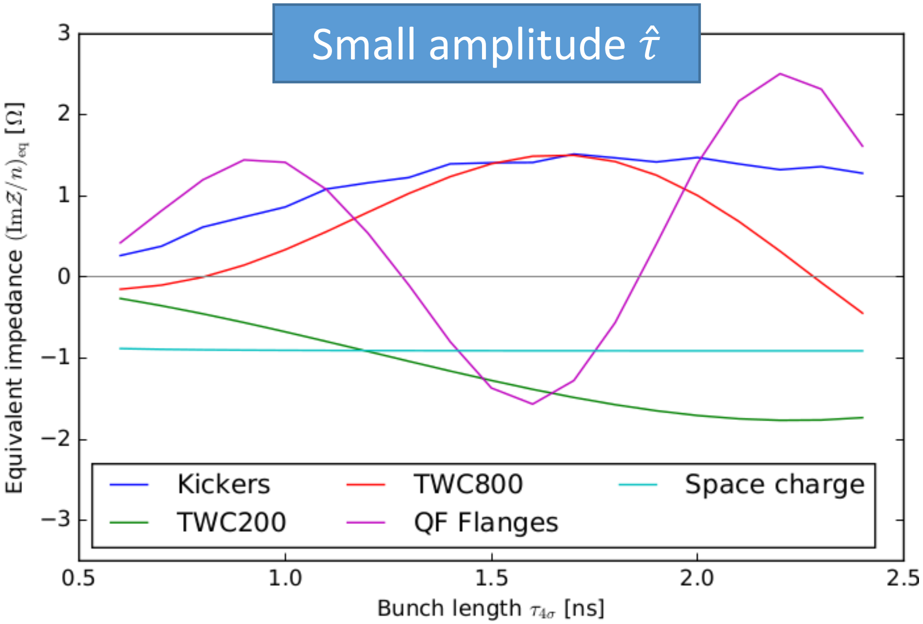
Behave as if no induced voltage!

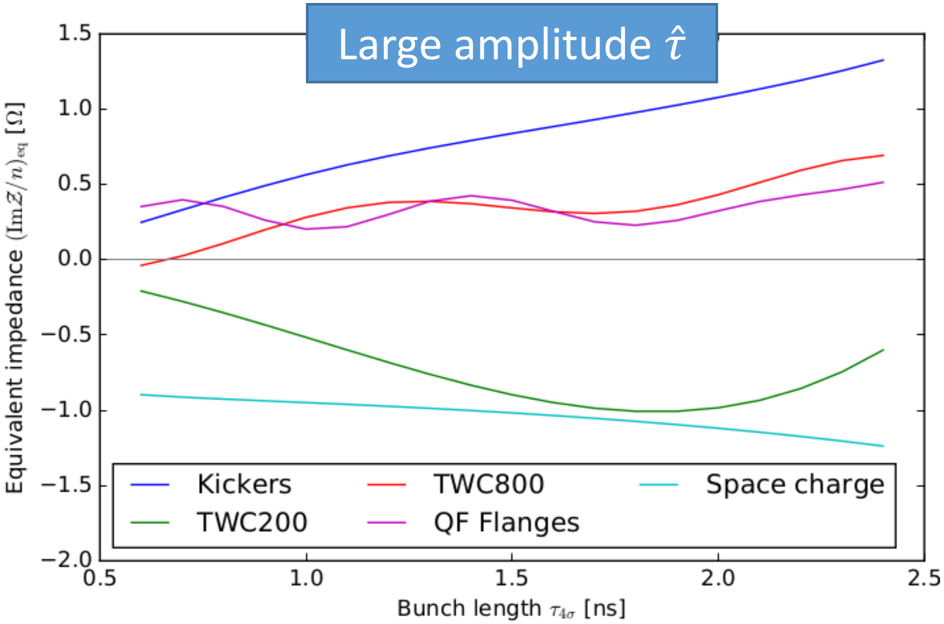
- The SPS impedance cannot be considered as a constant ImZ/n.
- The incoherent shift is computed numerically, and the "equivalent" impedance is given by:

Computed numerically

$$(\mathrm{Im}\mathcal{Z}/n)_{\mathrm{eq}} \overset{\mathrm{def}}{=} \frac{\omega_{\mathrm{rev}}^2 V_{\mathrm{RF}} h}{6q} \frac{\Delta f_{\mathrm{inc}}}{f_s^{(0)}} \frac{\tau_{4\sigma}^3}{N_b}$$

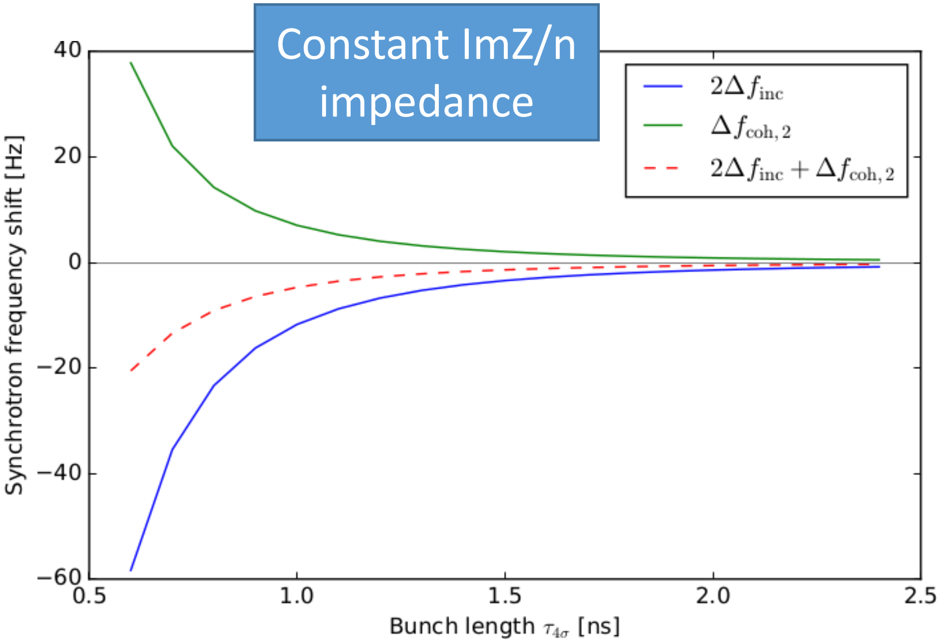
Effect of the various SPS impedance sources

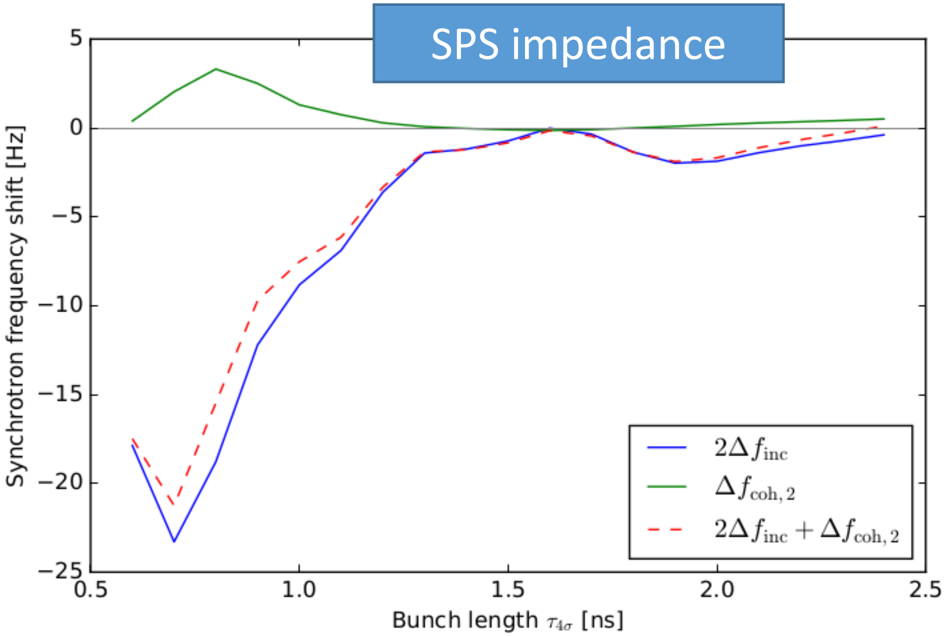

Small amplitude $\hat{\tau}$


Large amplitude $\hat{\tau}$

- Dependence on bunch length:
  - Depending on the frequency of the impedance source, the effective contribution is inductive (>0), or capacitive (<0).
  - The longitudinal space charge impedance has constant ImZ/n, and it is not negligible on the SPS flat bottom!
- The measured synchrotron frequency shift corresponds to particles with large amplitude of oscillations

Coherent synchrotron frequency shift


Constant ImZ/n impedance

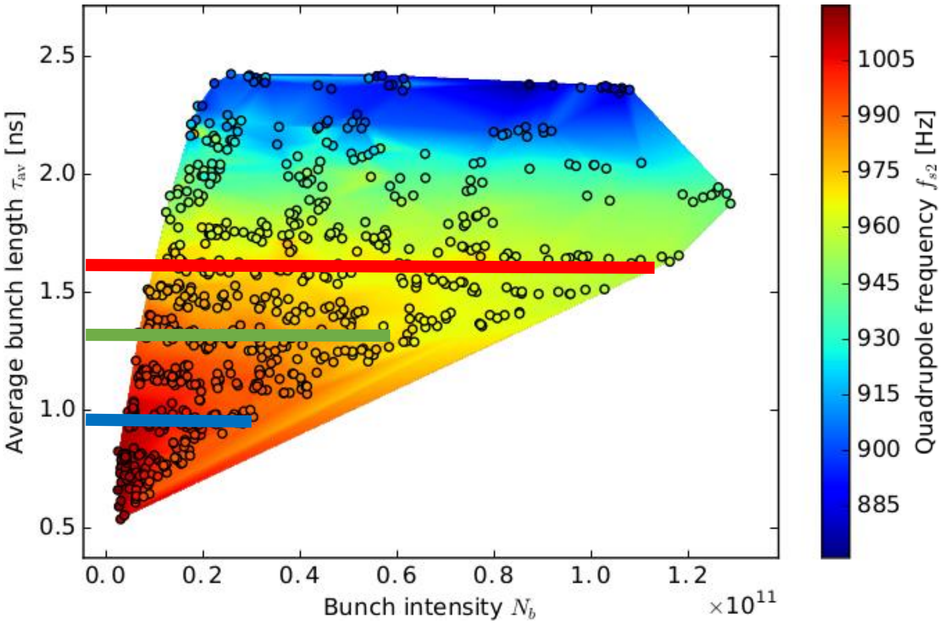

SPS impedance

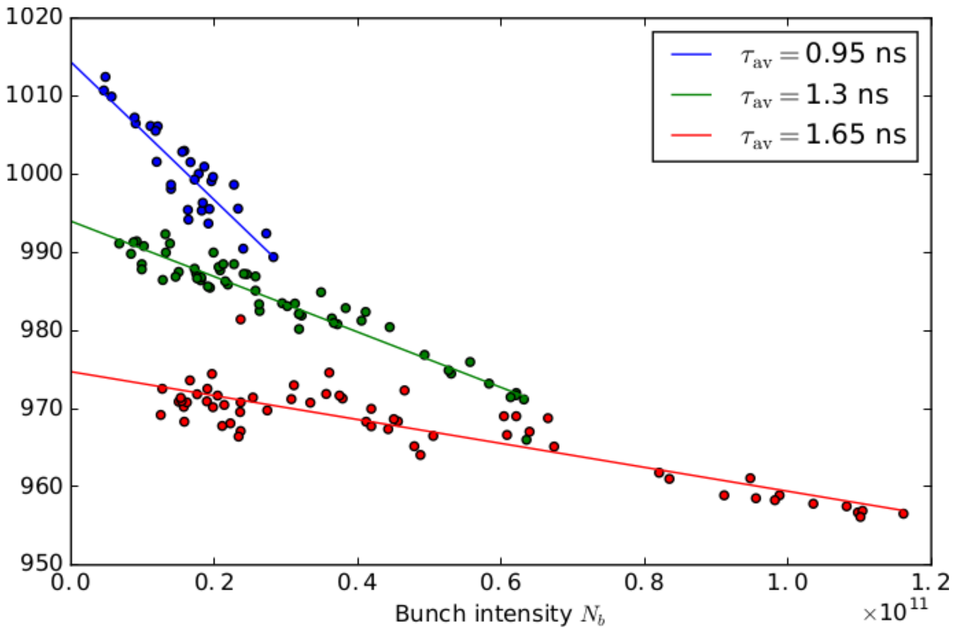
- The coherent motion of the bunch also contributes to the synchrotron frequency shift, which depends on the effective impedance of the perturbation spectrum

- For a constant ImZ/n impedance, the coherent shift reduces the total synchrotron frequency shift by ≈ a half

➢ For the SPS impedance, the coherent shift is small in comparison with the incoherent one

Measurements of the quadrupole frequency

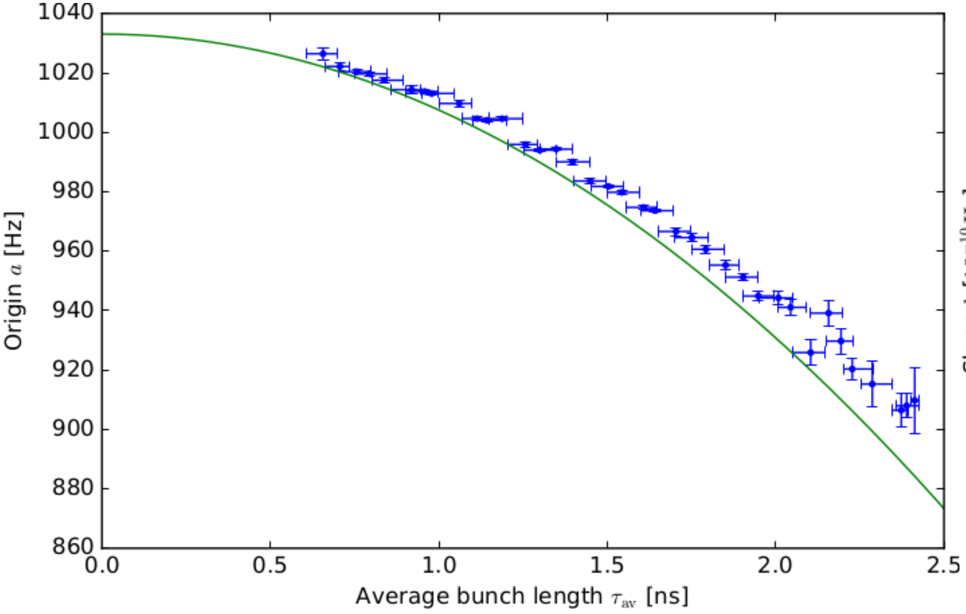


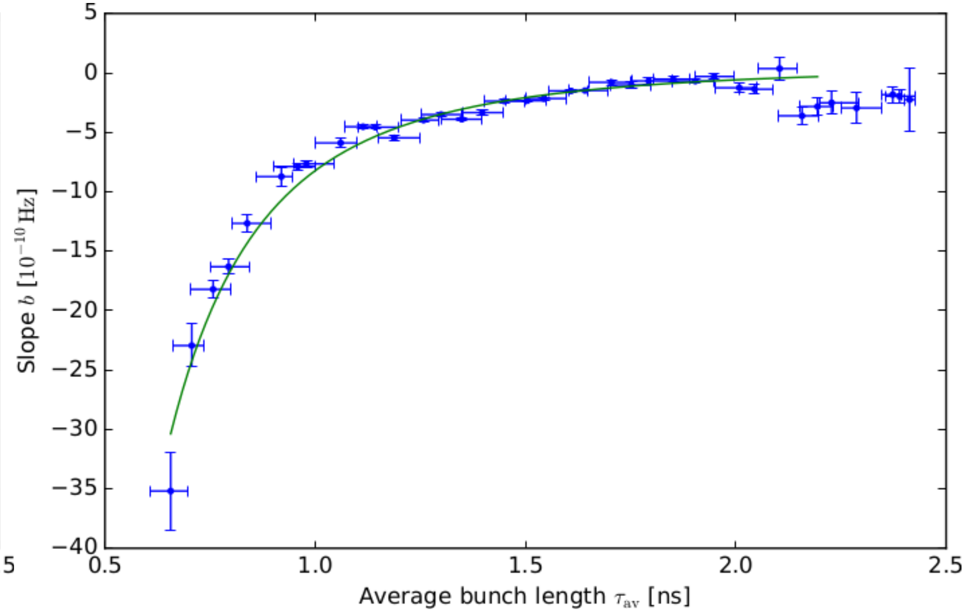


- Measurements done scanning both the average bunch length and the bunch intensity

- The data was organised in categories of bunch length, and fitted linearly ($f_{s2} = a + b\ N_b$)

Measured slope b and origin a





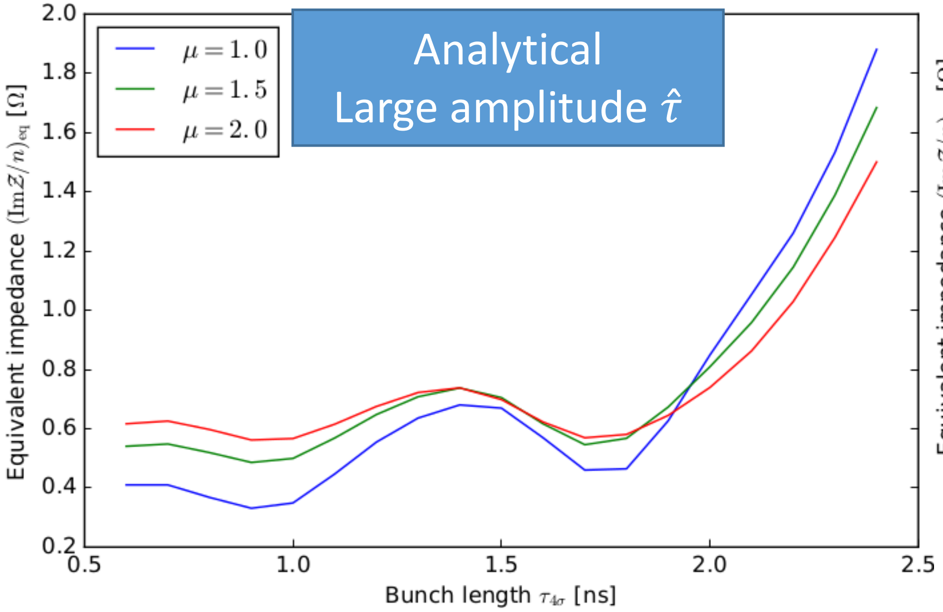
The origin a (non-linear synchrotron frequency without intensity effects) is

$$a \approx 2 f_{s0}\left(1 - \frac{(\omega_{RF}\tau_L)^2}{64}\right)$$

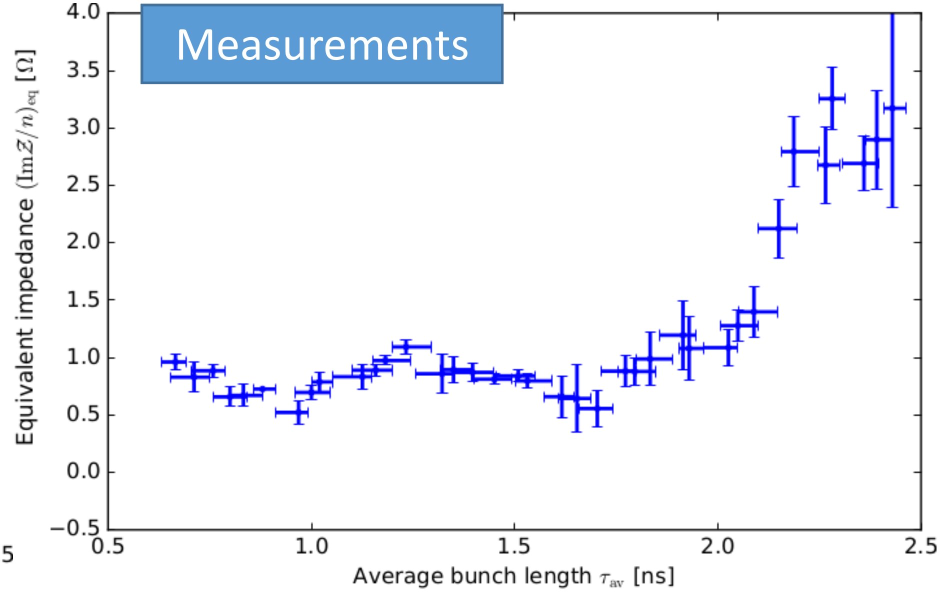
For constant ImZ/n, the synchrotron frequency shift scales as

$$b \propto \frac{1}{\tau_L^3}$$

Measured SPS equivalent impedance



Analytical
Large amplitude $\hat{\tau}$



Measurements

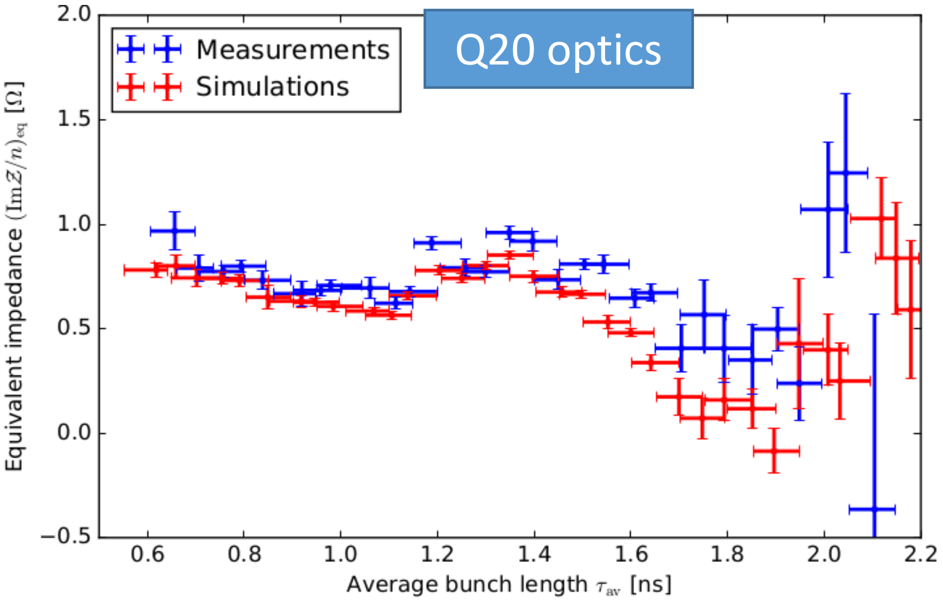
- Obtained with:

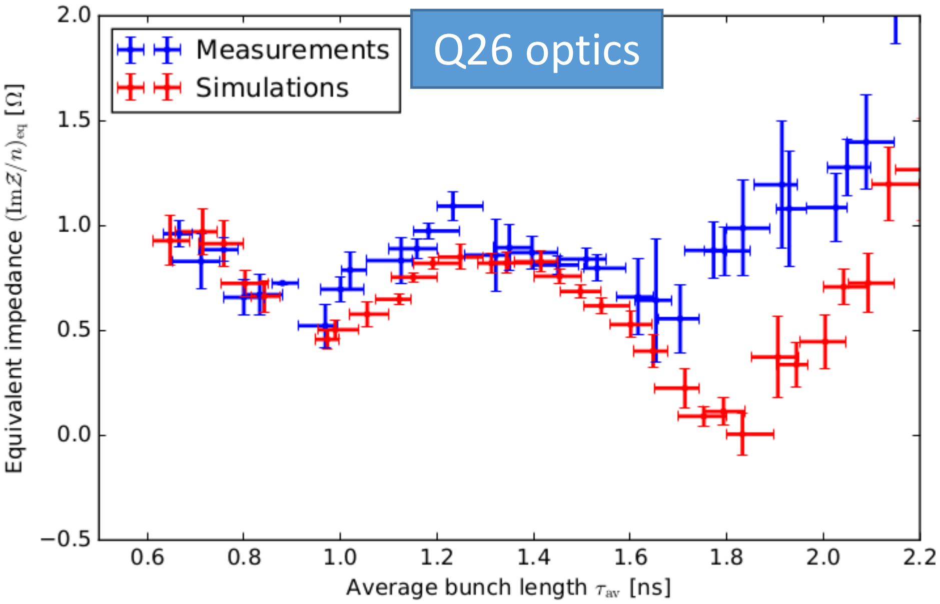
Measured $\approx \dfrac{\Delta f_{\mathrm{inc}}}{f_s^{(0)}}$

$$(\mathrm{Im}\mathcal{Z}/n)_{\mathrm{eq}} = \frac{\omega_{\mathrm{rev}}^2 V_{\mathrm{RF}}\, h}{6q} \frac{b}{a} \tau_{\mathrm{av}}^3$$

➢ Similar features to the equivalent impedance calculated for particles with large amplitude of oscillations

Comparison of measurements with simulations



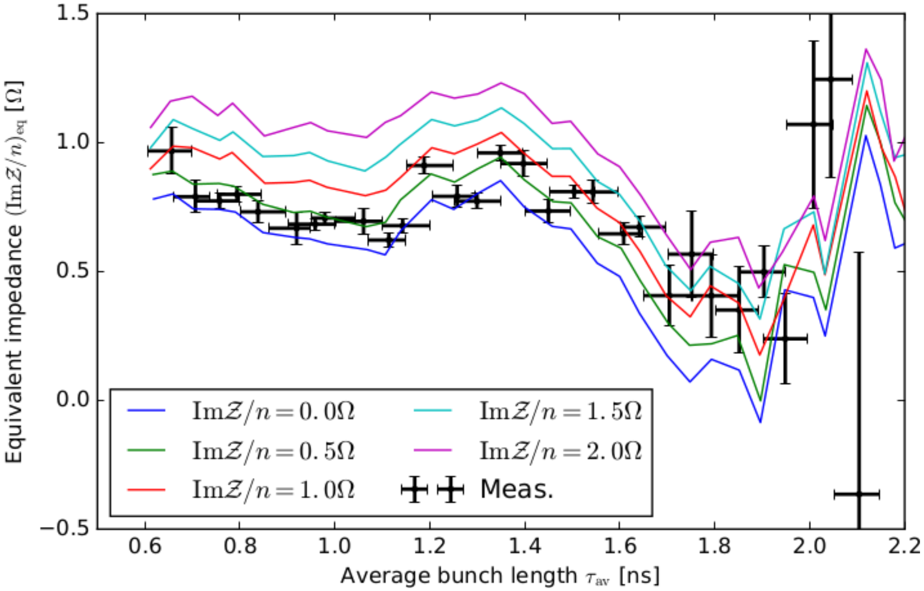


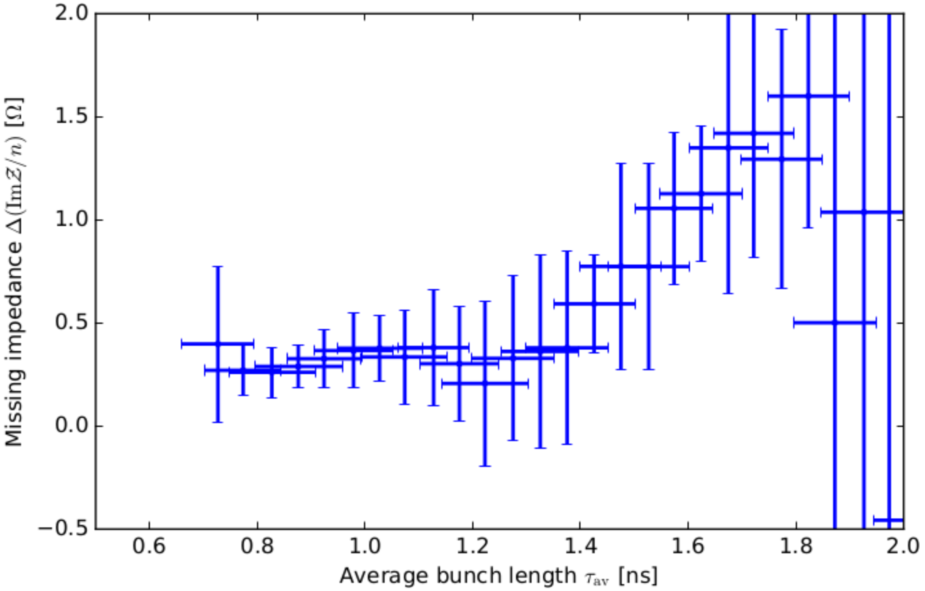
- Simulations were done with BLonD and the present SPS impedance model, using the machine parameters and bunch profiles from measurements (done for two different optics settings Q20 and Q26)

➢ Reasonable agreement, in pattern and amplitude

➢ Deviations may indicate some missing impedance source

Evaluating the missing impedance



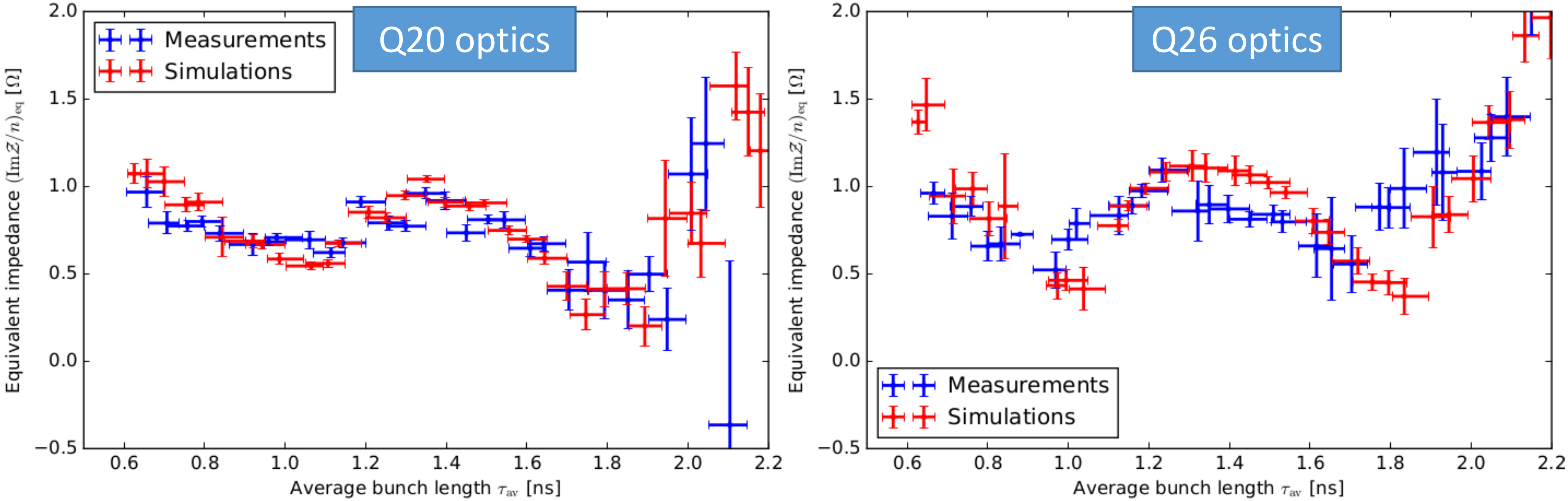


- Simulations were reiterated by adding a variable amount of ImZ/n, to determine for each bunch length the missing equivalent impedance

➢ The dependence of the missing equivalent impedance on bunch length suggests that a resonant impedance may be missing

➢ An accurate estimation of the longitudinal space charge impedance is essential

Adding a single resonator impedance



- Simulations done by scanning the R and f_r of the additional resonator

➢ The agreement is improved by adding a resonator at $f_r = 350$ MHz with $R/Q = 3$ kΩ (Q=1-10), the potential missing impedance source is now under investigation.

Conclusions

- The frequency characteristics of the SPS impedance was evaluated by measuring the synchrotron frequency shift with intensity as a function of bunch length

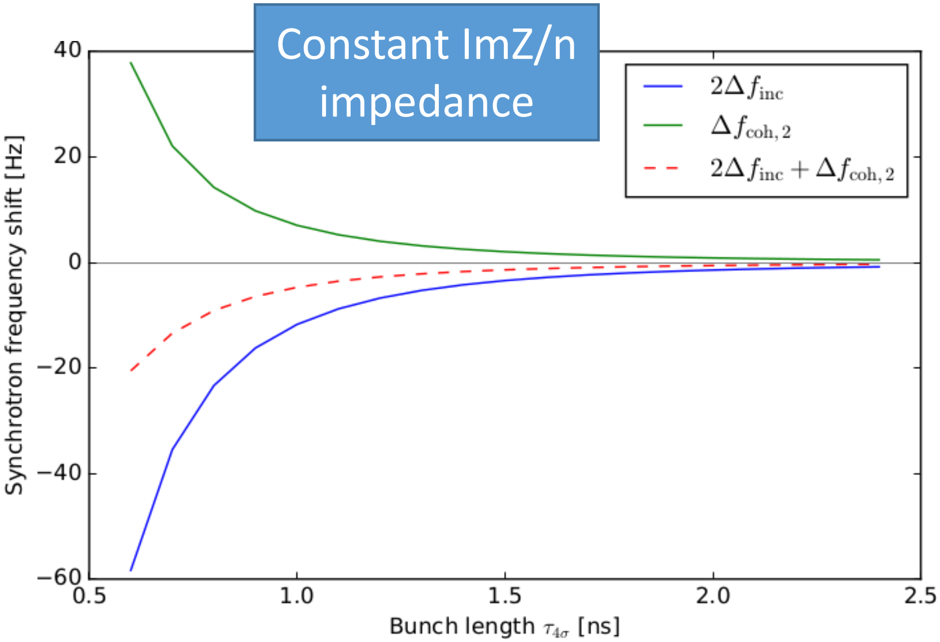
- Overall, particle simulations using the present SPS impedance model reproduces most of the features obtained in measurements

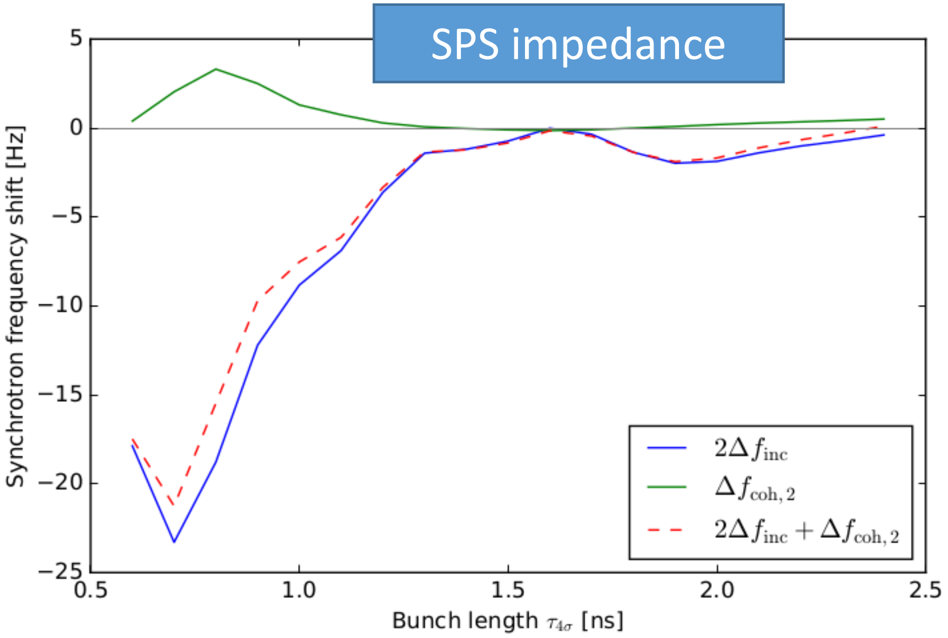
- The dependence of the missing impedance as a function of bunch length allows to get information about the frequency characteristics of that impedance.

➢ In this example, the SPS impedance model seems to miss an impedance source that can be modelled by a resonator impedance with $f_r = 350$ MHz and $R/Q = 3\ \mathrm{k\Omega}$ (Q=1-10).

Spare slides

Coherent synchrotron frequency shift


Constant ImZ/n impedance


SPS impedance

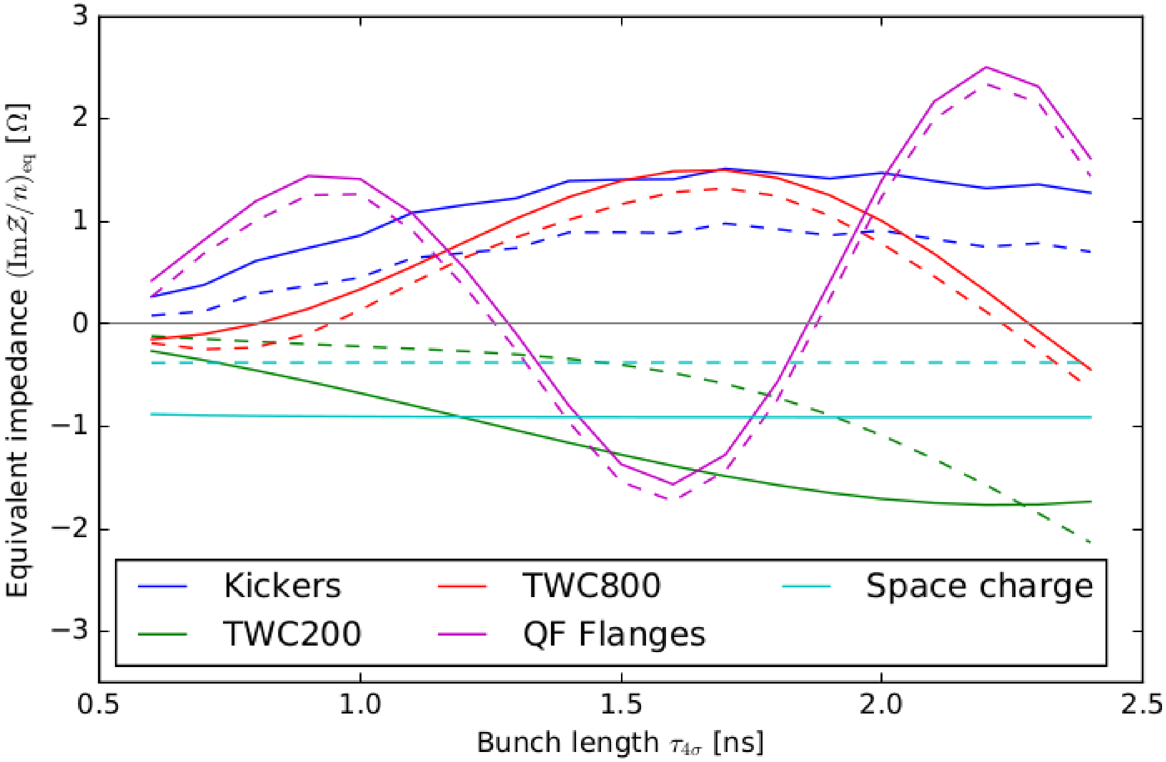
- The coherent synchrotron frequency shift is given by

$$\Delta f_{\mathrm{coh},2} = \frac{3\Gamma\left(5/2\right)}{8\pi^{5/2}} \frac{q^2\eta}{\beta^2 E f_{s0,\mathrm{inc}} \tau_L^3} \mathrm{Im}\mathcal{Z}_{\mathrm{coh},2} N_b$$

where for a parabolic bunch

$$\mathrm{Im}\mathcal{Z}_{\mathrm{coh},2} = \frac{\sum_{n=-\infty}^{\infty} \mathcal{S}_2\left(n\right)\left(\mathrm{Im}\mathcal{Z}/n\right)}{\sum_{n=-\infty}^{\infty} \mathcal{S}_2\left(n\right)} \qquad \mathcal{S}_2\left(f\right) = \frac{\left[J_{5/2}\left(2\pi f \tau_L\right)\right]}{2\pi f \tau_L}$$

Effect of the various impedance sources

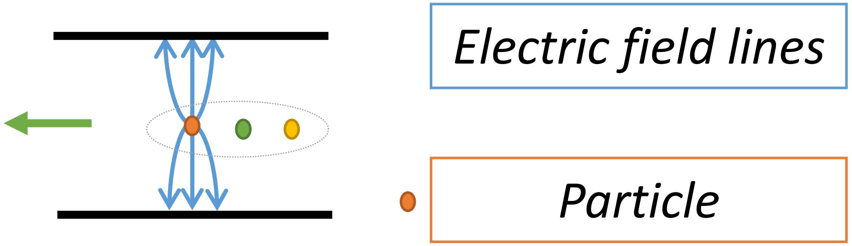


- The various impedance sources have a different effect on the coherent synchrotron frequency shift.

- Overall, the contribution of all impedance sources is that the coherent synchrotron frequency shift is small.

Longitudinal space charge in the SPS

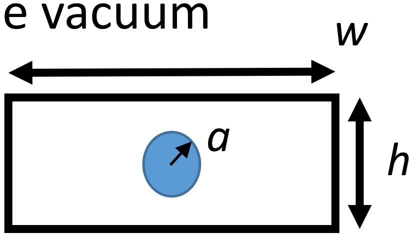
Longitudinal space charge



- First estimations of space charge impedance were done analytically (f_0 = revolution frequency, Z_0=free space impedance)

$$\frac{\mathrm{Im}Z}{n} = -\frac{iZ_0}{\beta\gamma^2}\,g \quad , \qquad n = \frac{f}{f_0}$$

- Geometrical factor g for a round uniform beam in a rectangle vacuum chamber (h=chamber height, w=width, a=beam size)



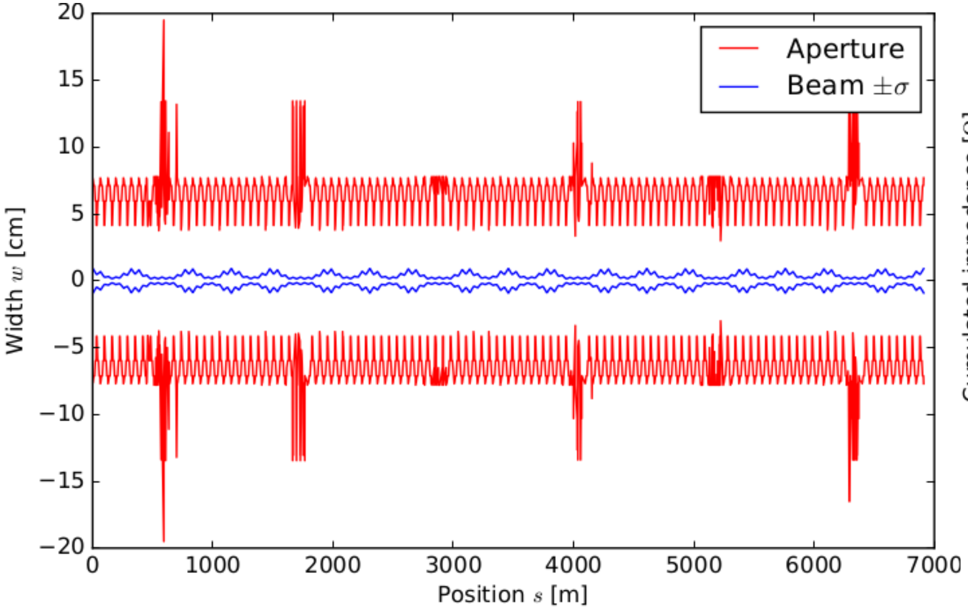
$$g = C + \ln\left[\frac{2h}{\pi a} tanh\left(\frac{\pi w}{2h}\right)\right]$$

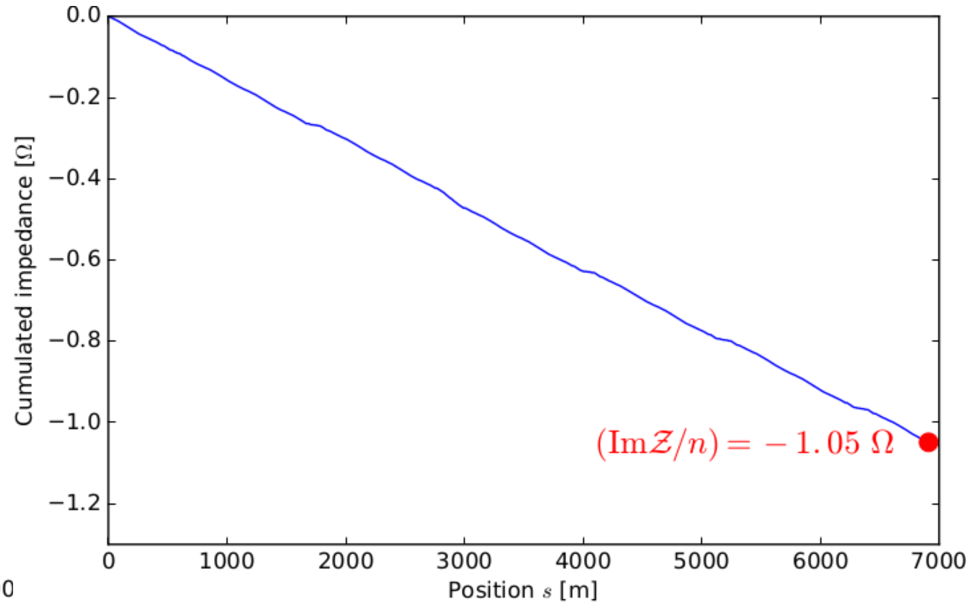
$$C = \frac{1}{2} \text{ on central axis ; } C = \frac{1}{4} \text{ if averaged over beam size}$$

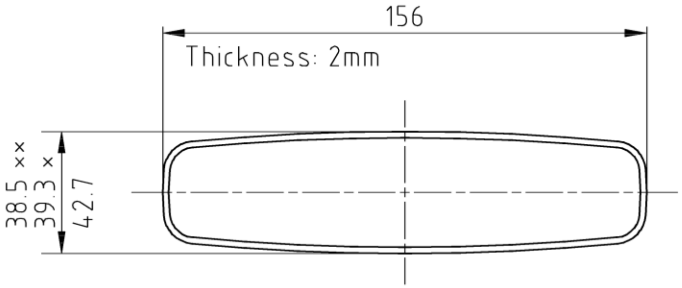
- Taking the average beam size and aperture over the ring
  - ImZ/n ~ 1.3-1.4 Ohm averaged over the beam size

- This value can be refined by taking into account the variations of the vacuum chamber geometry and the beam size.

Longitudinal space charge impedance







For MBA magnet
SPS 8095-0040

- Calculated taking into account the variation of the aperture geometry and beam size
- The obtained values are (for $\epsilon =$ 1.7μm and $dp/p = 1.1 \times 10^{-3}$):
  - **Q20 optics: -1.05**
  - **Q26 optics: -1.17**