Modeling magnetized plasma jets in electric propulsion

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Contents

- Introduction to Magnetic Nozzles (MNds)
  - Helicon Plasma Thrusters (HPT)
  - Electron-Cyclotron Resonance thrusters (ECRT)
- 2D two-fluid model of a MN
  - Near-region expansion and thrust generation
  - Far-region expansion and plasma detachment
  - Induced magnetic field effects
- 3D MN for contactless thrust vector control (TVC)
  - Fully-magnetized limit
  - TVC performance
- Advances on other MN topics
  - Propagation of waves into MN region in HPT and ECRT
  - Kinetic electron model in a MN (and in an unmagnetized jet)
  - Time-evolution of EVDF in a MN
Introduction: Magnetic nozzles
The magnetic nozzle

- A magnetic nozzle (MN) is a convergent-divergent magnetic field created by an axisymmetric set of coils or permanent magnets, to expand a hot plasma into a high velocity plume
  - Channel and accelerate a plasma jet like a de Laval nozzle
  - Generate magnetic thrust by converting thermal energy into directed kinetic energy

- Advantages:
  - Contactless operation → no erosion, lower plasma losses
  - Adaptable and throttleable by changing field shape and strength in-flight
Key operational principles

- Electrons must be well magnetized to follow magnetic tubes
  - Ions, however, need not be magnetized for MN operation
- Electron expansion sets up an electric field that opens ion trajectories and accelerates them downstream
  - This mechanism converts electron thermal energy into ion kinetic energy
- Other mechanisms for ion acceleration exist: Influence of the type of internal energy in the process (electron/ion, isotropic/ anisotropic, ...)
  - Different plasma sources → different MN physics
Key operational principles

- The azimuthal electric currents that exist in the plasma interact with the magnetic field applied by the coils to create magnetic thrust by the action-reaction principle
- The plasma must detach from the closed magnetic lines downstream to form a free plasma plume
- Other physics that can play a role in the expansion:
  - Electron thermodynamics (e.g., cooling rate) in the collisionless plasma
  - Plasma-induced magnetic field in high $\beta$ (dense) plasmas
  - Ion and/or electron energy ratio and anisotropy
  - Turbulence, instabilities.
- A model is needed to understand the main physics and scaling laws behind plasma acceleration, confinement, thrust generation, and detachment.
Helicon Plasma Thruster (HPT)

- Both the HPT and the ECRT are electrodeless RF plasma thrusters with MN.

- The Helicon Plasma Thruster (HPT) consists of a cylindrical chamber plus an antenna, coils/permanent magnets, and a gas injector. Typical operation is at a few MHz, and \( B = 200 \text{–} 2000 \text{ G} \).
  - High plasma densities can be achieved (up to \( 10^{19} \text{–} 10^{20} \text{ m}^{-3} \)).
  - Prototypes exist in the 50 W – 50 kW range, but still have low thrust efficiency: \( \eta_T < 0.2 \).
Electron cyclotron resonance thruster (ECRT)

- Similar to HPT in architecture, but resonant power absorption is the main heating mechanism. Typical operation is at a few GHz, and 900 G.

The ECRT of ONERA runs at 50-100 W. Up-scaling is contemplated in the MINOTOR H2020 project.

- Promising thrust efficiencies of 16% were reported in early tests at very low power.
Two-dimensional, two-fluid MN model
Key assumptions of basic model

Two-fluid, two-dimensional DIMAGNO plasma model:

- Plasma composed of **cold ions and hot Maxwellian electrons**
  - Representative of HPT and ECRT plasma regimes
- Known plasma properties at the magnetic throat (i.e. inlet)
- Steady state: $\partial / \partial t = 0$; axisymmetric: $\partial / \partial \theta = 0$ expansion
- **Collisionless** plasma: $\lambda_{mfp}/R, \nu/\Omega_e \ll 1$ (*)
- Quasineutral plasma: $\lambda_D/R \ll 1 \rightarrow n_e = n_i = n$
- Electron inertia neglected: $m_e/ m_i \ll 1$ (*)
- **Full electron magnetization**: $\ell_e/R \ll 1$
  - Ion magnetization number can be any order: $\Omega_i/(c_s R) = O(1)$
- Negligible induced magnetic field, $\beta = \mu_0 n T_e / B^2 \ll 1$ (*)
- Simple closure relation for electrons: isothermal or polytropic $T_e \propto n^{\gamma - 1}$ (*)

(*) These assumptions have been (partially) dropped in recent works
Basic model formulation

- The basic DIMAGNO model consists of the continuity and momentum equations of ions and electrons

\[ \nabla \cdot (n_1 u_i) = 0, \]
\[ \nabla \cdot (n_2 u_e) = 0, \]
\[ m_1 n_1 (u_i \cdot \nabla) u_i = -en \nabla \phi + en u_i \times B, \]
\[ 0 = -\nabla p_e + en \nabla \phi - en u_e \times B, \]
\[ \frac{T_e}{T_0} = \left( \frac{n}{n_0} \right)^{\gamma^{-1}} \]

- Magnetic streamfunction for solenoidal longitudinal B field

\[ \nabla \psi = r B_{1\perp} \]
\[ \frac{\partial \psi}{\partial z} = -r B_r, \quad \frac{\partial \psi}{\partial r} = r B_z, \]

- Electrons and ions admit streamfunctions too: \( \psi_i, \psi_e \)
Electron model properties

- Electron equations reduce to algebraic conservation laws

\[
\begin{align*}
  u_{\perp e} &= 0 \\
  nu_{\parallel e}/B &= G_e(\psi), \\
  h_e(n) - e\phi &= H_e(\psi). \\
  u_{\theta e} &= -\frac{E_{\perp}}{B} - \frac{1}{enB} \frac{\partial p_e}{\partial 1_{\perp}} \equiv -\frac{r}{e} \frac{dH_e}{d\psi},
\end{align*}
\]

- Electron and magnetic tubes coincide
- \(G_e\) and \(H_e\) known from initial conditions
- The azimuthal current \(j_{\theta e} = enu_{\theta e}\) on each magnetic/electron tube is the \(E \times B\) and diamagnetic drifts
- \(u_{\theta e}/r\) constant on each tube (isorotation)
Ion model properties

- Ion continuity and momentum equations lead to:

\[
\begin{align*}
    u_{ri} \frac{\partial \ln n}{\partial r} + u_{zi} \frac{\partial \ln n}{\partial z} + \frac{\partial u_{ri}}{\partial r} + \frac{\partial u_{zi}}{\partial z} &= -\frac{u_{ri}}{r}, \\
    u_{ri} \frac{\partial u_{ri}}{\partial r} + u_{zi} \frac{\partial u_{ri}}{\partial z} + c_s^2 \frac{\partial \ln n}{\partial r} &= -(u_{\theta e} - u_{\theta i}) \Omega_i \cos \alpha + \frac{u_{\theta i}^2}{r}, \\
    u_{ri} \frac{\partial u_{zi}}{\partial r} + u_{zi} \frac{\partial u_{zi}}{\partial z} + c_s^2 \frac{\partial \ln n}{\partial z} &= (u_{\theta e} - u_{\theta i}) \Omega_i \sin \alpha,
\end{align*}
\]

- Ion mechanical energy and canonical angular momentum about the axis are conserved on ion tubes:

\[
\begin{align*}
    \frac{1}{2} m_i u_i^2 + e\phi &= H_i(\psi_i) \\
    \frac{1}{2} r m_i u_{\theta i} + e\psi &= D_i(\psi_i)
\end{align*}
\]

- The supersonic ion equations constitute an hyperbolic system that can be integrated efficiently with the method of characteristics.

- Ion tubes ≠ magnetic tubes: separation occurs, depending on ion magnetization.
Near-region expansion

For isothermal case:

\[-e\phi \propto \ln n \to \infty \quad \text{at } |\vec{r}| \to \infty\]

which is nonphysical

For polytropic case:

\[-e\phi(\infty, 0) = \frac{\gamma}{\gamma - 1} T_0,\]

\[u_i(\infty, 0) = c_s0 \sqrt{\frac{M^2_0 + \frac{2}{\gamma - 1}}{}}\]
Thrust generation

- Plasma momentum equation (sum of ion plus electron momentum equations):

\[ \nabla \cdot \left( m_i n_i u_i u_i + p_e \bar{I} \right) = j \times B. \]

- Increment of axial plasma momentum is due to magnetic axial force. A globally-diamagnetic azimuthal electric current is necessary for positive thrust. At \( B = \text{const} \) sections:

\[ F(B) = F_0 + \int_{\gamma(B)} d\gamma (-j_\theta) B_r, \]

with \( F_0 \) the axial momentum gained inside the source upstream.

- Most magnetic thrust is generated early in the expansion, where most of the magnetic thrust force density concentrates.
Ion separation and plasma detachment

- Ions are weakly magnetized in most of the MN.
- Magnetic force on ions is then insufficient to deflect their trajectories to match magnetic lines, and the perpendicular electric field takes this task initially

\[ m_i n (u_i \cdot \nabla) u_i = -en \nabla \phi + en u_i \times B, \]

- As ions gain momentum, the electric field becomes insufficient too for full deflection (except at the plasma edge, where the quasineutrality assumption enforces a perfect match)
- As a result, ions separate inward from the magnetic field inside the plume
- Separation continues leads to the formation of longitudinal electric currents downstream.
Plasma detachment

- Ion separation increases dramatically after turning point of plasma jet → plasma detachment due to ion inertia
- When ions are hypersonic, the i-tubes become near-conical.
- Very small amount of plasma momentum turns back toward the thruster.
- A higher ion magnetization leads to a later onset of detachment and therefore more divergence losses: heavy propellants and moderate magnetic fields are preferred
A good detachment process is a requirement for high plume divergence efficiency, i.e. low jet power losses in the radial direction.

Plume divergence efficiency function at $B = \text{const}$ sections is defined as

$$\eta_{\text{plume}}(B) = \frac{\int_{S(B)} dA u_{zi}^3}{\int_{S(B)} dA u_{zi}^2 u_{zi}}.$$

MN operation improves for
- Low ion magnetization
- Low MN divergence rate
The diamagnetic nature of the hot plasma pushes against the applied magnetic field.

Induced field $B_p$ can be computed iteratively. $B_p$ tends to open and weaken magnetic nozzle.

Even if the plasma is low-$\beta$ initially, the local $\beta$ grows rapidly and the induced $B$ becomes important.

Separatrices may form downstream, setting a neat boundary to the magnetic influence of the MN.
3D MN for contactless thrust vector control
Thrust vector control

- Thrust vector control (TVC) is a necessity in electric propulsion (compensation of misalignments and drift of center of mass)
- Deflections of about 5 deg are desirable for station keeping; more deflection for enhanced mission flexibility
- Current solution: mount the thruster on a gimballed mechanism, and steer the whole device (heavy, cumbersome, expensive)

(Images by RUAG Space)
Thrust vector control with 3D MN

- Several ($N \geq 3$) coils tilted an angle $\alpha$ are intertwined to create a vector magnetic nozzle (VECMAN) [Patent P201331790]
- The current through each coil is independently controlled to generate a non-symmetric, variable magnetic field
  - The magnetic centerline can be directed in any direction inside the $N$-polygon of the figure (with negative currents, even outside of it)
- Centerline deflections of $>10$ deg are easily achievable
- Construction using a spool and simultaneous winding, or locking together three coils with a small offset
- First tests in EP2 lab planned for 2017-2018
Fully-magnetized 3D plasma model

- Fully-magnetized limit of DIMAGNO model makes it possible to compute 3D deflection in a simple way
  - We assume the ion and electron gyrofrequencies $\Omega_i, \Omega_e \rightarrow \infty$
  - This precludes any ion separation in the model
- Continuity and momentum equations boil down to 4 conservation laws along the magnetic streamlines:

\[
\nabla \cdot (n\mathbf{u}_i) = 0; \quad \nabla \cdot (n\mathbf{u}_e) = 0,
\]
\[
m_i (\mathbf{u}_i \cdot \nabla) \mathbf{u}_i = -e\nabla \phi + e\mathbf{u}_i \times \mathbf{B},
\]
\[
0 = -T_e \nabla \ln n + e\nabla \phi - e\mathbf{u}_e \times \mathbf{B},
\]

\[
nu_i / B = G_i \quad H_i = \frac{1}{2} m_i u_i^2 + e\phi,
\]
\[
nu_e / B = G_e \quad H_e = T_e \ln n - e\phi.
\]
TVC performance of a 3D MN

- Representative case with thruster solenoid + 3DMN made of 3 coils
- Magnetic thrust is deflected by up to 10 deg when coil angle is $\alpha = 15$ deg
- Larger deflections are possible with negative currents in one coil

<table>
<thead>
<tr>
<th>Simulation</th>
<th>Ampere-turn ratios</th>
<th>$F/F_0$</th>
<th>$\psi$ (deg)</th>
<th>$\theta$ (deg)</th>
<th>$\theta_B$ (deg)</th>
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<td>15 : 0.33 : 0.33  : 0.33</td>
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<td>5.61</td>
<td>5.70</td>
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</table>
Advances on other MN topics
Propagating RF into MN region

- In HPT and ECRT, ionization and heating is done with RF waves.
- Waves propagating downstream into MN region could be a source of power losses.
- A full-wave Yee scheme, Fourier transformed in $t$ and $\theta$, combined with a cold plasma dielectric tensor model is used to study wave propagation and (collisional) absorption.

PhD thesis of Bin Tian

Modeling magnetized plasma jets in electric propulsion
Mario Merino and Eduardo Ahedo
Kinetic electron model

- Plasma beam is nearly collisionless → no local thermodynamic equilibrium → no justification for isentropic/adiabatic behavior
- A kinetic electron model is required to understand collisionless electron cooling observed experimentally
- Kinetic model of paraxial convergent-divergent MN with collisionless, magnetized ions and electrons has been developed
- Particles conserve their total energy and their magnetic moment:

\[ E_\alpha = \frac{m_\alpha}{2} (w_{\alpha\parallel}^2 + w_{\alpha\perp}^2) + q_\alpha \phi, \quad \mu_\alpha = \frac{m_\alpha w_{\alpha\perp}^2}{2B} \]

- Effective potential for axial motion:

\[ U_{eff} = q_\alpha \phi(z) + \mu_\alpha B(z) \]

PhD thesis of Jaume Navarro
Kinetic electron cooling

- Individual electrons are reflected back when they reach
  \[ E_\parallel = U_{\text{eff}} \]
  (electron turning manifold)
- In MN divergent side, there are local extrema of that potential \( \rightarrow \) empty regions in the EVDF
- Three regions of phase space:
  - Free electrons
  - Reflected electrons
  - Doubly-trapped electrons

The existence of empty regions is related to cooling.
Kinetic electron cooling

- Interesting comparison with potential fall in Debye sheath:
  \[
  \frac{e\phi_{sh}}{T_e} \approx A \cdot \ln \frac{m_i}{m_e} - B
  \]
  with \(A, B\) constants

- Main differences in a MN with respect to a sheath:
  - It develops in an infinite region not in a very thin region
  - It is fully quasineutral
  - The doubly-trapped population is undefined but essential at a time.
  - The asymptotic behavior at infinity presents some issues

- Question remains: how are doubly-trapped electron regions populated?
  - Transient set-up of the plume
  - Occasional collisions
To answer the question of how doubly-trapped regions are populated, a first effort consisting in studying the transient set up of the plume is being carried out at EP2

- Similar kinetic formulation, but keeping $\partial/\partial t$ in Vlasov equation (far more computationally intensive)

Doubly-trapped region is partially populated during start-up

Thanks to Gonzalo Sánchez and Jiewei Zhou
Kinetic model of unmagnetized plasma plume

- In an unmagnetized plasma plume, electron motion is strongly 2D.
- However, radial electron motion is confined and has a short characteristic time:
  - An adiabatic invariant $J_r$ (action integral in $r$ orbits) can be found that plays a similar role to $\mu$ in magnetized plumes.
- Three conserved quantities: Energy $E$, angular momentum about the axis $p_\theta$, and $J_r$ (to order $\varepsilon^2$ when averaged).
  - More complex, but some analogies with magnetized case.
Thank you! Questions?

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