Modeling magnetized plasma jets in electric propulsion

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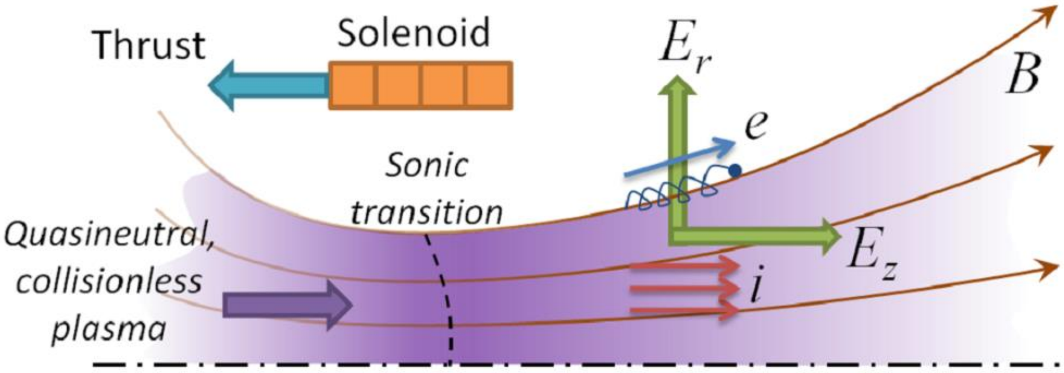
Contents

Introduction: Magnetic nozzles

The magnetic nozzle

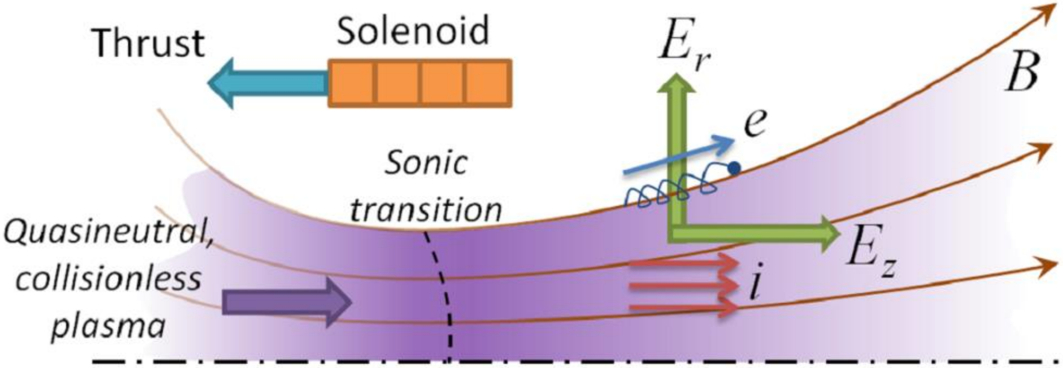
- A magnetic nozzle (MN) is a convergent-divergent magnetic field created by an axisymmetric set of coils or permanent magnets, to expand a hot plasma into a high velocity plume
  - Channel and accelerate a plasma jet like a de Laval nozzle
  - Generate magnetic thrust by converting thermal energy into directed kinetic energy



- Advantages:
  - Contactless operation → no erosion, lower plasma losses
  - Adaptable and throttleable by changing field shape and strength in-flight

Key operational principles

- Electrons must be well magnetized to follow magnetic tubes
  - ❑ Ions, however, need not be magnetized for MN operation
- Electron expansion sets up an electric field that opens ion trajectories and accelerates them downstream
  - ❑ This mechanism converts electron thermal energy into ion kinetic energy
- Other mechanisms for ion acceleration exist: Influence of the type of internal energy in the process (electron/ion, isotropic/ anisotropic, …)
  - ❑ Different plasma sources → different MN physics



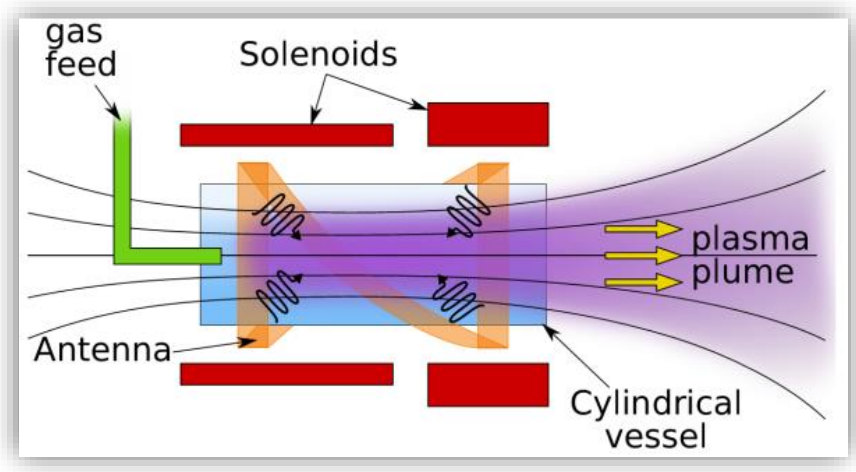
Key operational principles

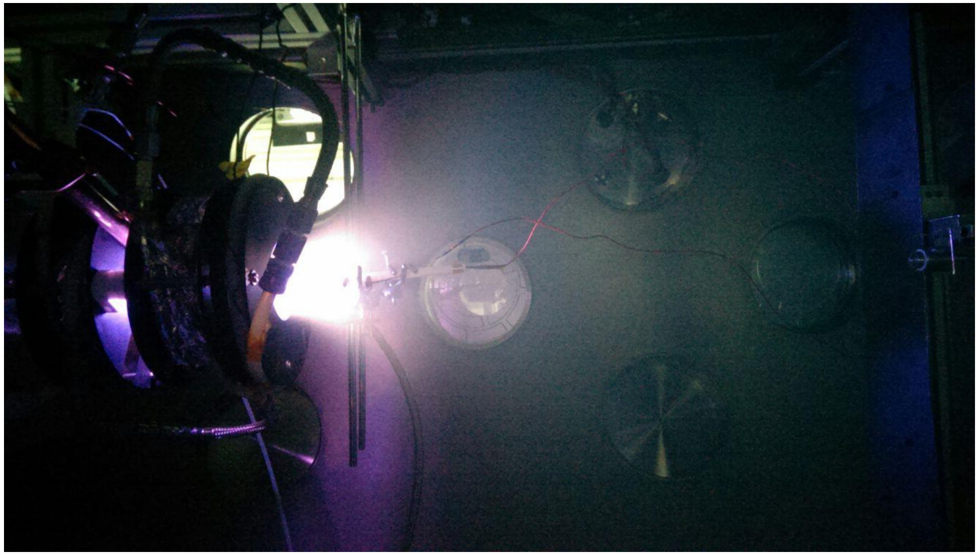
- The azimuthal electric currents that exist in the plasma interact with the magnetic field applied by the coils to create magnetic thrust by the action-reaction principle
- The plasma must detach from the closed magnetic lines downstream to form a free plasma plume
- Other physics that can play a role in the expansion:
  - Electron thermodynamics (e.g., cooling rate) in the collisionless plasma
  - Plasma-induced magnetic field in high β (dense) plasmas
  - Ion and/or electron energy ratio and anisotropy
  - Turbulence, instabilities.

- A model is needed to understand the main physics and scaling laws behind plasma acceleration, confinement, thrust generation, and detachment.

Helicon Plasma Thruster (HPT)

- Both the HPT and the ECRT are electrodeless RF plasma thrusters with MN



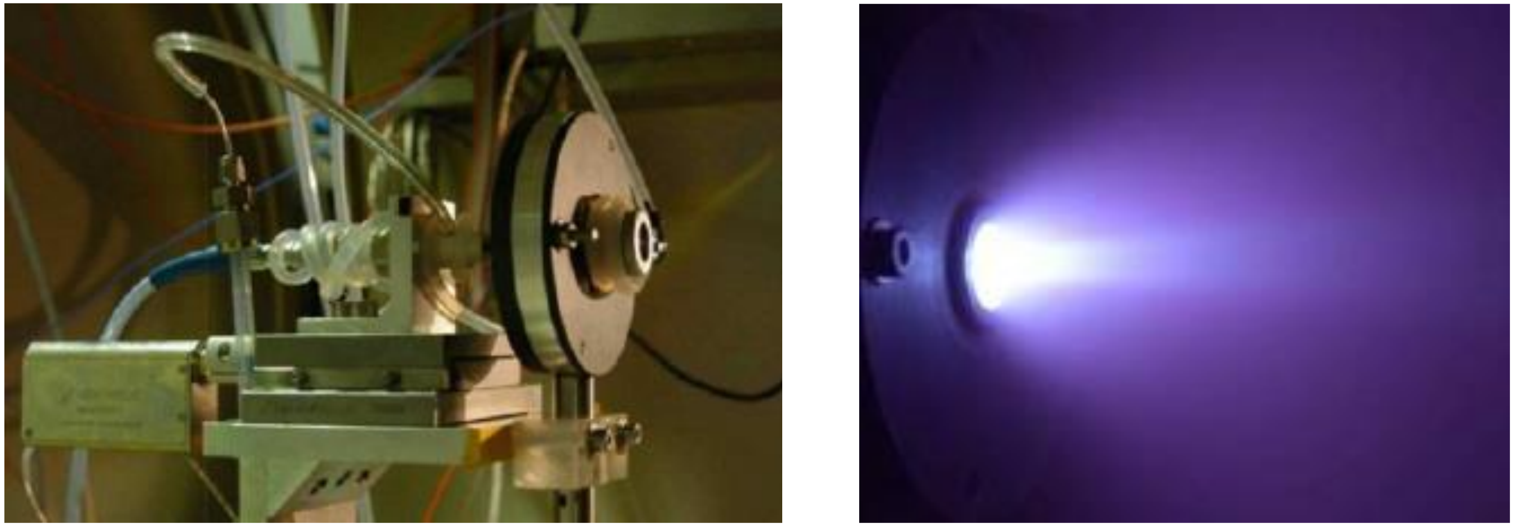


HPT05 prototype of SENER/EP2

- The Helicon Plasma Thruster (HPT) consists of a cylindrical chamber plus an antenna, coils/permanent magnets, and a gas injector. Typical operation is at a few MHz, and $B = 200 - 2000$ G
  - ❑ High plasma densities can be achieved (up to $10^{19} - 10^{20}\ \mathrm{m}^{-3}$)
  - ❑ Prototypes exist in the 50 W – 50 kW range, but still have low thrust efficiency: $\eta_T < 0.2$

Electron cyclotron resonance thruster (ECRT)

- Similar to HPT in architecture, but resonant power absorption is the main heating mechanism. Typical operation is at a few GHz, and 900 G.



ECRA (ONERA). Currently being developed further under MIINOTOR H2020 project with participation from EP2

- The ECRT of ONERA runs at 50-100 W. Up-scaling is contemplated in the MINOTOR H2020 project.
  - Promising thrust efficiencies of 16% were reported in early tests at very low power

Two-dimensional, two-fluid MN model

Key assumptions of basic model

Two-fluid, two-dimensional DIMAGNO plasma model:

- Plasma composed of **cold ions and hot Maxwellian electrons**
  - Representative of HPT and ECRT plasma regimes
- Known plasma properties at the magnetic throat (i.e. inlet)
- Steady state: $\partial/\partial t = 0$; axisymmetric: $\partial/\partial\theta = 0$ expansion
- **Collisionless** plasma: $\lambda_{mfp}/R\,, \nu/\Omega_e \ll 1$ (*)
- Quasineutral plasma: $\lambda_D/R \ll 1 \rightarrow n_e = n_i = n$
- Electron inertia neglected: $m_e/m_i \ll 1$ (*)
- **Full electron magnetization**: $\ell_e/R \ll 1$
  - Ion magnetization number can be any order: $\Omega_i/(c_s R) = O(1)$
- Negligible induced magnetic field, $\beta = \mu_0 n T_e/B^2 \ll 1$ (*)
- Simple closure relation for electrons:
  isothermal or polytropic $T_e \propto n^{\gamma-1}$ (*)

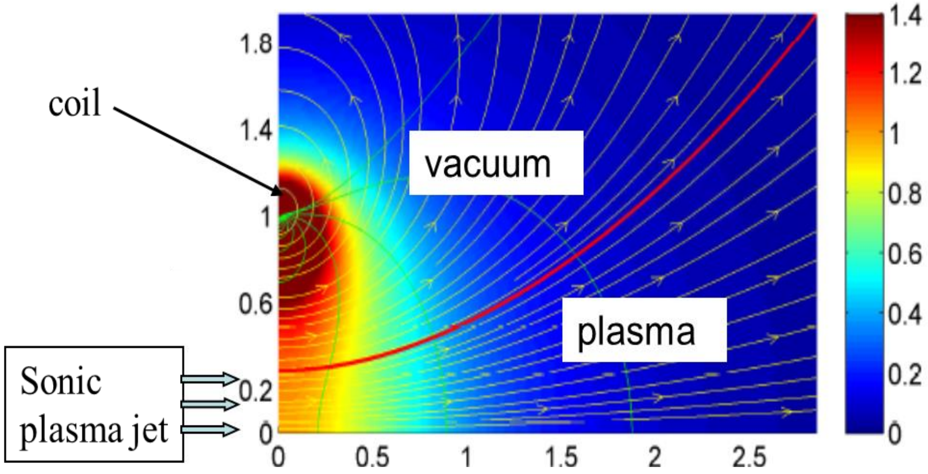
(*) These assumptions have been (partially) dropped in recent works

Basic model formulation

- The basic DIMAGNO model consists of the continuity and momentum equations of ions and electrons

$$
\begin{aligned}
&\nabla \cdot (n\boldsymbol{u}_i) = 0, \\
&\nabla \cdot (n\boldsymbol{u}_e) = 0, \\
&m_i n \left(\boldsymbol{u}_i \cdot \nabla\right) \boldsymbol{u}_i = -en\nabla\phi + en\boldsymbol{u}_i \times \boldsymbol{B}, \\
&0 = -\nabla p_e + en\nabla\phi - en\boldsymbol{u}_e \times \boldsymbol{B},
\end{aligned}
$$

$$\frac{T_e}{T_0} = \left(\frac{n}{n_0}\right)^{\gamma-1}$$

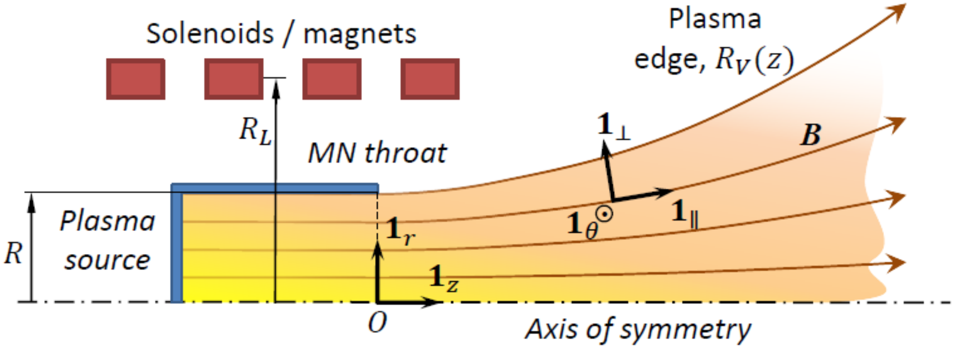


- Magnetic streamfunction for solenoidal longitudinal B field

$$\nabla\psi = rB\boldsymbol{1}_\perp$$

$$\partial\psi/\partial z = -rB_r, \quad \partial\psi/\partial r = rB_z,$$

- Electrons and ions admit streamfunctions too: ψ_i, ψ_e



Electron model properties

- Electron equations reduce to algebraic conservation laws

$$
\begin{aligned}
&u_{\perp e} = 0\\
&n u_{\| e}/B = G_e(\psi),\\
&h_e(n) - e\phi = H_e(\psi).\\
&u_{\theta e} = -\frac{E_\perp}{B} - \frac{1}{enB}\frac{\partial p_e}{\partial 1_\perp} \equiv -\frac{r}{e}\frac{dH_e}{d\psi},
\end{aligned}
$$

$$
\begin{aligned}
h_e(n) &= T_{e0}\ln n, && \text{for } \gamma_e = 1,\\
h_e(n) &= T_e\frac{\gamma}{\gamma - 1}, && \text{for } \gamma > 1.
\end{aligned}
$$

- Electron and magnetic tubes coincide
- G_e and H_e known from initial conditions
- The azimuthal current $j_{\theta e} = enu_{\theta e}$
  on each magnetic/electron tube
  is the $E \times B$ and diamagnetic drifts
- $u_{\theta e}/r$ constant on each tube (isorotation)

Ion model properties

- Ion continuity and momentum equations lead to:

$$u_{ri}\frac{\partial \ln n}{\partial r} + u_{zi}\frac{\partial \ln n}{\partial z} + \frac{\partial u_{ri}}{\partial r} + \frac{\partial u_{zi}}{\partial z} = -\frac{u_{ri}}{r},$$

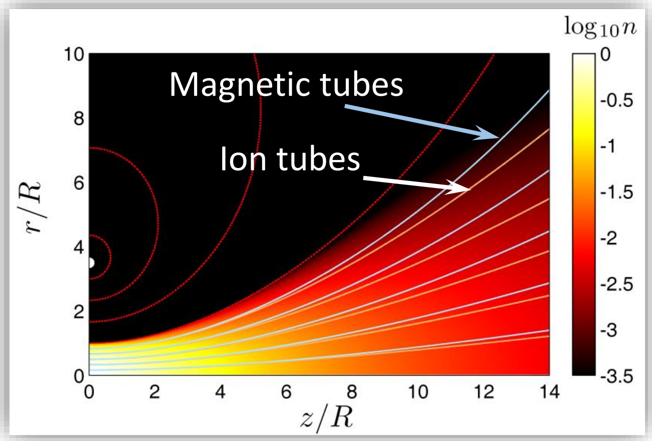
$$u_{ri}\frac{\partial u_{ri}}{\partial r} + u_{zi}\frac{\partial u_{ri}}{\partial z} + c_s^2\frac{\partial \ln n}{\partial r} = -(u_{\theta e} - u_{\theta i})\Omega_i \cos\alpha + \frac{u_{\theta i}^2}{r},$$

$$u_{ri}\frac{\partial u_{zi}}{\partial r} + u_{zi}\frac{\partial u_{zi}}{\partial z} + c_s^2\frac{\partial \ln n}{\partial z} = (u_{\theta e} - u_{\theta i})\Omega_i \sin\alpha,$$

- Ion mechanical energy and canonical angular momentum about the axis are conserved on ion tubes

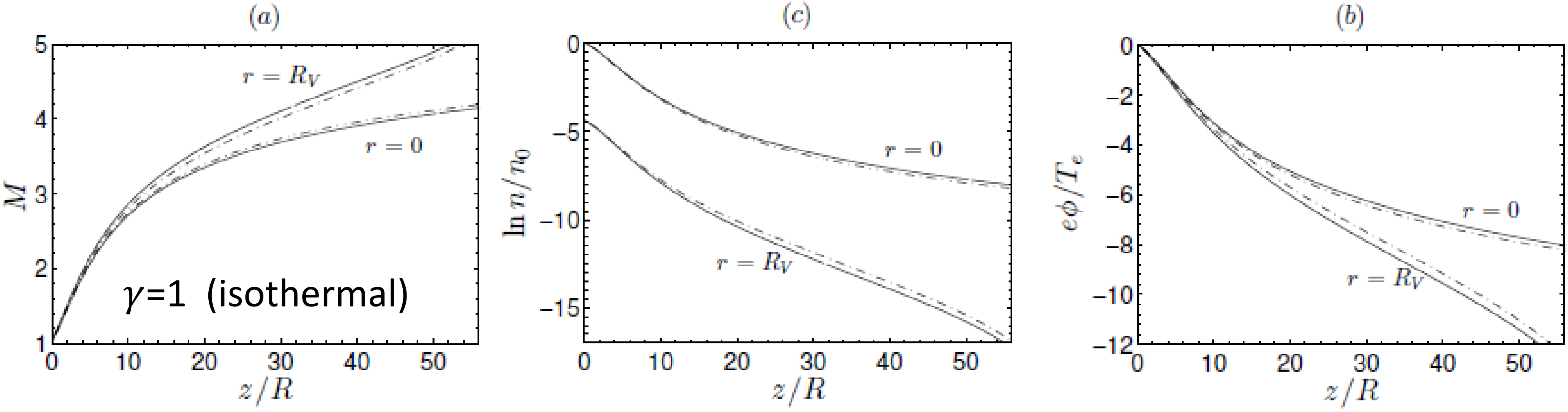
$$\frac{1}{2} m_i u_i^2 + e\phi = H_i(\psi_i)$$

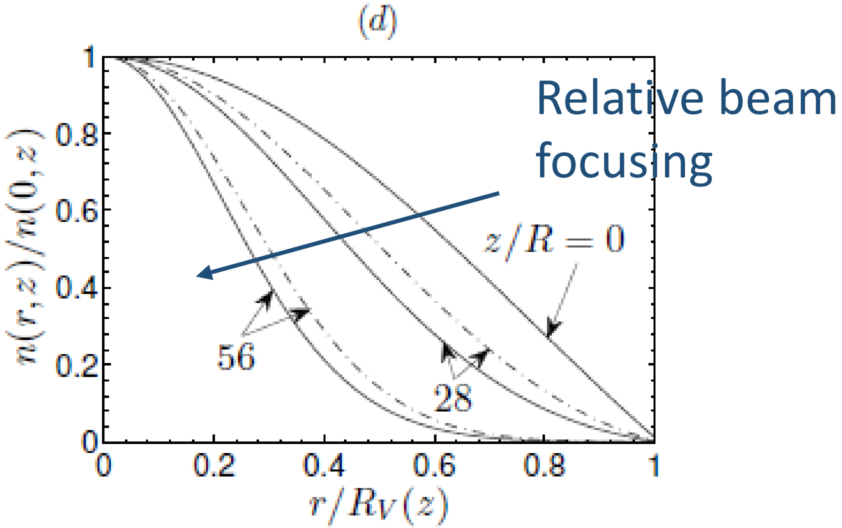
$$r m_i u_{\theta i} + e\psi = D_i(\psi_i)$$



- The supersonic ion equations constitute an hyperbolic system that can be integrated efficiently with the method of characteristics
- Ion tubes ≠ magnetic tubes: separation occurs, depending on ion magnetization

Near-region expansion





- For isothermal case:

$$-e\phi \propto \ln n \to \infty \quad \text{at } |\vec{r}| \to \infty$$

which is nonphysical

- For polytropic case:

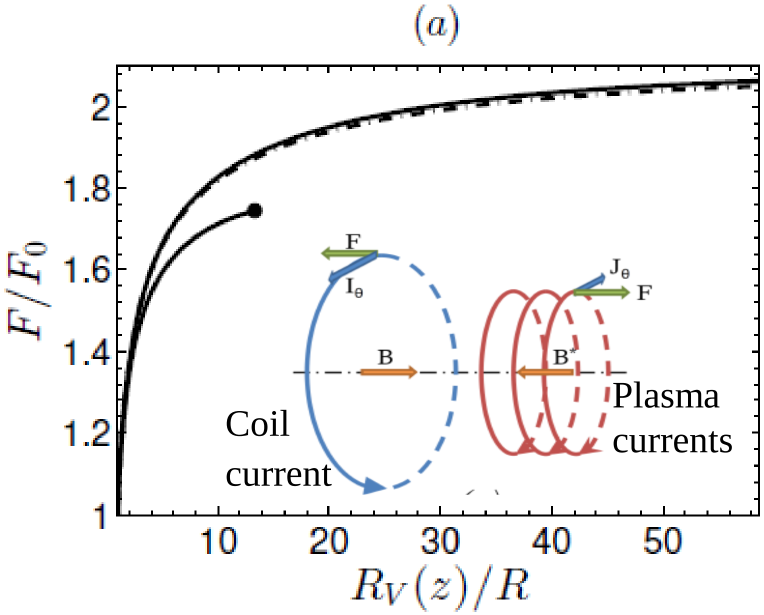
$$-e\phi(\infty, 0) = \frac{\gamma}{\gamma - 1} T_0,$$

$$u_i(\infty, 0) = c_{s0}\sqrt{M_0^2 + \frac{2}{\gamma - 1}},$$

Thrust generation

- Plasma momentum equation (sum of ion plus electron momentum equations):

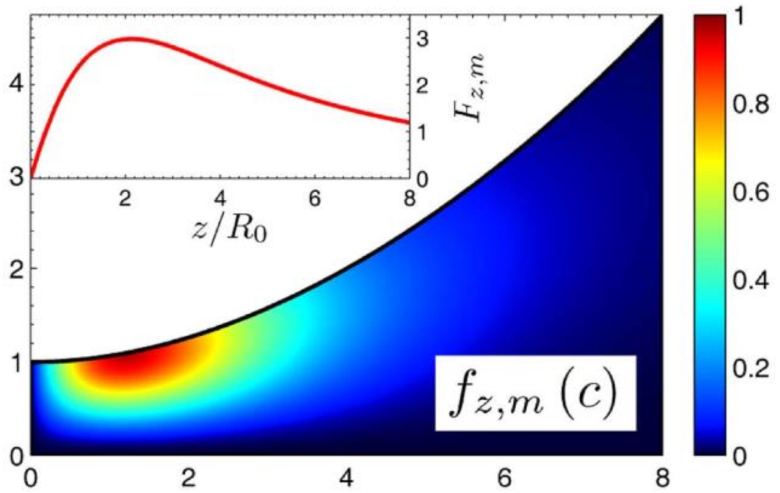
$$\nabla \cdot (m_i n \boldsymbol{u}_i \boldsymbol{u}_i + p_e \bar{\bar{I}}) = \boldsymbol{j} \times \boldsymbol{B}.$$



- Increment of axial plasma momentum is due to magnetic axial force. A globally-diamagnetic azimuthal electric current is necessary for positive thrust. At $B = \text{const}$ sections:

$$F(B) = F_0 + \int_{\mathcal{V}(\mathcal{B})} d\mathcal{V}(-j_\theta) B_r,$$

with F_0 the axial momentum gained inside the source upstream



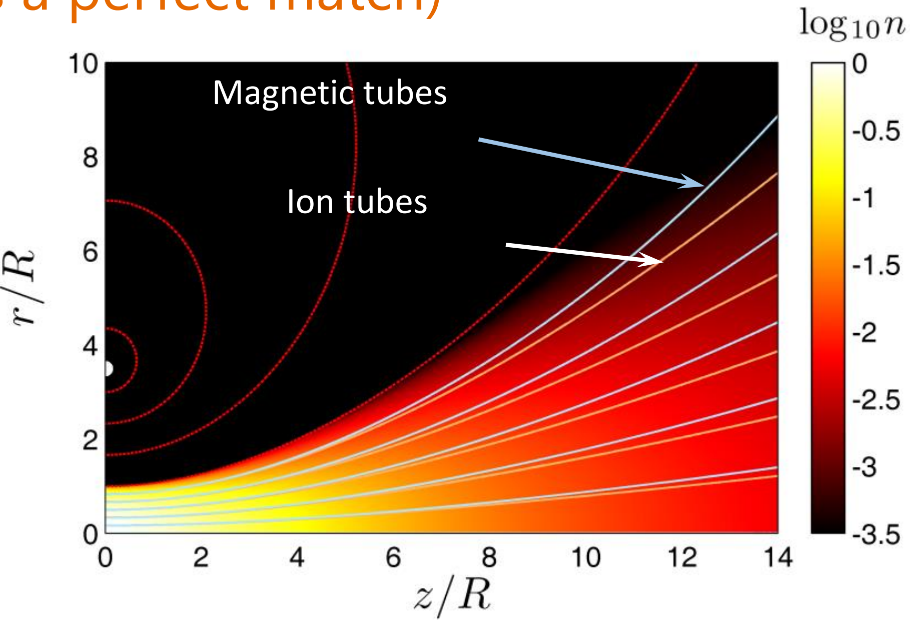
- Most magnetic thrust is generated early in the expansion, where most of the magnetic thrust force density concentrates.

Ion separation and plasma detachment

- Ions are weakly magnetized in most of the MN.
- Magnetic force on ions is then insufficient to deflect their trajectories to match magnetic lines, and the perpendicular electric field takes this task initially

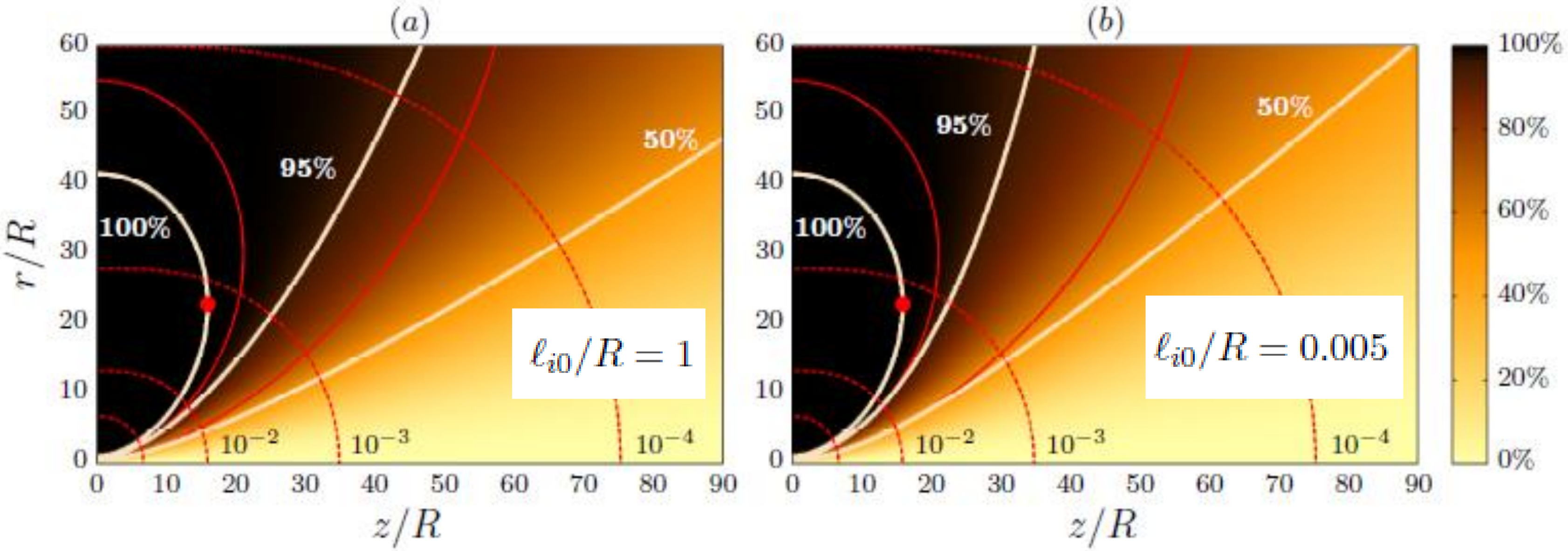
$$m_i n \left(\boldsymbol{u}_i \cdot \nabla\right) \boldsymbol{u}_i = -en\nabla\phi + en\boldsymbol{u}_i \times \boldsymbol{B},$$

- As ions gain momentum, the electric field becomes insufficient too for full deflection (except at the plasma edge, where the quasineutrality assumption enforces a perfect match)
- As a result, ions separate inward from the magnetic field inside the plume
- Separation continues leads to the formation of longitudinal electric currents downstream.



Plasma detachment

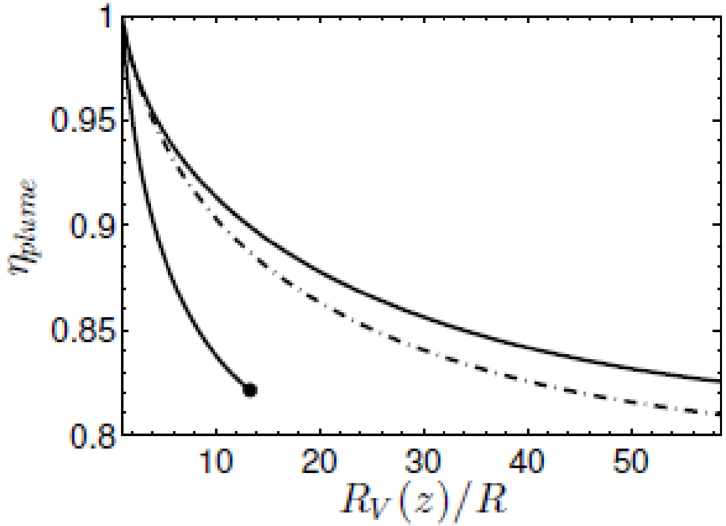
- Ion separation increases dramatically after turning point of plasma jet → plasma detachment due to ion inertia
- When ions are hypersonic, the i-tubes become near-conical.
- Very small amount of plasma momentum turns back toward the thruster.
- A higher ion magnetization leads to a later onset of detachment and therefore more divergence losses: heavy propellants and moderate magnetic fields are preferred



Plume divergence

- A good detachment process is a requirement for high plume divergence efficiency, i.e. low jet power losses in the radial direction
- Plume divergence efficiency function at $\boldsymbol{B} = \text{const}$ sections is defined as

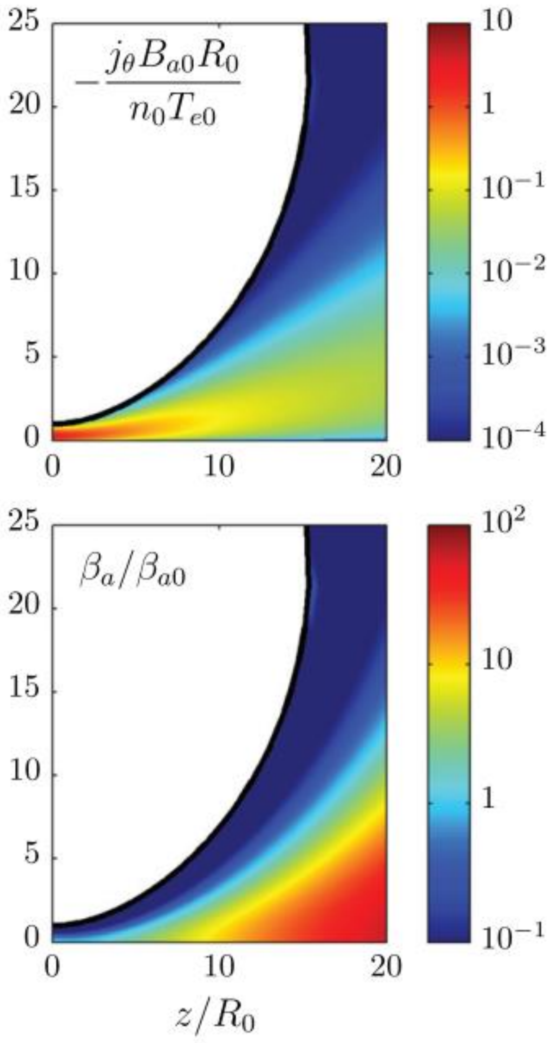
$$\eta_{plume}(B) = \frac{\int_{S(B)} dA n u_{zi}^3}{\int_{S(B)} dA n u_i^2 u_{zi}}.$$

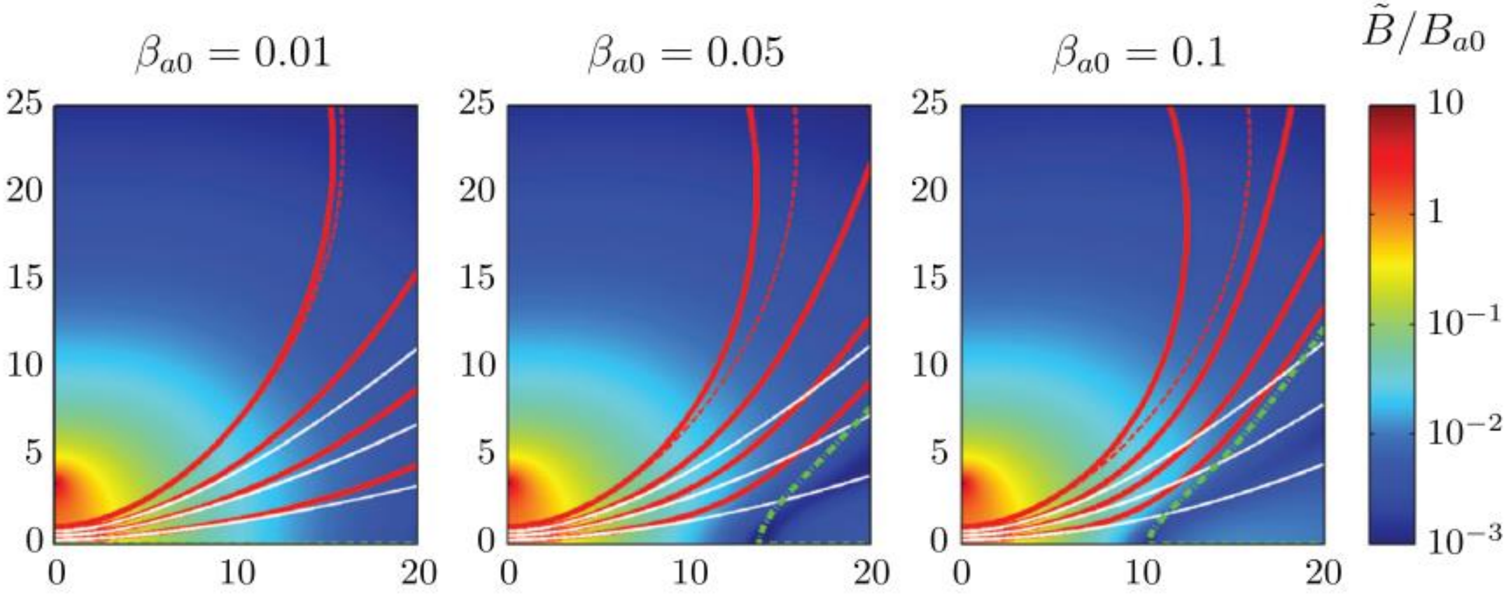


- MN operation improves for
  - ❑ Low ion magnetization
  - ❑ Low MN divergence rate

Effect of plasma-induced B field

- The diamagnetic nature of the hot plasma pushes against the applied magnetic field
- Induced field B_p can be computed iteratively. B_p tends to open and weaken magnetic nozzle
- Even if the plasma is low-β initially, the local β grows rapidly and the induced B becomes important
- Separatrices may form downstream, setting a neat boundary to de magnetic influence of the MN

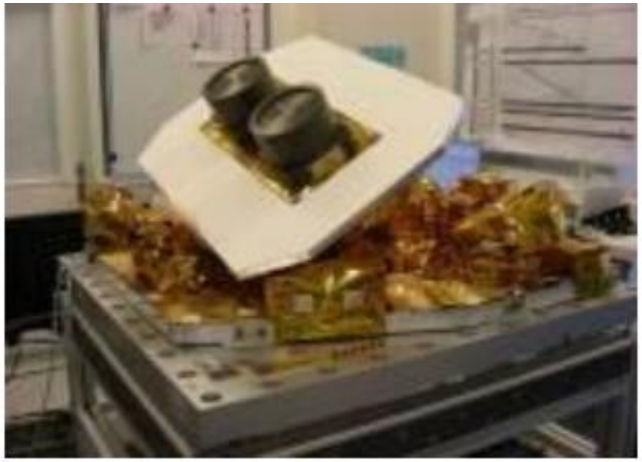


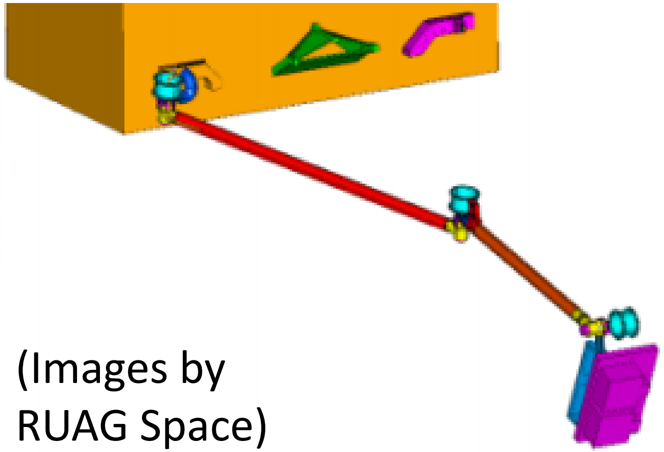


3D MN for contactless thrust vector control

Thrust vector control

- Thrust vector control (TVC) is a necessity in electric propulsion (compensation of misalignments and drift of center of mass)
- Deflections of about 5 deg are desirable for station keeping; more deflection for enhanced mission flexibility
- Current solution: mount the thruster on a gimballed mechanism, and steer the whole device (heavy, cumbersome, expensive)

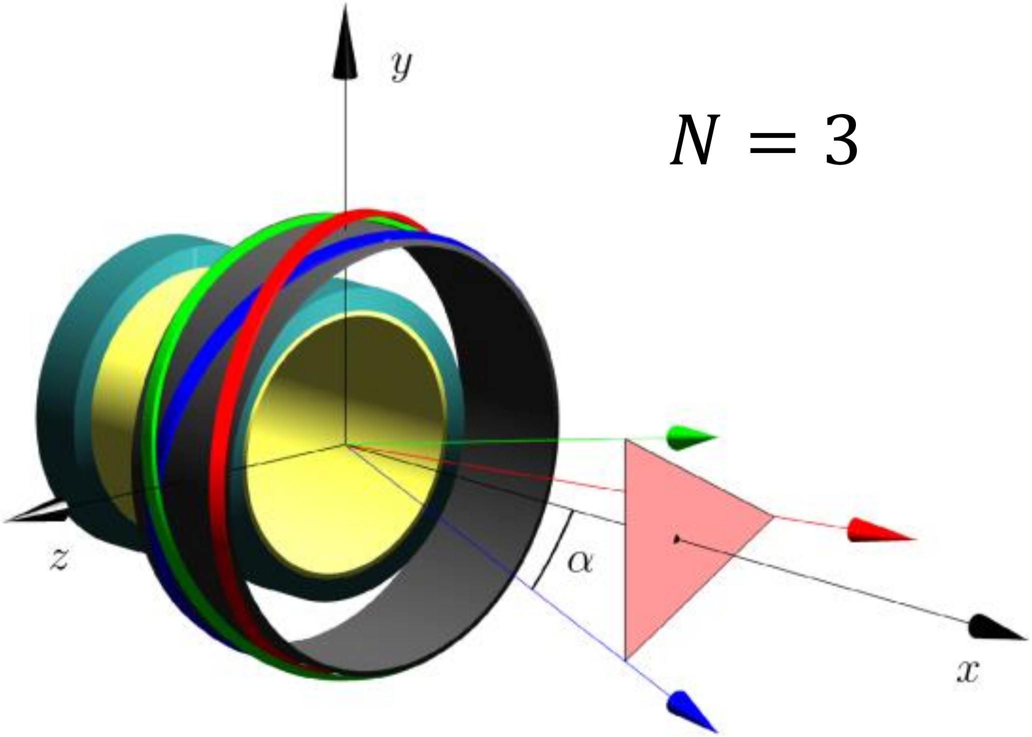




(Images by RUAG Space)

Thrust vector control with 3D MN

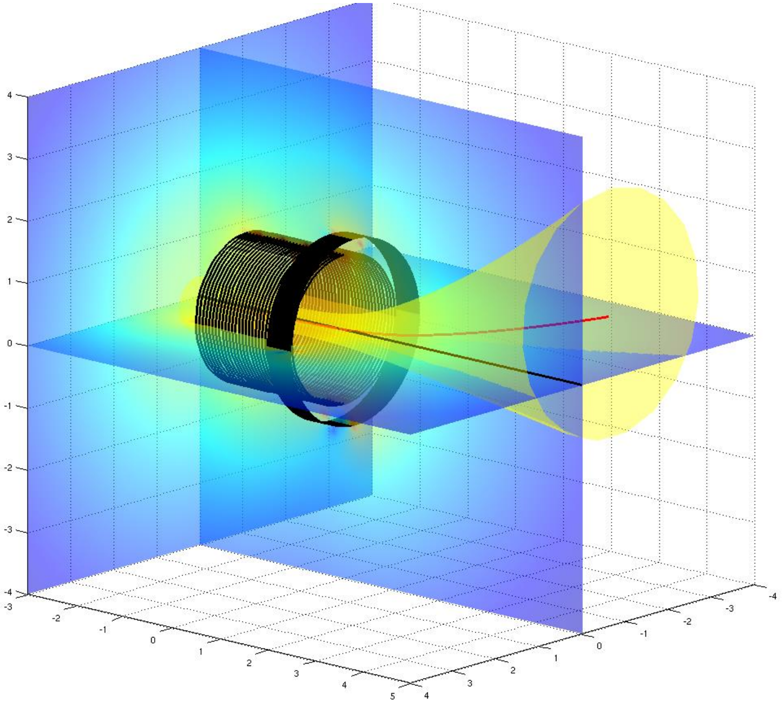
- Several ($N \geq 3$) coils tilted an angle α are intertwined to create a vector magnetic nozzle (VECMAN) [Patent P201331790]
- The current through each coil is independently controlled to generate a non-symmetric, variable magnetic field
  - ❑ The magnetic centerline can be directed in any direction inside the N-polygon of the figure (with negative currents, even outside of it)
- Centerline deflections of >10 deg are easily achievable
- Construction using a spool and simultaneous winding, or locking together three coils with a small offset
- First tests in EP2 lab planned for 2017-2018



$N = 3$

Fully-magnetized 3D plasma model

- Fully-magnetized limit of DIMAGNO model makes it possible to compute 3D deflection in a simple way
  - ❑ We assume the ion and electron gyrofrequencies $\Omega_i, \Omega_e \to \infty$
  - ❑ This precludes any ion separation in the model
- Continuity and momentum equations boil down to **4 conservation** laws **along the magnetic streamlines**:



$$\nabla \cdot (n\boldsymbol{u}_i) = 0; \quad \nabla \cdot (n\boldsymbol{u}_e) = 0,$$

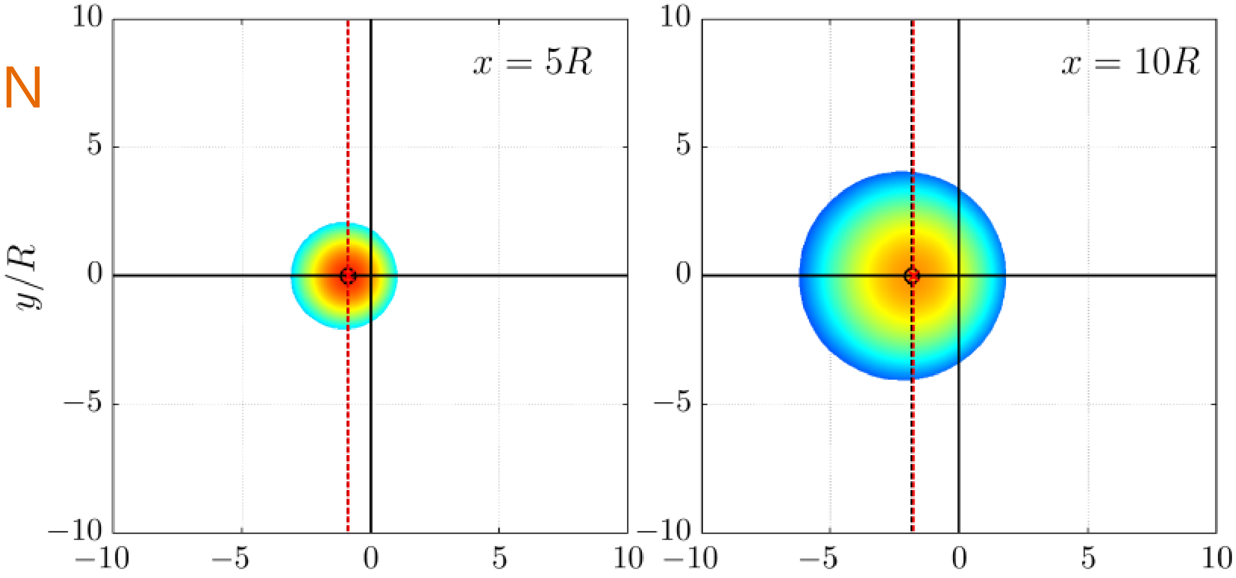
$$m_i (\boldsymbol{u}_i \cdot \nabla) \boldsymbol{u}_i = -e\nabla\phi + e\boldsymbol{u}_i \times \boldsymbol{B},$$

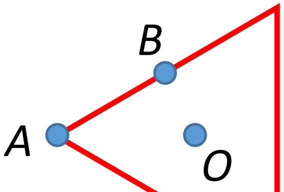
$$0 = -T_e \nabla \ln n + e\nabla\phi - e\boldsymbol{u}_e \times \boldsymbol{B},$$

$$\boxed{\begin{aligned} nu_i/B &= G_i & \qquad H_i &= \frac{1}{2}m_i u_i^2 + e\phi, \\ nu_e/B &= G_e & \qquad H_e &= T_e \ln n - e\phi. \end{aligned}}$$

TVC performance of a 3D MN

- Representative case with thruster solenoid + 3DMN made of 3 coils
- Magnetic thrust is deflected by up to 10 deg when coil angle is $\alpha = 15$ deg
- Larger deflections are possible with negative currents in one coil



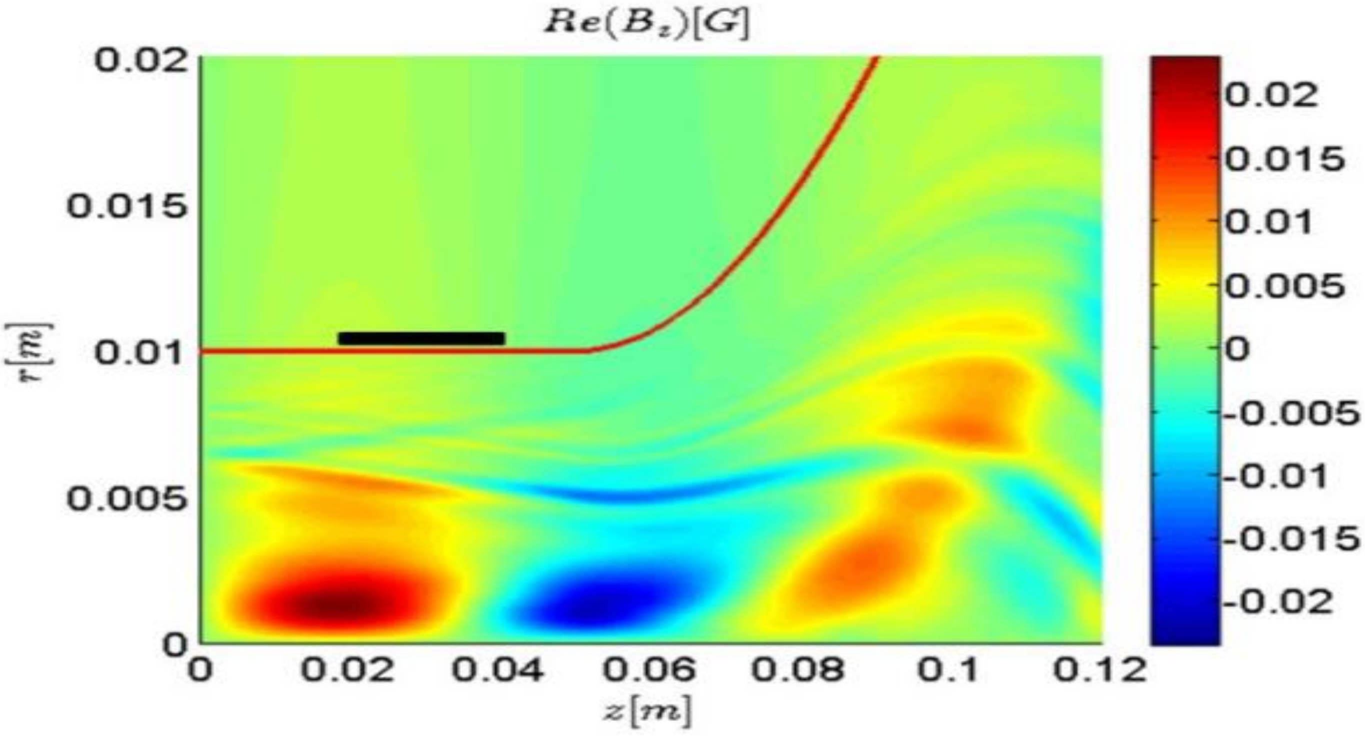


Simulation	Ampere-turn ratios	F/F_0	ψ (deg)	θ (deg)	θ_B (deg)
O	$15:0.33:0.33:0.33$	1.44	–	0.00	0.00
A	$15:1:0:0$	1.44	−180.00	5.66	5.76
B	$15:0.5:0.5:0$	1.44	−120.00	2.86	2.91
A'	$15:5:0:0$	1.34	−180.00	11.06	11.24
B'	$15:2.5:2.5:0$	1.34	−120.00	5.61	5.70

Advances on other MN topics

Propagation of RF into MN region

- In HPT and ECRT, ionization and heating is done with RF waves
- Waves propagating downstream into MN region could be a source of power losses
- A full-wave Yee scheme, Fourier transformed in t and θ, combined with a cold plasma dielectric tensor model is used to study wave propagation and (collisional) absorption



PhD thesis of Bin Tian

Kinetic electron model

- Plasma beam is nearly collisionless → no local thermodynamic equilibrium → no justification for isentropic/adiabatic behavior
- A kinetic electron model is required to understand collisionless electron cooling observed experimentally
- Kinetic model of paraxial convergent-divergent MN with collisionless, magnetized ions and electrons has been developed
- Particles conserve their total energy and their magnetic moment:

$$E_\alpha = \frac{m_\alpha}{2}(w_{\alpha\parallel}^2 + w_{\alpha\perp}^2) + q_\alpha \phi, \qquad \mu_\alpha = \frac{m_\alpha w_{\alpha\perp}^2}{2B}$$

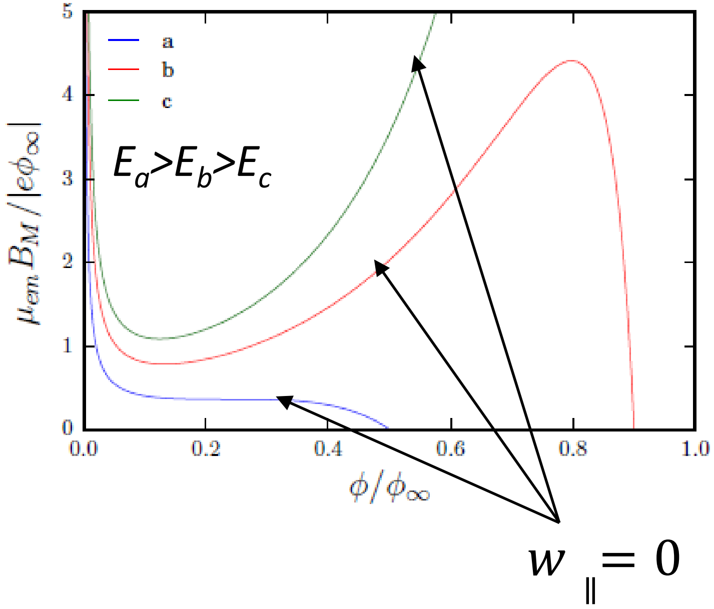
- Effective potential for axial motion:

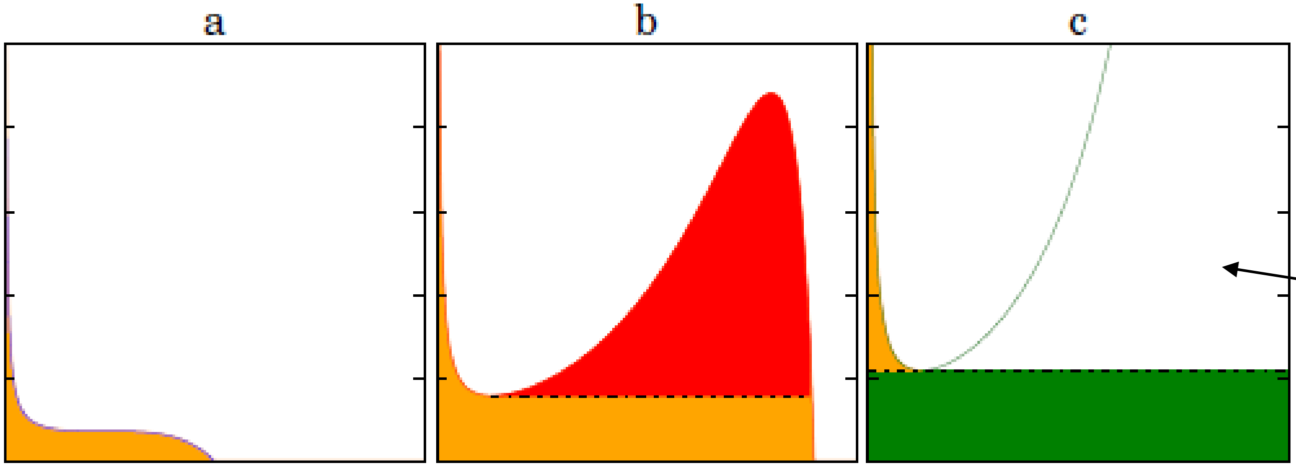
$$U_{eff} = q_\alpha \phi(z) + \mu_\alpha B(z)$$

PhD thesis
of Jaume Navarro

Kinetic electron cooling

- Individual electrons are reflected back when they reach
$$E_{\parallel} = U_{eff}$$
(electron turning manifold)
- In MN divergent side, there are local extrem
empty regions in the EVDF
- Three regions of phase space:
  - ☐ Free electrons
  - ☐ Reflected electrons
  - ☐ Doubly-trapped electrons





The existence of empty regions is related to cooling

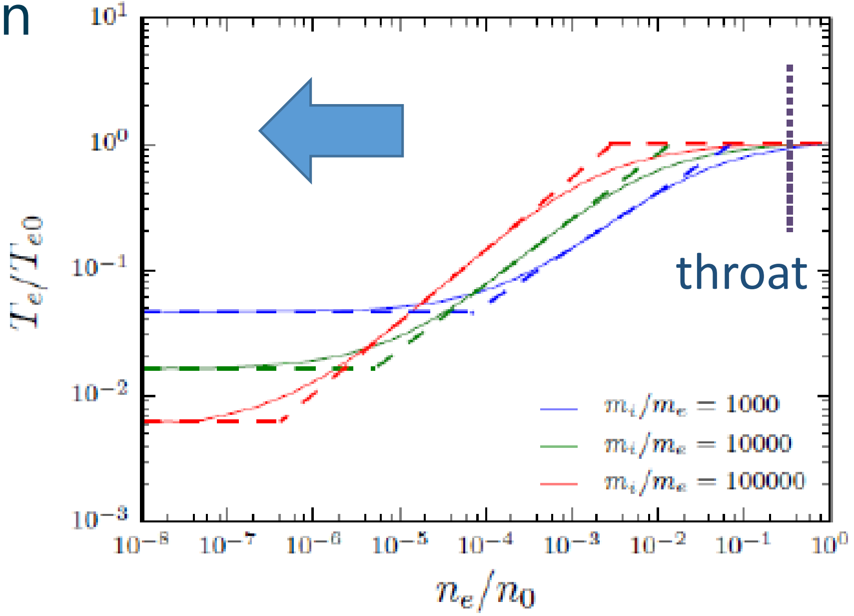
Kinetic electron cooling

- Interesting comparison with potential fall in Debye sheath:

$$\frac{e\phi_{sh}}{T_e} \approx A \cdot \ln \frac{m_i}{m_e} - B$$

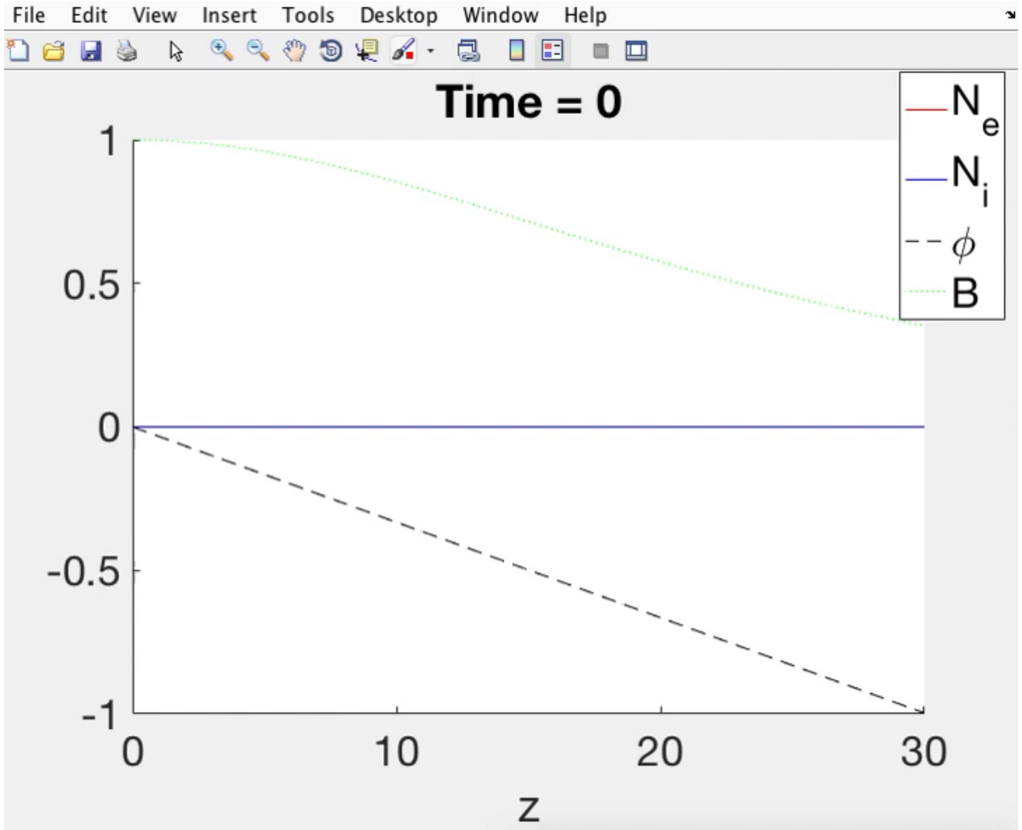
  with A, B constants

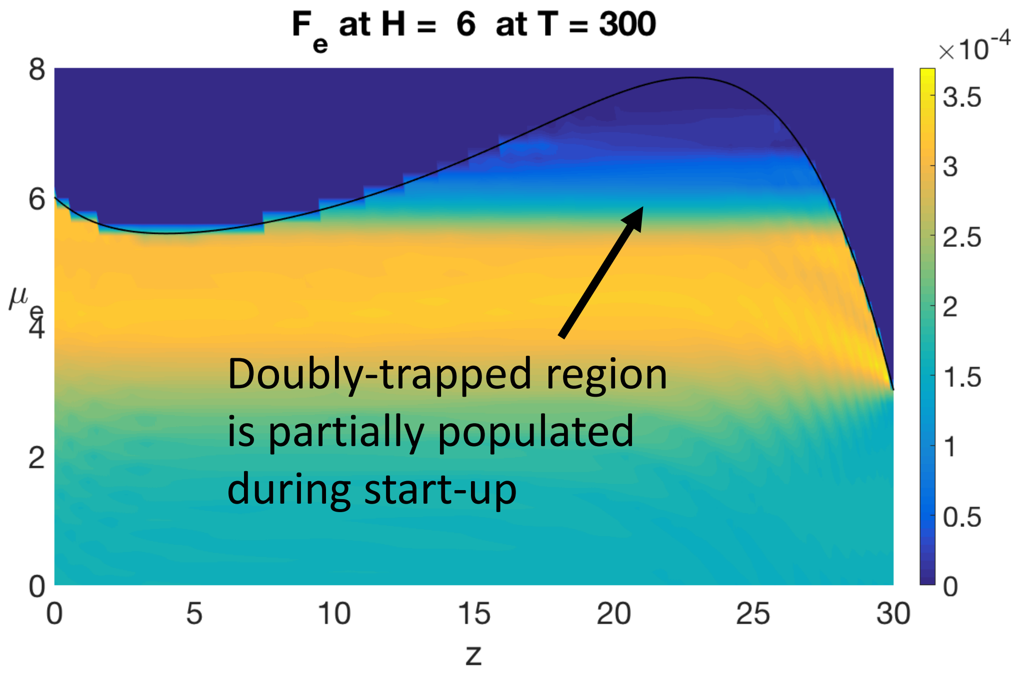
- Main differences in a MN with respect to a sheath:
  - ❑ It develops in an infinite region not in a very thin region
  - ❑ It is fully quasineutral
  - ❑ The doubly-trapped population is undefined but essential at a time.
  - ❑ The asymptotic behavior at infinity presents some issues
- Question remains: how are doubly-trapped electron regions populated?
  - ❑ Transient set-up of the plume
  - ❑ Occasional collisions



Time evolution of EVDF

- To answer the question of how doubly-trapped regions are populated, a first effort consisting in studying the transient set up of the plume is being carried out at EP2
  - ☐ Similar kinetic formulation, but keeping $\partial/\partial t$ in Vlasov equation (far more computationally intensive)



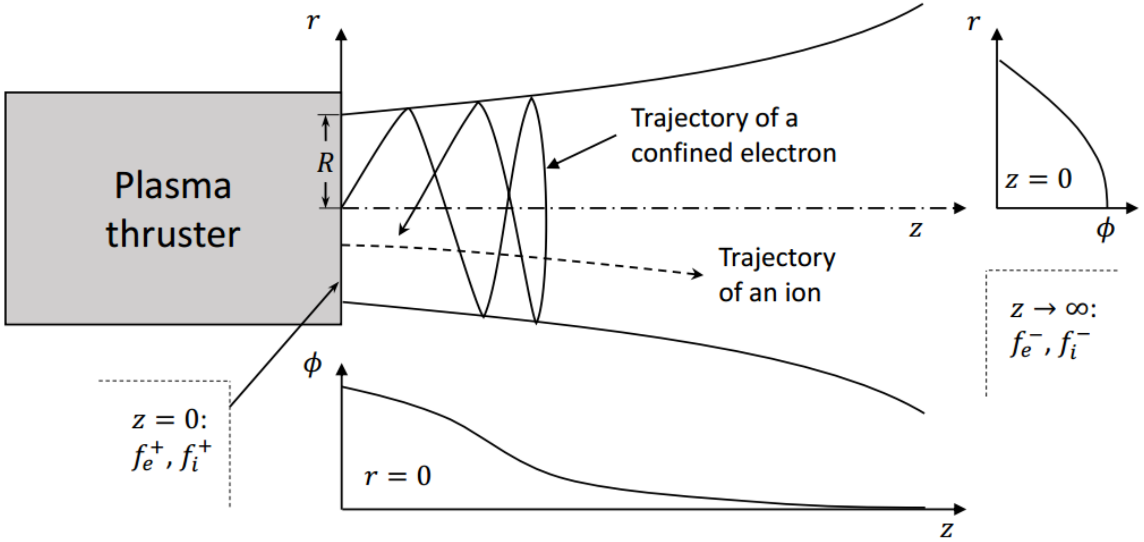


Doubly-trapped region is partially populated during start-up

Thanks to Gonzalo Sánchez and Jiewei Zhou

Kinetic model of unmagnetized plasma plume

- In an unmagnetized plasma plume, electron motion is strongly 2D
- However, radial electron motion is confined and has a short characteristic time:
  - An adiabatic invariant J_r (action integral in r orbits) can be found that plays a similar role to μ in magnetized plumes
- Three conserved quantitites: Energy E, angular momentum about the axis p_θ, and J_r (to order ε^2 when averaged)
  - More complex, but some analogies with magnetized case



Under study in the ESA-ADS project MODEXVAL

Thank you! Questions?

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