

# **A one and a half hours walk through the physics of neutron stars**

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**Rewriting Nuclear Physics Textbooks: Basic nuclear  
interactions and their applications to nuclear processes in the  
Cosmos and on Earth  
24<sup>th</sup>-28<sup>th</sup> July, Pisa (Italy)**

## The final message of this lecture



Neutron stars are excellent observatories to test fundamental properties of matter under extreme conditions and offer an interesting interplay between nuclear processes and astrophysical observables

# Road Map of this Walk



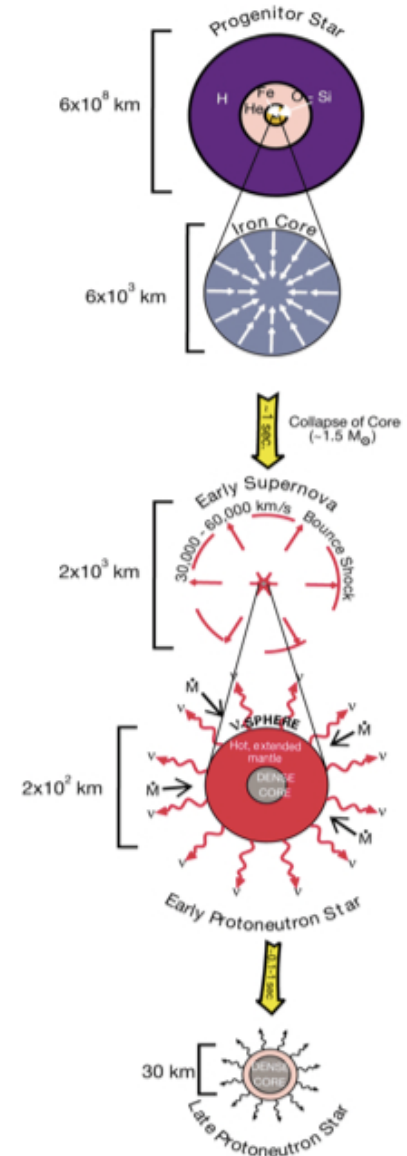
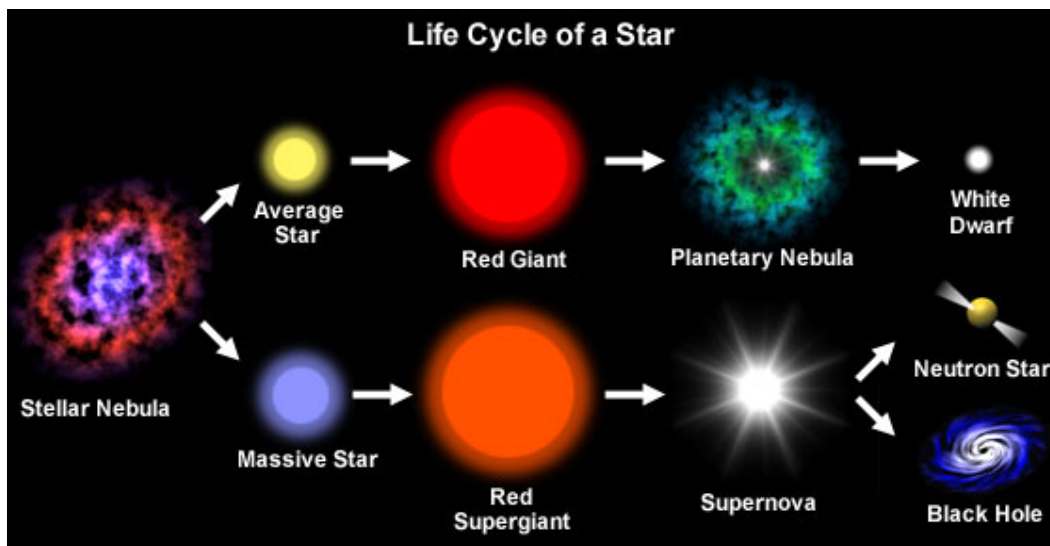
- ❖ Very general introduction to neutron stars
- ❖ A brush-stroke on the role of hyperons in neutron stars

# Neutron stars are different things for different people

- ✧ For **astronomers** are very **little stars** “visible” as radio pulsars or sources of X- and  $\gamma$ -rays.
- ✧ For **particle physicists** are **neutrino sources** (when they born) and probably the only places in the Universe where **deconfined quark matter** may be abundant.
- ✧ For **cosmologists** are “almost” **black holes**.
- ✧ For **nuclear physicists** are the **biggest neutron-rich nuclei** of the Universe ( $A \sim 10^{56}$ - $10^{57}$ ,  $R \sim 10$  km,  $M \sim 1$ - $2 M_{\odot}$  ).

But everybody agrees that ...

Neutron stars are a type of stellar compact remnant that can result from the gravitational collapse of a massive star ( $8 M_{\odot} < M < 25 M_{\odot}$ ) during a Type II, Ib or Ic supernova event.

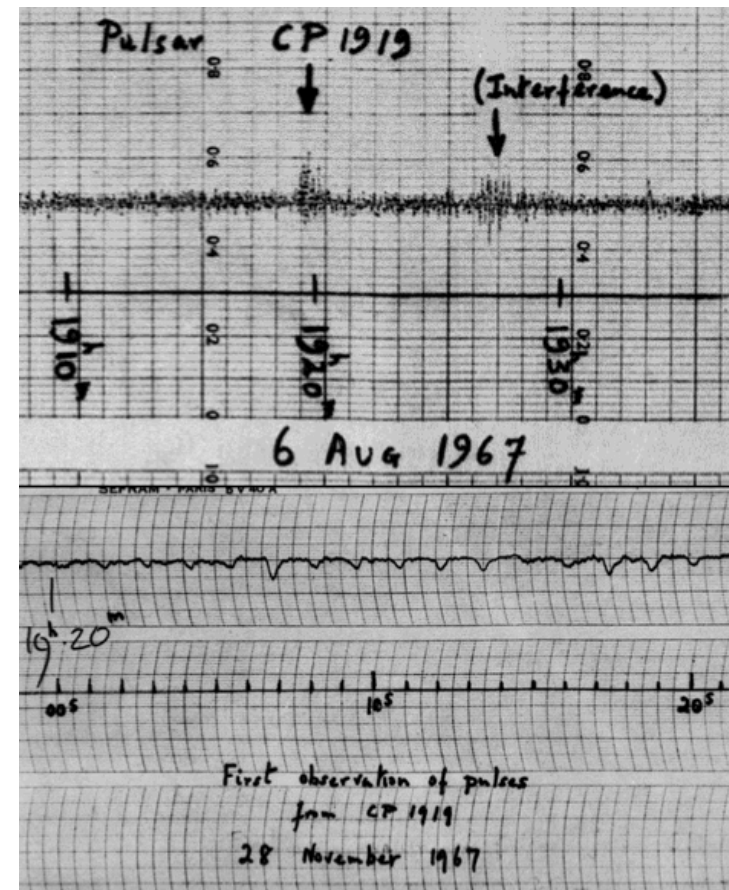


## 50 years of the discovery of the first radio pulsar

- ✧ radio pulsar at 81.5 MHz
- ✧ pulse period  $P=1.337$  s



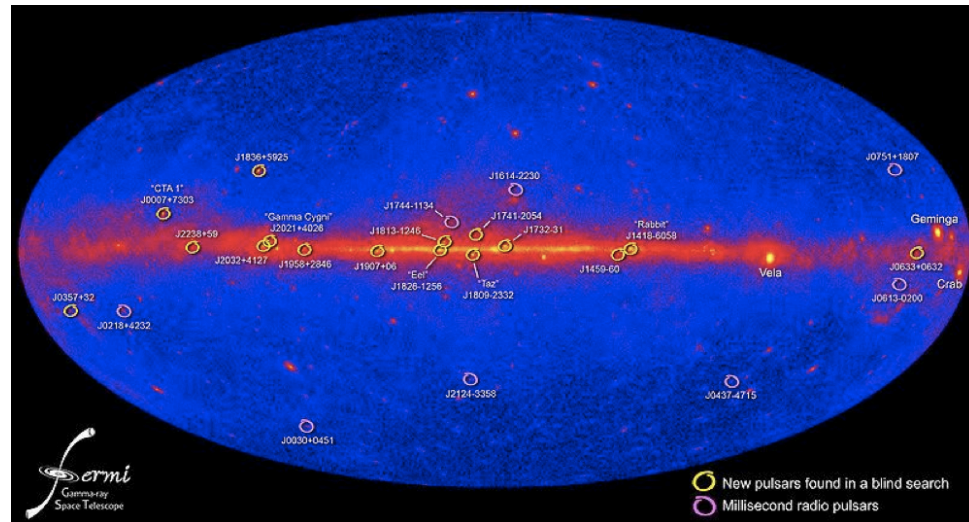
Most NS are observed as pulsars. In 1967 Jocelyn Bell & Anthony Hewish discover the first radio pulsar, soon identified as a rotating neutron star (1974 Nobel Prize for Hewish but not for Jocelyn)



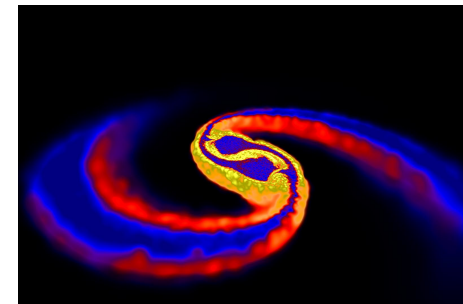
Nowadays more than 2000 pulsars are known  
(~ 1900 Radio PSRs (141 in binary systems), ~  
40 X-ray PSRs & ~ 60  $\gamma$ -ray PSRs)

## Observables

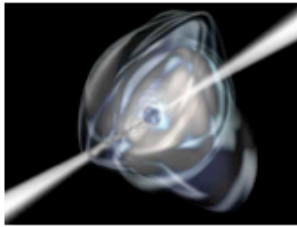
- Period ( $P$ ,  $dP/dt$ )
- Masses
- Luminosity
- Temperature
- Magnetic Field
- Gravitational Waves (NS-NS, BH-NS mergers,  
NS oscillation modes)



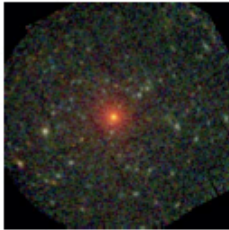
[http://www.phys.ncku.edu.tw/~astrolab/mirrors/apod\\_e/ap090709.html](http://www.phys.ncku.edu.tw/~astrolab/mirrors/apod_e/ap090709.html)



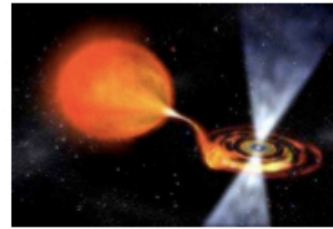
# The 1001 Astrophysical Faces of Neutron Stars



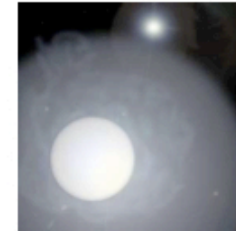
*Anomalous X-ray Pulsars*



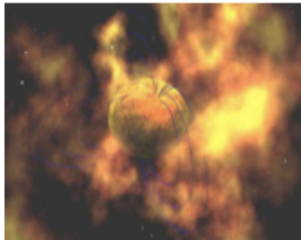
*dim isolated  
neutron stars*



*X-ray binaries*



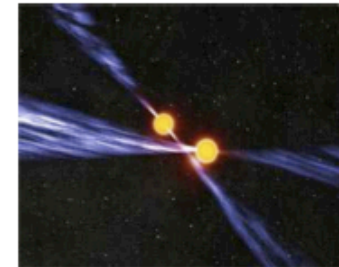
*bursting pulsars*



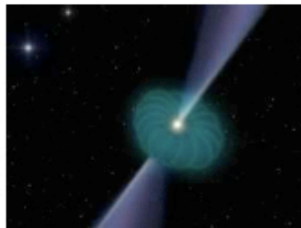
*Soft Gamma Repeaters*



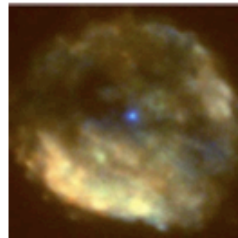
*pulsars*



*binary pulsars*



*Rotating Radio Transients*



*Compact Central Objects*



*planets around pulsar*



# Observation of Neutron Stars

## X- and $\gamma$ -ray telescopes



Chandra

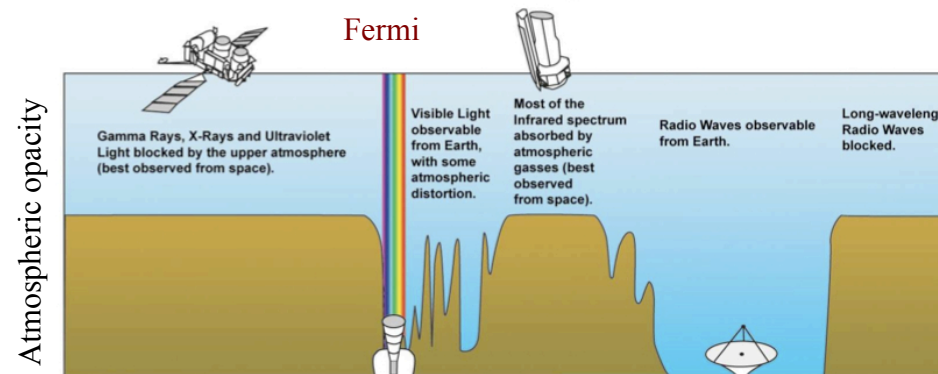


Fermi

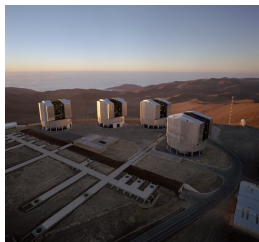
## Space telescopes



HST (Hubble)



## Optical telescopes



VLT (Atacama, Chile)



Arecibo (Puerto Rico):  $d= 305$  m

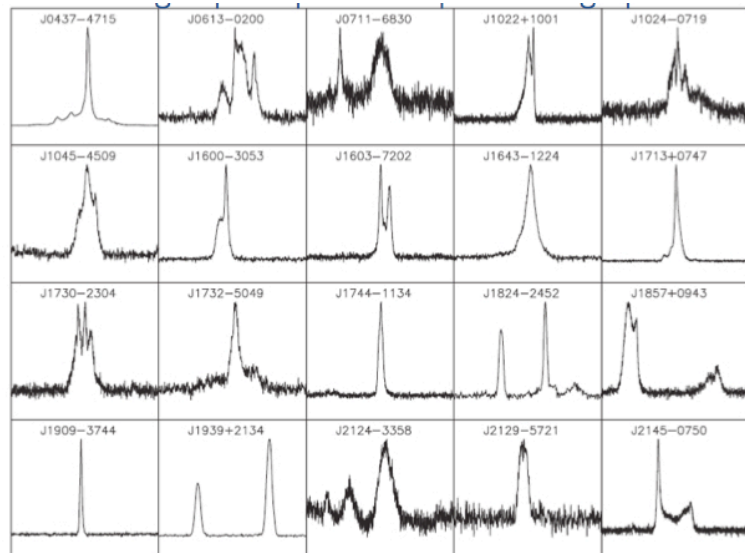
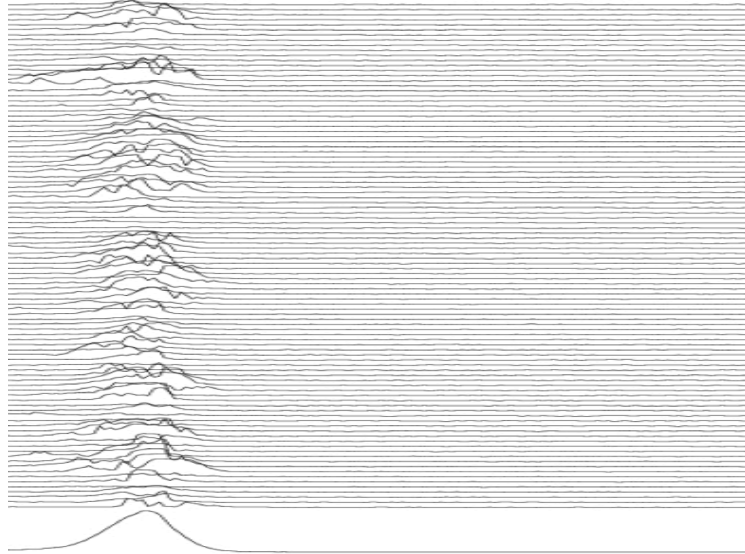


Green Banks (USA):  $d= 100$  m



Nançay (France):  $d \sim 94$  m

# The Fingerprint of a Pulsar

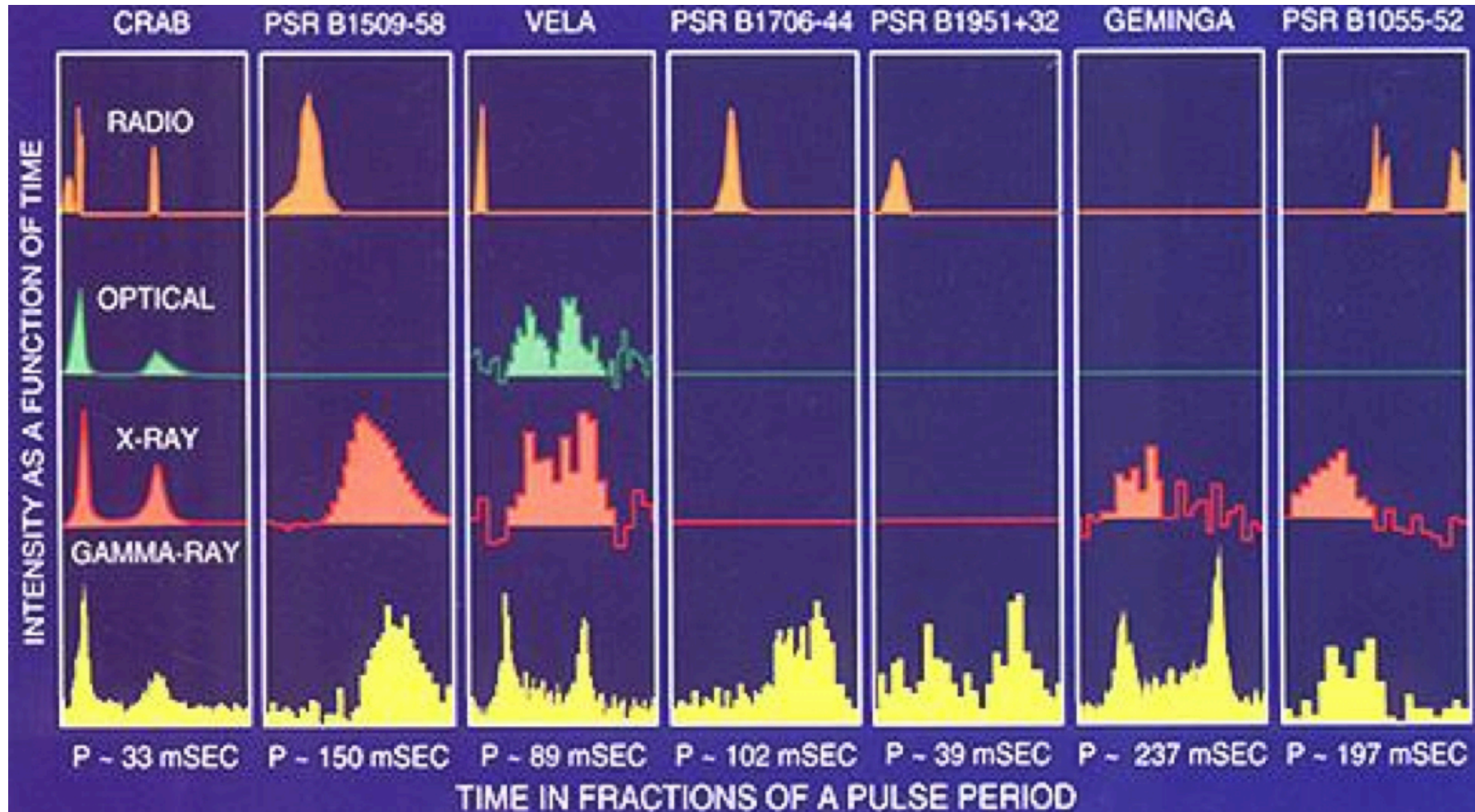


Individual pulses are very different. But the average over 100 or more pulses is **extremely stable and specific** of each pulsar

✧ **Top:** 100 single pulses from the pulsar PSR B0950+08 ( $P=0.253$  s) showing the pulse-to-pulse variability in shape and intensity

✧ **Bottom:** Average profiles of several pulsars

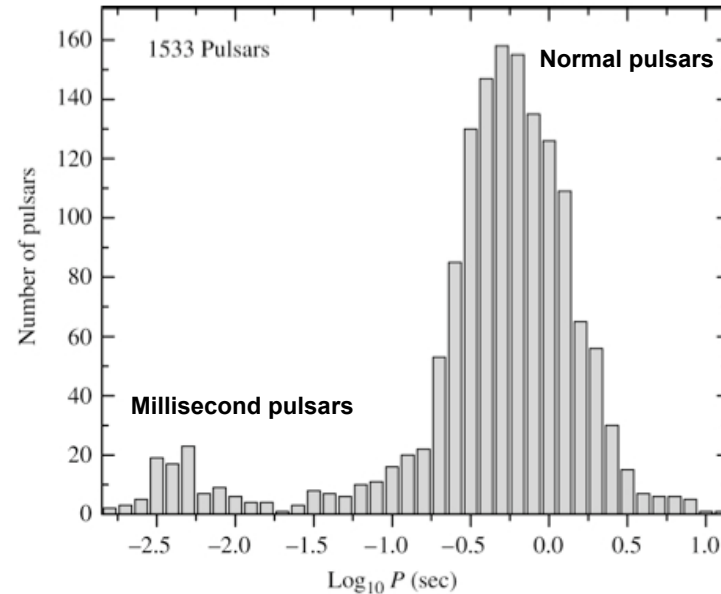
# Pulsar shape at different wavelength



# Pulsar Rotational Period

The distribution of the rotational period of pulsars shows two clear peaks that indicate the existence of two types of pulsars

- normal pulsars with  $P \sim s$
- millisecond pulsars with  $P \sim ms$



Globular cluster Terzan 5

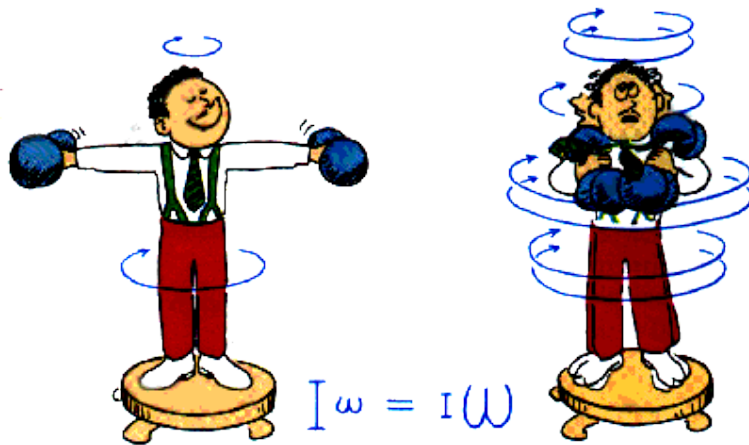
- First millisecond pulsar discovered in 1982 (Arecibo)
- Nowadays more than 200 millisecond pulsars are known
- PSR J1748-2446ad discovered in 2005 is until know the fastest one with  $P=1.39$  ms (716 Hz)

# Why Pulsars spin so fast ?

Conservation of the angular momentum and mass during the gravitational collapse of the iron core that will form the neutron star

If the initial iron core and the final neutron star are assumed to be rigid spheres with moment of inertia  $I=(2/5)MR^2$

$$J_i = J_f \Rightarrow P_f = P_i \left( \frac{R_f}{R_i} \right)^2$$



Taking  $P_i \sim 10^3$  s and  $R_f/R_i \sim 10^{-2}$   
one gets  $P_f \sim 10^{-3}$  s

# Minimum Rotational Period of a Neutron Star

Pulsar **cannot spin arbitrarily fast.**  
The absolute minimum rotational period is obtained when

Centrifugal Force = Gravitational Force



Keplerian Frequency



"... And that, Jimmy, is what we call 'centrifugal force'."

In Newtonian Gravity

$$P_{\min} = 2\pi \sqrt{\frac{R^3}{GM}} \approx 0.55 \left( \frac{M_{\text{sun}}}{M} \right)^{1/2} \left( \frac{R}{10\text{km}} \right)^{3/2} \text{ ms}$$

In General Relativity

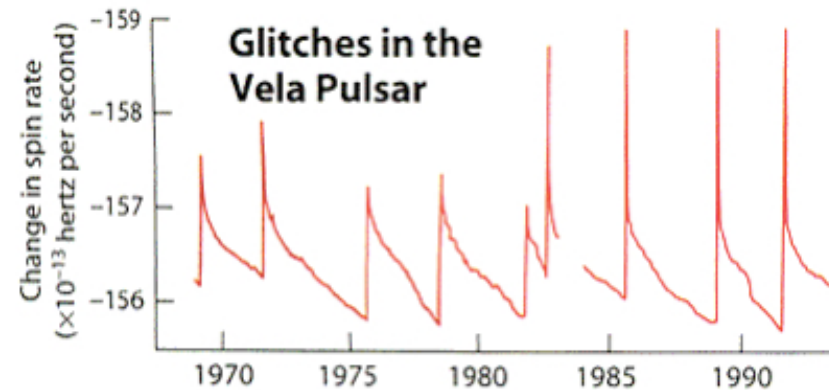
$$P_{\min} = 0.96 \left( \frac{M_{\text{sun}}}{M} \right)^{1/2} \left( \frac{R}{10\text{km}} \right)^{3/2} \text{ ms}$$

Actual record: PSR J1748-2446ad → **P=1.39595482 ms**

# Pulsar glitches

Sometimes the period  $P$  of a pulsar decreases suddenly. These variations (glitches), although small, are observable

$$\frac{\Delta\Omega}{\Omega} \approx 10^{-9} - 10^{-5}$$



Small glitches are interpreted as starquakes  $\rightarrow$  indirect proof of the existence of NS crust

Big glitches and the observation of long relaxation times are a proof of the existence of superfluid matter in the NS interiors

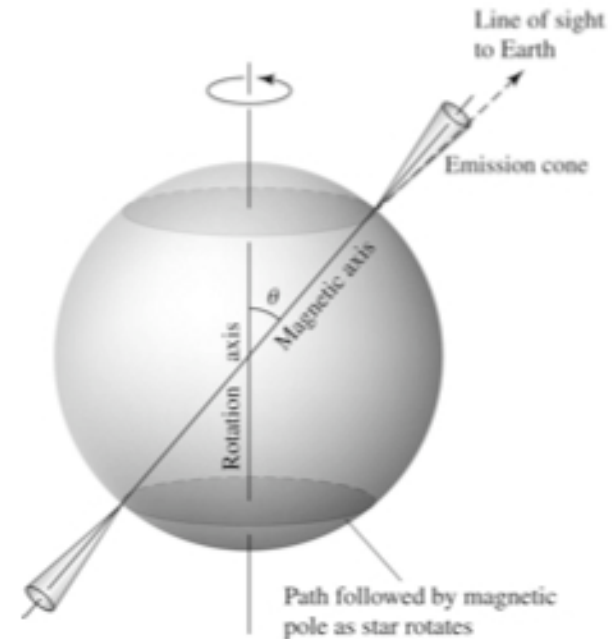


## Basic Model of a Pulsar: Magnetic Dipole

Pulsars are believed to be **highly magnetized rotating neutron stars** radiating at the expense of their rotational energy

$$\dot{E}_{mag} = -\frac{2}{3c^3} |\ddot{\vec{\mu}}|^2 = \dot{E}_{rot}$$

$\vec{\mu} \equiv$  Magnetic dipole moment



Pacini, Nature 216 (1967), 219 (1968)



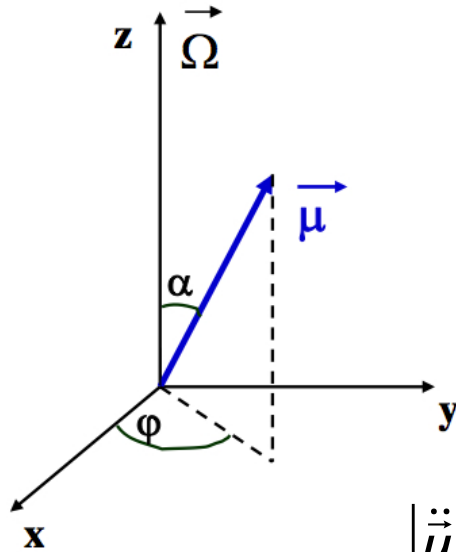
Gold, Nature 218 (1968), 221 (1969)



Ostriker & Gunn, ApJ 157 (1969)



# Basic Model of a Pulsar: Magnetic Dipole



$$\vec{\mu} = \mu \sin \alpha \cos \varphi \hat{e}_x + \mu \sin \alpha \sin \varphi \hat{e}_y + \mu \cos \alpha \hat{e}_z$$

supposing:  $\alpha = \text{const}$ ,  $\mu = |\vec{\mu}| = \text{const}$      $\Omega = \frac{d\varphi}{dt} = \dot{\varphi}$

$$\dot{\Omega}^2 \ll \Omega^4$$

$$|\ddot{\vec{\mu}}| = \mu^2 (\sin \alpha)^2 [\Omega^4 + \dot{\Omega}^2] \longrightarrow |\ddot{\vec{\mu}}| \approx \mu^2 (\sin \alpha)^2 \Omega^4$$

Therefore

$$\dot{E}_{mag} = -\frac{2}{3c^3} \mu^2 (\sin \alpha)^2 \Omega^4 = \dot{E}_{rot}$$

For a sphere with a pure dipole magnetic field

$$\mu = \frac{1}{2} B_p R^3$$

- ✓  $B_p$ : magnetic field at the poles
- ✓  $R$ : radius of the sphere

## Basic Model of a Pulsar: Magnetic Dipole

Then  $\dot{E}_{mag} = -\frac{1}{6c^3} R^6 B_P^2 (\sin \alpha)^2 \Omega^4 = \dot{E}_{rot}$

On the other hand  $E_{rot} = \frac{1}{2} I \Omega^2 \xrightarrow{\dot{I} = 0} \dot{E}_{rot} = I \Omega \dot{\Omega}$

One arrives to the **PSR evolution differential equation**

$$\dot{\Omega} = -K \Omega^3 \quad \text{or} \quad P\dot{P} = (2\pi)^2 K, \quad K = \frac{1}{6c^3} \frac{R^6}{I} (B_P \sin \alpha)^2$$

More generally, one can write the **PSR evolution differential equation** as

$$\dot{\Omega} = -K\Omega^n \quad \text{or} \quad P^{n-2}\dot{P} = (2\pi)^{n-1} K, \quad K = \frac{1}{6c^3} \frac{R^6}{I} (B_p \sin \alpha)^2$$

with solution

$$\Omega(t) = \frac{\Omega_0}{\left[ (n-1)K\Omega_0^{n-1}t + 1 \right]^{1/(n-1)}}, \quad P(t) = P_0 \left[ (n-1)K\Omega_0^{n-1}t + 1 \right]^{1/(n-1)}$$

Differentiating it assuming **K=const**, one obtains

$$n = \frac{\Omega\ddot{\Omega}}{\dot{\Omega}^2} = 2 - \frac{P\ddot{P}}{\dot{P}^2} \quad \text{braking index}$$

**n=3** within the magnetic dipole model

The three quantities **P**,  **$\dot{P}$**  &  **$\ddot{P}$**  have been measured for few PSRs

# The Pulsar Age

The solution of the PSR evolution differential equation can be rewritten as

$$t = -\frac{1}{n-1} \frac{\Omega(t)}{\dot{\Omega}(t)} \left[ 1 - \left( \frac{\Omega(t)}{\Omega_0} \right)^{n-1} \right]$$

or

$$t = \tau - \left[ (n-1)K\Omega_0^{n-1} \right]^{-1}$$

“True” Pulsar Age

(valid under the assumption  $K=\text{const.}$ )

with

$$\tau = -\frac{1}{n-1} \frac{\Omega(t)}{\dot{\Omega}(t)} = \frac{1}{n-1} \frac{P(t)}{\dot{P}(t)} \xrightarrow{n=3} \tau = -\frac{1}{2} \frac{\Omega(t)}{\dot{\Omega}(t)} = \frac{1}{2} \frac{P(t)}{\dot{P}(t)}$$

Pulsar Dipole Age

if

$$\Omega(t) \ll \Omega_0 \longrightarrow t \approx \tau$$

(t=present time)

The measure of  $P$  and  $\dot{P}$  gives the pulsar dipole age

## Example: the age of the Crab Pulsar

SN explosion: 1054 AD

$P=0.0330847$  s,  $\dot{P}=4.22765 \times 10^{-13}$  s/s

Braking index:  $n=2.515 \pm 0.005$



$$t_{\text{Crab}} = (2014 - 1054) \text{ yr} = 960 \text{ yr,}$$

$$\tau = 1238 \text{ yr (dipole age)}$$

Assuming the validity of the pulsar dipole mode, using the previous equation for the true pulsar age we can infer the initial spin period of the Crab pulsar

$$P_0 = P \left( 1 - \frac{t_{\text{Crab}}}{\tau} \right)^{1/2} \cong 0.016 \text{ s}$$

But  $n \neq 3$

# Measured value of the braking index $n$

PSR	$n$	P (s)	P dot ( $10^{-15}$ s/s)	Dipole age (yr)
PSR B0531+21 (Crab)	2.512 +/- 0.005	0.03308	422.765	1238
PSR B0833-45 (Vela)	1.4 +/- 0.2	0.08933	125.008	11000
PSR B0540-69	2.839 +/- 0.005	0.1506	1536.5	1554
PSR B0540-69	2.01 +/- 0.02	0.0505	478.924	1672
PSR J1119-6127	2.91 +/- 0.05	0.40077	4021.782	1580

Deviations of **braking index  $n$  from 3** probably due to:

- ✓ Torque on the pulsar from outflow particles
- ✓ Change with  $t$  of “constant”  $K$ , i.e.,  $I(t)$ ,  $B(t)$ ,  $\alpha(t)$

# Pulsar evolutionary path on the P- $\dot{P}$ plane

Taking the logarithm of

$$P\dot{P} = (2\pi)^2 K, \quad K = \frac{1}{6c^3} \frac{R^6}{I} (B_p \sin \alpha)^2$$

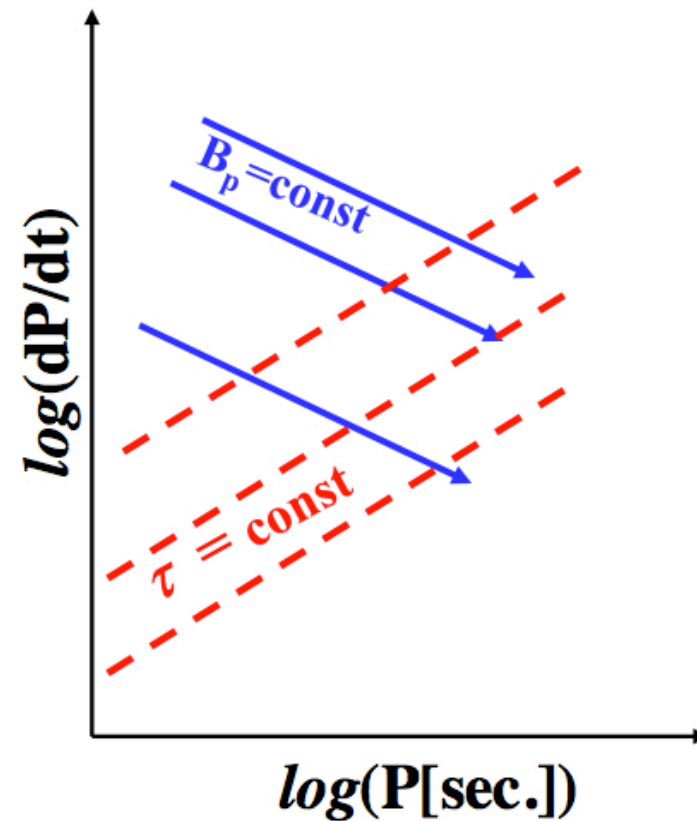
and

$$\tau = \frac{P}{2\dot{P}}$$

we get

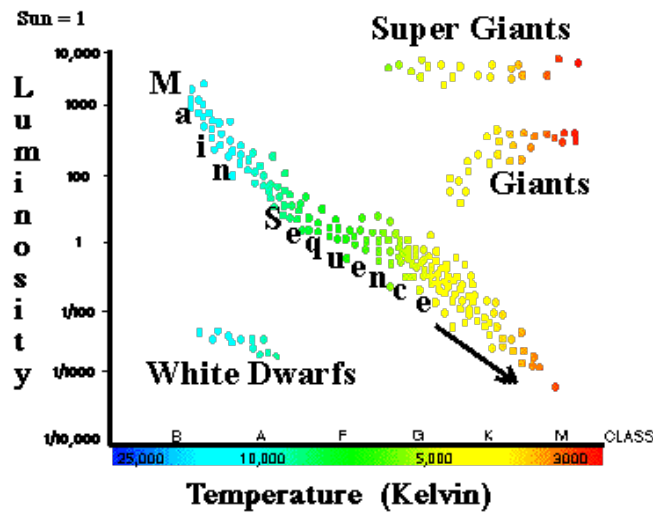
$$\log \dot{P} = \log \left[ \frac{(2\pi)^2 R^6}{6c^3 I} B_p^2 \sin^2 \alpha \right] - \log P$$

$$\log \dot{P} = \log P - \log(2\tau)$$



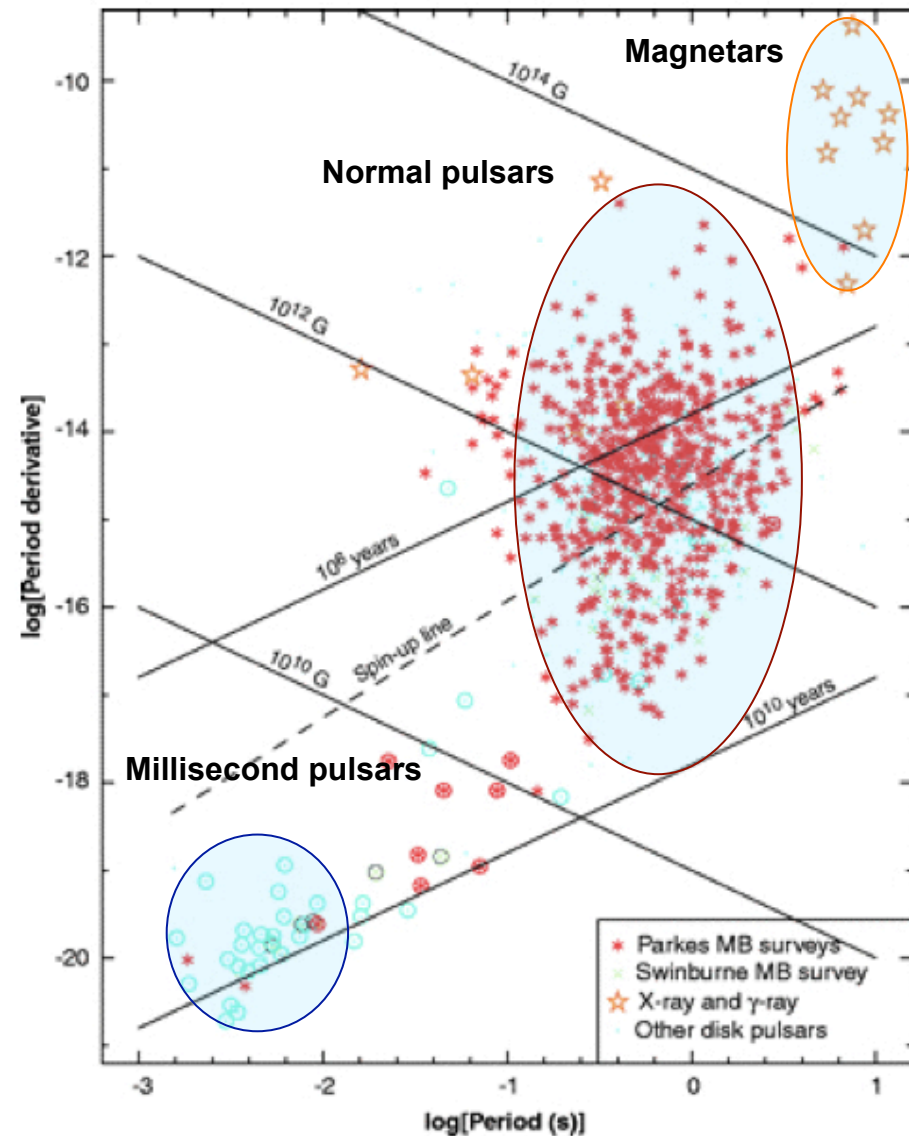
# Pulsar distribution in the P- $\dot{P}$ plane

Pulsar equivalent of the  
Hertzprung-Russell diagram  
for ordinary stars



$$\log \dot{P} = \log \left[ \frac{(2\pi)^2 R^6}{6c^3 I} B_p^2 \sin^2 \alpha \right] - \log P$$

$$\log \dot{P} = \log P - \log(2\tau)$$



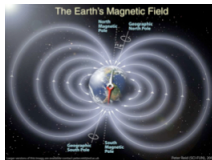


# Magnetic Field of a Pulsar

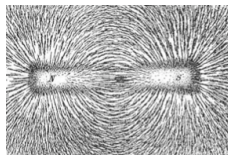
Type of Pulsar	Surface magnetic field
Millisecond	$10^8 - 10^9$ G
Normal	$10^{12}$ G
Magnetar	$10^{14} - 10^{15}$ G

Extremely high compared to ...

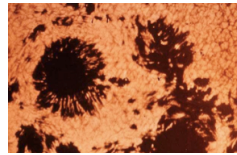
Earth  
0.3 – 0.5 G



Magnet  
 $10^3 - 10^4$  G



Sun spots  
 $10^5$  G



Largest continuous field in lab. (USA)

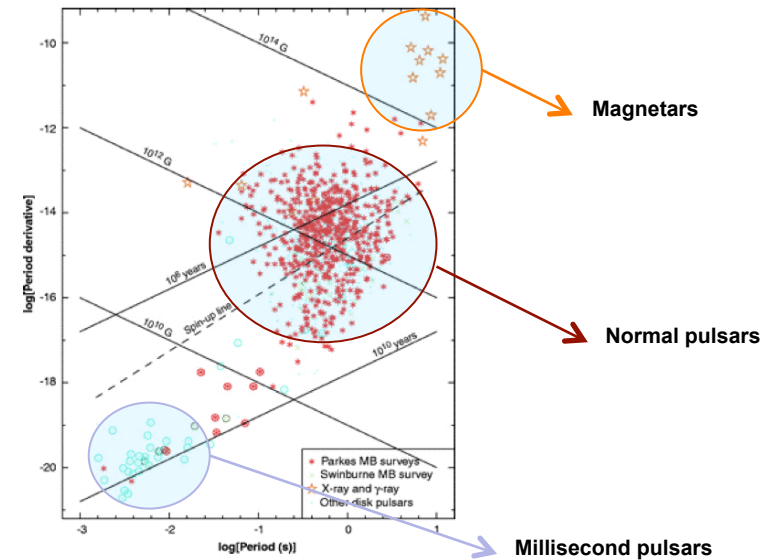
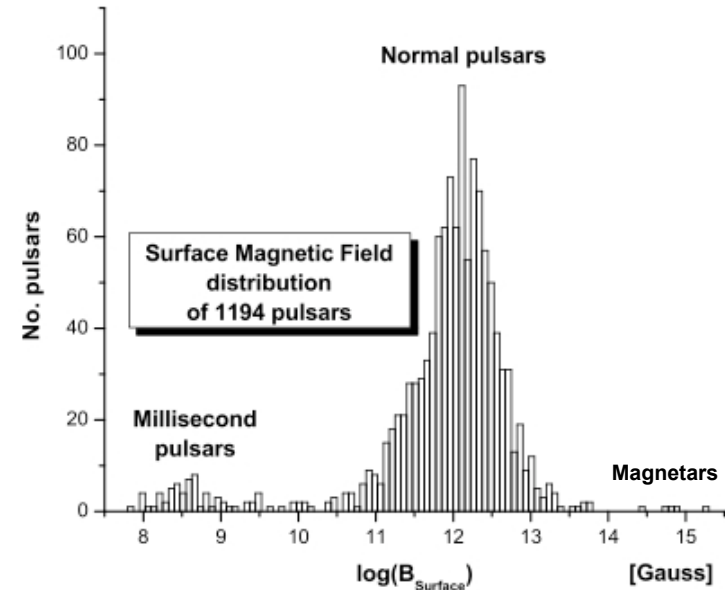


$4.5 \times 10^5$  G

Largest magnetic pulse in lab. (Russia)



$2.8 \times 10^7$  G



## Where the NS magnetic field comes from ?

A satisfactory answer does not exist yet. Several possibilities have been considered:

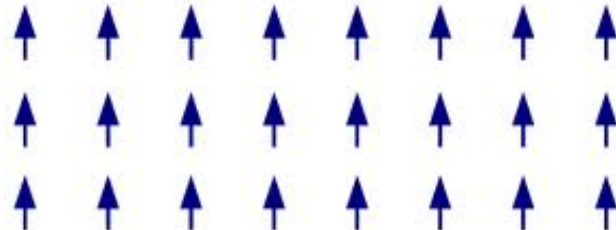
- ✧ **Conservation of the magnetic flux** during the gravitational collapse of the iron core

$$\phi_i = \phi_f \Rightarrow B_f = B_i \left( \frac{R_i}{R_f} \right)^2$$

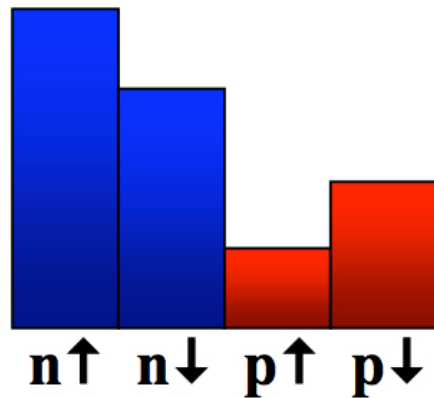
For a progenitor star with  $B_i \sim 10^2$  G  
&  $R_i \sim 10^6$  km we have  $B_f \sim 10^{12}$  G

- ✧ **Electric currents** flowing in the highly conductive NS interior

- ✧ **Spontaneous transition to a ferromagnetic state** due to the nuclear interaction



# Spin-polarized Isospin Asymmetric Nuclear Matter



## ✧ Densities & Asymmetries

$$\checkmark \quad \rho_n = \rho_{n\uparrow} + \rho_{n\downarrow}, \quad \rho_p = \rho_{p\uparrow} + \rho_{p\downarrow}$$

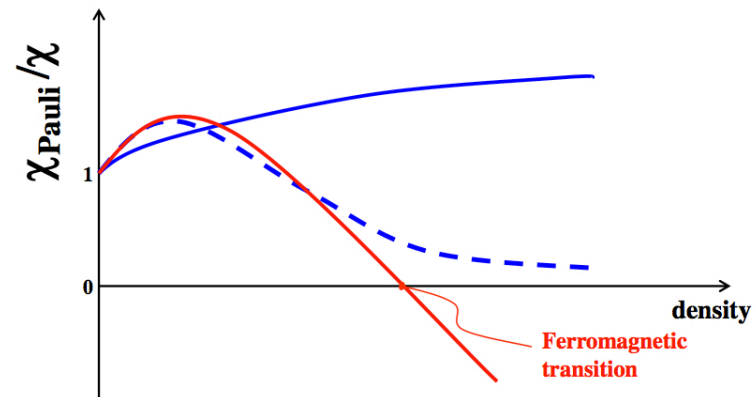
$$\checkmark \quad \rho = \rho_n + \rho_p, \quad \beta = \frac{\rho_n - \rho_p}{\rho}$$

$$\checkmark \quad S_n = \frac{\rho_{n\uparrow} - \rho_{n\downarrow}}{\rho_n}, \quad S_p = \frac{\rho_{p\uparrow} - \rho_{p\downarrow}}{\rho_p}$$

## ✧ Magnetic Susceptibility

$$\frac{1}{\chi_{ij}} = \frac{\rho}{\mu_i \rho_i \mu_j \rho_j} \frac{\partial^2 (E/A)}{\partial S_i \partial S_j}$$

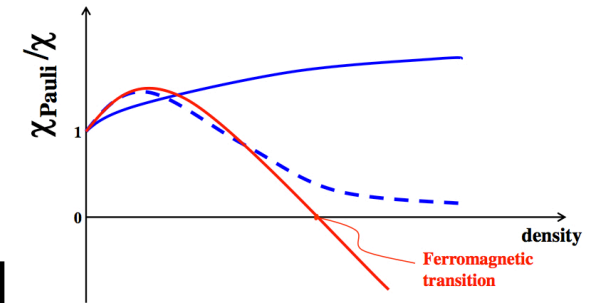
Stability against spin fluctuations if  $\chi > 0$



# Ferromagnetic Transition

Considered by many authors with contradictory results:

Year	Autor/Model	Ferromagnetic Transition ?
1969	Brownell, Callaway, Rice (hard sphere gas)	Yes, $k_F > 2.3 \text{ fm}^{-1}$
1969	Clark & Chao	No
1970	Ostgard	Yes, $k_F > 4.1 \text{ fm}^{-1}$
1972	Pandharipande et al., (variational)	No
1975	Backman, Kallaman, Haensel (BHF)	No
1984	Vidaurre (Skyrme)	Yes, $k_F > 1.7-2.0 \text{ fm}^{-1}$
1991	S. Marcos et al., (DBHF)	No
2001	Fantoni et at. (AFDMC)	No
2002/2005	I.V., et al. (BHF)	No
2005/2006	I.V. et al., (Skyrme, Gogny)	Yes, $k_F > 2-3.4 \text{ fm}^{-1}$
2007-2011	F. Sammarruca (DBHF)	No



- ✧ Calculations based on **phenomenological interactions** (e.g., Skyrme, Gogny) predict the transition to occur at  $(1-4)\rho_0$
- ✧ Calculations based on **realistic NN & NNN forces** (e.g., Monte Carlo, BHF, DBHF, LOCV) exclude such a transition

# Neutron Star Structure: General Relativity or Newtonian Gravity ?

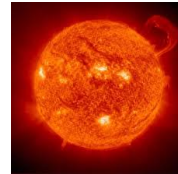
Surface gravitational potential tell us how much compact an object is

$$\frac{2GM}{c^2 R}$$

→ Relativistic effects are very important in Neutron Stars and General Relativity must be used to describe their structure



$$\sim 10^{-10}$$



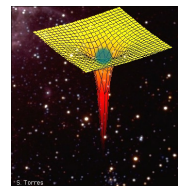
$$\sim 10^{-5}$$



$$\sim 10^{-4} - 10^{-3}$$



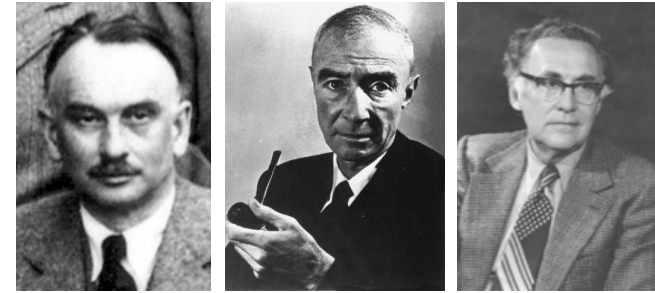
$$\sim 0.2 - 0.4$$



$$1$$

# The Tolman-Oppenheimer-Volkoff Equations

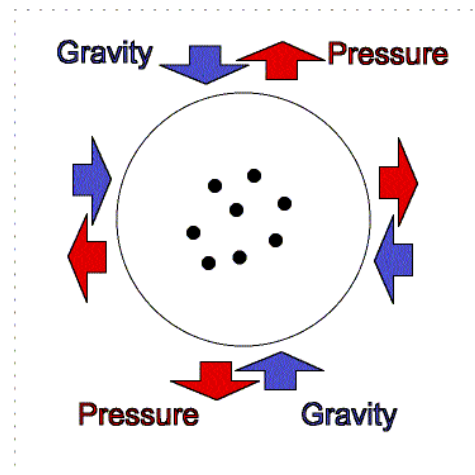
In 1939 Tolman, Oppenheimer & Volkoff obtain the equations that describe the structure of a static star with spherical symmetry in General Relativity (Chandrasekhar & von Neumann obtained them in 1934 but they did not published their work)



Tolman, Phys. Rev. 55, 364 (1939)



Oppenheimer & Volkoff, Phys. Rev. 55, 374 (1939)



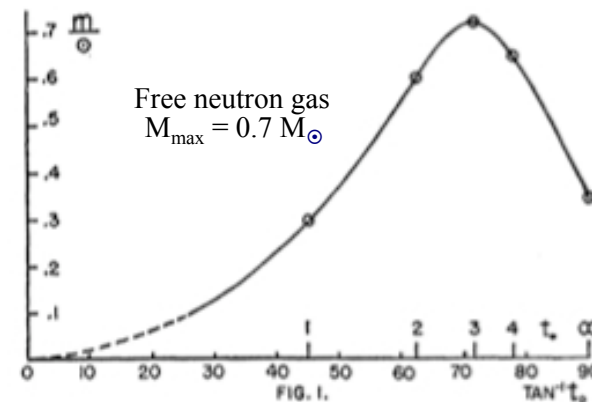
$$\frac{dP}{dr} = -G \frac{m(r)\epsilon(r)}{r^2} \left( 1 + \frac{P(r)}{c^2 \epsilon(r)} \right) \left( 1 + \frac{4\pi r^3 P(r)m(r)}{c^2} \right) \left( 1 - \frac{2Gm(r)}{c^2 r} \right)^{-1}$$

$$\frac{dm}{dr} = 4\pi r^2 \epsilon(r)$$

boundary conditions

$$P(0) = P_o, \quad m(0) = 0$$

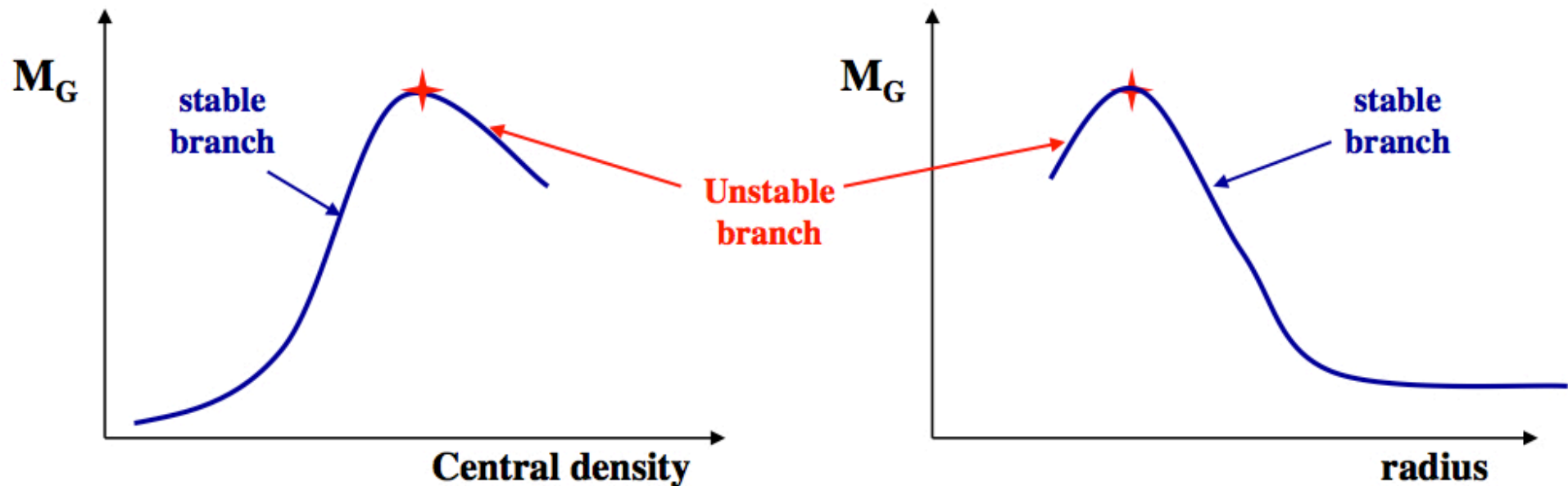
$$P(R) = 0, \quad m(R) = M$$



# Stability solutions of the TOV equations

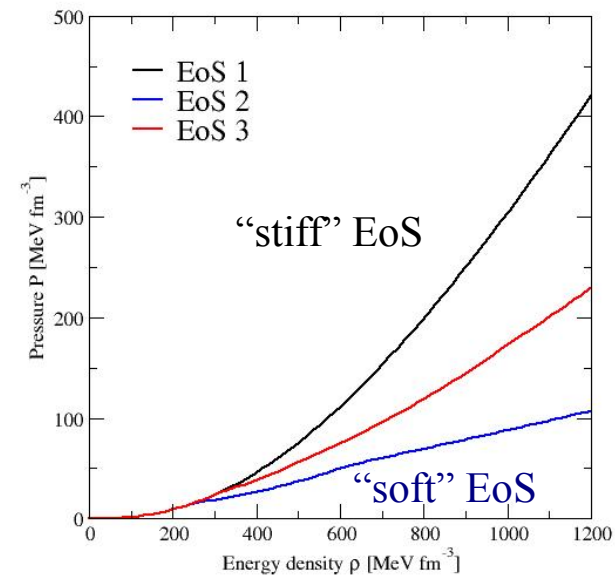
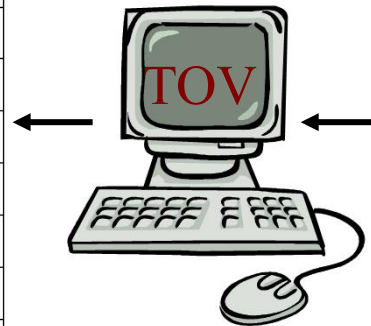
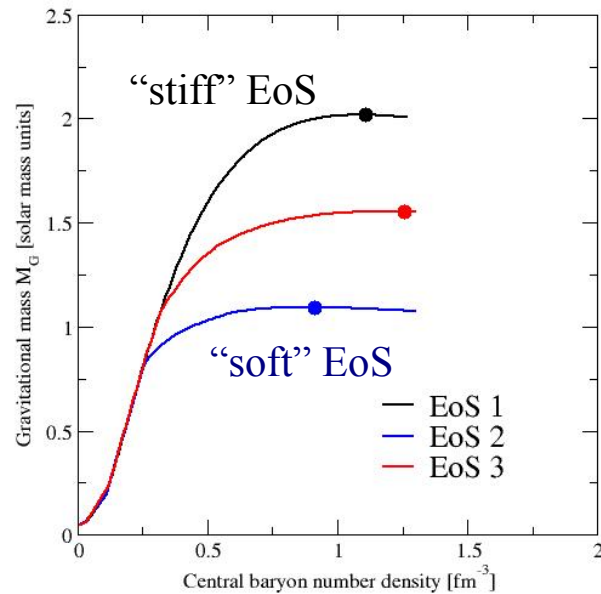
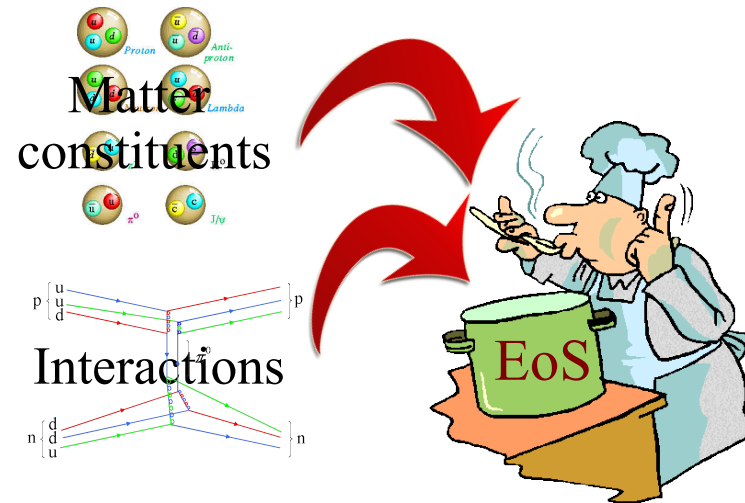
- ✧ The solutions of the TOV eqs. represent **static equilibrium configurations**
- ✧ Stability is required with respect to **small perturbations**

$$\frac{dM_G}{d\rho_c} > 0, \text{ or } \frac{dM_G}{dr} < 0$$



# The role of the Equation of State

The only ingredient needed to solve the TOV equations is the (poorly known) EoS (i.e.,  $p(\epsilon)$ ) of dense matter





# General Features of a “realistic” neutron star matter EoS

Any “realistic” neutron star matter EoS must satisfy:

## ✧ Saturation Properties of Symmetric Matter

$$n_0 = 0.16 - 0.18 \text{ fm}^{-3}, \quad \left( \frac{E}{A} \right)_0 = -16 \pm 1 \text{ MeV}$$

## ✧ Nuclear Symmetry Energy $E_{\text{sym}}(n_0) = 28 - 32 \text{ MeV}$

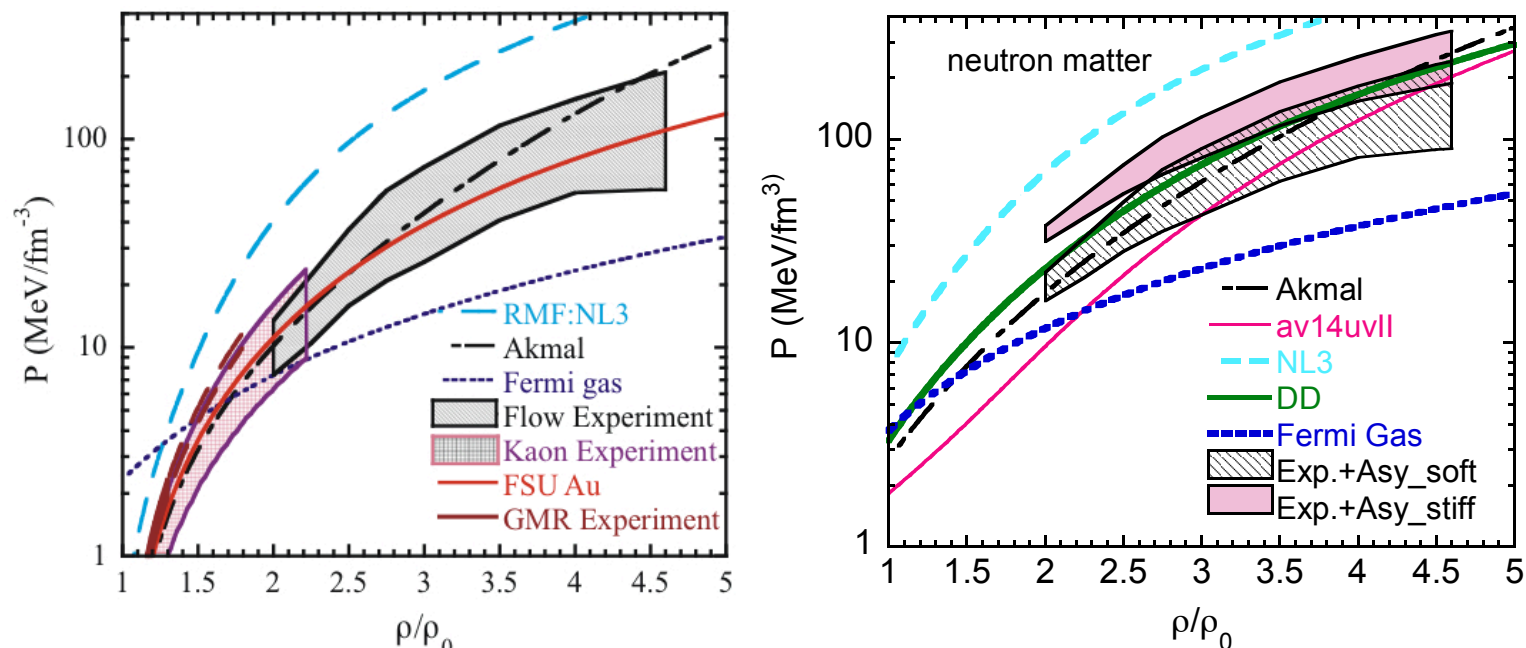
$E_{\text{sym}}$  must be “well behaved” at high densities

## ✧ Nuclear Incompressibility $K_0 = 240 \pm 20 \text{ MeV}$

## ✧ Causal Condition

$$c_s^2 = \frac{dP}{d\rho} \leq c^2$$

# Constraints of the Nuclear EoS from HIC



- ❖ Collective flow constraints confirms the softening of the EoS at high densities
- ❖ Constraints from kaon production are consistent with the flow constraints and bridge gap to GMR constraints
- ❖ Symmetry energy dominates the uncertainty in the neutron matter EoS

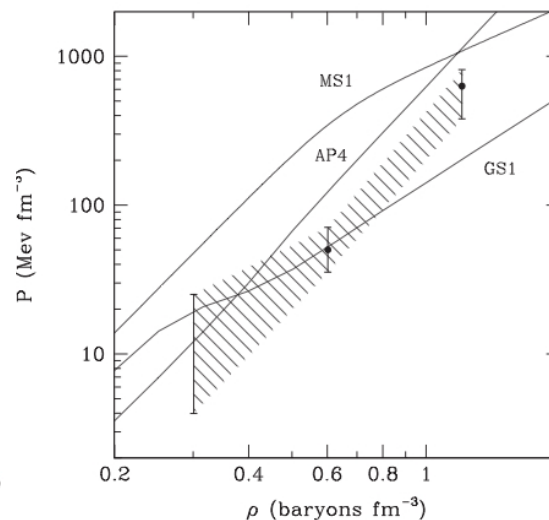
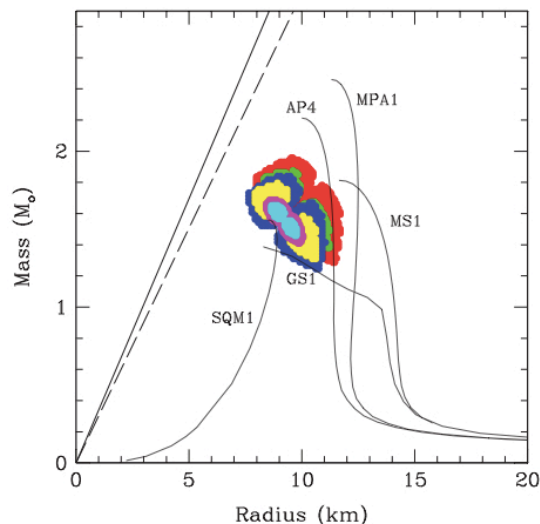


# Astrophysical determination of the Nuclear EoS

✧ Piecewise polytropic EoS  
above  $\rho_0$  from mass-radius  
relation of 3 type-I X-ray  
bursts

❖ SLy below  $\rho_0$

❖ Piecewise polytropic above  $\rho_0$



$$\rho_{i-1} < \rho \leq \rho_i, \quad \varepsilon = \alpha_i \rho + \beta_i \rho^{\Gamma_i}, \quad P = (\Gamma_i - 1) \beta_i \rho^{\Gamma_i}$$

$\log P_0 (0.37\rho_{\text{ns}})$	$\log P_1 (1.85\rho_{\text{ns}})$	$\log P_2 (3.7\rho_{\text{ns}})$	$\log P_3 (7.4\rho_{\text{ns}})$
-0.64	[0.6-1.4]	$1.70^{+0.15}_{-0.15}$	$2.8^{+0.1}_{-0.2}$



F. Ozel & D. Psaltis, PRD 80, 103003 (2009)

F. Ozel, G. Baym & T. Guver, PRD 82, 101301(R) (2010)

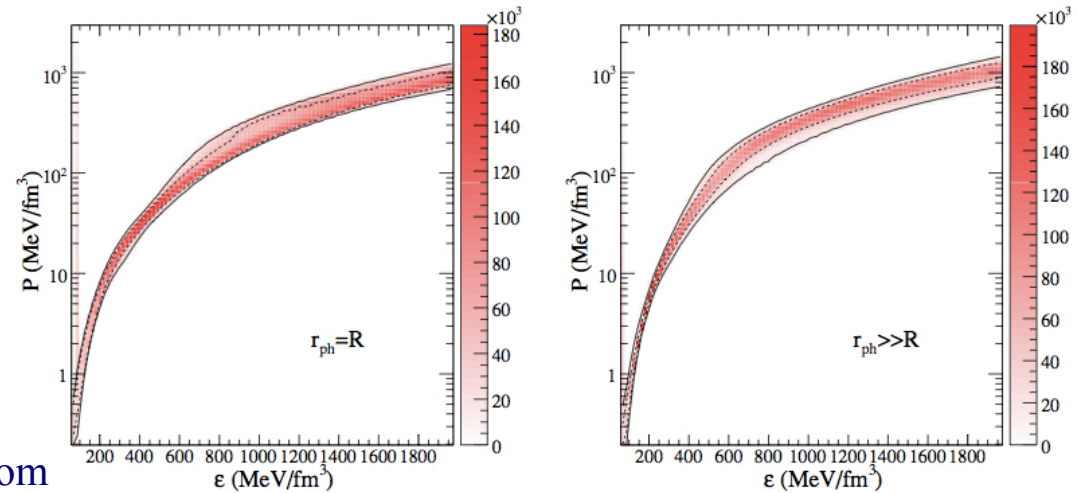
# Astrophysical determination of the Nuclear EoS

✧ Nuclear parameters determined in a Bayesian data analysis of:

- ❖ 3 type-I X-ray burst
- ❖ 3 transient low mass X-ray binaries
- ❖ Cooling of 1 isolated NS, RX J1856-3754

Parameters in the range expected from nuclear systematics & lab. experiments

Quantity	Lower Limit	Upper Limit
$K$ (MeV)	180	280
$K'$ (MeV)	-1000	-200
$S_v$ (MeV)	28	38
$\gamma$	0.2	1.2
$n_1$ (fm $^{-3}$ )	0.2	1.5
$n_2$ (fm $^{-3}$ )	0.2	2.0
$\epsilon_1$ (MeV fm $^{-3}$ )	150	600
$\epsilon_2$ (MeV fm $^{-3}$ )	$\epsilon_1$	1600



$$\epsilon = n_B \left\{ m_B + B + \frac{K}{18}(u-1)^2 + \frac{K'}{162}(u-1)^3 + (1-2x)^2 [S_k u^{2/3} + S_p u^\gamma] + \frac{3}{4} \hbar c x (3\pi^2 n_{bx})^{1/3} \right\}$$



# Theoretical Approaches to the Nuclear EoS

## Phenomenological approaches

Based on effective density-dependent interactions with parameters adjusted to reproduce nuclear observables and compact star properties

- ❖ Liquid drop type: BPS, BBP, LS, OFN
- ❖ Thomas-Fermi: Shen
- ❖ HF: NV, Sk, BSk, PAL, RMF, RHF, QMC
- ❖ Statistical models: HWN, RG, HS



I apologize for all  
those approaches  
I have missed

## Microscopic ab-initio approaches

Based on two- & three-body realistic interactions. The EoS is obtained by “solving” the complicated many-body problem

- ❖ Variational: APS, CBF, FHNC, LOVC
- ❖ Monte-Carlo: VMC, DMC, GFMC, AFDMC
- ❖ Diagrammatic: BBG (BHF), SCGF
- ❖ RG methods:  $V_{\text{low } k}$  & SRG from  $\chi\text{EFT}$  potentials
- ❖ DBHF

## Upper limit of the Maximum Mass

$M_{\max}$  depends mainly on the behaviour of EoS,  $P(\epsilon)$ , at high densities. Any realistic EoS must satisfy two conditions:

$$\blacksquare \text{ Causality: } \frac{dP}{d\rho} \leq c^2 \quad \blacksquare \text{ Stability: } \frac{dP}{d\rho} > 0$$

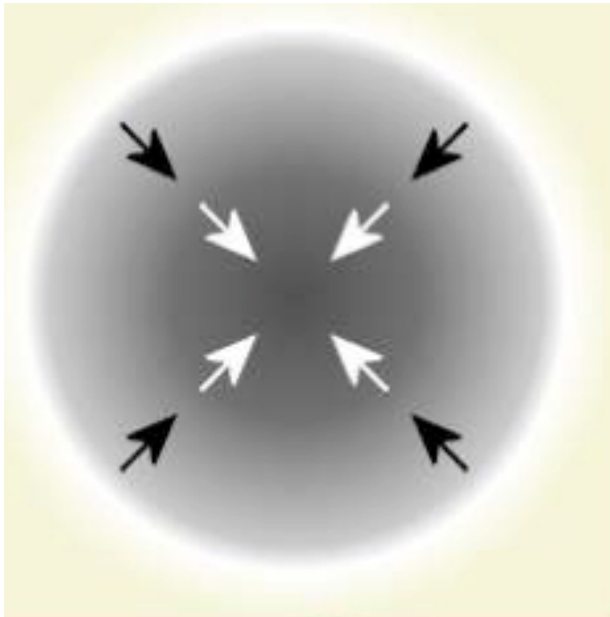
If the EoS is known up to  $\rho_r$ , these conditions imply:

$$M_{\max} \leq 3M_{\odot} \left( \frac{5 \times 10^{14} \text{ g / cm}^3}{\rho_r} \right)^{1/2}$$

If rotation is taken into account  $M_{\max}$  can increase up to 20%:

$$M_{\max} \leq 3.89M_{\odot} \left( \frac{5 \times 10^{14} \text{ g / cm}^3}{\rho_r} \right)^{1/2}$$

# Estimation of Neutron Star Mass & Radius



Imagine that a neutron star is:

- ✓ a sphere of uniform density
- ✓ made only of neutrons
- ✓ in addition to the nuclear force neutrons feel also the gravity

Idea: use Bethe-Weizsäcker semi-empirical mass formula including the gravitational force

$$B(Z, A) = a_v A - a_s A^{2/3} - a_{coul} \frac{Z^2}{A^{1/3}} - a_{sim} \frac{(Z - N)^2}{A} + \delta a_p A^{-1/2}$$

Only Neutrons ( $Z=0$ ) + Gravitational Energy (sphere with  $M=Nm_n$  &  $R$ )

$$B(Z = 0, A = N) = a_v N - a_s N^{2/3} - a_{sim} N + \delta a_p N^{-1/2} + \frac{3}{5} \frac{G(Nm_n)^2}{R}$$

Since  $N > N^{2/3}$  &  $N^{-1/2}$

$$B(Z = 0, A = N) \approx (a_v - a_{sim}) N + \frac{3}{5} \frac{Gm_n^2}{r_0} N^{5/3}$$

$$R = r_0 N^{1/3} = 1.15 \times 10^{-15} N^{1/3} m$$

The **minimum number of neutrons** needed to **bound gravitationally** is obtained imposing

$$B > 0$$



The  $B > 0$  tell us that:

$$(a_v - a_{sim})N + \frac{3}{5} \frac{Gm_n^2}{r_0} N^{5/3} > 0 \Rightarrow N > \left( \frac{5 (a_{sim} - a_v) r_0}{3 Gm_n^2} \right)^{3/2}$$

Using the values:

$$a_v = 16 \text{ MeV}, \quad a_{sim} = 30 \text{ MeV}, \quad G = 6.707 \times 10^{-39} \hbar c \left( \frac{c^4}{\text{GeV}^2} \right), \quad m_n = 0.939 \frac{\text{GeV}}{c^2}$$

We finally arrive to:

$$N \sim 10^{56} - 10^{57} \quad M \sim 1 M_{\odot} \quad R \sim 10 \text{ km}$$

Which gives an average density of:

$$\rho \sim 10^{14} - 10^{15} \text{ g/cm}^3$$

$$N \sim 10^{56} - 10^{57}$$

$$M \sim 1 M_{\odot}$$

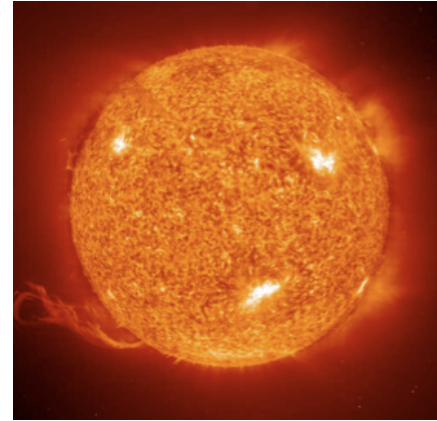
A neutron star  
is a kind of **GIANT  
ATOMIC NUCLEUS**  
in which particles are  
gravitationally bound



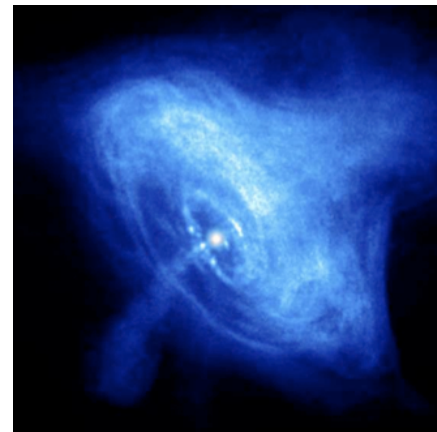
$$R \sim 10 \text{ km}$$

$$\rho \sim 10^{14} - 10^{15} \text{ g/cm}^3$$

A neutron star  
has a mass similar to that  
of the Sun, but with a  
radius about  
70.000 smaller !!!



Radius: ~ 700.000 km  
Mass:  $1.989 \times 10^{30}$  kg



Radius ~ 10 km  
Mass ~  $1.989 \times 10^{30}$  kg

# How to Measure Neutron Star Masses

Use Doppler variations in spin period to measure orbital velocity changes along the line-of-sight

- 5 Keplerian parameters can normally be determined:

$P, a \sin i, \epsilon, T_0$  &  $\omega$

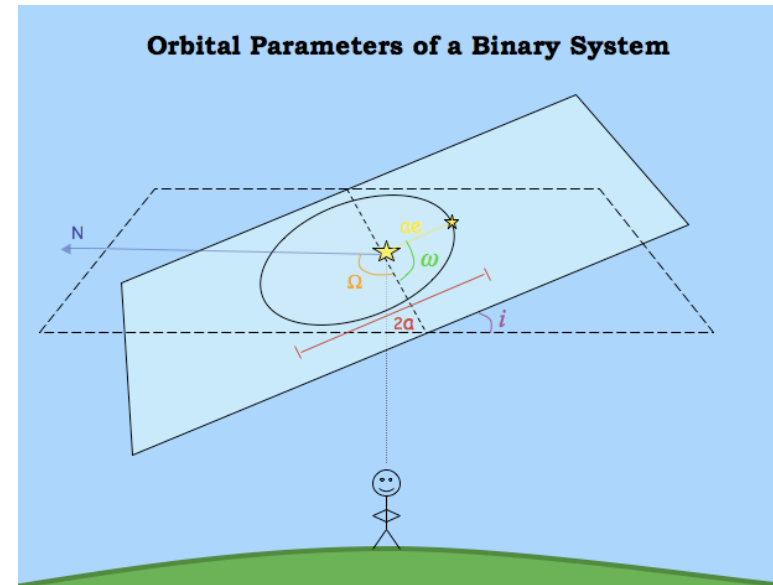
- 3 unknowns:  $M_1, M_2, i$

Kepler's 3<sup>rd</sup> law

$$\frac{G(M_1 + M_2)}{a^3} = \left(\frac{2\pi}{P}\right)^2 \rightarrow$$

$$f(M_1, M_2, i) \equiv \frac{(M_2 \sin i)^3}{(M_1 + M_2)^2} = \frac{Pv^3}{2\pi G}$$

mass function



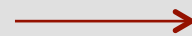
# In few cases small deviations from Keplerian orbit due to GR effects can be detected

Measure of at least 2 post-Keplerian parameters



High precision NS mass determination

$$\dot{\omega} = 3T_{\otimes}^{2/3} \left( \frac{P_b}{2\pi} \right)^{-5/3} \frac{1}{1-\varepsilon} (M_p + M_c)^{2/3}$$



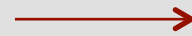
Periastron precession

$$\gamma = T_{\otimes}^{2/3} \left( \frac{P_b}{2\pi} \right)^{1/3} \varepsilon \frac{M_c (M_p + 2M_c)}{(M_p + M_c)^{4/3}}$$



Time dilation and grav. redshift

$$r = T_{\otimes} M_c$$



Shapiro delay “range”

$$s = \sin i = T_{\otimes}^{-1/3} \left( \frac{P_b}{2\pi} \right)^{-2/3} x \frac{(M_p + M_c)^{2/3}}{M_c}$$



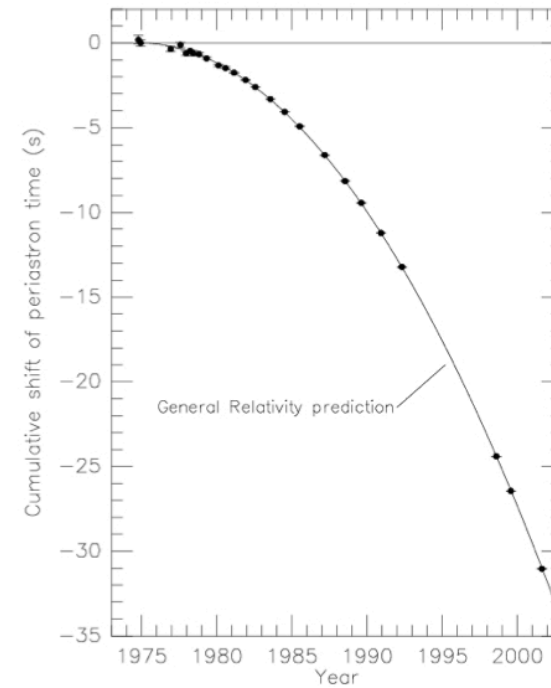
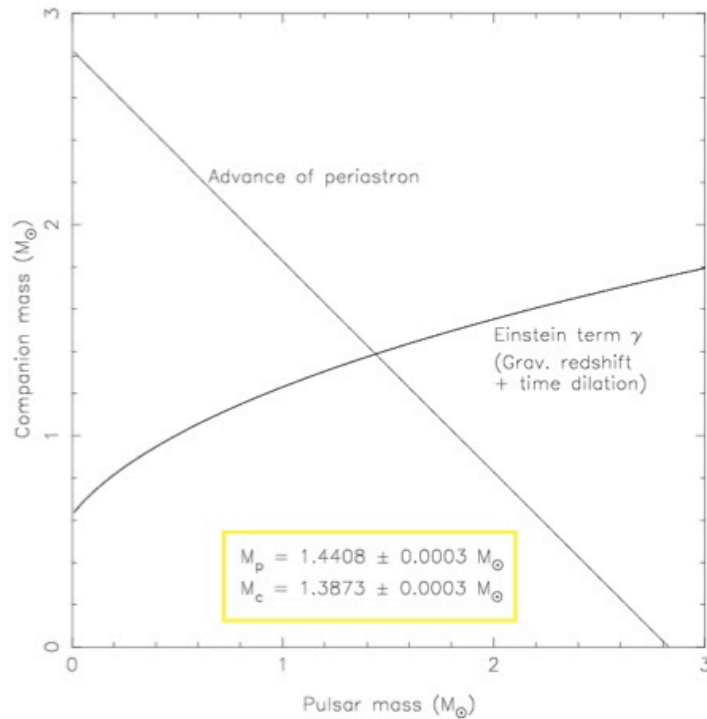
Shapiro delay “shape”

$$\dot{P}_b = -\frac{192\pi}{5} T_{\otimes}^{5/3} \left( \frac{P_b}{2\pi} \right)^{-5/3} f(\varepsilon) \frac{M_p M_c}{(M_p + M_c)^{1/3}}$$



Orbit decay due to GW emission

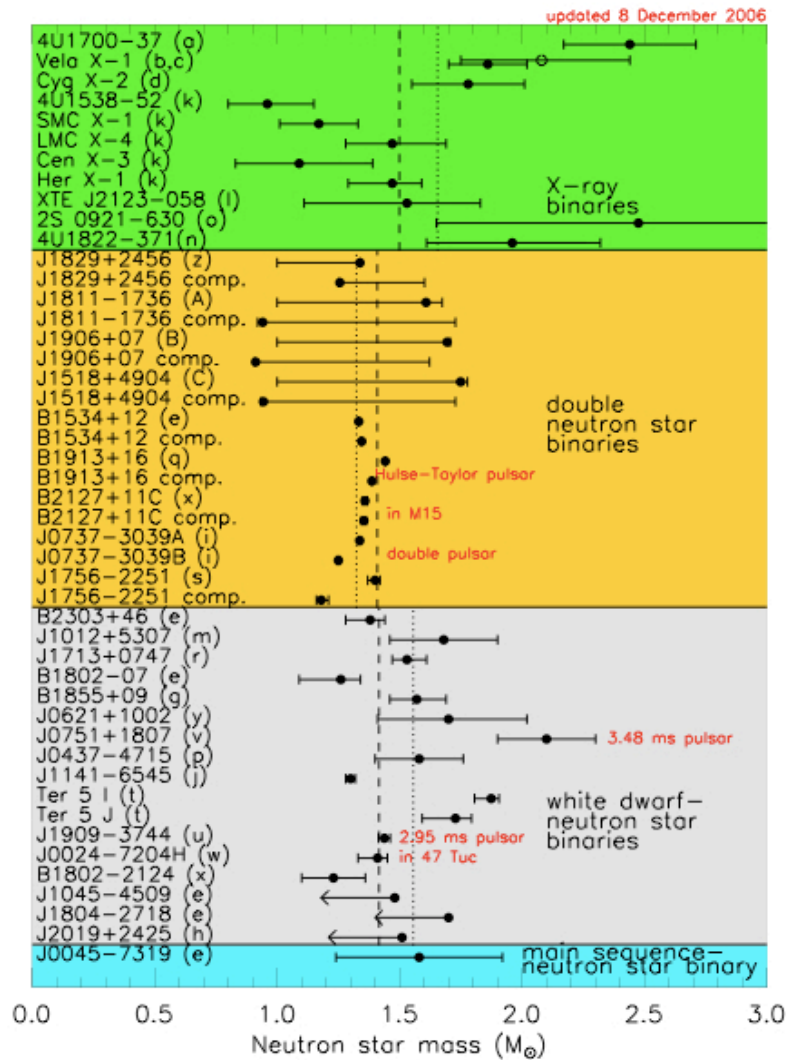
# An example: the mass of the Hulse-Taylor pulsar (PSR J1913+16)



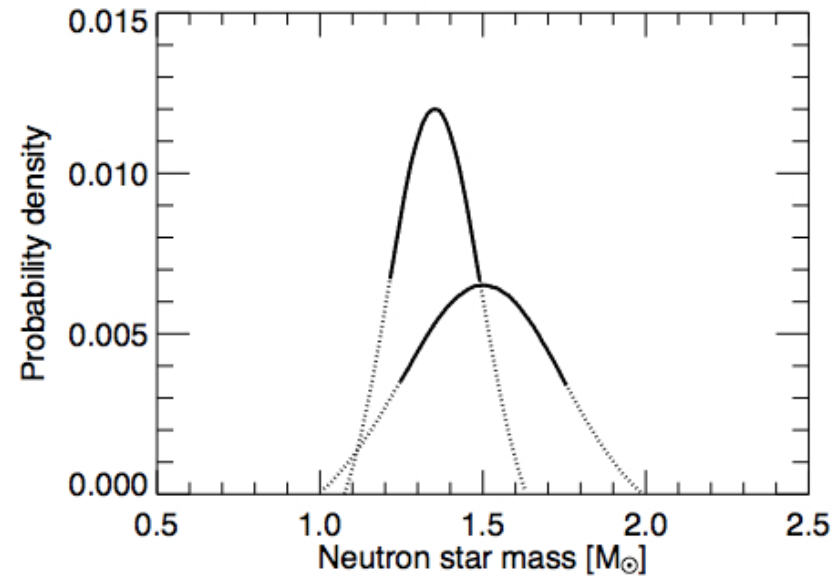
Parameter	Value
Orbital period $P_b$ (d)	0.322997462727(5)
Projected semi-major axis $x$ (s)	2.341774(1)
<u>Eccentricity <math>e</math></u>	<u>0.6171338(4)</u>
Longitude of periastron $\omega$ (deg)	226.57518(4)
Epoch of periastron $T_0$ (MJD)	46443.99588317(3)
Advance of periastron $\dot{\omega}$ (deg yr $^{-1}$ )	4.226607(7)
Gravitational redshift $\gamma$ (ms)	4.294(1)
Orbital period derivative $(\dot{P}_b)^{obs}$ ( $10^{-12}$ )	-2.4211(14)



# Measured Neutron Star Masses (up to ~ 2006-2008)



(Lattimer & Prakash 2007)



up to ~ 2006-2008 any valid  
 EoS should predict

$$M_{\max} [EoS] > 1.4 - 1.5 M_{\odot}$$

N.B. I will comment on more recent measurements later when talking about the “hyperon problem”

# Limits on the Neutron Star Radius

The radius of a neutron star with mass  $M$  cannot be arbitrarily small

General Relativity:  
a Neutron Star is not a  
Black Hole

$$R > \frac{2GM}{c^2}$$

Finite Pressure:  
Neutron Star matter cannot  
be arbitrarily compressed

$$R > \frac{9}{4} \frac{GM}{c^2}$$

Causality:  
speed of sound must  
be smaller than  $c$

$$R > 2.9 \frac{GM}{c^2}$$

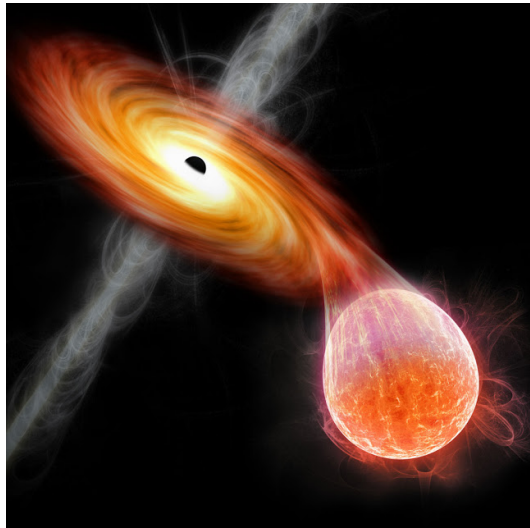


# How to measure Neutron Star Radii

Radii are very difficult to measure because NS:

- ✧ are very small ( $\sim 10$  km)
- ✧ are far from us (e.g., the closest NS, RX J1856.5-3754, is at  $\sim 400$  ly)

A possible way to measure it is to use the thermal emission of low mass X-ray binaries:



NS radius can be obtained from

- ✧ Flux measurement + Stefan-Boltzmann's law
- ✧ Temperature (Black body fit+atmosphere model)
- ✧ Distance estimation (difficult)
- ✧ Gravitational redshift  $z$  (detection of absorption lines)

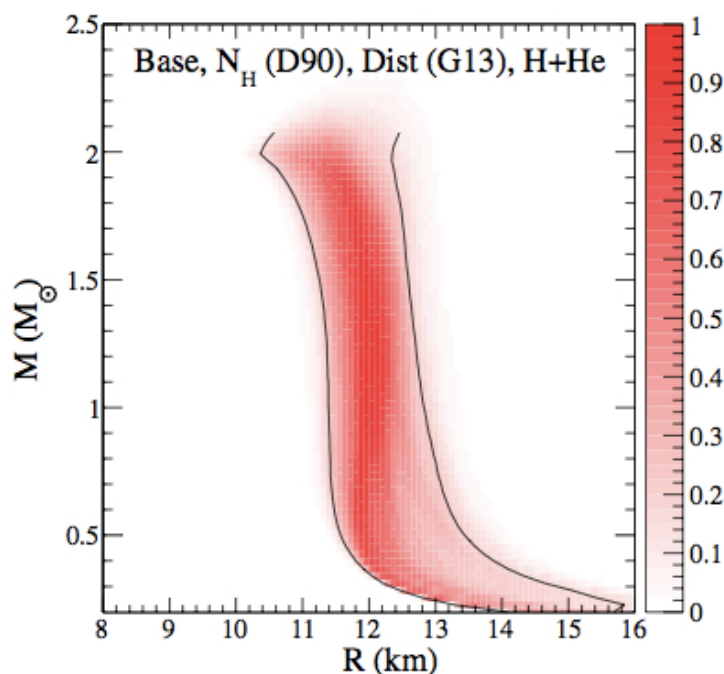
$$R_{\infty} = \sqrt{\frac{FD^2}{\sigma_{SB}T^4}} \rightarrow R_{NS} = \frac{R_{\infty}}{1+z} = R_{\infty} \sqrt{1 - \frac{2GM}{R_{NS}c^2}}$$

# Recent Estimations of Neutron Star Radii

The recent analysis of the thermal spectrum from 5 quiescent LMXB in globular clusters is still controversial



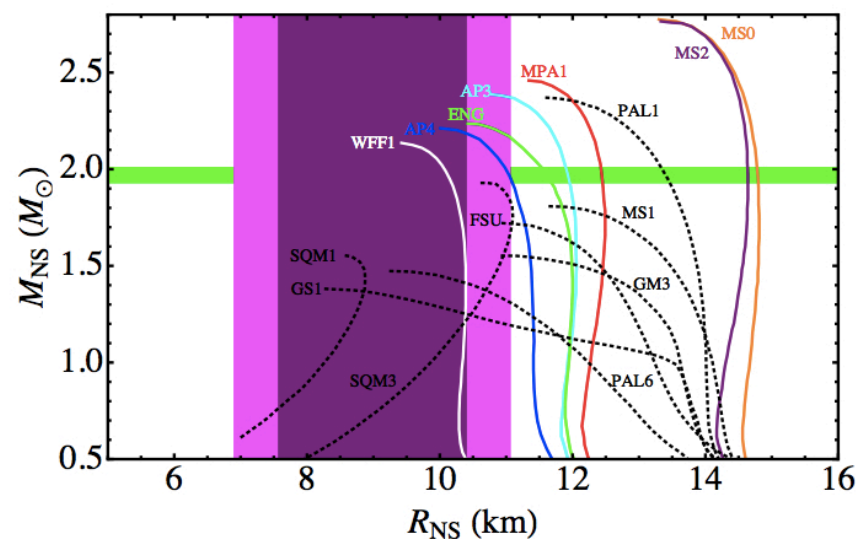
Steiner et al. (2013, 2014)



$$R = 12.0 \pm 1.4 \text{ km}$$



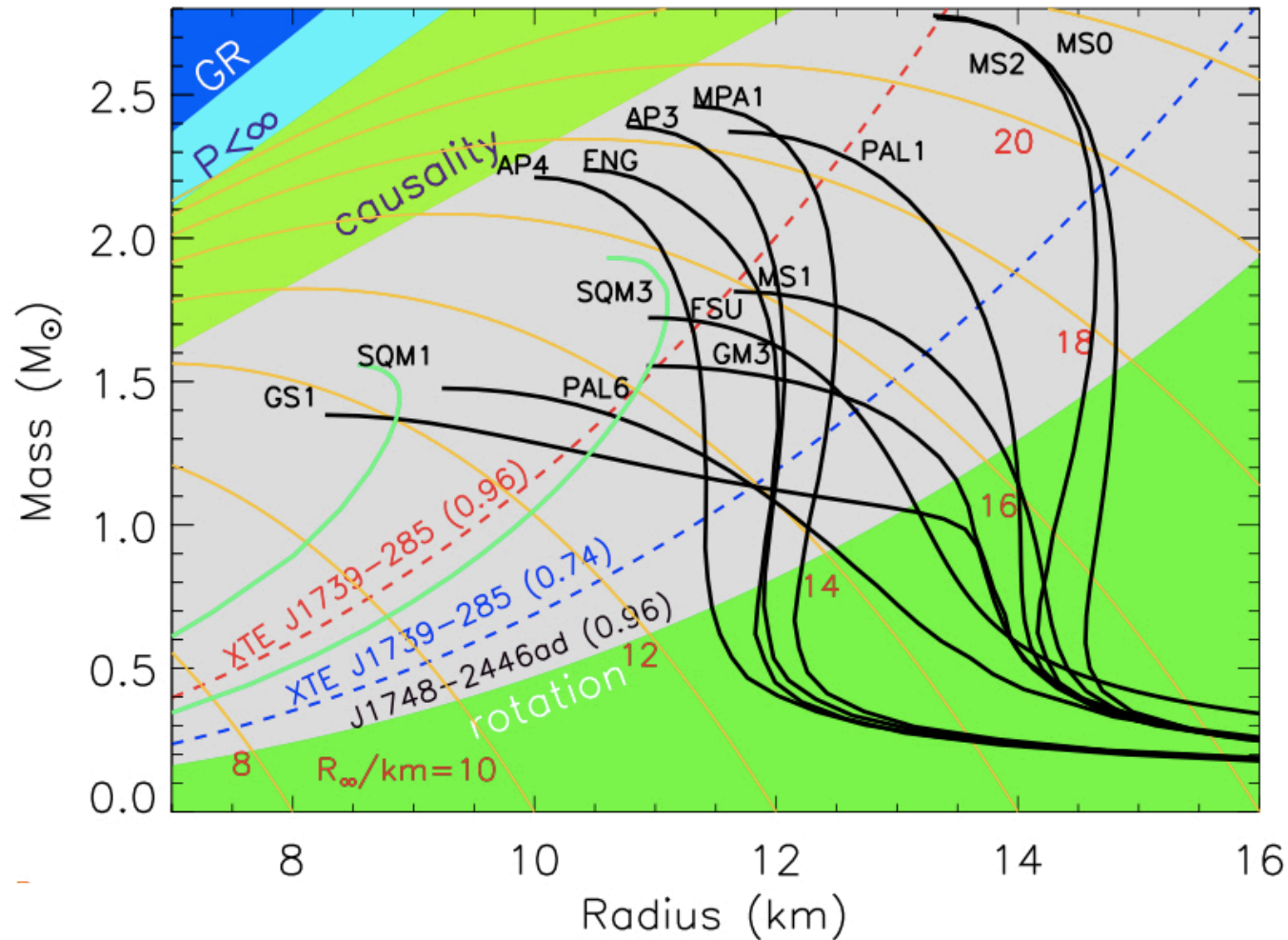
Guillot et al. (2013, 2014)



$$R = 9.1^{+1.3}_{-1.5} \text{ km} \text{ 2013 analysis}$$

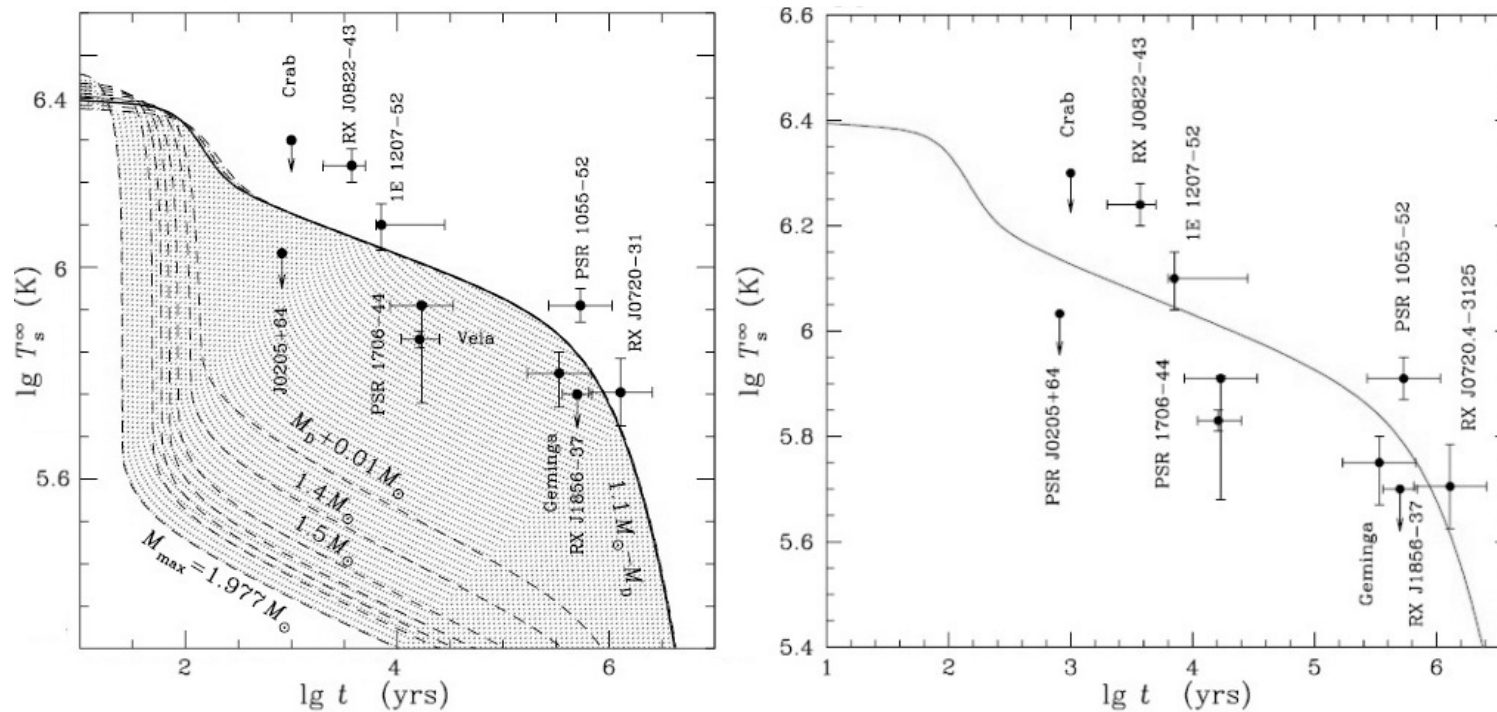
$$R = 9.4 \pm 1.2 \text{ km} \text{ 2014 analysis}$$

# Limits of the Mass & Radius of a Neutron Star



# Thermal Evolution of Neutron Stars

Information, complementary to that from mass & radius, can be also obtained from the measurement of the **temperature (luminosity) of neutron stars**



# Neutron Star Cooling in a Nutshell



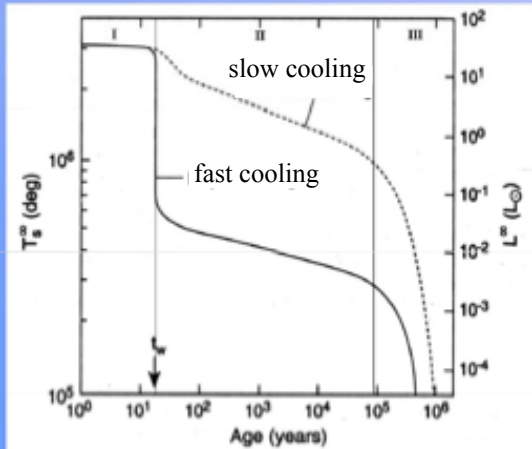
## Two cooling regimes

Slow

Low NS mass

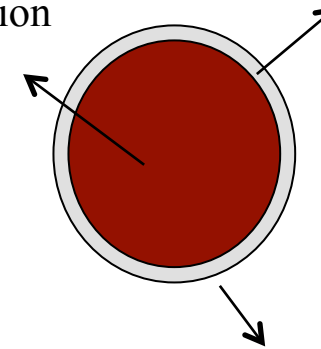
Fast

High NS mass



- I. Core relaxation epoch
- II. Neutrino cooling epoch
- III. Photon cooling epoch

Core cools by  
neutrino emission

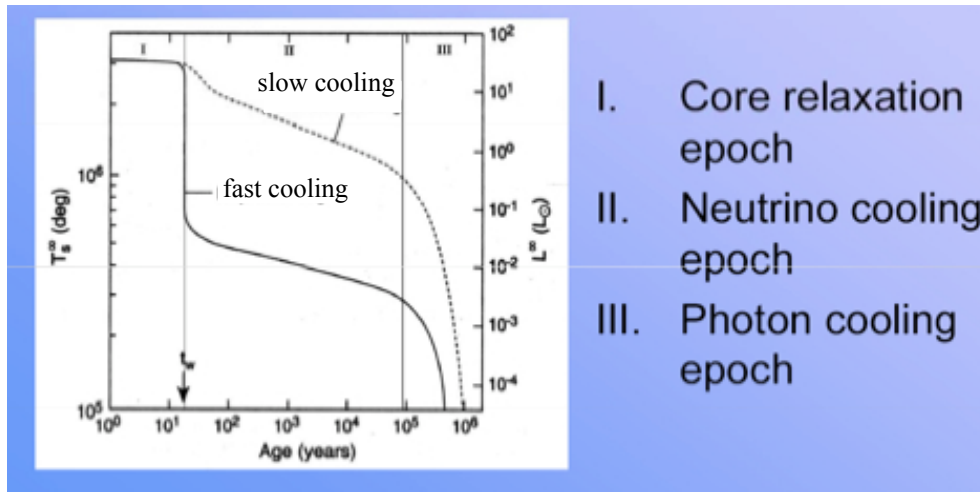


Surface photon emission  
dominates at  $t > 10^6$  yrs

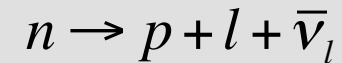
$$\frac{dE_{th}}{dt} = C_v \frac{dT}{dt} = -L_\gamma - L_\nu + H$$

- ✓  $C_v$ : specific heat
- ✓  $L_\gamma$ : photon luminosity
- ✓  $L_\nu$ : neutrino luminosity
- ✓  $H$ : “heating”

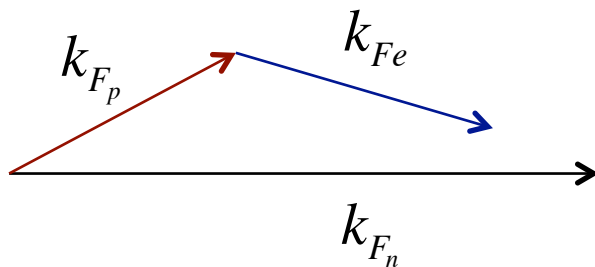
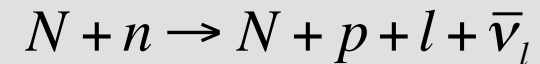
# Neutron Star Cooling & Symmetry Energy



- Fast: e.g., Direct URCA



- Slow: e.g., Modified URCA



Direct URCA cannot occur unless  $x_p > 11\%-15\%$

Larger Symmetry Energy  $\rightarrow$  Larger  $x_p \rightarrow$  Earlier onset of Direct URCA

$$\mu_n - \mu_p = 4(1 - 2x_p)S_2(\rho) = \mu_l - \mu_{\nu_l} \Rightarrow \frac{x_p}{1 - 2x_p} = \frac{4S_2(\rho)}{\hbar c(3\pi^2\rho)^{1/3}}$$

# Neutrino Emission

Name	Process	Emissivity (erg cm <sup>-3</sup> s <sup>-1</sup> )	
Modified Urca cycle (neutron branch)	$n + n \rightarrow n + p + e^- + \bar{\nu}_e$	$\sim 2 \times 10^{21} R T_9^8$	Slow
	$n + p + e^- \rightarrow n + n + \nu_e$		
Modified Urca cycle (proton branch)	$p + n \rightarrow p + p + e^- + \bar{\nu}_e$	$\sim 10^{21} R T_9^8$	Slow
	$p + p + e^- \rightarrow p + n + \nu_e$		
Bremsstrahlung	$n + n \rightarrow n + n + \nu + \bar{\nu}$	$\sim 10^{19} R T_9^8$	Slow
	$n + p \rightarrow n + p + \nu + \bar{\nu}$		
	$p + p \rightarrow p + p + \nu + \bar{\nu}$		
Cooper pair formations	$n + n \rightarrow [nn] + \nu + \bar{\nu}$	$\sim 5 \times 10^{21} R T_9^7$	Medium
	$p + p \rightarrow [pp] + \nu + \bar{\nu}$	$\sim 5 \times 10^{19} R T_9^7$	
Direct Urca cycle (nucleons)	$n \rightarrow p + e^- + \bar{\nu}_e$	$\sim 10^{27} R T_9^6$	Fast
	$p + e^- \rightarrow n + \nu_e$		
Direct Urca cycle ( $\Lambda$ hyperons)	$\Lambda \rightarrow p + e^- + \bar{\nu}_e$	$\sim 10^{27} R T_9^6$	Fast
	$p + e^- \rightarrow \Lambda + \nu_e$		
Direct Urca cycle ( $\Sigma^-$ hyperons)	$\Sigma^- \rightarrow n + e^- + \bar{\nu}_e$	$\sim 10^{27} R T_9^6$	Fast
	$n + e^- \rightarrow \Sigma^- + \nu_e$		
$\pi^-$ condensate	$n + \langle \pi^- \rangle \rightarrow n + e^- + \bar{\nu}_e$	$\sim 10^{26} R T_9^6$	Fast
$K^-$ condensate	$n + \langle K^- \rangle \rightarrow n + e^- + \bar{\nu}_e$	$\sim 10^{25} R T_9^6$	Fast

Anything beyond just neutrons & protons results in an **enhancement**  
of the neutrino emission

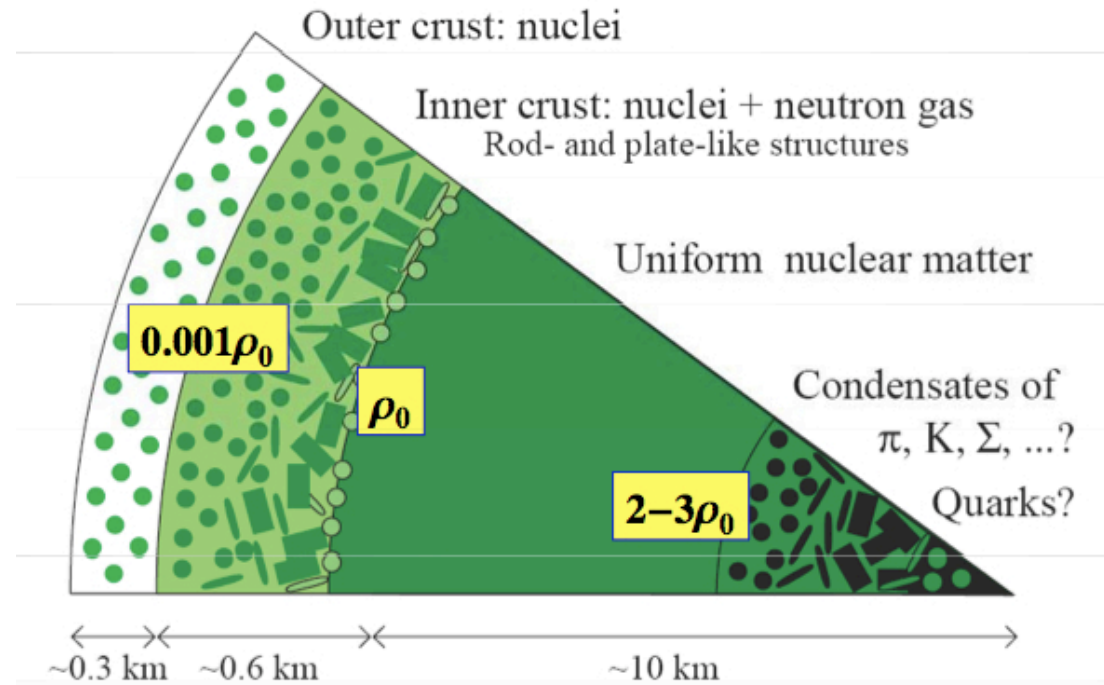
# Anatomy of a Neutron Star

Equilibrium composition  
determined by

- ✓ Charge neutrality

$$\sum_i q_i \rho_i = 0$$

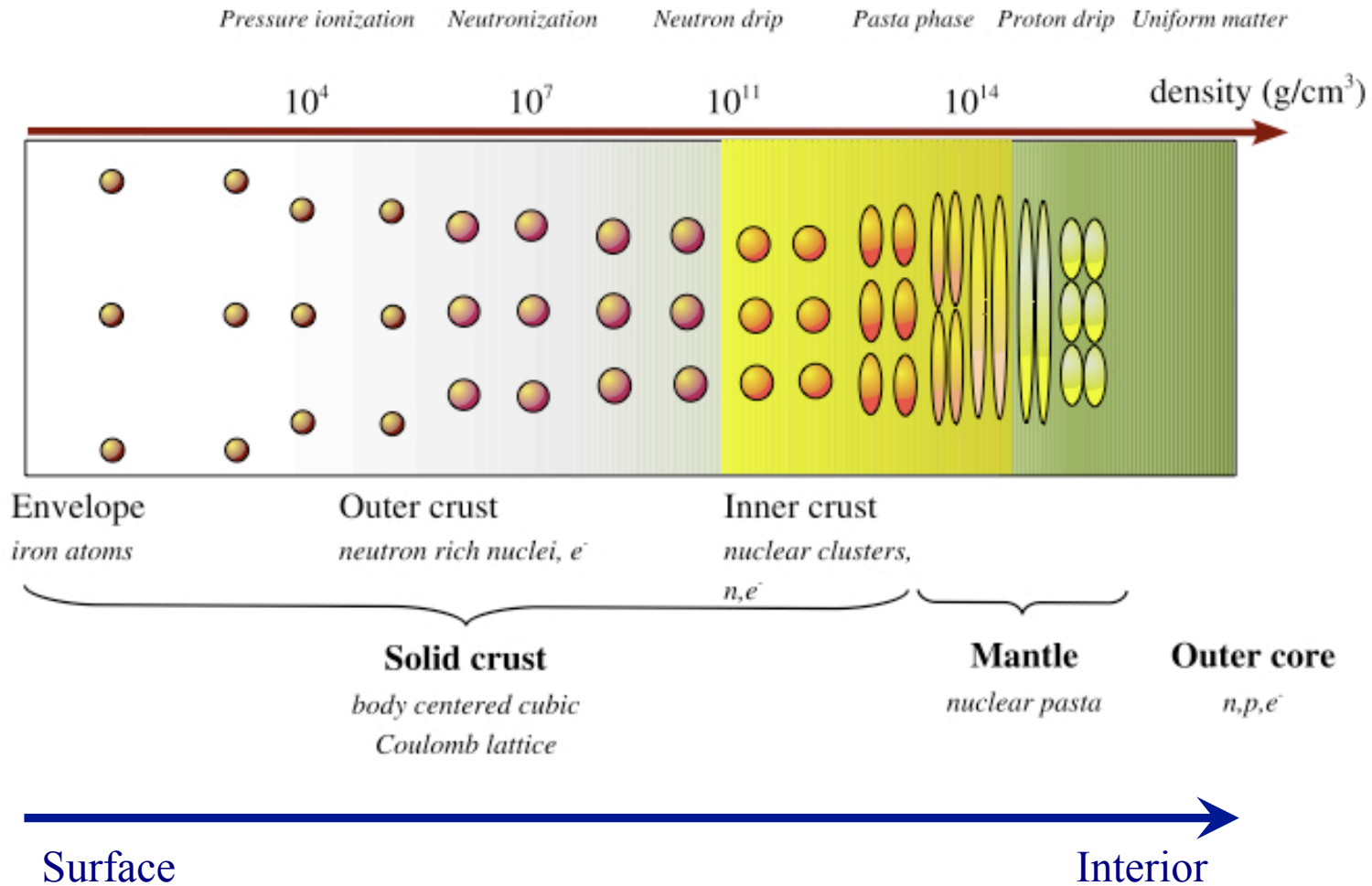
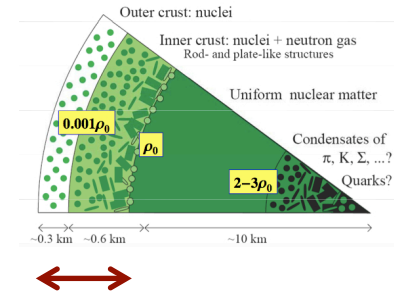
- ✓ Equilibrium with respect to weak interacting processes



$$\begin{array}{l}
 b_1 \rightarrow b_2 + l + \bar{\nu}_l \\
 b_2 + l \rightarrow b_1 + \nu_l
 \end{array}
 \longrightarrow
 \mu_i = b_i \mu_n - q_i (\mu_e - \mu_{\nu_e}), \quad \mu_i = \frac{\partial \varepsilon}{\partial \rho_i}$$



# Crust of a Neutron Star

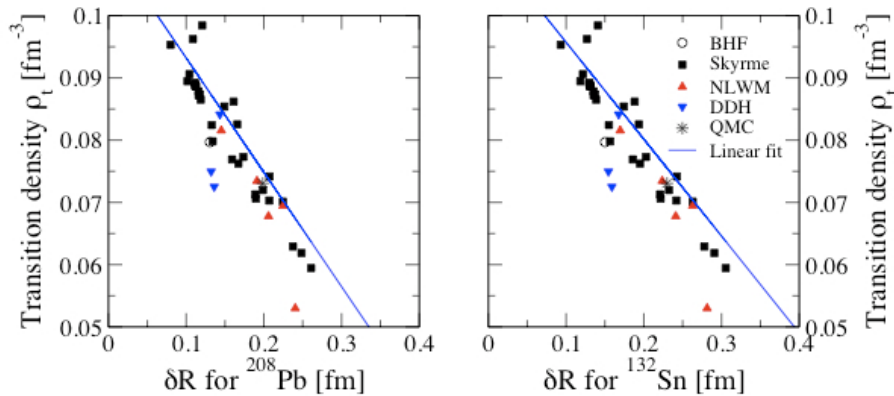
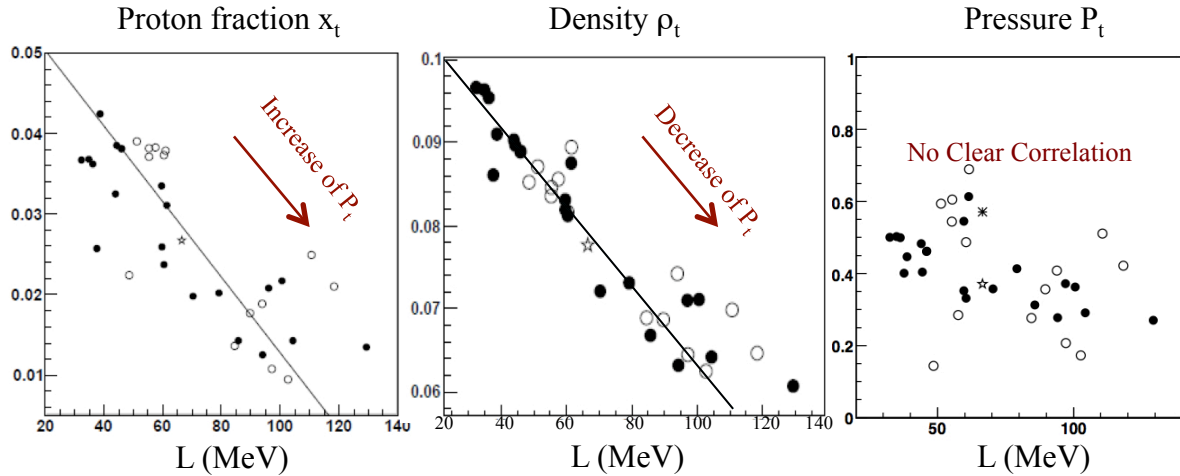
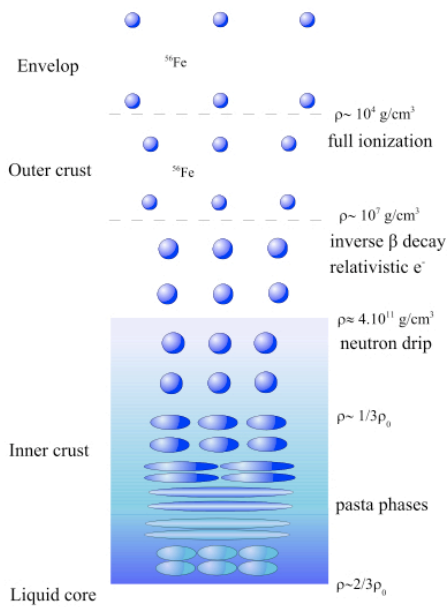


# Crust-core Transition & Symmetry Energy

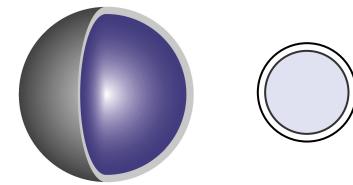
$$P(\rho, \beta) = \frac{\rho^2}{3\rho_0} \left( L\beta^2 + (K_0 + K_{sym}\beta^2) \frac{\rho - \rho_0}{3\rho_0} + \dots \right)$$

surface

interior



Inverse correlation between  $\delta R$  and  $\rho_t$   
(Horowitz & Piekarewicz)



Neutron Star

Heavy nucleus

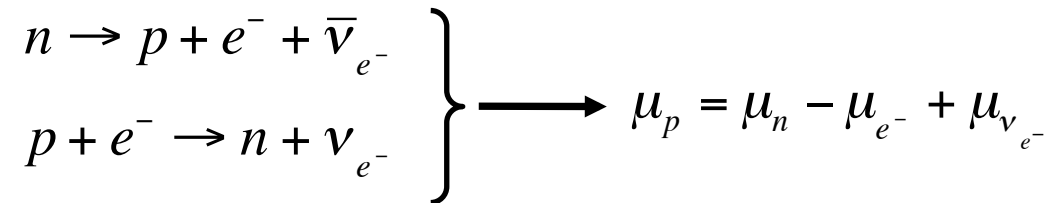
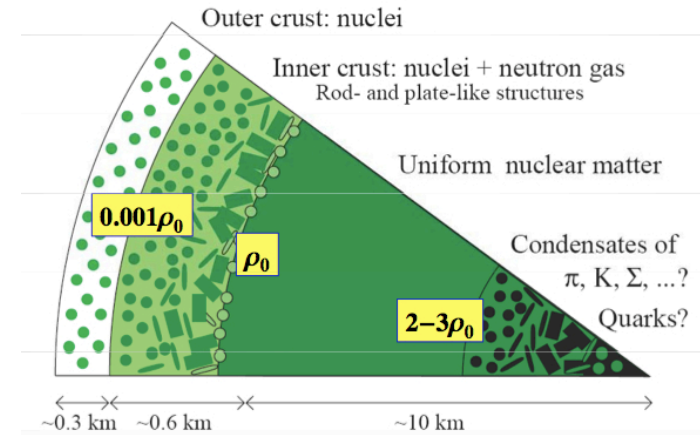
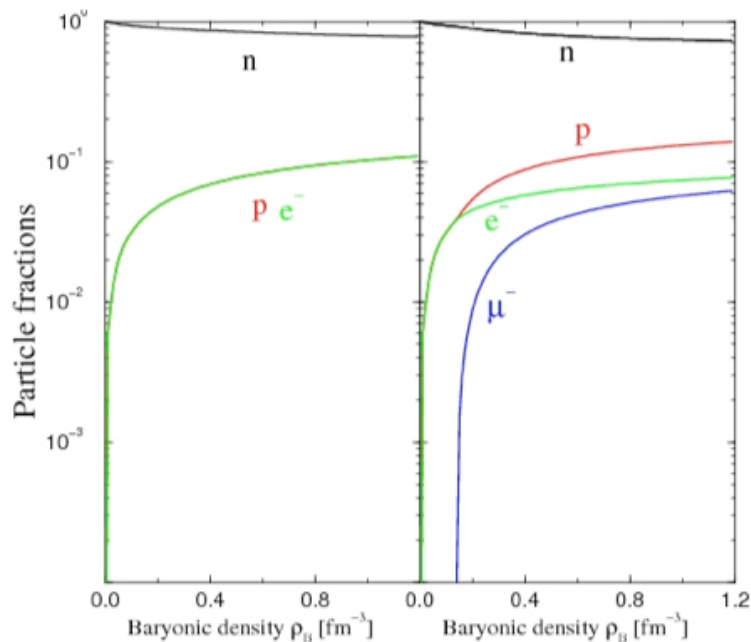
Crust & Neutron Skin  
made out of neutron  
rich matter at similar  
densities



Both governed by EoS  
at  $\rho < \rho_0$  (particularly by  
 $S_2(\rho)$  & its derivatives)

# External Core of a Neutron Star

The external core of a neutron star is mainly a fluid of neutron-rich matter in equilibrium with respect to weak interaction processes ( **$\beta$ -stable matter**)



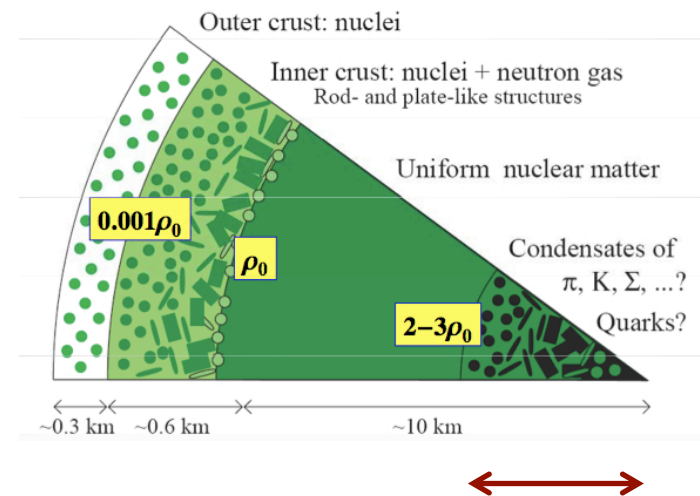
# Internal Core of a Neutron Star

Since:

- ✧ The value of the central density is very high:  $\rho_c \sim (4-8)\rho_0$

$$(\rho_0 = 0.17 \text{ fm}^{-3} = 2.8 \times 10^{14} \text{ g/cm}^3)$$

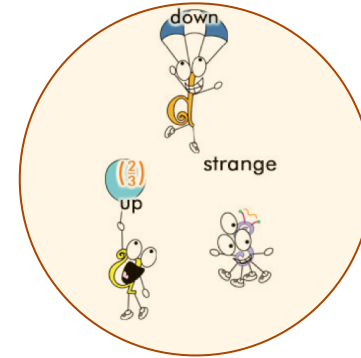
- ✧ Nucleon chemical potential increases rapidly with the density  $\rho$



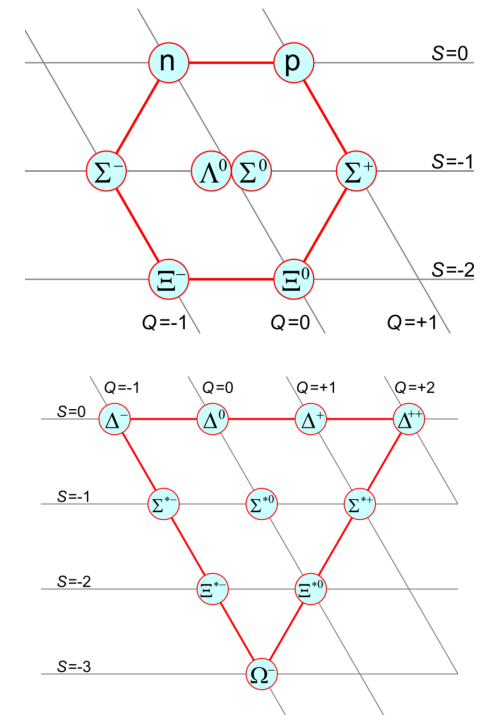
The presence of exotic degrees of freedom is expected in the Neutron Star interior ( $\pi$ , K<sup>-</sup> condensates, hyperons, quarks,...)

# What is a hyperon ?

✧ A **hyperon** is a baryon made of one , two or three **strange quarks**



Hyperon	Quarks	I(J <sup>P</sup> )	Mass (MeV)
$\Lambda$	uds	0(1/2 <sup>+</sup> )	1115
$\Sigma^+$	uus	1(1/2 <sup>+</sup> )	1189
$\Sigma^0$	uds	1(1/2 <sup>+</sup> )	1193
$\Sigma^-$	dds	1(1/2 <sup>+</sup> )	1197
$\Xi^0$	uss	1/2(1/2 <sup>+</sup> )	1315
$\Xi^-$	dss	1/2(1/2 <sup>+</sup> )	1321
$\Omega^-$	sss	0(3/2 <sup>+</sup> )	1672



# Hyperons in Neutron Stars

Hyperons in NS considered by many authors since the pioneering work of Ambartsumyan & Saakyan (1960)



## Phenomenological approaches

- ✧ **Relativistic Mean Field Models:** Glendenning 1985; Knorren et al. 1995; Shaffner-Bielich & Mishustin 1996, Bonano & Sedrakian 2012, ...
- ✧ **Non-relativistic potential model:** Balberg & Gal 1997
- ✧ **Quark-meson coupling model:** Pal et al. 1999, ...
- ✧ **Chiral Effective Lagrangians:** Hanauske et al., 2000
- ✧ **Density dependent hadron field models:** Hofmann, Keil & Lenske 2001



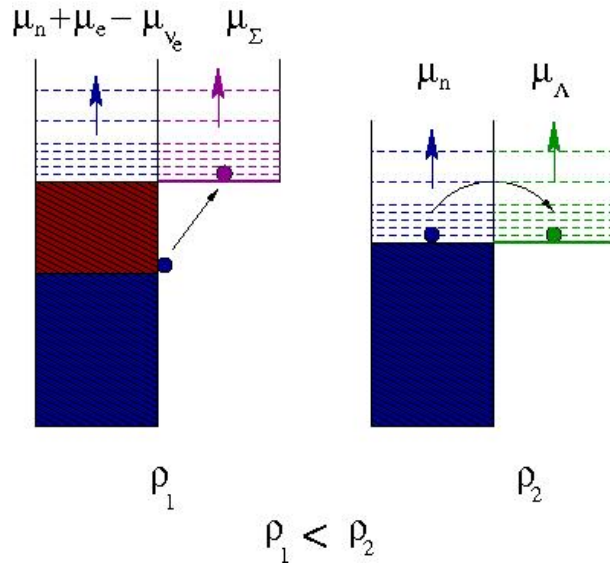
## Microscopic approaches

- ✧ **Brueckner-Hartree-Fock theory:** Baldo et al. 2000; I. V. et al. 2000, Schulze et al. 2006, I.V. et al. 2011, Burgio et al. 2011, Schulze & Rijken 2011
- ✧ **DBHF:** Sammarruca (2009), Katayama & Saito (2014)
- ✧  $V_{\text{low } k}$ : Djapo, Schaefer & Wambach, 2010
- ✧ **Quantum Monte Carlo:** Lonardonì et al., (2014)



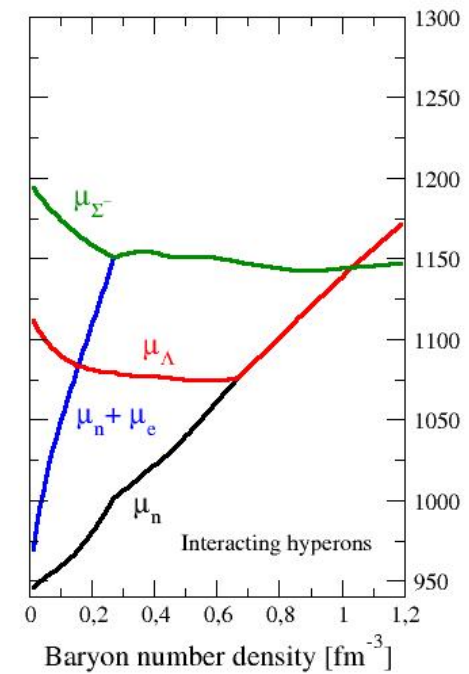
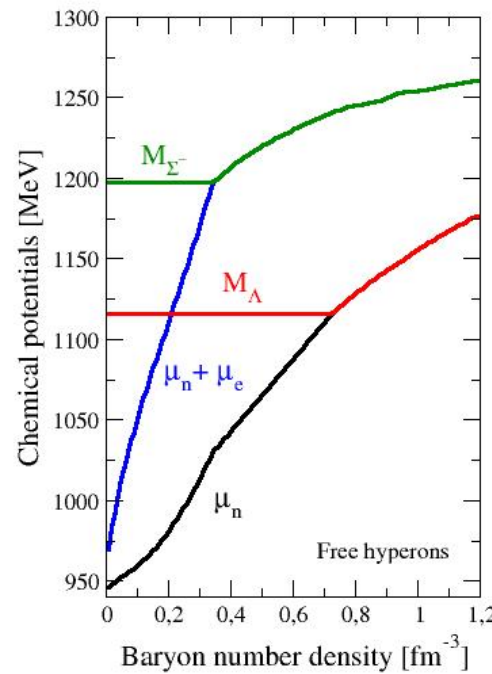
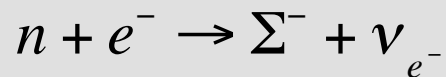
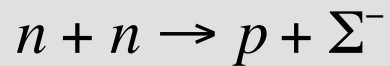
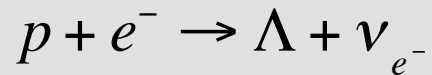
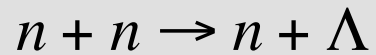
Sorry if I missed somebody

Hyperons are expected to appear in the core of neutron stars at  $\rho \sim (2-3)\rho_0$  when  $\mu_N$  is large enough to make the conversion of N into Y energetically favorable.

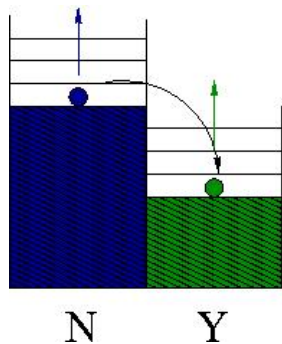
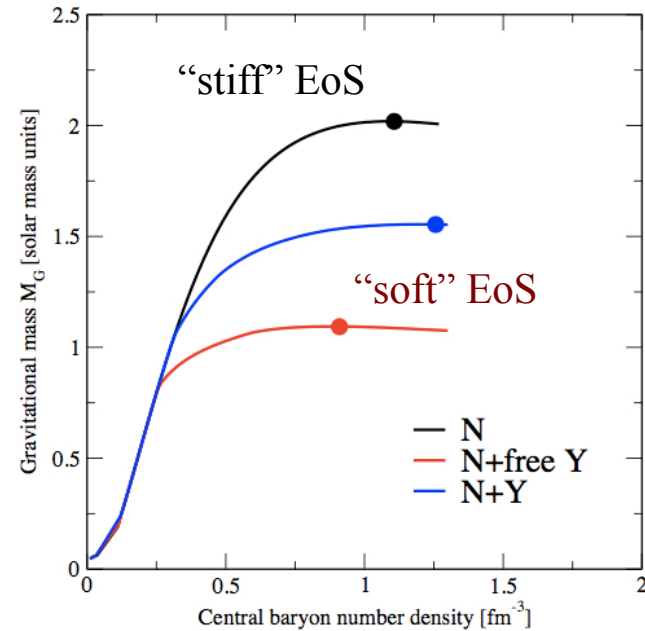
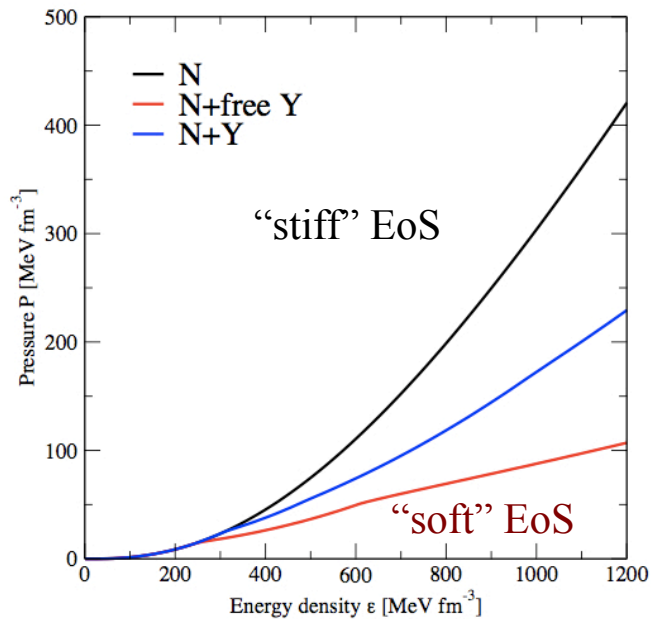


$$\mu_{\Sigma^-} = \mu_n + \mu_{e^-} - \mu_{\nu_{e^-}}$$

$$\mu_{\Lambda} = \mu_n$$



# Effect of Hyperons in the EoS and Mass of Neutron Stars

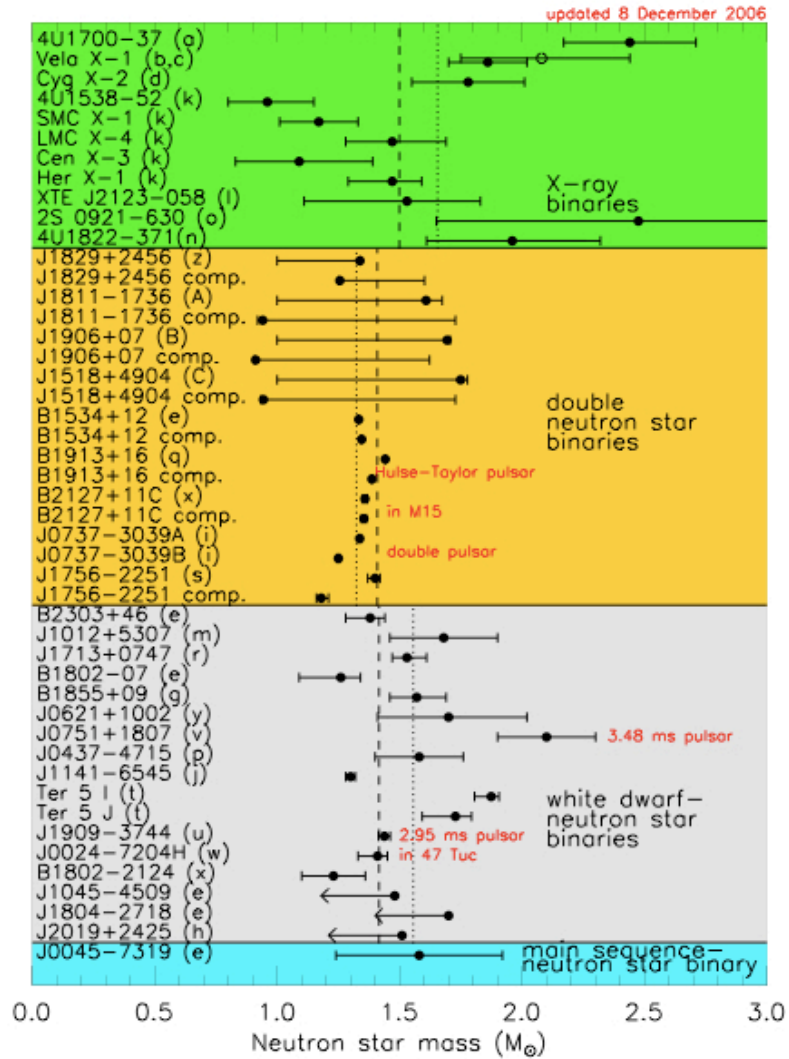


Relieve of Fermi pressure due to the appearance of hyperons →  
EoS softer → reduction of the mass



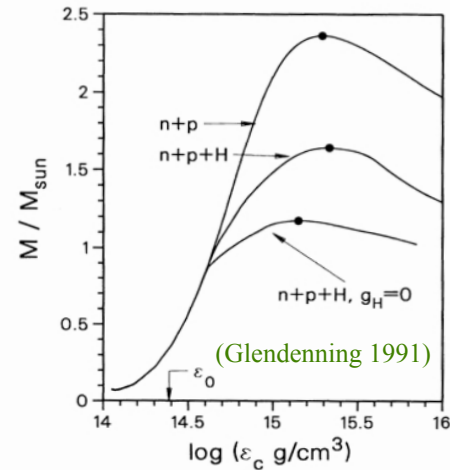
# Hyperons in NS

(up to ~ 2006-2008)

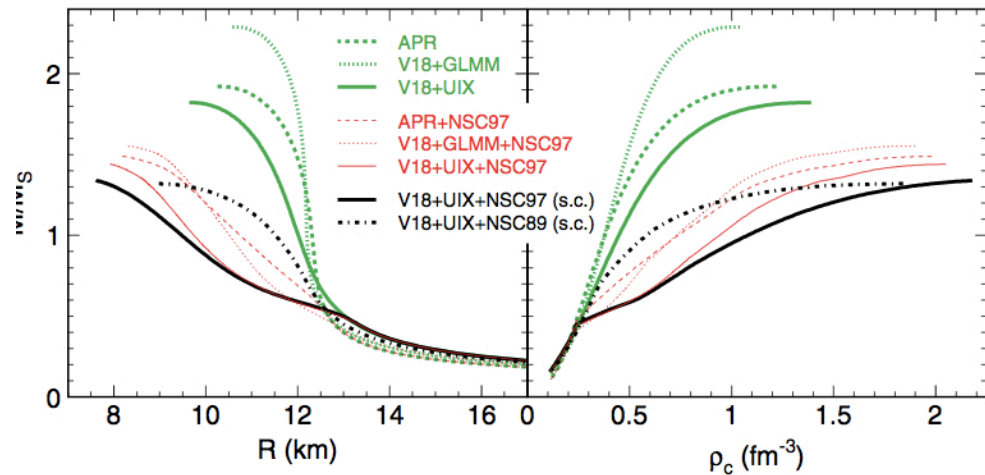


(Lattimer & Prakash 2007)

Phenomenological:  
 $M_{\max}$  compatible with 1.4-1.5  $M_{\odot}$



Microscopic :  $M_{\max} < 1.4-1.5 M_{\odot}$

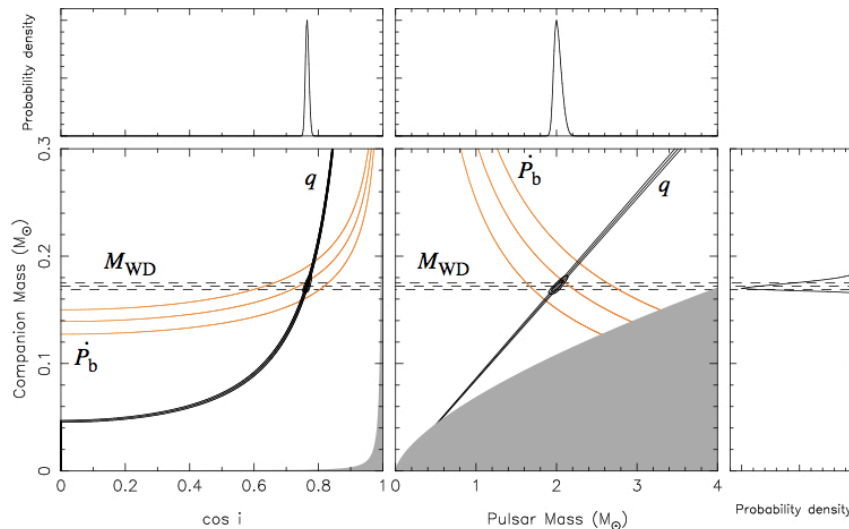
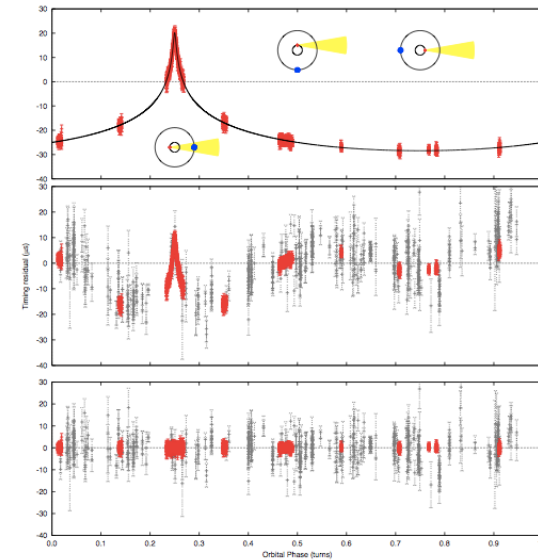


(Schulze, Polls, Ramos & IV 2006)

Recent measurements of high masses → life of hyperons (and theoreticians)  
 more difficult

■ PSR J164-2230 (Demorest et al. 2010)

- ✓ binary system (P=8.68 d)
- ✓ low eccentricity ( $\epsilon=1.3 \times 10^{-6}$ )
- ✓ companion mass:  $\sim 0.5M_{\odot}$
- ✓ pulsar mass:  $M = 1.928 \pm 0.017M_{\odot}$



■ PSR J0348+0432 (Antoniadis et al. 2013)

- ✓ binary system (P=2.46 h)
- ✓ very low eccentricity
- ✓ companion mass:  $0.172 \pm 0.003M_{\odot}$
- ✓ pulsar mass:  $M = 2.01 \pm 0.04M_{\odot}$

# Formation of Binary Systems

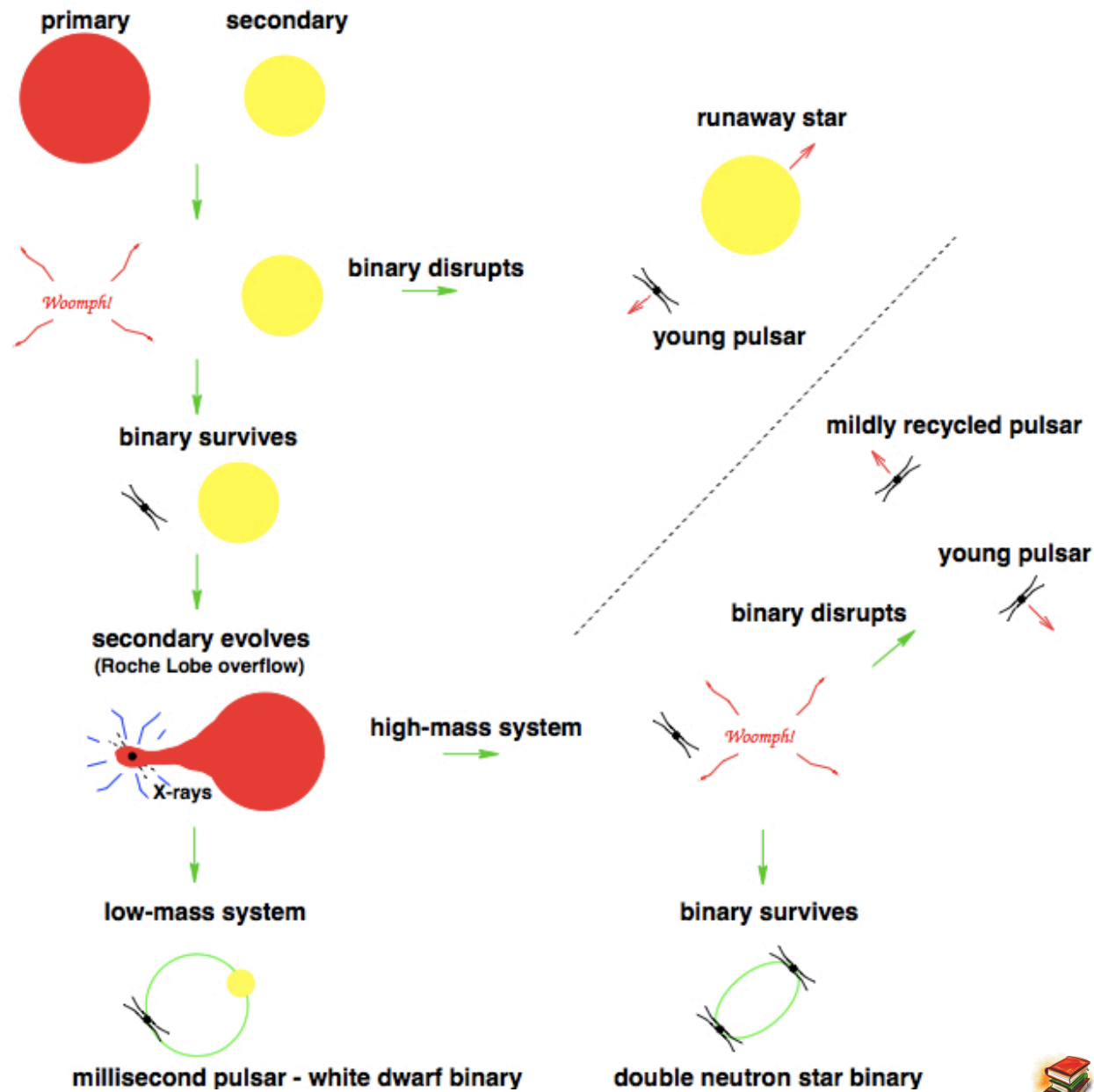
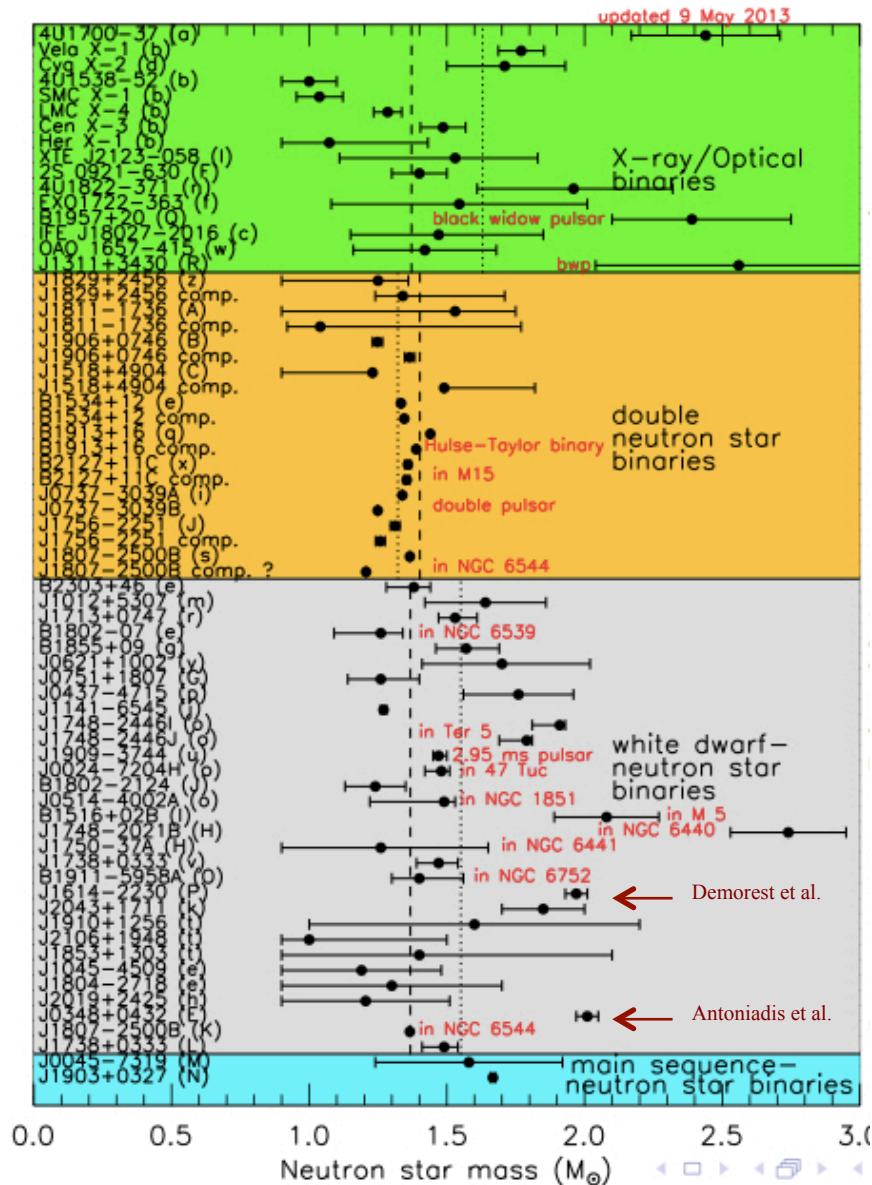


Figure by P.C.C. Freire

# Measured Neutron Star Masses (2017)



Observation of  $\sim 2 M_{\odot}$  neutron stars



Dense matter EoS stiff enough is required such that

$$M_{\max} [EoS] > 2M_{\odot}$$

A natural question arises:

Can hyperons, or strangeness in general, still be present in the interior of neutron stars in view of this constraint?

## The Hyperon Puzzle



“Hyperons → “soft (or too soft) EoS” not compatible (mainly in microscopic approaches) with measured (high) masses. However, the presence of hyperons in the NS interior seems to be unavoidable.”



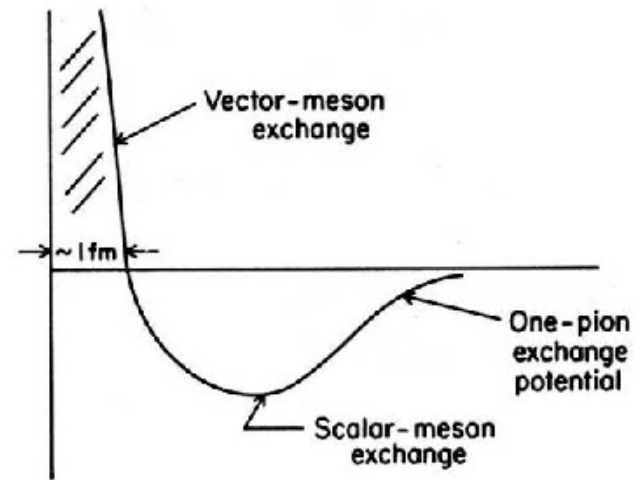
- ✓ can YN & YY interactions still solve it ?
- ✓ or perhaps hyperonic three-body forces ?
- ✓ what about quark matter ?

# Solution I: YY vector meson repulsion

(explored in the context of RMF models)

## General Feature:

Exchange of scalar mesons generates attraction (softening), but the exchange of vector mesons generates repulsion (stiffening)



Add vector mesons with hidden strangeness ( $\phi$ ) **coupled to hyperons** yielding a strong repulsive contribution at high densities

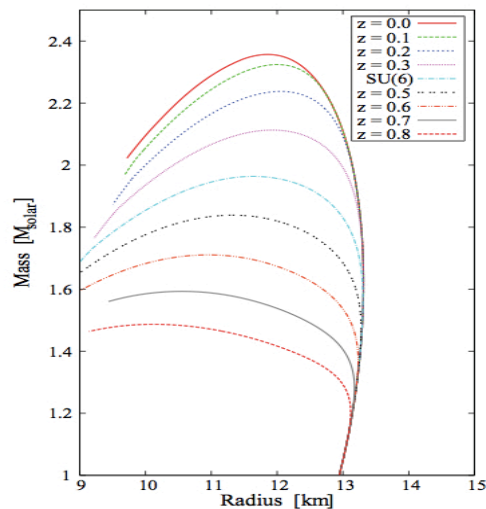


Dexhamer & Schramm (2008), Bednarek et al, (2012), Weissenborn et al., (2012)  
Oertel et al. (2014), Maslov et al. (2015)



## Weissenborn et al. (2012)

- ✓  $\sigma^2, \sigma^3, \sigma^4$  terms
- ✓  $\rho^2, \omega^2, \omega^4$  terms
- ✓ “hidden strangeness” mesons:  $\sigma^*, \phi$   
( $\sigma^{*2}, \phi^2$ )
- ✓  $g_{YV}$  couplings: from SU(6) to SU(3)  
vary  $z=g_8/g_1$  &  $\alpha=F/(F+D)$
- ✓  $g_{YS}$  couplings adjusted by fitting  $U_B^{(N)}$   
( $U_\Lambda^{(N)}=-30, U_\Sigma^{(N)}=+30, U_\Xi^{(N)}=-28$  MeV)

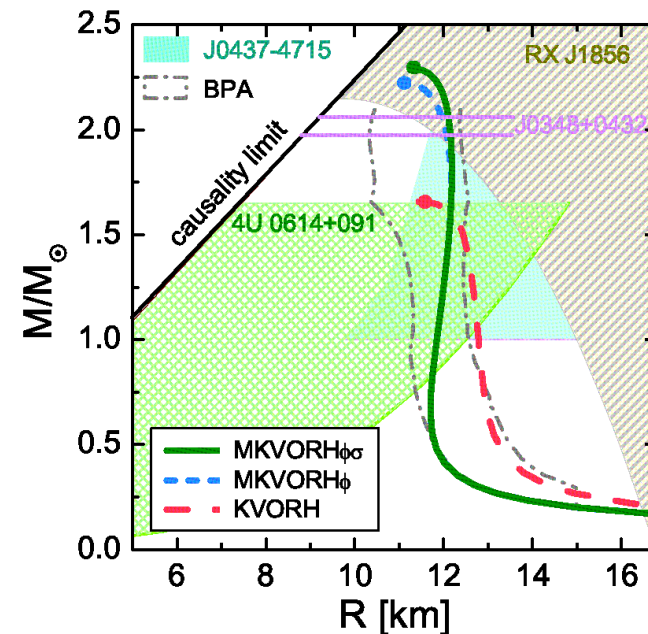


$M_{\max}$  compatible  
with  $2M_\odot$



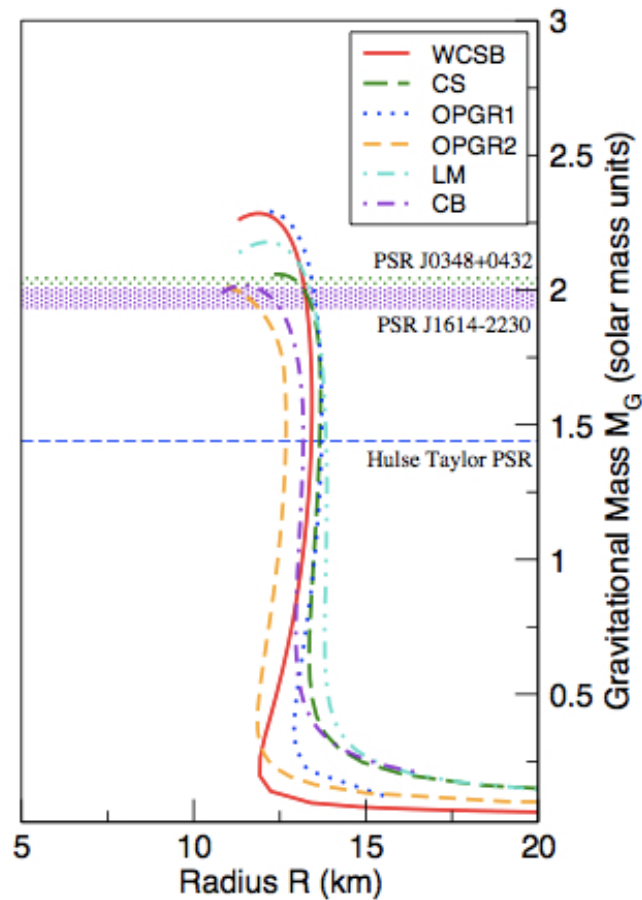
## Maslov et al. (2015)

- ✓ RMF with scaled hadron masses (universal) & coupling constants (not universal)
- ✓ Model flexible enough to satisfy constraints from HIC & astrophysical data
- ✓ Hyperon puzzle partially solved if a reduction of  $\phi$  meson mass is included





Although these and other similar models are able to reconcile the presence of hyperons in the NS interior with the existence of  $2M_{\odot}$  NS, one must be cautious !!



D. Chatterjee & I. V. (2015)

✧ These models contain several **free parameters** which most of the times are **arbitrarily chosen** being the only **justification** our still “scarce” knowledge of the YY interaction.

Hence:

In absence of sufficient experimental data on multi-strange hypernuclei and YY scattering the validity of these models is still questionable.

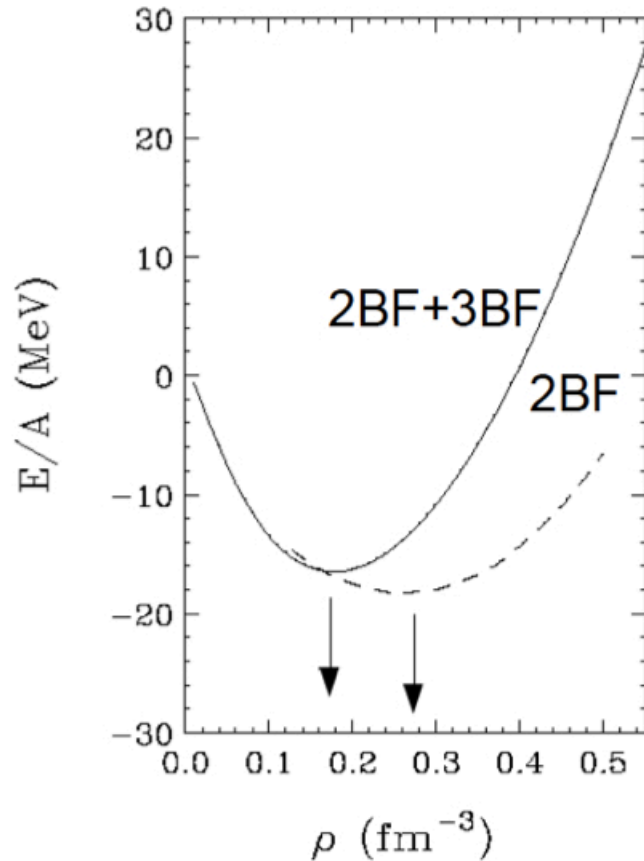


# Solution II: can Hyperonic TBF solve this puzzle ?

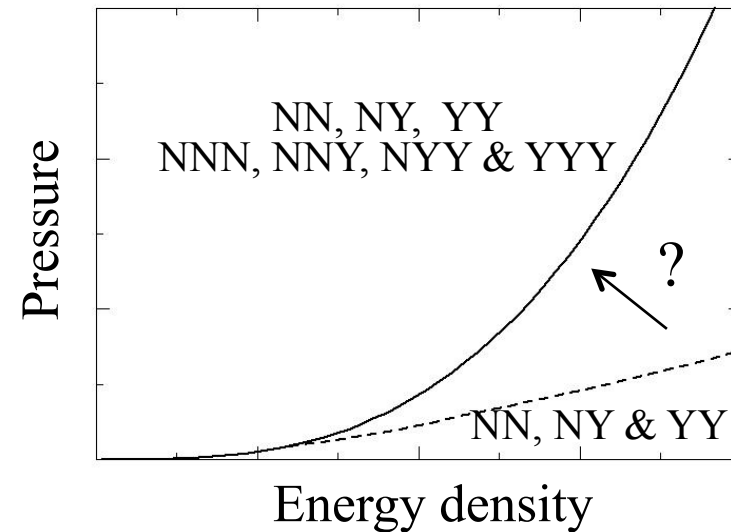
Natural solution based on: **Importance of NNN force in Nuclear Physics**

(Considered by several authors: Chalk, Gal, Usmani, Bodmer, Takatsuka, Loiseau, Nogami, Bahaduri, IV)

## NNN Force

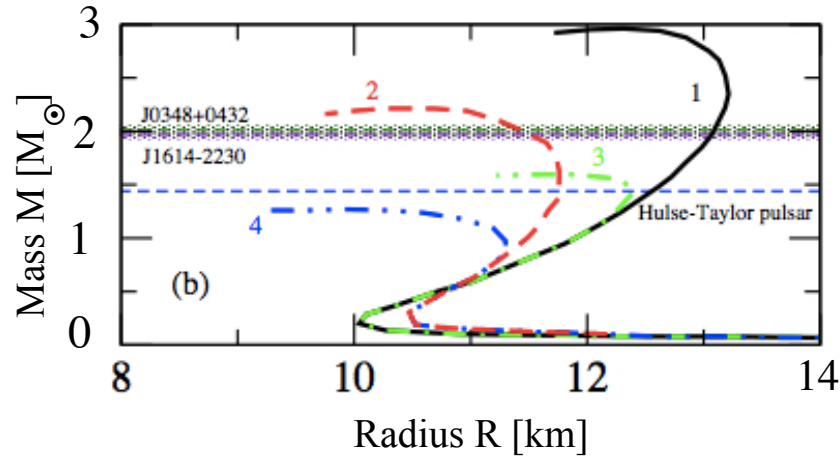


## NNY, NYN & YYY Forces



Can hyperonic TBF provide enough repulsion at high densities to reach  $2M_{\odot}$ ?

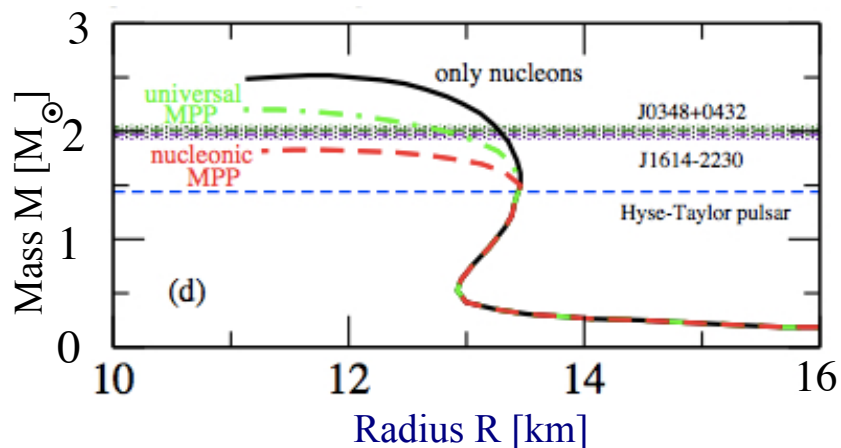
## The results are contradictory



I. V. et al. (2011)

BHF with NN+YN+phenomenological YTBF. Different strength of YTBF including the case of universal TBF

$$1.27 < M_{\max} < 1.6 M_{\odot}$$

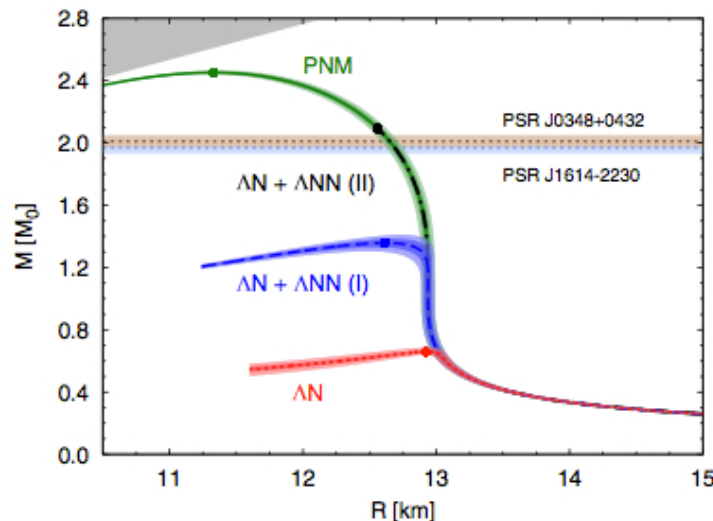
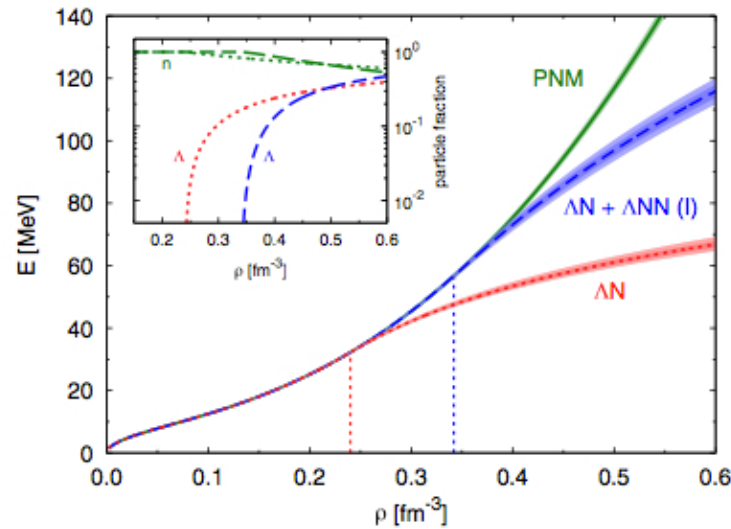


Yamamoto et al. (2015)

BHF with NN+YN+universal repulsive TBF (multipomeron exchange mechanism)

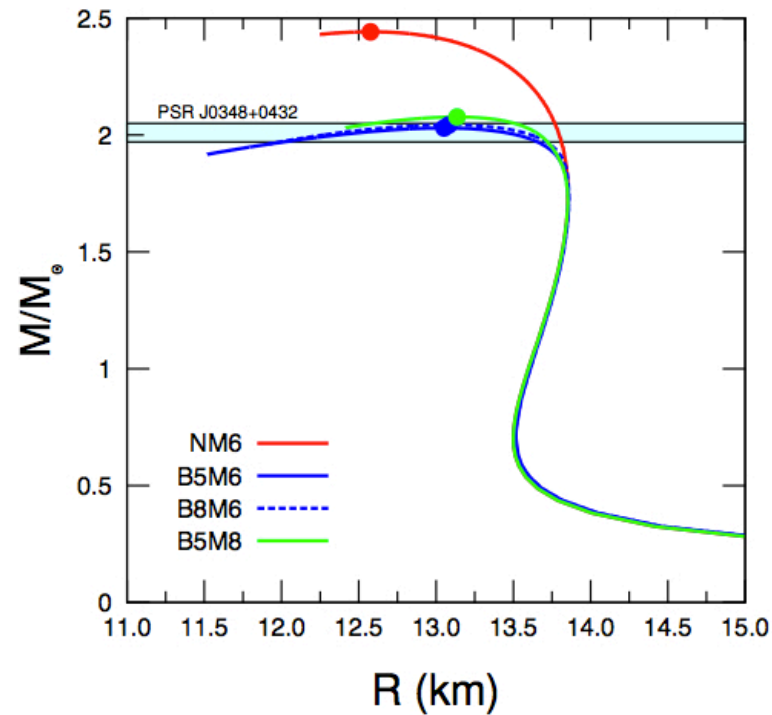
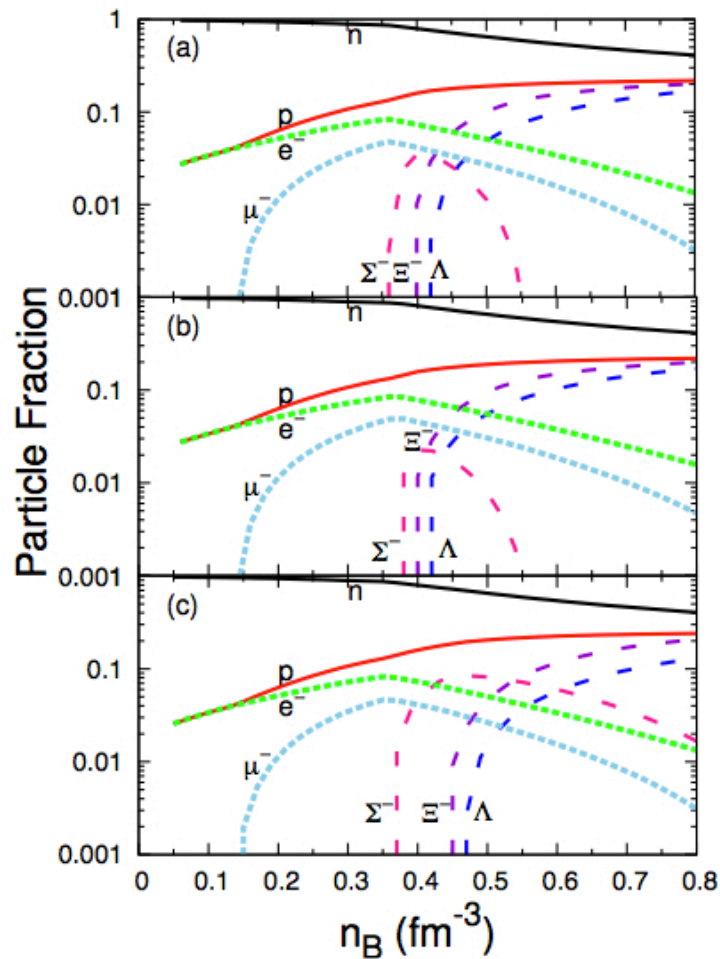
$$M_{\max} > 2 M_{\odot}$$

It should be mentioned also the recent **Quantum Monte Carlo** calculation by **Lonardonì et al. (2015)**



- ❖ First **Quantum Monte Carlo** calculation on **neutron+ $\Lambda$**  matter
- ❖ Strong dependence of  $\Lambda$  onset on  **$\Lambda_{nn}$**  force
- ❖ Some of the parametrizations of the  **$\Lambda_{nn}$**  force give maximum masses compatible with  $2M_{\odot}$  but the onset of  $\Lambda$  is above the maximum density considered ( $\sim 0.56 \text{ fm}^{-3}$ ). So in fact, **no  $\Lambda$ s** are present in NS interior

and the recent DBHF calculation of hyperonic matter by Katayama & Saito (2014)



- DBHF includes some TBF effects in a natural way
- $M_{\max}$  compatible with  $2M_{\odot}$
- But the construction of YN is a bit obscure in this work

# Take Away Message



- ✧ It is still an open question whether hyperonic TBFs can, by themselves, solve completely the hyperon puzzle or not.
- ✧ It seems, however, that even if they are not the full solution, most probably they can contribute to it in an important way.

## Solution III: Quark Matter Core

### General Feature:

Some authors have suggested an early phase transition to deconfined quark matter as solution to the hyperon puzzle. Massive neutron stars could actually be hybrid stars with a stiff quark matter core.

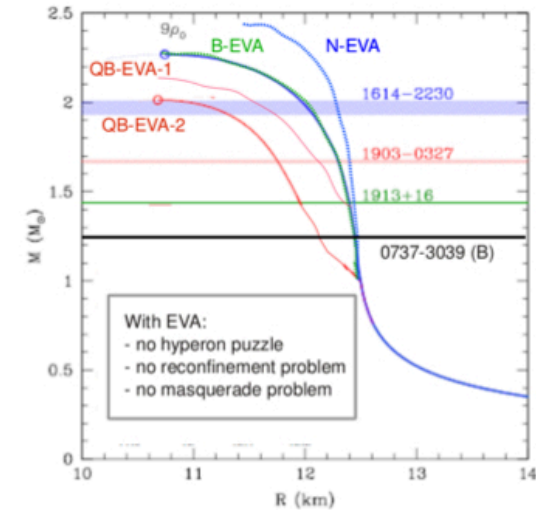
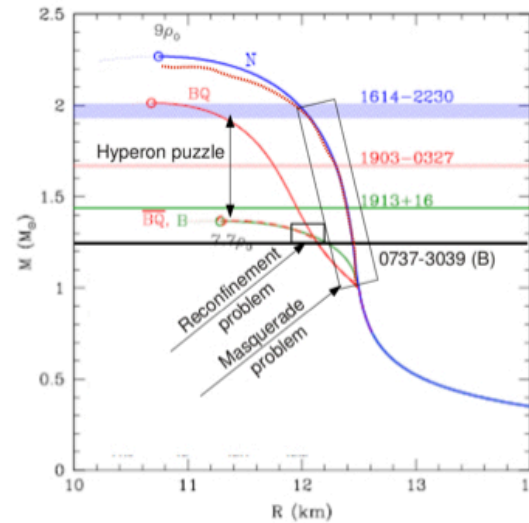
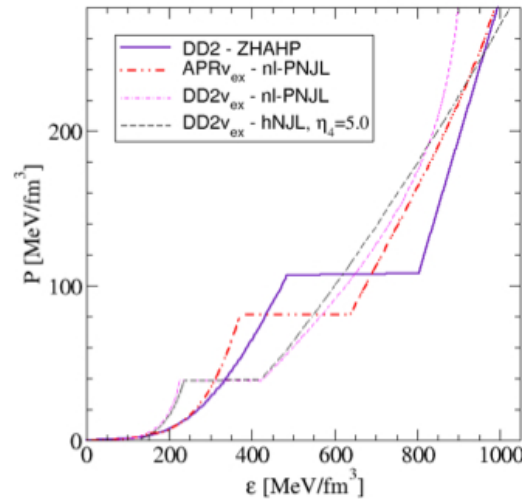
To yield  $M_{\max} > 2M_{\odot}$  Quark Matter should have:

- significant overall quark repulsion  $\longrightarrow$  stiff EoS
- strong attraction in a channel  $\longrightarrow$  strong color superconductivity



Ozel et al., (2010), Weissenborn et al., (2011), Klaehn et al., (2011), Bonano & Sedrakian (2012), Lastowiecki et al., (2012), Zdunik & Haensel (2012)

## A recent work by D. Blaschke & D. Alvarez-Castillo (2015)

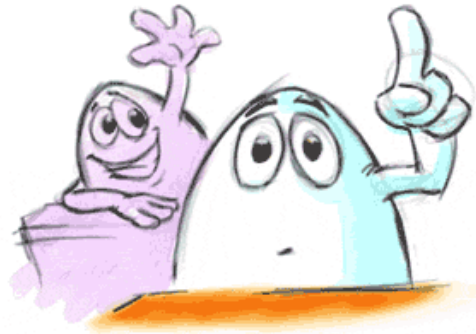


Compositeness of baryons (by excluded volume and/or quark Pauli blocking) on the hadronic side + **confinement** and **stiffening effects** on the quark matter:

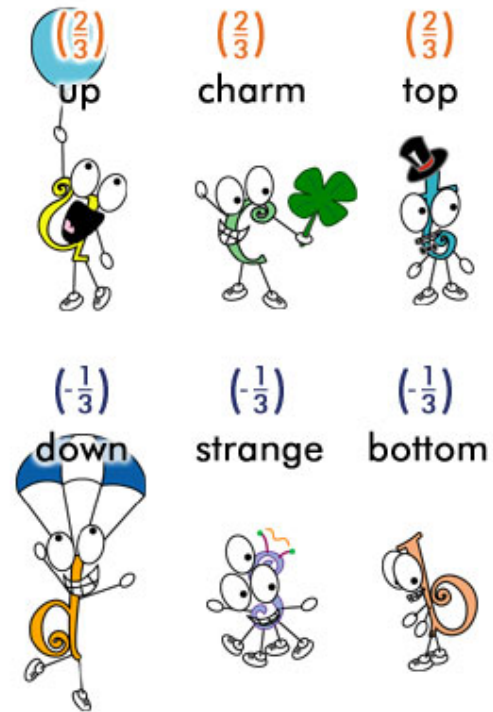


Earlier phase transition to QM with sufficient **stiffening at high densities** to solve: hyperon puzzle, masquerade problem & reconfinement puzzle

What quark flavors are expected in a Neutron Stars ?



Flavor	Mass	Charge [e]
u	~ 5 MeV	2/3
d	~ 10 MeV	-1/3
s	~ 200 MeV	-1/3
c	~ 1.3 GeV	2/3
b	~ 4.3 GeV	-1/3
t	~ 175 GeV	2/3





Suppose:  $\checkmark$  u, d, s non-interacting  $\longrightarrow$  i.e., ideal ultra-relativistic Fermi gas (\*)  
 $\checkmark$   $m_u=m_d=m_s=0$

❖ Threshold density for the c quark (similar for b & t)

$$s \rightarrow c + e^- + \bar{\nu}_e \Rightarrow \mu_s = \mu_c + \mu_e + \mu_{\bar{\nu}_e}$$

but  $\checkmark$  u, d, s in  $\beta$ -equilibrium  
 $\checkmark$   $Q_{\text{tot}}=0$

$$n_B = n_u = n_d = n_s$$

$$n_e = n_{\bar{\nu}_e} = 0$$

then

$$\mu_s = E_{F_s} = \hbar c \left( \pi^2 n_s \right)^{1/3} = \hbar c \left( \pi^2 n_B \right)^{1/3} \geq m_c = 1.3 \text{ GeV}$$

$$\Rightarrow n_B \geq 29 \text{ fm}^{-3} \sim 180 n_0$$

Only u,d,s quarks are expected in Neutron Stars

# The Equation of State for Hybrid Stars

## ✧ Hadronic phase :

RMF Models

Microscopic BHF

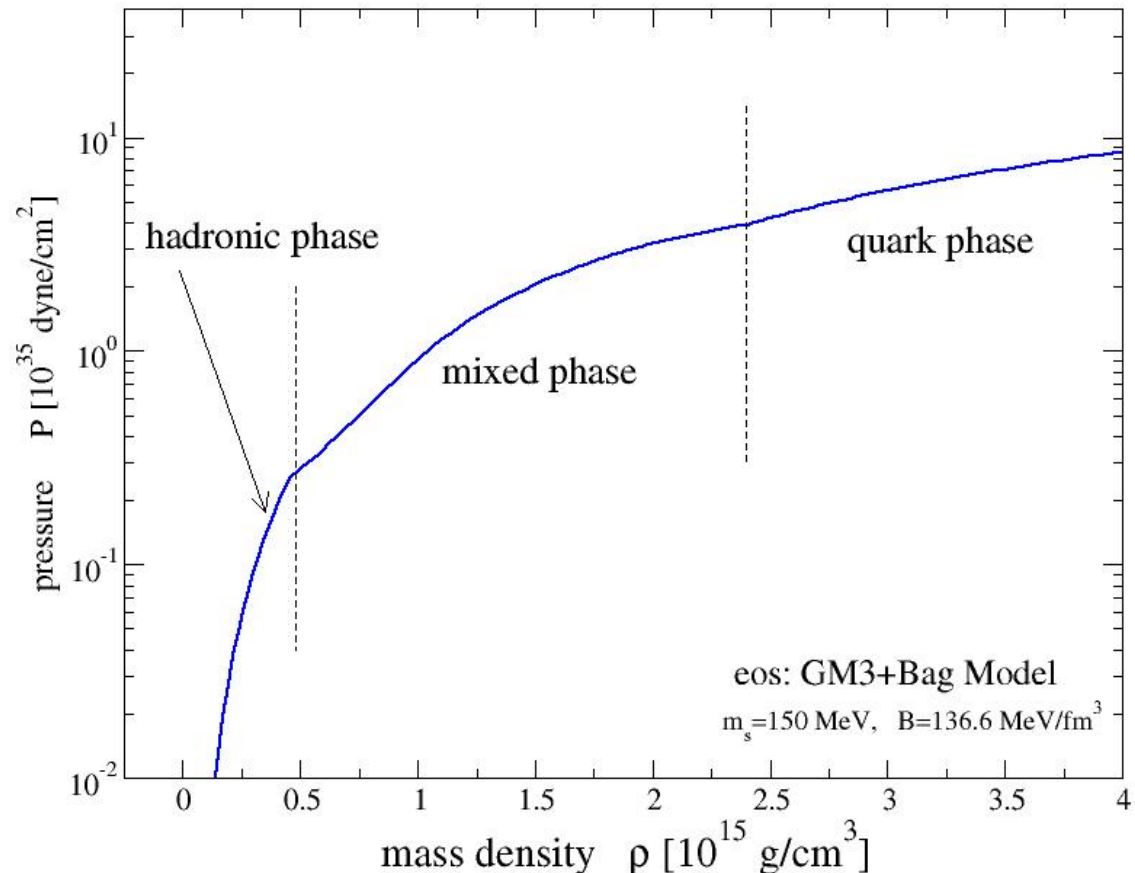
## ✧ Quark phase :

EOS based on the MIT bag model for hadrons.

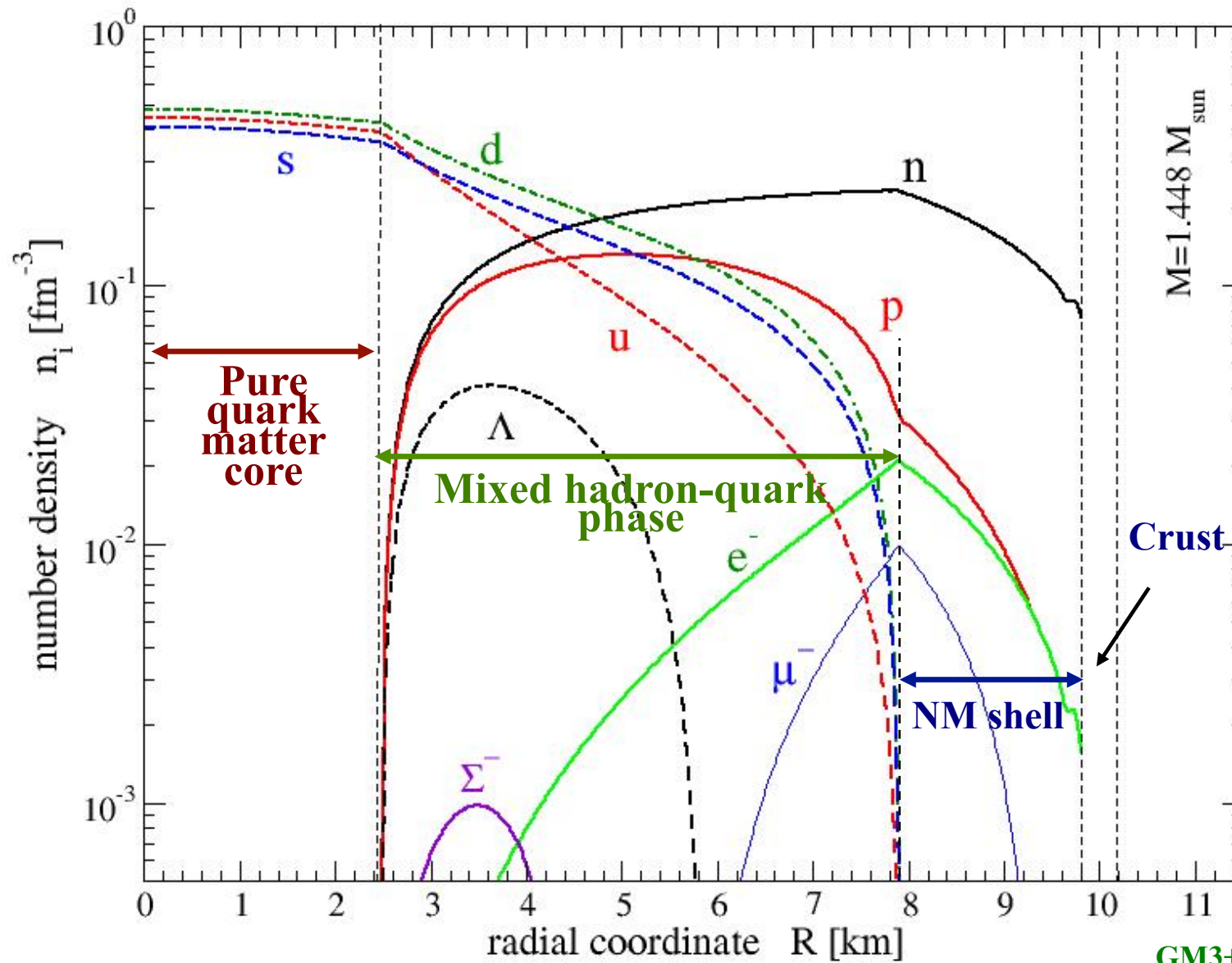
[Farhi, Jaffe, Phys. Rev. D46(1992)]

## ✧ Mixed phase :

Gibbs construction for a multicomponent system with two conserved “charges”. [Glendenning, Phys. Rev. D46 (1992)]



# Hybrid Star Composition



GM3+Bag model  
 $m_s = 150 \text{ MeV}$ ,  $B = 13.6.6 \text{ MeV}/\text{fm}^3$

# The Strange Matter Hypothesis

Bodmer (1971), Terezawa (1979) & Witten (1984)

Three-flavour **u,d,s quark** matter in equilibrium with respect to the weak interactions, could be the **true ground state of strongly interacting mater**, rather than  $^{56}\text{Fe}$

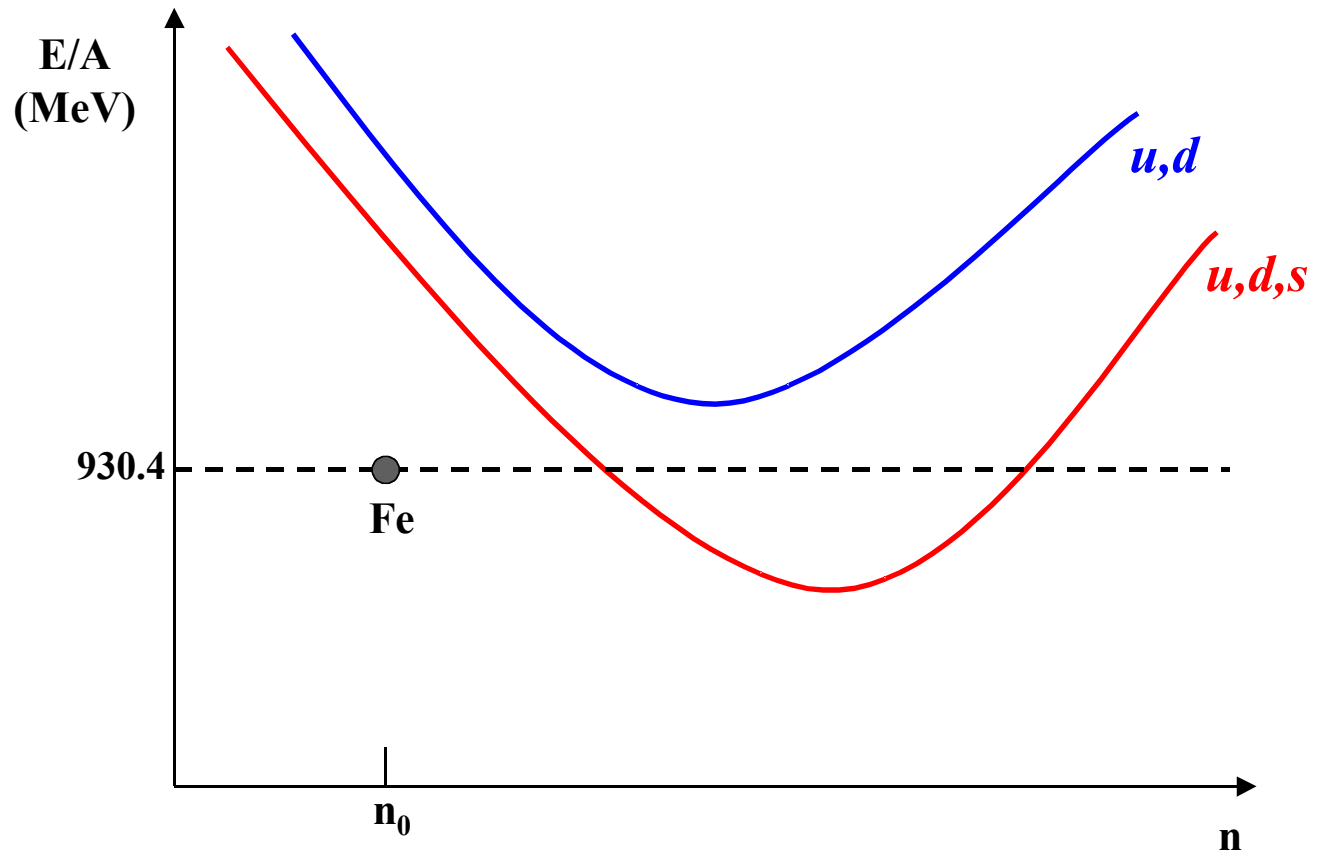
$$E/A|_{\text{SQM}} < E(^{56}\text{Fe})/56 \sim 930 \text{ MeV}$$

Stability of nuclei with respect to u,d quark matter

The success of traditional nuclear physics provides a clear indication that **quarks in the atomic nuclei are confined within neutrons and protons**

$$E/A|_{\text{ud}} > E(^{56}\text{Fe})/56 \sim 930 \text{ MeV}$$

# Schematically

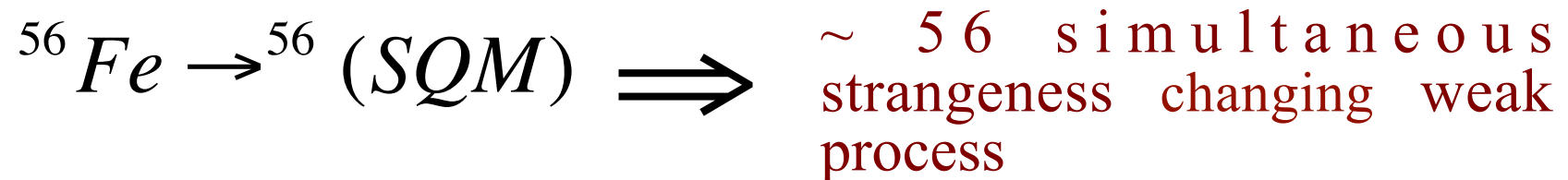


- If the SQM hypothesis is true, **why nuclei do not decay into SQM droplets (strangelets) ?**
- One should explain the **existence of atomic nuclei** in Nature



## Stability of Nuclei with respect to SQM

- Direct decay of  $^{56}\text{Fe}$  to a SQM droplet



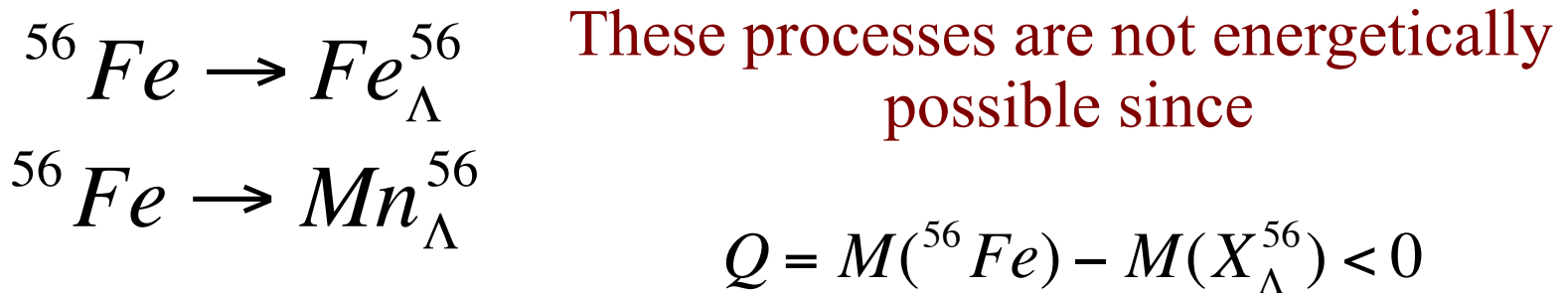
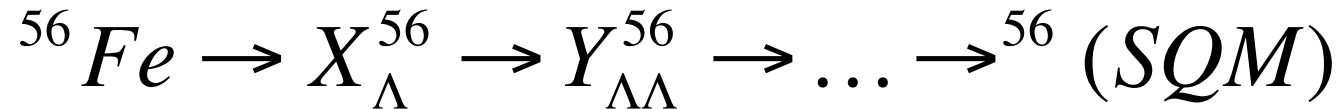
$$u \rightarrow s + e^+ + \nu_e$$

$$d + u \rightarrow s + u$$

The probability for the direct decay is  $P \sim (G_F^2)^{56} \sim 0$   
and the **mean**-life time of  $^{56}\text{Fe}$  with respect to the  
direct decay to a drop of SQM is

$\tau \gg$  age of the Universe

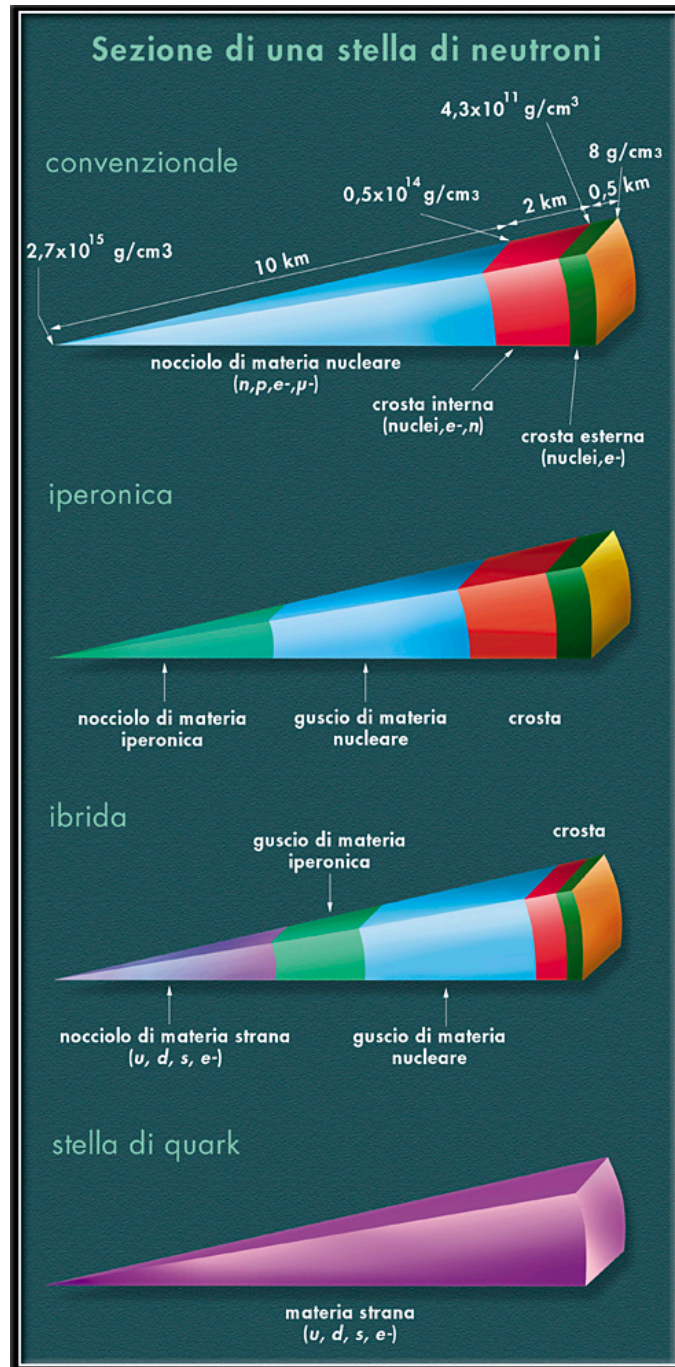
- Step by step decay of  $^{56}\text{Fe}$  to a SQM droplet



Thus, according with the **Bodmer-Terezawa-Witten hypothesis**, nuclei are metastable states of strong interacting matter with a mean-life time

$$\tau \gg \text{age of the Universe}$$





## Two families of Neutron Stars

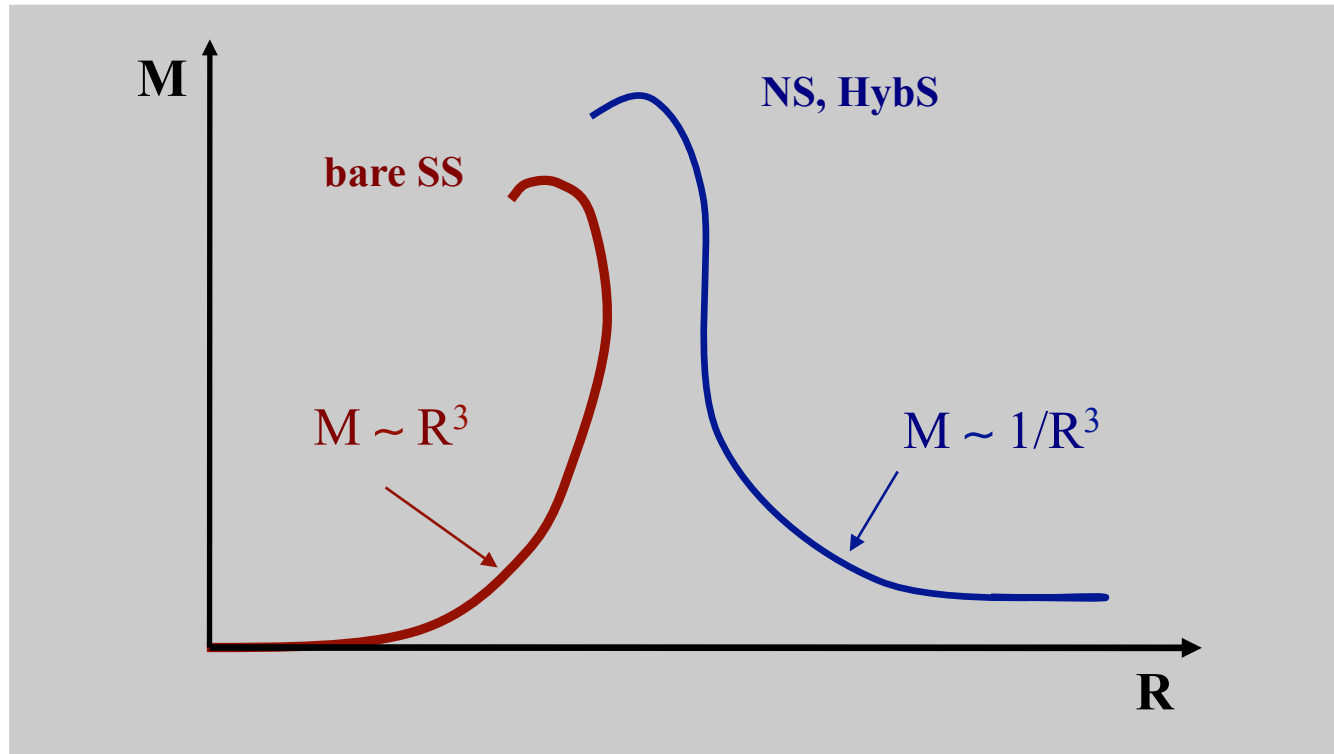
### Hadron Stars (HS)

- Nucleonic Stars
- Hyperonic Stars

### Quark Stars (QS)

- Hybrid Stars
- Strange Stars

# Mass-radius relation



- ✧ **Strange Stars** are self-bound bodies i.e., bound by the strong interactions
- ✧ **Hadronic or Hybrid Stars** are bound by gravity.

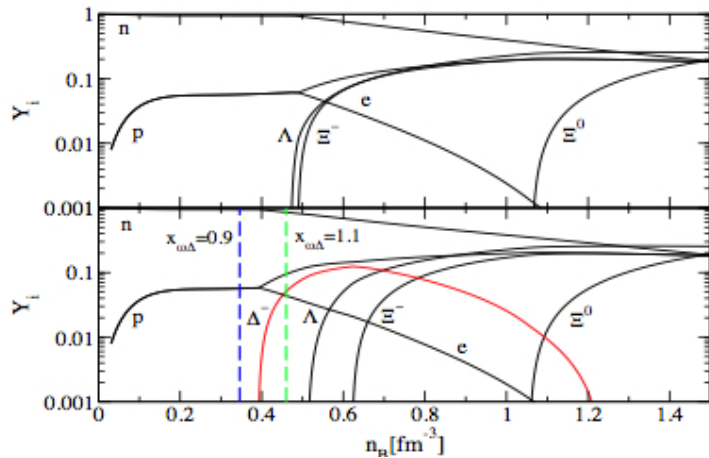
But also in this case we must **pay attention**



Currently theoretical descriptions of quark matter at high density rely on phenomenological models which are constrained using the few available experimental information on high density baryonic matter from heavy-ion collisions.

# Is there also a $\Delta$ isobar puzzle ?

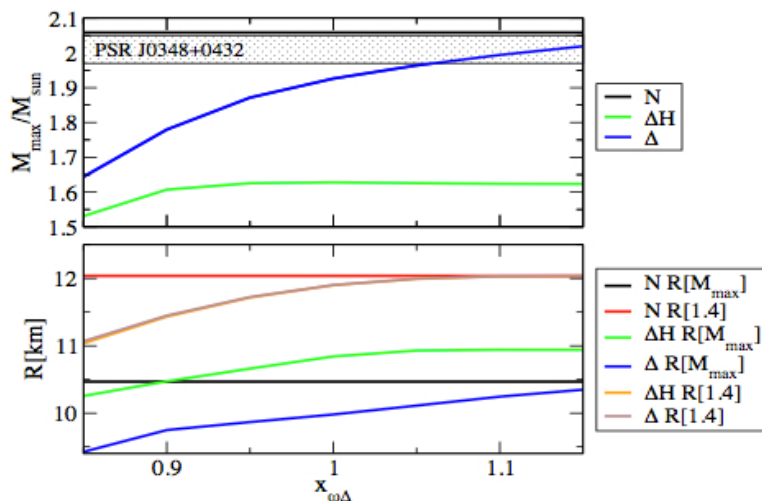
The recent work by Drago et al. (2014) calculation have studied the role of the  $\Delta$  isobar in neutron star matter



❖ Constraints from L indicate an early appearance of  $\Delta$  isobars in neutron stars matter at  $\sim 2-3 \rho_0$  (same range as hyperons)

❖ Appearance of  $\Delta$  isobars modify the composition & structure of hadronic stars

❖  $M_{\max}$  is dramatically affected by the presence of  $\Delta$  isobars



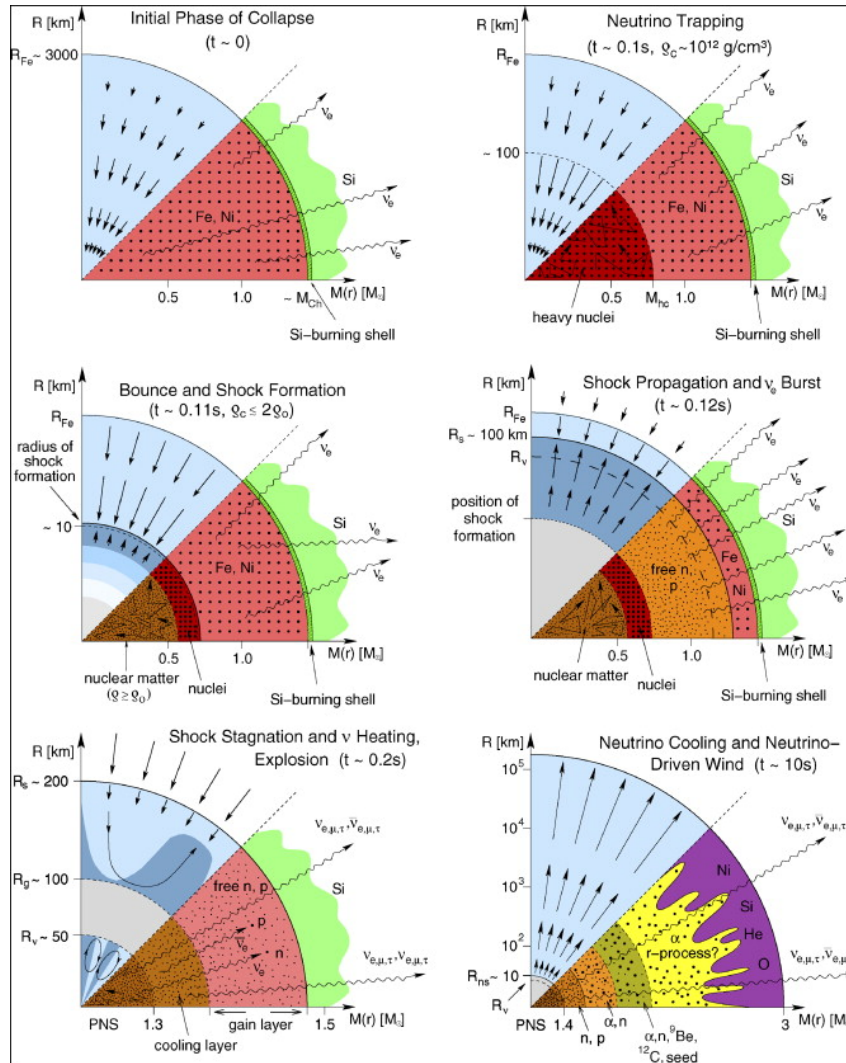
If  $\Delta$  potential is close to that indicated by  $\pi^-$ , e-nucleus or photoabsorption nuclear reactions then EoS is too soft  $\rightarrow$   $\Delta$  puzzle similar to the hyperon one



# Hyperon Stars at Birth

*David Lloyd Glover*

# Proto-Neutron Stars



New effects on PNS matter:

- Thermal effects

$$T \approx 30 - 40 \text{ MeV}$$

$$S / A \approx 1 - 2$$

- Neutrino trapping

$$\mu_\nu \neq 0$$

$$Y_e = \frac{\rho_e + \rho_{\nu_e}}{\rho_B} \approx 0.4$$

$$Y_\mu = \frac{\rho_\mu + \rho_{\nu_\mu}}{\rho_B} \approx 0$$

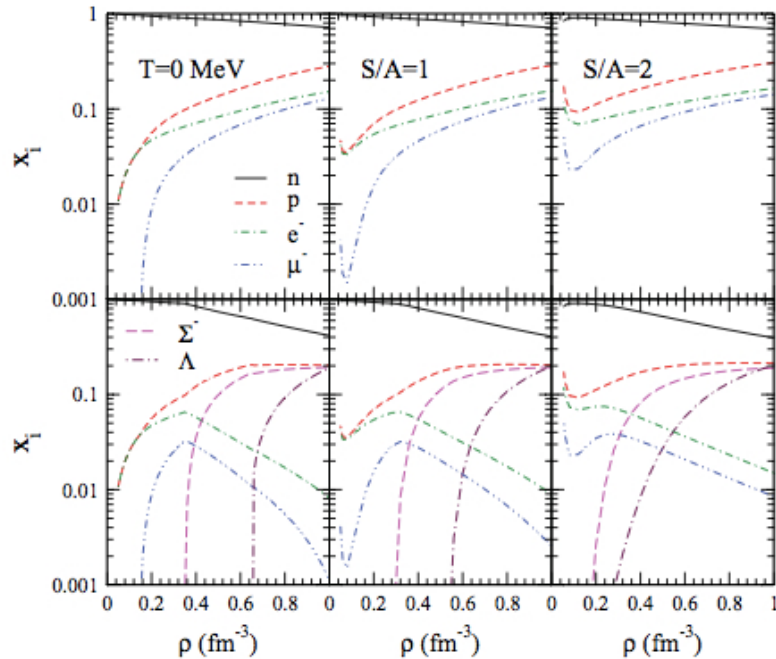
(Janka, Langanke, Marek, Martinez-Pinedo & Muller 2006)

# Proto-Neutron Stars: Composition

- Neutrino free

$$\mu_\nu = 0$$

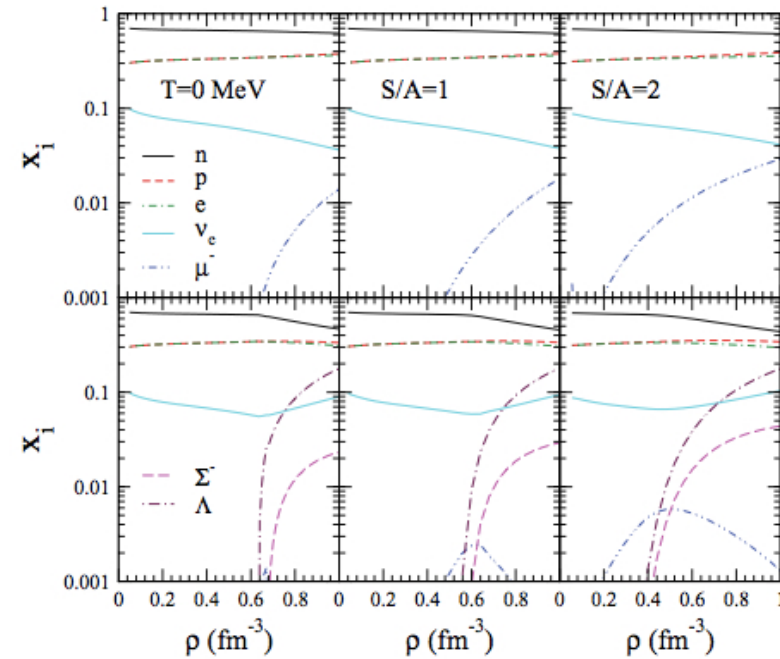
(Burgio & Schulze 2011)



- Neutrino trapped

$$\mu_\nu \neq 0$$

(Burgio & Schulze 2011)



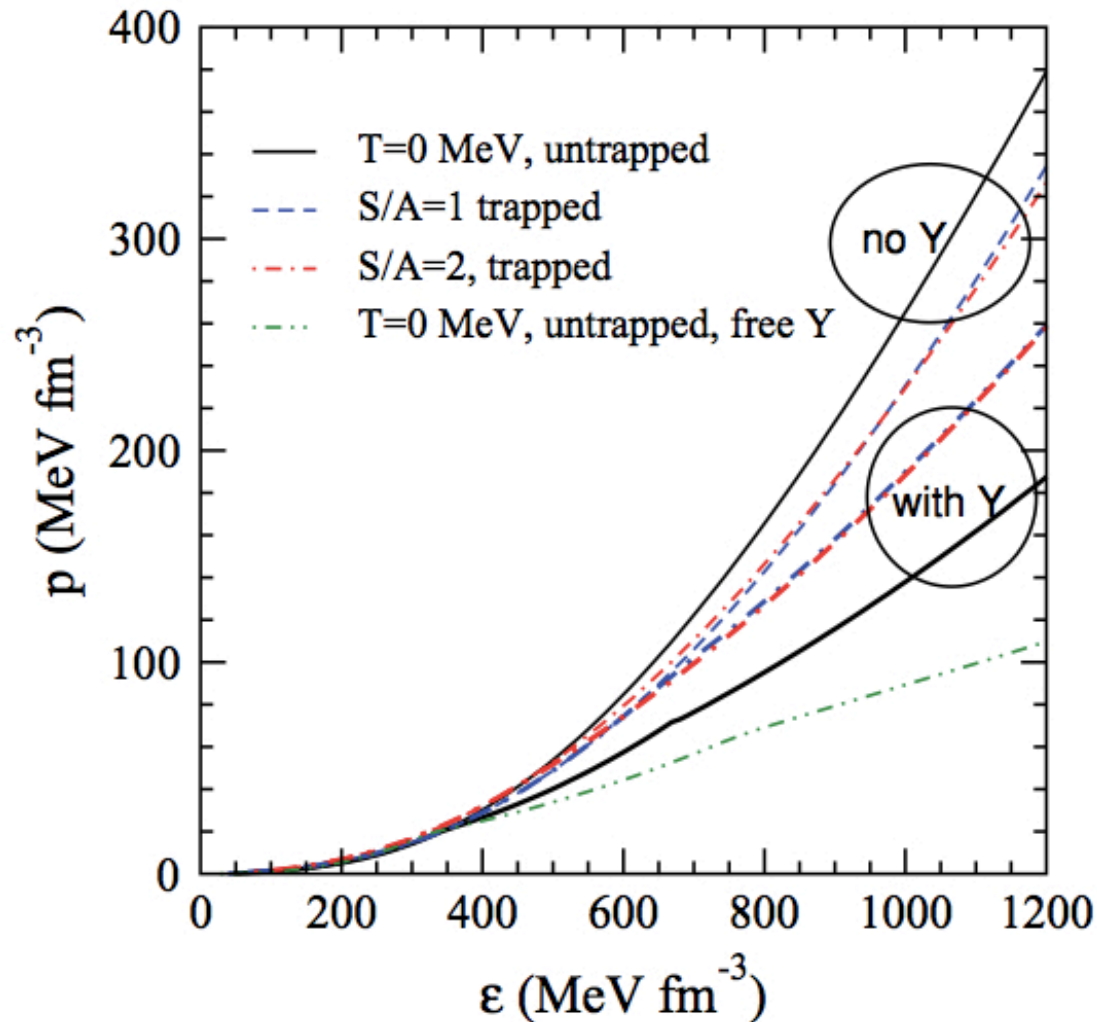
Neutrino trapped



- ✓ Large proton fraction
- ✓ Small number of muons
- ✓ Onset of  $\Sigma^-(\Lambda)$  shifted to higher (lower) density
- ✓ Hyperon fraction lower in  $\nu$ -trapped matter

# Proto-Neutron Stars: EoS

(Burgio & Schulze 2011)



## ■ Nucleonic matter

- ✧  $\nu$ -trapping + temperature  
→ softer EoS

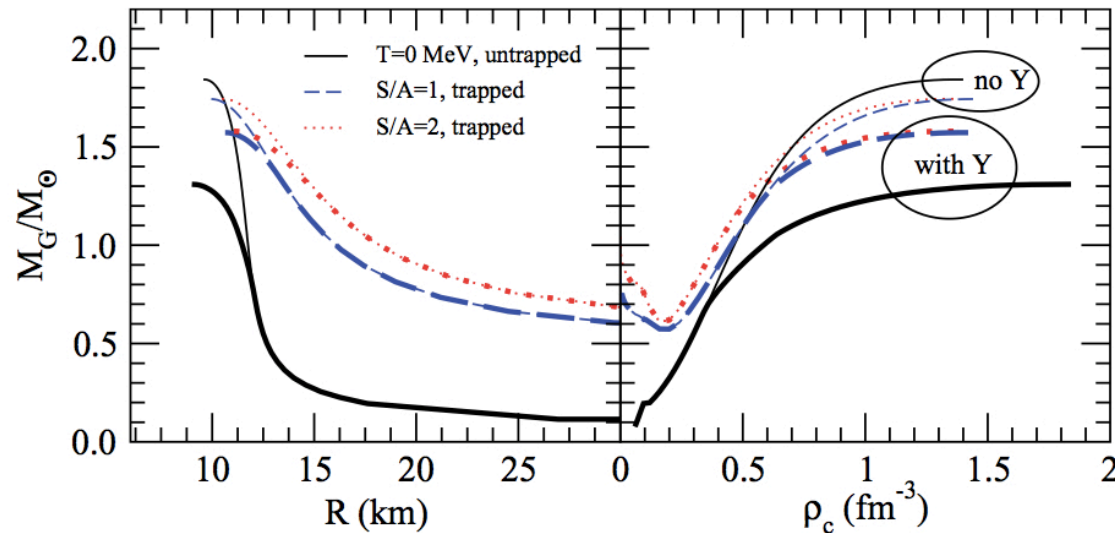
## ■ Hyperonic matter

- ✧  $\nu$ -trapping + temperature  
→ stiffer EoS
- ✧ More hyperon softening in  $\nu$ -untrapped matter (larger hyperon fraction)



# Proto-Neutron Stars: Structure

(Burgio & Schulze 2011)



## Hyperonic matter

$\nu$ -trapping + T:  
increase of  $M_{\text{max}}$

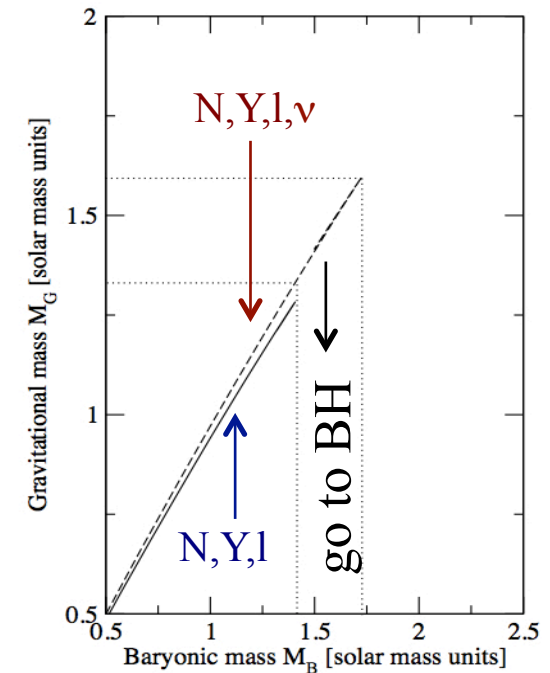


delayed formation  
of a low mass BH

## Nucleonic matter

$\nu$ -trapping + T:  
reduction of  $M_{\text{max}}$

(IV et al. 2003)





# Hyperons & Neutron Star Cooling

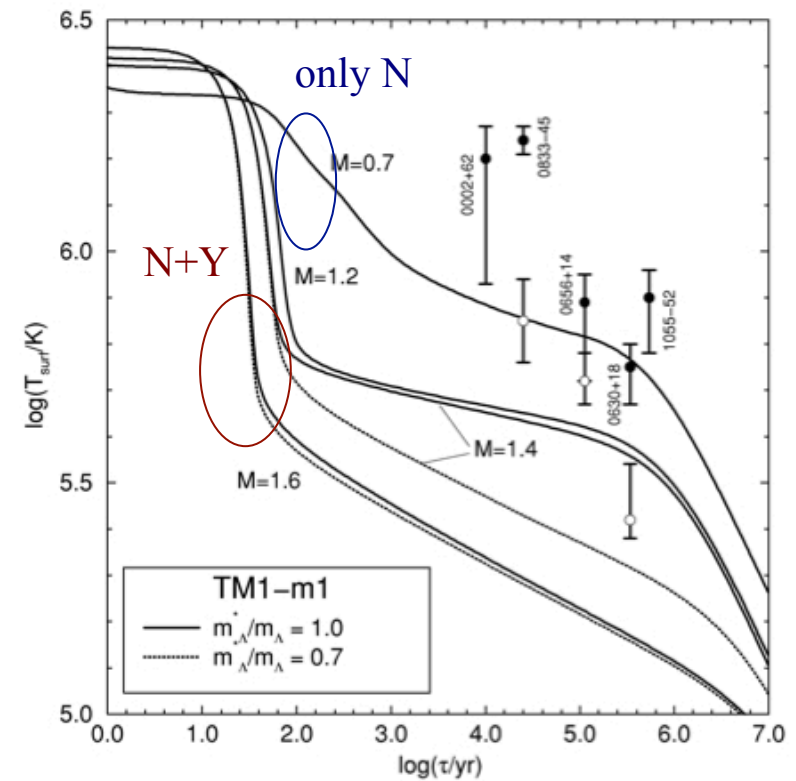
Hyperonic DURCA processes possible  
 as soon as hyperons appear  
 (nucleonic DURCA requires  $x_p > 11-15\%$ )

➔ Additional  
 Fast Cooling  
 Processes

Process	R
$\Lambda \rightarrow p + l + \bar{\nu}_l$	0.0394
$\Sigma^- \rightarrow n + l + \bar{\nu}_l$	0.0125
$\Sigma^- \rightarrow \Lambda + l + \bar{\nu}_l$	0.2055
$\Sigma^- \rightarrow \Sigma^0 + l + \bar{\nu}_l$	0.6052
$\Xi^- \rightarrow \Lambda + l + \bar{\nu}_l$	0.0175
$\Xi^- \rightarrow \Sigma^0 + l + \bar{\nu}_l$	0.0282
$\Xi^0 \rightarrow \Sigma^+ + l + \bar{\nu}_l$	0.0564
$\Xi^- \rightarrow \Xi^0 + l + \bar{\nu}_l$	0.2218

+ partner reactions generating neutrinos,  
 Hyperonic MURCA, ...

(Schaab, Shaffner-Bielich & Balberg 1998)

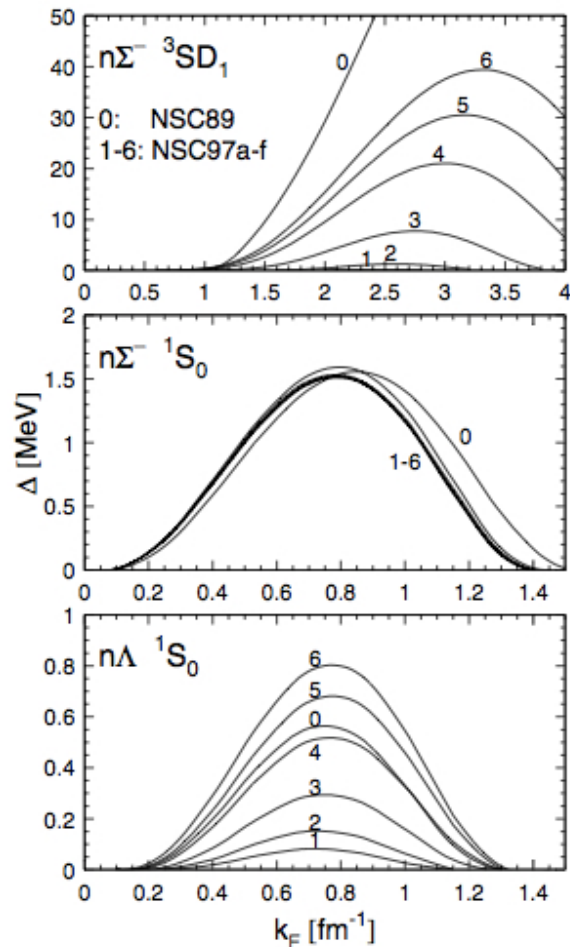


R: relative emissivity w.r.t. nucleonic DURCA

Pairing Gap  $\longrightarrow$  suppression of  $C_v$  &  $\epsilon$  by

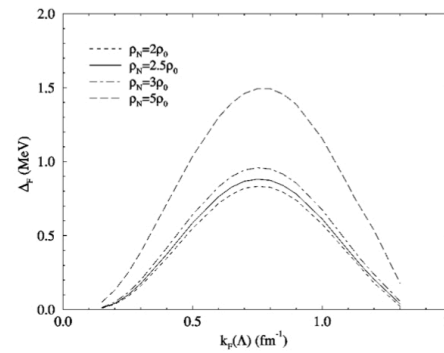
$$\sim e^{(-\Delta/k_B T)}$$

■  $^1S_0$ ,  $^3SD_1$   $\Sigma N$  &  $^1S_0$   $\Lambda N$  gap

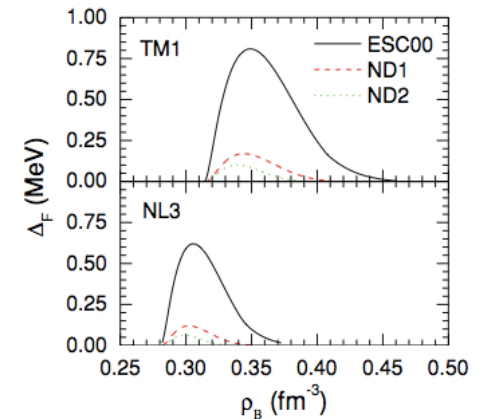


(Zhou, Schulze, Pan & Draayer 2005)

■  $^1S_0$   $\Lambda\Lambda$  gap

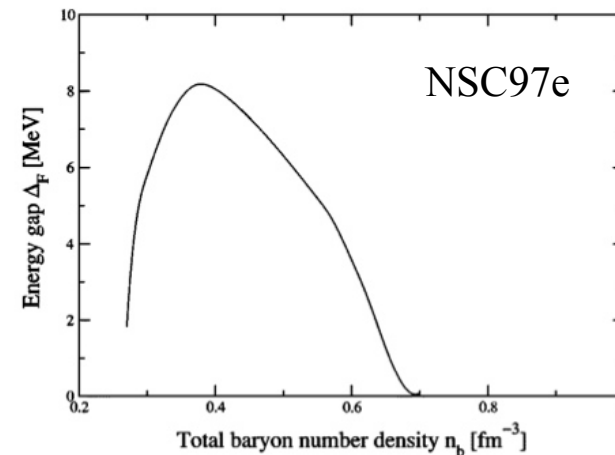


(Balberg & Barnea 1998)



(Wang & Shen 2010)

■  $^1S_0$   $\Sigma\Sigma$  gap



(IV & Tolós 2004)

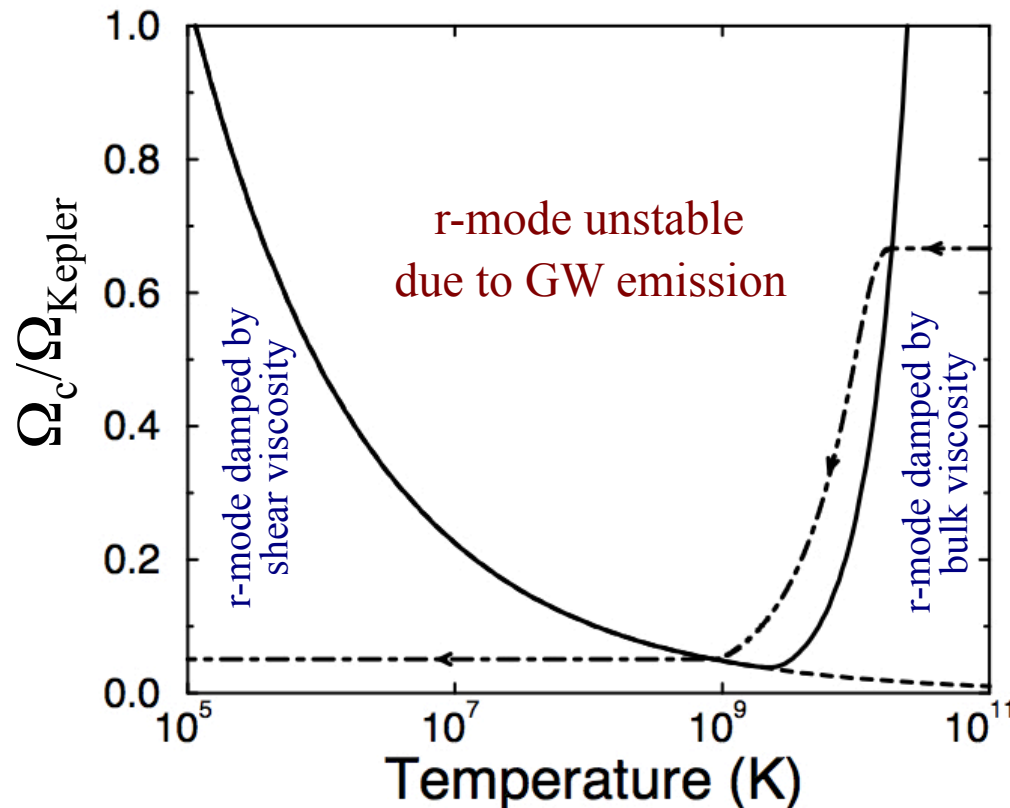
The background of the slide is a reproduction of the Japanese woodblock print 'The Great Wave off Kanagawa' by Katsushika Hokusai. It depicts a massive, curling blue wave with white foam, threatening three small boats on the sea. In the distance, the snow-capped Mount Fuji is visible under a pale, hazy sky. The overall color palette is muted, with soft blues, greys, and a pale yellowish-tan background.

# **Hyperons & the R-mode instability of Neutron Stars**

# The r-mode Instability

$\Omega_{\text{Kepler}}$  : Absolute Upper Limit  
of Rot. Freq.

Instabilities prevent NS  
to reach  $\Omega_{\text{Kepler}}$



r-mode Instability : toroidal mode  
of oscillation

- ✓ restoring force: Coriolis
- ✓ emission of GW in hot & rapidly rotating NS (CFS mechanism)
  - GW makes the mode unstable
  - Viscosity stabilizes the mode

$$A \propto A_0 e^{-i\omega(\Omega)t - t/\tau(\Omega, T)}$$

$$\frac{1}{\tau(\Omega, T)} = -\frac{1}{\tau_{\text{GW}}(\Omega)} + \frac{1}{\tau_{\text{Viscosity}}(\Omega, T)}$$

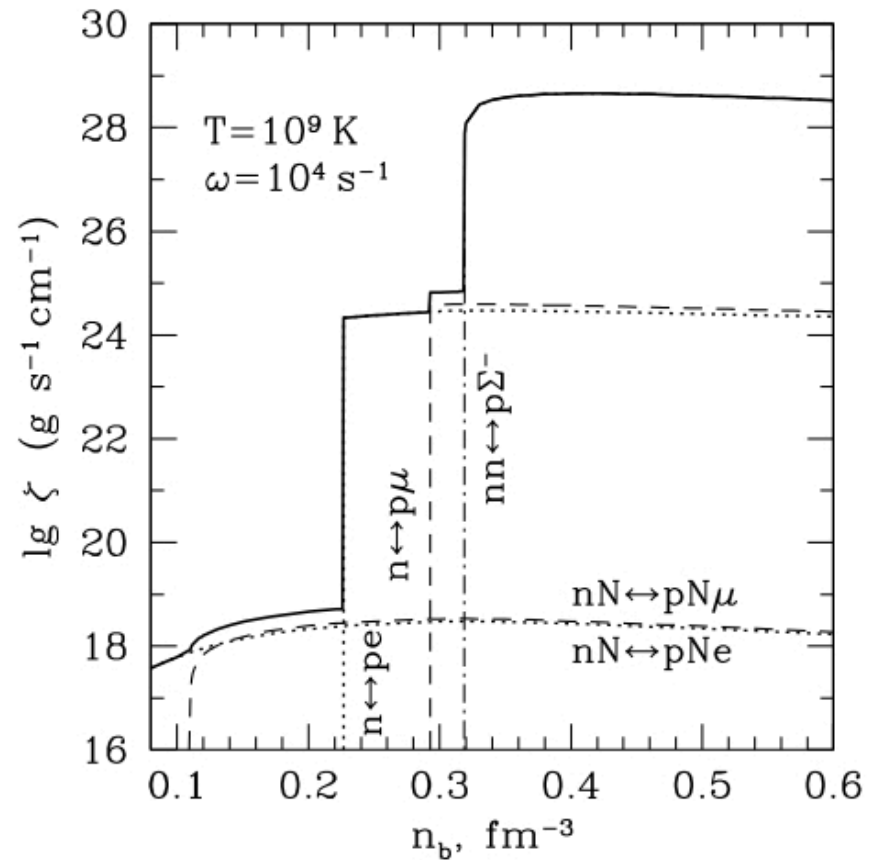
# Hyperon Bulk Viscosity $\xi_Y$

(Lindblom et al. 2002, Haensel et al 2002, van Dalen et al. 2002, Chatterjee et al. 2008, Gusakov et al. 2008, Shina et al. 2009, Jha et al. 2010,...)

## Sources of $\xi_Y$ :

non-leptonic weak reactions	$N + N \leftrightarrow N + Y$ $N + Y \leftrightarrow Y + Y$
Direct & Modified URCA	$Y \rightarrow B + l + \bar{\nu}_l$ $B' + Y \rightarrow B' + B + l + \bar{\nu}_l$
strong reactions	$N + Y \leftrightarrow N + Y$ $N + \Xi \leftrightarrow Y + Y$ $Y + Y \leftrightarrow Y + Y$

(Haensel, Levenfish & Yakovlev 2002)



Reaction Rates &  $\xi_Y$  reduced by  
Hyperon Superfluidity

# Critical Angular Velocity of Neutron Stars

- r-mode amplitude:  $A \propto A_o e^{-i\omega(\Omega)t - t/\tau(\Omega)}$

$$\frac{1}{\tau(\Omega, T)} = -\frac{1}{\tau_{GW}(\Omega)} + \frac{1}{\tau_{\xi}(\Omega, T)} + \frac{1}{\tau_{\eta}(T)}$$

→  $\frac{1}{\tau(\Omega_c, T)} = 0$  r-mode instability region

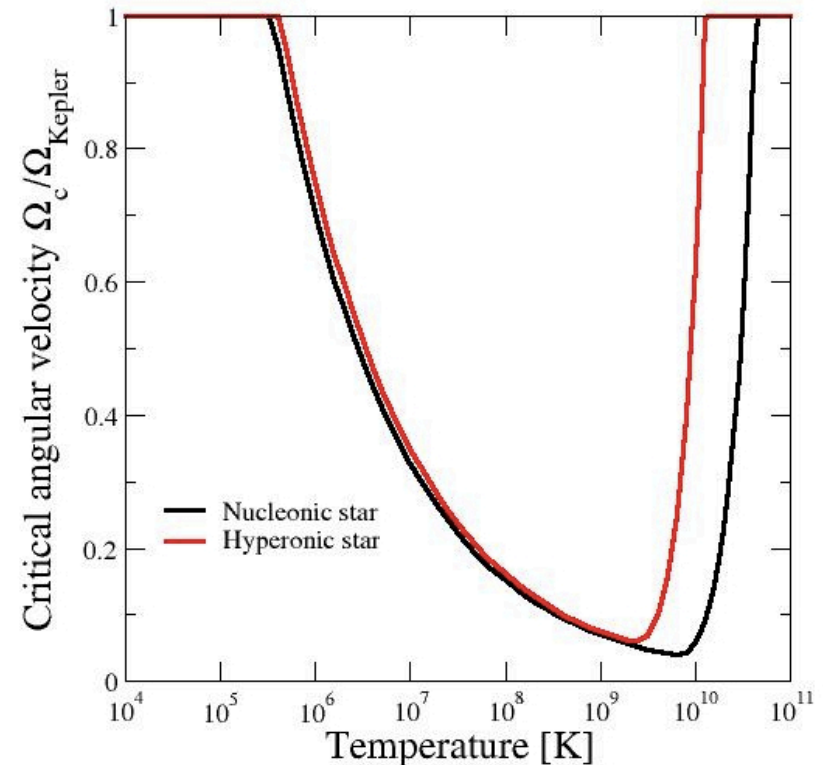
$$\Omega < \Omega_c \quad \text{stable}$$

$$\Omega > \Omega_c \quad \text{unstable}$$



As expected:  
smaller r-mode instability region  
due to hyperons

(I.V. & C. Albertus in preparation)



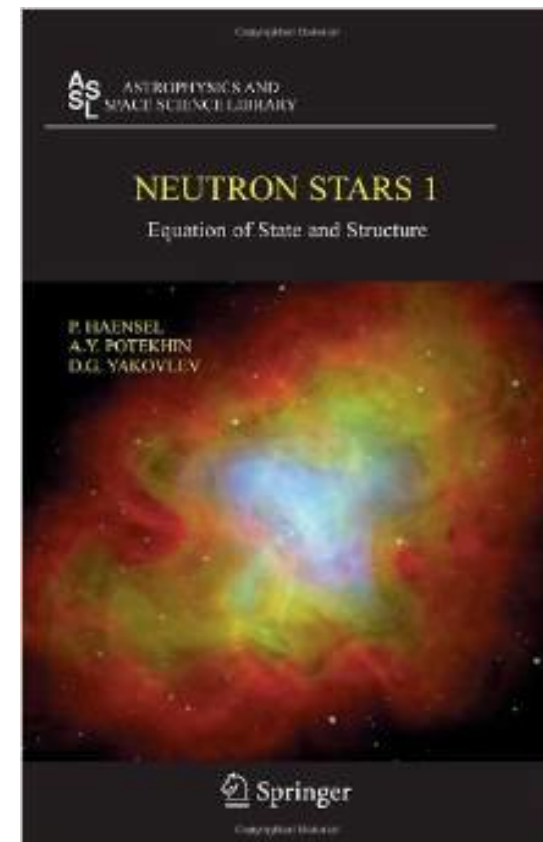
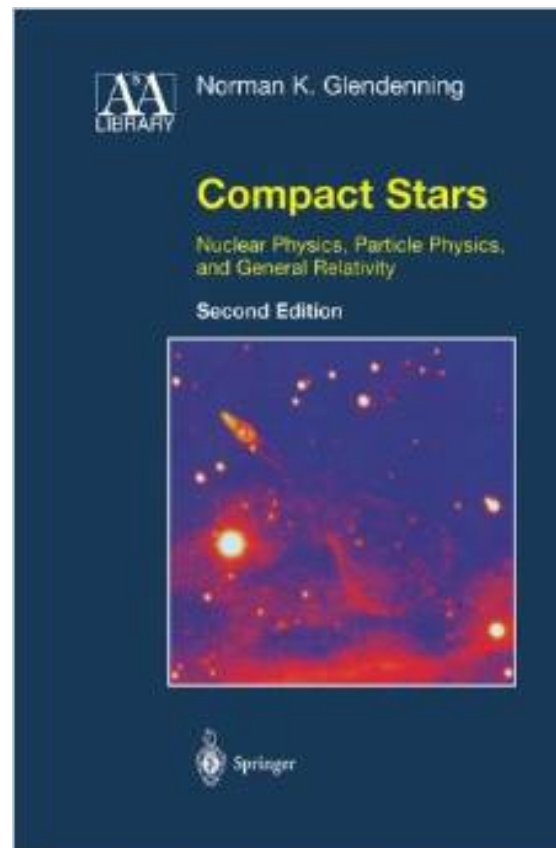
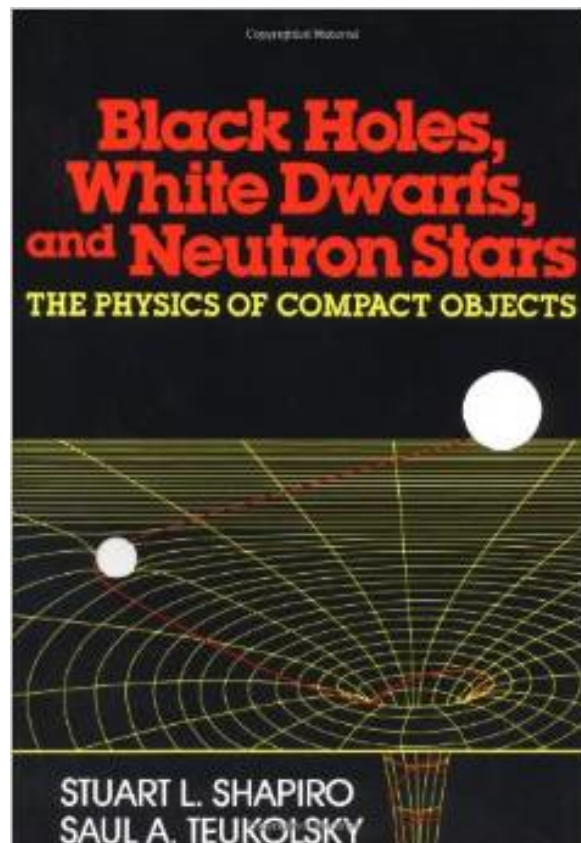
BHF: NN (Av18)+NY (NSC89)  
( $M=1.27M_{\odot}$ )



This short talk is just a brush-stroke on the physics of neutron stars. Three excellent monographs on this topic for interested readers are:



Images are copyrighted. Contact the CSLP at 1-866-637-6338 or [info@csllp.org](mailto:info@csllp.org) for more information.



# Incomplete list of other general references



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- ✧ P. C. C. Freire, arXiv:0907.3219v1 340, n 6131 (2013)
- ✧ I. Vidaña, *Nucl. Phys. A* 914, 367 (2013)
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- ✧ I. Bednared et al, *Astron & Astrophys.* 543, A157 (2012)
- ✧ A. V. Astashenok et al. *Phys. Rev. D* 89, 103509 (2014)
- ✧ ...

## The final message of this talk



Neutron stars are excellent observatories to test fundamental properties of matter under extreme conditions and offer an interesting interplay between nuclear processes and astrophysical observables

- You for your time & attention
- The organizers for their invitation

