Light and unbound nuclei

Rewriting Nuclear Physics Textbooks: Basic nuclear interactions and their link to nuclear processes in the cosmos and on earth

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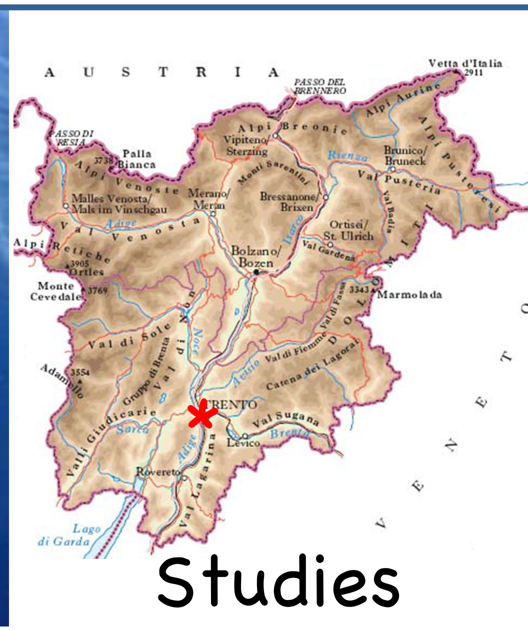
Pisa, July 25, 2017

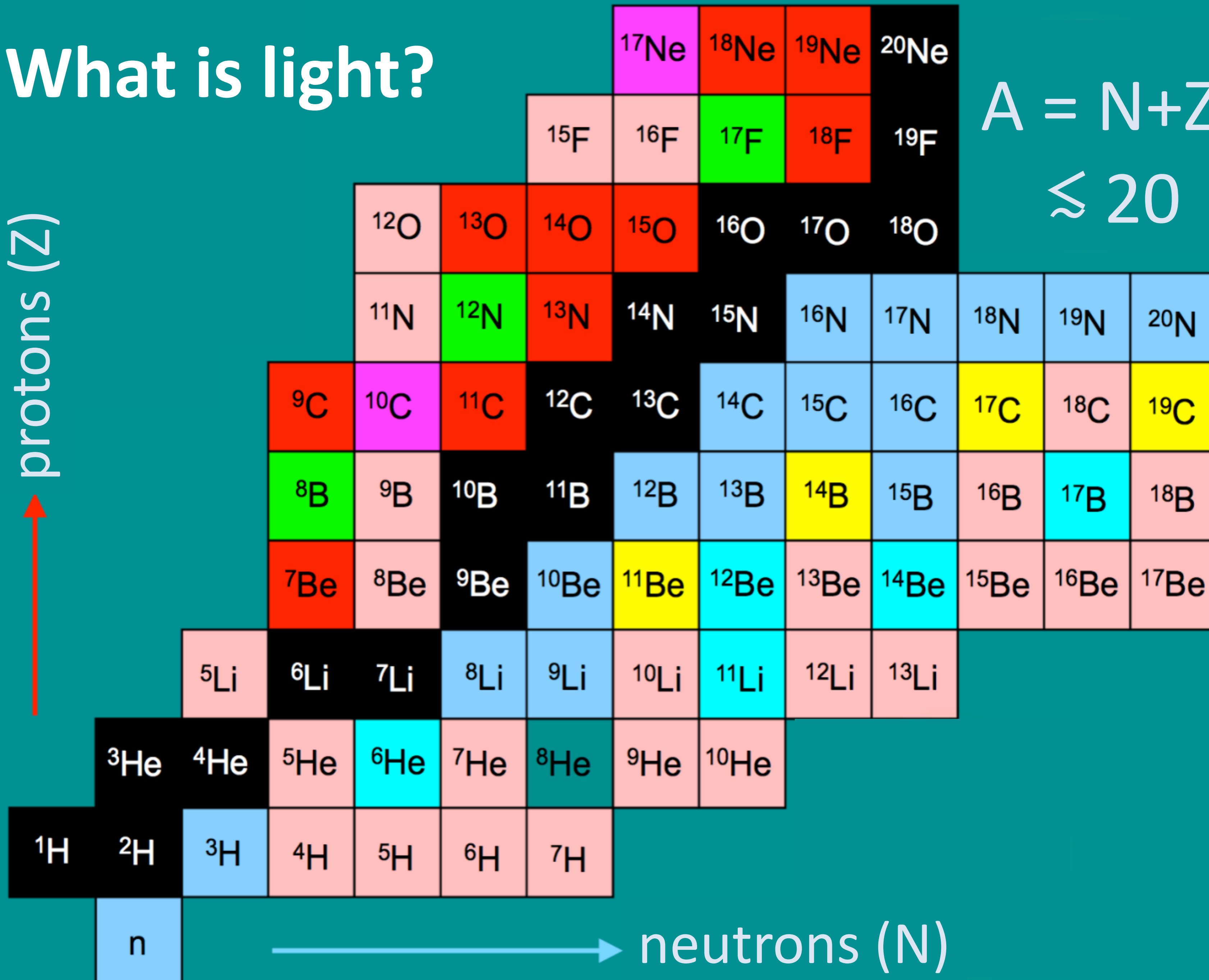


Content

- What are light and unbound nuclei?
- What is the role of light and unbound nuclei in the Cosmos and on Earth?
- How can we learn about the basic nuclear interactions?
- Can we describe exotic nuclei and the phenomena of low-energy nuclear reactions?

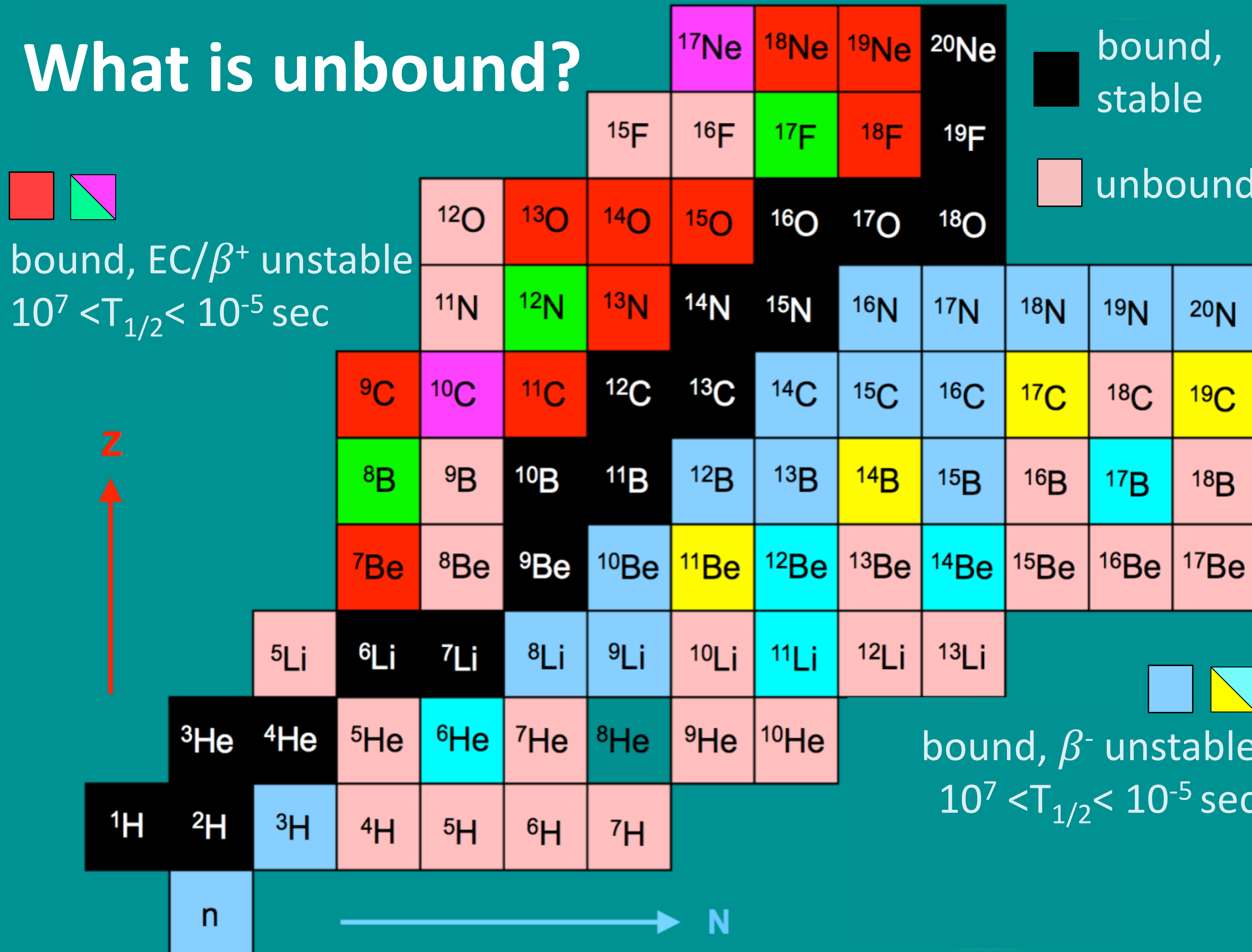
What are light and unbound nuclei?

What is light?



$$A = N+Z \lesssim 20$$

What is unbound?



Binding energy (BE)

- The energy required to disintegrate a nucleus into its components

$$BE(Z,N) = Z\, m_p\, c^2 + N\, m_n\, c^2 - M(Z,N)\, c^2$$

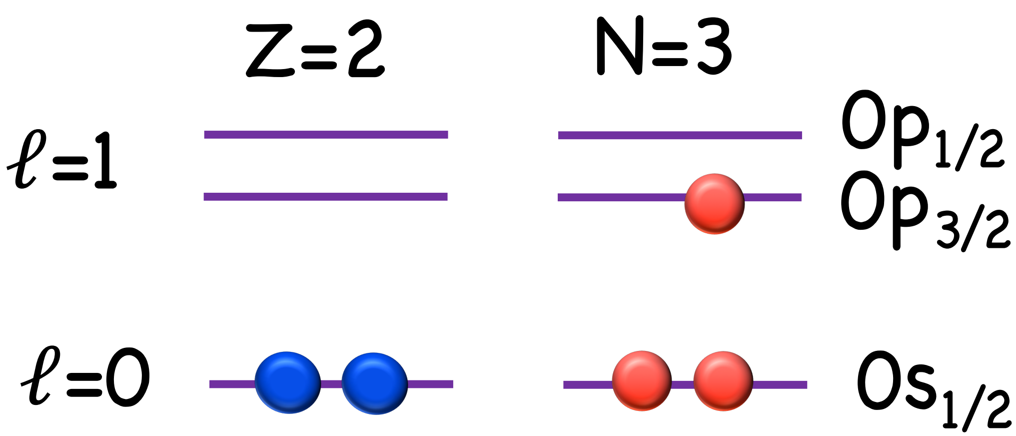
- Progressively adding neutrons (protons) drives the binding energy to zero: driplines

The case of ^5He

- ^4He tightly bound (BE = 28.30 MeV)
- ^5He is not bound. Why?!?

The case of ^{5}He

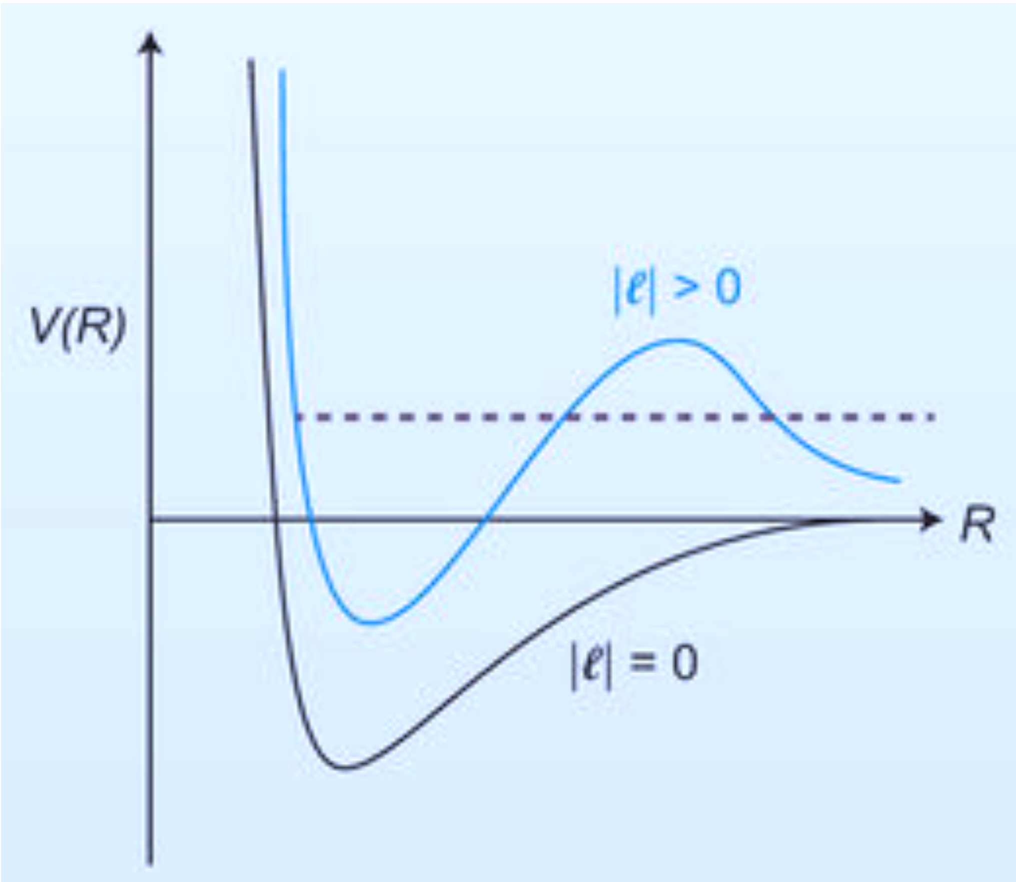
1) Pauli exclusion principle



s-shell is full, extra neutron must be in p shell

The case of ^{5}He

2) Centrifugal barrier



Overall potential is attractive but not enough to bind the system

Unbound nuclear systems, resonances

$$\psi(t, \boldsymbol{r}) = \exp\left(-\frac{iE}{\hbar}t\right)\psi(0, \boldsymbol{r})$$

Solution of time-dependent Schrodinger equation

Solution of time-independent Schrodinger equation

Unbound nuclear systems, resonances

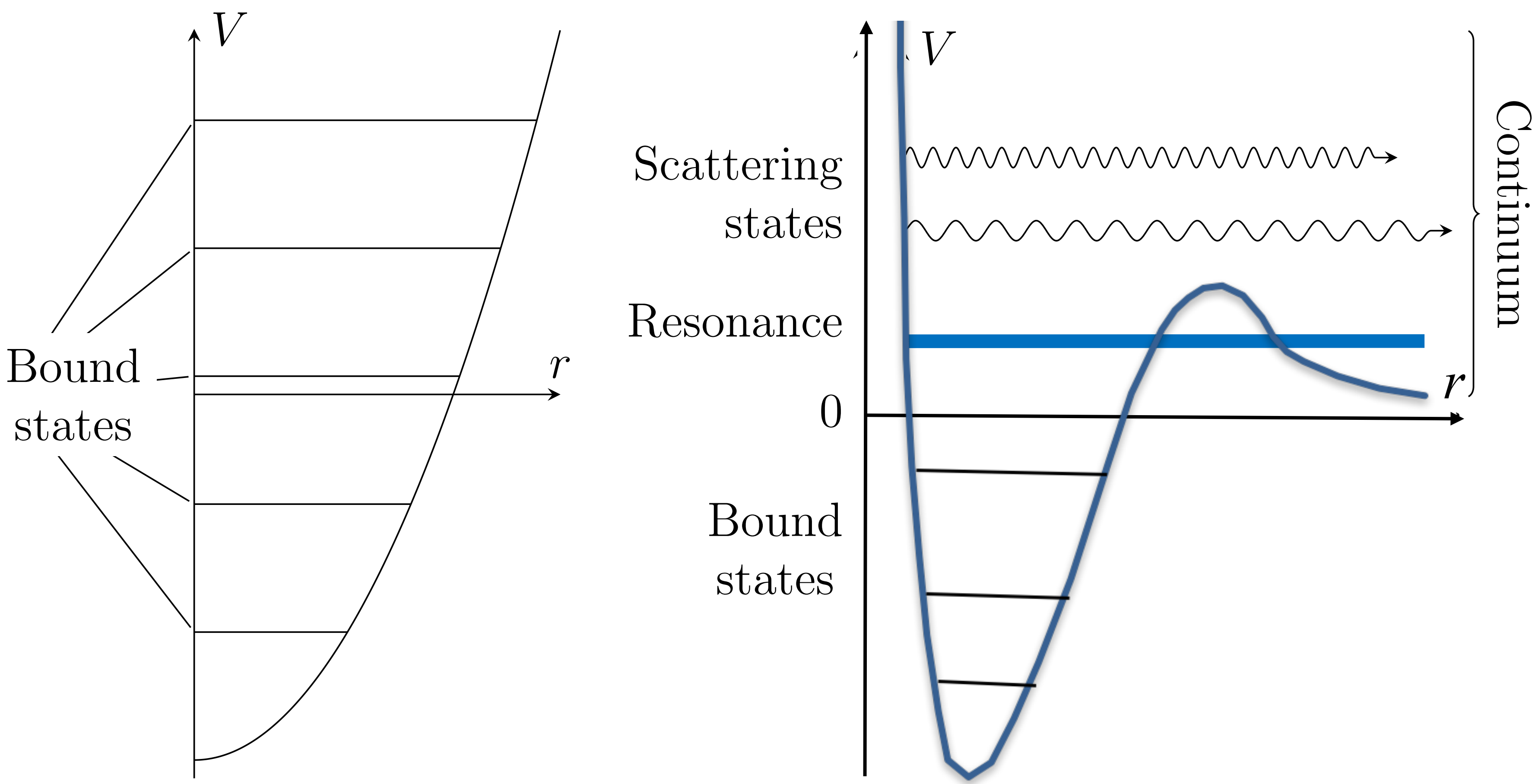
$$\psi(t,\boldsymbol{r}) = \exp\left(-\frac{iE}{\hbar}t\right)\psi(0,\boldsymbol{r})$$

- Energy E is a real number: $|\psi(t,\boldsymbol{r})|^2 = |\psi(0,\boldsymbol{r})|^2$
- Energy E is a complex complex: $E = E_0 - i\frac{\Gamma}{2}$

$$|\psi(t,\boldsymbol{r})|^2 = \exp\left(-\frac{\Gamma}{\hbar}t\right)|\psi(0,\boldsymbol{r})|^2$$

Resonance state
decaying exponentially
$T_{1/2} = \ln 2/\Gamma$

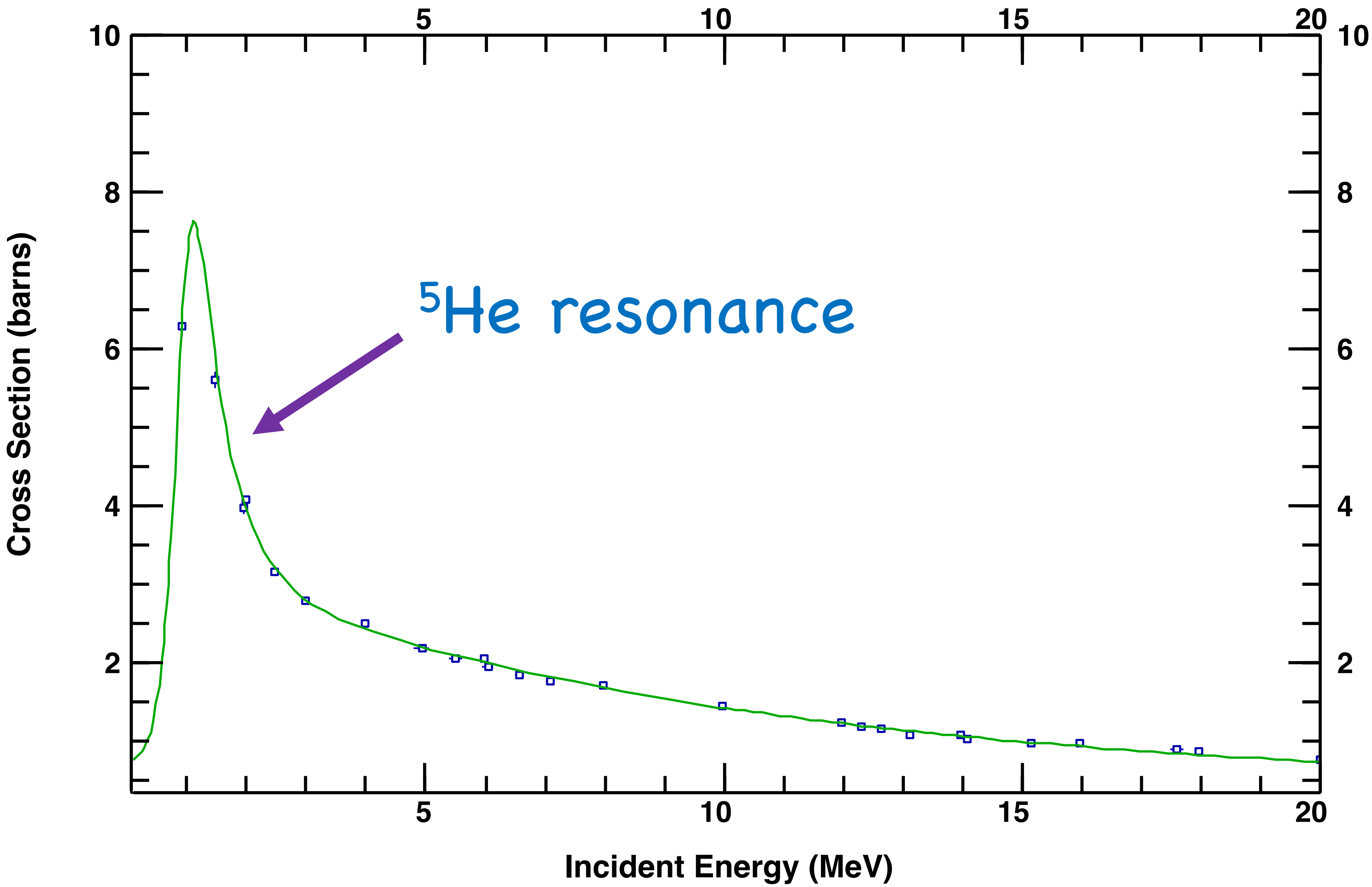
Unbound nuclear systems, resonances

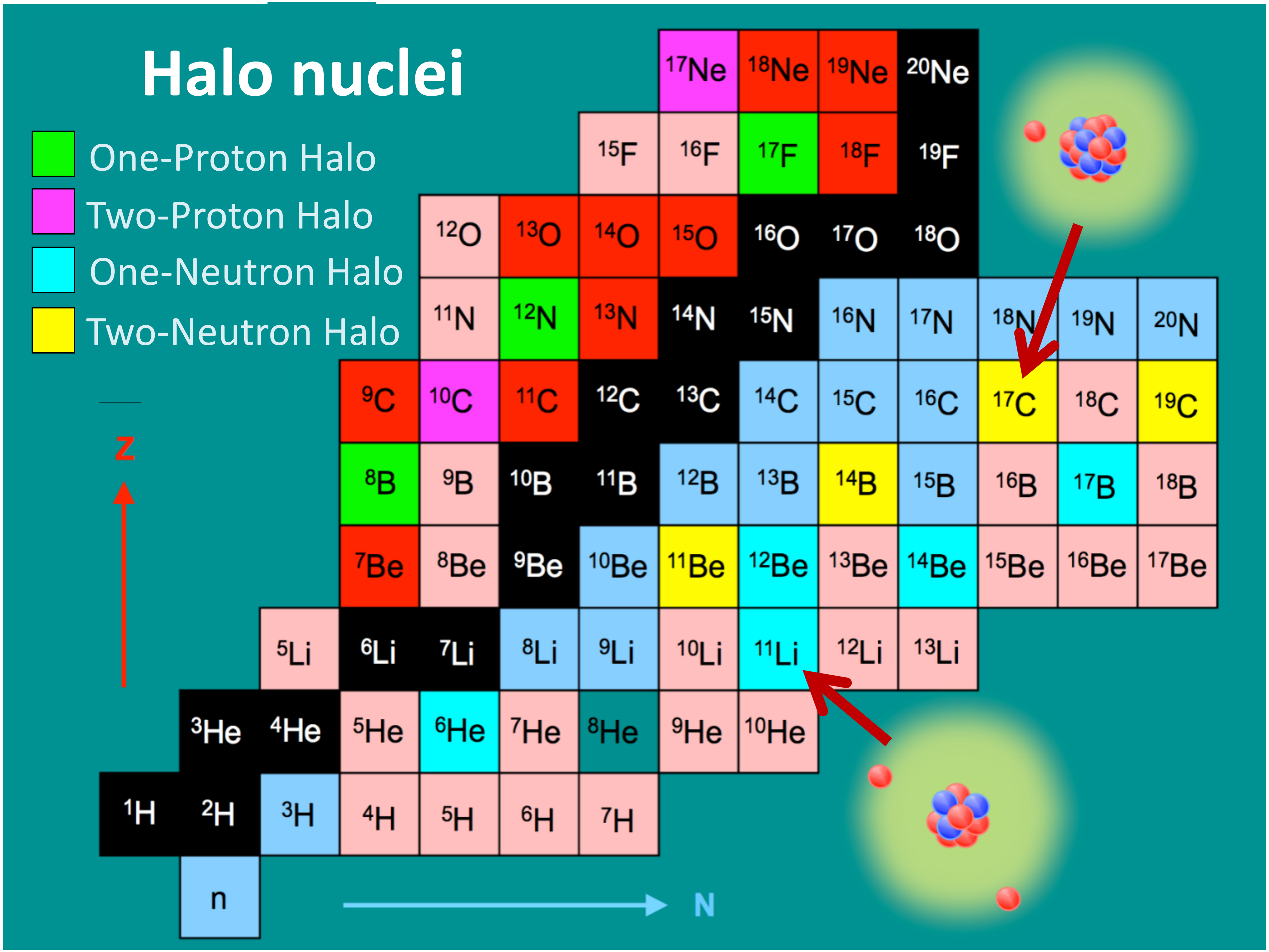


(a) A closed quantum system (b) An open quantum system

Adapted from J. Bengtsson, BS Thesis, Chalmers University

Elastic scattering of neutrons on ^{4}He

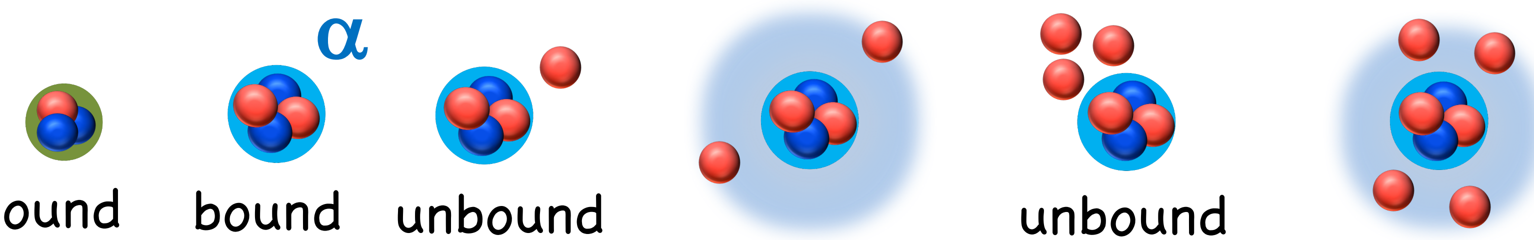




Halo nuclei
One-Proton Halo
Two-Proton Halo
One-Neutron Halo
Two-Neutron Halo
^{17}Ne ^{18}Ne ^{19}Ne ^{20}Ne
^{15}F ^{16}F ^{17}F ^{18}F ^{19}F
^{12}O ^{13}O ^{14}O ^{15}O ^{16}O ^{17}O ^{18}O
^{11}N ^{12}N ^{13}N ^{14}N ^{15}N ^{16}N ^{17}N ^{18}N ^{19}N ^{20}N
^{9}C ^{10}C ^{11}C ^{12}C ^{13}C ^{14}C ^{15}C ^{16}C ^{17}C ^{18}C ^{19}C
^{8}B ^{9}B ^{10}B ^{11}B ^{12}B ^{13}B ^{14}B ^{15}B ^{16}B ^{17}B ^{18}B
^{7}Be ^{8}Be ^{9}Be ^{10}Be ^{11}Be ^{12}Be ^{13}Be ^{14}Be ^{15}Be ^{16}Be ^{17}Be
^{5}Li ^{6}Li ^{7}Li ^{8}Li ^{9}Li ^{10}Li ^{11}Li ^{12}Li ^{13}Li
^{3}He ^{4}He ^{5}He ^{6}He ^{7}He ^{8}He ^{9}He ^{10}He
^{1}H ^{2}H ^{3}H ^{4}H ^{5}H ^{6}H ^{7}H
n
Z
N

The helium isotopes chain

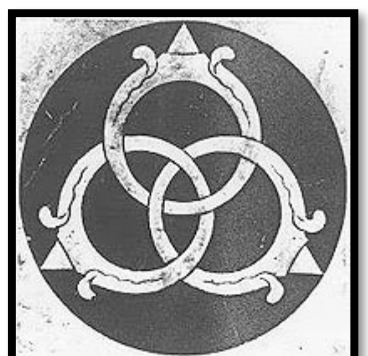
^{3}He ^{4}He ^{5}He ^{6}He ^{7}He ^{8}He



- ^{3}He: bound
- ^{4}He: bound
- ^{5}He: unbound
- ^{6}He: bound halo (806 ms)
- ^{7}He: unbound
- ^{8}He: bound halo (119 ms)

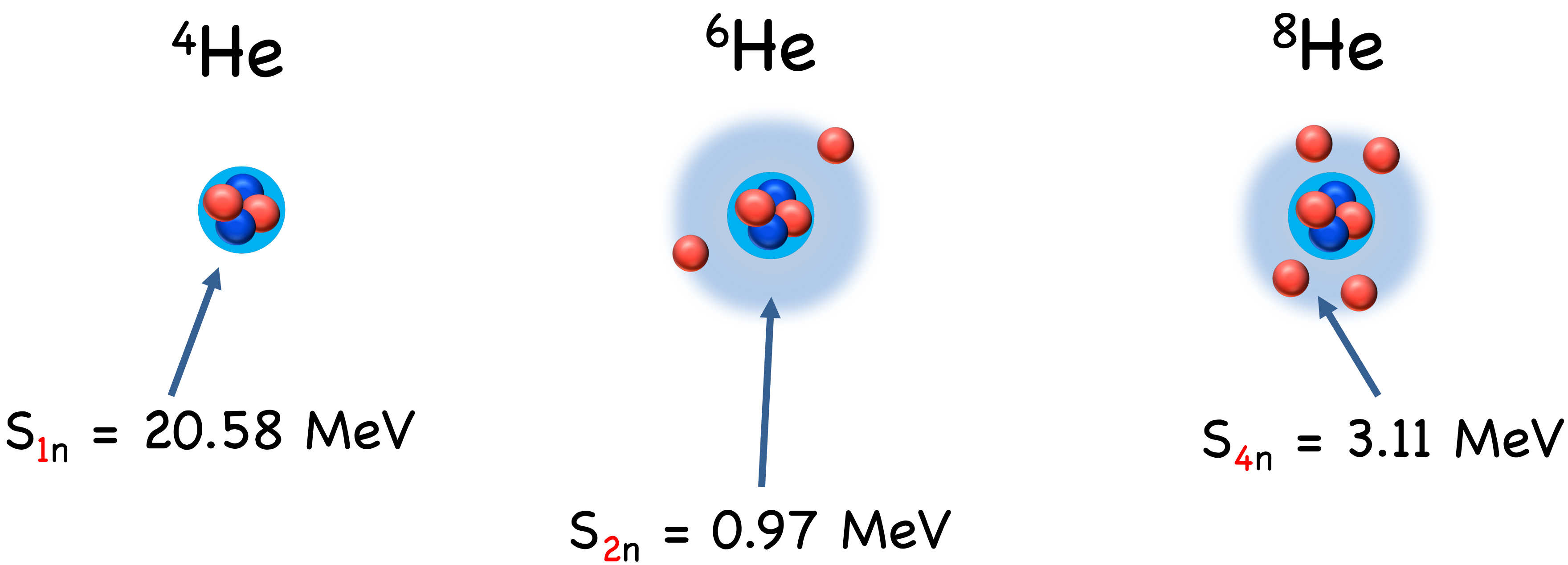
N/Z = 3 !

Borromean Halo

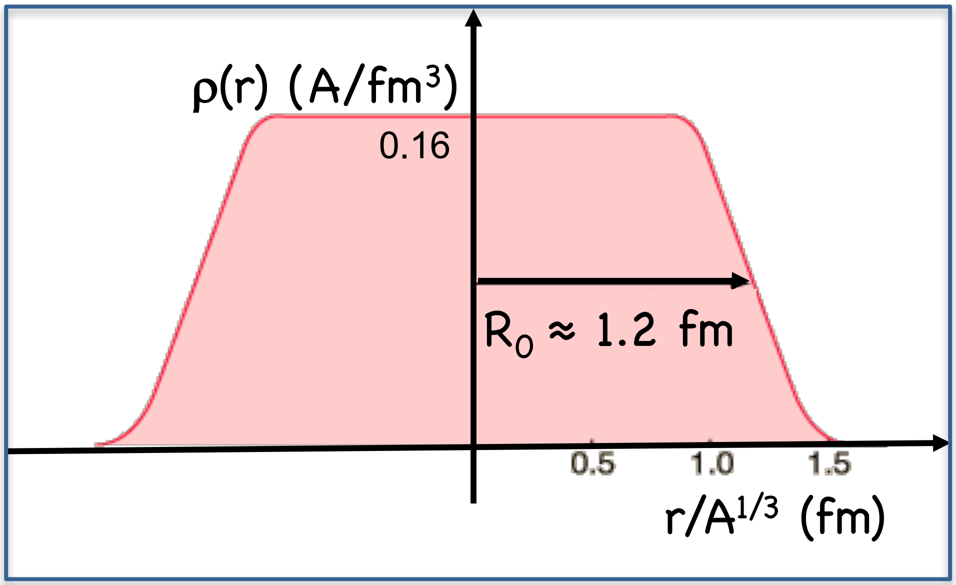


Separation energy: energy required to separate particle(s)

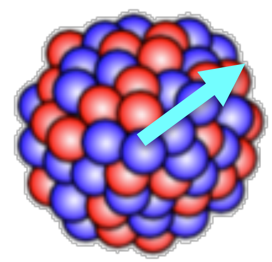
$$S_{an}(Z,N) = BE(Z,N) - BE(Z,N-a)$$



Nuclear sizes



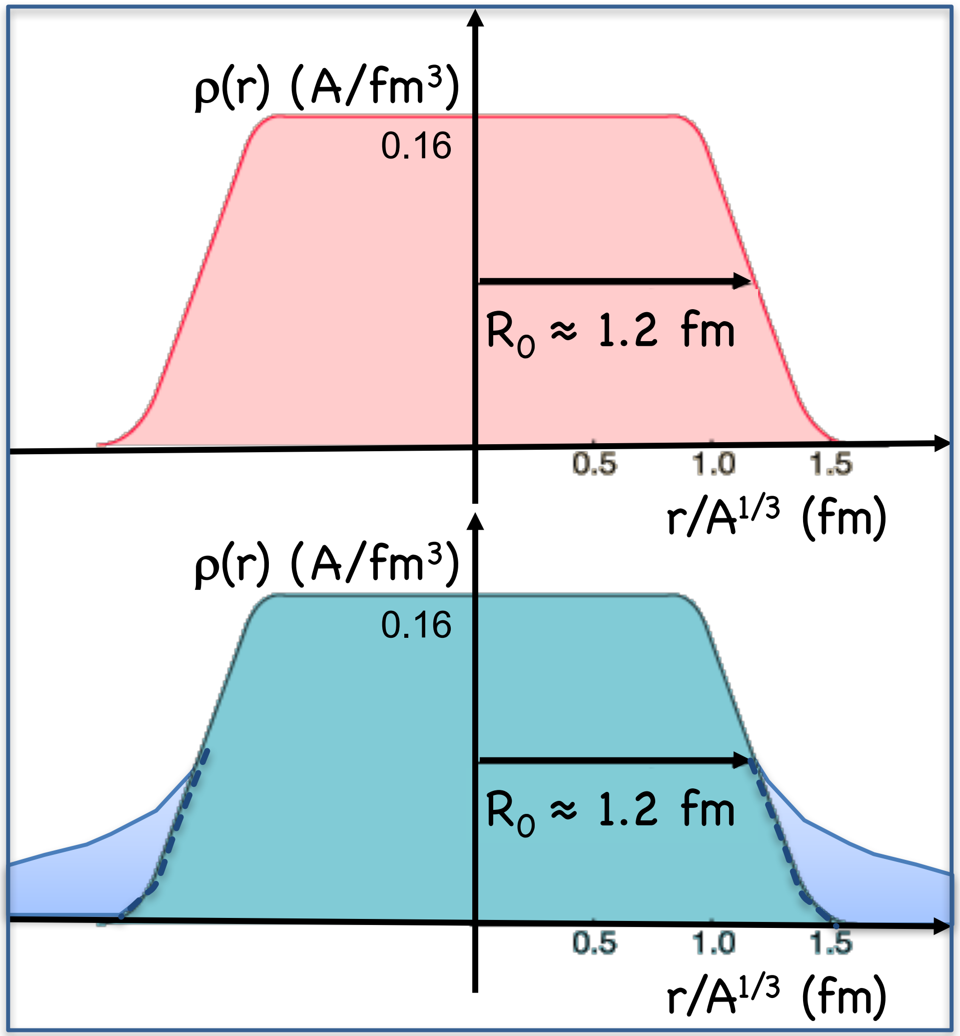
^{208}Pb



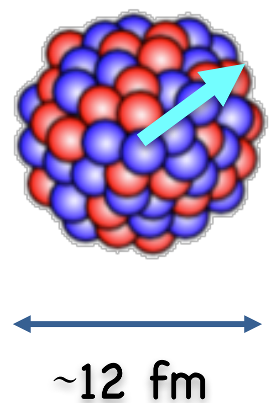
$\rho(r) \approx A/(4\pi r^3/3)$

$r \approx R_0 A^{1/3}$

Nuclear sizes



^{208}Pb



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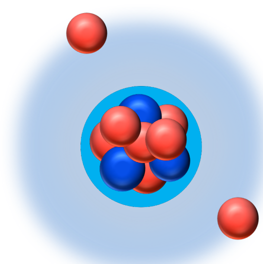
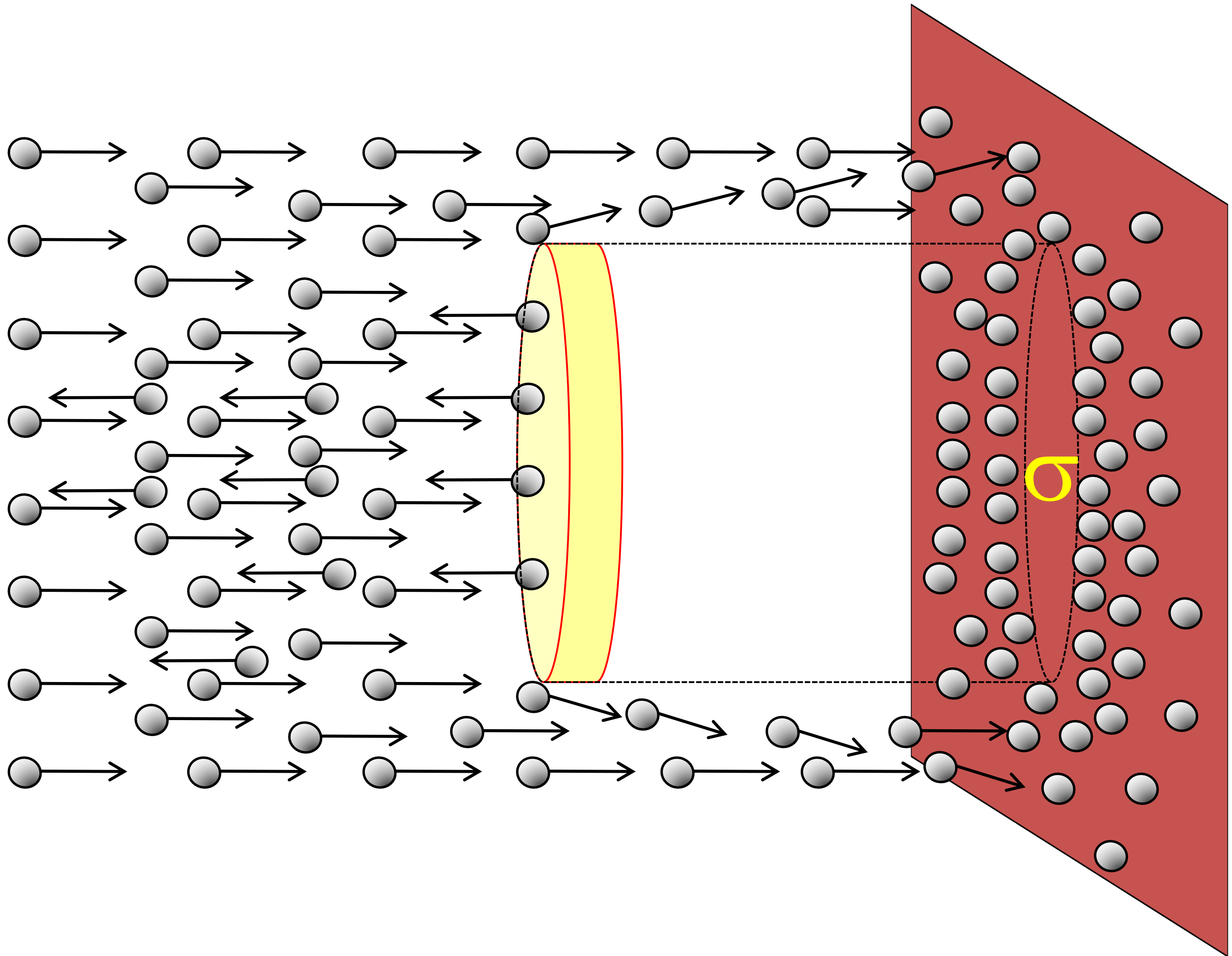


^{11}Li

Borromean
Halo

How do we measure nuclear sizes? Cross section (σ): a classical view



As big as a barn …

$$1 \text{ barn} = 10^{-28} \text{ m}^2 = 100 \text{ fm}^2$$

Summary

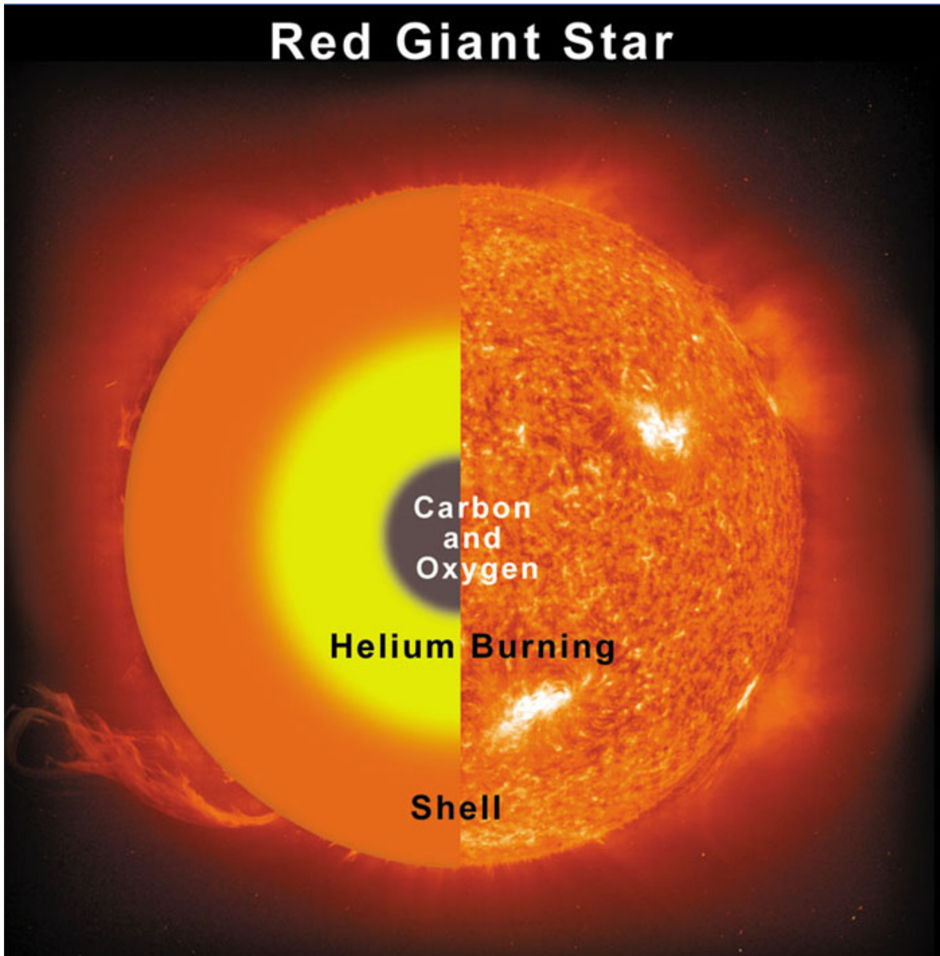
- The vast majority of light nuclei are either unstable or not bound
- Nuclear physics does not stop at binding energies and radii
- **All** observed nuclear phenomena can help us understand the basic nuclear interactions
- Light nuclei already display a wide variety of phenomena

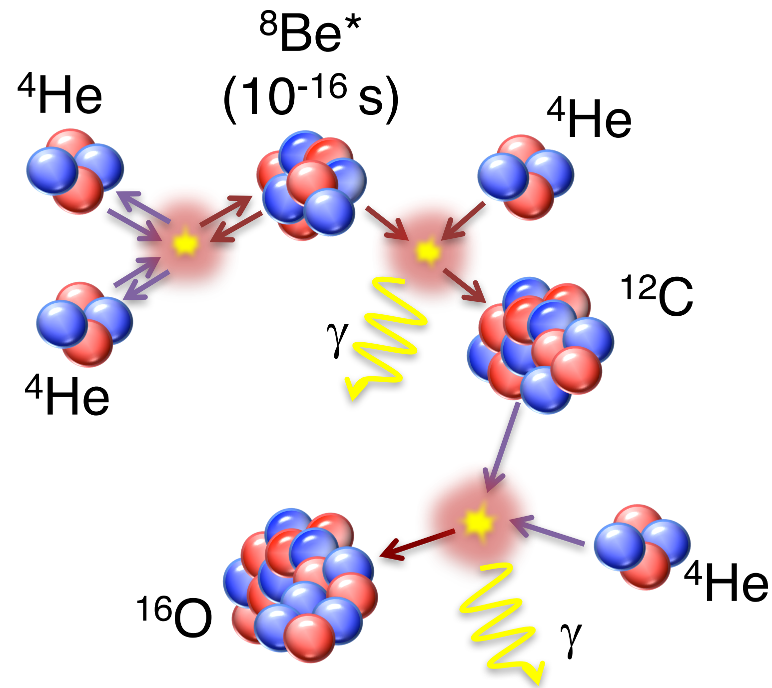
Questions

- Do you have any question so far?
- Form a group of 2 or 3 people and take a couple of minutes to discuss …

What is the role of light and unbound nuclei in the Cosmos and on Earth?

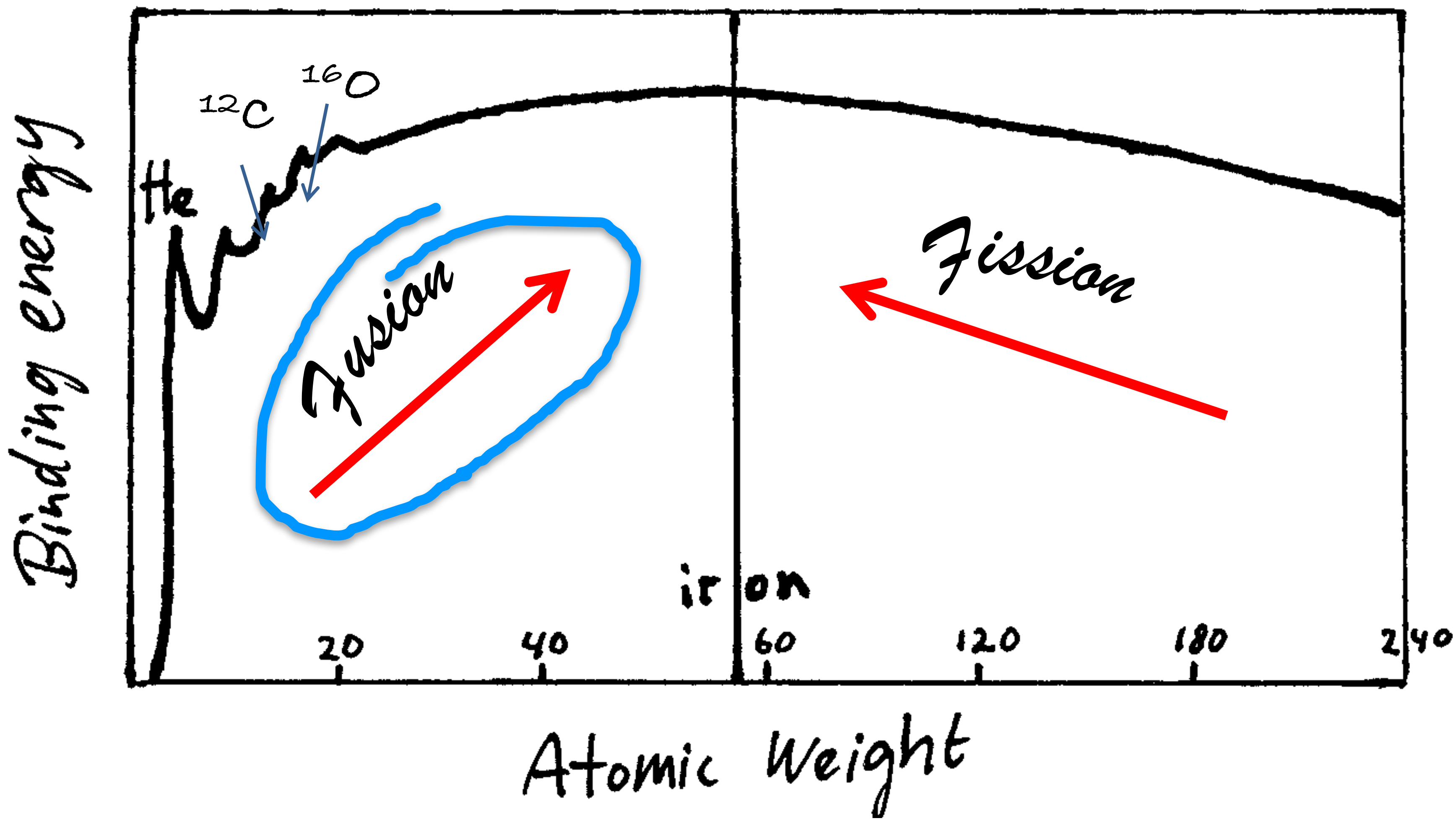
Reactions 'R' Us





From light and unbound nuclei to the the chemical building blocks of life, to the processes that shaped our Universe

Stars are powered by thermonuclear fusion reactions

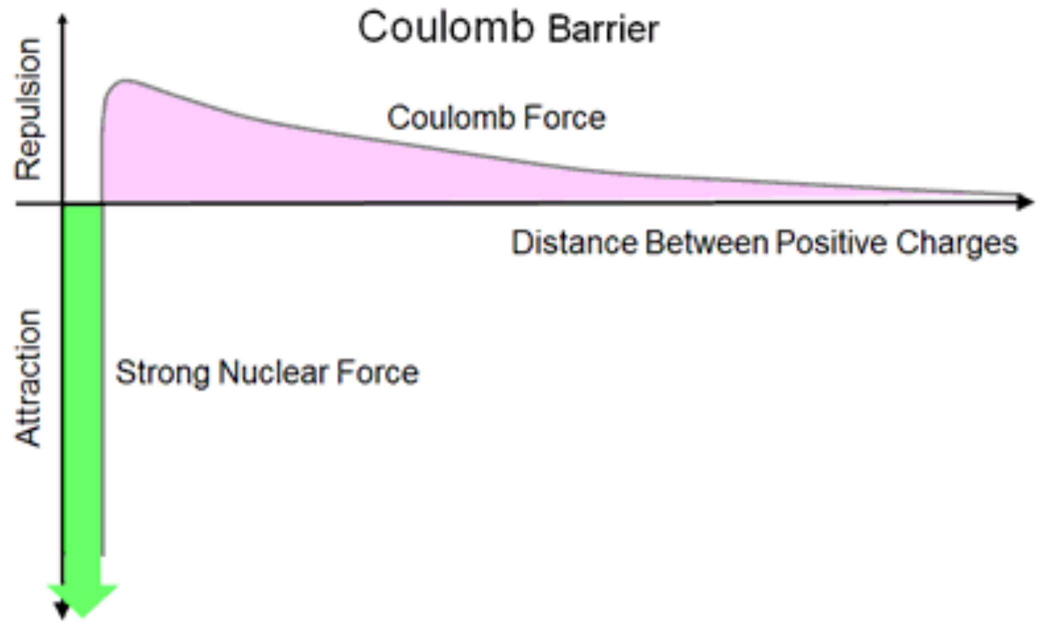


Cross section of fusion reactions at stellar energies are very small !

- Positively-charged colliding nuclei electrically repel each other

Quantum tunneling

- Fusion process operates mainly by tunneling through the Coulomb barrier



Fusion cross sections drop nearly exponentially with decreasing energy

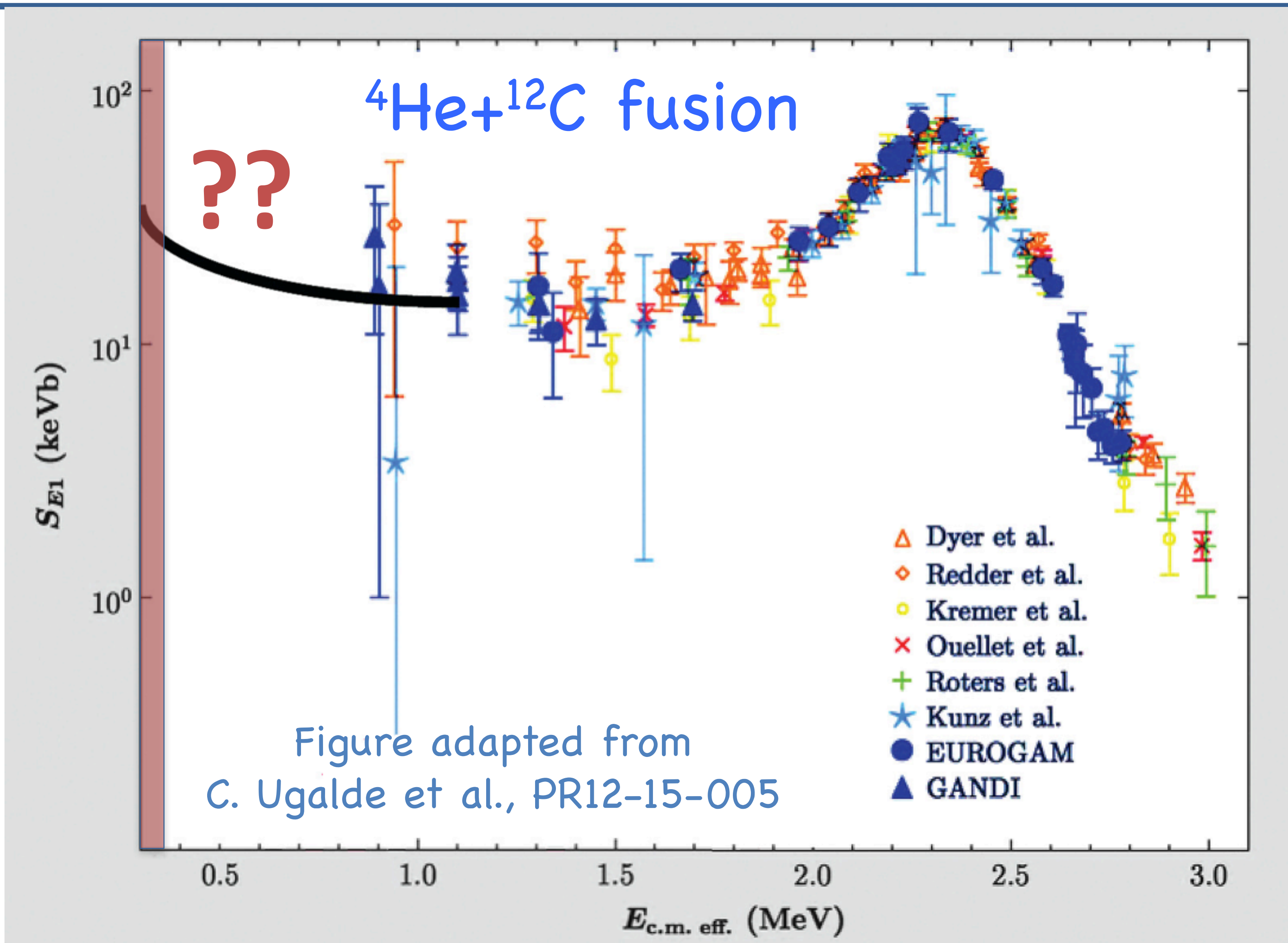
Astrophysical S-factor: nuclear contribution

Fusion cross section

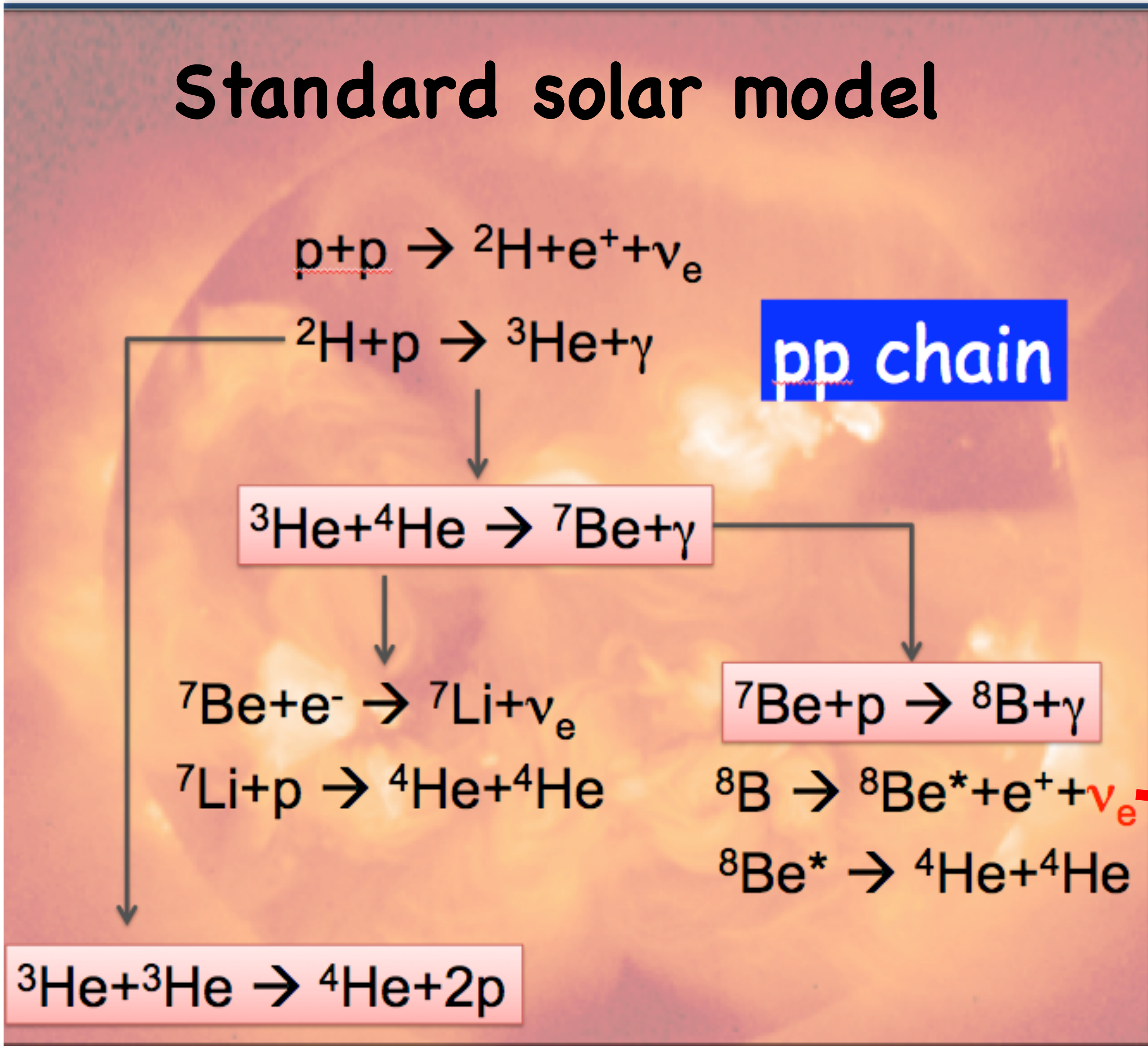
$$\sigma(E) = \frac{S(E)}{E} \exp\left(-\frac{2\pi Z_1 Z_2 e^2}{\hbar\sqrt{2E/m}}\right)$$

'Coulomb' Contribution (tunneling)

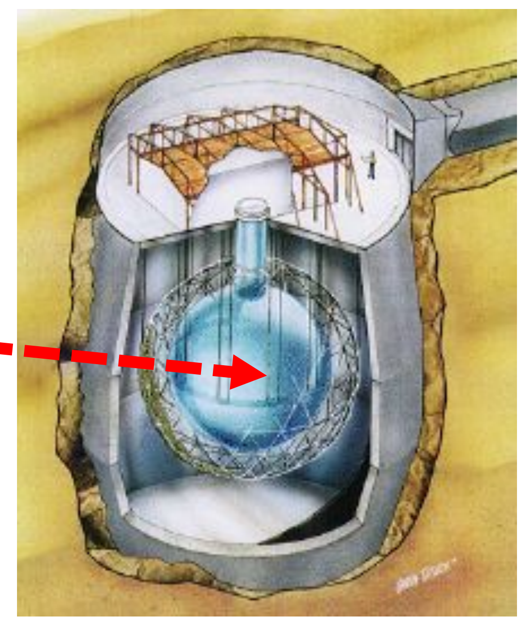
We need reliable theory to estimate the S-factor at stellar energies



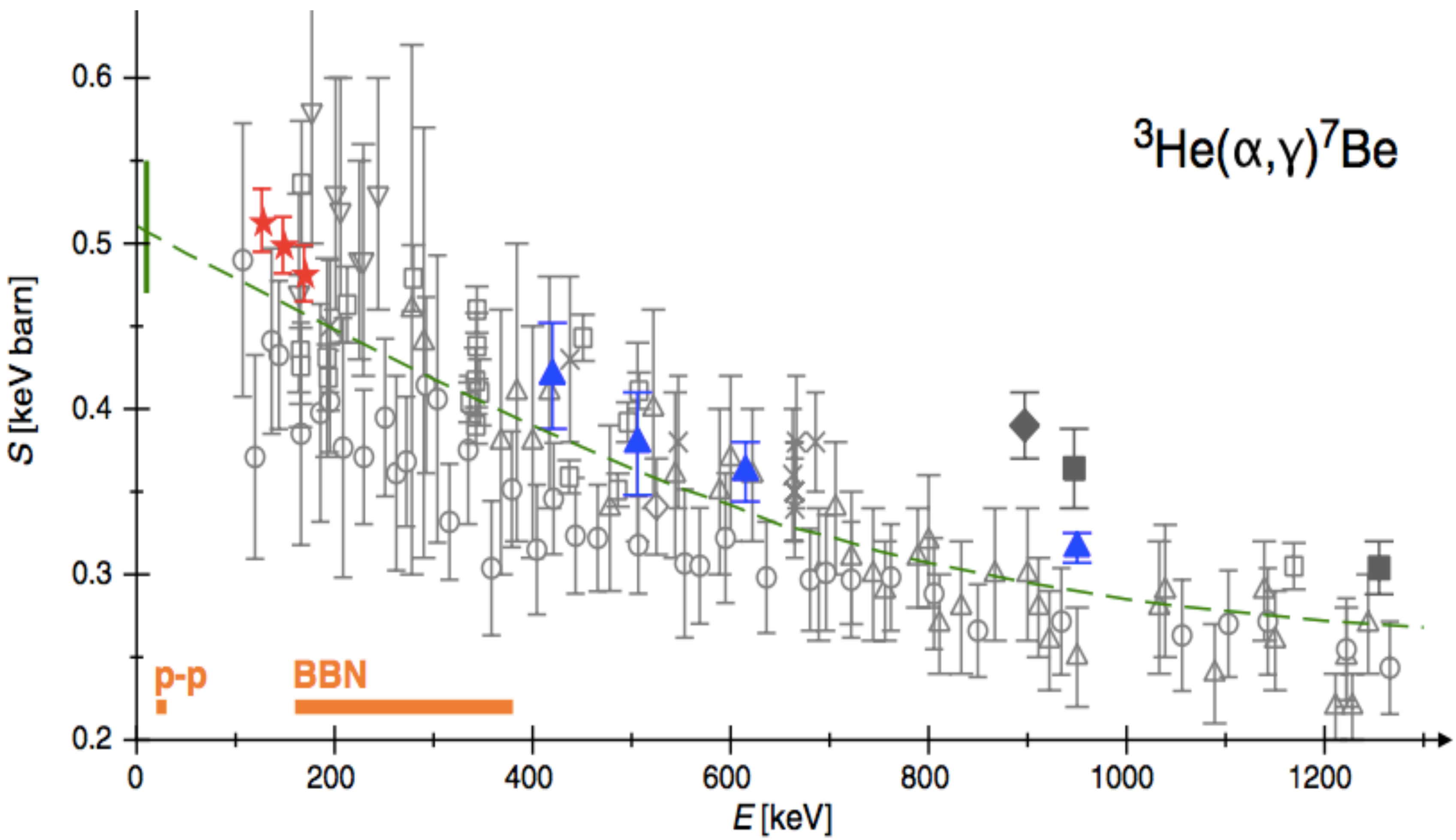
Our Sun: one of the best tools for studying neutrinos



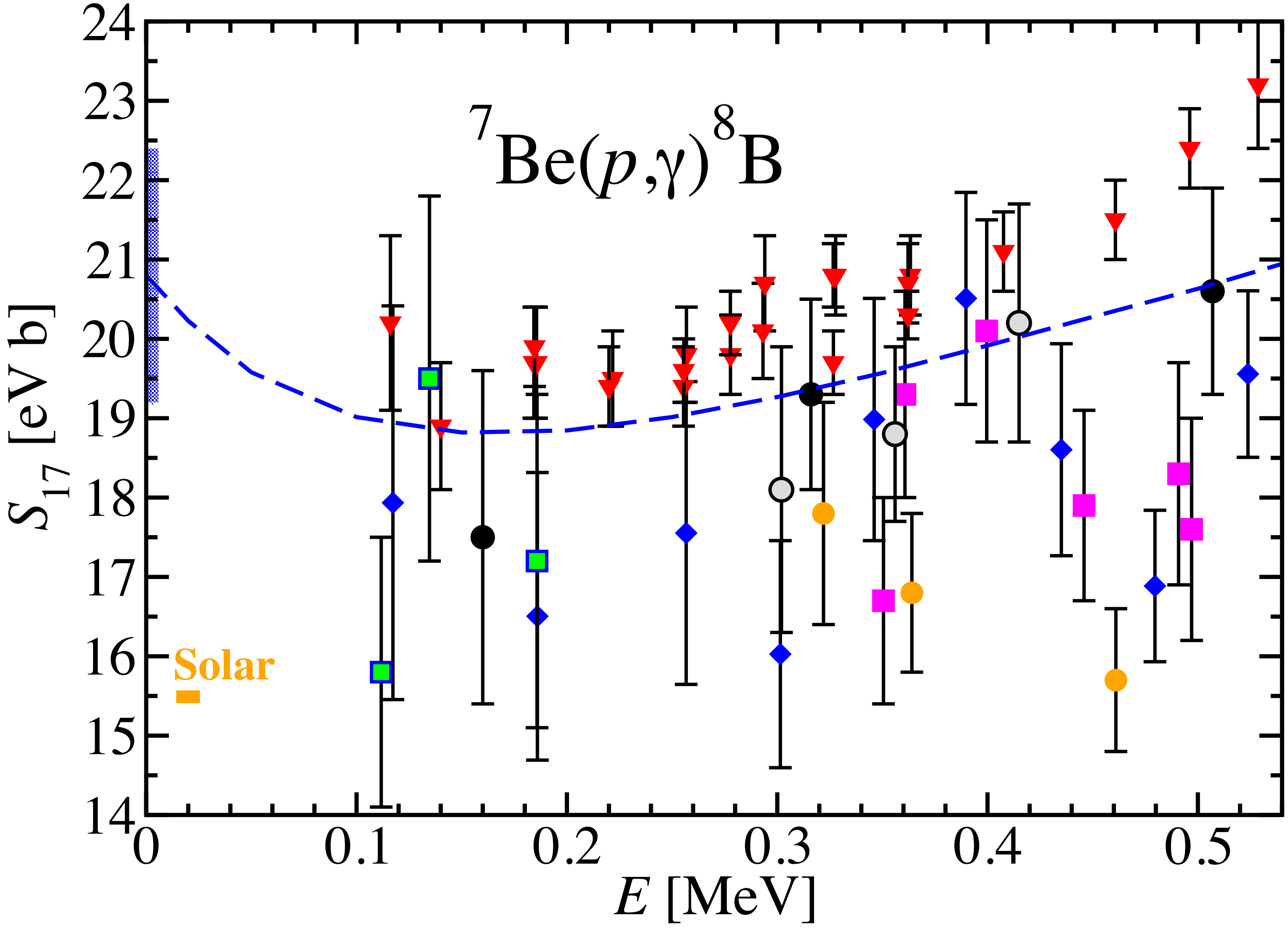
Neutrino oscillations
2015 Noble Prize in Physics



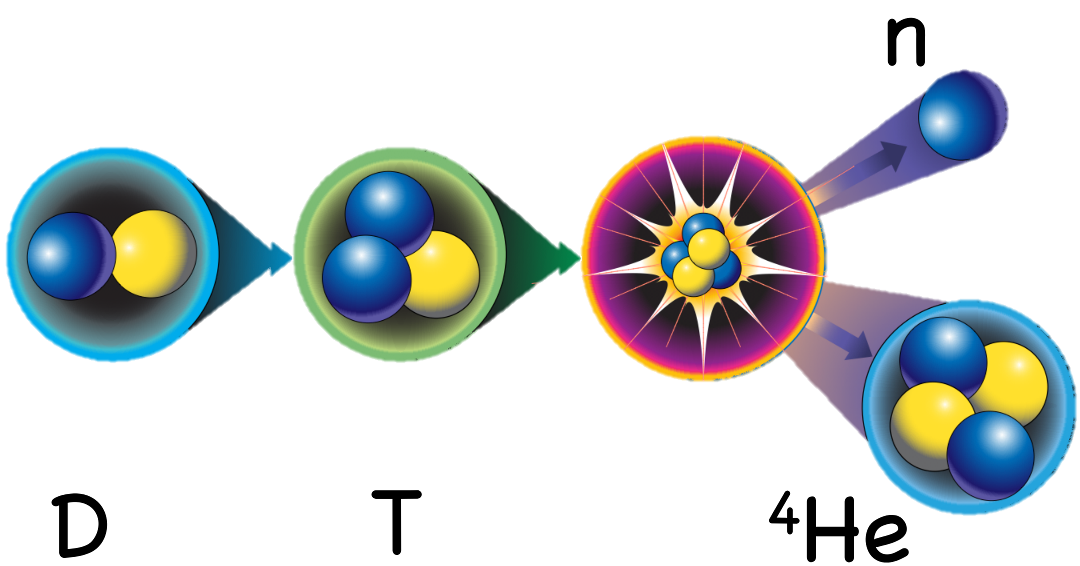
Uncertainties in solar fusion S-factors



Uncertainties in solar fusion S-factors



Fusion energy generation

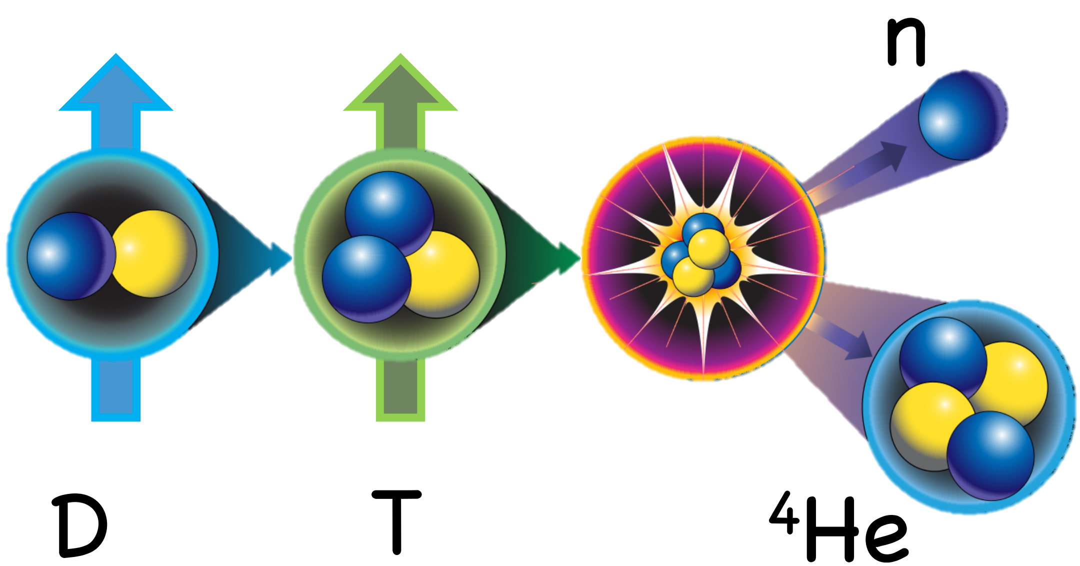


Symbol	BE (MeV)
^{4}He	28.30
^{3}H or T	– 8.48
^{2}H or D	– 2.22

= +17.6 !

- Laser confinement experiments
- Magnetic confinement experiments (ITER)

Fusion energy generation



Symbol	BE (MeV)
^{4}He	28.30
^{3}H or T	– 8.48
^{2}H or D	– 2.22

= +17.6 !

- By perfectly aligning the spins of D and T estimated 50% enhancement of reaction rate
- How does the rate depend on polarization?

Summary

- Light nuclei are the building blocks of life and the universe as we know it
- Ongoing attempts to harness energy from thermonuclear fusion reactions
- Fusion reactions are extremely difficult to measure at stellar energies
- Predictive theory of fusion reactions needed to help extrapolate down to stellar energies

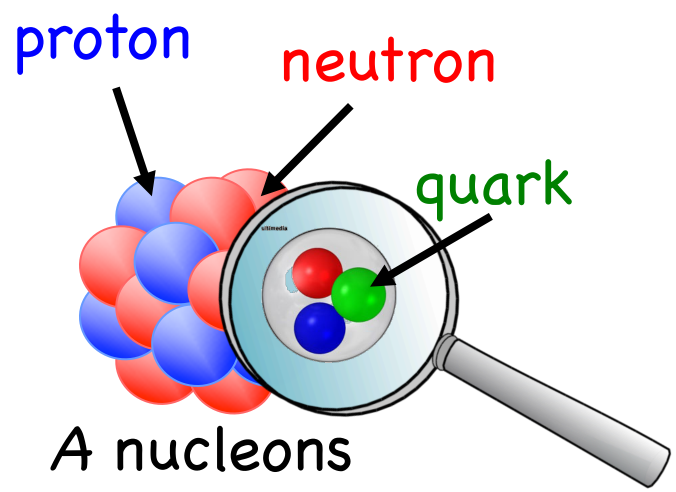
Questions

- Do you have any questions on this section?
- Question for you:
  - How much energy is released from the fusion of two ^2H nuclei?
- Form a group of 2 or 3 people and take a couple of minutes to discuss …

How can we learn about the basic nuclear interactions?

Can we accurately explain …

- … how stable nuclei and rare isotopes are put together from the neutron and proton constituents?
- In terms of:
  a) The laws of quantum mechanics
  b) The underlying theory of the strong force (quantum chromodynamics)



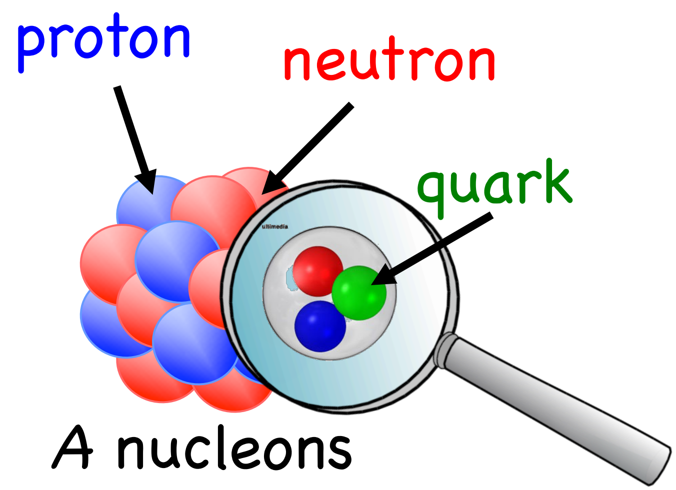
The problem

How do we describe these A-nucleon wave functions?

$$H\Psi = E\Psi$$

How do we describe the interactions among nucleons in this A-nucleon Hamiltonian?

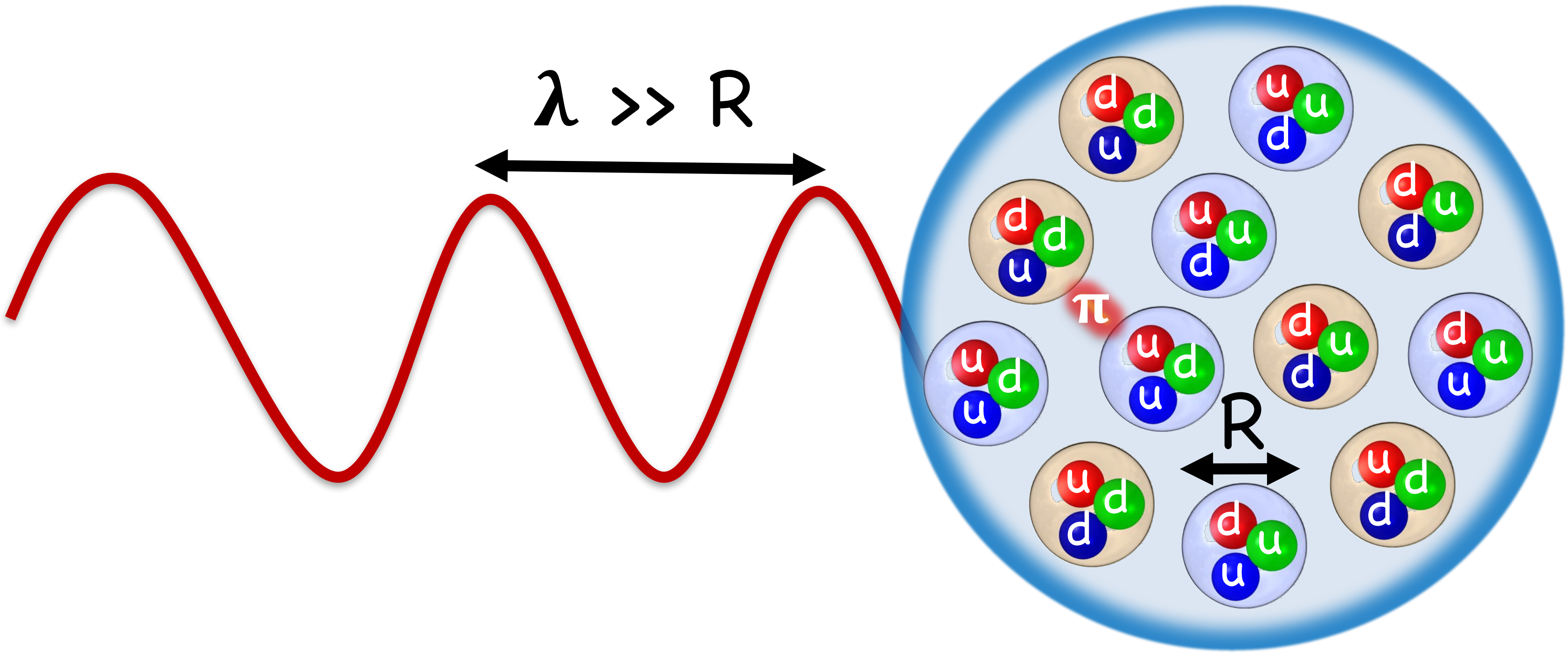
How do we solve this equation ``exactly" ?



How to describe the interactions among nucleons?

Separation of scales

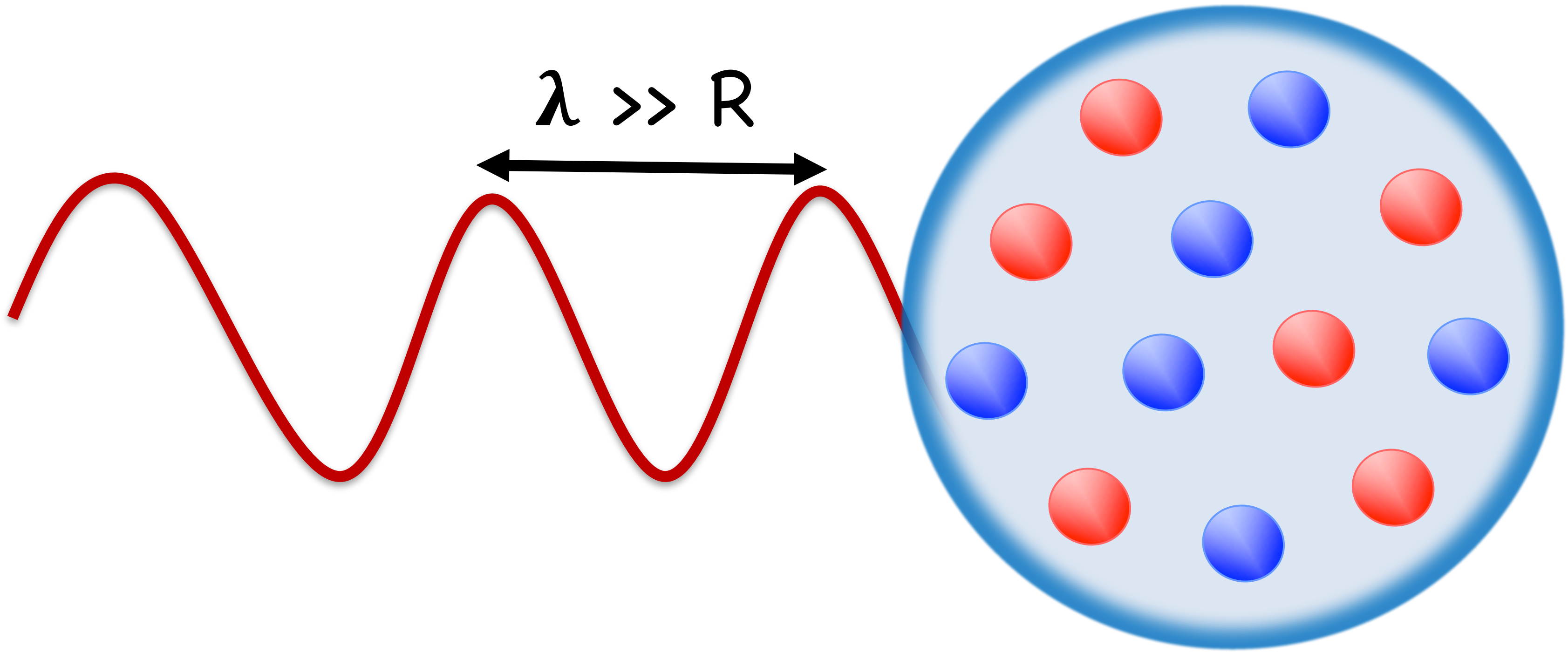
- At low energy short-distance physics is not resolved



$$1/\lambda = Q \ll \Lambda = 1/R$$

Separation of scales

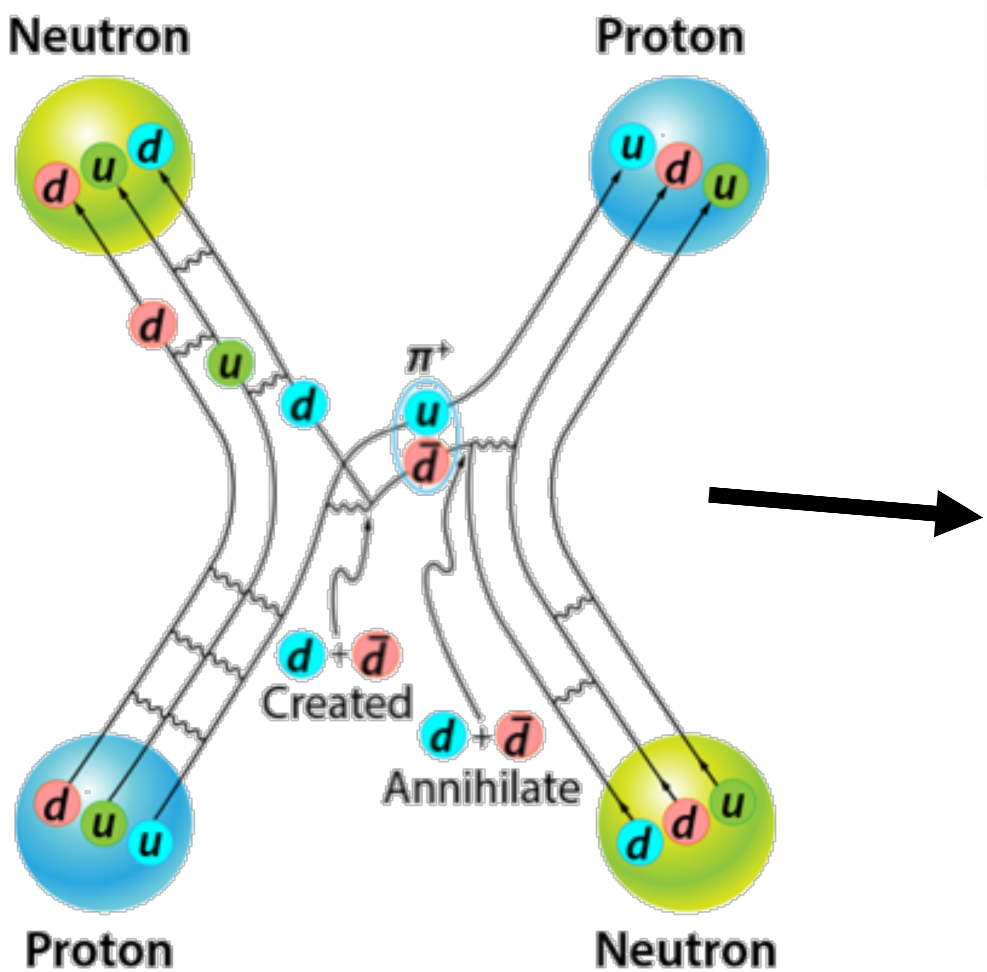
- At low energy short-distance physics is not resolved



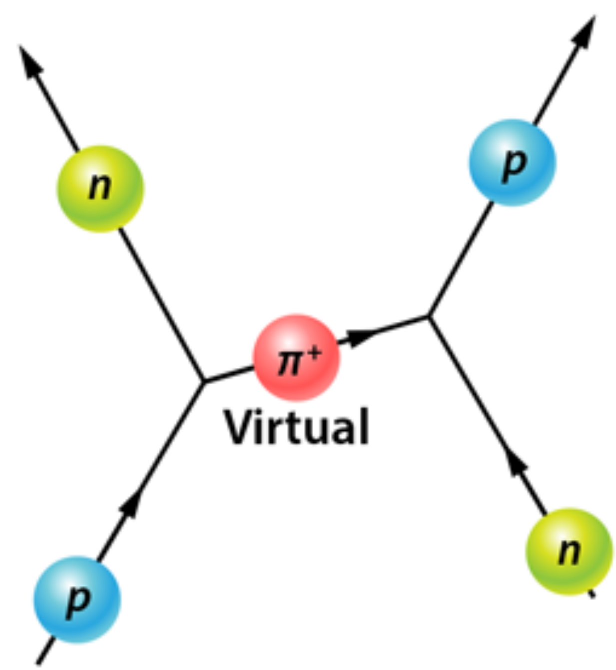
$1/\lambda = Q \ll \Lambda = 1/R$

Nucleon-nucleon interaction

Quantum Chromodynamics



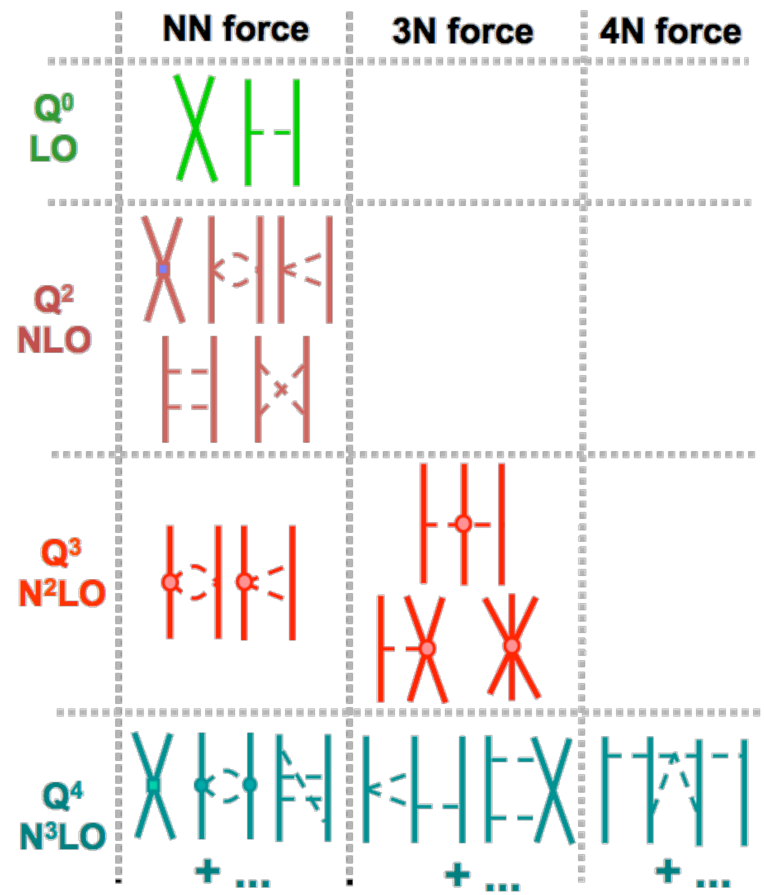
Chiral Effective Field Theory



Chiral effective field theory has transformed the way we think about and treat nuclear forces

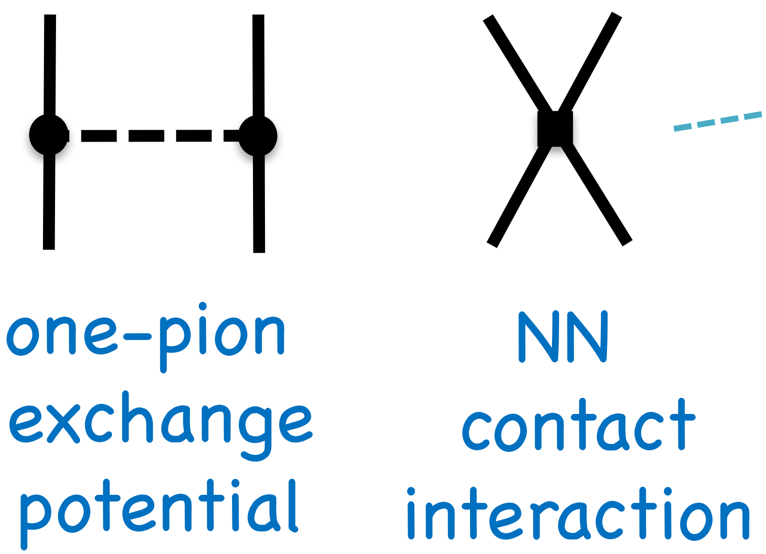
Links the nuclear forces to the fundamental theory of quantum chromodynamics (QCD)

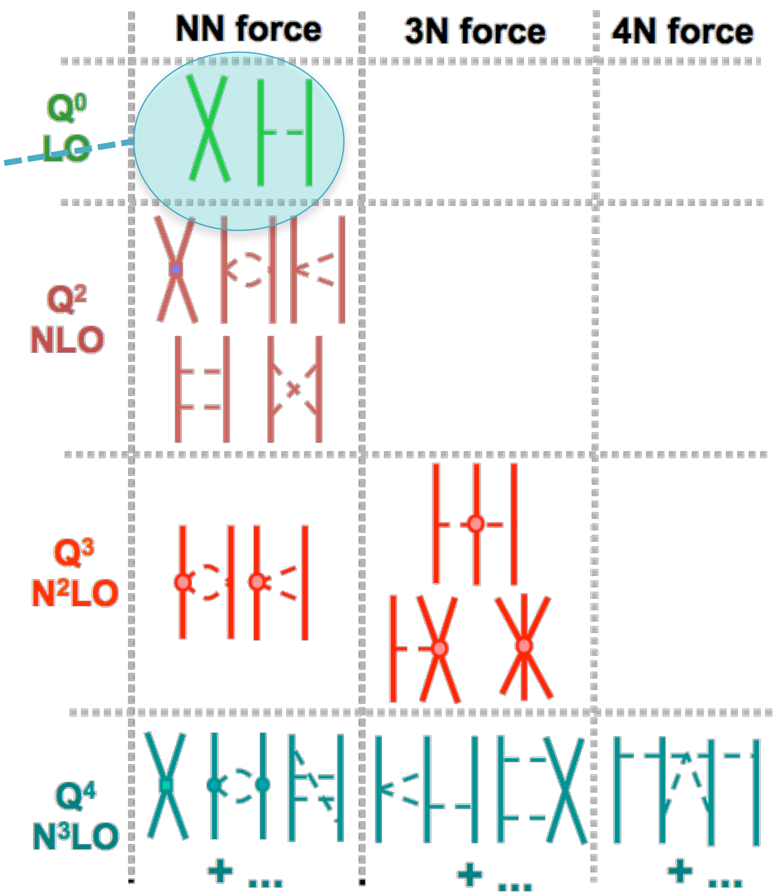
- organization in systematically improvable expansion: $(Q/\Lambda)^{\nu}$
- empirically constrained parameters capture unresolved short-distance physics



Worked out by Van Kolck, Keiser, Meissner, Epelbaum, Machleidt, ...

At leading order long-range NN interaction is usual one-pion exchange





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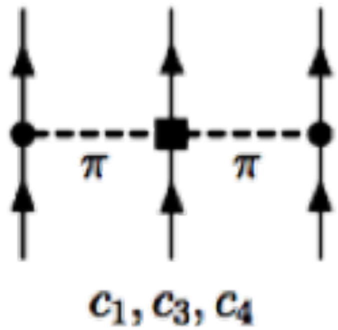
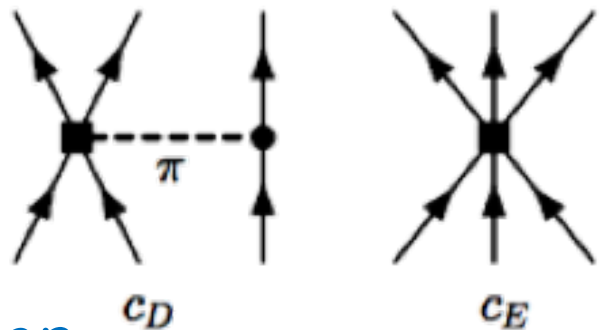
Three-nucleon forces appear at N^2LO

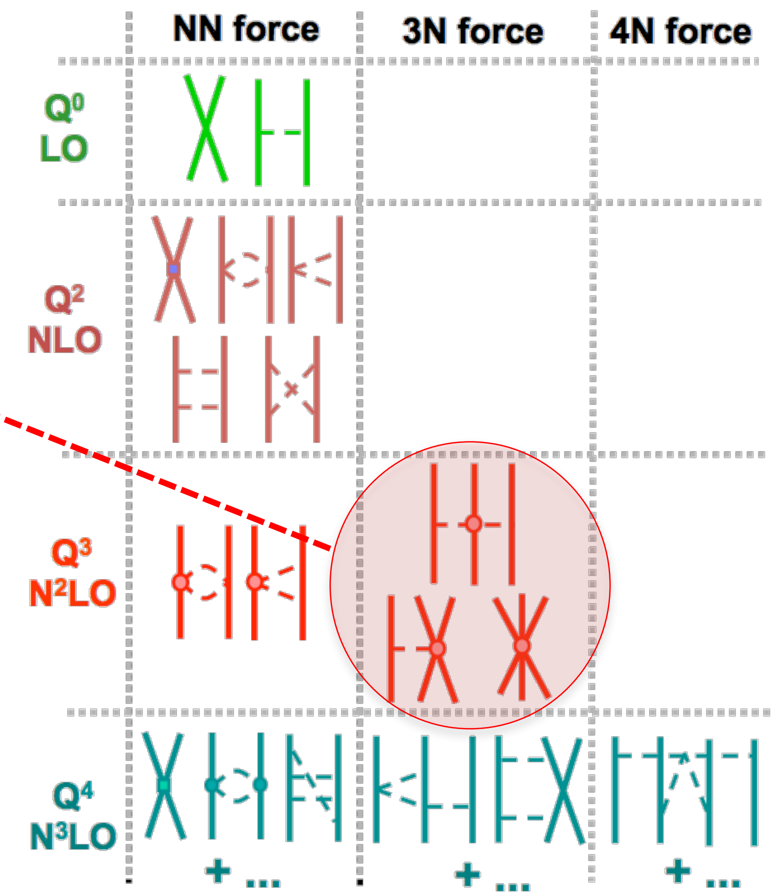
two-pion exchange potential





one-pion exchange + NN contact

NNN contact interaction

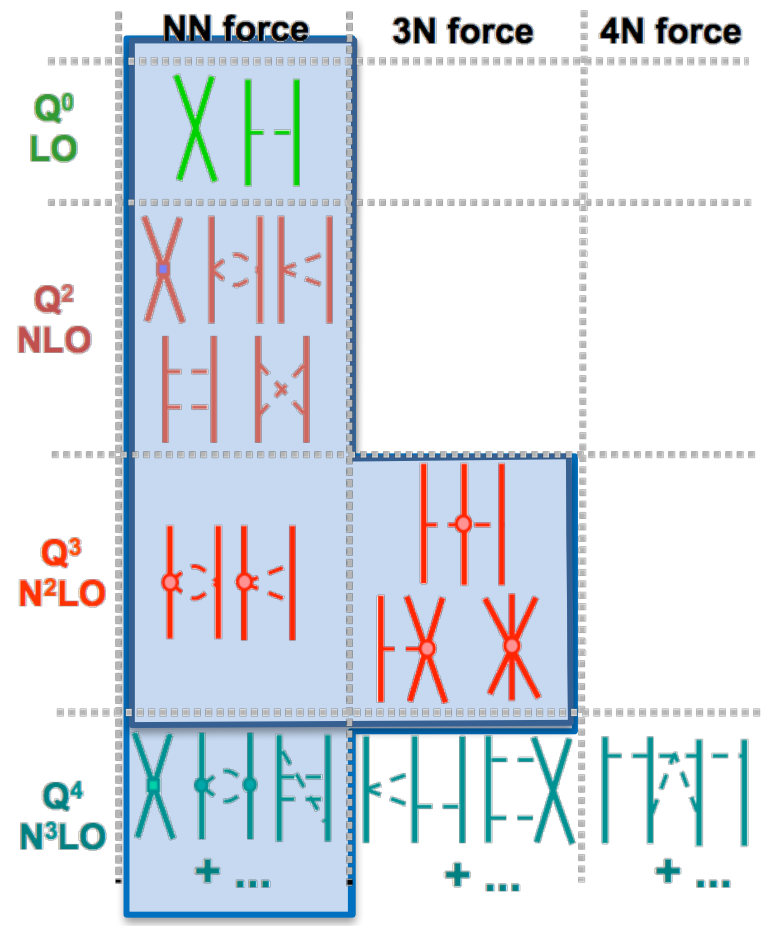


Worked out by Van Kolck, Keiser, Meissner, Epelbaum, Machleidt, ...

How to best implement the theory and constrain it is an active topic or research

Nomenclature/Parameterizations:

- **NN:** potential at N^3LO, 500 MeV cutoff (by Entem & Machleidt)
- **NN+3N(500):** NN plus 3N force at N^2LO, 500 MeV cutoff (local form by Navrátil)
- **NN+3N(400):** NN plus 3N forace at N^2LO, 400 MeV cutoff (local form by Navrátil)
- **N^2LOsat :** NN+3N at N^2LO, fitted simultaneously (by Ekström et al.)
- …



Worked out by Van Kolck, Keiser, Meissner, Epelbaum, Machleidt, ...

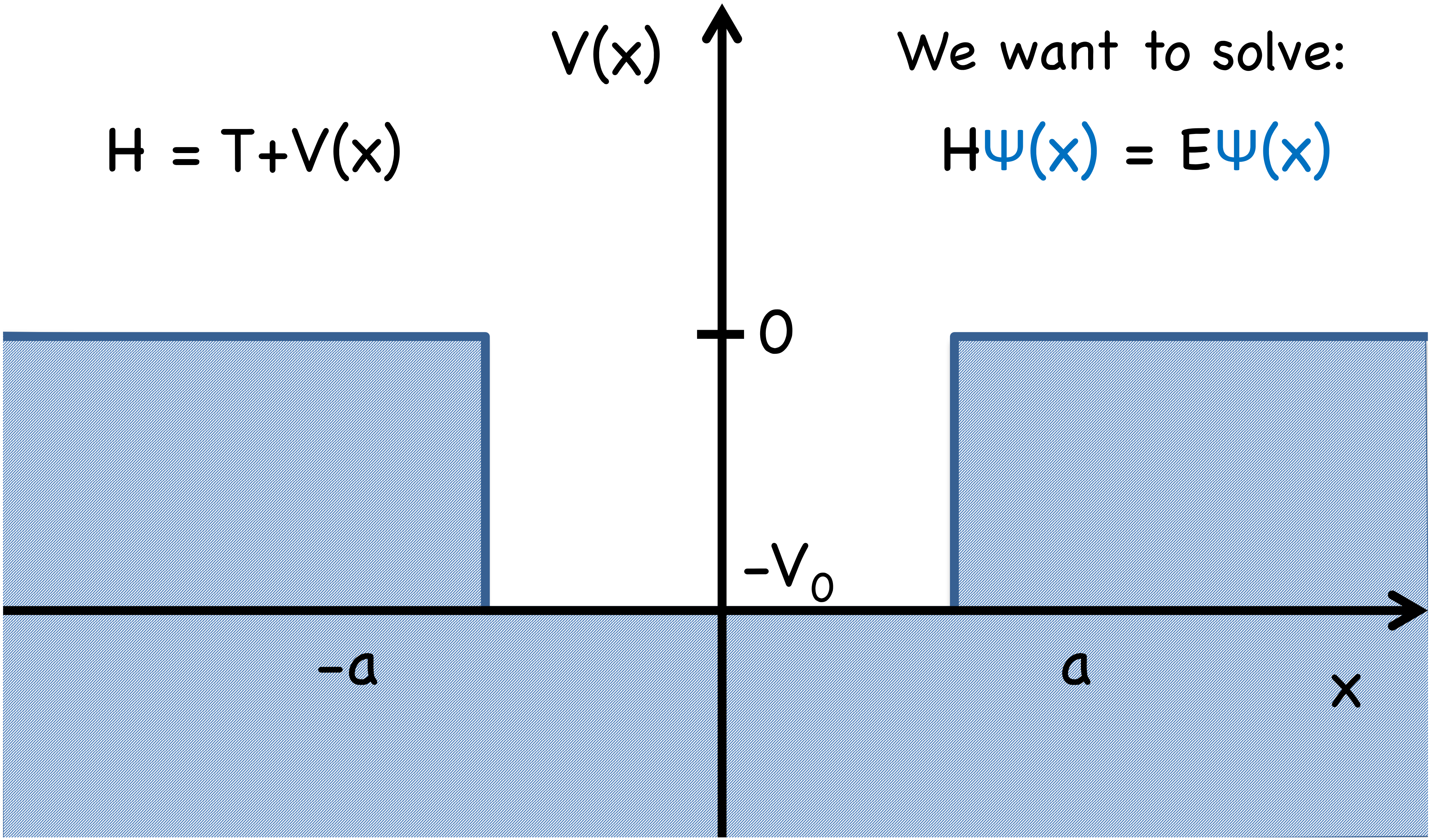
How to solve the Schrödinger equation?

Well-known example: one particle in 1D, finite square-well potential

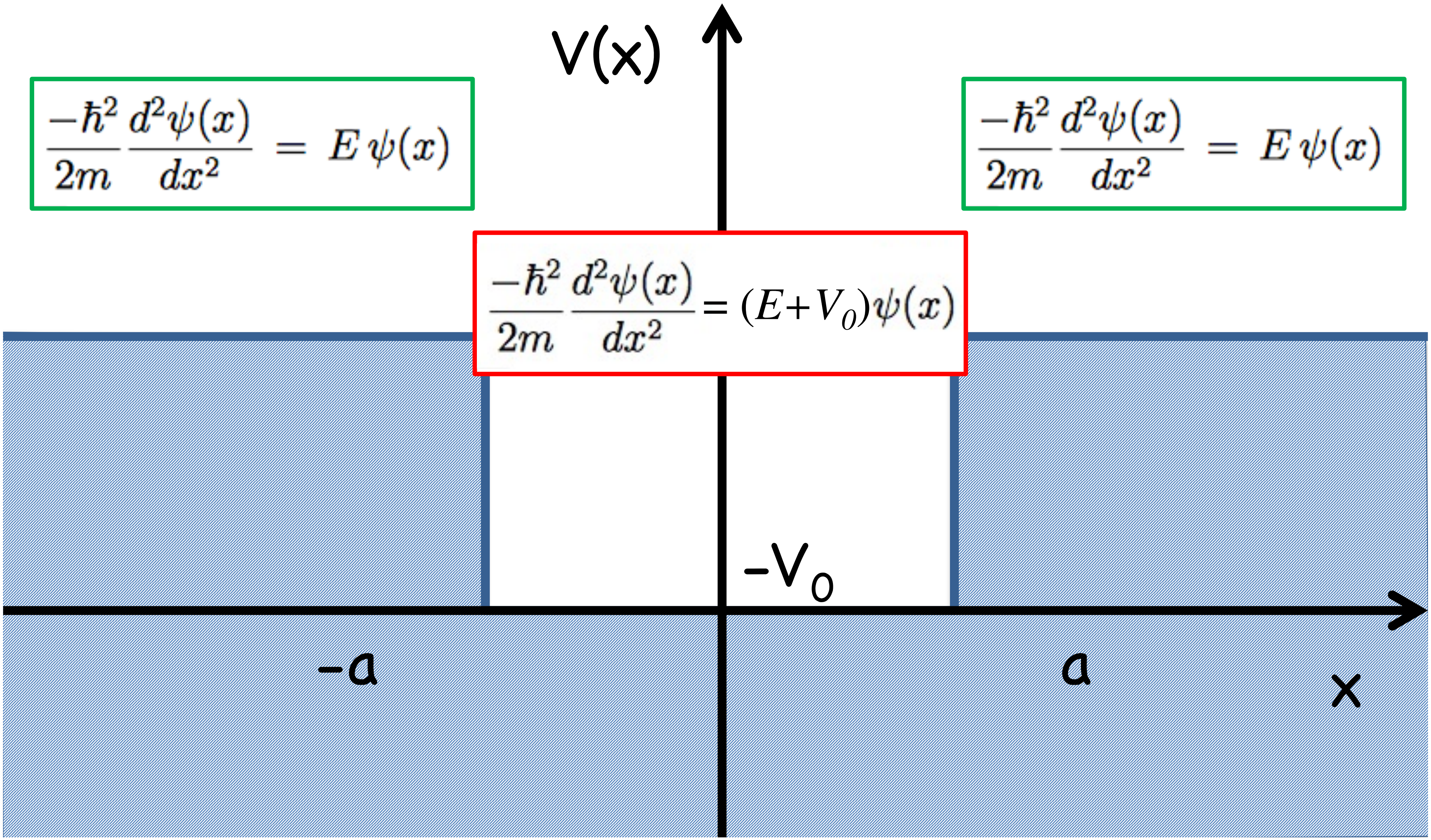
$H = T + V(x)$

We want to solve:

$H\Psi(x) = E\Psi(x)$



Well-known example: one particle in 1D, finite square-well potential

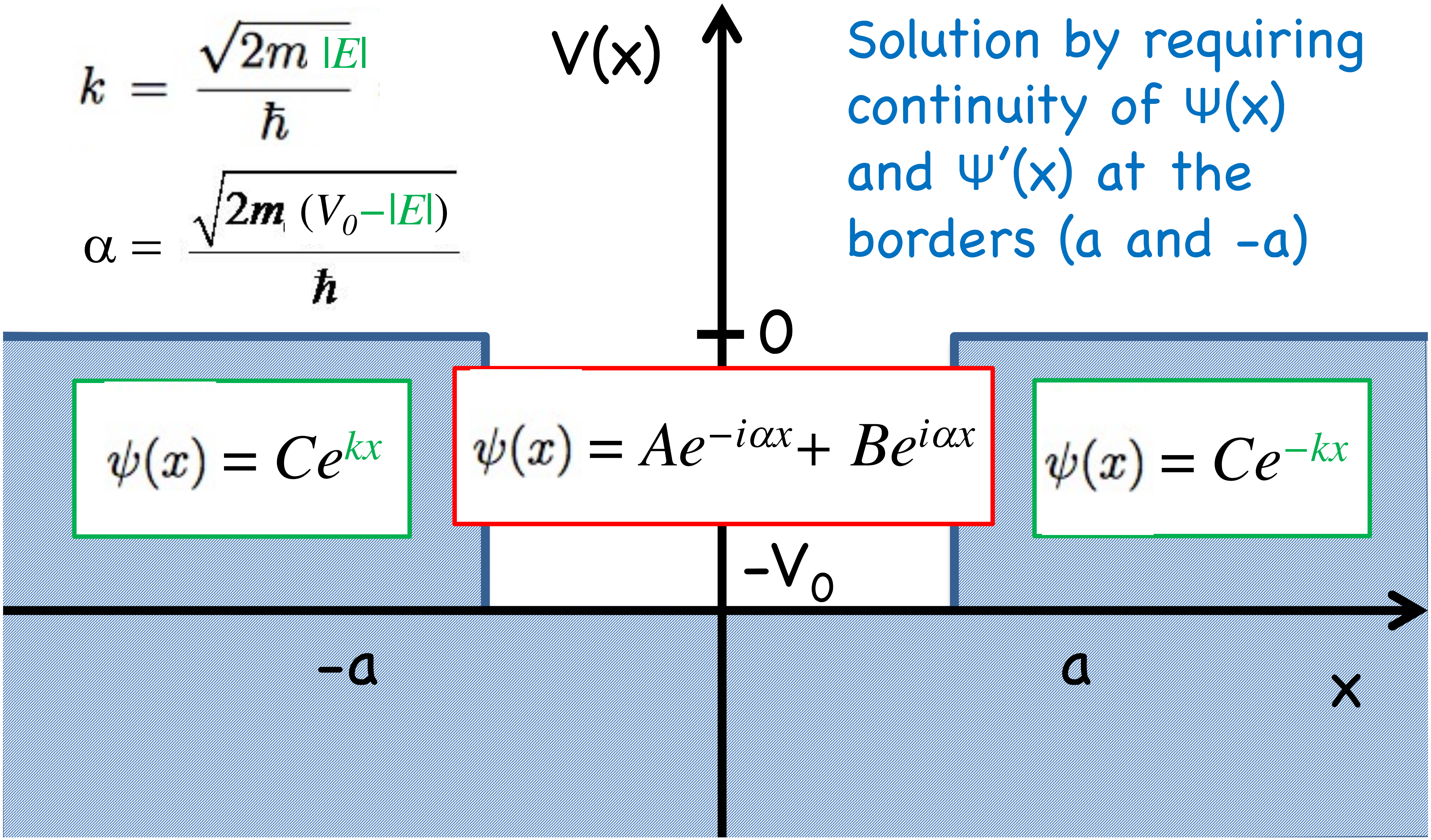


1D finite square-well potential: E<0

$$k = \frac{\sqrt{2m\,|E|}}{\hbar}$$

$$\alpha = \frac{\sqrt{2m\,(V_0 - |E|)}}{\hbar}$$

Solution by requiring continuity of Ψ(x) and Ψ′(x) at the borders (a and -a)

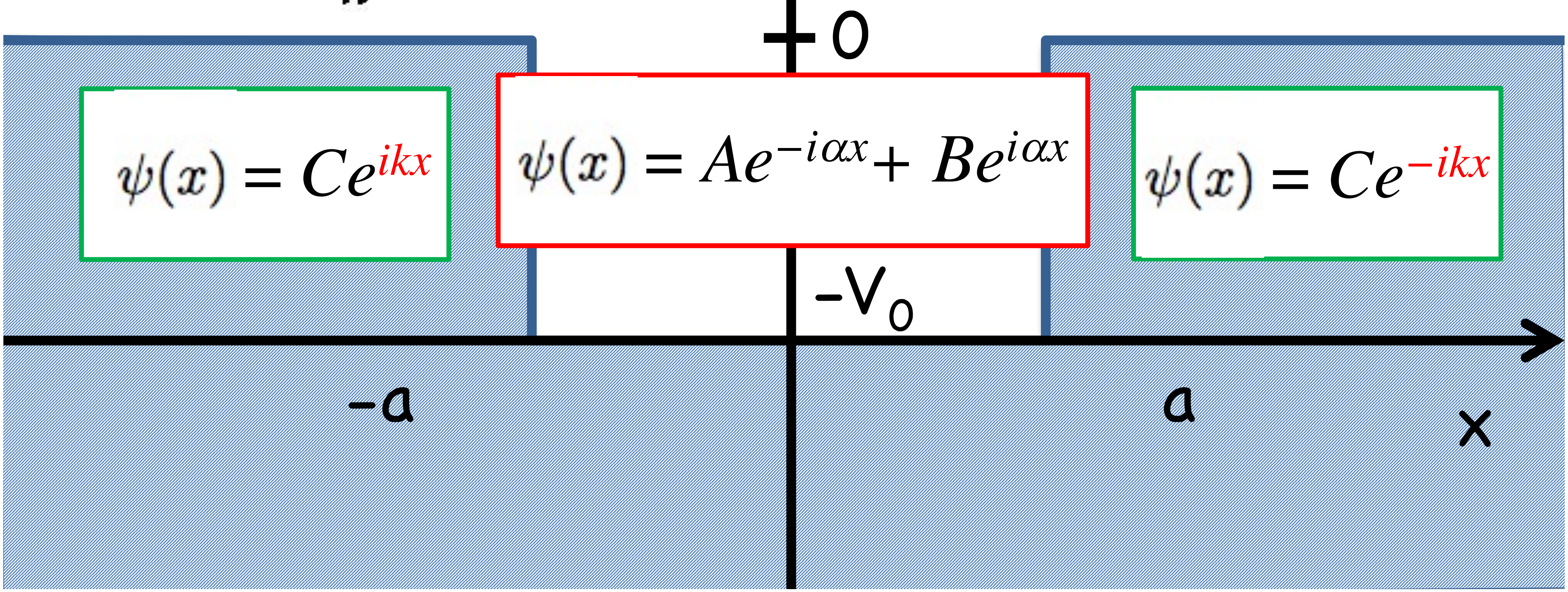


1D finite square-well potential: E>0

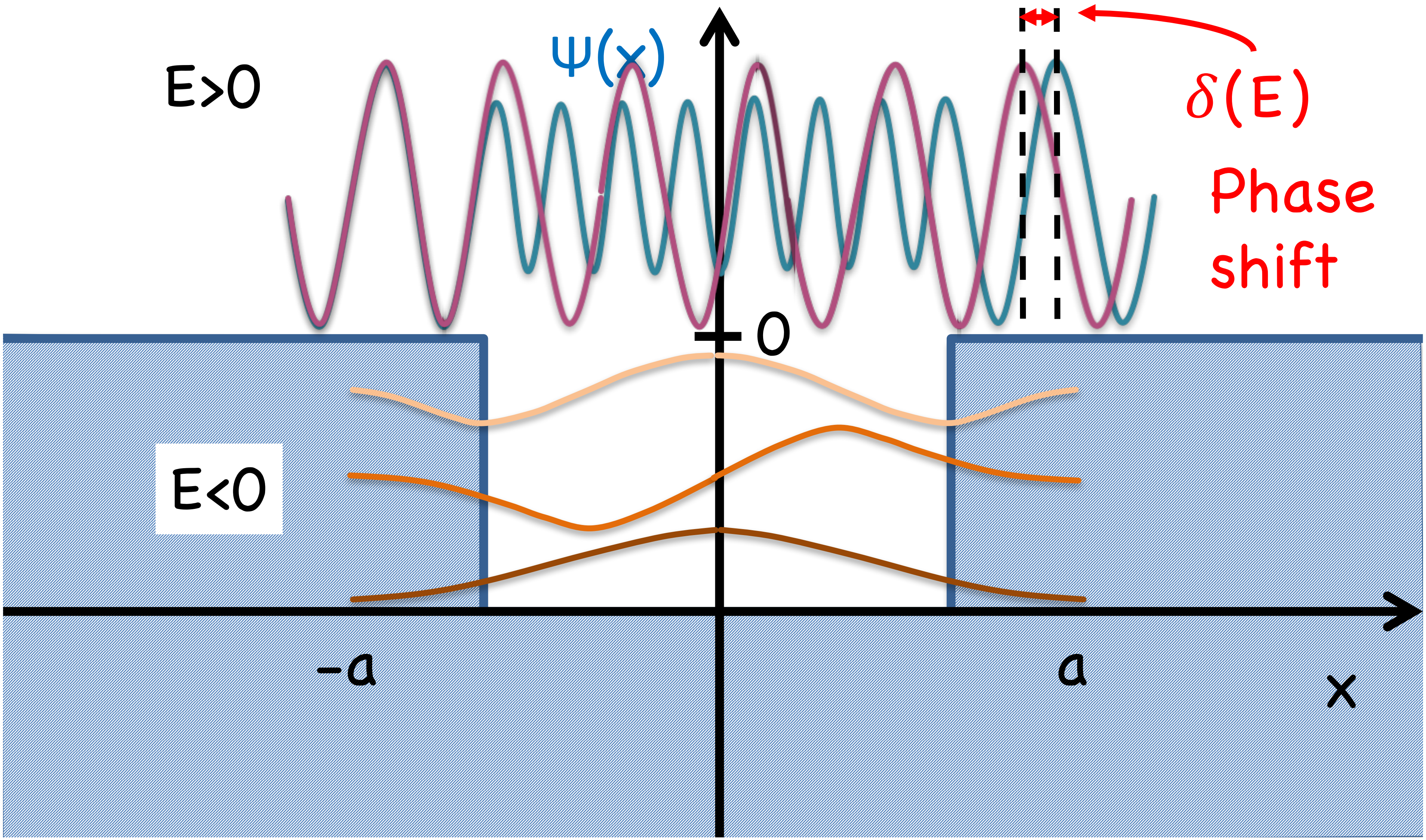
$$k = \frac{\sqrt{2mE}}{\hbar}$$

$$\alpha = \frac{\sqrt{2m\,(V_0+E)}}{\hbar}$$

Solution by requiring continuity of Ψ(x) and Ψ′(x) at the borders (a and -a)

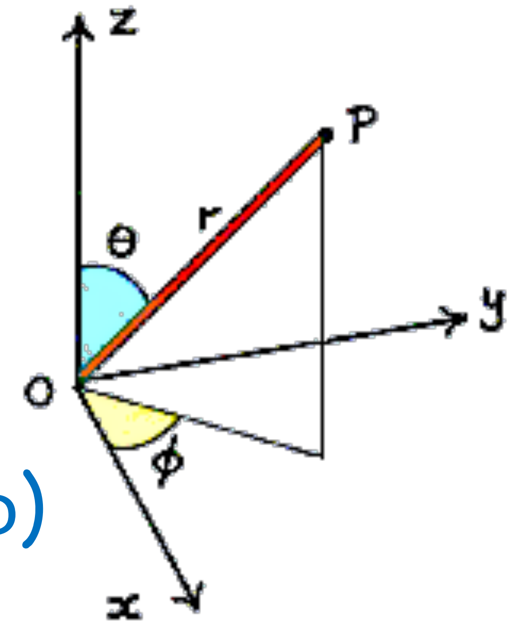


1D finite square-well potential: solutions



One particle in 3D more complicated, can be solved analytically in some case

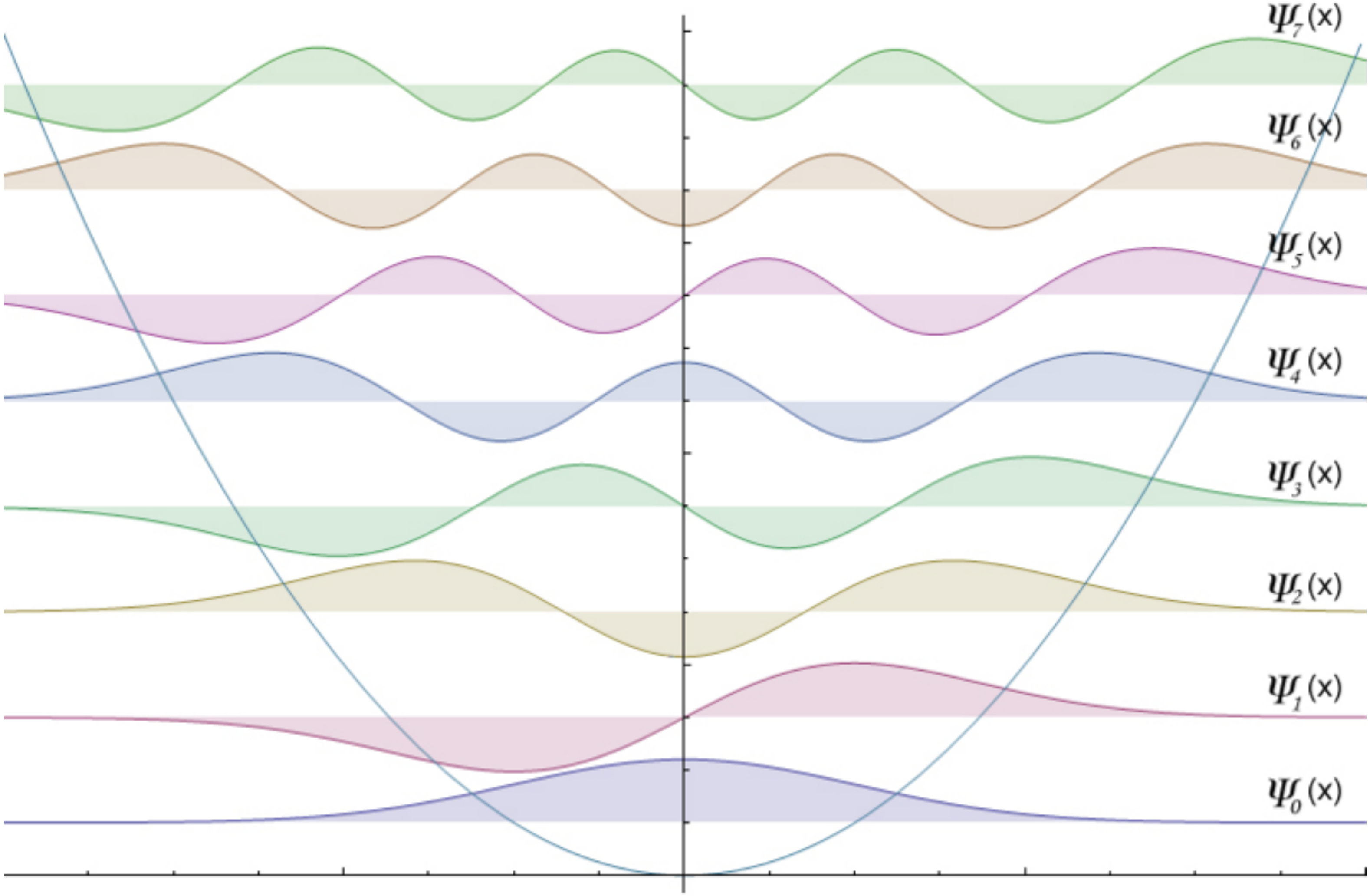
- Particle in spherically symmetric potential
  - —Spherical 'square-well' potential
  - —Harmonic Oscillator potential
  - —Hydrogen-like atoms
  - — …



$$\Psi(\mathbf{r}) = R(r)\, Y_{\ell m}(\theta,\phi)$$

- Reduce to 1D problem using spherical coordinates, spherical harmonics

Harmonic oscillator potential



Two-nucleon problem

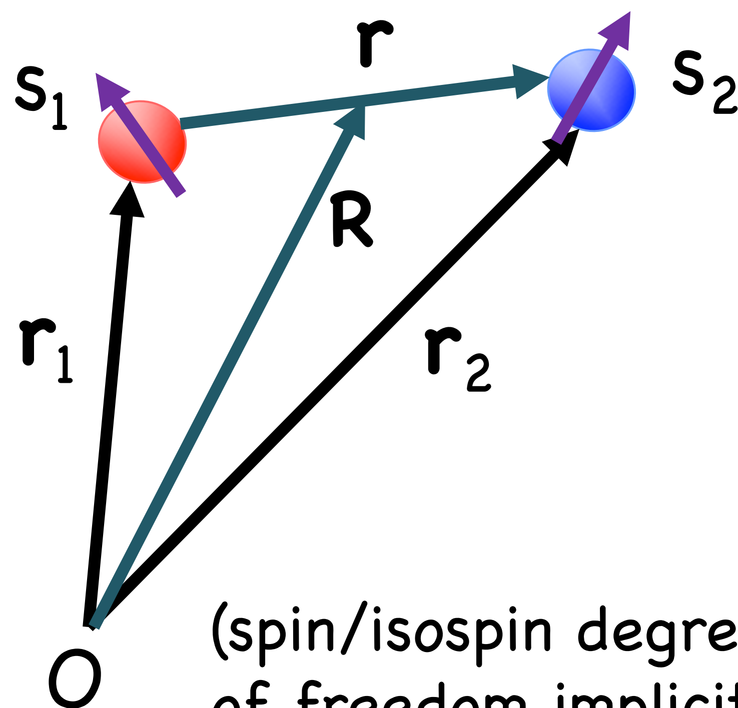
$$H^{(2)} = T_1 + T_2 + V(|\mathbf{r}_1 - \mathbf{r}_2|)$$

$$\mathbf{r} = \mathbf{r}_1 - \mathbf{r}_2$$

$$\mathbf{R} = (\mathbf{r}_1 + \mathbf{r}_2)/2$$

$$\mathbf{k} = (\mathbf{p}_1 - \mathbf{p}_2)/2$$

$$\mathbf{P} = \mathbf{p}_1 + \mathbf{p}_2$$

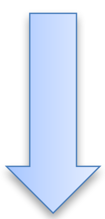


(spin/isospin degrees of freedom implicit)

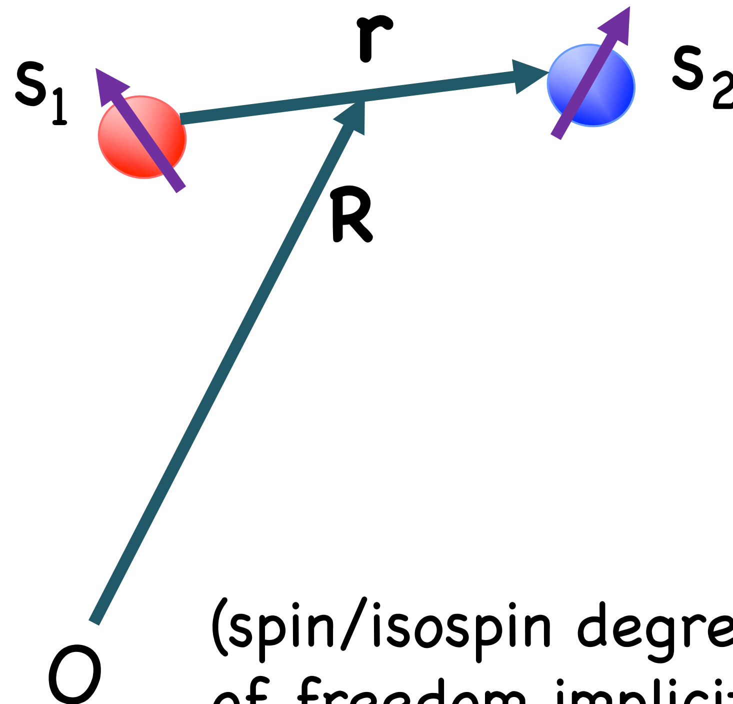
Two-nucleon problem

$$H^{(2)} = T_{cm} + T_k + V(r)$$

$$= T_{cm} + H_{int}$$



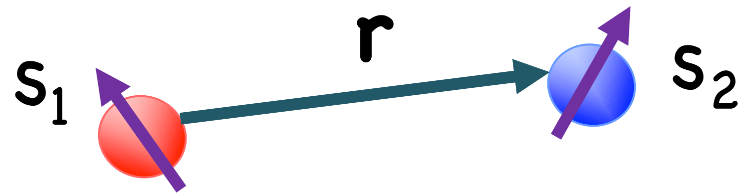
$$\Psi(\mathbf{r},\mathbf{R}) = e^{-i\mathbf{PR}}\, \Psi_{int}(\mathbf{r})$$



(spin/isospin degrees of freedom implicit)

Two-nucleon problem

- Reduce to one-body 3D problem for the intrinsic motion …



$$H_{int}\Psi_{int}(\mathbf{r}) = E\Psi_{int}(\mathbf{r})$$

$$S = 1/2+1/2 = 0,\ 1$$

$$T = 1/2+1/2 = 0,\ 1$$

- … and to 1D problem

$$\Psi_{int}(\mathbf{r}) = \Sigma_{\kappa}\ c_{\kappa}\ R_{n\ell}(r)\ Y_{\ell m}(\theta,\phi)\ \chi_{S\upsilon}(1,2)\ \chi_{T\tau}(1,2)$$

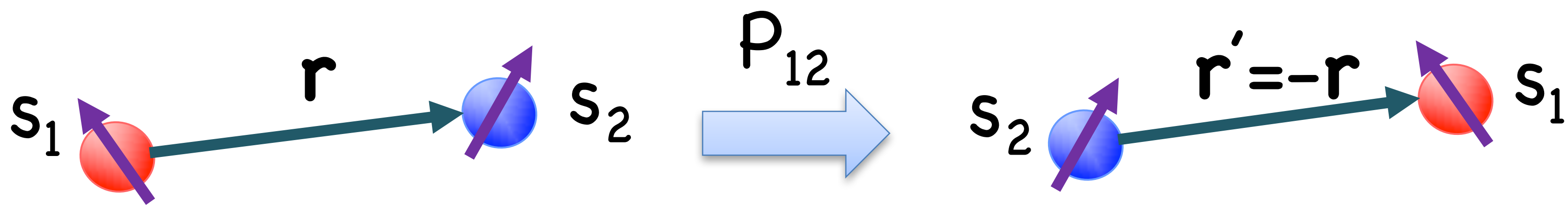
all quantum numbers (→ κ) — radial — orbital angular — spin — isospin

Two-nucleon problem

- Nucleons are identical particles

- Spin statistic theorem:
  - —Bosons = integer spin ➔ $P_{12}\Psi = \Psi$ (symmetric)
  - —Fermions = half-integer spin ➔ $P_{12}\Psi = -\Psi$ (antisymmetric)

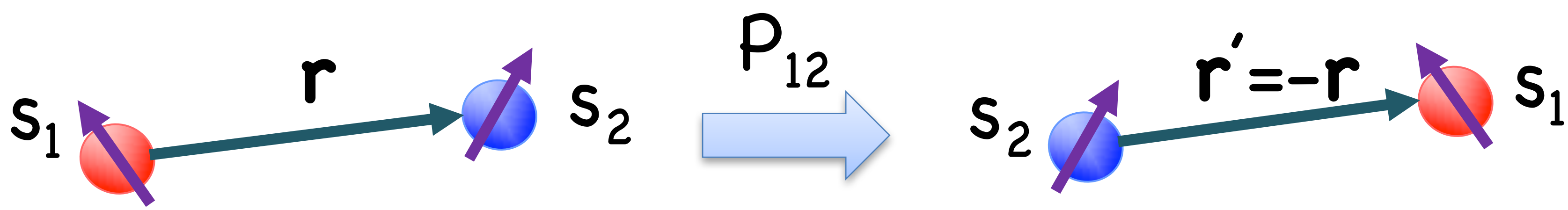
Two-nucleon problem



$R_{n\ell}(r)\, Y_{\ell m}(\theta,\phi)$ $\chi_{S\upsilon}(1,2)\ \chi_{T\tau}(1,2)$

$R_{n\ell}(r)\, Y_{\ell m}(\pi-\theta, 2\pi+\phi)$ $\chi_{S\upsilon}(2,1)\ \chi_{T\tau}(2,1)$

Two-nucleon problem



$R_{n\ell}(r)\ Y_{\ell m}(\theta,\phi)$
$\chi_{S\upsilon}(1,2)\ \chi_{T\tau}(1,2)$

$(-1)^{\ell+S+T}\ R_{n\ell}(r)\ Y_{\ell m}(\pi,\phi)$
$\chi_{S\upsilon}(1,2)\ \chi_{T\tau}(1,2)$

Only the components for which ℓ+S+T is odd are physical two-nucleon configurations

Two-nucleon problem: To summarize

- Reduces to 1D 1-body problem by
  1) Moving to relative coordinates
  2) Using expansion in spherical harmonics
- Solutions have to be antisymmetric under nucleon exchange (Pauli exclusion principle)
- Can be solved analytically only in a few cases (e.g., harmonic oscillator potential)
- With chiral forces need to solve numerically

Questions

- Do you have any questions?

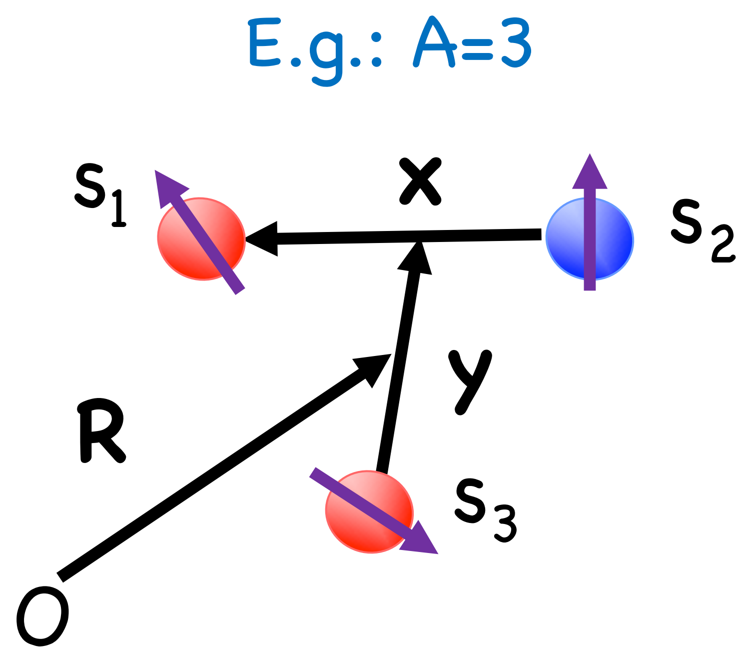
- Question for you:
  - What are the physical two-nucleon channels?

Some physical two-nucleon channels

- ~~$\ell = 0$, S = 0, J = 0, T = 0?~~
- $\ell = 0$, S = 0, J = 0, T = 1?
- $\ell = 0$, S = 1, J = 1, T = 0?
- ~~$\ell = 0$, S = 1, J = 1, T = 1?~~
- $\ell = 1$, S = 0, J = 1, T = 0?
- ~~$\ell = 1$, S = 0, J = 1, T = 1?~~
- …

Few-nucleon problem: A = 3,4,5 …

- Use 'Jacobi' coordinates (generalization of 2-body relative coordinates)
- Use expansion in hyperspherical harmonics (generalization of 1D spherical harmonics)
- Hard to antisymmetrize!

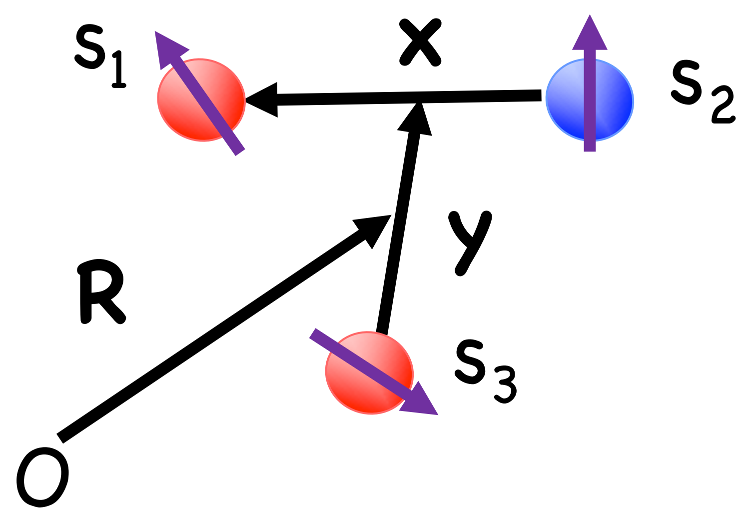


Few-nucleon problem: A = 3,4,(5), 6 …

Main examples:

- Faddeev equations (A=3)
- Faddeev-Yacubovsky (A=4,5*), Alt-Grassberger-Sandhas equations (A=4)
- Jacobi-coordinate no-core shell model (A = 3,4)
- Hyperspherical harmonics expansions (A = 3, 4, 6)

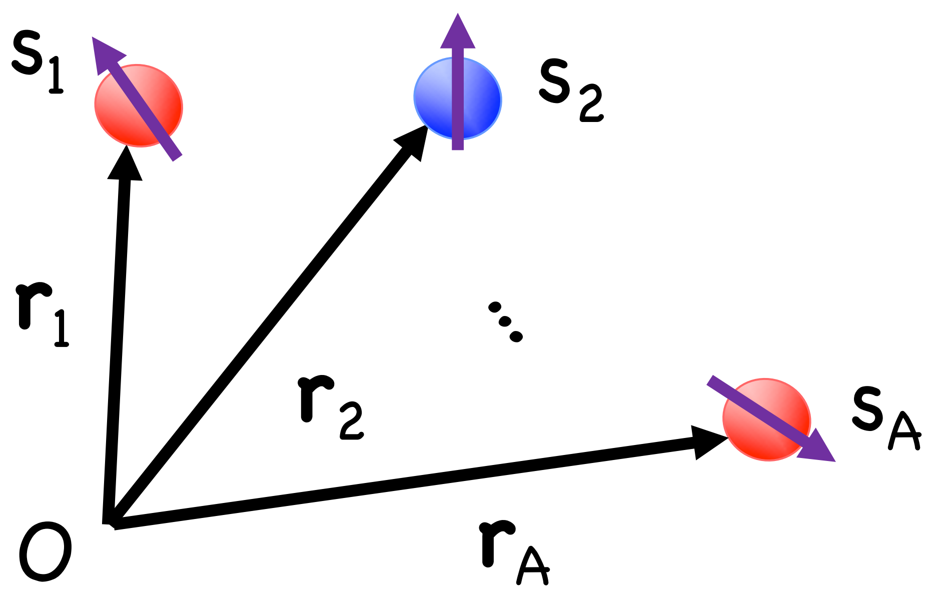
E.g.: A=3



* New development!

A-nucleon problem

$$H^{(A)} = \sum_{i=1}^{A} T_i + \sum_{i<j=1}^{A} V^{NN}(|\mathbf{r}_i - \mathbf{r}_j|) + \sum_{i<j<k=1}^{A} V_{ijk}^{3N}$$



A-nucleon problem

- A position coordinates (A-1 without C.M.)
- A spin coordinates
- A isospin coordinates
- The solution has to be antisymmetric under exchange of any two nucleons
- Way to complicated to solve as before!

What to do???

… search under the lamp post!

"I'm searching for my keys."

We know how to solve the independent-particle problem

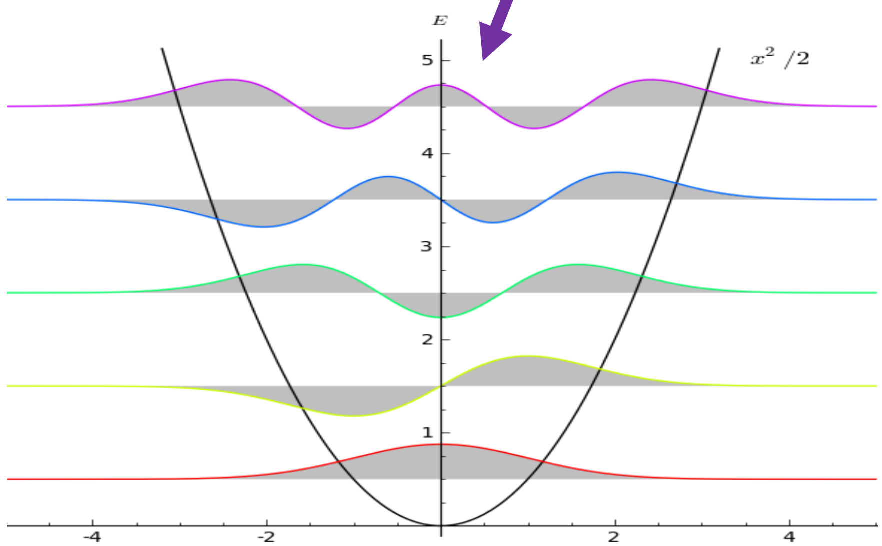
$$\tilde{H}^{(A)} = \sum_{i=1}^{A} T_i + U(r_i) = \sum_{i=1}^{A} h_i$$

Single-particle Hamiltonian

1) Solve single-particle problem:

e.g.: $U(r) = \frac{1}{2} m \Omega^2 r^2$

$$h\, \varphi_n(\mathbf{r}) = \varepsilon_n\, \varphi_n(\mathbf{r})$$



We know how to solve the independent-particle problem

2) The antisymmetric A-nucleon solutions can be build as

$$\Phi_k = \frac{1}{\sqrt{A!}} \det \begin{pmatrix} \varphi_{k1}(\mathbf{r}_1) & \varphi_{k1}(\mathbf{r}_2) & \cdots & \varphi_{k1}(\mathbf{r}_A) \\ \varphi_{k2}(\mathbf{r}_1) & \varphi_{k2}(\mathbf{r}_2) & \cdots & \varphi_{k2}(\mathbf{r}_A) \\ \vdots & \vdots & & \vdots \\ \varphi_{kA}(\mathbf{r}_1) & \varphi_{kA}(\mathbf{r}_2) & \cdots & \varphi_{kA}(\mathbf{r}_A) \end{pmatrix}$$

We have that:

Degeneracy

$$H^{(A)} \Phi_k = E_k \Phi_k \quad \text{with} \quad E_k = \sum_{i=1}^{A} \varepsilon_{ki} \, d_{ki}$$

What about our original A-nucleon problem?

3) Use the independent-particle model solutions as `basis states' to **build an ansatz** for the A-nucleon wave function

$$\Psi(\mathbf{r}_1,\mathbf{r}_2,\ldots,\mathbf{r}_A) = \sum_{k}^{N} c_k\, \phi_k(\mathbf{r}_1,\mathbf{r}_2,\ldots,\mathbf{r}_A)$$

Maximum number of excitations

$$(H^{(A)}-E)\,\Psi = 0 \rightarrow \sum_{k}^{N} c_k\, (H^{(A)}-E)\,\phi_k = 0$$

What about our original A-nucleon problem?

4) Project the equation on the basis states (from the left)

$$\sum_{k}^{N} c_k \int \phi_m^*(\mathbf{r}_1,\ldots,\mathbf{r}_A)\, H^{(A)}\, \phi_k(\mathbf{r}_1,\ldots,\mathbf{r}_A)\, d\mathbf{r}_1\ldots d\mathbf{r}_A$$

$$= E \sum_{k}^{N} c_k \int \phi_m^*(\mathbf{r}_1,\ldots,\mathbf{r}_A)\phi_k(\mathbf{r}_1,\ldots,\mathbf{r}_A)\, d\mathbf{r}_1\ldots d\mathbf{r}_A$$

δ_{mk}

$$\Rightarrow \quad \sum_{k}^{N} \mathbf{H}_{mk}\, c_k = E\, c_m$$

What about our original A-nucleon problem?

The A-nucleon Schrödinger equation becomes a linear algebra eigenvalue problem

$$\mathbf{H}\,\mathbf{c} = E\,\mathbf{c}$$

uknown

- The elements of the N×N Hamiltonian matrix are

$$\mathbf{H}_{mk} = \int \phi_m^*(\mathbf{r}_1,\ldots,\mathbf{r}_A)\, H^{(A)}\, \phi_k(\mathbf{r}_1,\ldots,\mathbf{r}_A)\, d\mathbf{r}_1 \ldots d\mathbf{r}_A$$

- And the unknown expansion coefficients c_k are the elements of the eigenvector **c**

What about our original A-nucleon problem?

The A-nucleon Schrödinger equation becomes a linear algebra eigenvalue problem

$$\mathbf{H}\, \mathbf{c} = E\, \mathbf{c}$$

- The elements of the N×N Hamiltonian matrix are

$$\mathbf{H}_{mk} = \langle \phi_m \mid H^{(A)} \mid \phi_k \rangle$$

Short-hand notation

- And the unknown expansion coefficients c_k are the elements of the eigenvector **c**

Some notes

- This is an ‘expansion’ technique: uses large (but finite!) expansion on A-body basis states
- Convergence to the exact result is approached (variationally) by increasing N (i.e., basis size)
- Antisymmetrization is trivial
- Did we forget about anything?

What about the center of mass motion?

- In the independent-particle problem, in general the c.m. motion is mixed with intrinsic motion, giving rise to spurious effects
- Exception: harmonic oscillator (HO) potential is exactly separable

$$\tilde{H}^{(HO)} = \sum_{i=1}^{A} T_i + \frac{1}{2} m \Omega^2 r^2$$

What about the center of mass motion?

- In the independent-particle problem, in general the c.m. motion is mixed with intrinsic motion, giving rise to spurious effects
- Exception: harmonic oscillator (HO) potential is exactly separable

$$\tilde{H}^{(HO)} = \underbrace{T_{int} + \sum_{i<j=1}^{A} \frac{m\Omega^2}{2A}(\mathbf{r}_i - \mathbf{r}_j)^2}_{H_{int}} + \underbrace{T_{cm} + \frac{1}{2}Am\Omega^2 R^2}_{H_{cm}}$$

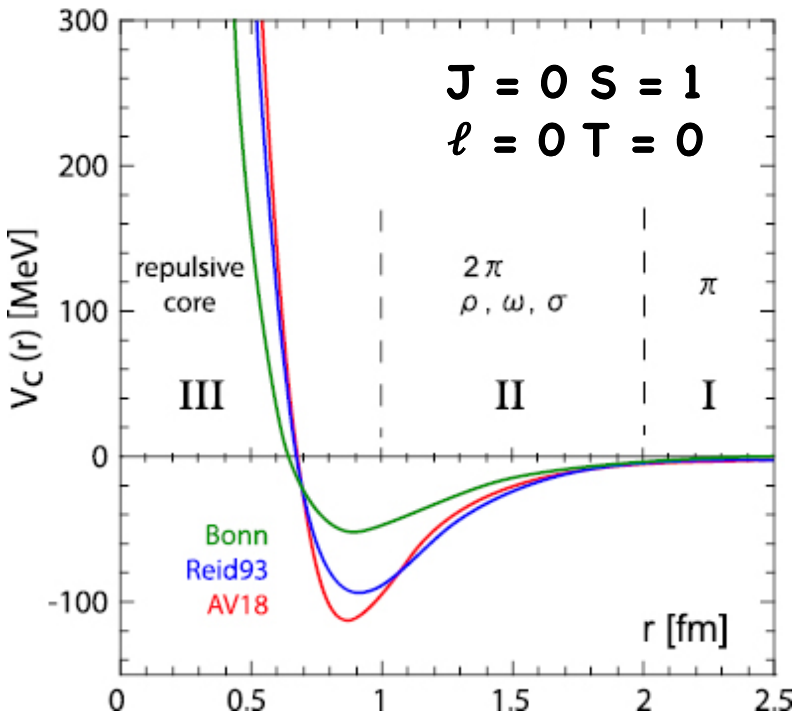
Hard core of nuclear interaction scatters nucleons to high momenta

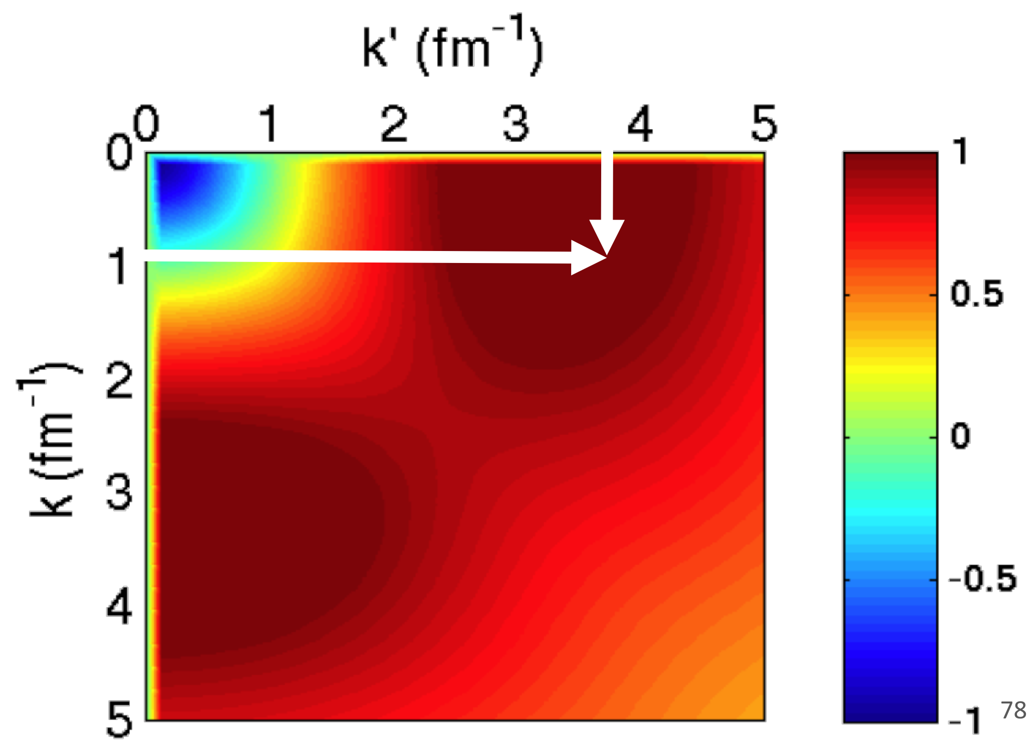
$V(r)$

Fourier Transform

$$V_{kk'} = \int e^{ik'r}\, V(r)\, e^{-ikr} d\mathbf{r}$$

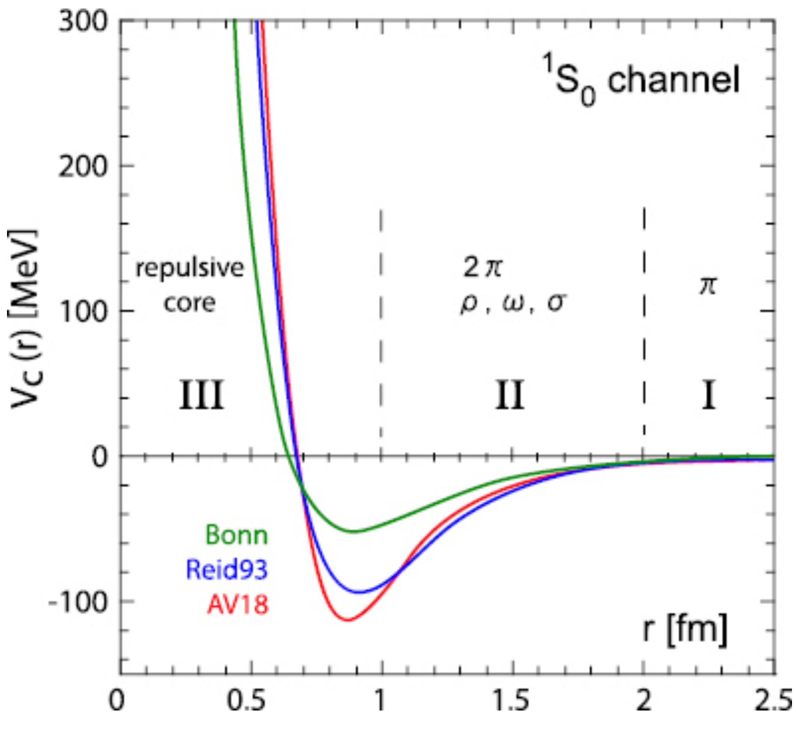
$$= \langle k|\, V\, |k'\rangle$$

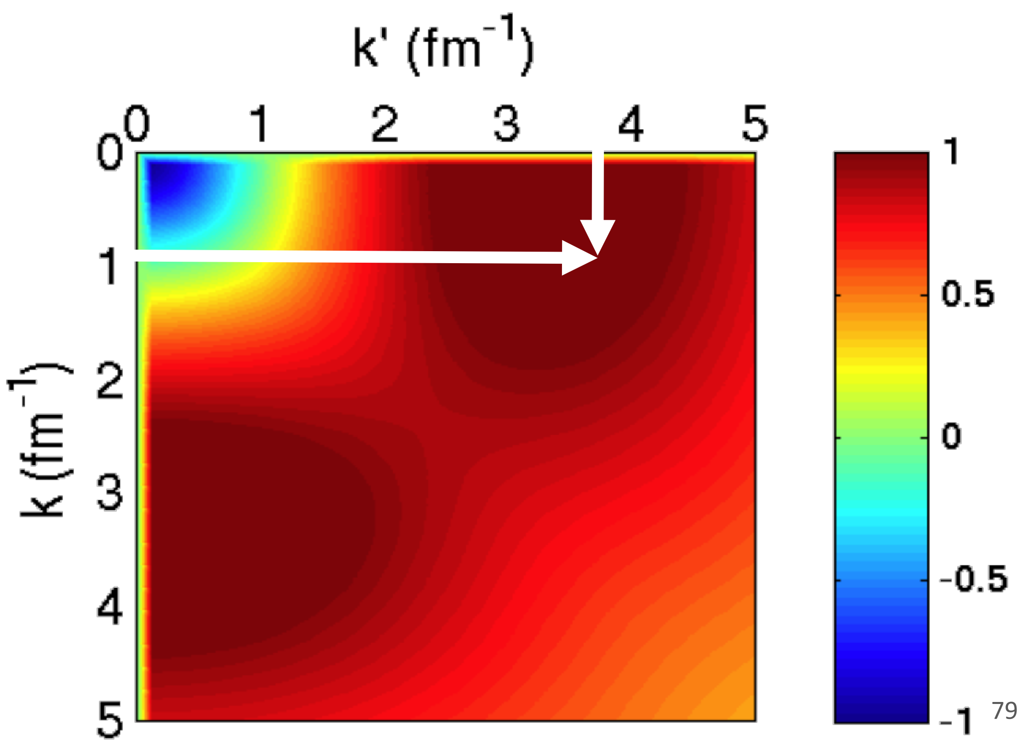




Hard core of nuclear interaction scatters nucleons to high momenta

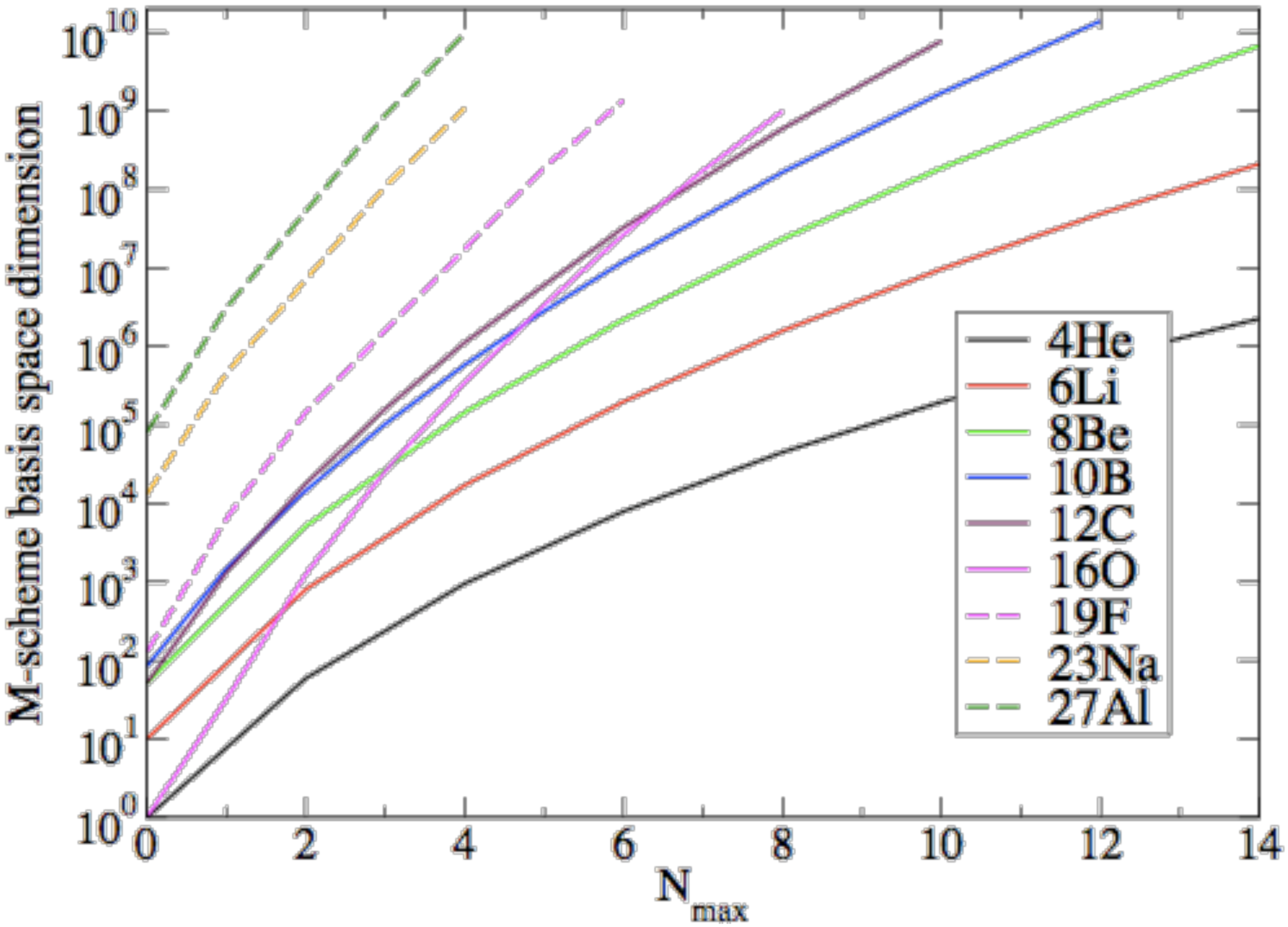
Very large N values (basis sizes) are required to reach convergent solution!





Basis dimension grows rapidly with A!

Convergence can be a challenge!



Effective interactions from unitary transformations of bare Hamiltonian

- Introduce unitary transformation: $\mathcal{U}$ $(\mathcal{U}^{\dagger}\mathcal{U} = \mathbb{1})$

$$E = \langle\Psi|\, H^{(A)}\, |\Psi\rangle$$

Bare Hamiltonian, wave function

$$= \langle\Psi|\, \mathcal{U}^{\dagger}\mathcal{U}\, H^{(A)}\, \mathcal{U}^{\dagger}\mathcal{U}\, |\Psi\rangle$$

$$= (\langle\Psi|\mathcal{U}^{\dagger})\, \mathcal{U}H^{(A)}\mathcal{U}^{\dagger}\, (\mathcal{U}|\Psi\rangle)$$

Effective Hamiltonian, wave function

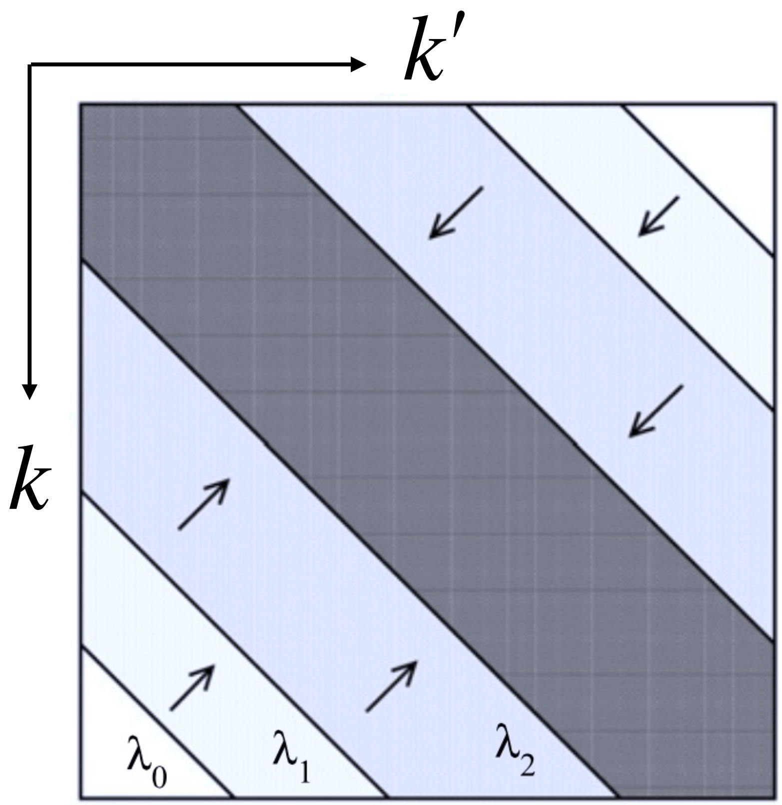
$$= \langle\widetilde{\Psi}|\, \widetilde{H}^{(A)}\, |\widetilde{\Psi}\rangle$$

Example: Similarity renormalization group (SRG) transformation

$$\tilde{H}_\lambda = \mathcal{U}_\lambda H \, \mathcal{U}^{+}_\lambda$$

$$\frac{d\tilde{H}_\lambda}{d\lambda} = -\frac{4}{\lambda^5}[\eta(\lambda), \tilde{H}_\lambda]$$

Flow parameter



Two-body Hamiltonian in momentum space

$$\langle k | \tilde{H}^{(2)}_\lambda | k' \rangle$$

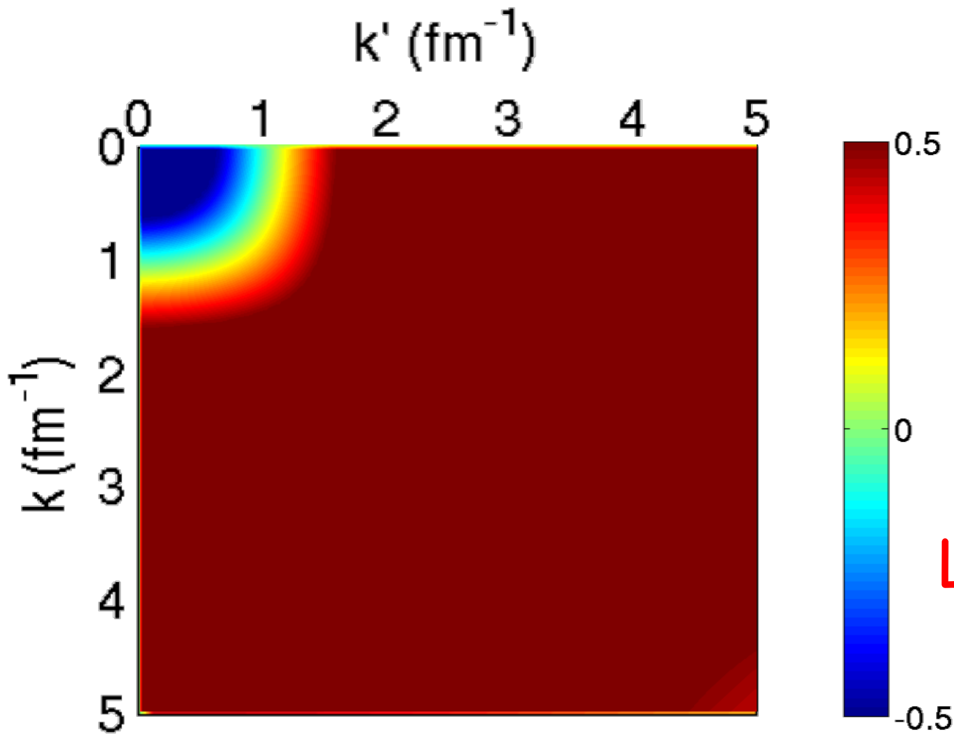
Plane wave

$$\lambda_0 > \lambda_1 > \lambda_2 \ldots$$

Example: Similarity renormalization group (SRG) transformation

$\tilde{H}_\lambda = \mathcal{U}_\lambda H \mathcal{U}^\dagger_\lambda$

$$\frac{d\tilde{H}_\lambda}{d\lambda} = -\frac{4}{\lambda^5}[\eta(\lambda),\tilde{H}_\lambda]$$



$\langle k|\ \tilde{H}^{(2)}_\lambda\ |k'\rangle$

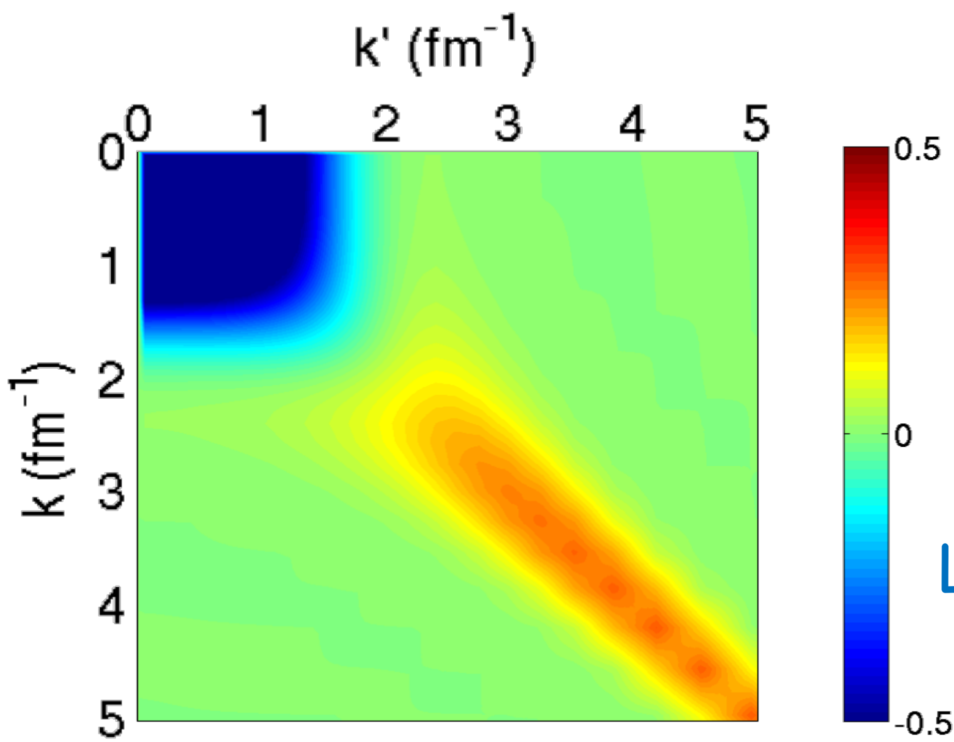
$\lambda = 20\ \text{fm}^{-1}$

Low and high momentum components coupled

Example: Similarity renormalization group (SRG) transformation

$\tilde{H}_\lambda = \mathcal{U}_\lambda H \mathcal{U}^{+}_\lambda$

$$\frac{d\tilde{H}_\lambda}{d\lambda} = -\frac{4}{\lambda^5}[\eta(\lambda), \tilde{H}_\lambda]$$



$\langle k | \tilde{H}^{(2)}_\lambda | k' \rangle$

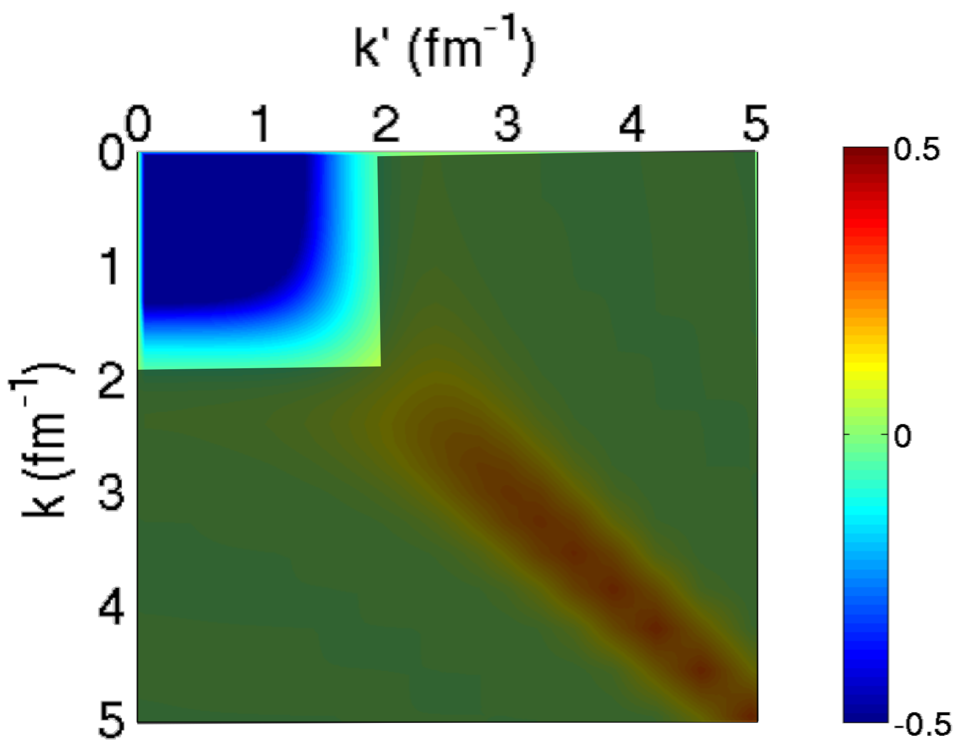
$\lambda = 2\ fm^{-1}$

Low and high momentum components de-coupled

Example: Similarity renormalization group (SRG) transformation

$\tilde{H}_\lambda = \mathcal{U}_\lambda H \mathcal{U}^\dagger_\lambda$

$$\frac{d\tilde{H}_\lambda}{d\lambda} = -\frac{4}{\lambda^5}[\eta(\lambda),\tilde{H}_\lambda]$$



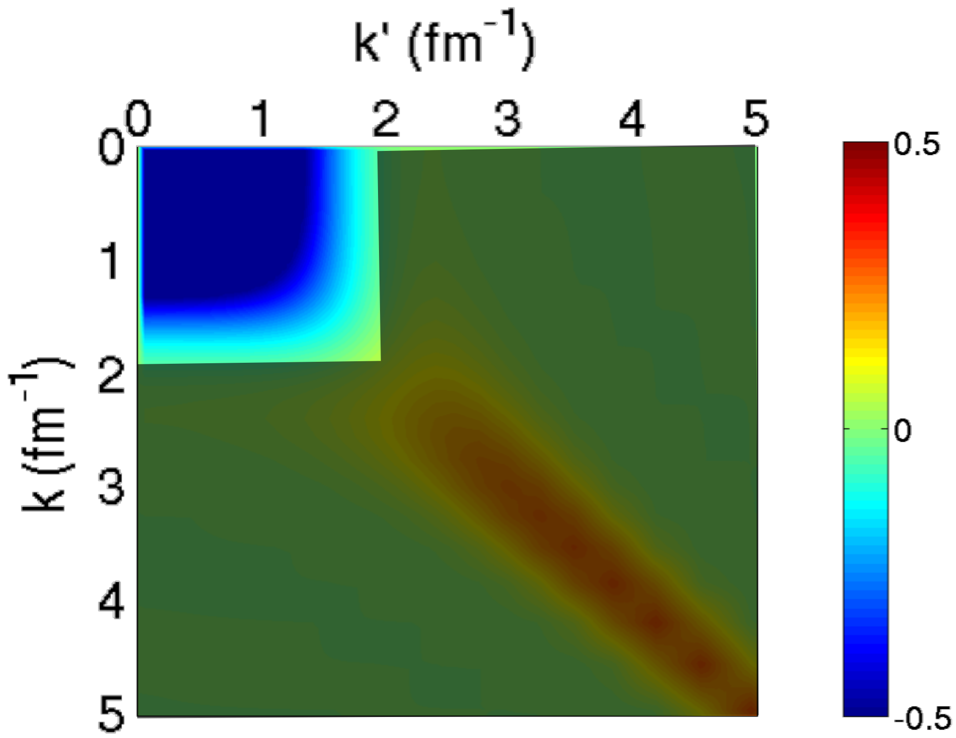
$\langle k|\ \tilde{H}^{(2)}_\lambda\ |k'\rangle$

$\lambda = 2\ \text{fm}^{-1}$

Can work with smaller N values (basis sizes)!

Example: Similarity renormalization group (SRG) transformation

$$\tilde{H}_\lambda = \mathcal{U}_\lambda H \, \mathcal{U}^{\dagger}_\lambda \qquad \frac{d\tilde{H}_\lambda}{d\lambda} = -\frac{4}{\lambda^5}[\eta(\lambda), \tilde{H}_\lambda]$$



$\langle k | \tilde{H}^{(2)}_\lambda | k' \rangle$

$\lambda = 2 \text{ fm}^{-1}$

See: Bogner, Furnstahl, Schwenk, Prog. Part. Nucl. Phys. 65 (2010)

Question

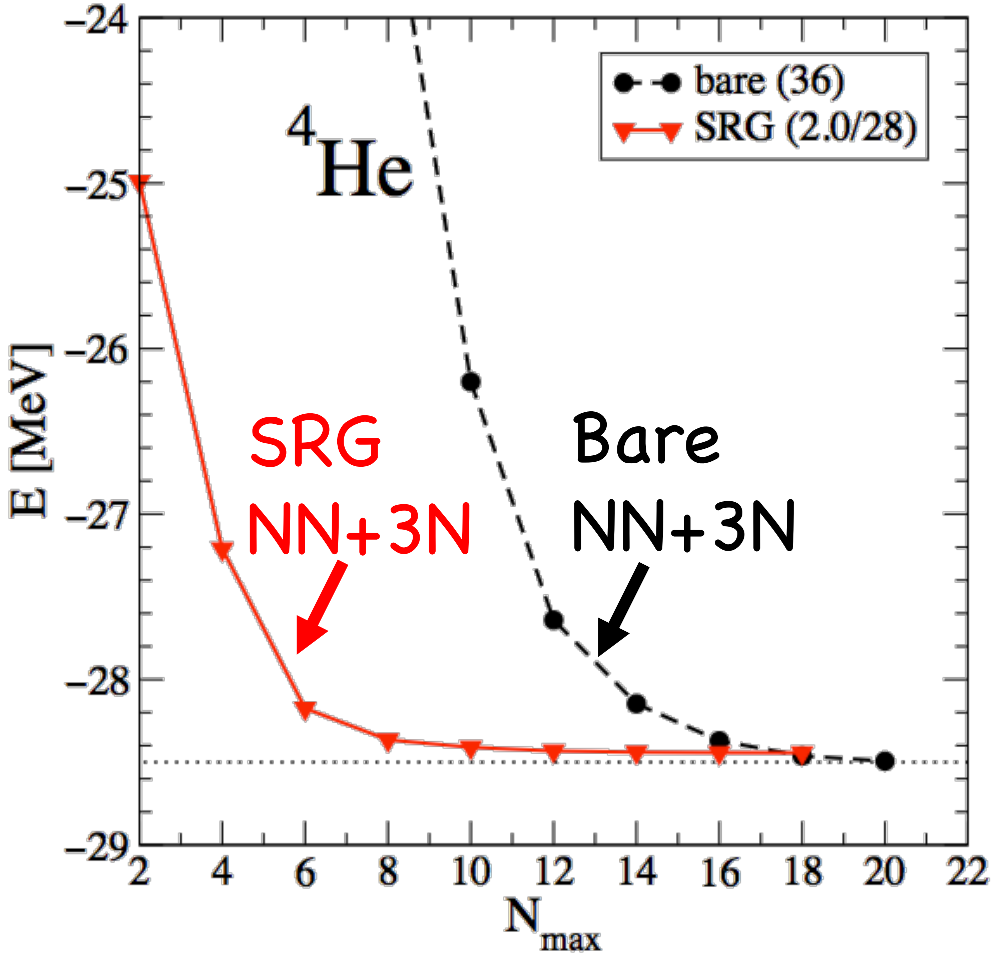
- This sounds to good to be true …
- What's the catch?

Notes on effective interactions

- The transformation (e.g., SRG) generates a 'new', softer NN interaction
- Unitarily equivalent to the bare NN potential in the two-nucleon sector only!
- Induces 3-body and, in general, up to A-body forces even starting from an NN potential

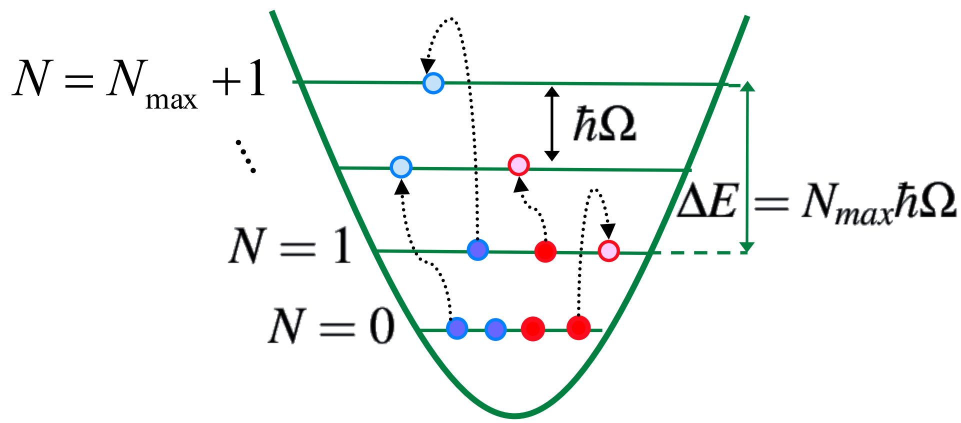
Example: convergence of ^{4}He ground-state energy with chiral NN+3N forces

Jurgenson et al., PRL **103**, 082501



Ab initio no-core shell model (NCSM)

- Superposition of Harmonic Oscillator (HO) wave functions
- Bare/effective (e.g., SRG) NN+3N forces
- ‘Diagonalizes’ Hamiltonian matrix
- A $\lesssim$ 16



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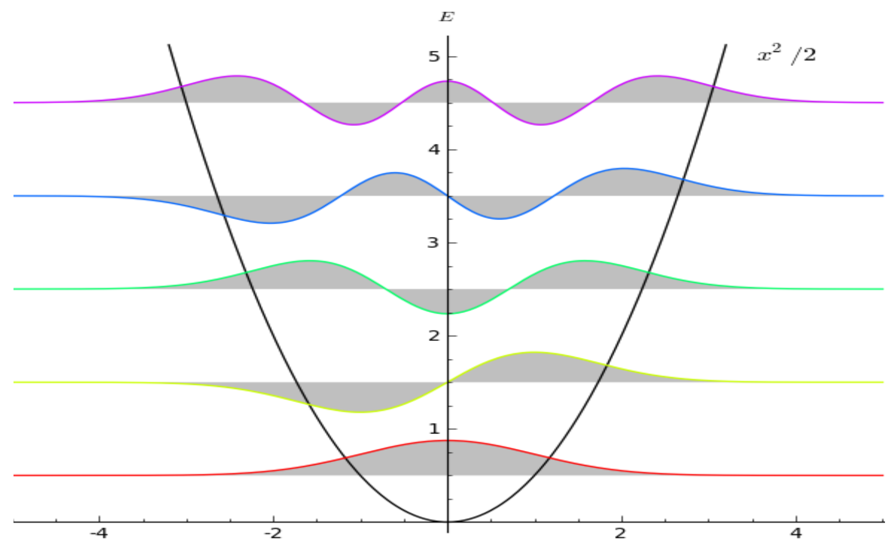

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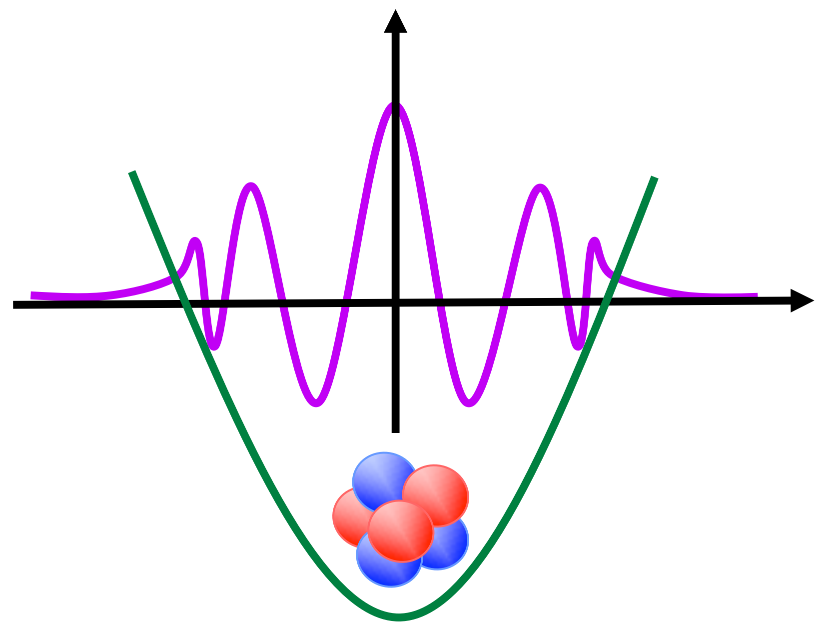
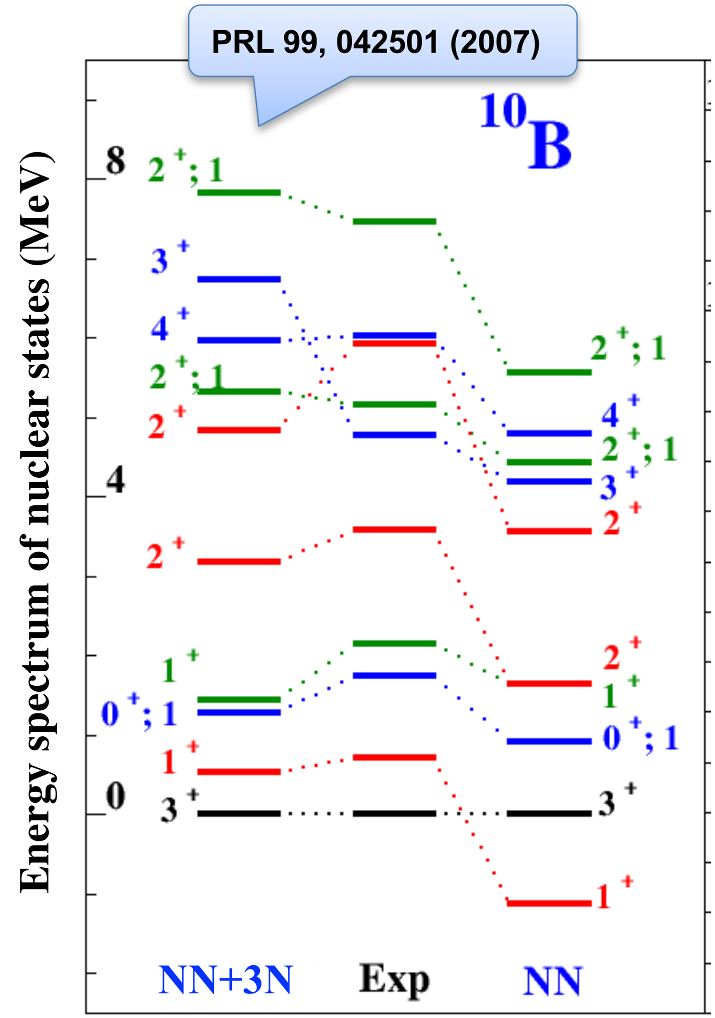


Works well if wave function is localized (well-bound states)

Ab initio no-core shell model (NCSM)

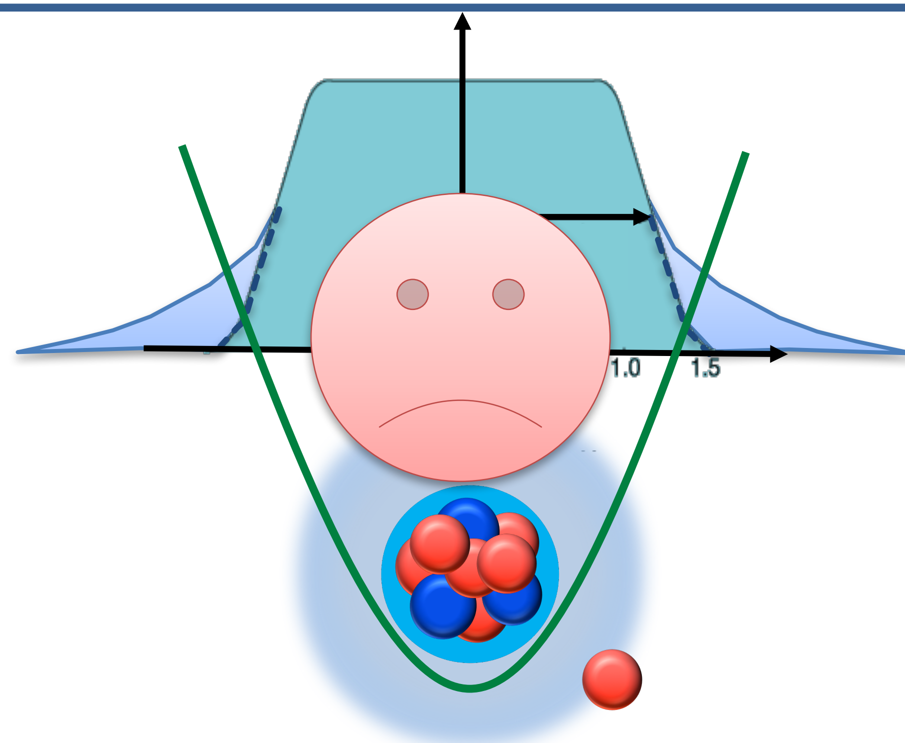
Example: energy spectrum of nuclear states of the ^{10}B nucleus

Helped to point out the fundamental importance of 3N forces in structure calculations



Ab initio no-core shell model (NCSM)

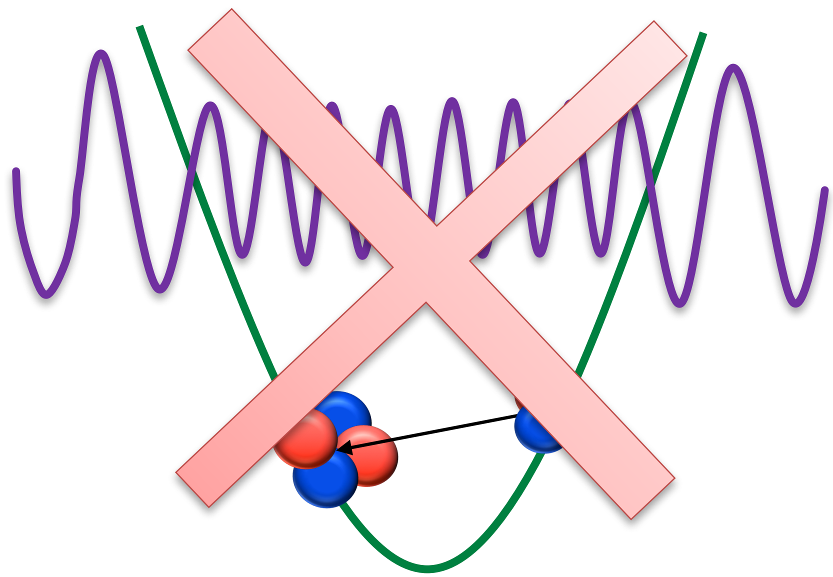
- Superposition of Harmonic Oscillator (HO) wave functions
- Bare/effective (e.g., SRG) NN+3N forces
- 'Diagonalizes' Hamiltonian matrix
- A ≲ 16



Does not works as well for nuclei with exotic densities (halo nuclei)

Ab initio no-core shell model (NCSM)

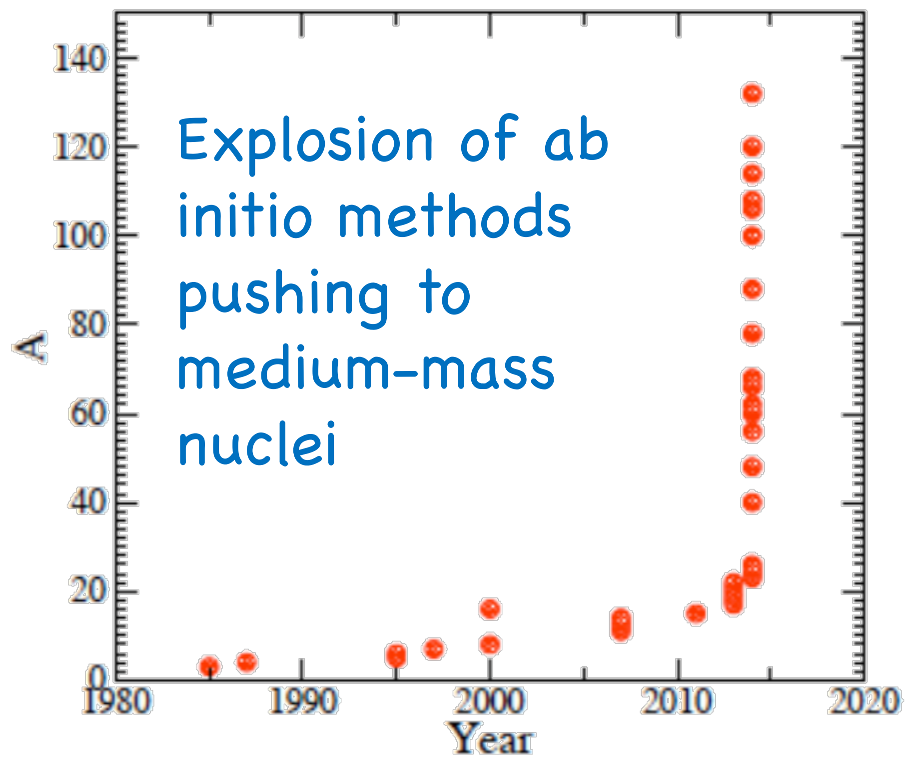
- Superposition of Harmonic Oscillator (HO) wave functions
- Bare/effective (e.g., SRG) NN+3N forces
- ‘Diagonalizes’ Hamiltonian matrix
- A $\lesssim$ 16



Definitively not adapted to the description of scattering wave functions!

Ab initio community extremely successful in describing the static properties of nuclei

- Green's function Monte Carlo
- Nuclear Lattice Effective Field Theory
- Coupled Cluster theory
- In-Medium SRG
- Gorkov-Green function theory
- Many-Body Perturbation Theory
- Ab initio valence-space shell model



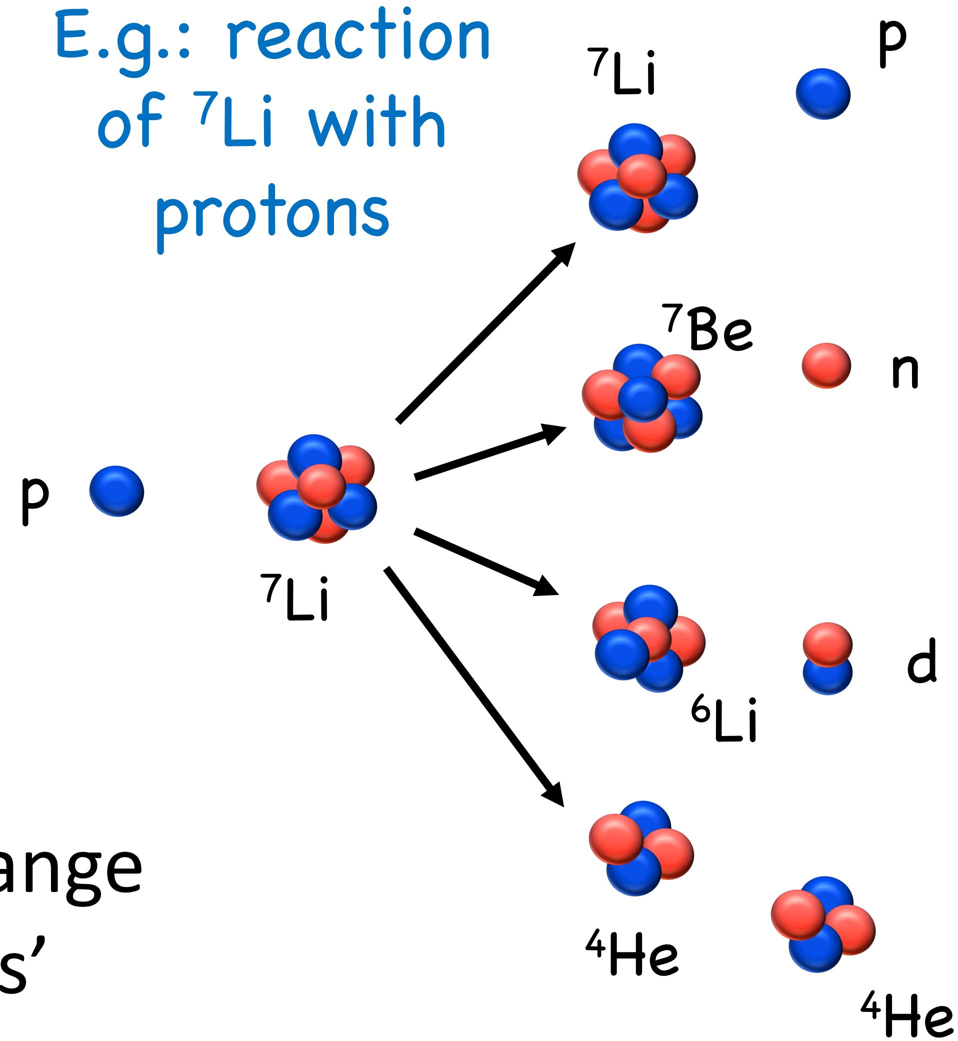
What about the dynamics between nuclei (scattering States with E>0)?

How to describe the phenomena of low-energy nuclear reactions based on colliding nuclei made of interacting nucleons?

Problem of nuclear collisions in A-nucleon systems even harder to solve!

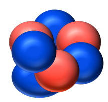
- In collisions wave functions extend all over the place
- Simultaneously A-nucleon and projectile-target problem
- Nucleons can re-arrange in different 'channels'

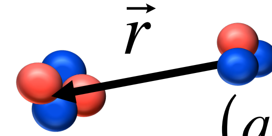
E.g.: reaction of ^{7}Li with protons



At low-energy usually only a few reaction channels are open …

$$\Psi = \sum_{\lambda} c_{\lambda} \left| {}^{(A)}, \lambda \right\rangle + \sum_{\nu} \int d\vec{r}\, u_{\nu}(\vec{r})\, \hat{A}_{\nu} \left|_{(A-a)} \vec{r} \; (a), \nu \right\rangle$$

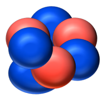
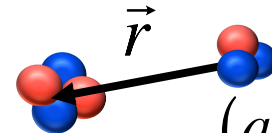




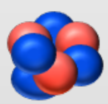
Unknowns

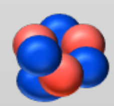
- We can improve our ansatz for the A-nucleon wave function by further adding ‘microscopic cluster states’ for the relevant reaction channels

Ab initio no-core shell model with continuum (NCSMC)

$\Psi = \sum_{\lambda} c_{\lambda} \Big|^{(A)}$  $, \lambda \Big\rangle + \sum_{\nu} \int d\vec{r}\; u_{\nu}(\vec{r}\,)\; \hat{A}_{\nu} \Big|$  $, \nu \Big\rangle$

Localized A-nucleon solutions (eigenstates) computed with the NCSM

$\Big|^{(A)}$  $, \lambda \Big\rangle = \sum_{k}^{N} b_k^{(\lambda)} \phi_k(\mathbf{r}_1, \mathbf{r}_2, \ldots, \mathbf{r}_A)$

$(H^{(A)} - E_{\lambda}) \Big|^{(A)}$  $, \lambda \Big\rangle = 0$

Ab initio no-core shell model with continuum (NCSMC)

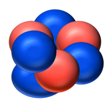
$$\Psi = \sum_{\lambda} c_{\lambda} \left| {}^{(A)} \cdots , \lambda \right\rangle + \sum_{\nu} \int d\vec{r}\; u_{\nu}(\vec{r}\,)\; \hat{A}_{\nu} \left| {}_{(A-a)} \cdots \overset{\vec{r}}{\longleftarrow} \cdots {}_{(a)} , \nu \right\rangle$$

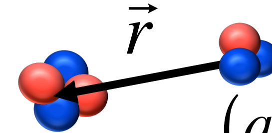
$$\left| {}_{(A-a)} \cdots \overset{\vec{r}}{\longleftarrow} \cdots {}_{(a)} , \nu \right\rangle = \left| \overset{(A-a)}{\cdots}, \nu_1 \right\rangle \left| \overset{(a)}{\cdots}, \nu_2 \right\rangle \delta(\vec{r} - \vec{r}_{A-a,a})$$

Continuous microscopic cluster states made of projectile-target pairs in relative motion

Ab initio no-core shell model with continuum (NCSMC)

$$\Psi = \sum_{\lambda} c_{\lambda} \left| {}^{(A)} \ldots , \lambda \right\rangle + \sum_{\nu} \int d\vec{r}\; u_{\nu}(\vec{r}\,)\; \hat{A}_{\nu} \left|_{(A-a)} \ldots \vec{r} \ldots {}^{(a)} , \nu \right\rangle$$



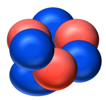


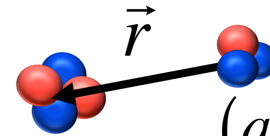
Sum over relevant reaction channels (mass partitions)

Antisymmetrizes exchanges of nucleons between projectile and target

Ab initio no-core shell model with continuum (NCSMC)

$$\Psi = \sum_{\lambda} c_{\lambda} \left| {}^{(A)} \; , \lambda \right\rangle + \sum_{\nu} \int d\vec{r} \; u_{\nu}(\vec{r}\,) \; \hat{A}_{\nu} \left| {}_{(A-a)} \; {}^{(a)} \; , \nu \right\rangle$$





Describe efficiently the wave function when all A nucleons are close together

Describe efficiently the wave function when the reactants/ reaction products are far apart

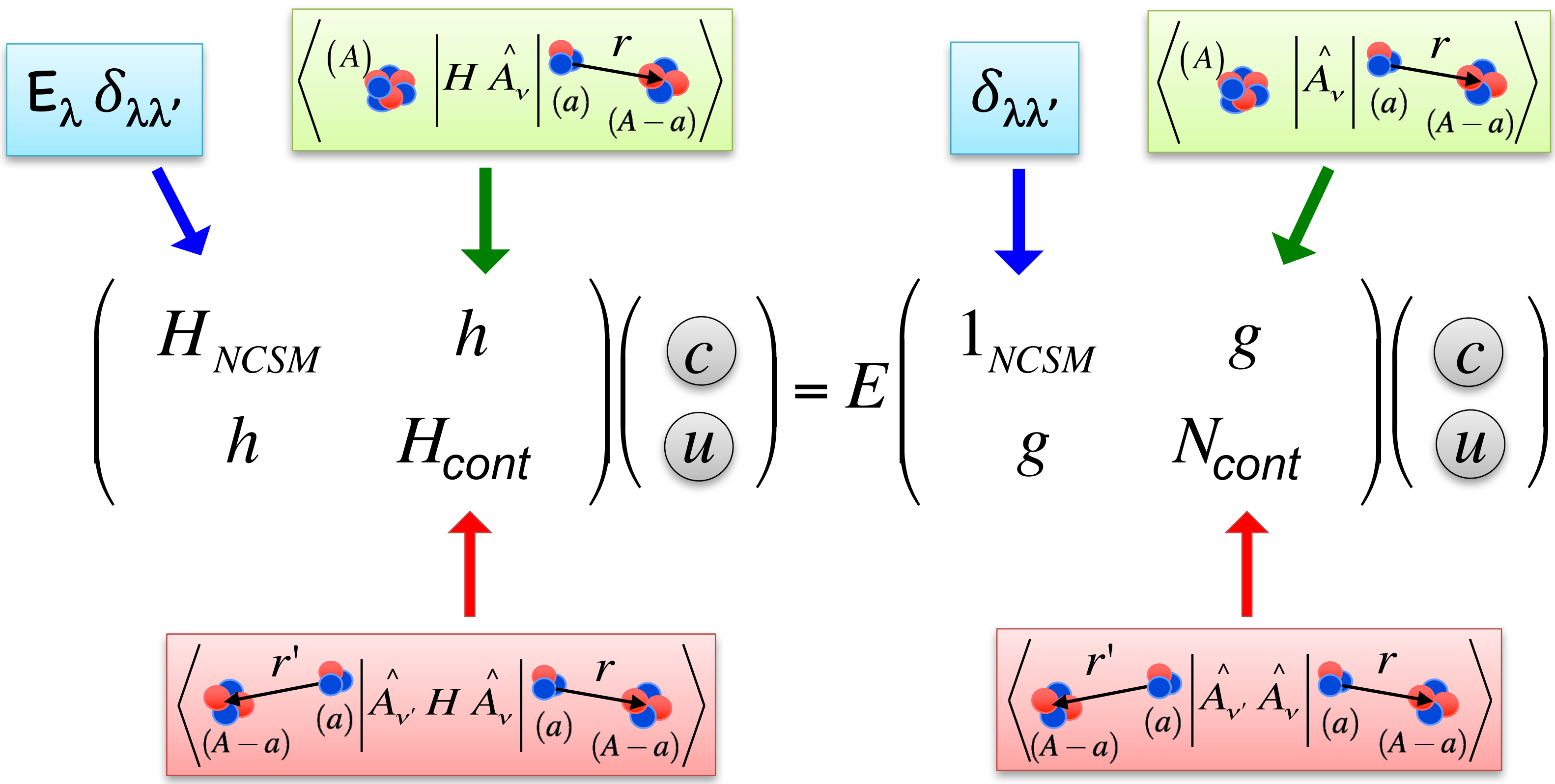
Ab initio no-core shell model with continuum (NCSMC)

$$\Psi = \sum_{\lambda} c_{\lambda} \left| {}^{(A)}, \lambda \right\rangle + \sum_{\nu} \int d\vec{r}\; u_{\nu}(\vec{r}\,)\; \hat{A}_{\nu} \left| {}_{(A-a)} \overset{\vec{r}}{\longleftarrow} {}_{(a)}, \nu \right\rangle$$

Works well for describing clustering in nuclei (halo nuclei)

Works well for describing both bound and scattering state

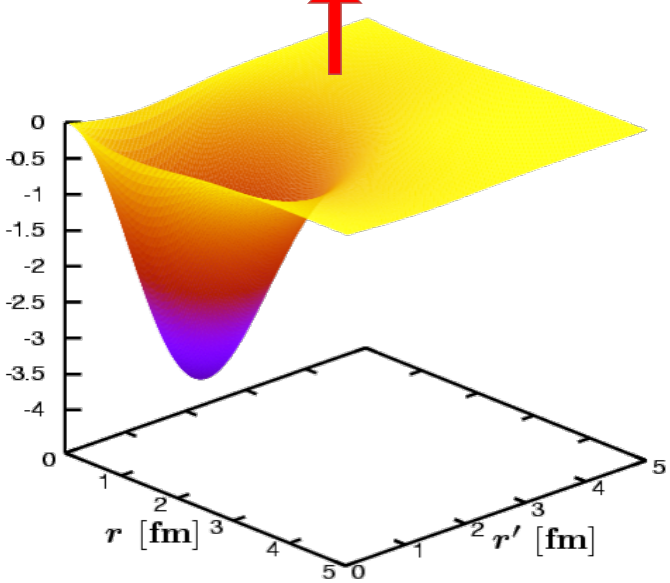
A-nucleon Schrödinger equation again reduces to an eigenvalue problem

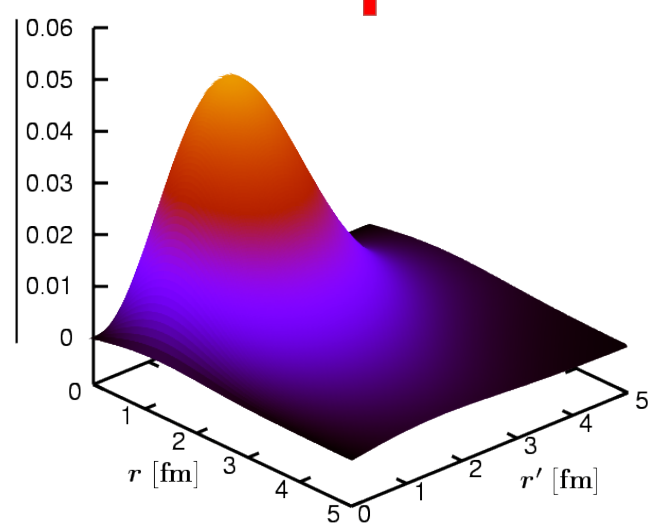


A-nucleon Schrödinger equation again reduces to an eigenvalue problem

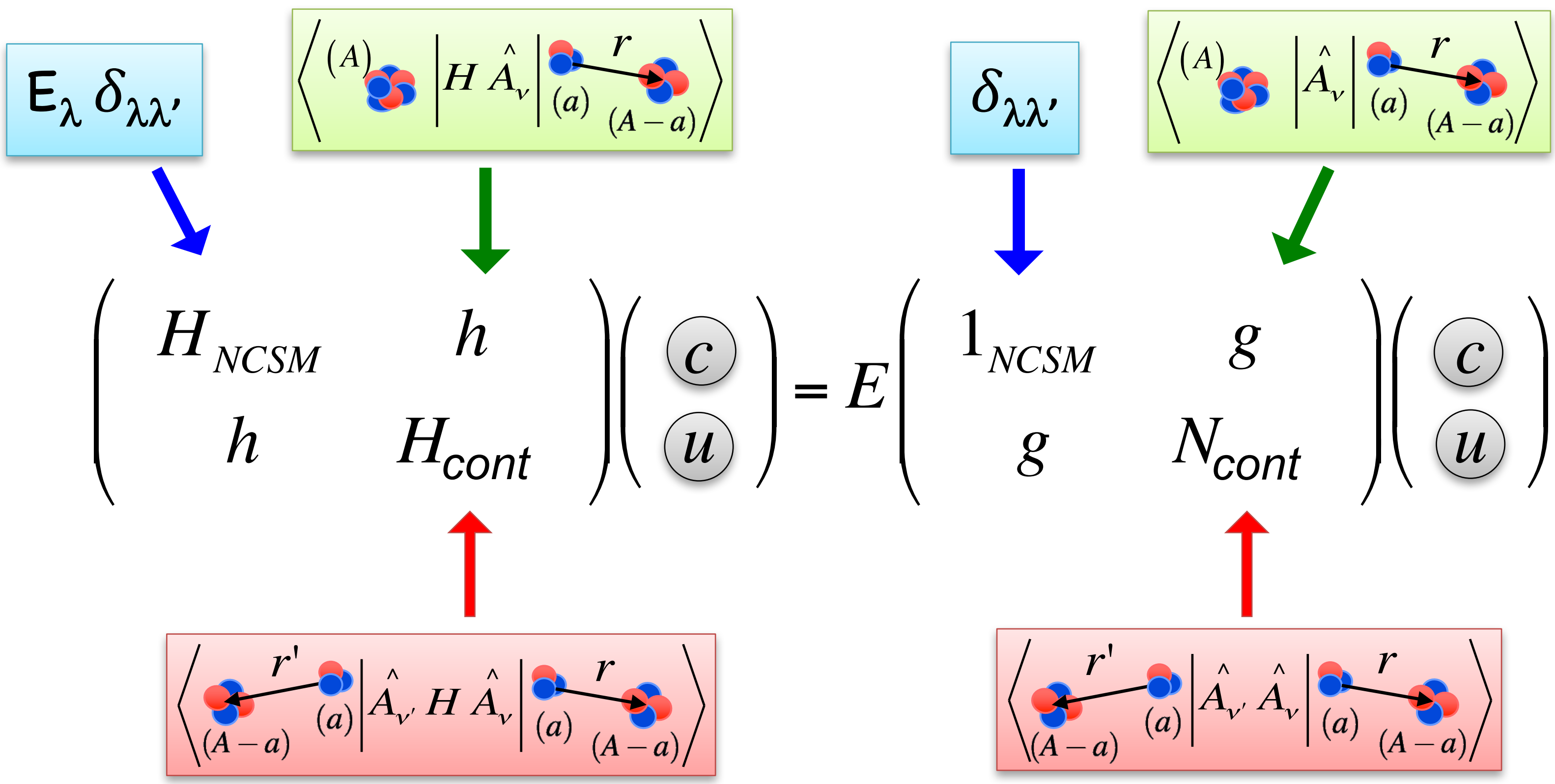
$E_\lambda \, \delta_{\lambda\lambda'}$ $\quad$ $\left\langle (A) \middle| H \hat{A}_\nu \middle| (a) \; r \; (A-a) \right\rangle$ $\quad$ $\delta_{\lambda\lambda'}$ $\quad$ $\left\langle (A) \middle| \hat{A}_\nu \middle| (a) \; r \; (A-a) \right\rangle$

$$\begin{pmatrix} H_{NCSM} & h \\ h & H_{cont} \end{pmatrix} \begin{pmatrix} c \\ u \end{pmatrix} = E \begin{pmatrix} 1_{NCSM} & g \\ g & N_{cont} \end{pmatrix} \begin{pmatrix} c \\ u \end{pmatrix}$$





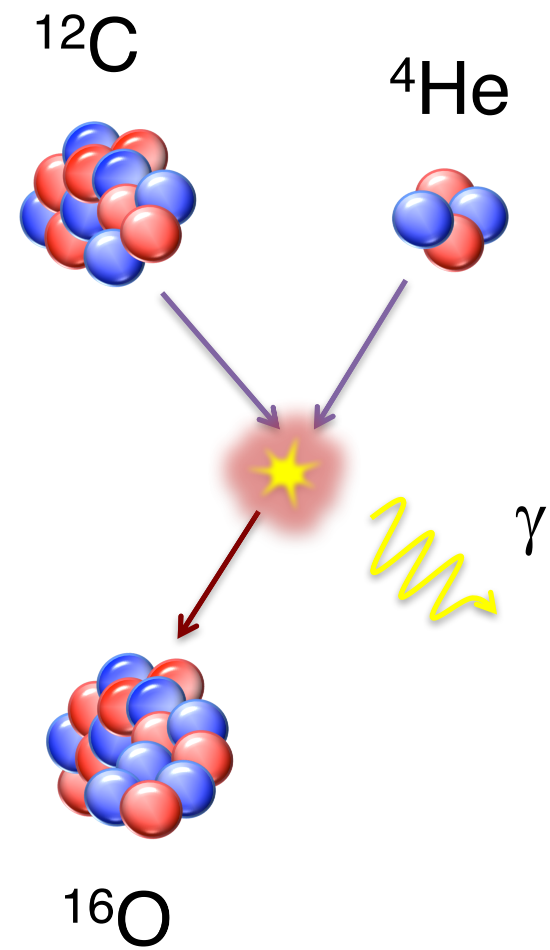
A-nucleon Schrödinger equation again reduces to an eigenvalue problem



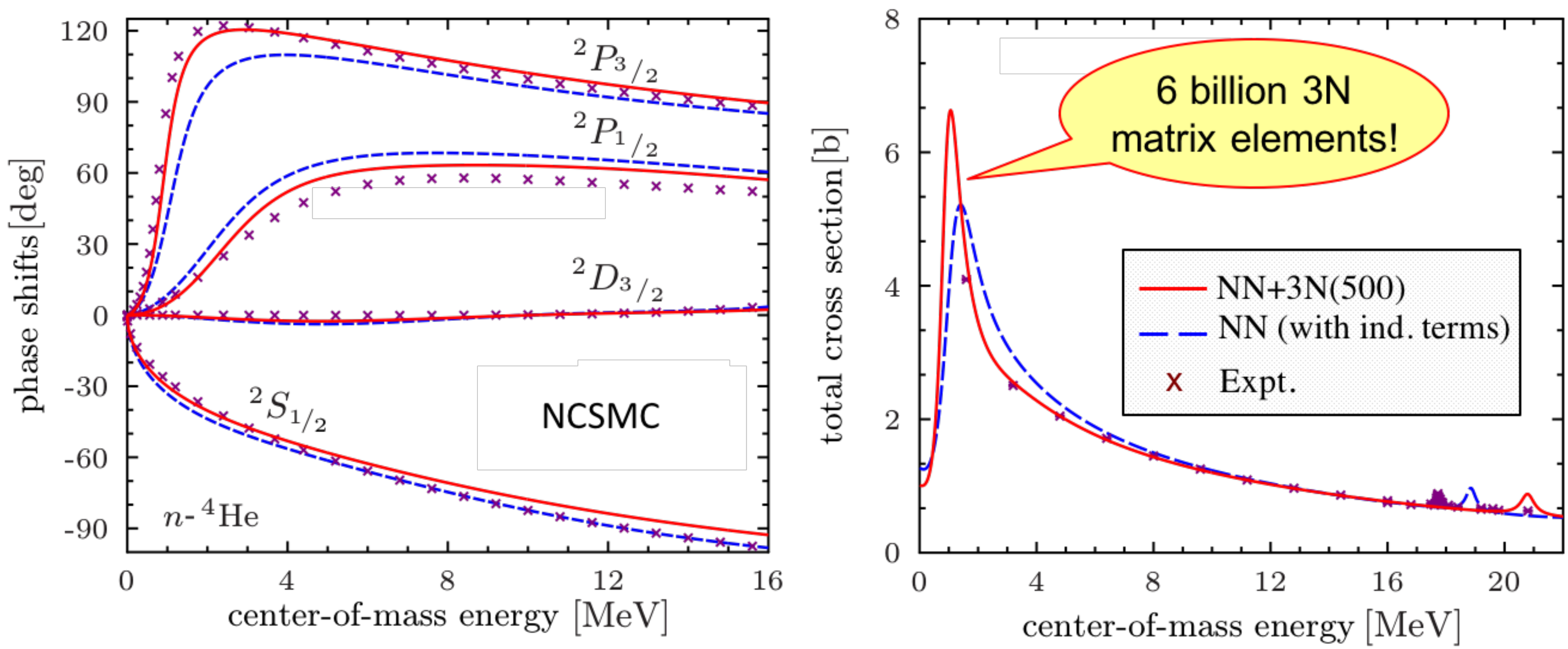
- The spurious center-of mass motion can again be separated exactly (a bit more complicated)

Question II: Can we predict …

- … the phenomena of low-energy nuclear reactions based on colliding nuclei made of interacting nucleons?



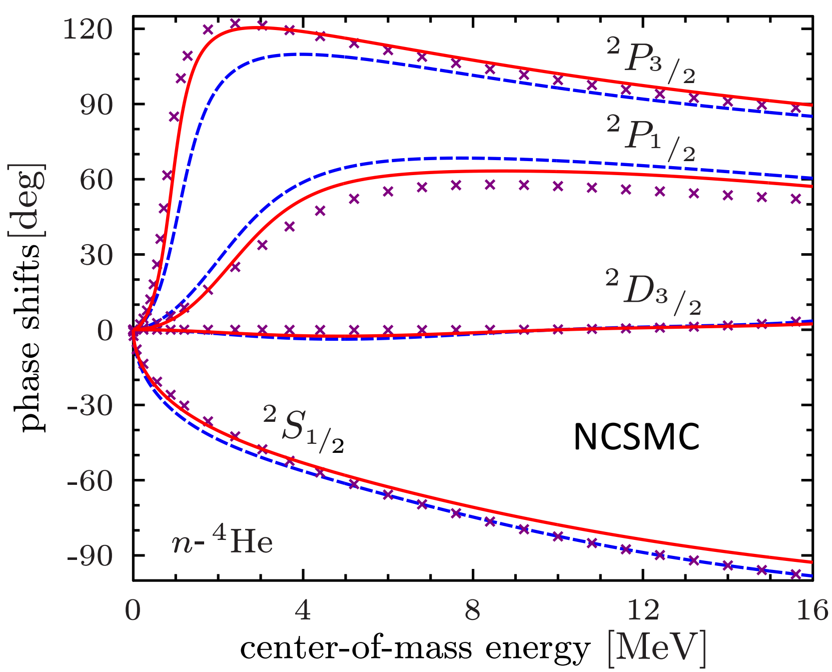
A good starting point: elastic scattering of neutrons on ^{4}He

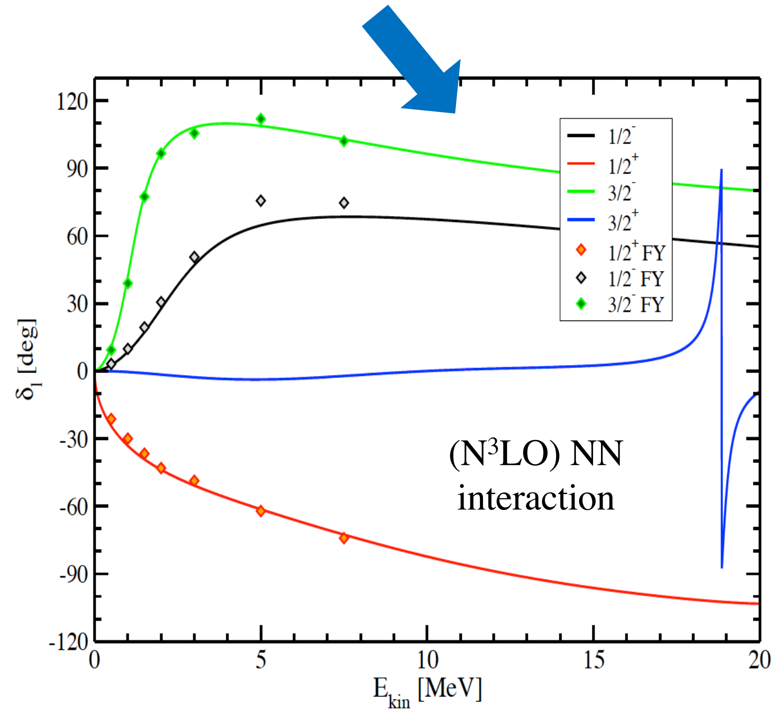


G. Hupin, S. Quaglioni, and P. Navratil, JPC Conf. Proc. (2015)

A good starting point: elastic scattering of neutrons on ^4He

- New 5-body Faddeev-Yacubovsky (FY, symbols) calculations from R. Lazauskas (ongoing work), in very good agreement with the NCSMC results (solid lines)

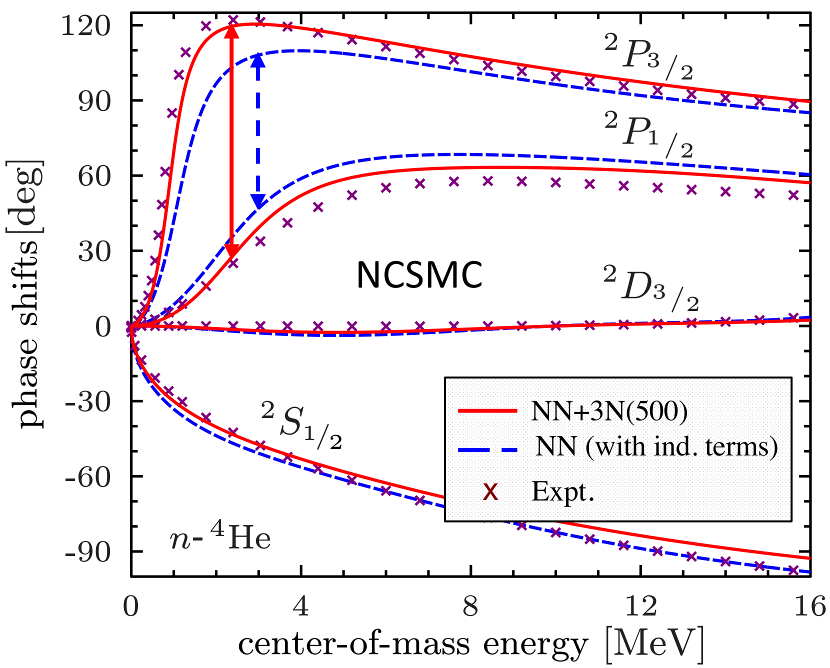




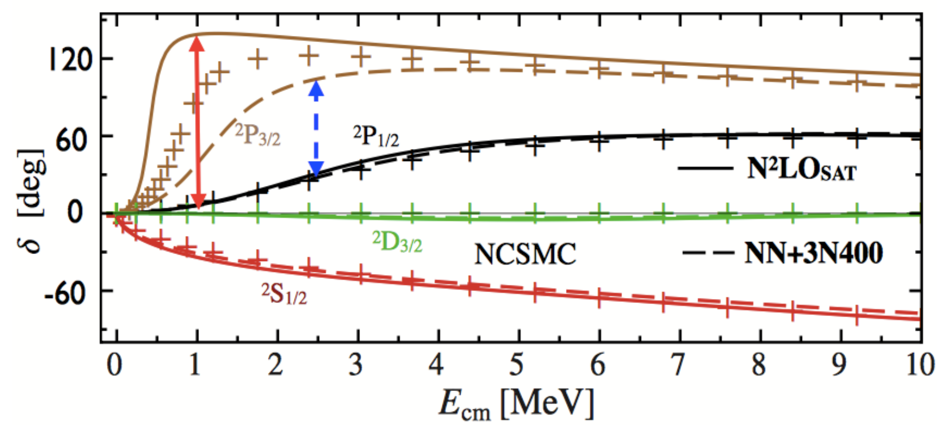
R. Lazauskas, INT Program 17-1a

A good starting point: elastic scattering of neutrons on ^{4}He

G. Hupin, S. Quaglioni, and P. Navratil, JPC Conf. Proc. (2015)



- The 3N force enhances the splitting between the 1/2- and 3/2- phase shifts



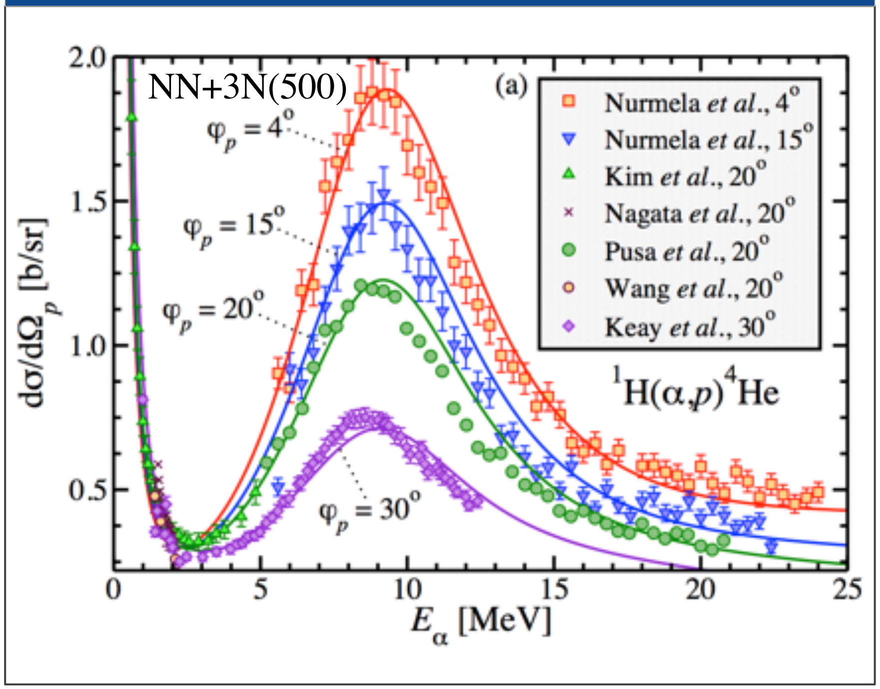
n-^{4}He scattering represents a stringent test for nuclear interaction models, and can be used in the future to better constrain chiral NN+3N forces

We can now predict nucleon and deuterium scattering on ^{4}He based on chiral NN+3N forces

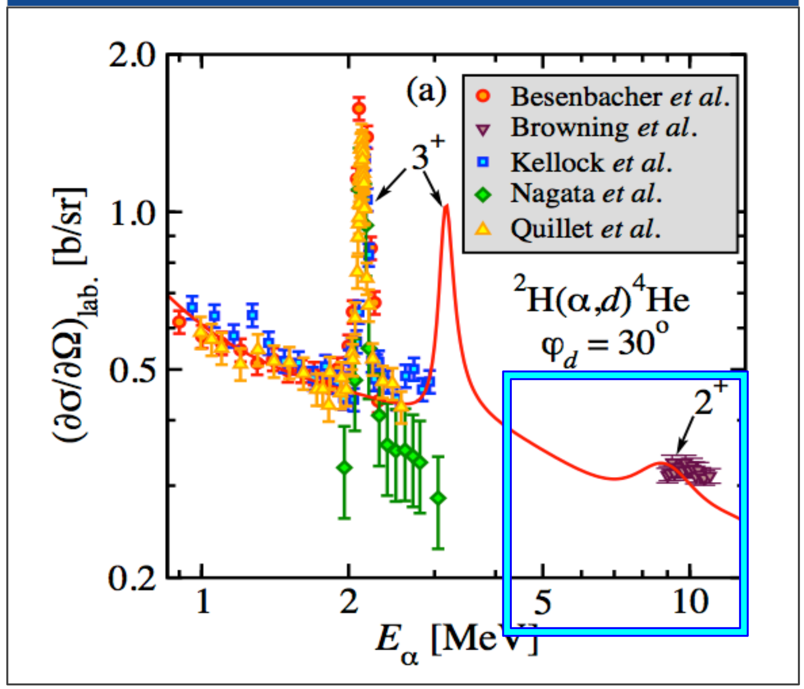
G. Hupin, S.Q., and P. Navratil,
Phys. Rev. C **90**, 061601(R) (2014)

G. Hupin, S.Q, and P. Navratil,
Phys. Rev. Lett. **114**, 212502 (2015)

Proton elastic recoil off helium



Elastic d-^{4}He Scattering

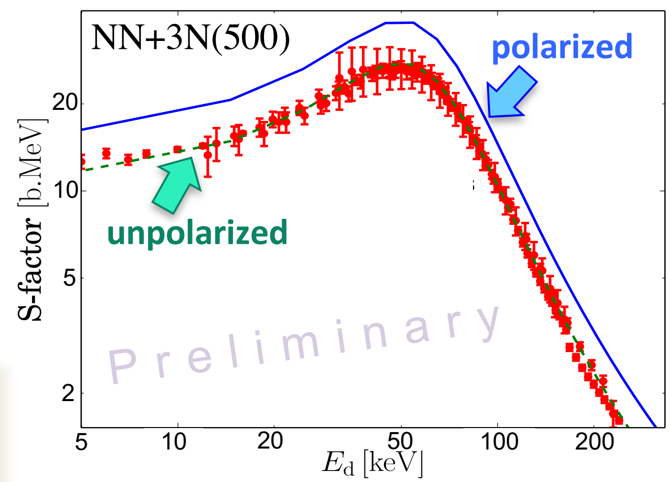


Chiral NN+3N forces works well, but not everywere!

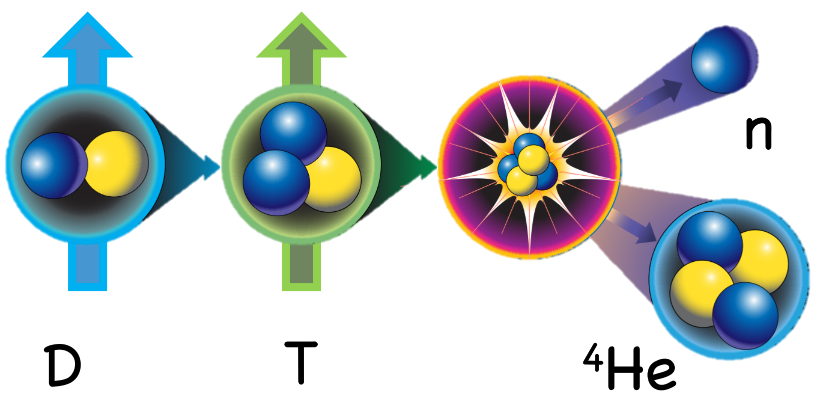
With the same NN+3N forces, we can also make predictions for more complex transfer reactions

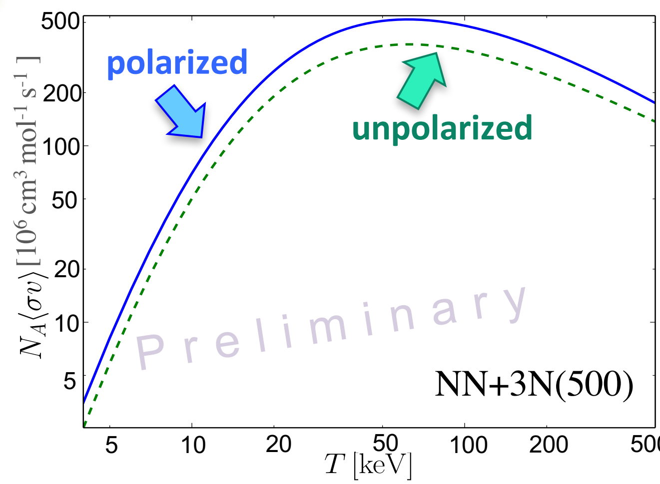
- What is the effect of spin polarization on the DT reaction rate?


G. Hupin



$$N_A\langle\sigma v\rangle = \sqrt{\frac{8}{\pi\mu}}\frac{N_A}{(k_BT)^{3/2}}\int_0^\infty dE\ S(E)\ \exp\left(-\frac{2\pi Z_1 Z_2 e^2}{\hbar\sqrt{2mE}} - \frac{E}{k_BT}\right)$$

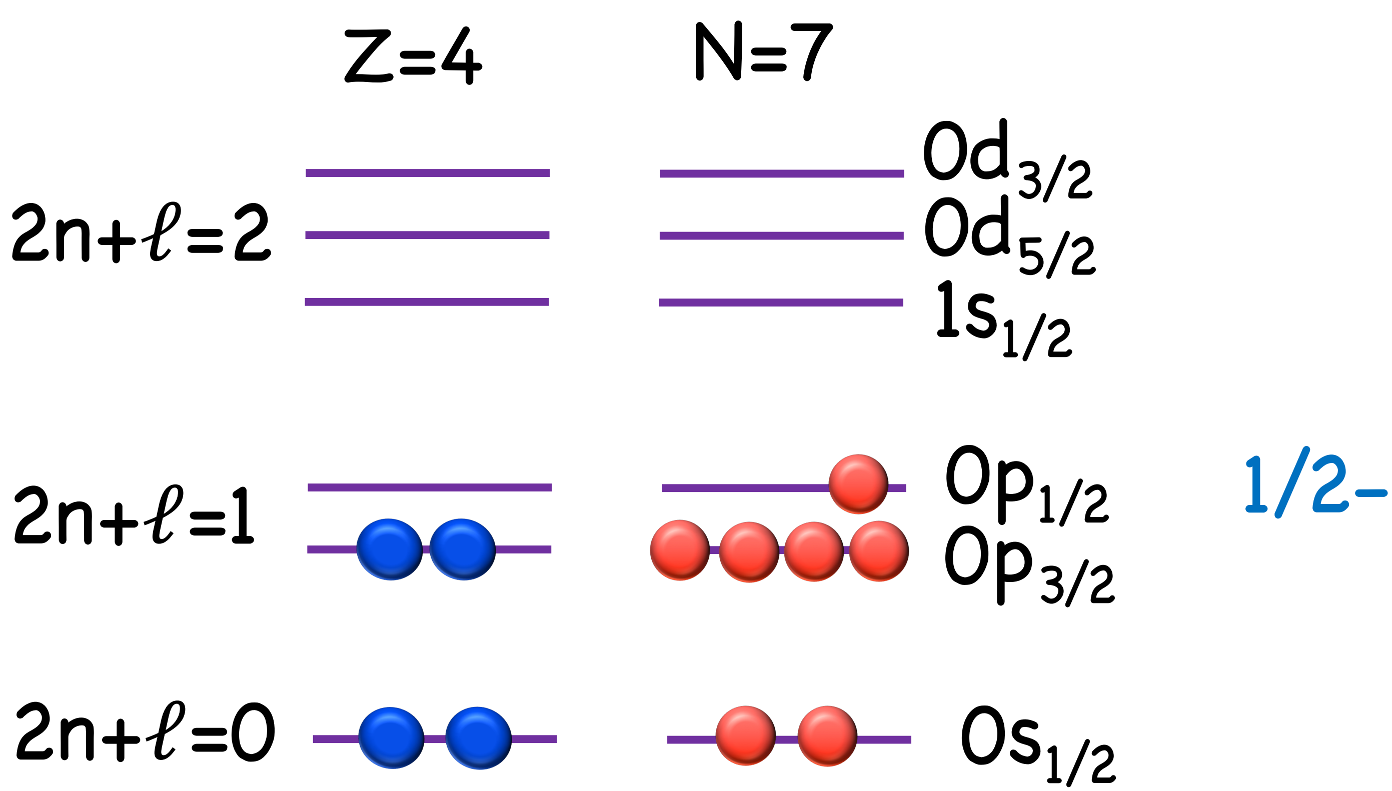




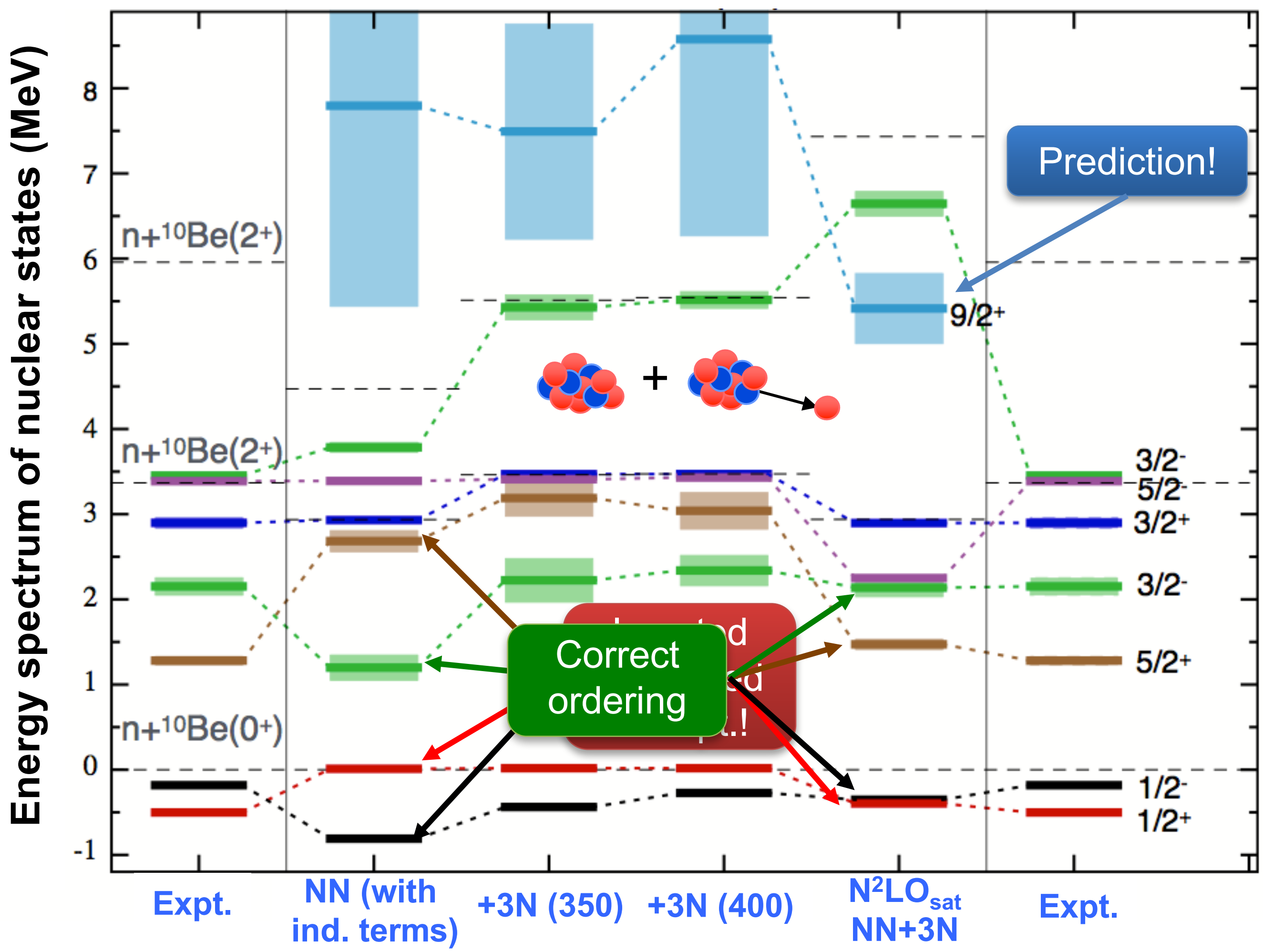
G. Hupin, S. Quaglioni, and P. Navrátil, in progress

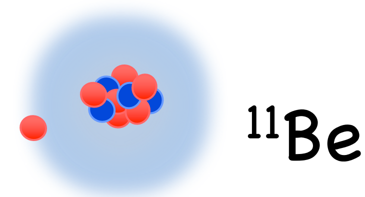
Problem

- What is the spin-parity of the ground state of the ^{11}B nucleus?

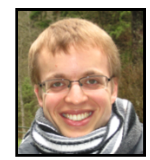


Can ab initio theory explain the phenomenon of parity inversion in ^{11}Be?





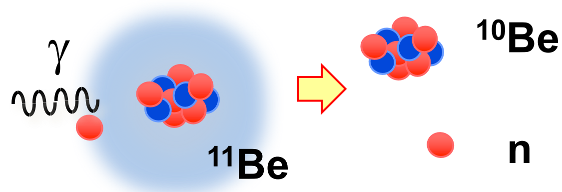
A. Calci

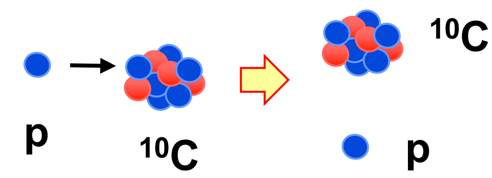


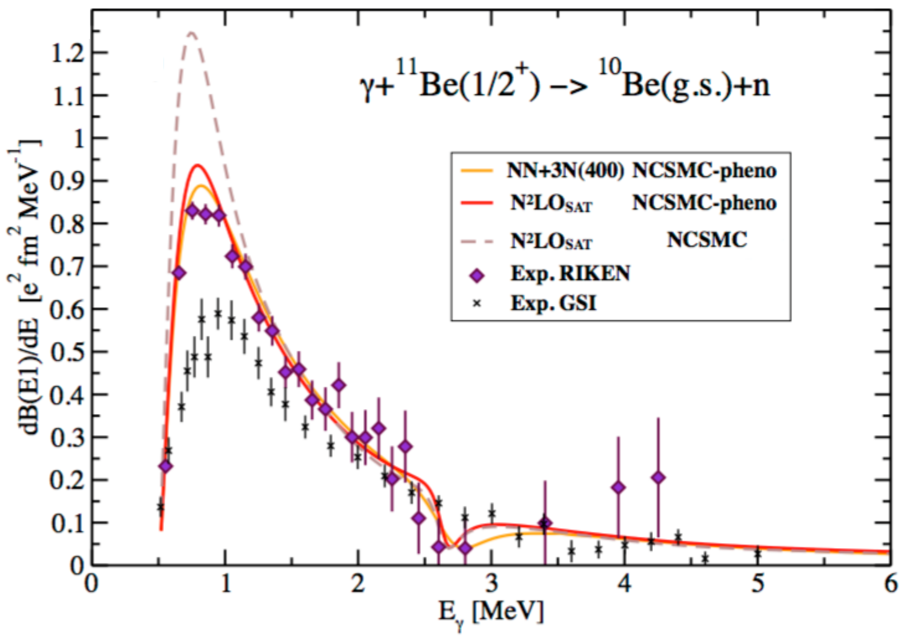
J. Dohet-Eraly

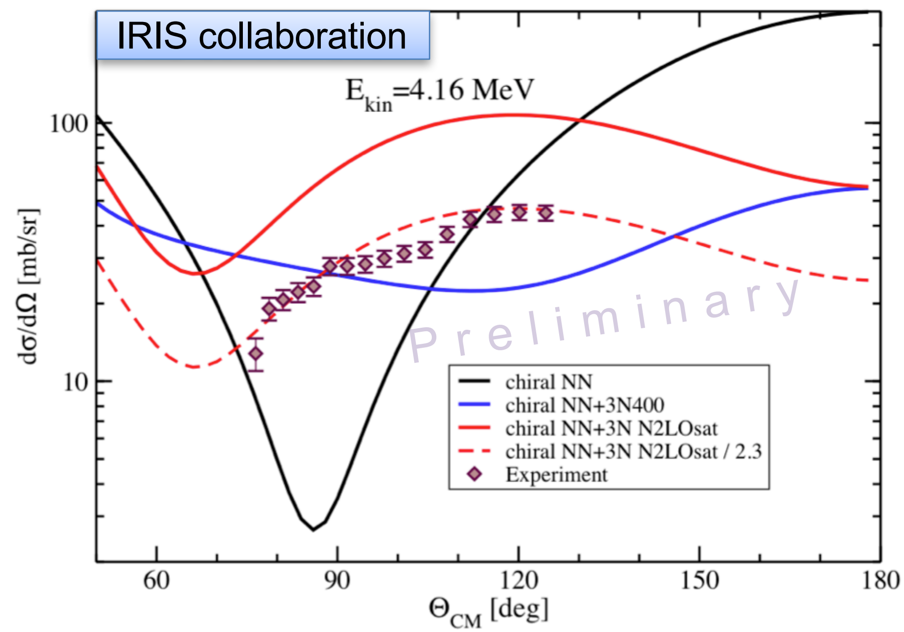
A. Calci, P. Navratil, R. Roth, **J. Dohet-Eraly**, S.Q., and G. Hupin, Phys. Rev. Lett. 117, 242501 (2016)

Scattering and reactions in A = 11





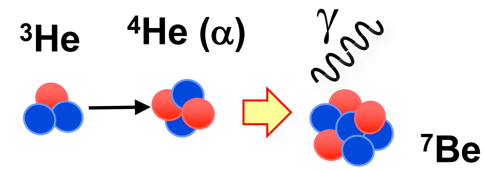




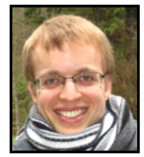
A. Calci, P. Navratil, R. Roth, **J. Dohet-Eraly**, S.Q., and G. Hupin, Phys. Rev. Lett. 117, 242501 (2016)

A. Kumar, R. Kanungo, **A. Calci**, P. Navratil et al., to appear shortly in Phys. Rev. Lett.

Now gradually building up capability to describe solar pp-chain reactions

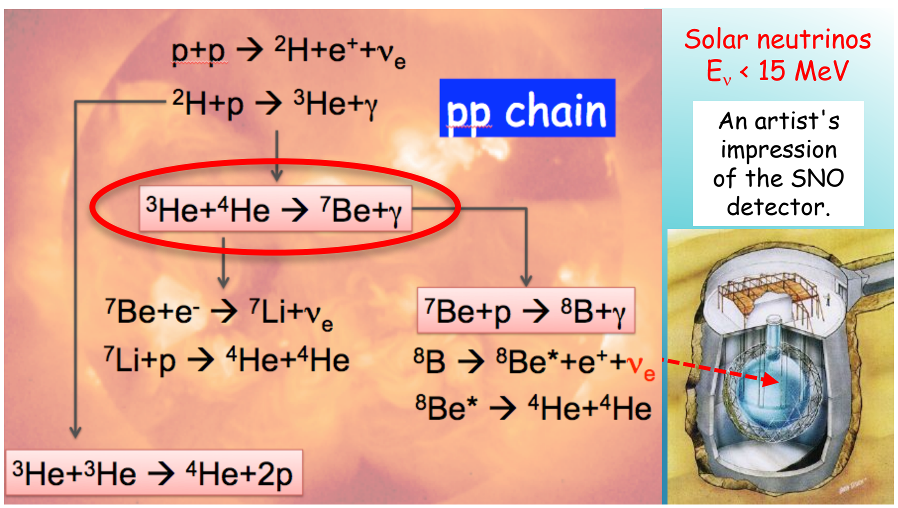


The $^3He(\alpha,\gamma)^7Be$ fusion essential to simulate the flux of solar neutrinos

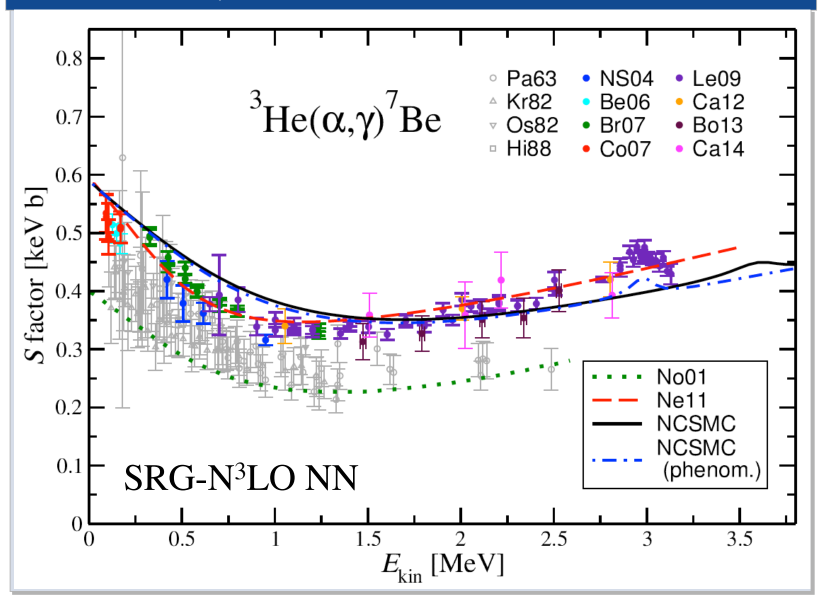


J. Dohet-Eraly

J. Dohet-Eraly, P. Navrátil, S.Q., W. Horiuchi, and F. Raimondi, Physics Letters B **757**, 430 (2016)



$^3He(\alpha,\gamma)^7Be$ Astrophysical S-factor



Quantitative comparison still requires inclusion of 3N forces

What about helium burning reactions?

Nuclear Lattice EFT with the Adiabatic Projection Method

- Use projection Monte Carlo to ‘dress’ clusters

$$|\vec{R}\rangle_\tau = \exp(-H\tau)|\vec{R}\rangle$$

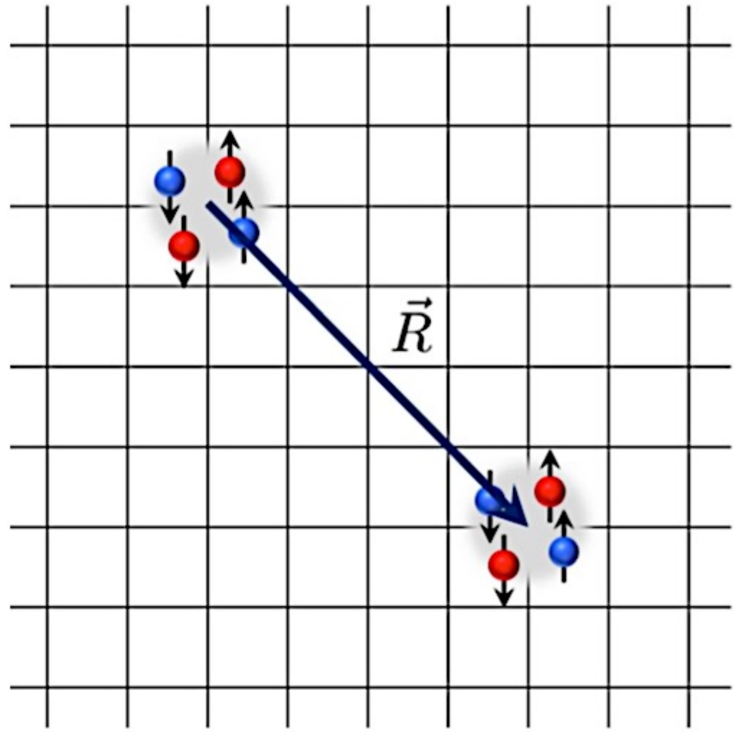
- Evaluate adiabatic inter-cluster Hamiltonian, norm matrix elements

$$[H_\tau]_{\vec{R},\vec{R}'} = {}_\tau\langle\vec{R}|H|\vec{R}'\rangle_\tau$$

$$[N_\tau]_{\vec{R},\vec{R}'} = {}_\tau\langle\vec{R}|\vec{R}'\rangle_\tau$$

- Solve for relative scattering wave function

$$|\vec{R}\rangle = \sum_{\vec{r}} |\vec{r}+\vec{R}\rangle_1 \otimes |\vec{r}\rangle_2$$

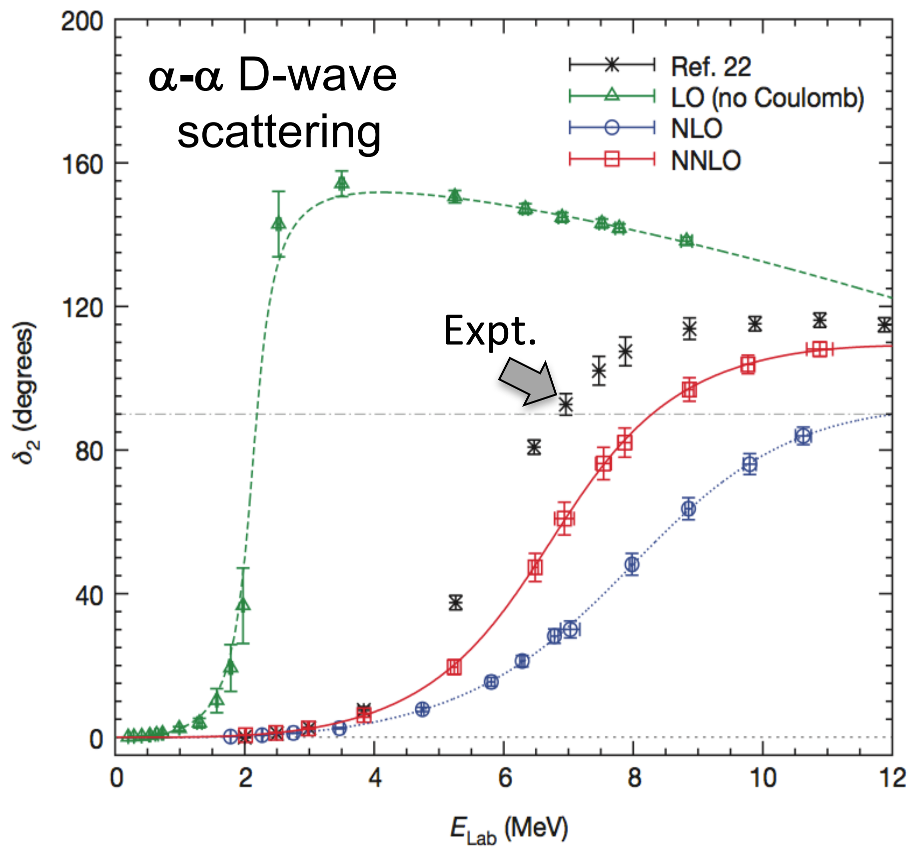


Elhatisari, Lee, Rupak, Epelbaum, Krebs, Lähde, Luu, Meißner, Nature 528, 111 (2015)

Towards ab initio calculations of helium burning

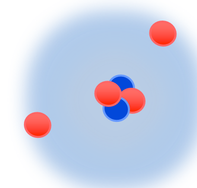
Nuclear Lattice EFT with the Adiabatic Projection Method

- Promising results for α-α scattering
- Quantitative predictions still require calculation at N^3LO
- Computational scaling ~A^2
- $^{12}C(\alpha,\gamma)^{16}O$ becoming possible!
- Extensions to enable treatment of three-cluster dynamics required before the method can be applied to the $^4He(\alpha\alpha,\gamma)^{12}C$ process

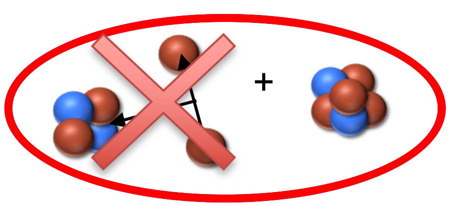


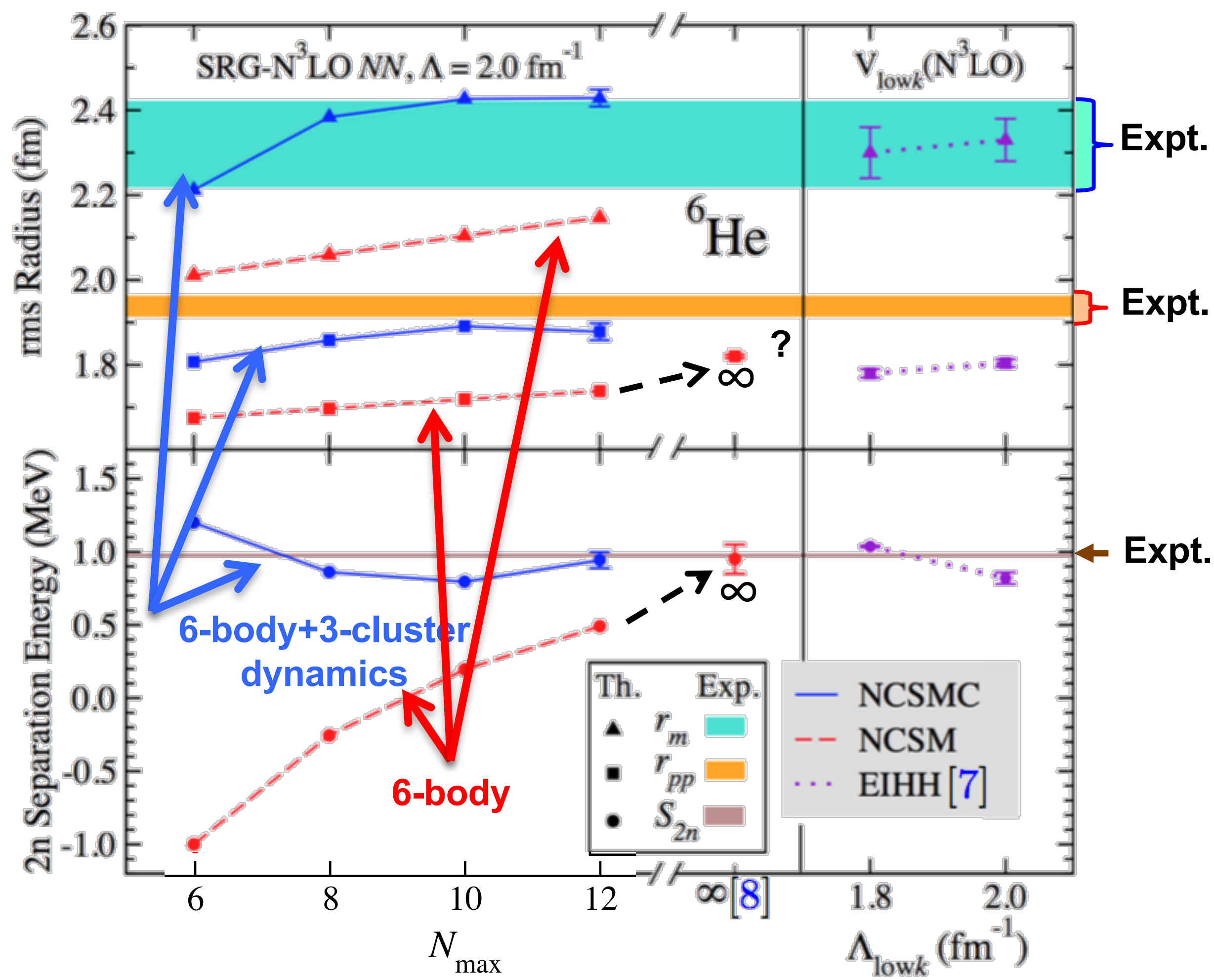
Elhatisari, Lee, Rupak, Epelbaum, Krebs, Lähde, Luu, Meißner, Nature 528, 111 (2015)

How many-body correlations and α-clustering shape ^{6}He



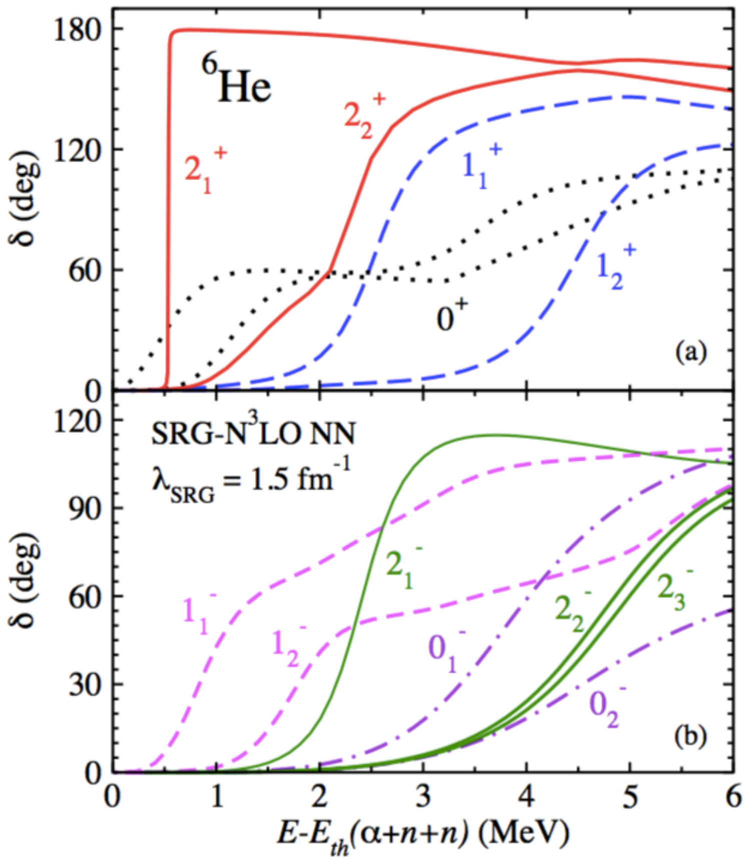
- Stringent tests for *ab initio* theory
- 3-cluster NCSMC

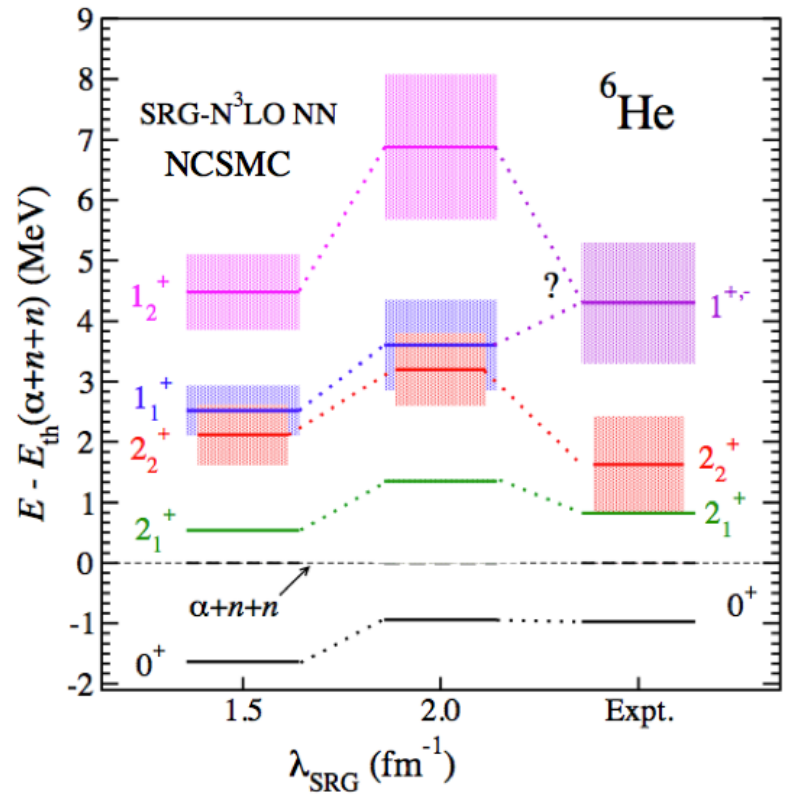




C. Romero-Redondo, S.Q., P.Navratil, and G. Hupin,
Phys. Rev. Lett. 117, 222501 (2016)

Results for ^{6}He low-energy continuum





For now, qualitative agreement with experiment. Inclusion of 3N forces (currently underway) remains crucial to arrive at accurate description of the spectrum as a whole.

Conclusions and Prospects

- In recent years ab initio theory has made great strides in its description of light-nuclei scattering and reactions as well as of the structure of loosely bound and unbound exotic nuclei
- We are on the verge of predicting Solar fusion and Helium burning cross sections from chiral NN+3N forces
- This will aid in solving long-standing problems in stellar nucleosynthesis
- These developments are also allowing to further expose and will help overcome deficiencies in chiral NN+3N forces