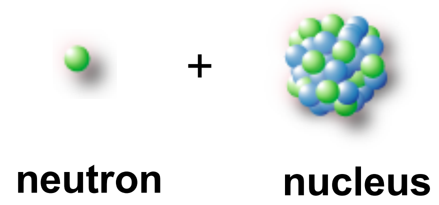
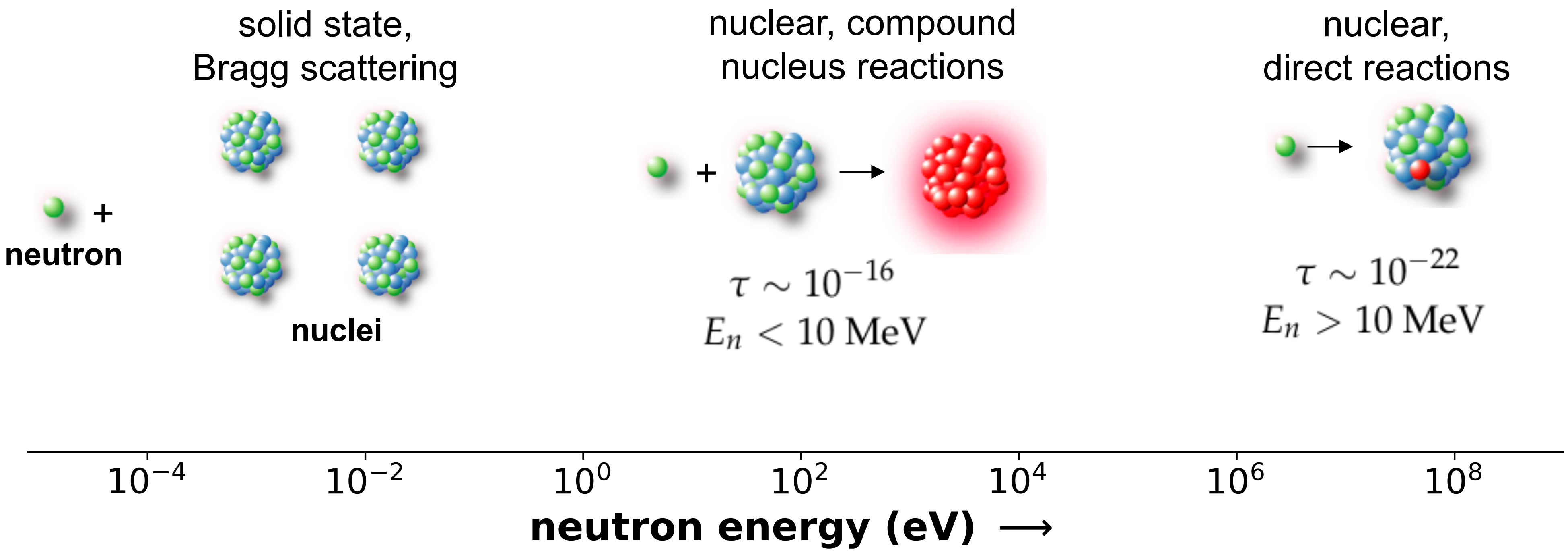
Neutron-induced nuclear reactions

Frank Gunsing
CEA Saclay, Irfu
F-91191 Gif-sur-Yvette
France

Neutron-induced nuclear reactions

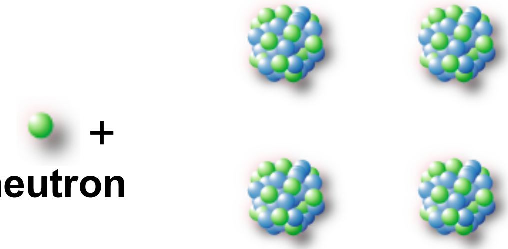


Neutron-induced nuclear reactions



Neutron-induced nuclear reactions

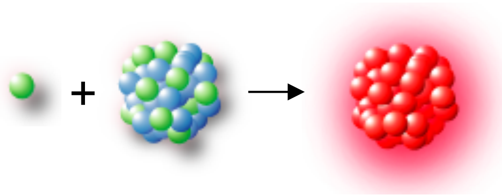
solid state,
Bragg scattering



neutron

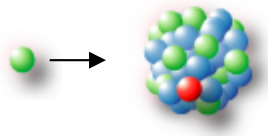
nuclei

nuclear, compound
nucleus reactions

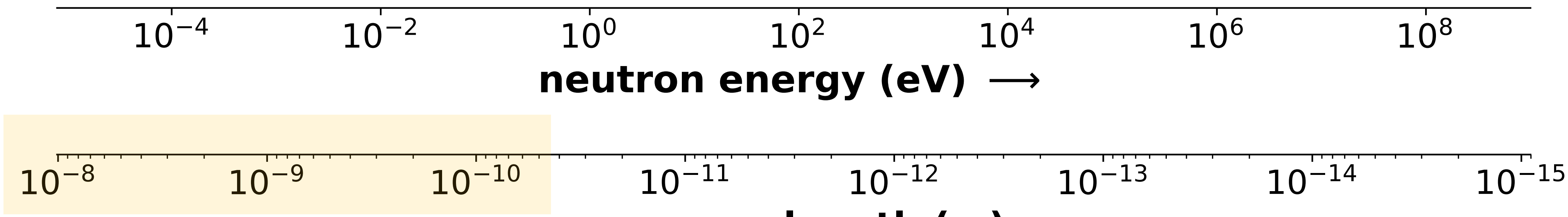


$\tau \sim 10^{-16}$
$E_n < 10$ MeV

nuclear,
direct reactions



$\tau \sim 10^{-22}$
$E_n > 10$ MeV



1 Å = 10^{-10} m

de Broglie wavelength: $$\lambda = \frac{h}{\sqrt{2mE_k}}$$

Neutron-induced nuclear reactions

- Reaction notations:

$^{10}B + {}^{1}n \rightarrow {}^{7}Li + {}^{4}He$

$^{10}B + n \rightarrow {}^{7}Li + \alpha$

$^{10}B(n,\alpha)$

$^{238}U + n \rightarrow {}^{239}U^{*}$

$^{238}U + n \rightarrow {}^{239}U + \gamma$

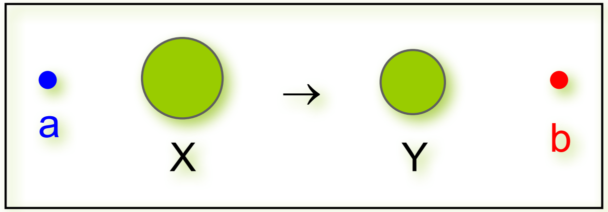
$^{238}U(n,\gamma)$

- Neutron induced nuclear reactions:
  - elastic scattering (n,n)
  - inelastic scattering (n,n')
  - capture (n,γ)
  - fission (n,f)
  - particle emission (n,α), (n,p), (n,xn)
  - total cross section σ_{tot}: sum of all partial reactions

- Cross section σ, expressed in barns, $1\ b = 10^{-28}\ m^2$

Neutron-induced nuclear reactions

- neutron reaction X(a,b)Y



- neutron cross section:

  function of the kinetic energy of the particle a

$$\sigma(E_a) = \int\int \frac{d^2\sigma(E_a, E_b, \Omega)}{dE_b d\Omega} dE_b d\Omega$$

- differential cross section:

  function of the kinetic energy of the particle a
  and function of the kinetic energy or the angle
  of the particle b

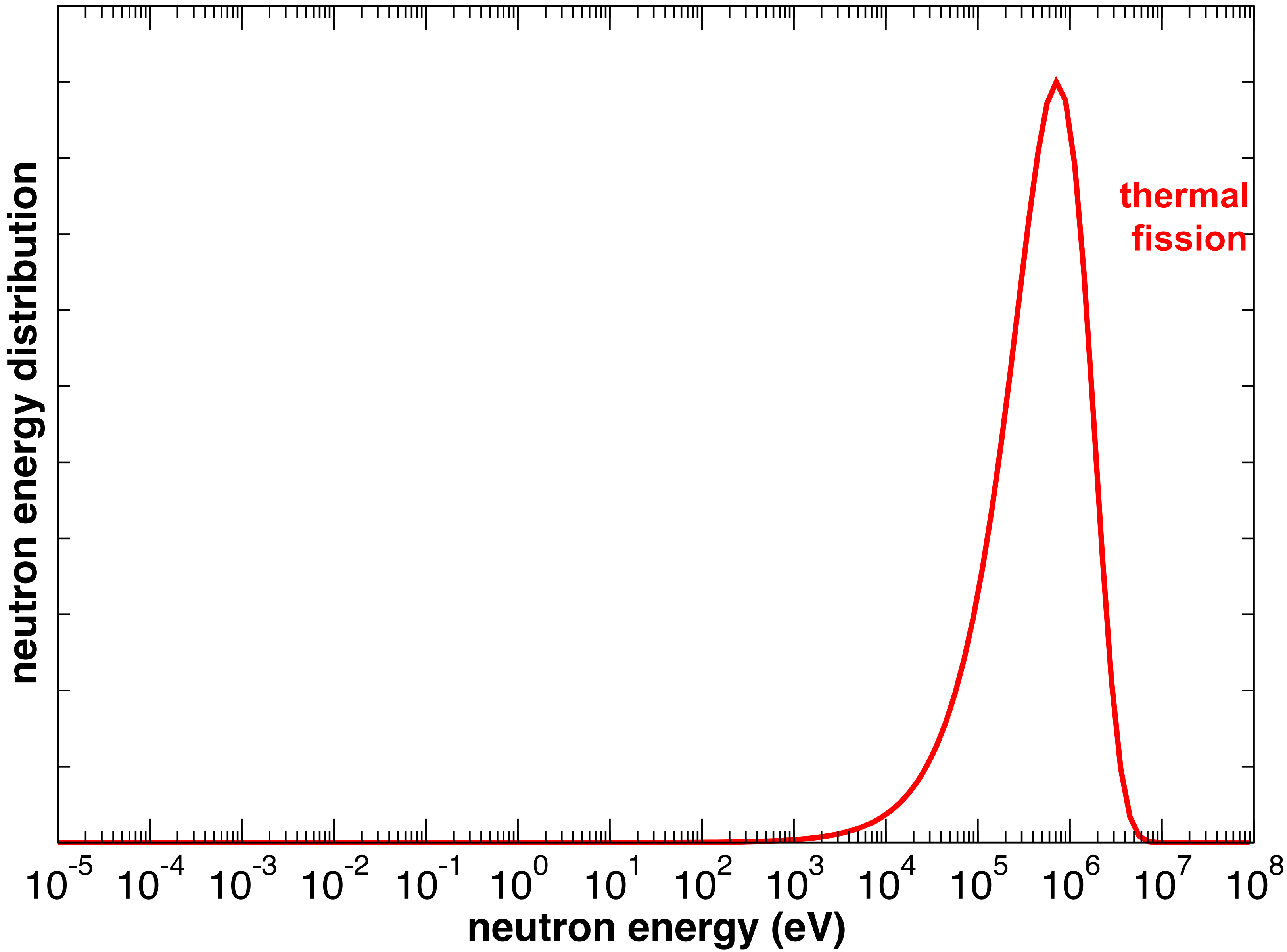
$$\frac{d\sigma(E_a, E_b)}{dE_b} \qquad \frac{d\sigma(E_a, \Omega)}{d\Omega}$$

- double differential cross section:

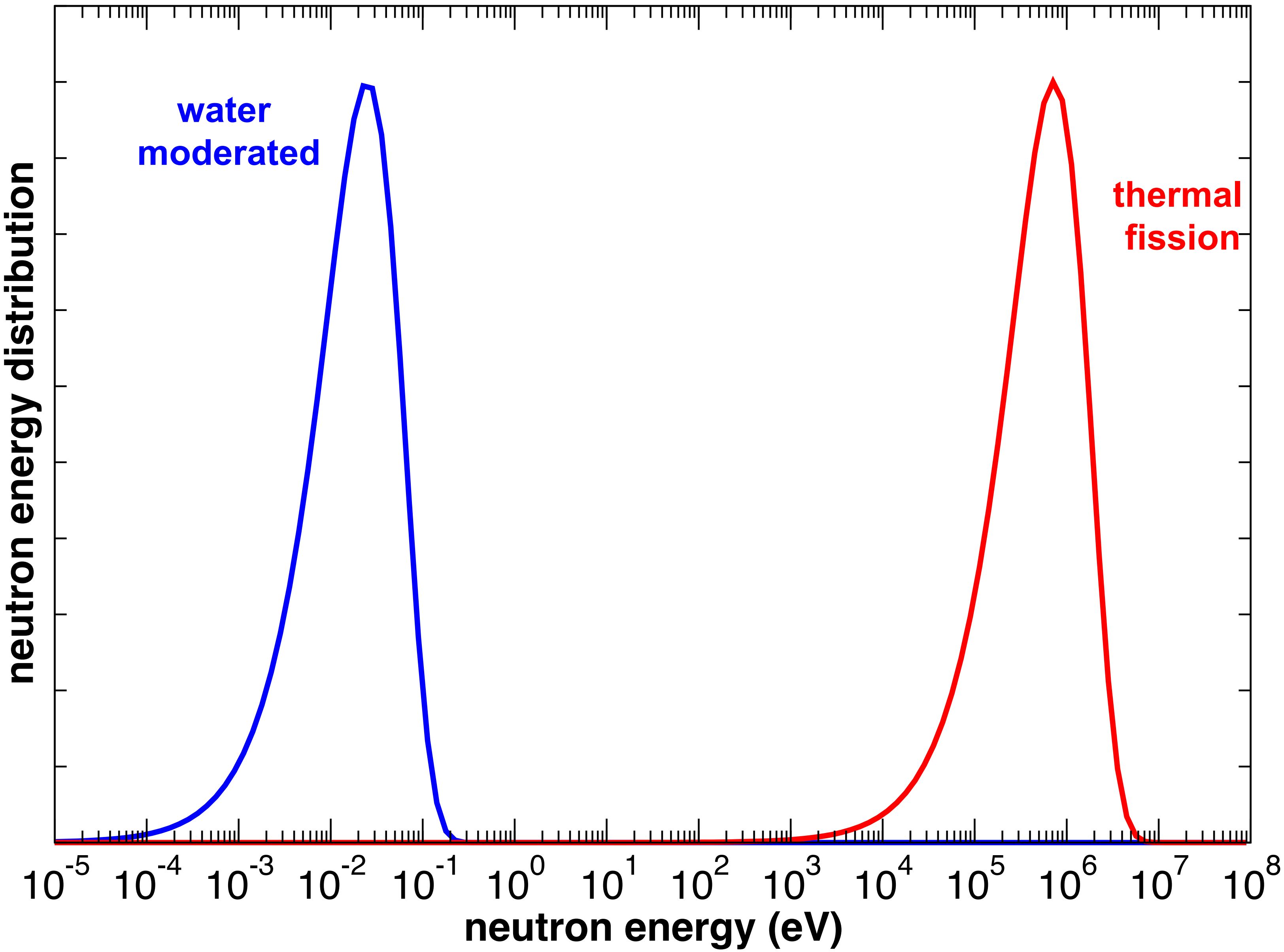
  function of the kinetic energy of the particle a
  and function of the kinetic energy and the angle
  of the particle b

$$\frac{d^2\sigma(E_a, E_b, \Omega)}{dE_b d\Omega}$$

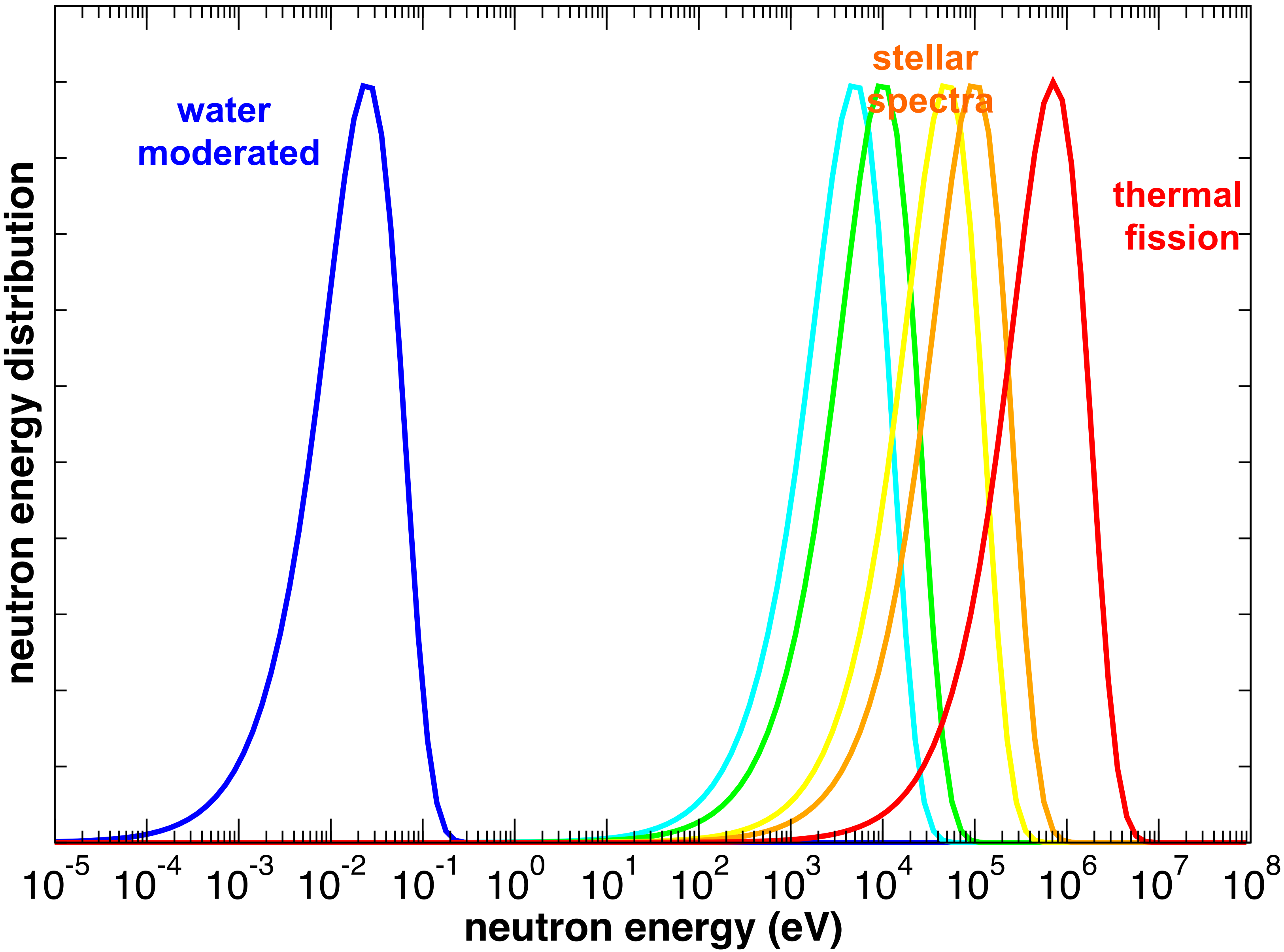
neutron energy distributions



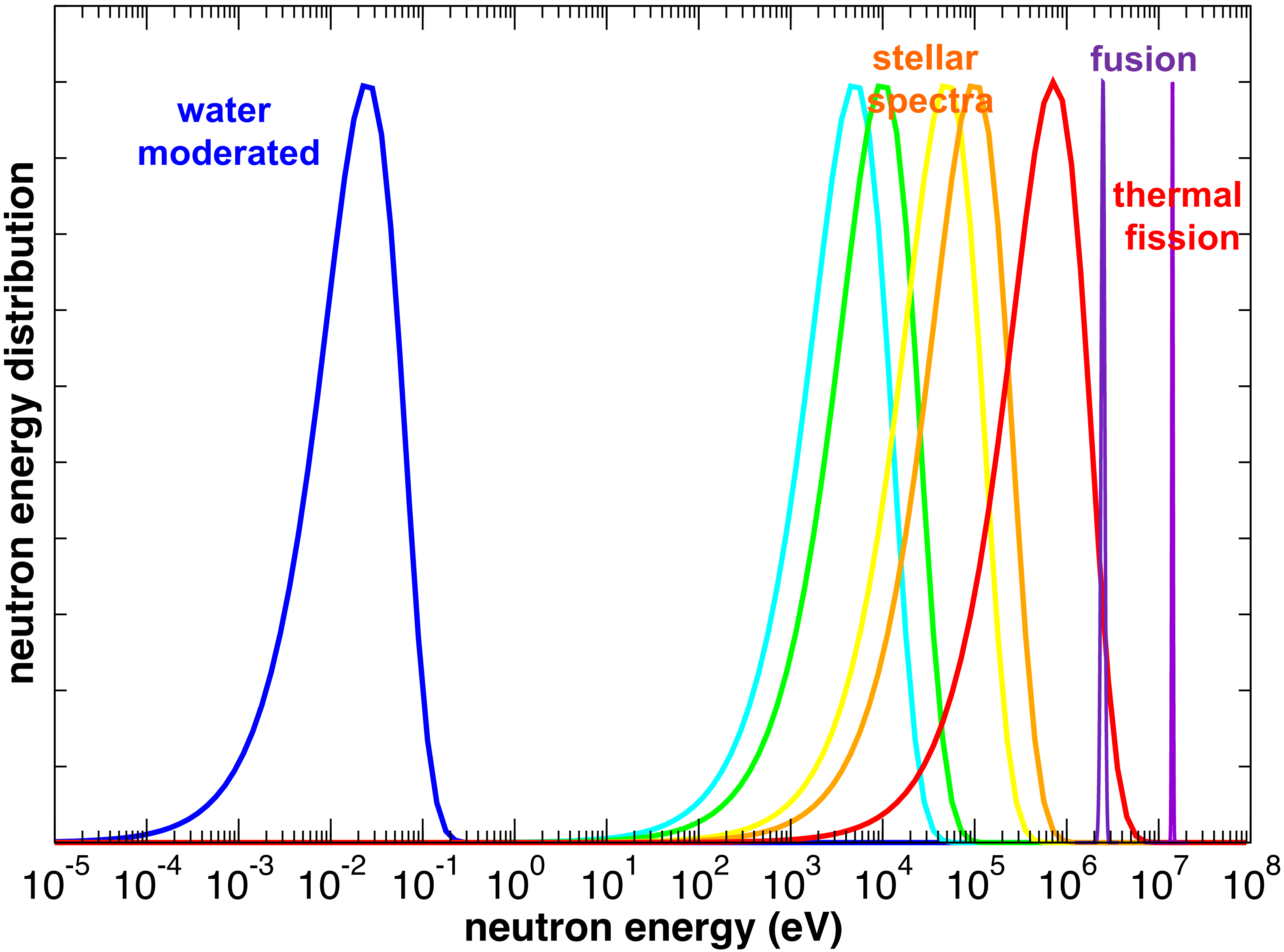
neutron energy distributions



neutron energy distributions



neutron energy distributions



Maxwell-Boltzmann distribution

- Maxwell-Boltzmann statistics describe neutron spectra from
  - thermal-neutron induced fission
  - water moderated neutrons (infinite moderator)
  - stellar spectra (sources $^{22}Ne(\alpha,n)^{25}Mg$, $^{13}C(\alpha,n)^{16}O$)

- Velocity distribution at temperature T

$$n_v(v) = 4\pi \left(\frac{m}{2\pi kT}\right)^{3/2} v^2 \exp\left(-\frac{mv^2}{2kT}\right)$$

has maximum at $$v_{\mathrm{max}} = \sqrt{2kT/m}$$

- At velocity v = 2200 m/s (used as thermal neutron reference)
  E_{max} = **25.3** meV,   T = **293.6** K, λ = **0.18** nm

Maxwell-Boltzmann distribution

- Distributions of kinetic energy, wavelength or time-of-flight can be converted into each other

$$n_v(v)dv = n_E(E)dE = n_t(t)dt = n_\lambda(\lambda)d\lambda$$

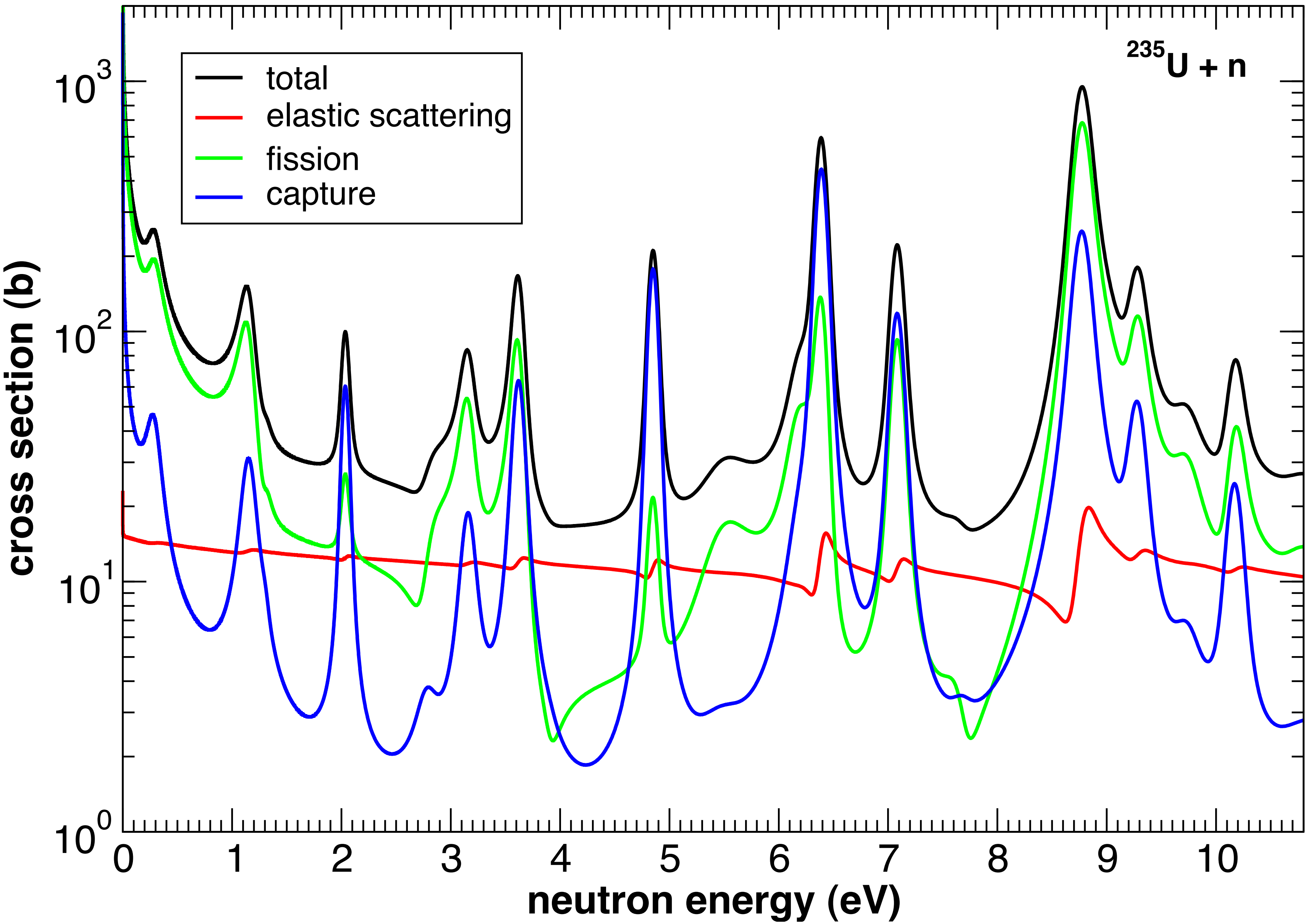
- For neutron beams, a "flux"-like distribution is more appropriate

$$\varphi_v(v) \propto v \times n_v(v)$$

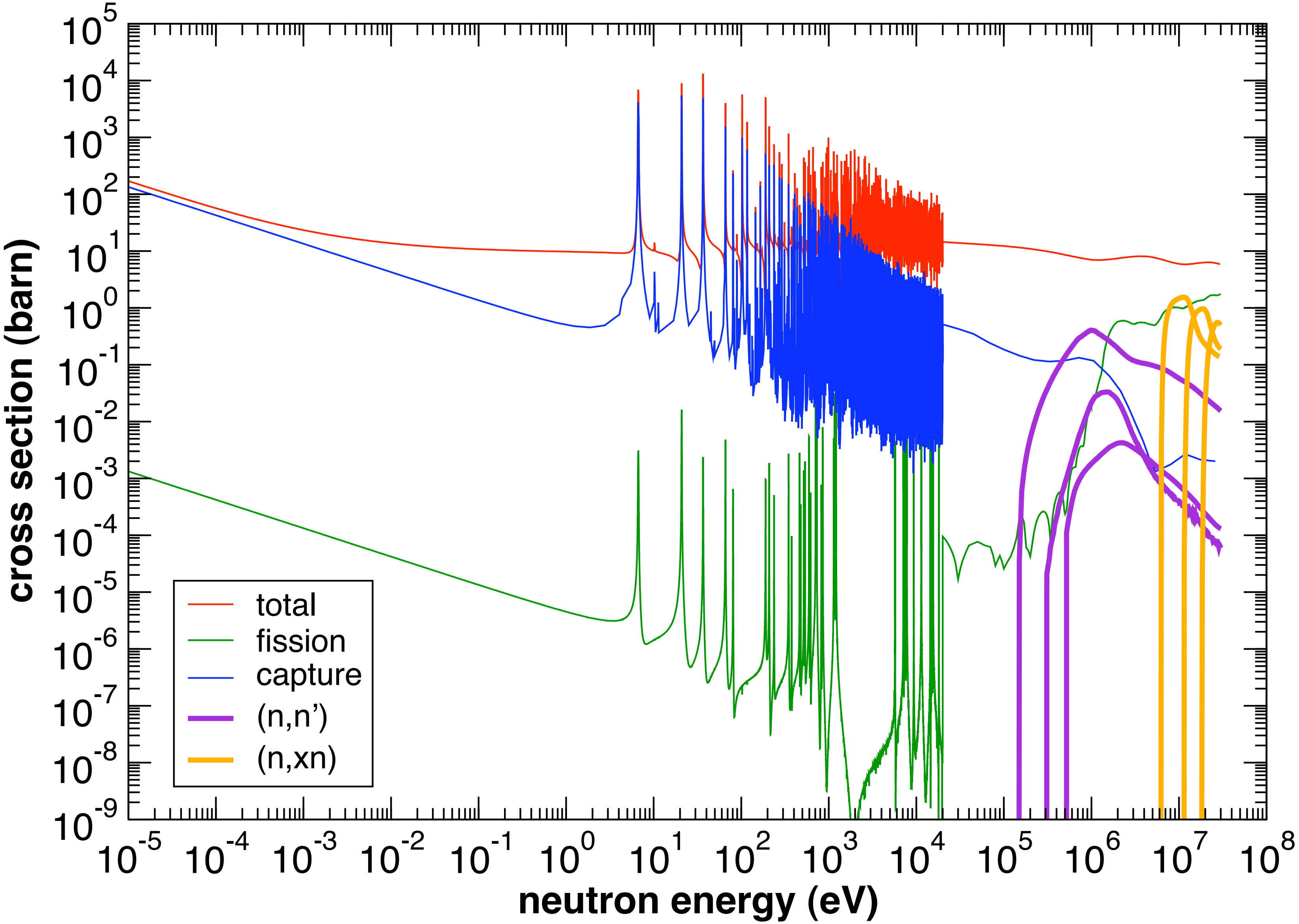
with conversions

$$\varphi_v(v)dv = \varphi_E(E)dE = \varphi_t(t)dt = \varphi_\lambda(\lambda)d\lambda$$

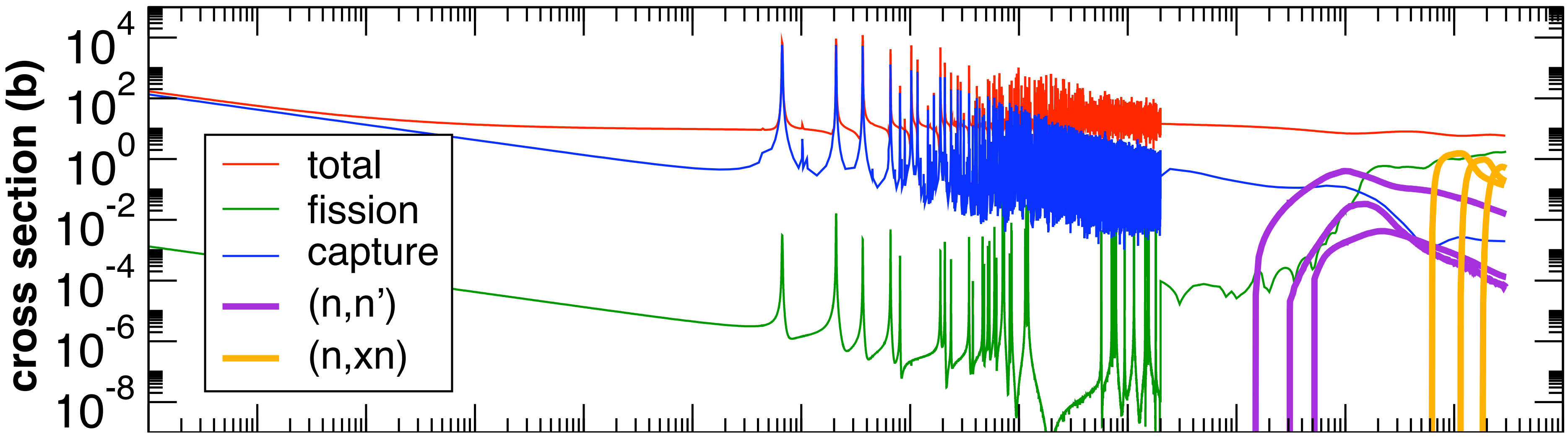
Neutron cross sections



Neutron cross sections

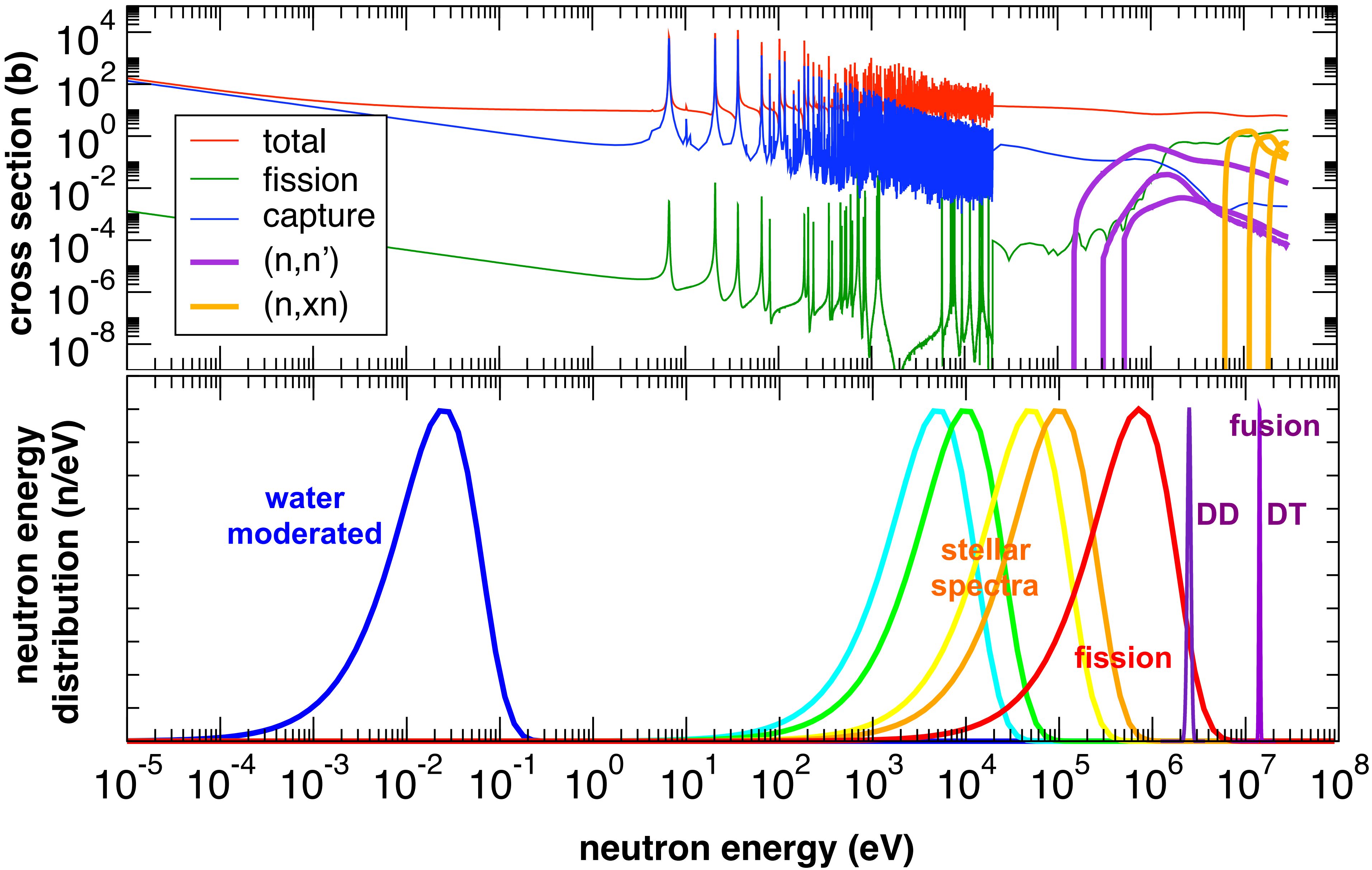


Neutron cross sections

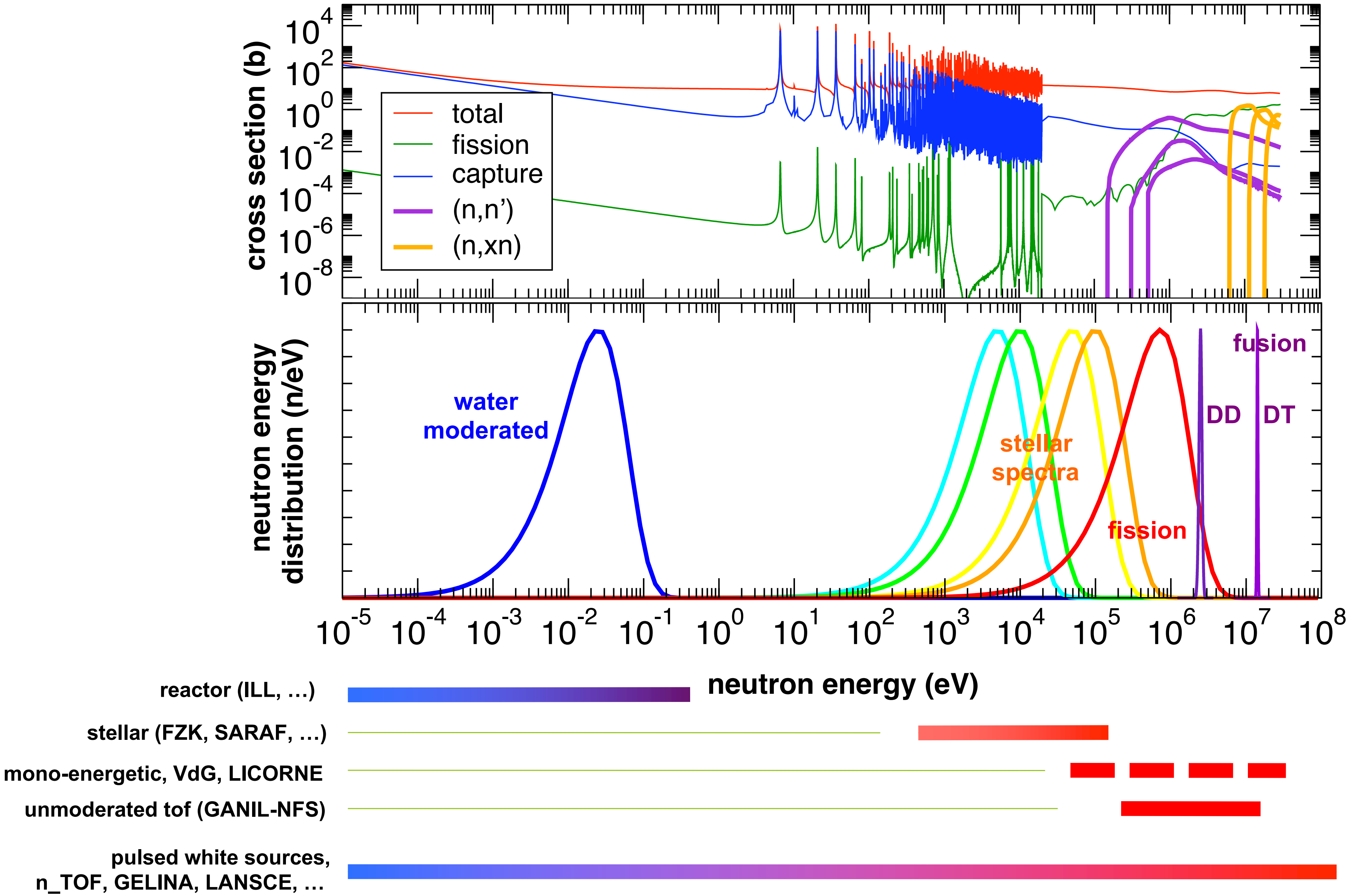


neutron energy (eV)

Neutron cross sections



Neutron cross sections



Applications of neutron-induced reactions

Different fields for applications

- Stellar nucleo-synthesis, neutron capture, elements > Fe
- Nuclear technology, reactors, fuel cycles, waste transmutation
- Resonance spectroscopy, level densities
- Reaction mechanisms (fission) and model development
- Others

Need for "evaluated" data for simulations

- Historically developed for nuclear reactors
- Nowadays general purpose (Nuclear Data)

Evaluated nuclear data libraries

Libraries:
- JEFF - Europe
- JENDL - Japon
- ENDF/B - US
- BROND - Russia
- CENDL - China

Common format:
ENDF-6

Contents:
Data for particle-induced reactions (neutrons, protons, gamma, other)
but also radioactive decay data

Data are indentified by “materials”
(isotopes, isomeric states, (compounds))

ex.	^{16}O:	mat = 825
	^{nat}V:	mat = 2300
	^{242m}Am:	mat = 9547

Files for a material

from report ENDF-102

1 General information
2 Resonance parameter data
3 Reaction cross sections
4 Angular distributions for emitted particles
5 Energy distributions for emitted particles
6 Energy-angle distributions for emitted particles
7 Thermal neutron scattering law data
8 Radioactivity and fission-product yield data
9 Multiplicities for radioactive nuclide production
10 Cross sections for photon production
12 Multiplicities for photon production
13 Cross sections for photon production
14 Angular distributions for photon production
15 Energy distributions for photon production
23 Photo-atomic interaction cross sections
27 Atomic form factors or scattering functions for photo-atomic interactions
30 Data Covariances obtained from parameter covariances and sensitivities
31 Data covariances for nubar
32 Data covariances for resonance parameters
33 Data covariances for reaction cross sections
34 Data covariances for angular distributions
35 Data covariances for energy distributions
39 Data covariances for radionuclide production yields
40 Data covariances for radionuclide production cross sections

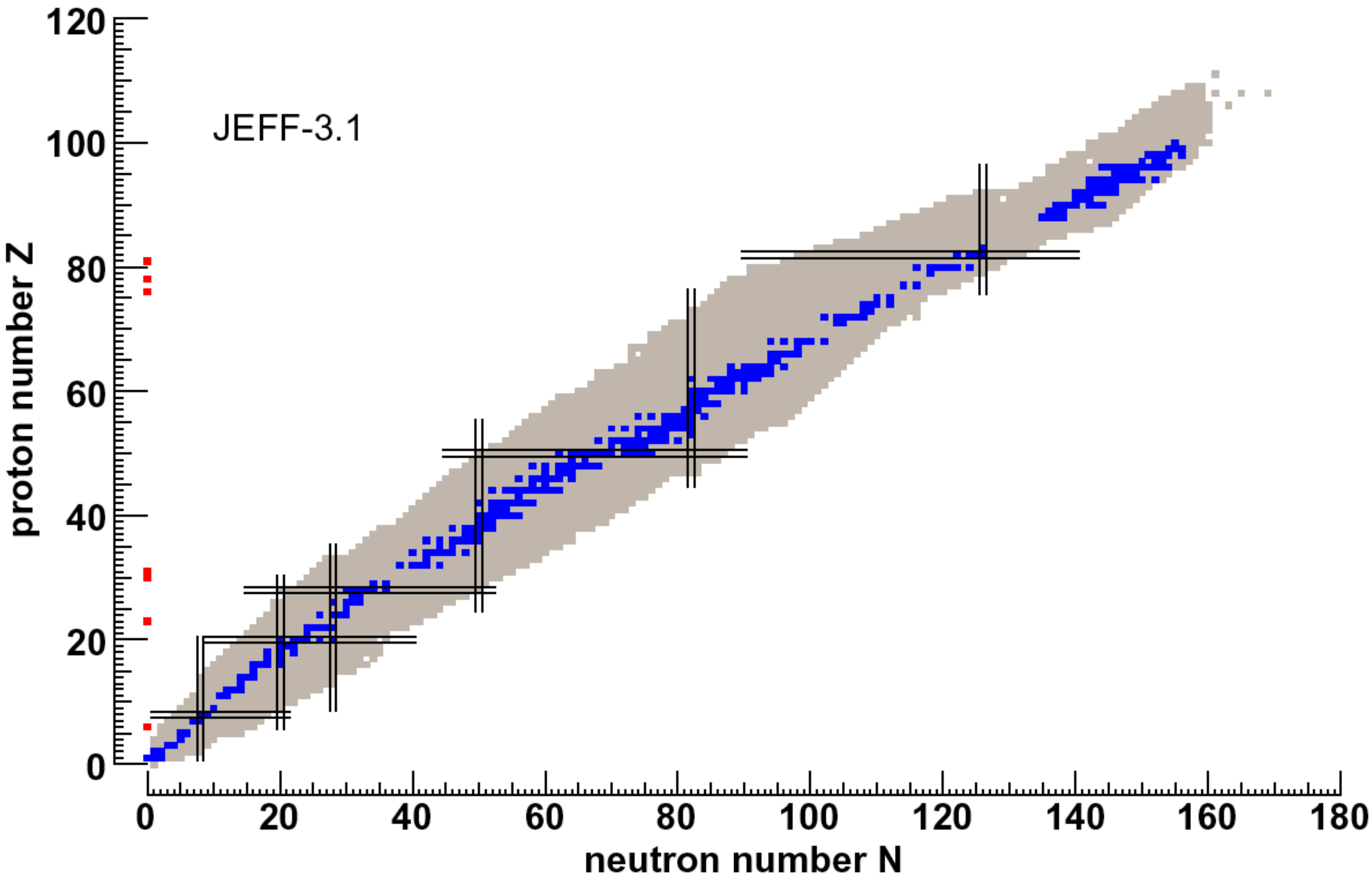
Example: part of an evaluated data file

Z and A values — nuclear mass — formalism flag — number of resonances — material number — MF number — MT number

```
 7.919700+4 1.952740+2          0          0          1          07925 2151     1
 7.919700+4 1.000000+0          0          0          1          07925 2151     2
 1.000000-5 5.000000+3          1          2          0          07925 2151     3
 1.500000+0 9.800000-1          0          0          1          07925 2151     4
 1.952740+2 0.000000+0          0          0       1578       2637925 2151     5
-3.380000+1 2.000000+0 2.562000-1 1.562000-1 1.000000-1 0.000000+07925 2151     6
 4.906000+0 2.000000+0 1.377000-1 1.520000-2 1.225000-1 0.000000+07925 2151     7
 4.645000+1 1.000000+0 1.241300-1 1.300000-4 1.240000-1 0.000000+07925 2151     8
 5.810000+1 1.000000+0 1.164000-1 4.400000-3 1.120000-1 0.000000+07925 2151     9
```

resonance energy — spin — total width — neutron width — gamma width — fission width — line number

The library JEFF-3.1

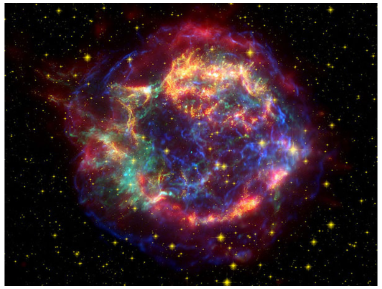


Nuclear Data

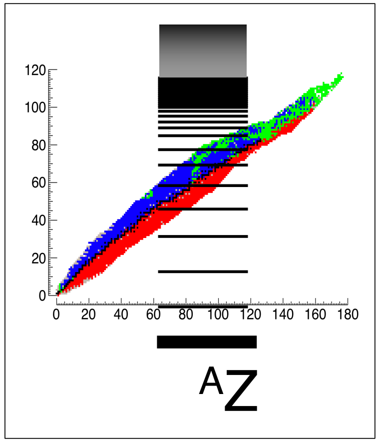
Processing / Calculations / Simulations

Users

nuclear energy technology



astrophysics



nuclear science

Nuclear Data

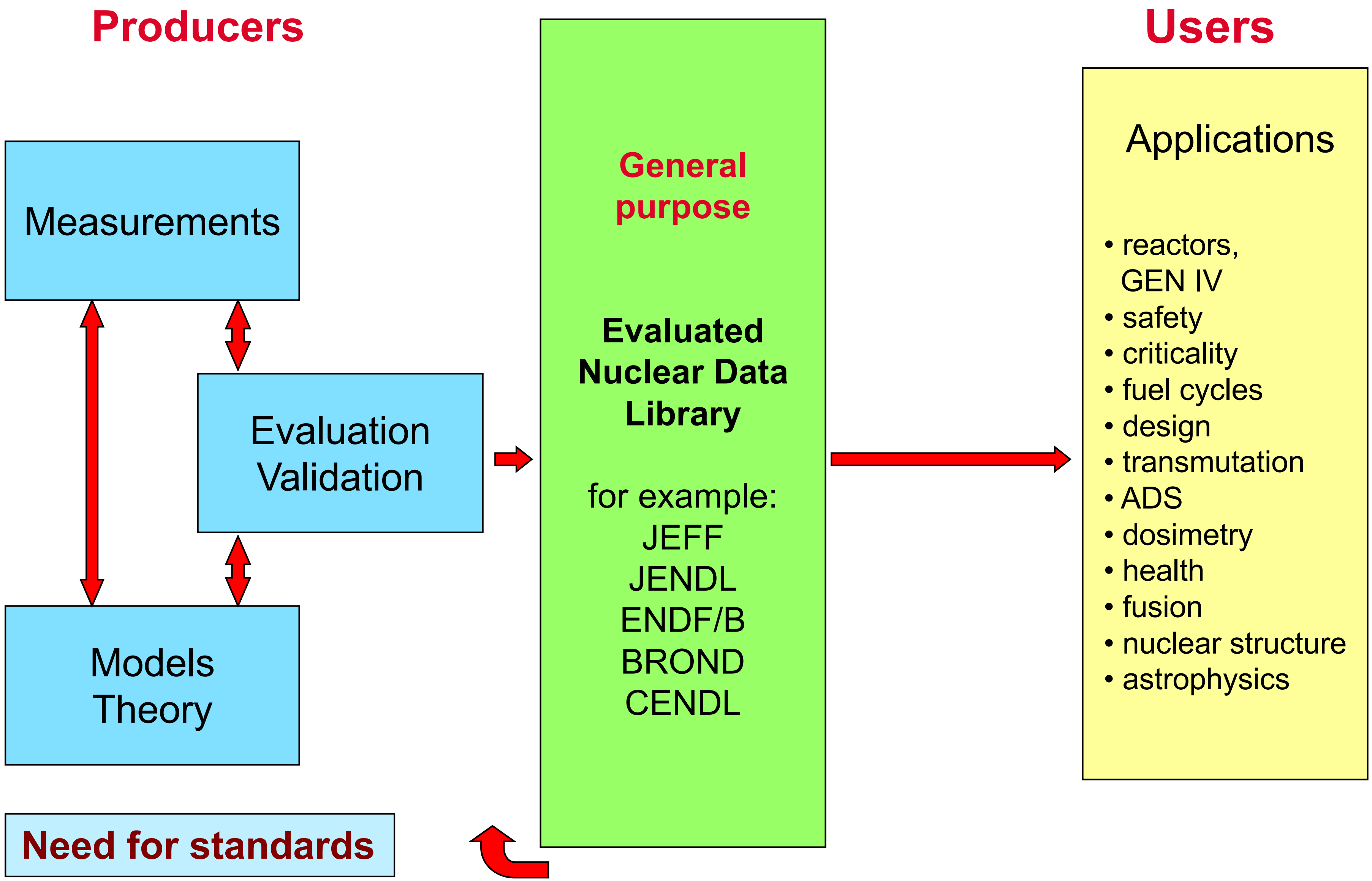
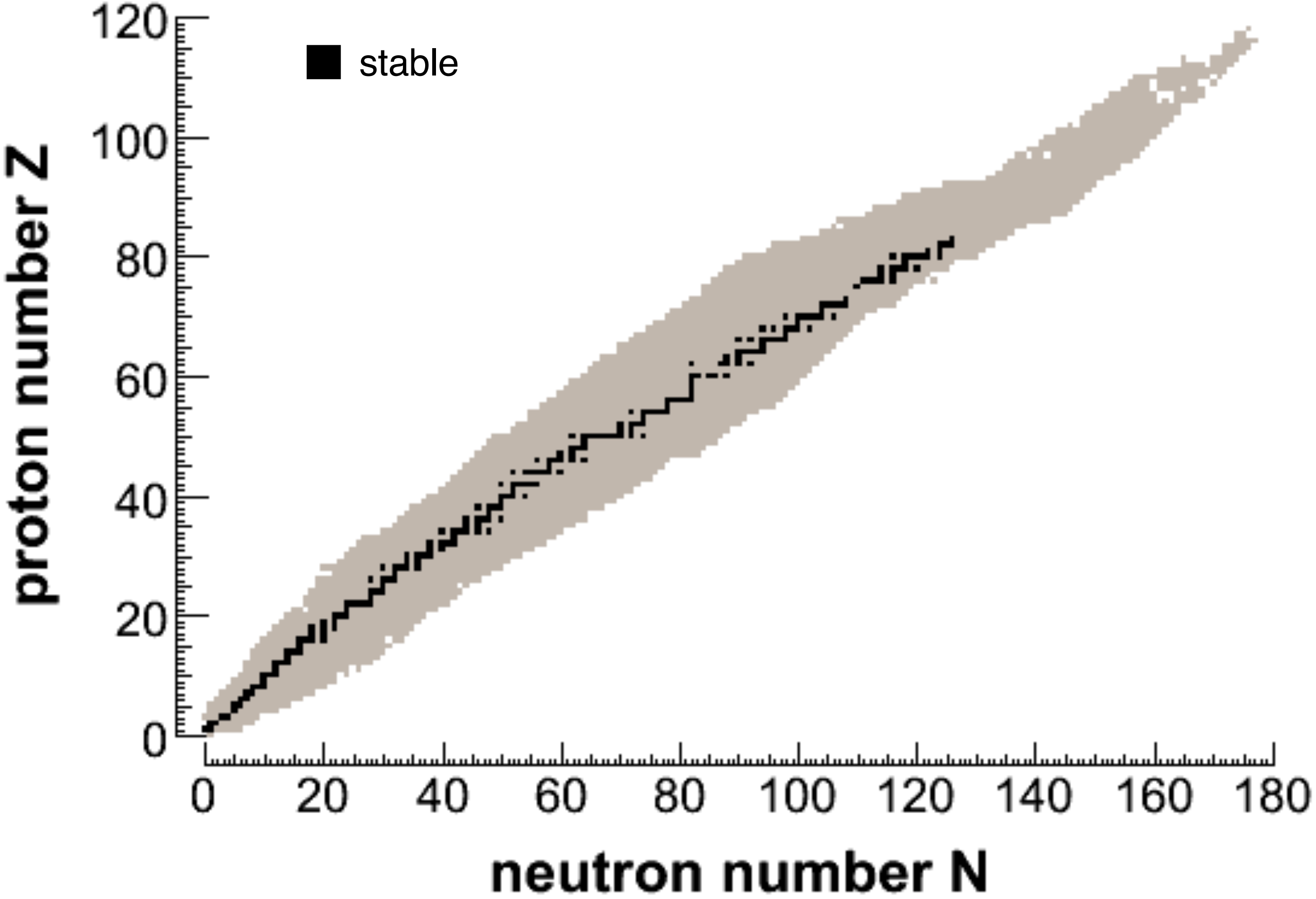
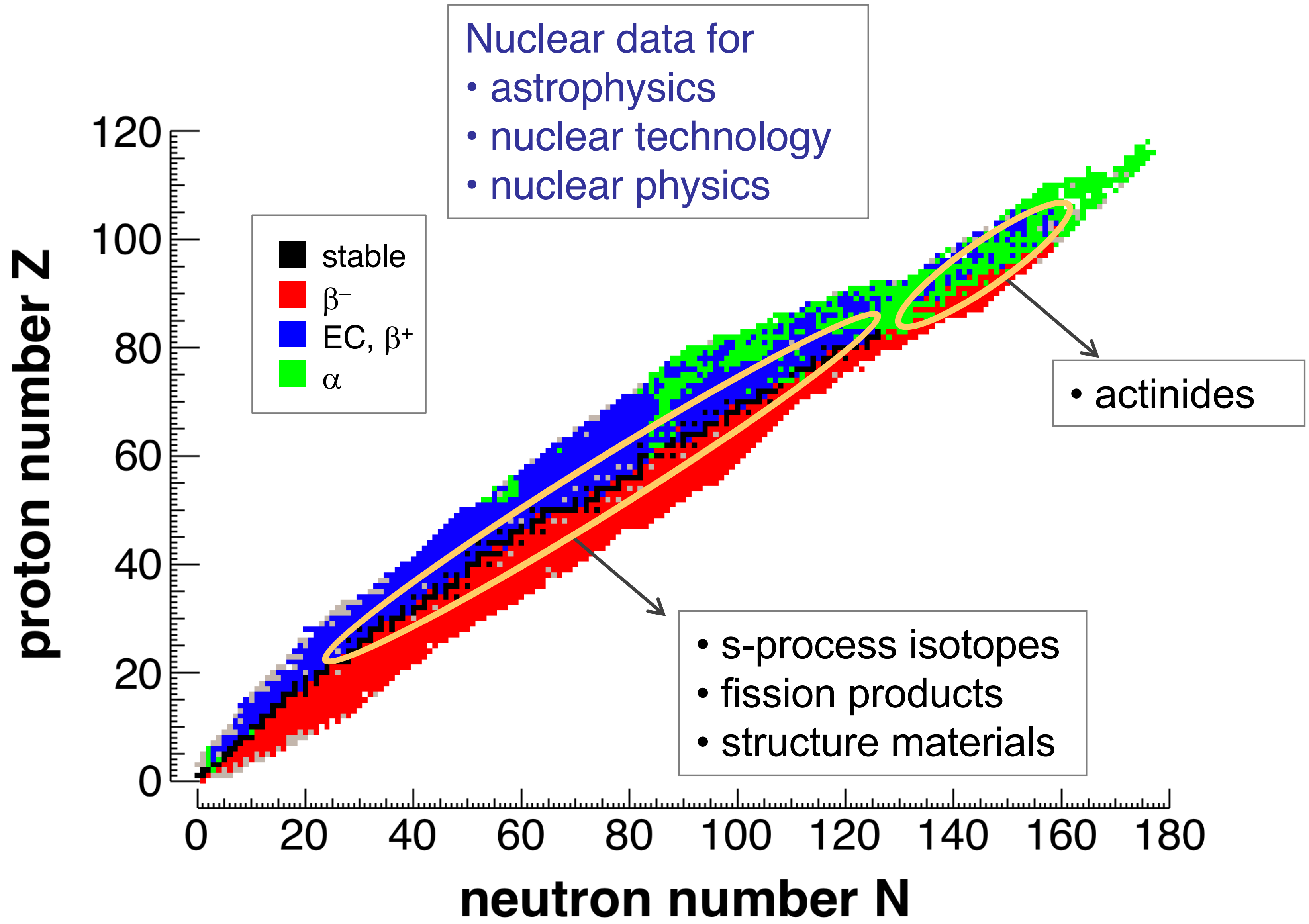


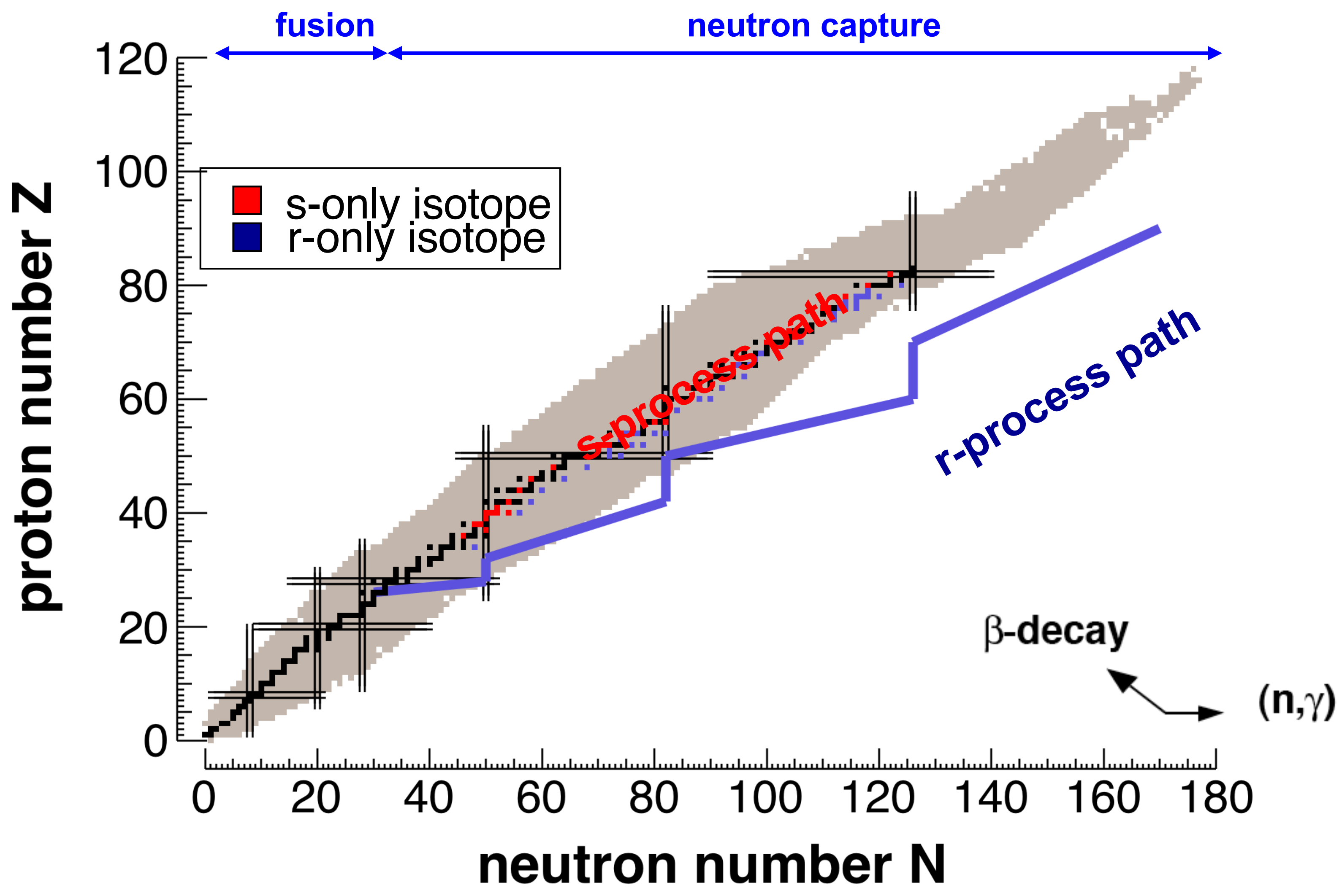
Chart of nuclides



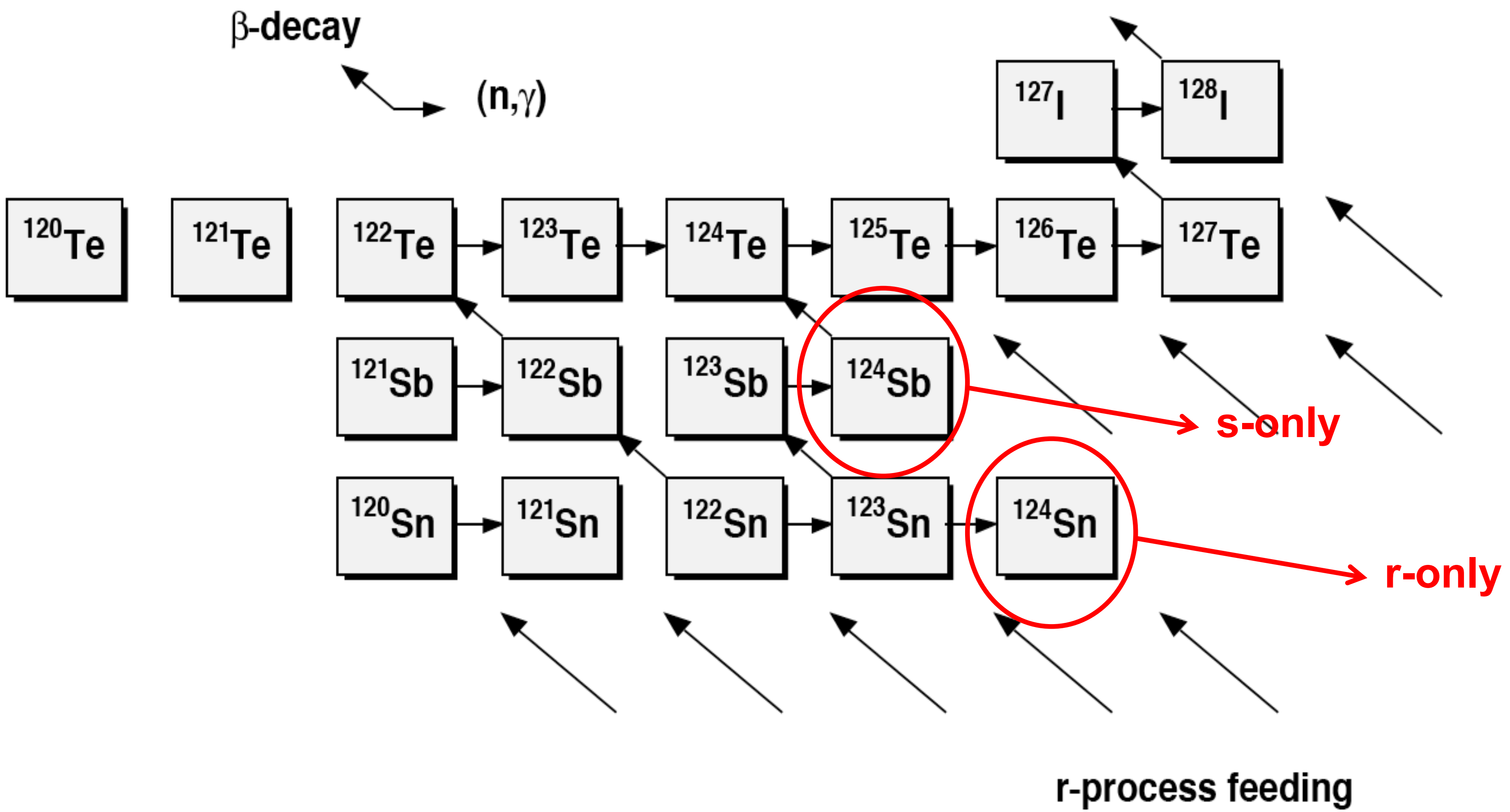
Nuclei of interest for neutron induced reactions



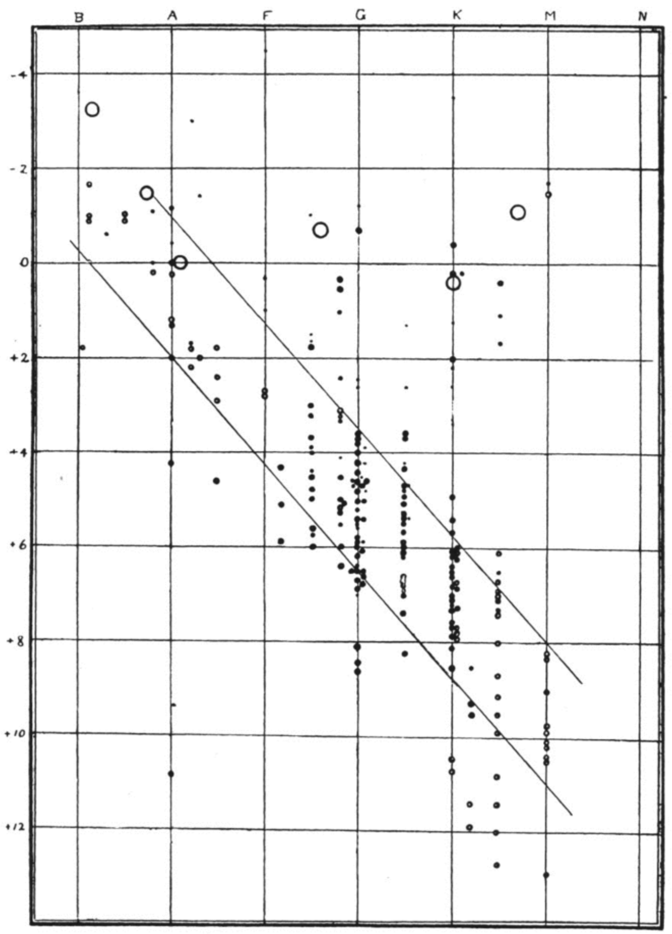
Stellar nucleosynthesis (s-, r-process)



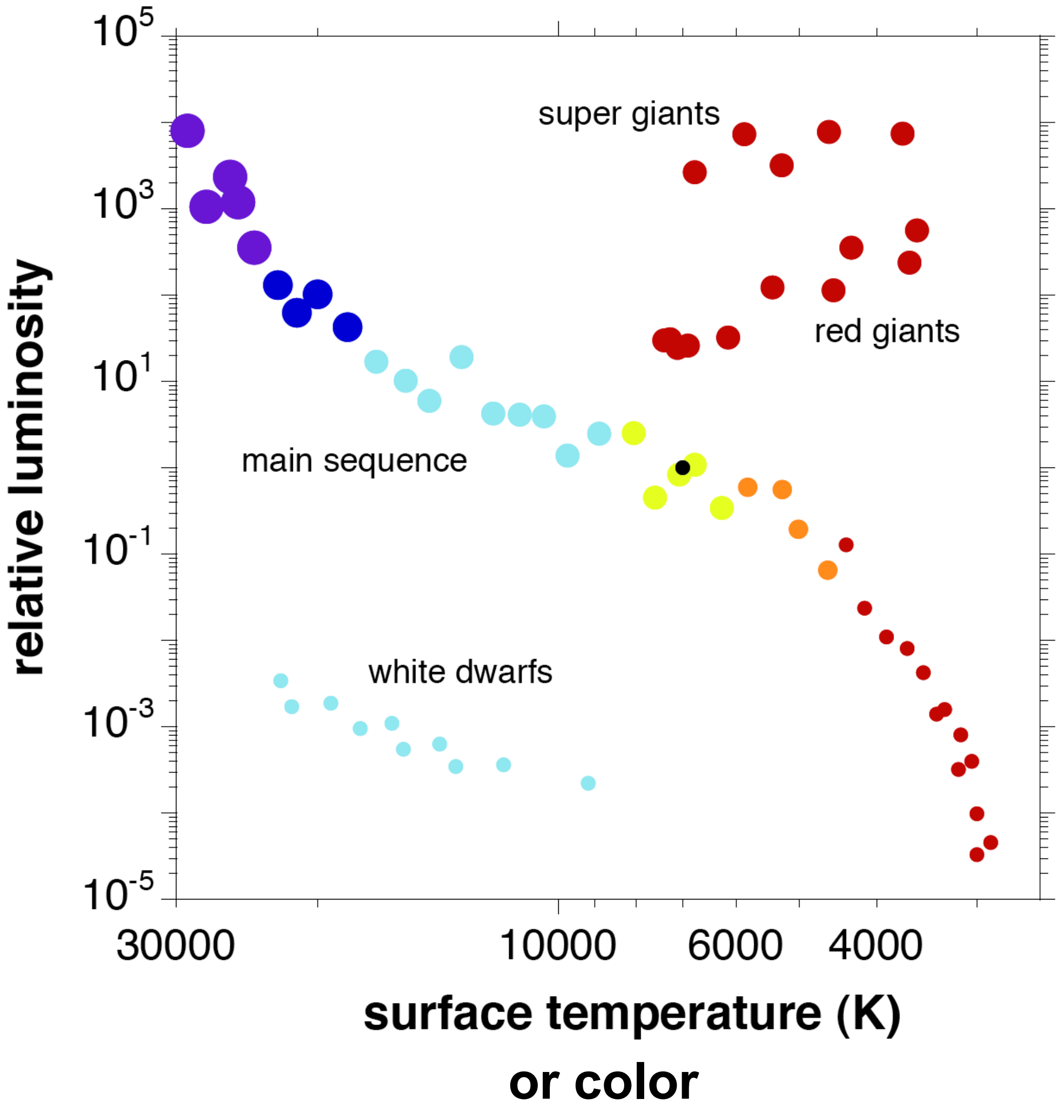
Stellar nucleosynthesis (s-, r-process)



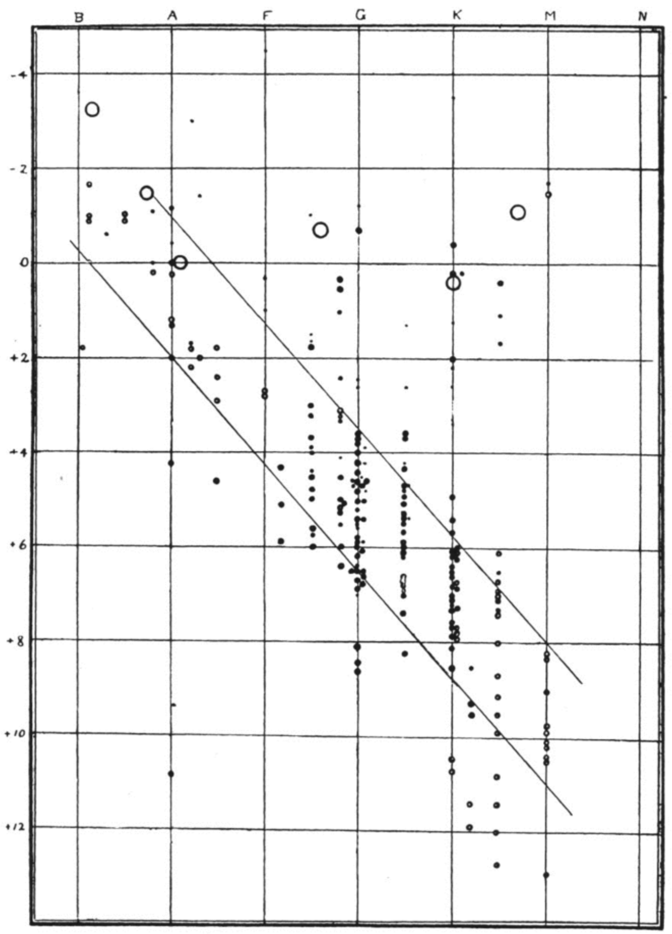
Hertzsprung-Russell diagram



Russell, Nature 93 (1914) 252

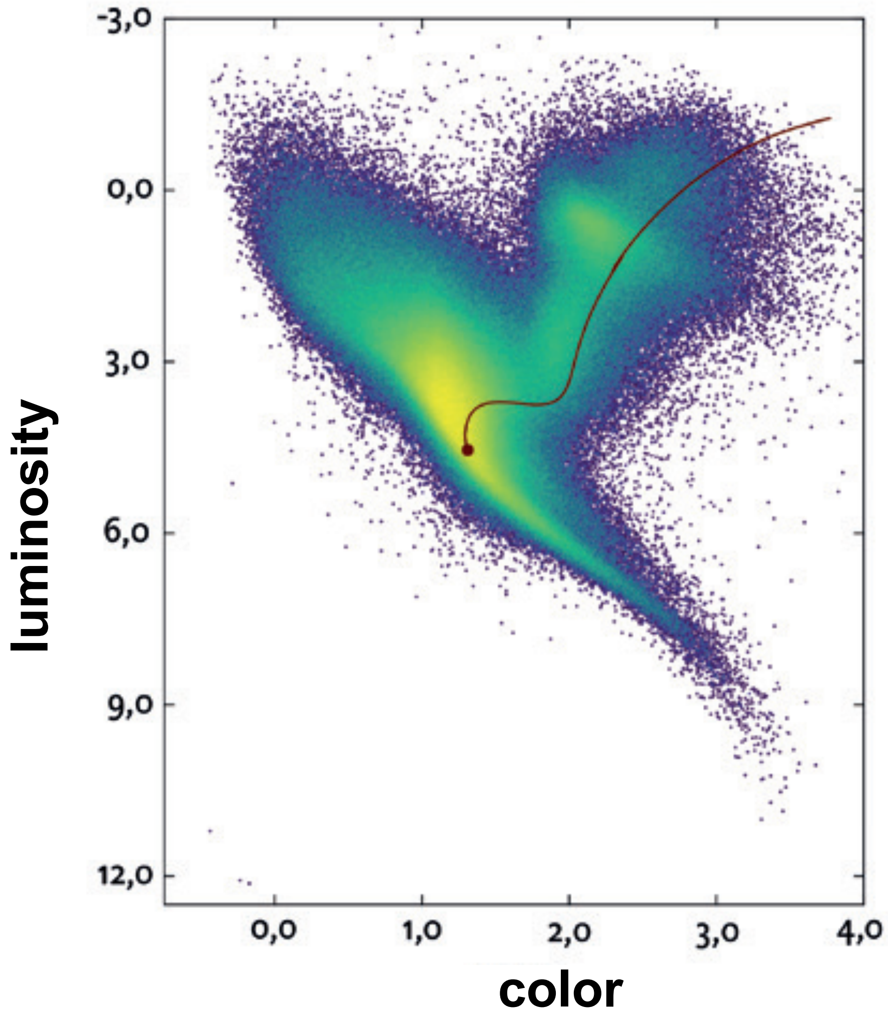


Hertzsprung-Russell diagram



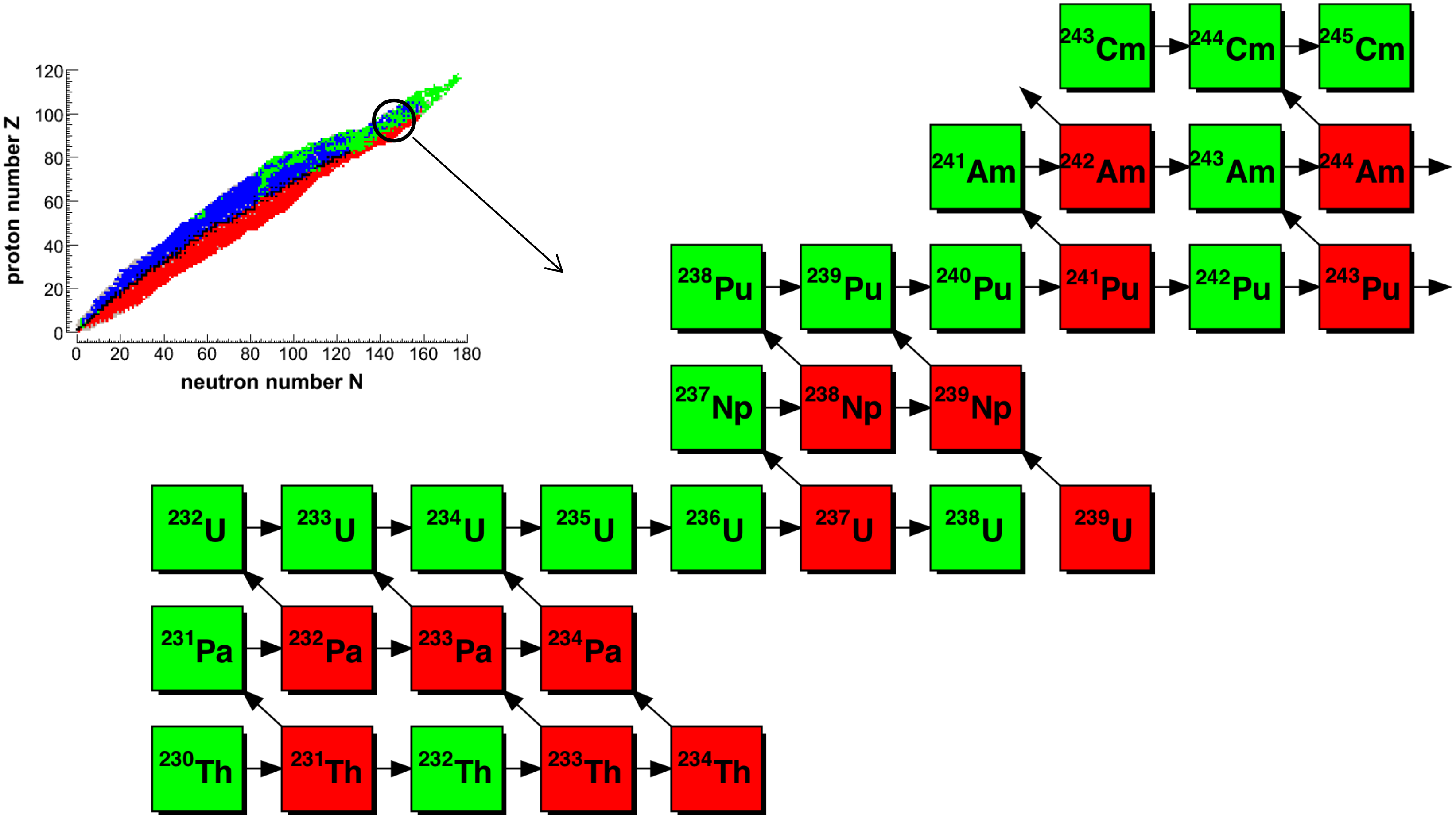
Russell, Nature 93 (1914) 252

10^6 stars observed so far with GAIA satellite



NVTN 83:5 (2016)

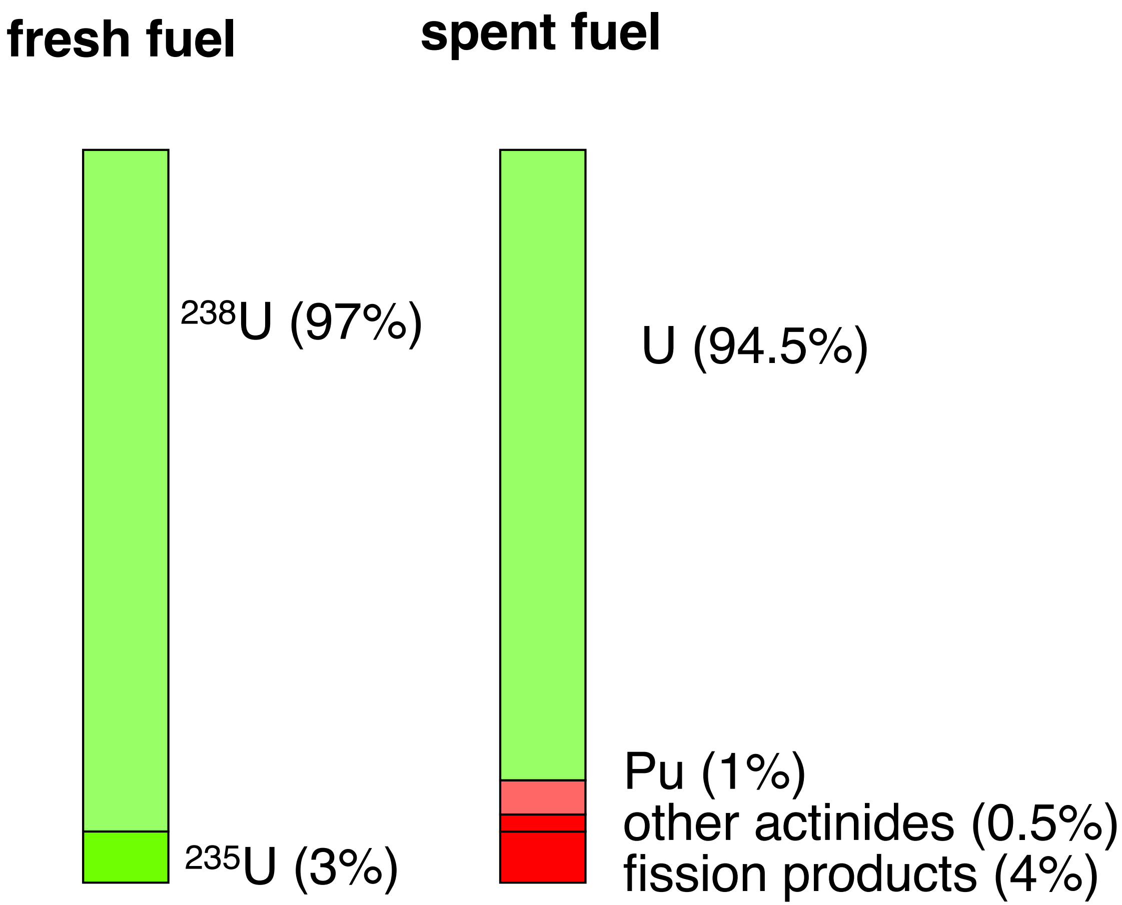
Actinide build-up in reactors ("w" - process)



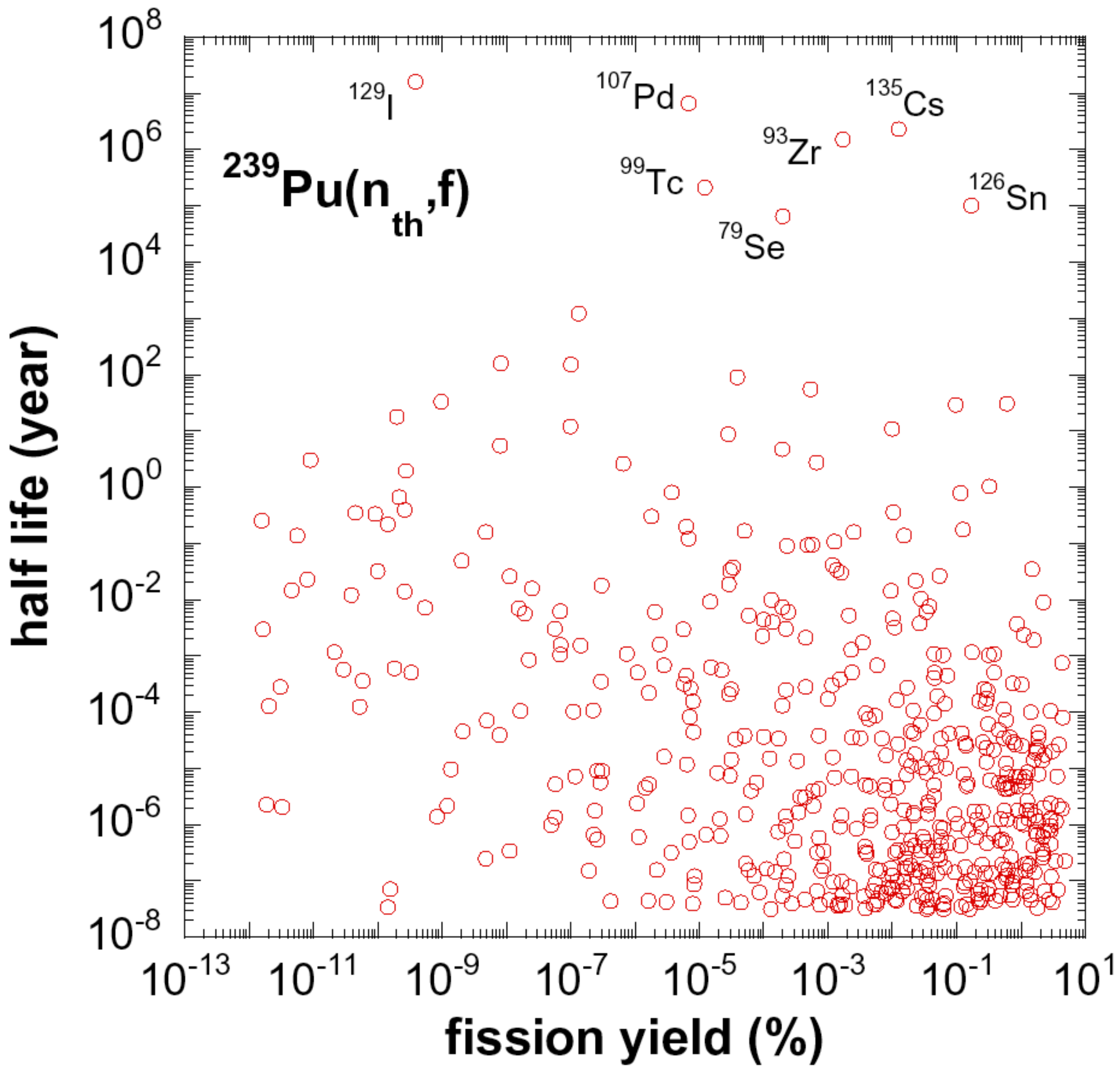
$$^{238}\mathrm{U} + \mathrm{n} \longrightarrow {}^{239}\mathrm{U} \xrightarrow{\beta^- \ (23\ min)} {}^{239}\mathrm{Np} \xrightarrow{\beta^- \ (2.4\ d)} {}^{239}\mathrm{Pu}$$

$$^{232}\mathrm{Th} + \mathrm{n} \longrightarrow {}^{233}\mathrm{Th} \xrightarrow{\beta^- \ (22\ min)} {}^{233}\mathrm{Pa} \xrightarrow{\beta^- \ (27\ d)} {}^{233}\mathrm{U}$$

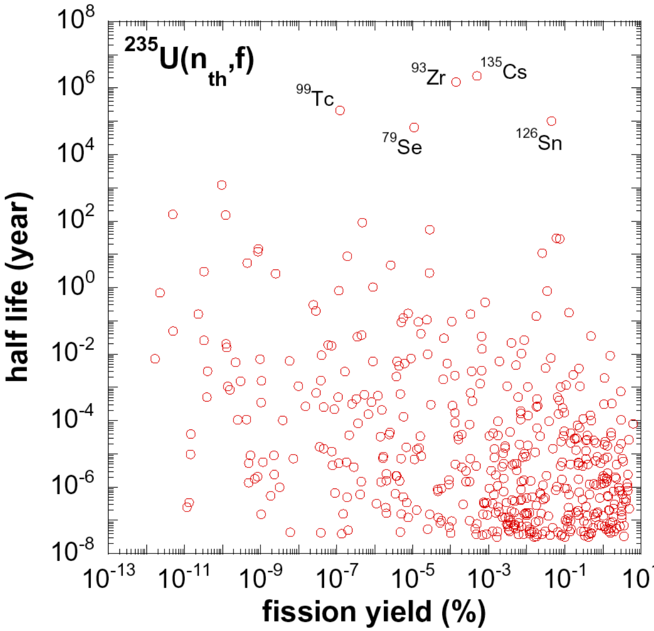
nuclear waste from spent fuel

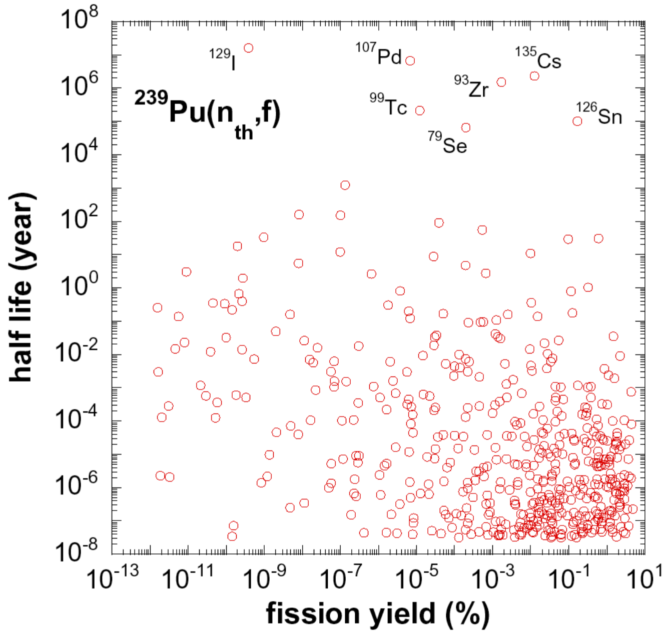


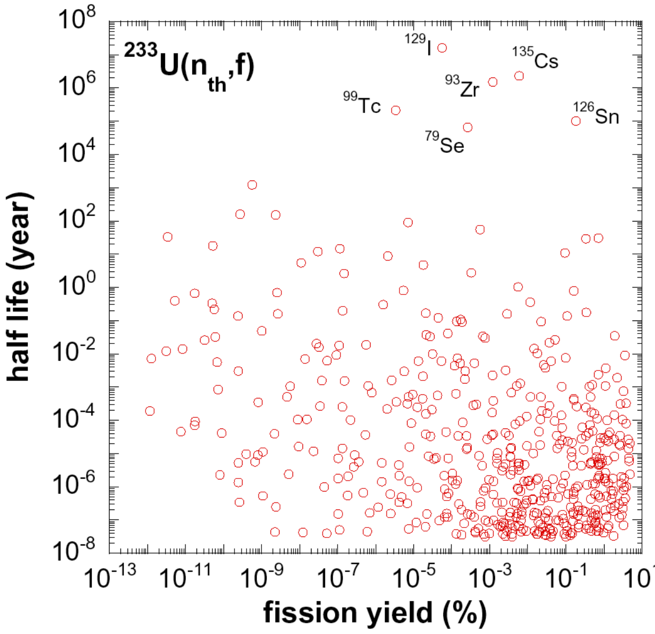
Fission yield and fission product half lifes



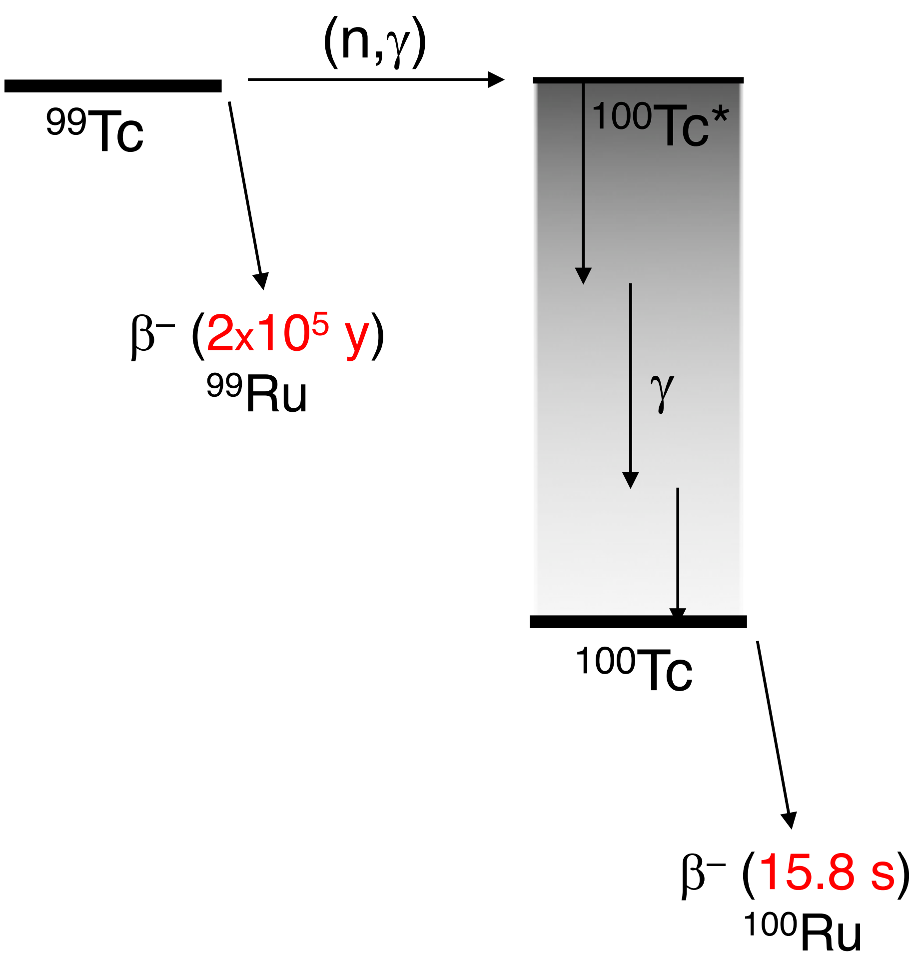
Fission yield and fission product half lifes



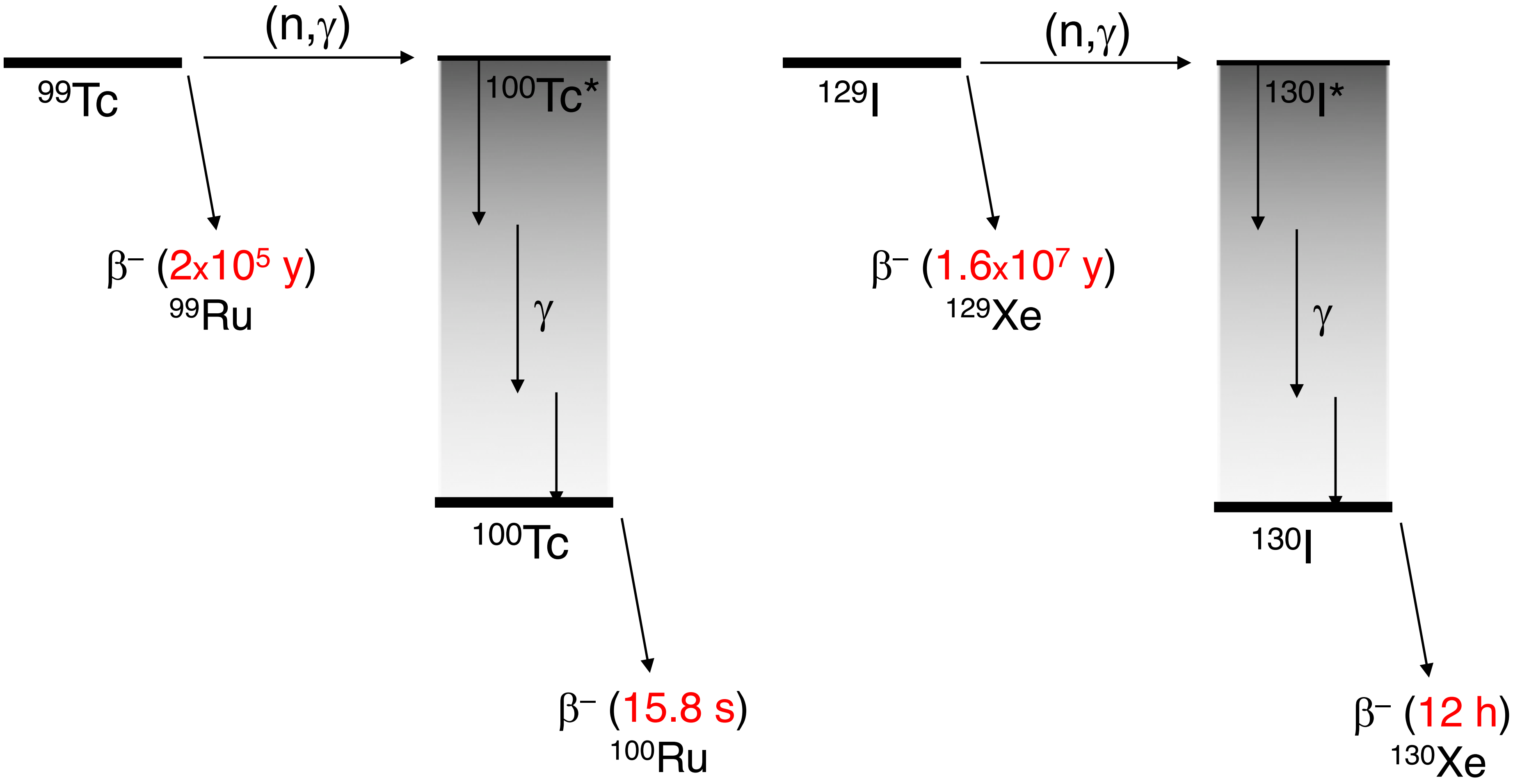




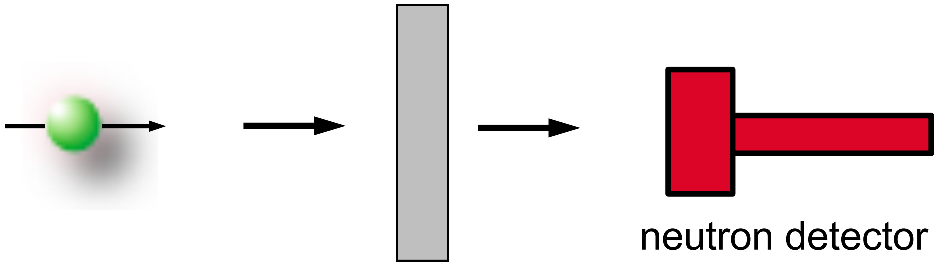
Transmutation of long-lived fission products

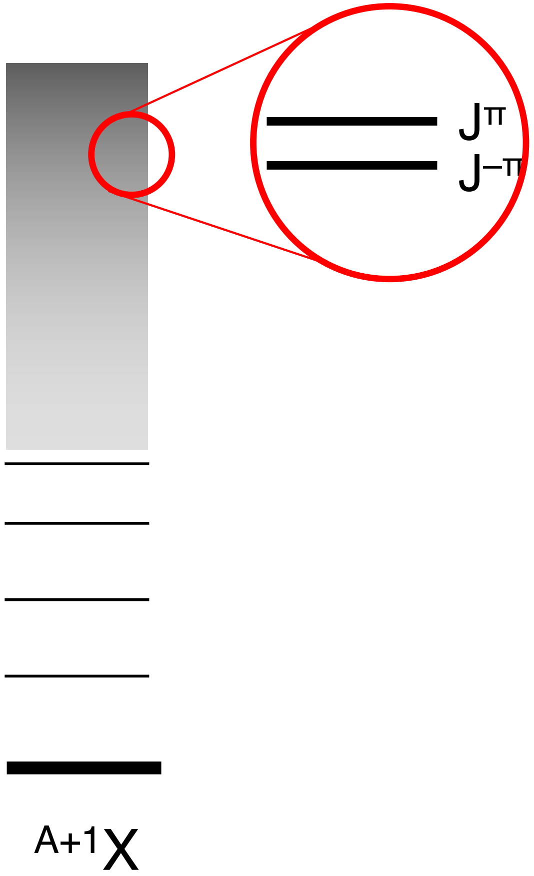


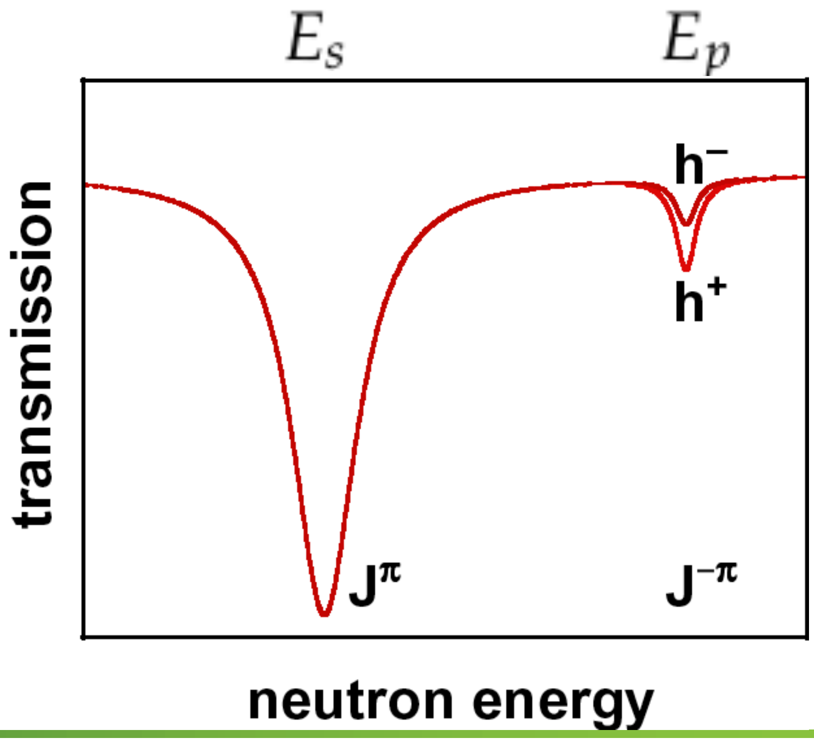
Transmutation of long-lived fission products



Parity non-conservation







Parity non-conservation

- Parity non-converation observed in neutron resonances (TRIPLE Collaboration)

- Helicity dependence of transmission of polarized neutrons
  Observed asymmetries up to several percent.

$$A = \frac{\sigma_+ - \sigma_-}{\sigma_+ + \sigma_-} = \frac{2V_{sp}^J}{E_s - E_p}\sqrt{\frac{\Gamma_{n,s}}{\Gamma_{n,p}}}$$

- **CPT** invariance

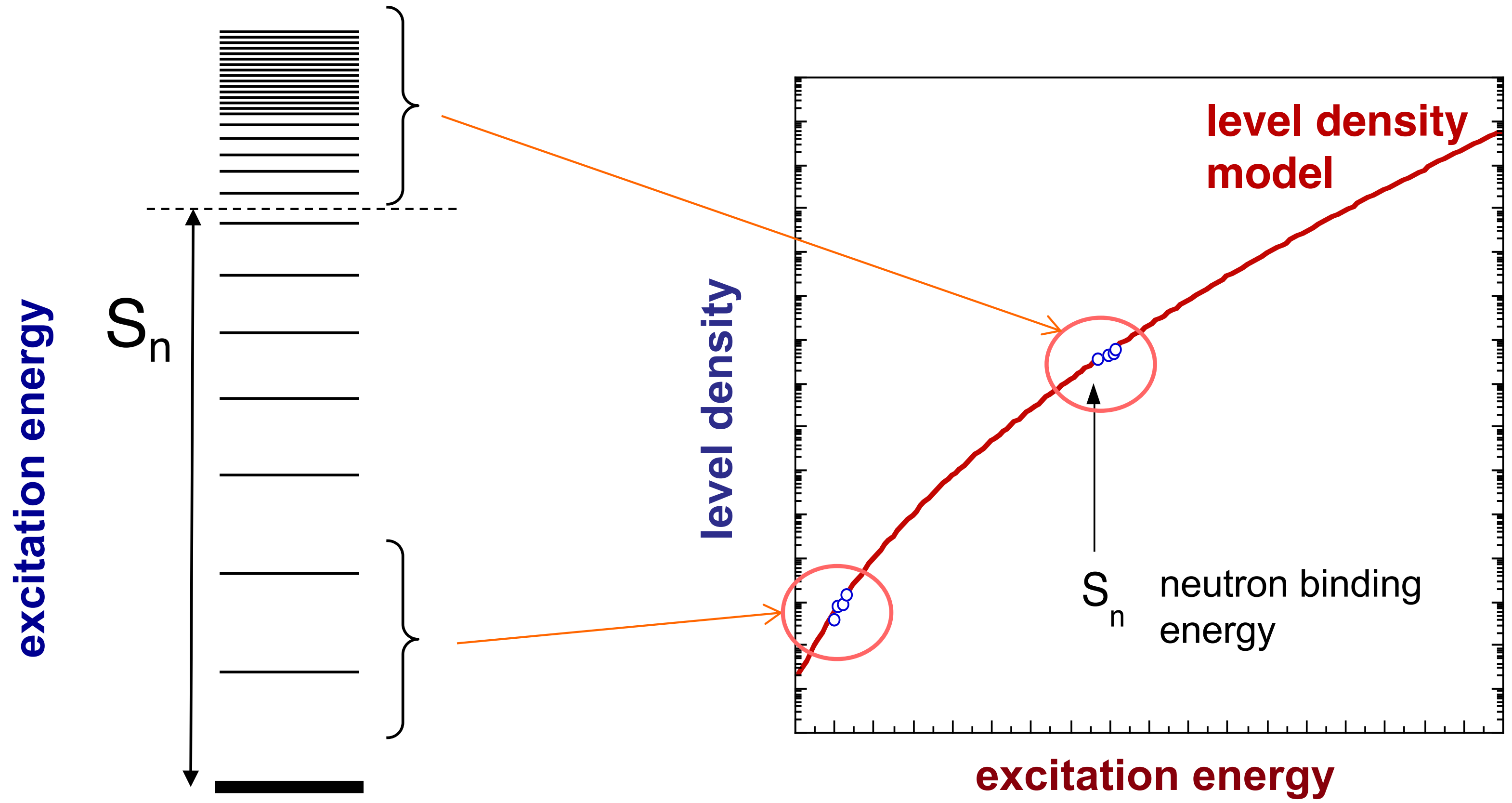
- Asymmetries due to weak interaction
  with

$$\Psi = \Psi^{\pi} + F\Psi^{-\pi}$$

$$F \sim 10^{-7}$$

- Amplification of 10^6 due to factors $\sqrt{\frac{\Gamma_{n,s}}{\Gamma_{n,p}}}$ $\frac{1}{E_s - E_p}$

Nuclear level densities

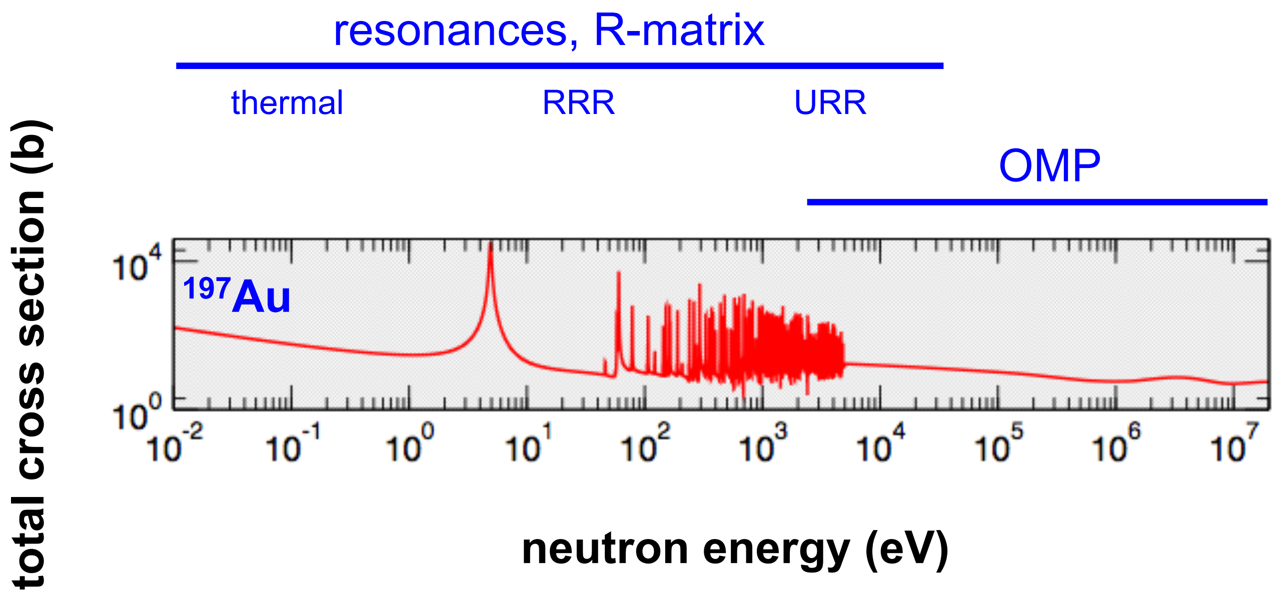


low-lying levels:
Count levels, all J^π

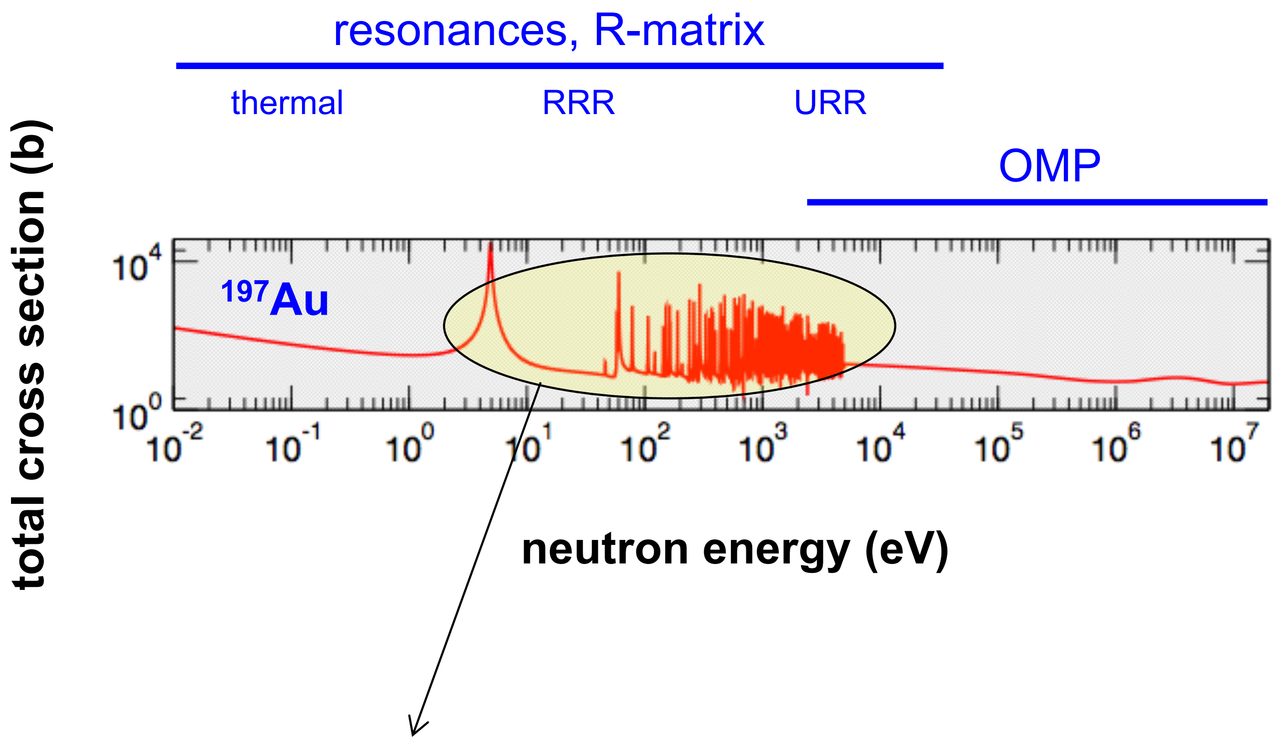
neutron resonances:
Count levels, selected J^π,
extract $\mathbf{D_0}$

- **All level density models reproduce the low-lying levels and D_0 at S_n**

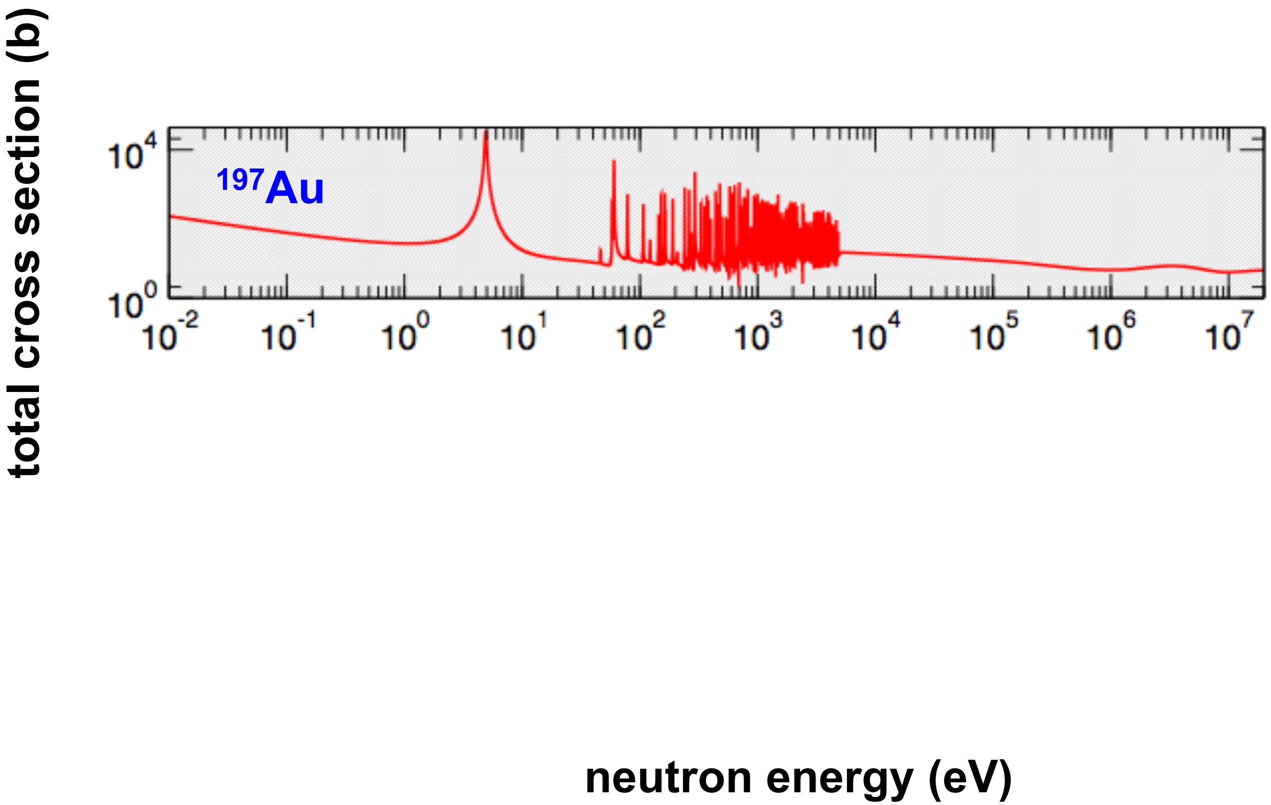
Nuclear level densities



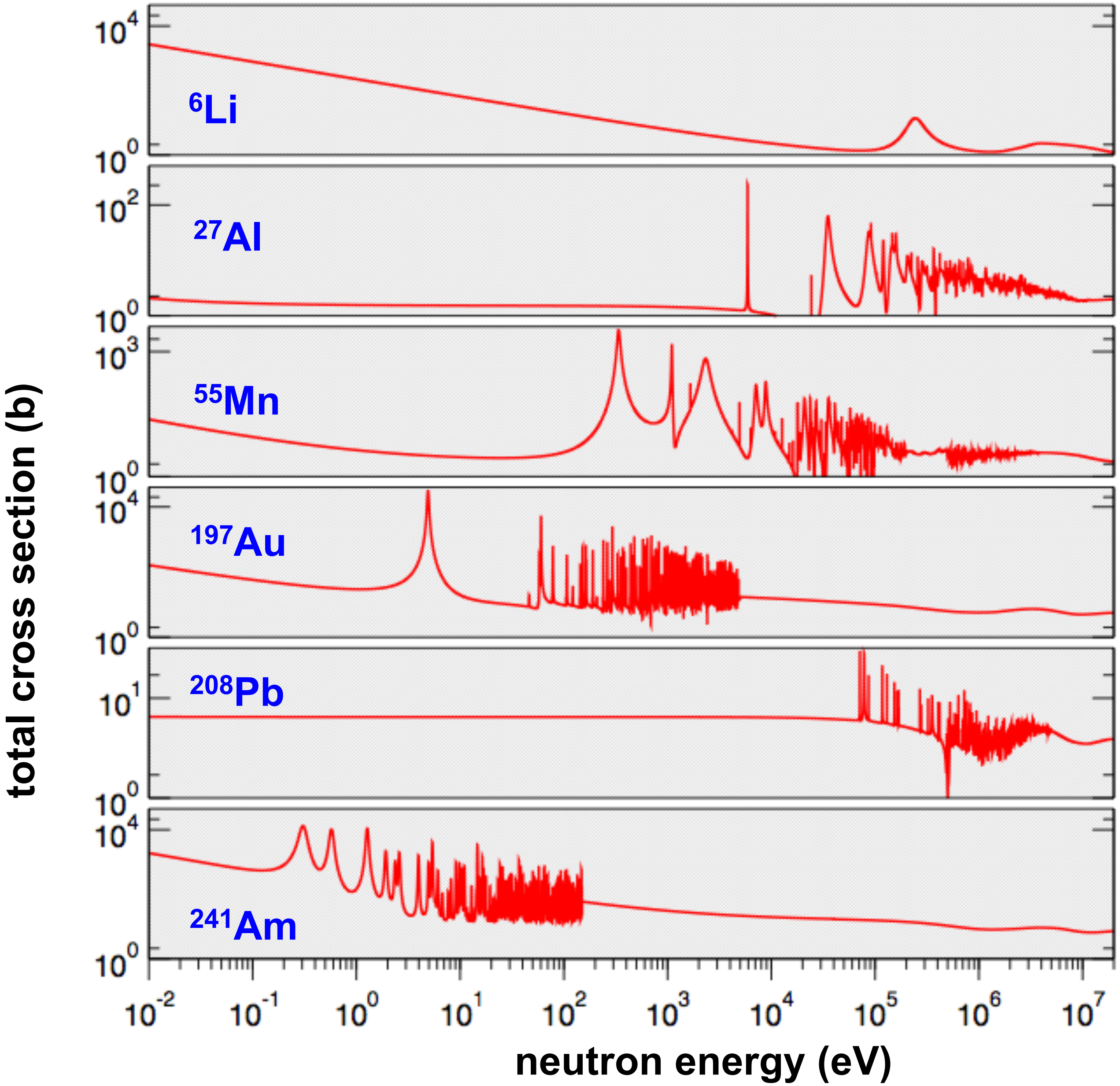
Nuclear level densities



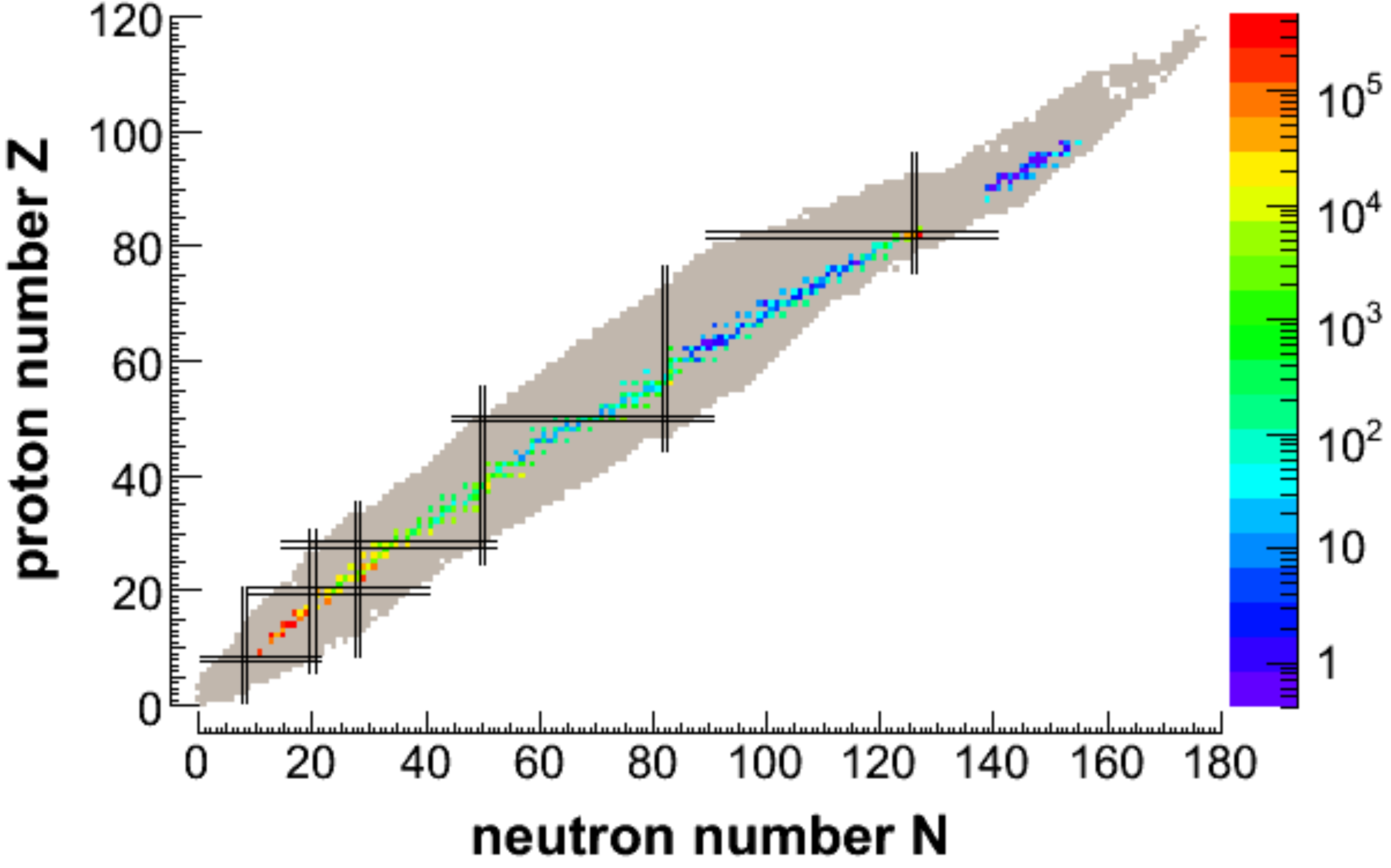
Count the number of levels in the energy interval → level density


total cross section (b)
10^4
10^0
197Au
10^-2
10^-1
10^0
10^1
10^2
10^3
10^4
10^5
10^6
10^7
neutron energy (eV)

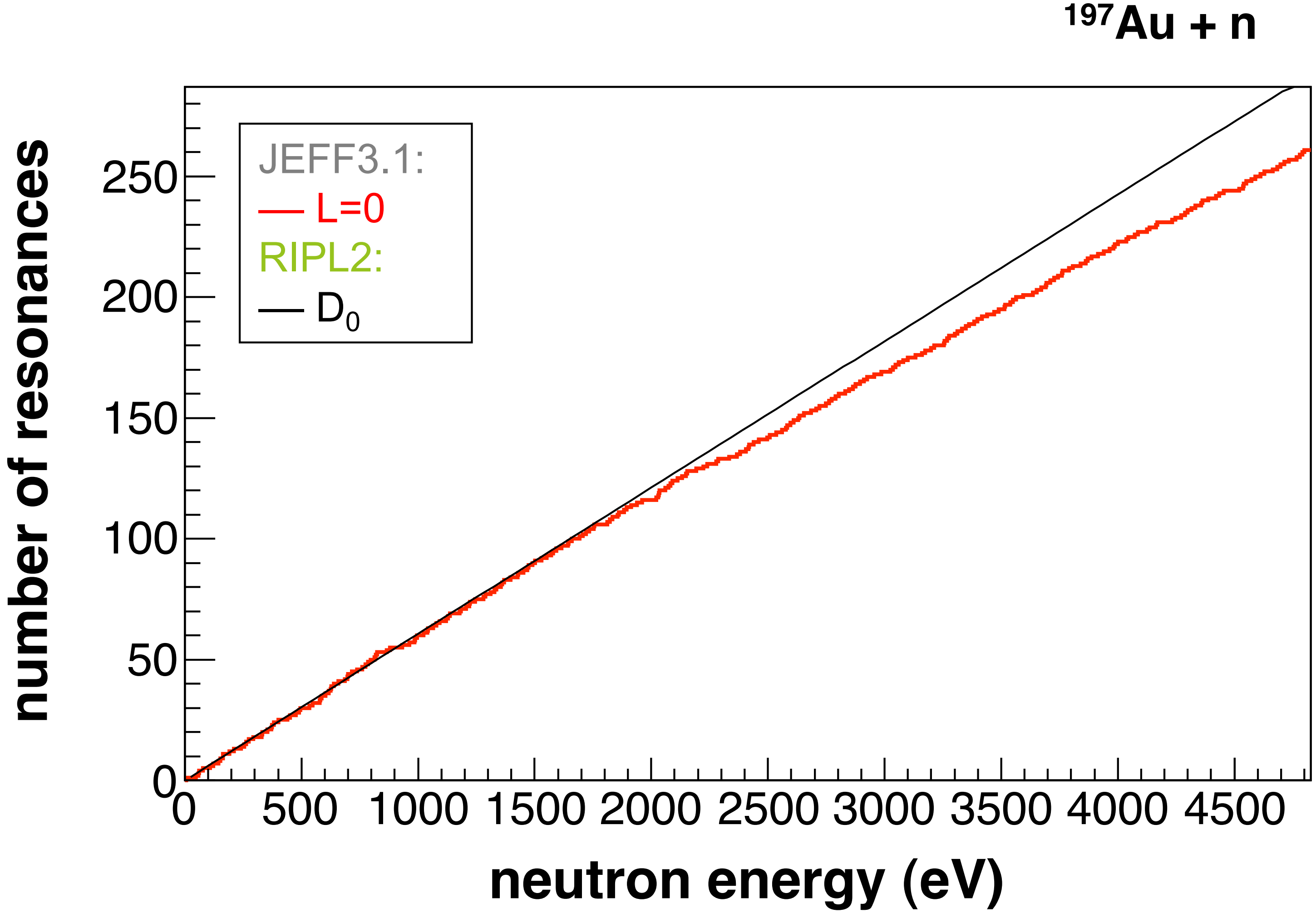
Nuclear level densities



Level spacing D_0



Level density by counting levels: missing levels



What is the statistical model for a nucleus

- Neutron resonances correspond to states in a compound nucleus, which is a nucleus in a highly excited state above the neutron binding energy.

- The compound nucleus corresponds to a very complex particle-hole configuration.
  → **Gaussian Orthogonal Ensemble (GOE)**

- The transition probability between two levels is related to the matrix elements of the interaction between two levels.

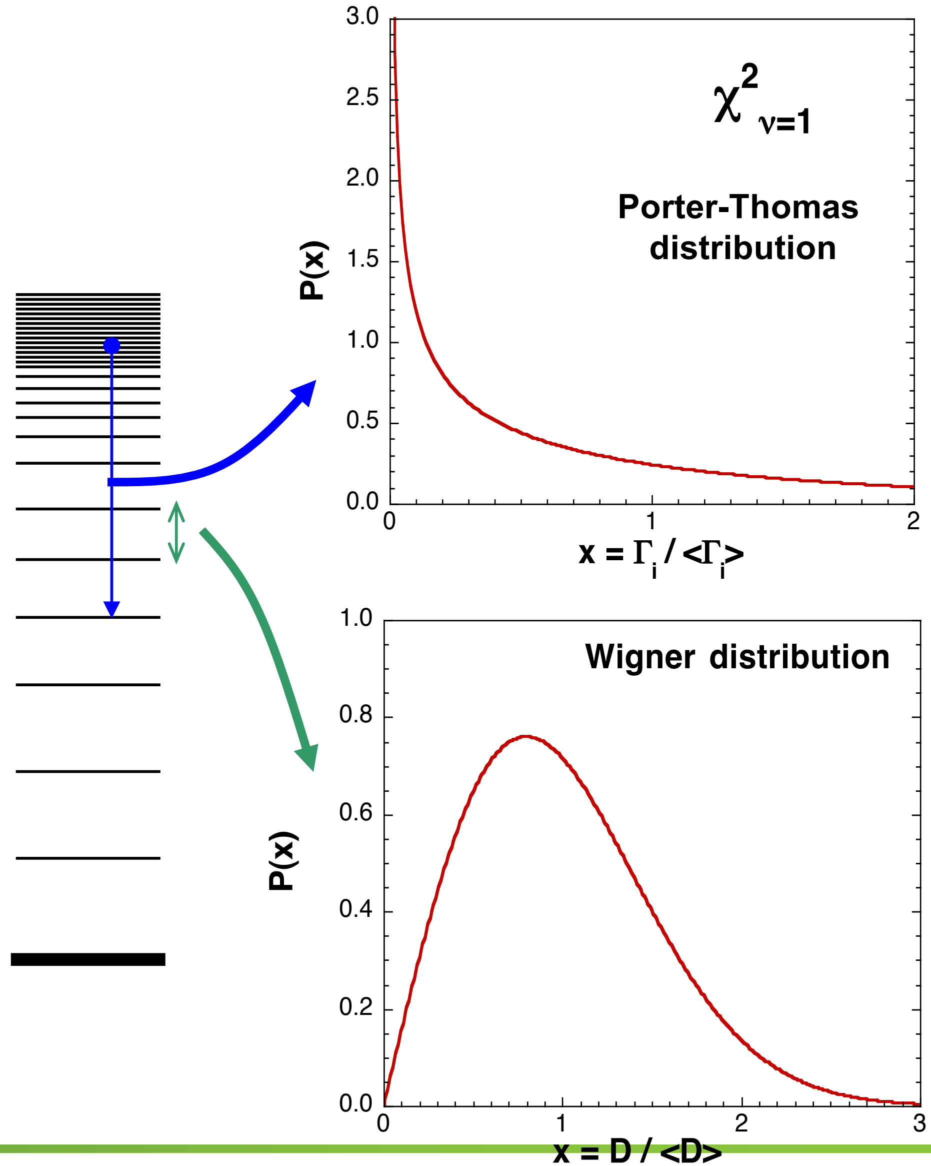
- Matrix elements (amplitudes γ) are Gaussian random variables with zero mean.
  Observables are widths $\Gamma \sim \gamma^2$.

The statistical model

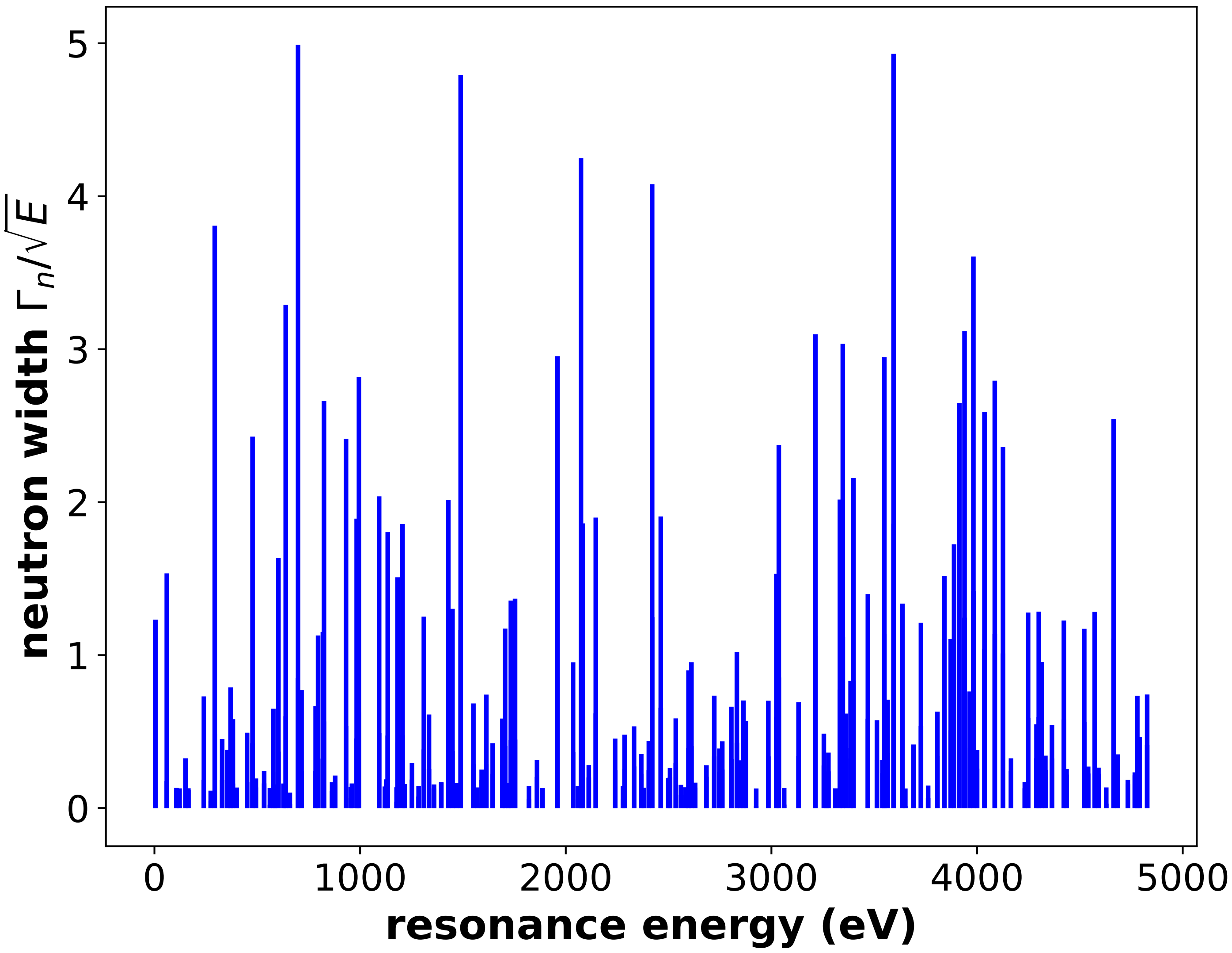
The nucleus at energies around S_n can be described by the
Gaussian Orthogonal Ensemble (GOE)

The matrix elements governing the nuclear transitions are random variables with a Gaussian distribution with zero mean.

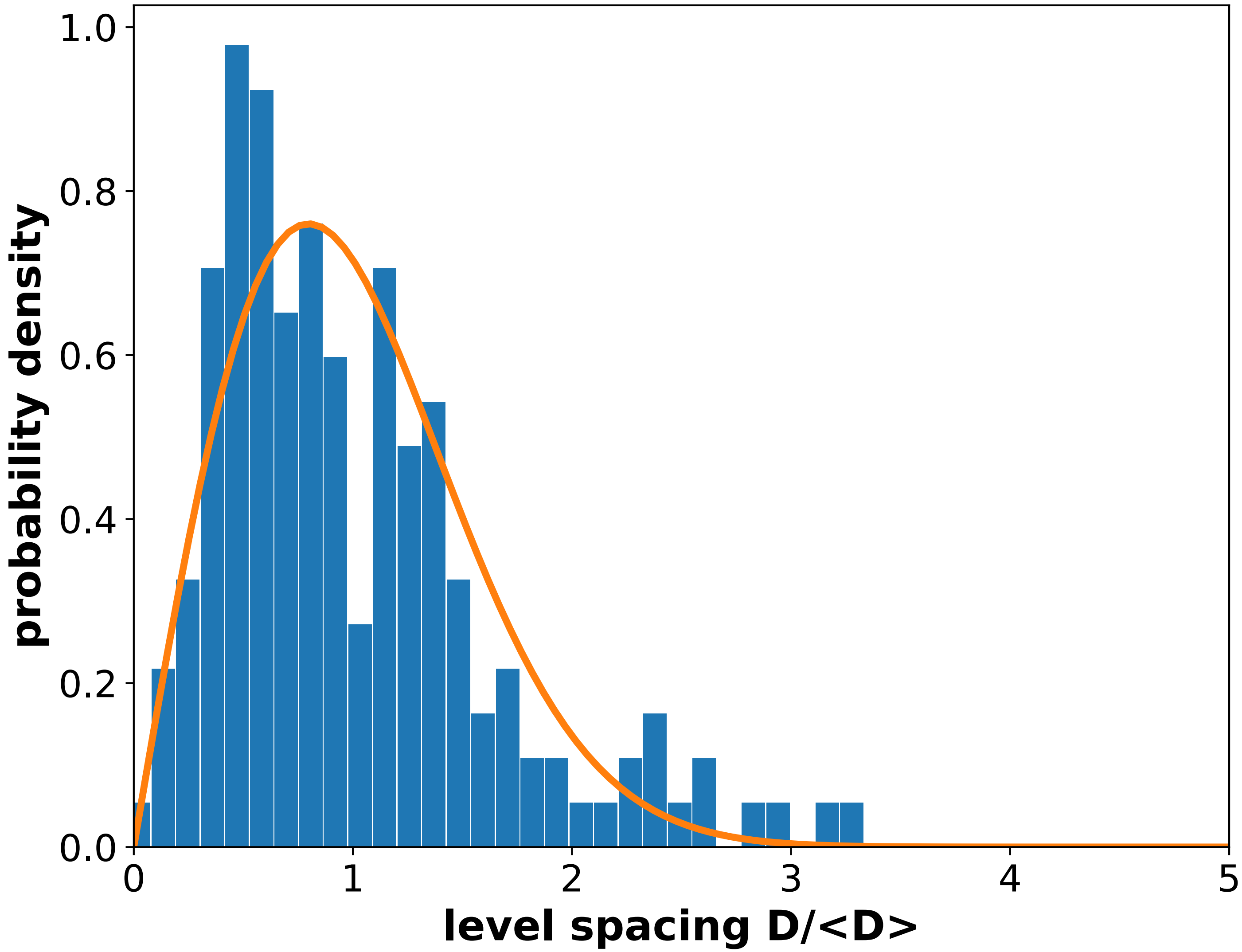
- Consequences:
  - The partial widths have a Porter-Thomas distribution.
  - The spacing of levels with the same J^π have approximately a Wigner distribution.



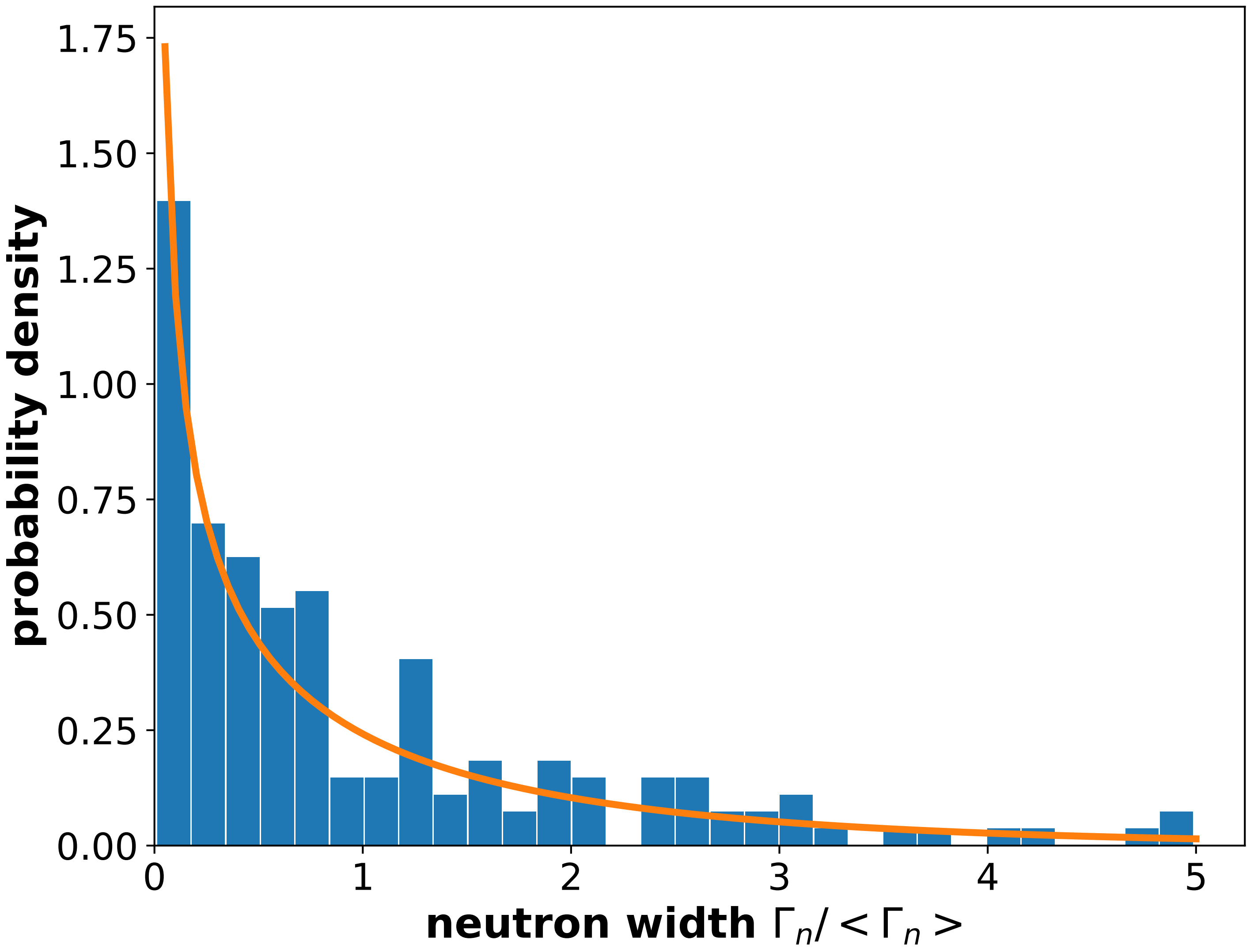
Resolved resonances of ^{197}Au s-waves (J=2)



Resolved resonances of ^{197}Au s-waves (J=2)



Resolved resonances of ^{197}Au s-waves (J=2)



Chi-square distribution

$$x = \frac{\gamma^2}{< \gamma^2 >} \qquad P_{\mathrm{PT}}(x) = \frac{1}{\sqrt{2\pi x}} \exp\left(-\frac{x}{2}\right)$$

For neutron widths (s-waves), use the effective reduced neutron width

$$\Gamma_n^0 = \Gamma_n / \sqrt{(E)}$$

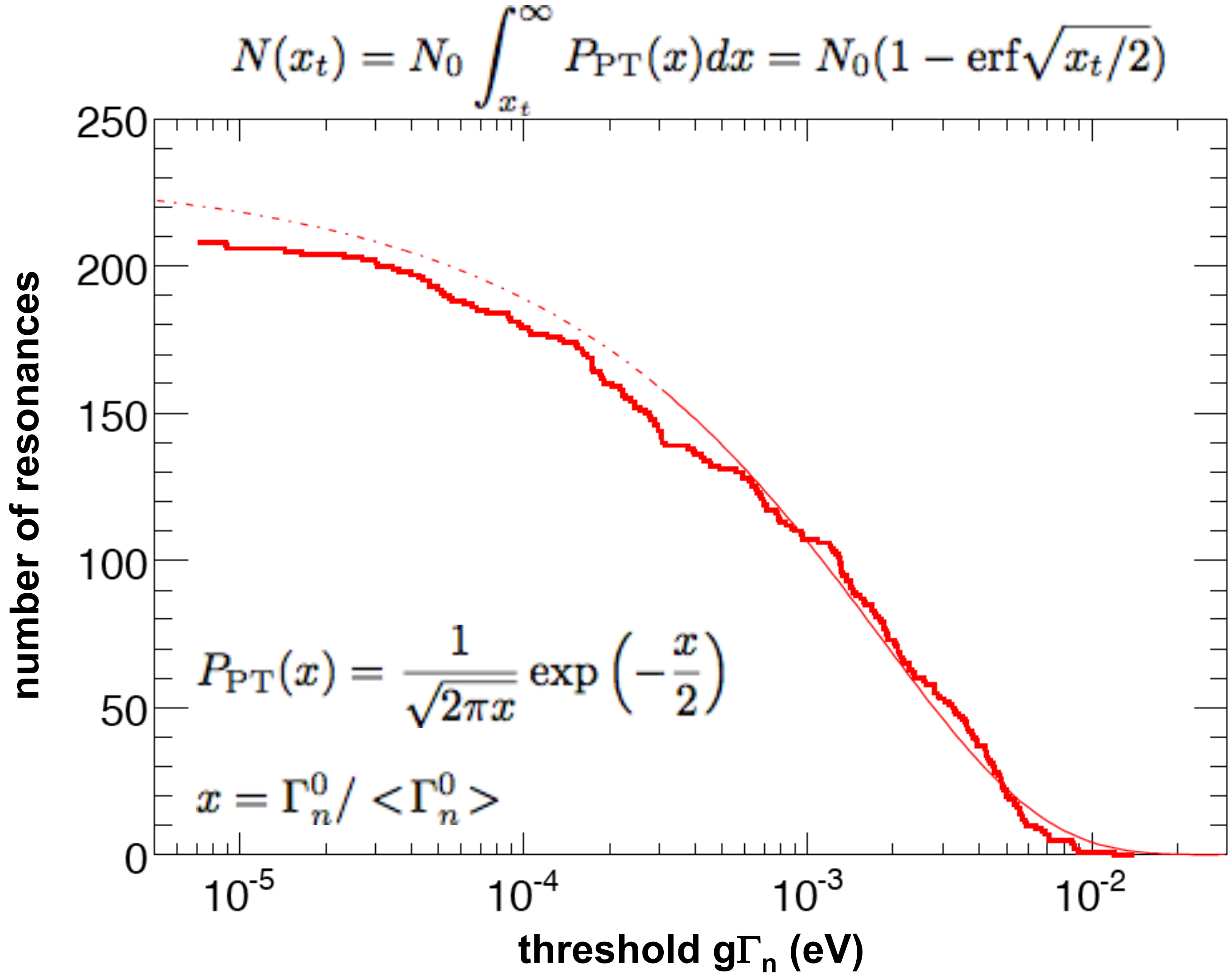
and

$$x = \frac{g\Gamma_n^0}{< g\Gamma_n^0 >}$$

and for easy handling use

$$\int_{x_t}^{\infty} P_{\mathrm{PT}}(x)$$

Missing level correction



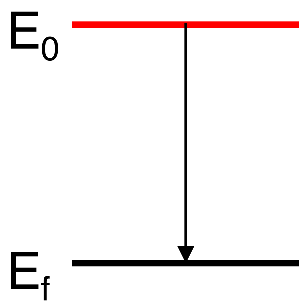
Introduction R-matrix theory

- Formalism to decribe (neutron) reactions
- For resolved resonances, full cross sections can be constructed from only a few resonance parameters
- Standard way of storage for evaluated nuclear data

Decay of a (nuclear) quantum state

state with a life time τ:

$$\Psi(t) = \Psi_0 e^{-iE_0 t/\hbar} e^{-t/2\tau}$$



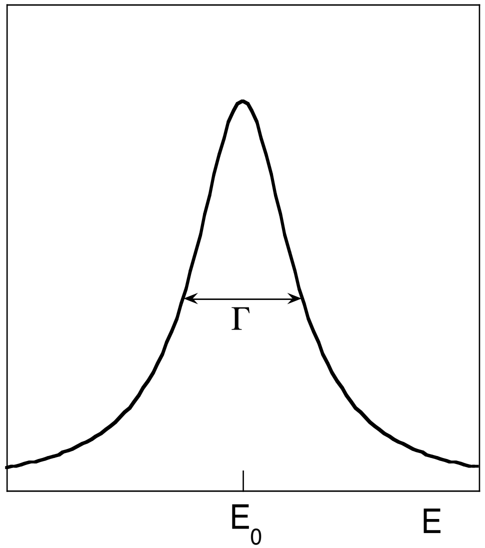
eigen state,
transitition $E_0 \rightarrow E_f$
life time τ

definition (Heisenberg):

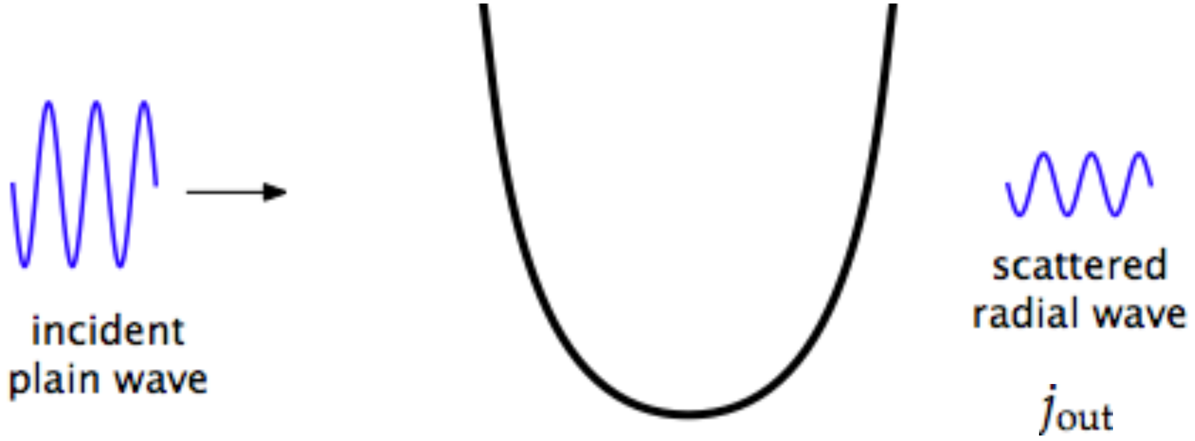
$$\Gamma = \frac{\hbar}{\tau}$$

Fourier transform gives energy profile:

$$I(E) = \frac{\Gamma / 2\pi}{(E - E_0)^2 + \Gamma^2 / 4}$$



Neutron-nucleus reactions



$$j_{\text{inc}} = \frac{\hbar}{2mi}(\psi^*\nabla\psi - \psi\nabla\psi^*)$$

nucleus as
potential well

Conservation of probability density: $\sigma(\Omega) = \dfrac{r^2 j_{\text{out}}(r,\Omega)}{j_{\text{inc}}}$

Solve Schrödinger equation of system to get cross sections.
Shape of wave functions of in- and outgoing particles are known,
potential is unknown. Two approaches:

- **calculate potential (optical model calculations, smooth cross section)**
- **use eigenstates (R-matrix, resonances)**

Quantum system: the finite well

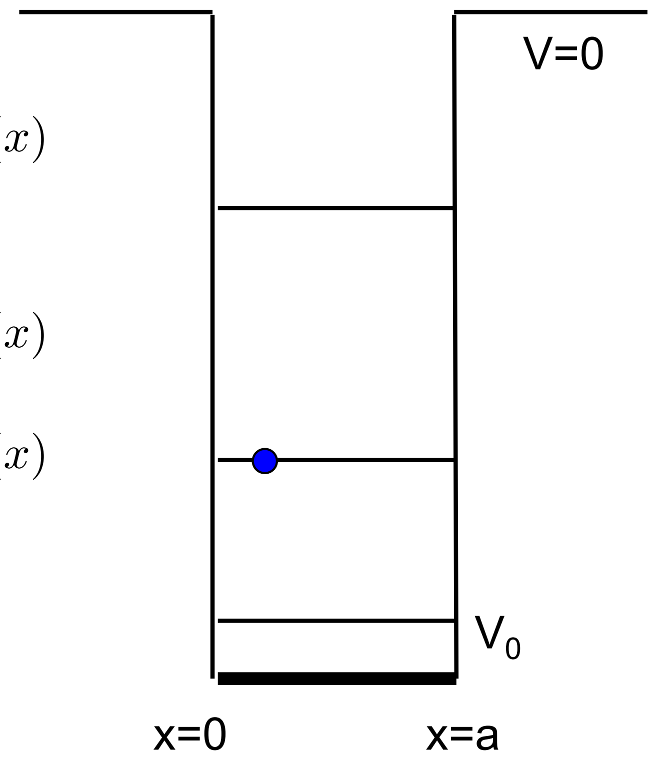
Solve Schrödinger equation in two regions:
- inside and outside the well
- normalize solutions to match value and derivative and borders x=0 and x=a

Now the wave function exists also outside the well at x<0 and x>a

$$-\frac{\hbar^2}{2m}\frac{d^2\psi(x)}{dx^2} + V(x)\psi(x) = E\psi(x)$$

$$0 < x < a \qquad -\frac{\hbar^2}{2m}\frac{d^2\psi(x)}{dx^2} + V_0\psi(x) = E\psi(x)$$

$$x < 0, x > a \qquad -\frac{\hbar^2}{2m}\frac{d^2\psi(x)}{dx^2} = E\psi(x)$$



Quantum system: the finite well

Solve Schrödinger equation in two regions:

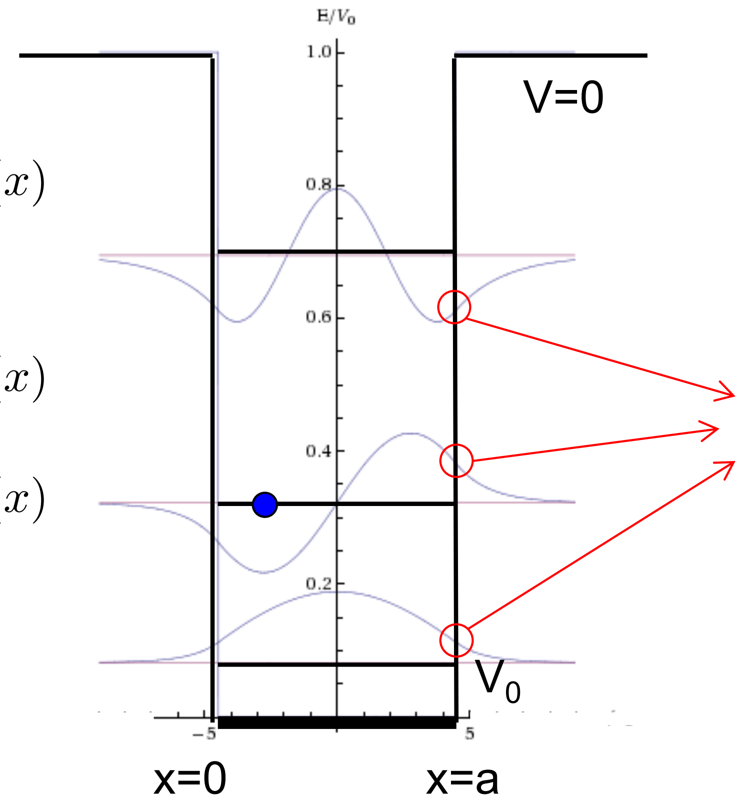
- inside and outside the well
- normalize solutions to match value and derivative and borders x=0 and x=a

Now the wave function exists also outside the well at x<0 and x>a

$$-\frac{\hbar^2}{2m}\frac{d^2\psi(x)}{dx^2} + V(x)\psi(x) = E\psi(x)$$

$$0 < x < a \qquad -\frac{\hbar^2}{2m}\frac{d^2\psi(x)}{dx^2} + V_0\psi(x) = E\psi(x)$$

$$x < 0, x > a \qquad -\frac{\hbar^2}{2m}\frac{d^2\psi(x)}{dx^2} = E\psi(x)$$



match value and derivative

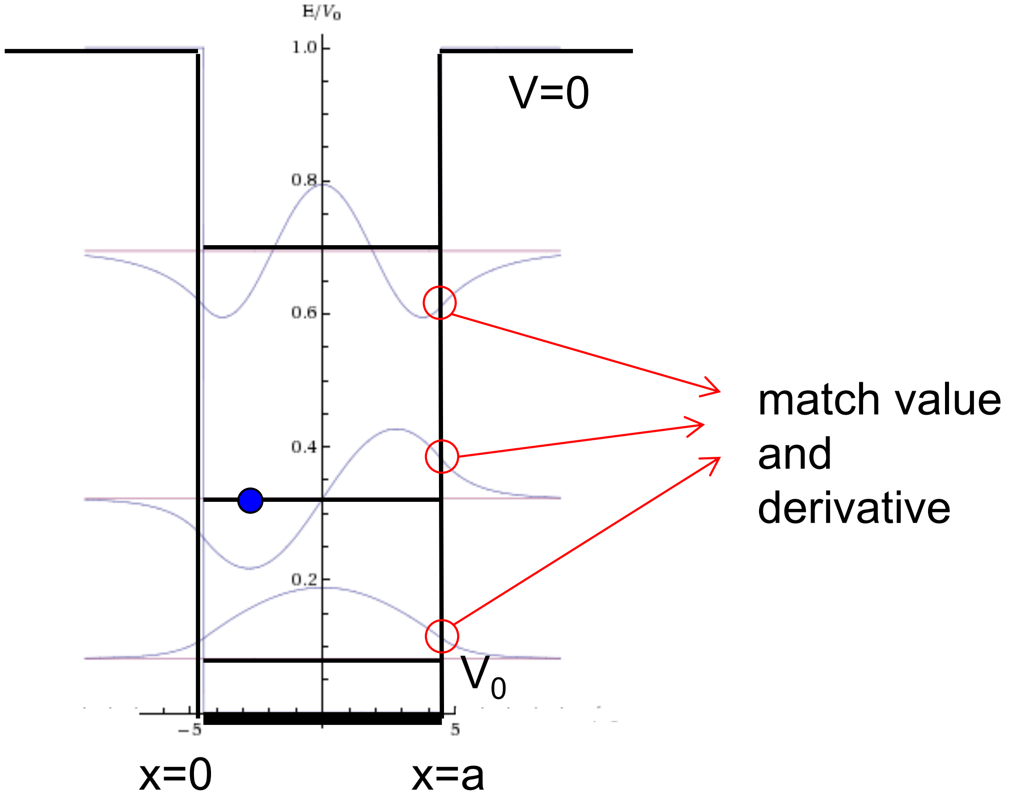
Quantum system: the finite well

Solve Schrödinger equation in two regions:
• inside and outside the well
• normalize solutions to match value and derivative and borders x=0 and x=a

Now the wave function exists also outside the well at x<0 and x>a

In general, a generic state can be written as a linear expansion of it eigenstates:

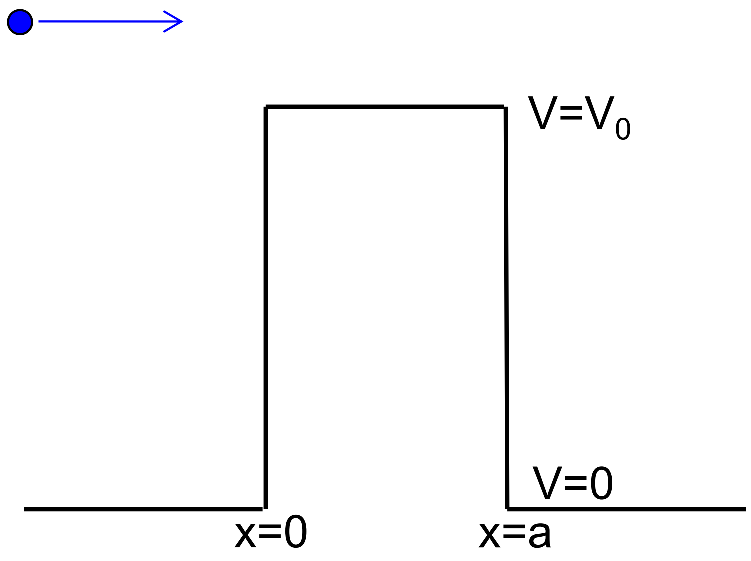
$$\psi(x) = \sum_k c_k \psi_k(x)$$



Quantum system: the potential barrier

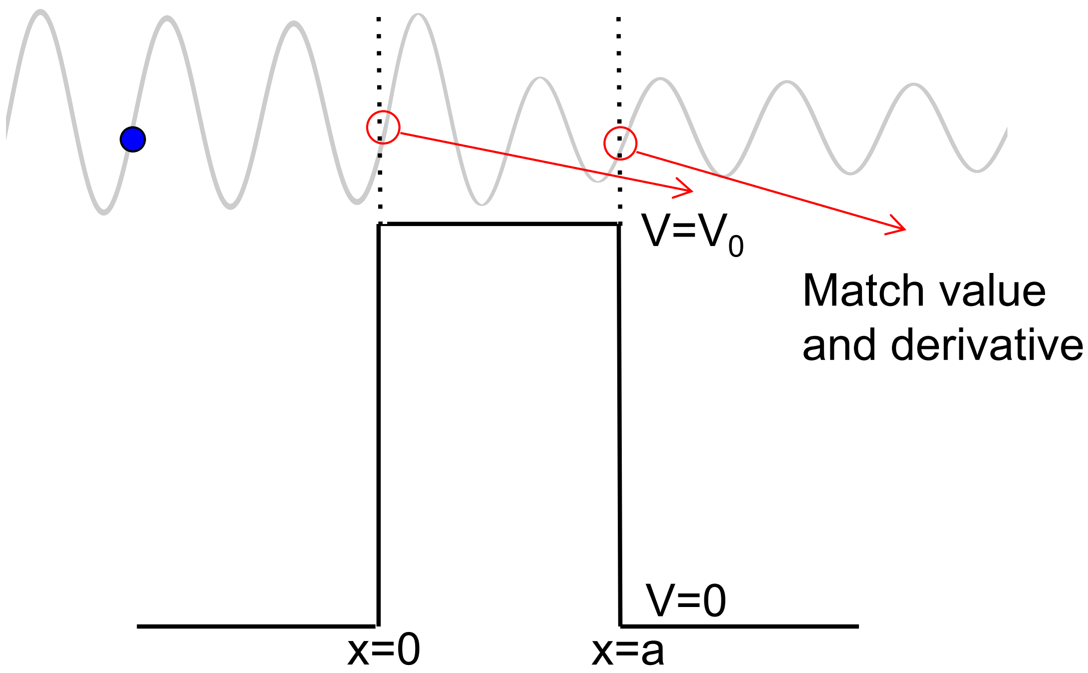
Solve Schrödinger equation in three regions:

- free travelling particle of energy E
- inside and outside the well
- normalize solutions to match value and derivative and borders x=0 and x=a
- **transmission and reflection**



Quantum system: the potential barrier

Solve Schrödinger equation in three regions:
• free travelling particle of energy E
• inside and outside the well
• normalize solutions to match value and derivative and borders x=0 and x=a
• **transmission and reflection**



Quantum system: the potential barrier

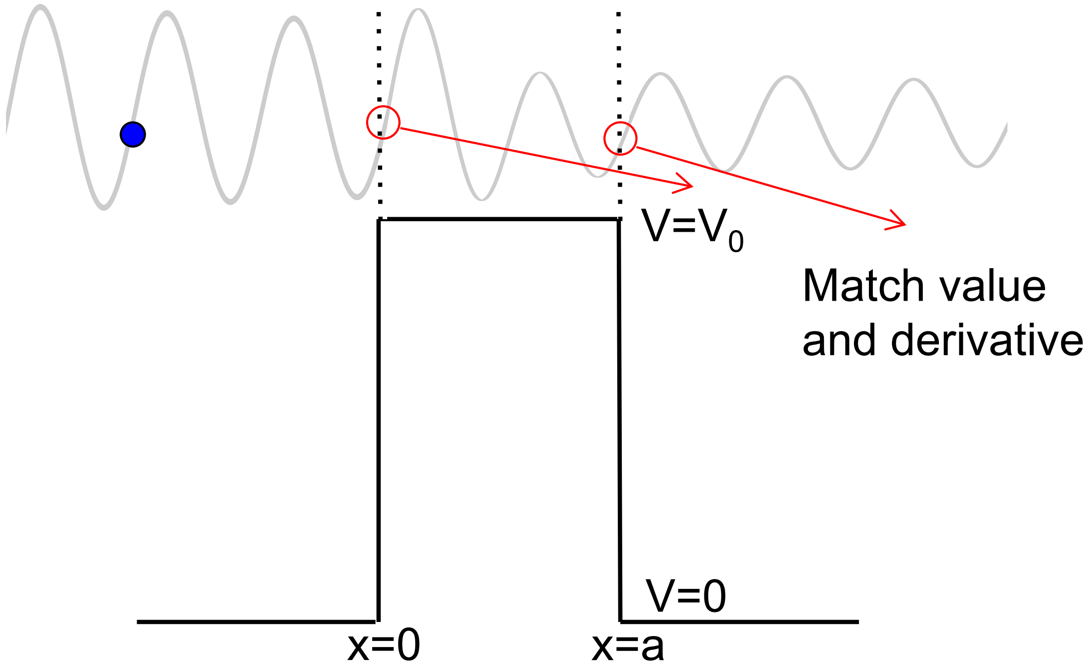
Solve Schrödinger equation in three regions:
- free travelling particle of energy E
- inside and outside the well
- normalize solutions to match value and derivative and borders x=0 and x=a
- **transmission and reflection**

$$\psi_1(x) = Ae^{ik(x)x} + Be^{-ik(x)x}$$

$$\psi_2(x) = Ce^{ik(x)x} + De^{-ik(x)x}$$

$$\psi_3(x) = Ee^{ik(x)x} + Fe^{-ik(x)x}$$

$$k(x) = \sqrt{2m(E - V_0)/\hbar^2}$$



Quantum system: the potential barrier

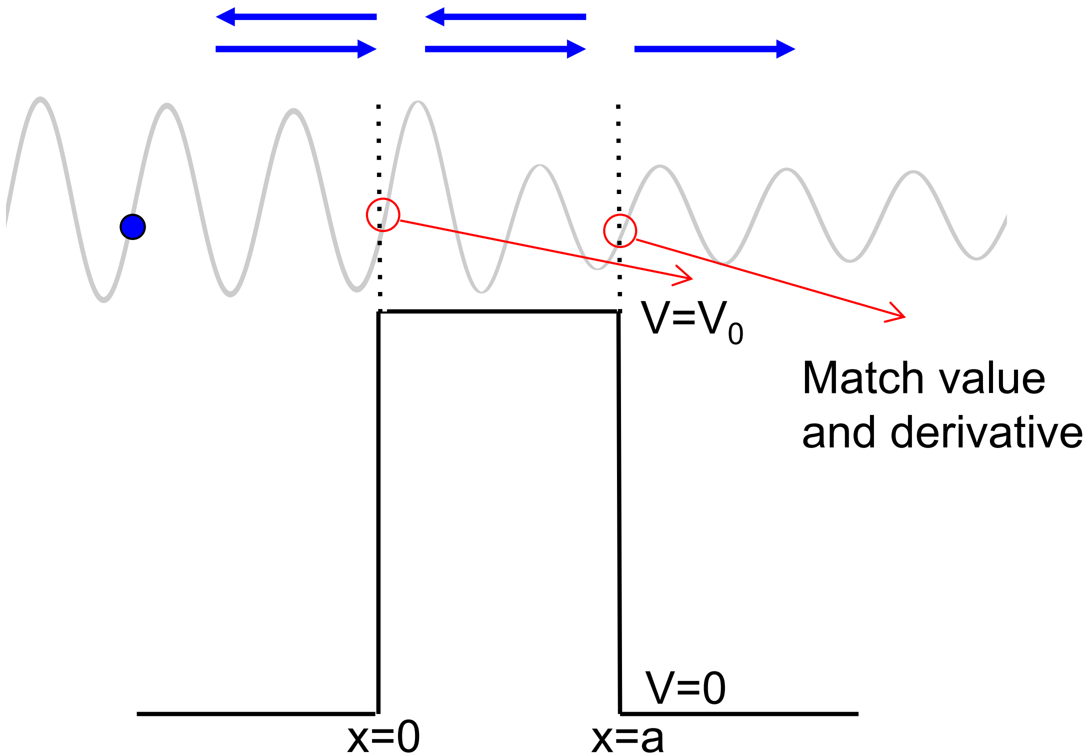
Solve Schrödinger equation in three regions:
- free travelling particle of energy E
- inside and outside the well
- normalize solutions to match value and derivative and borders x=0 and x=a
- **transmission and reflection**

$$\psi_1(x) = Ae^{ik(x)x} + Be^{-ik(x)x}$$

$$\psi_2(x) = Ce^{ik(x)x} + De^{-ik(x)x}$$

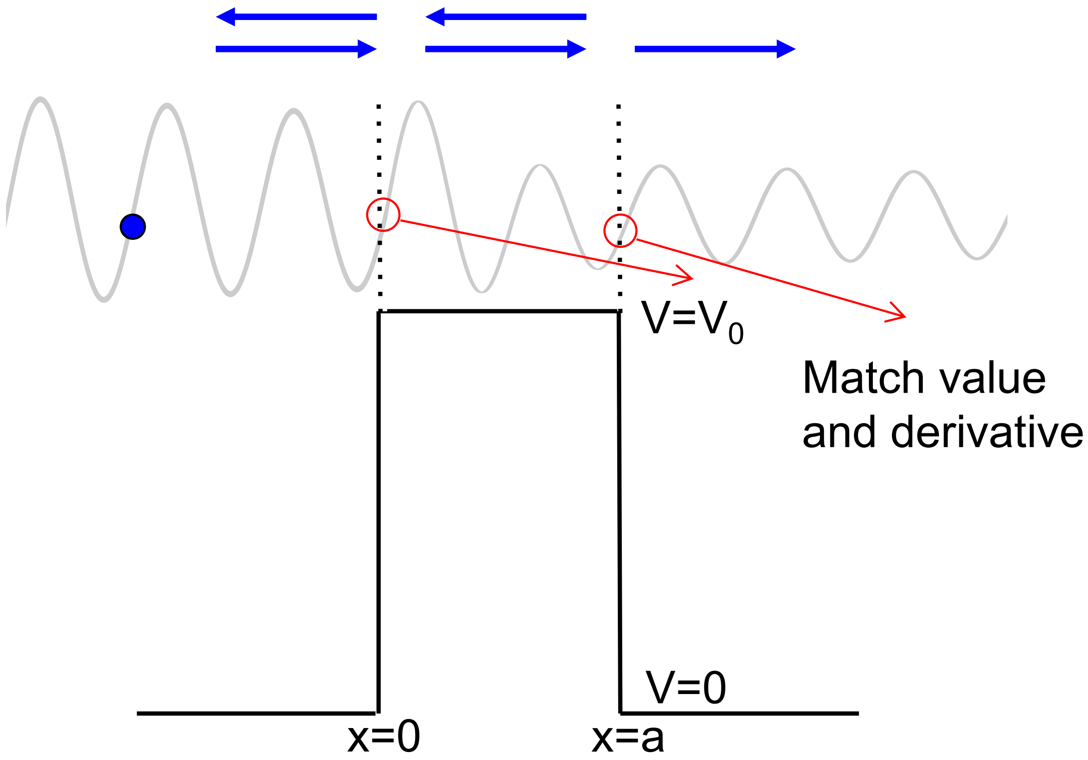
$$\psi_3(x) = Ee^{ik(x)x} + Fe^{-ik(x)x}$$

$$k(x) = \sqrt{2m(E - V_0)/\hbar^2}$$



Solve Schrödinger equation in three regions:

- free travelling particle of energy E
- inside and outside the well
- normalize solutions to match value and derivative and borders x=0 and x=a
- **transmission and reflection**



$$\psi_1(x) = Ae^{ik(x)x} + Be^{-ik(x)x}$$

$$\psi_2(x) = Ce^{ik(x)x} + De^{-ik(x)x}$$

$$\psi_3(x) = Ee^{ik(x)x} + Fe^{-ik(x)x}$$

$$k(x) = \sqrt{2m(E - V_0)/\hbar^2}$$

$$j = \frac{\hbar}{2mi}\left(\psi^*\nabla\psi - \nabla\psi^*\psi\right)$$

transmission $T = |F|^2/|A|^2 = j_{\text{trans}}/j_{\text{inc}}$

Quantum systems

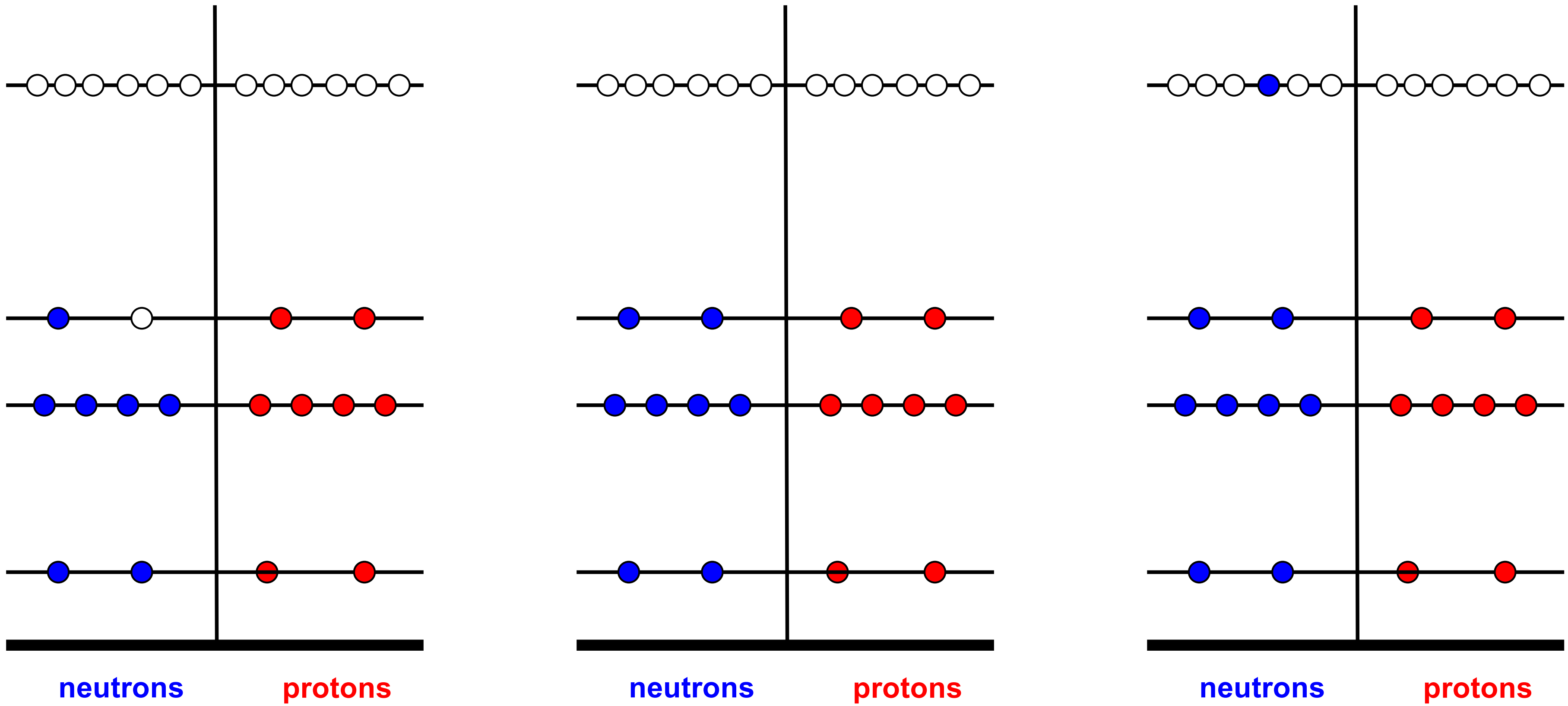
Other interesting excercises in 1D:
• barrier potential
• finite potential well
• harmonic oscillator

More complicated in 3D, V=V(r), more particles, degeneracy:
• cartesian well
• spherical well
• harmonic oscillator
• realistic potentials (Wood-Saxon),

→No analytical solution possible,
numerical solutions

Apply to real quantum systems:
atoms (hydrogen) but also to nuclei.

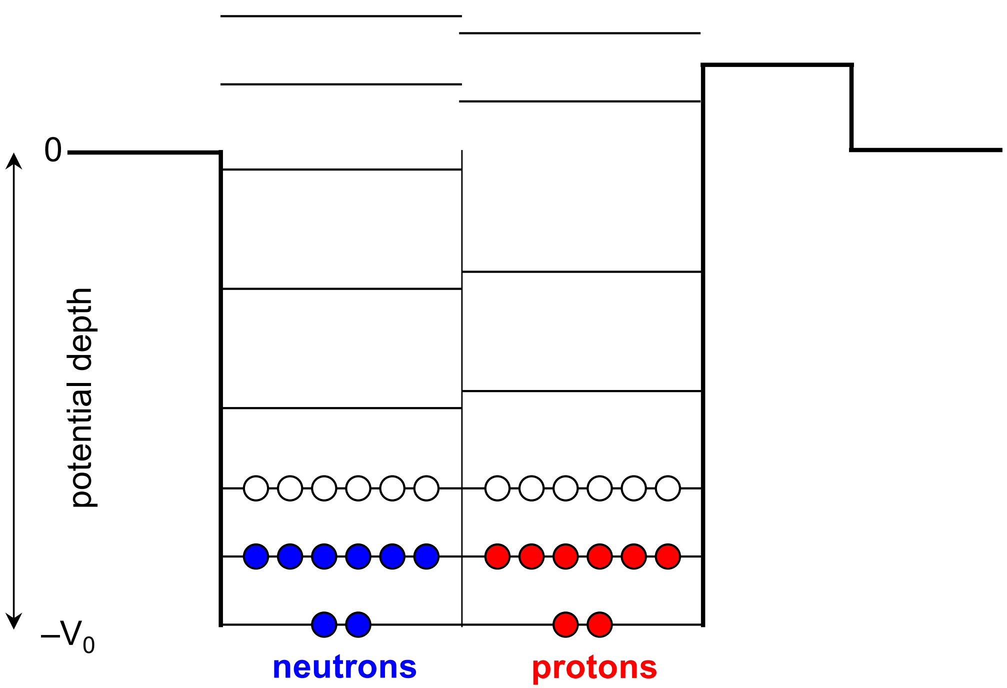
The nucleus as a quantum system



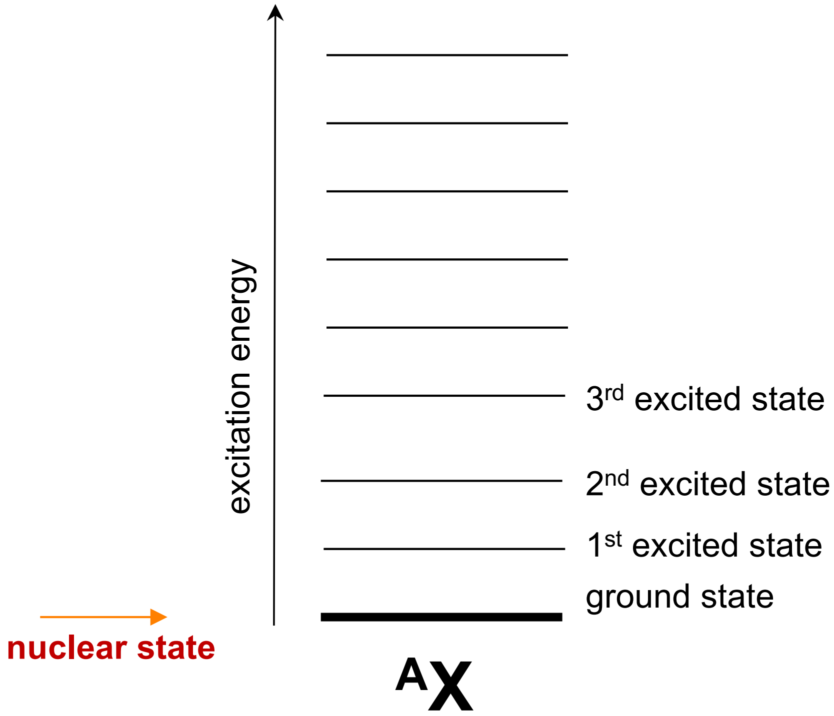
$^{15}_{8}\mathrm{O}$ $^{16}_{8}\mathrm{O}$ $^{17}_{8}\mathrm{O}$

The nucleus as a quantum system

shell model representation:
configuration of nucleons in their potential



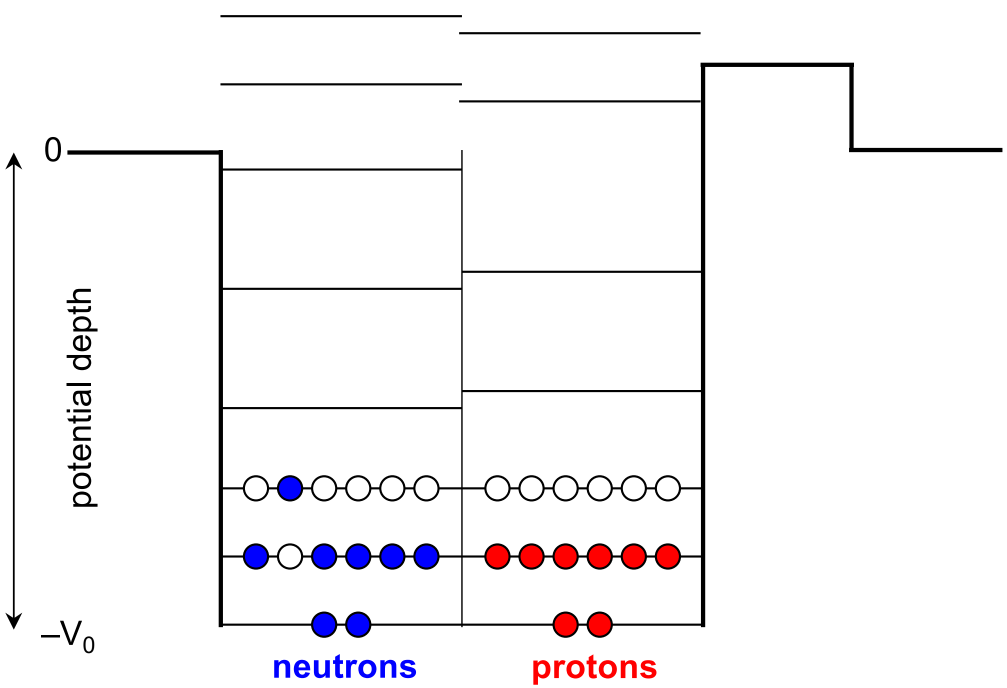
level scheme representation:
excited states of a nucleus
(shell model and other states)

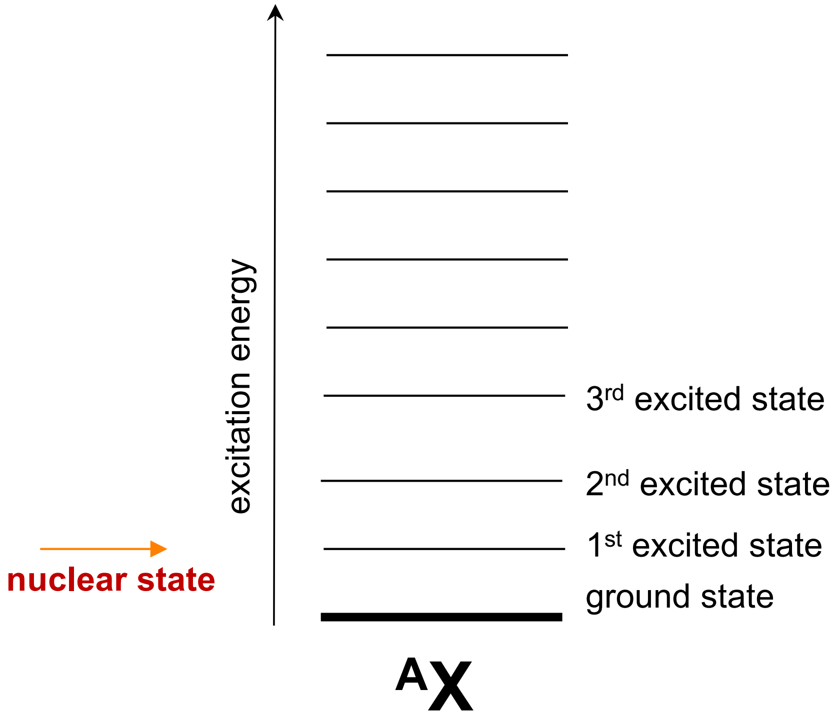


The nucleus as a quantum system

shell model representation:
configuration of nucleons in their potential

level scheme representation:
excited states of a nucleus
(shell model and other states)

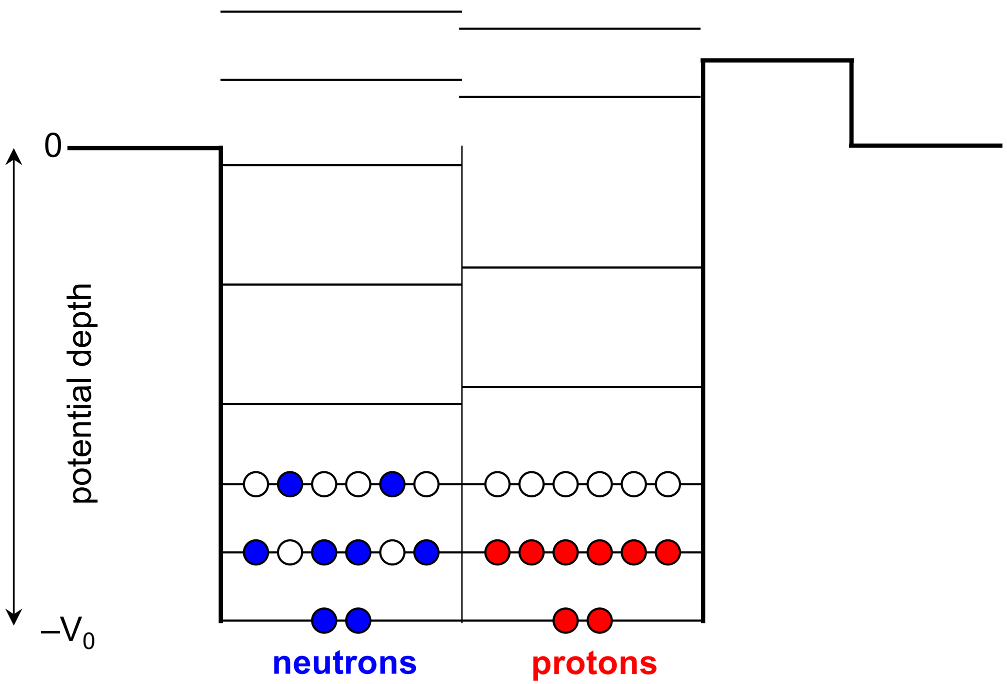


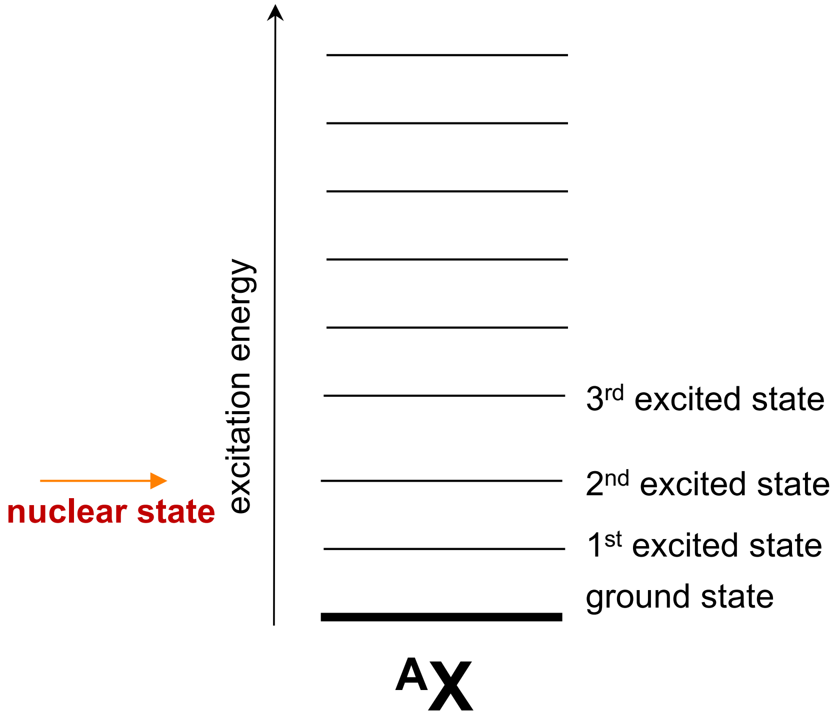


The nucleus as a quantum system

shell model representation:
configuration of nucleons in their potential

level scheme representation:
excited states of a nucleus
(shell model and other states)

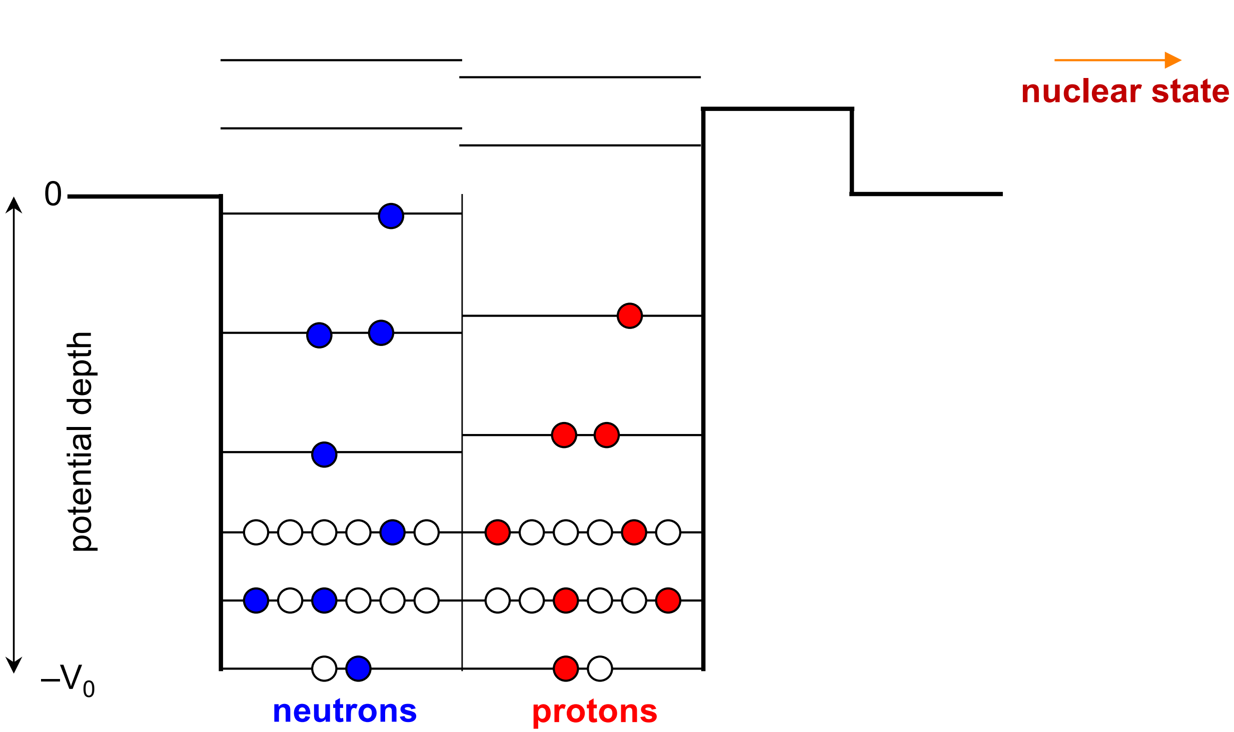


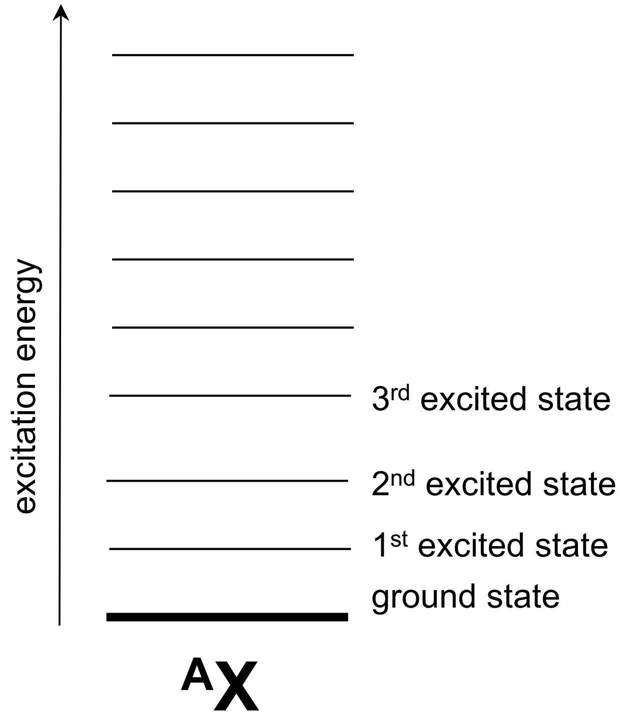


The nucleus as a quantum system

shell model representation:
configuration of nucleons in their potential

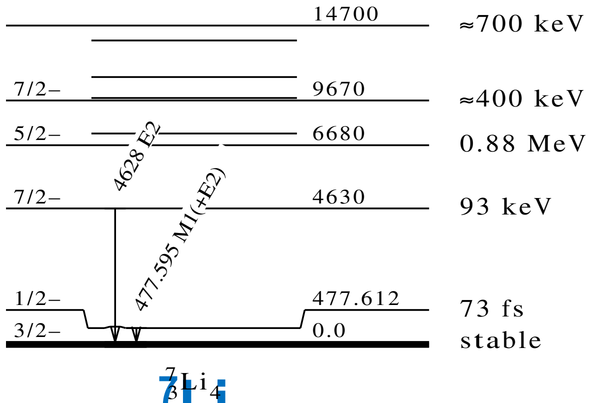
level scheme representation:
excited states of a nucleus
(shell model and other states)



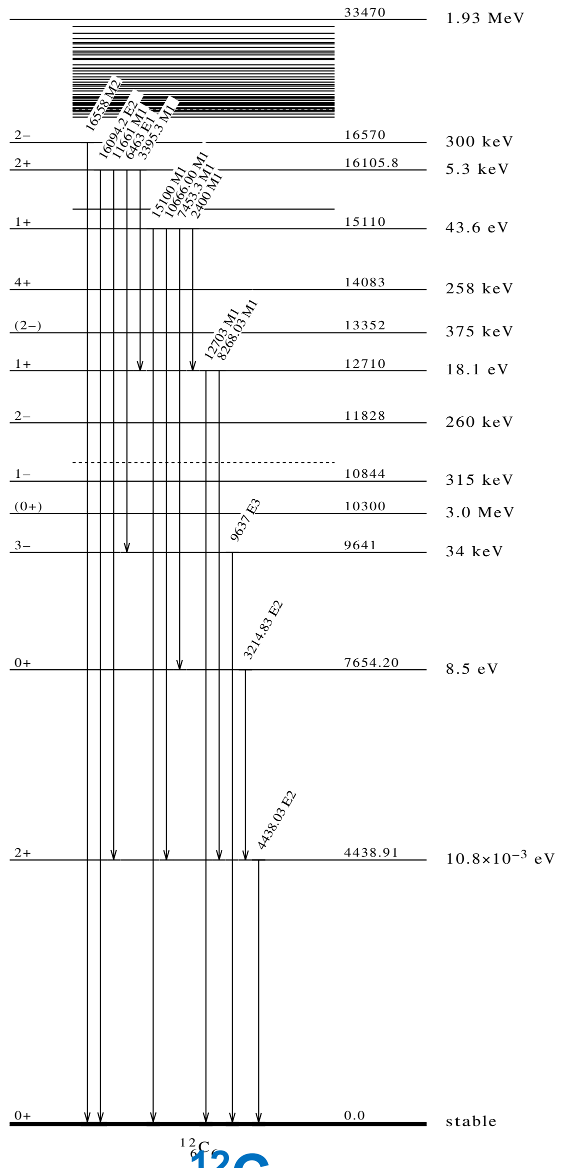


The nucleus as a quantum system

Level schemes from ENSDF
www.nndc.bnl.gov/ensdf

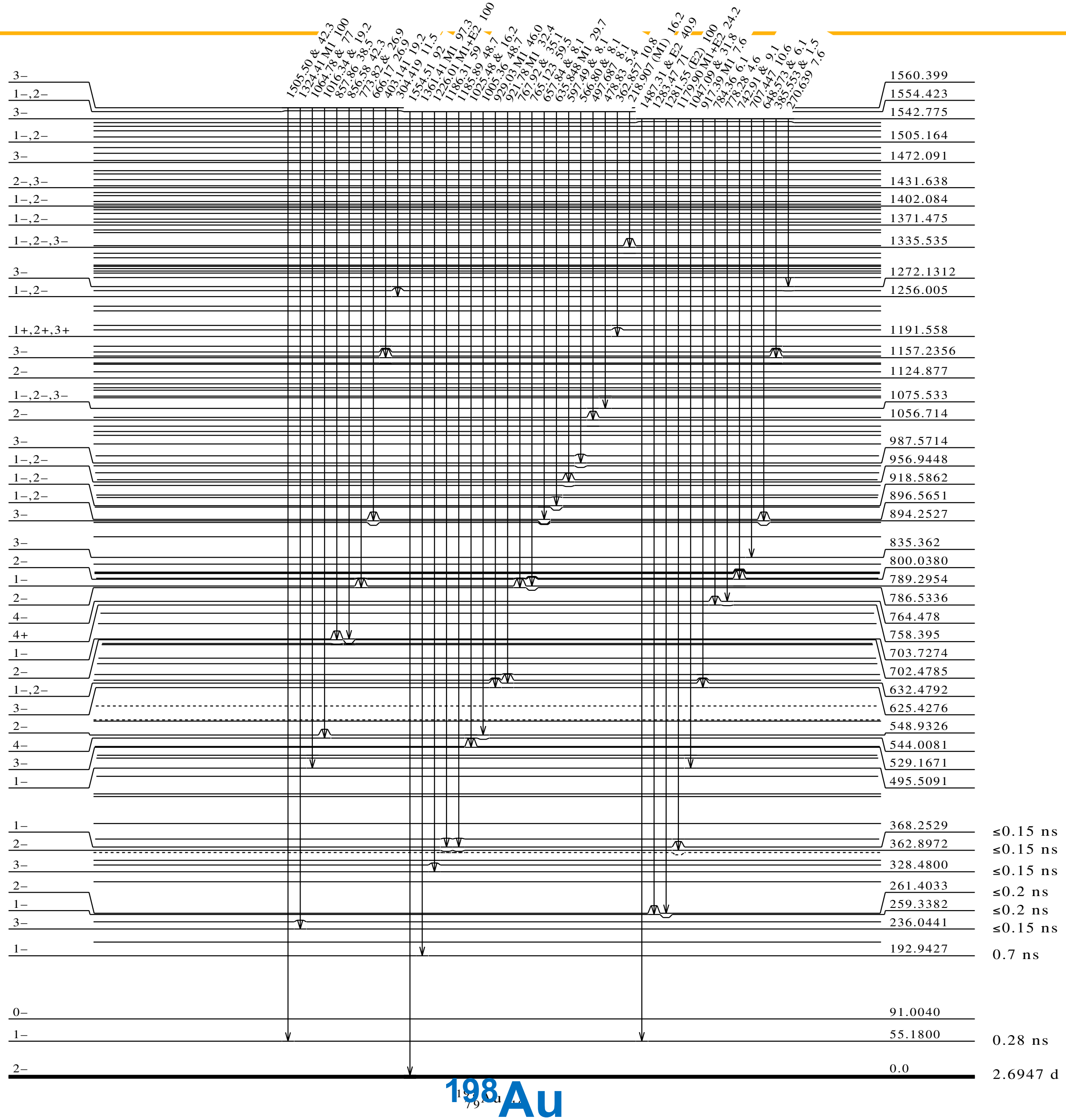


^{7}Li

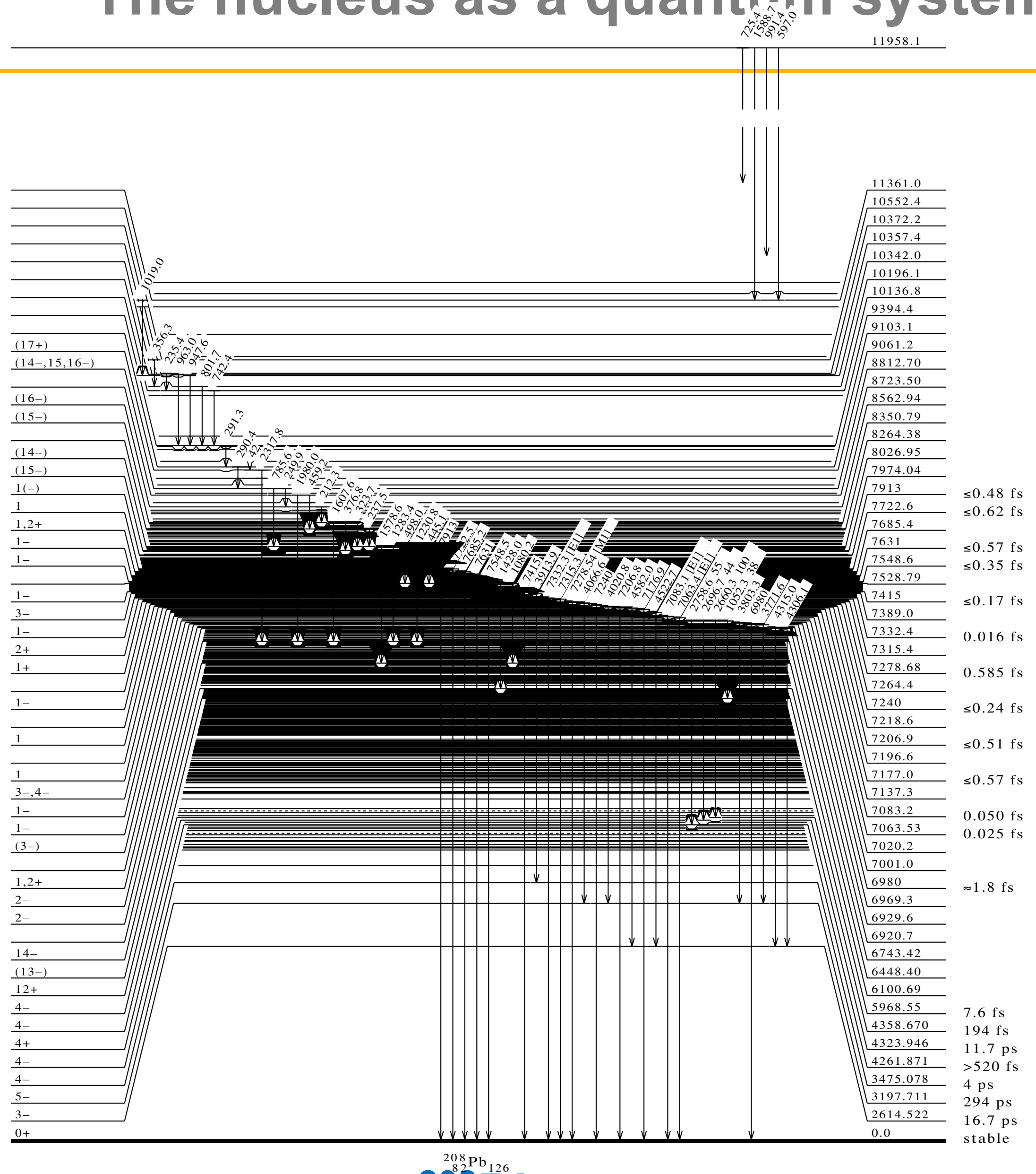


^{12}C

The nucleus as a quantum system

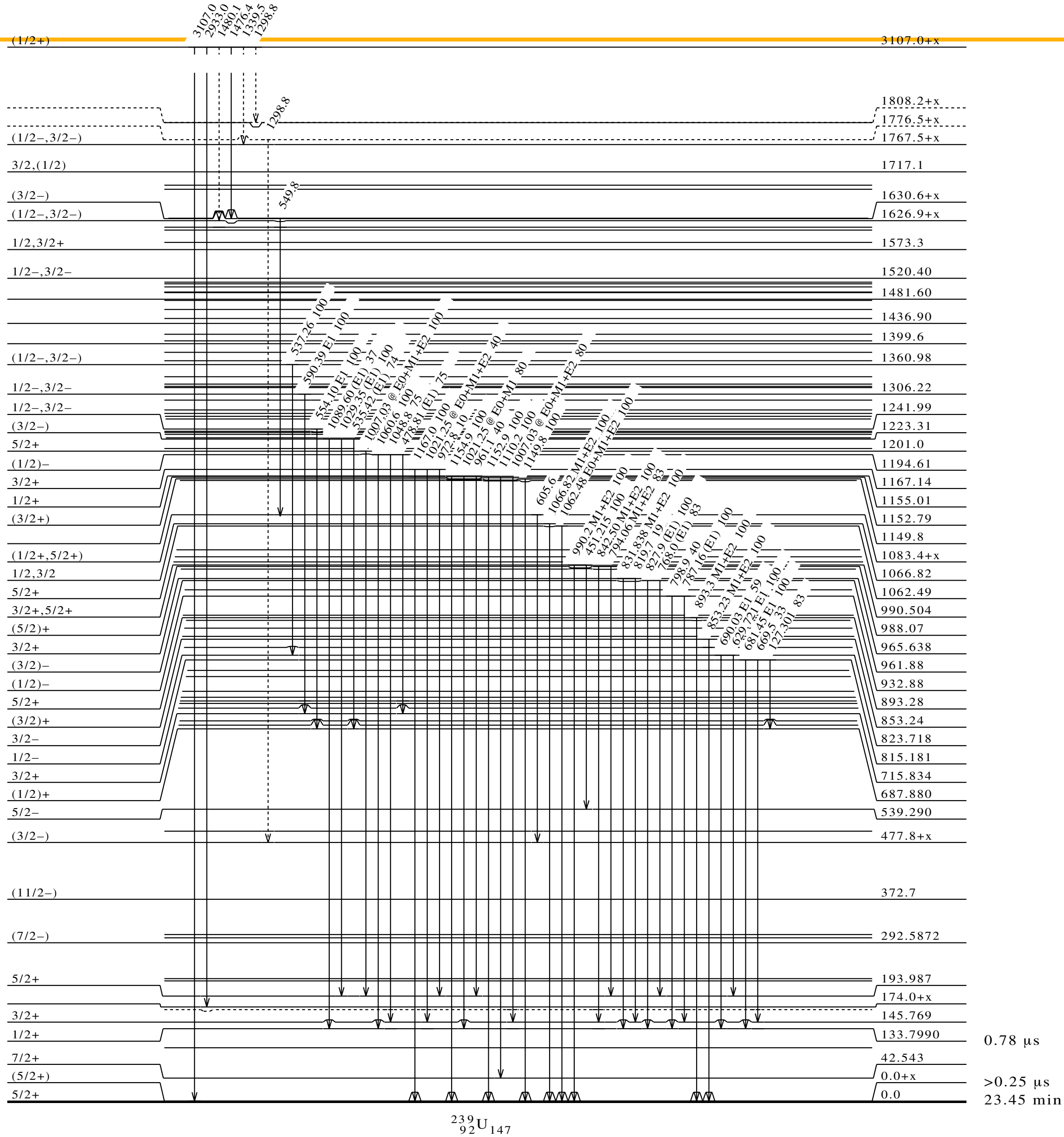


The nucleus as a quantum system



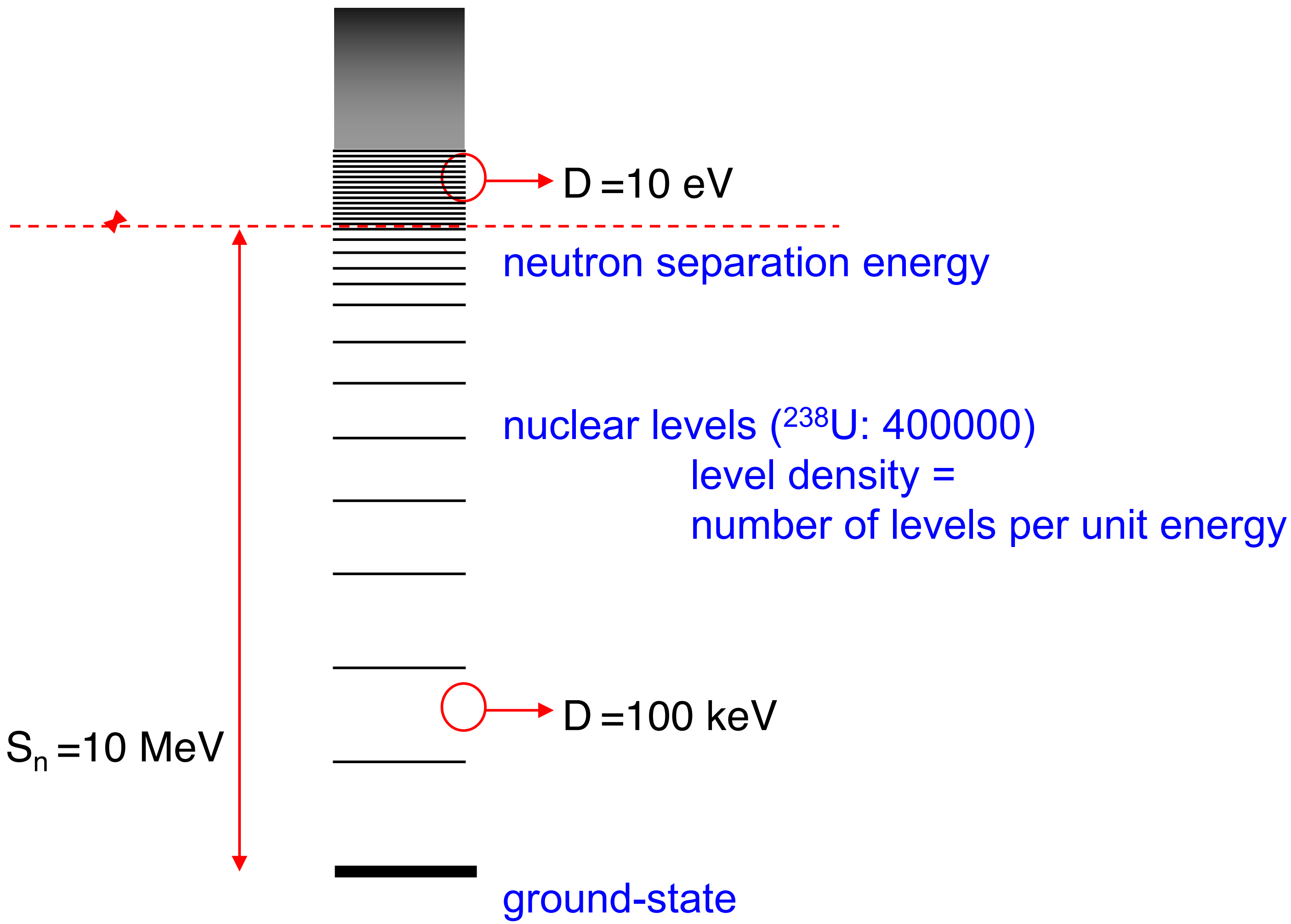
^{208}Pb

The nucleus as a quantum system

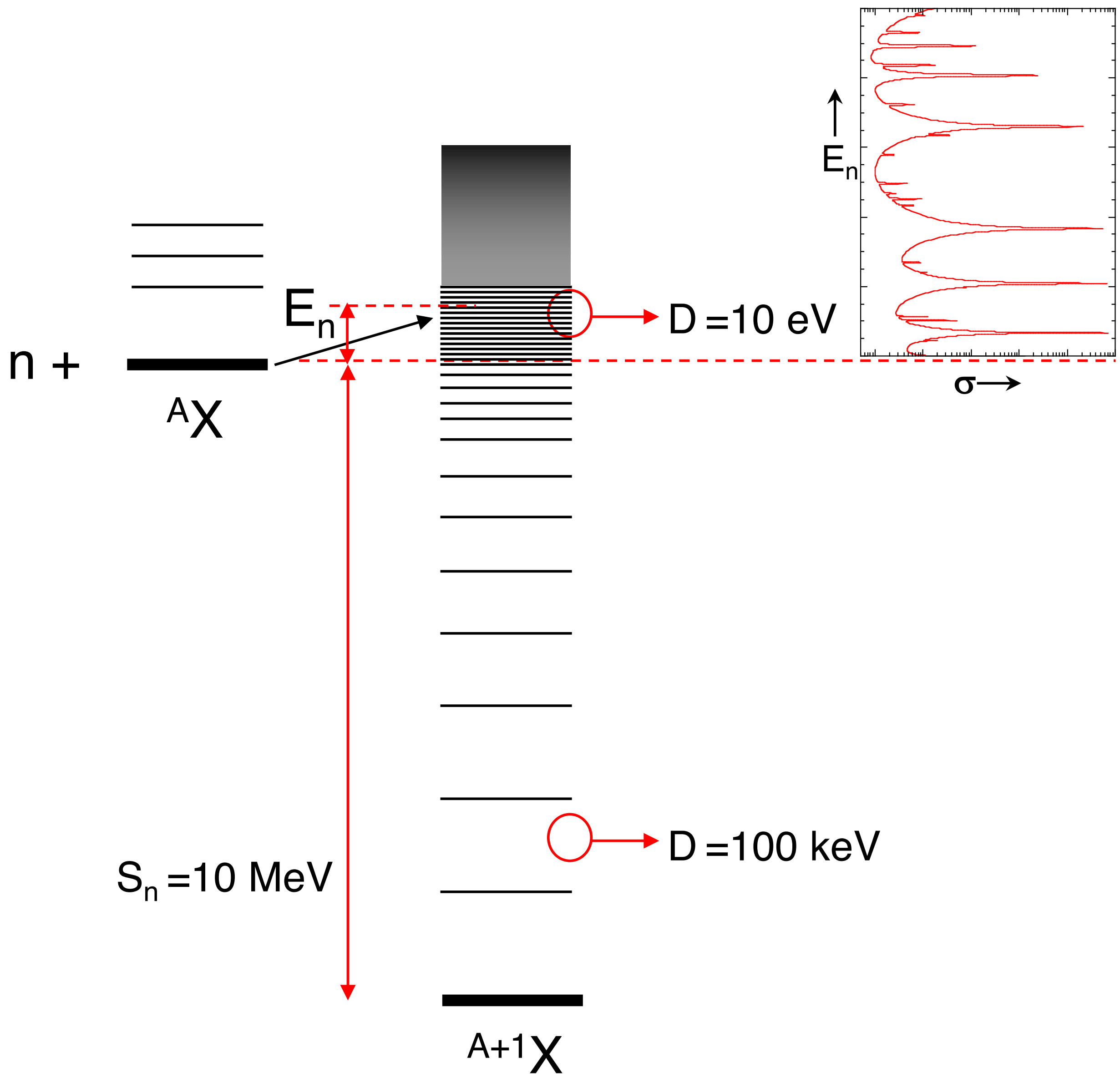


^{239}U

Nuclear levels



Compound neutron-nucleus reactions



R-matrix formalism

partial incoming wave functions: $\mathcal{I}_c$

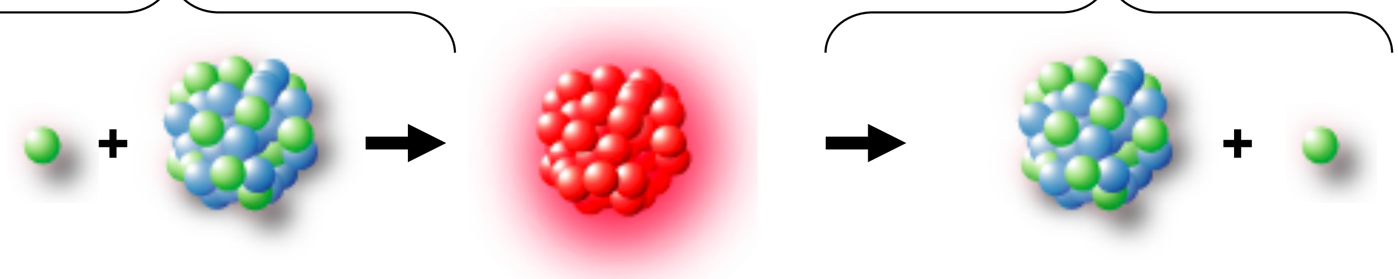
partial outgoing wave functions: $\mathcal{O}_{c'}$

related by collision matrix: $U_{cc'}$

cross section:

$$\sigma_{cc'} = \pi \lambda_c^2 |\delta_{c'c} - U_{c'c}|^2$$

External region (r>a_c, well separated particles):
• no interaction, Schrödinger equation solvable.

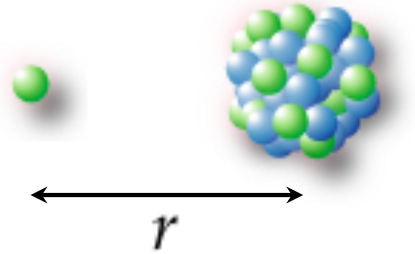


entrance channel
$c = \{\alpha, \ell, j, J, m_J\}$

compound nucleus

exit channel
$c' = \{\alpha', \ell', j', J', m_J'\}$

Internal region (r<a_c compound nucleus):
• wave function is expansion of eigenstates λ.



separation distance

$r > a_c$ external region

$r < a_c$ internal region

$r = a_c$ match value and derivate of Ψ

$$\left[\frac{d^2}{dr^2} - \frac{\ell(\ell+1)}{r^2} - \frac{2m_c}{\hbar^2}(V-E)\right] rR(r) = 0$$

External region: **easy**, solve Schrödinger equation
central force, separate radial and angular parts.

$$\psi(r,\theta,\phi) = R(r)\Theta(\theta)\Phi(\phi)$$

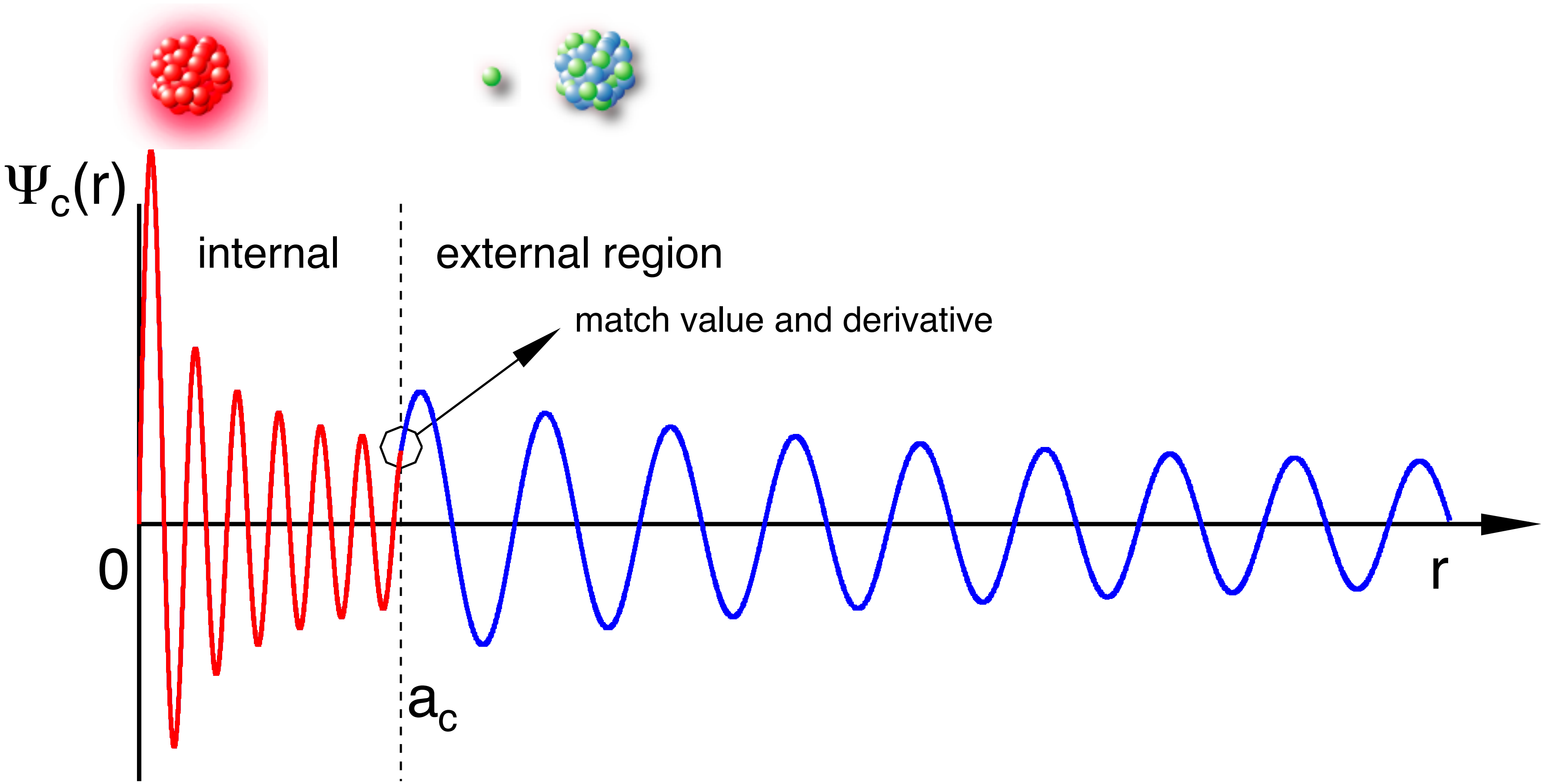
solution: solve Schrödinger equation of relative motion:

- Coulomb functions
- special case of neutron particles (neutrons): fonctions de Bessel

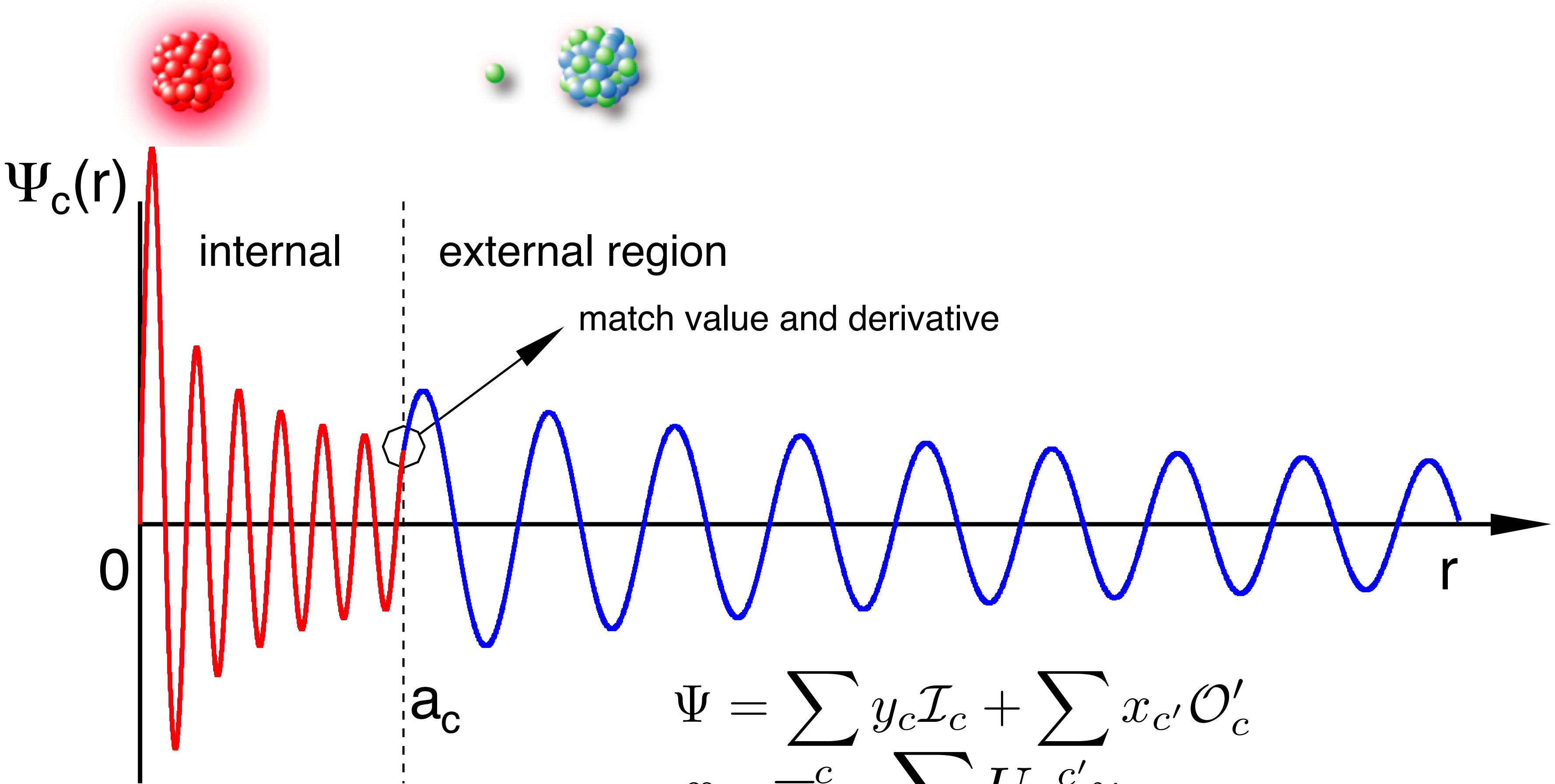
Internal region: **very difficult**, Schrödinger equation cannot be solved directly
solution: expand the wave function as a linear combination of its eigenstates.
using the R-matrix:

$$R_{cc'} = \sum_{\lambda} \frac{\gamma_{\lambda c}\gamma_{\lambda c'}}{E_\lambda - E}$$

The R-matrix formalism



The R-matrix formalism



$$\Psi = \sum y_c \mathcal{I}_c + \sum x_{c'} \mathcal{O}'_c$$

$$x_{c'} \equiv -\sum U_{c'c} y_c$$

$$\mathcal{I}_c = I_c r^{-1} \varphi_c i^\ell Y^\ell_{m_\ell}(\theta, \phi)/\sqrt{v_c}$$

$$\mathcal{O}_c = O_c r^{-1} \varphi_c i^\ell Y^\ell_{m_\ell}(\theta, \phi)/\sqrt{v_c}$$

$$\Psi = \Psi(R_{cc'})$$

$$R_{cc'} = \sum_\lambda \frac{\gamma_{\lambda c}\gamma_{\lambda c'}}{E_\lambda - E}$$

The R-matrix formalism

The wave function of the system is a superposition of incoming and outgoing waves:

$$\Psi = \sum_c y_c \mathcal{I}_c + \sum_{c'} x_{c'} \mathcal{O}'_c$$

Incoming and outgoing wavefunctions have form:

$$\mathcal{I}_c = I_c r^{-1} \varphi_c i^\ell Y^\ell_{m_\ell}(\theta, \phi)/\sqrt{v_c}$$

$$\mathcal{O}_c = O_c r^{-1} \varphi_c i^\ell Y^\ell_{m_\ell}(\theta, \phi)/\sqrt{v_c}$$

The physical interaction is included in the collision matrix **U**:

$$x_{c'} \equiv -\sum_c U_{c'c} y_c$$

The wave function depends on the R-matrix, which depends on the widths and levels of the eigenstates.

$$\Psi = \Psi(R_{cc'})$$

$$R_{cc'} = \sum_\lambda \frac{\gamma_{\lambda c}\gamma_{\lambda c'}}{E_\lambda - E}$$

The R-matrix formalism

The relation between the R-matrix and the collision matrix:

$$\mathbf{U} = \mathbf{\Omega P}^{1/2}[\mathbf{1} - \mathbf{R}(\mathbf{L} - \mathbf{B})]^{-1}[\mathbf{1} - \mathbf{R}(\mathbf{L}^{*} - \mathbf{B})]\mathbf{P}^{-1/2}\mathbf{\Omega}$$

with: $L_c = S_c + iP_c = \left(\frac{\rho}{O_c}\frac{dO_c}{d\rho}\right)_{r=a_c}$

The relation between the collision matrix and cross sections:

channel to one other channel: $\sigma_{cc'} = \pi\lambda_c^2|\delta_{c'c} - U_{c'c}|^2$

channel to any other channel: $\sigma_{cr} = \pi\lambda_c^2(1 - |U_{cc}|^2)$

channel to same channel: $\sigma_{cc} = \pi\lambda_c^2|1 - U_{cc}|^2$

channel to any channel (total): $\sigma_{c,T} = \sigma_c = 2\pi\lambda_c^2(1 - \mathrm{Re}\, U_{cc})$

The R-matrix formalism

The Breit-Wigner Single Level approximation:

total cross section:

$$\sigma_c = \pi \bar{\lambda}_c^2 g_c \left(4 \sin^2 \phi_c + \frac{\Gamma_\lambda \Gamma_{\lambda c} \cos 2\phi_c + 2(E - E_\lambda - \Delta_\lambda)\Gamma_{\lambda c} \sin 2\phi_c}{(E - E_\lambda - \Delta_\lambda)^2 + \Gamma_\lambda^2/4} \right)$$

neutron channel: $c = n$

only capture, scattering, fission: $\Gamma_\lambda = \Gamma = \Gamma_n + \Gamma_\gamma + \Gamma_f$

other approximations: $\ell = 0 \qquad \cos\phi_c = 1 \qquad \sin\phi_c = \rho = k a_c \qquad \Delta_\lambda = 0$

total cross section:

interference elastic capture fission

potential

$$\sigma_T(E) = \underbrace{4\pi R'^2}_{\text{potential}} + \pi \bar{\lambda}^2 g \left(\frac{\overbrace{4\Gamma_n (E - E_0) R'/\bar{\lambda}}^{\text{interference}} + \overbrace{\Gamma_n^2}^{\text{elastic}} + \overbrace{\Gamma_n \Gamma_\gamma}^{\text{capture}} + \overbrace{\Gamma_n \Gamma_f}^{\text{fission}}}{(E - E_0)^2 + \underbrace{(\Gamma_n + \Gamma_\gamma + \Gamma_f +)^2}_{\text{total width}}/4} \right)$$

total width

Average cross sections

The relation between the energy averaged collision matrix and energy averaged cross sections:

average scattering: $\overline{\sigma_{cc}} = \pi \lambda_c^2 g_c \overline{|1 - U_{cc}|^2}$

shape elastic (potential) $\overline{\sigma_{cc}^{\mathrm{se}}} = \pi \lambda_c^2 g_c |1 - \overline{U_{cc}}|^2$

compound elastic $\overline{\sigma_{cc}^{\mathrm{ce}}} = \pi \lambda_c^2 g_c \left(\overline{|U_{cc}|^2} - |\overline{U_{cc}}|^2 \right)$

average any reaction $\overline{\sigma_{cr}} = \pi \lambda_c^2 g_c (1 - \overline{|U_{cc}|^2})$

average total $\overline{\sigma_{c,T}} = 2\pi \lambda_c^2 g_c (1 - \mathrm{Re}\, \overline{U_{cc}})$

average single reaction $\overline{\sigma_{cc'}} = \pi \lambda_c^2 g_c \overline{|\delta_{cc'} - U_{cc'}|^2}$

average compound nucleus formation $\overline{\sigma_c} = \pi \lambda_c^2 g_c (1 - |\overline{U_{cc}}|^2)$

Average cross sections

- From optical model calculations one can calculate $\overline{U_{cc}}$ but not $\overline{|U_{cc}|^2}$

- Therefore, only $\overline{\sigma_{c,T}}, \quad \overline{\sigma^{\mathrm{se}}_{cc}}, \quad \overline{\sigma_c}$ can be calculated, of which only the total average cross section can be compared with measurements.

- In OMP one uses transmission coefficients $T_c = 1 - |\overline{U_{cc}}|^2$

- Average single reaction cross section (Hauser-Feshbach):

$$\overline{\sigma_{cc'}} = \overline{\sigma^{\mathrm{se}}_{cc}}\delta_{cc'} + \pi \lambda_c^2 g_c \frac{T_c T_{c'}}{\Sigma T_i} W_{cc'}$$

- Average single reaction cross section (Hauser-Feshbach):

$$W_{cc'} = \overline{\left(\frac{\Gamma_c \Gamma_{c'}}{\Gamma}\right)} \frac{\overline{\Gamma}}{\overline{\Gamma_c}\,\overline{\Gamma_{c'}}}$$

Measured quantities for resolved resonances

- **Experimental quantities are not cross sections but reaction yields and transmission factors**

reaction yield: $$Y(E_n) = \mu(E_n)\left(1 - e^{-n\sigma_T(E_n)}\right) \cdot \frac{\sigma_\gamma(E_n)}{\sigma_T(E_n)}$$

transmission: $$T(E_n) = e^{-n\sigma_T(E_n)}$$

- **Cross sections are functions of the resonance parameters**

cross section:

$$\sigma_{cr} = \pi \lambda_c^2 g_c (1 - |U_{cc}|^2)$$

$$\sigma = \sigma(\{E_r, J^\pi, \Gamma, \Gamma_r\}, \ldots)$$

Measured quantities for unresolved resonances

- **Experimental quantities are average yields and average transmission factors**

reaction yield: $$\langle Y \rangle = \left\langle \mu (1 - e^{-n\sigma_T}) \frac{\sigma_\gamma}{\sigma_T} \right\rangle = f_r \times n \times < \sigma_\gamma >$$

transmission: $$\langle T \rangle = \left\langle e^{-n\sigma_T} \right\rangle = e^{-n\langle \sigma_T \rangle} \cdot \left\langle e^{-n(\sigma_T - \langle \sigma_T \rangle)} \right\rangle =$$

$$f_T \times e^{-n\langle \sigma_T \rangle}$$

- **Change of parameters describing the cross section**

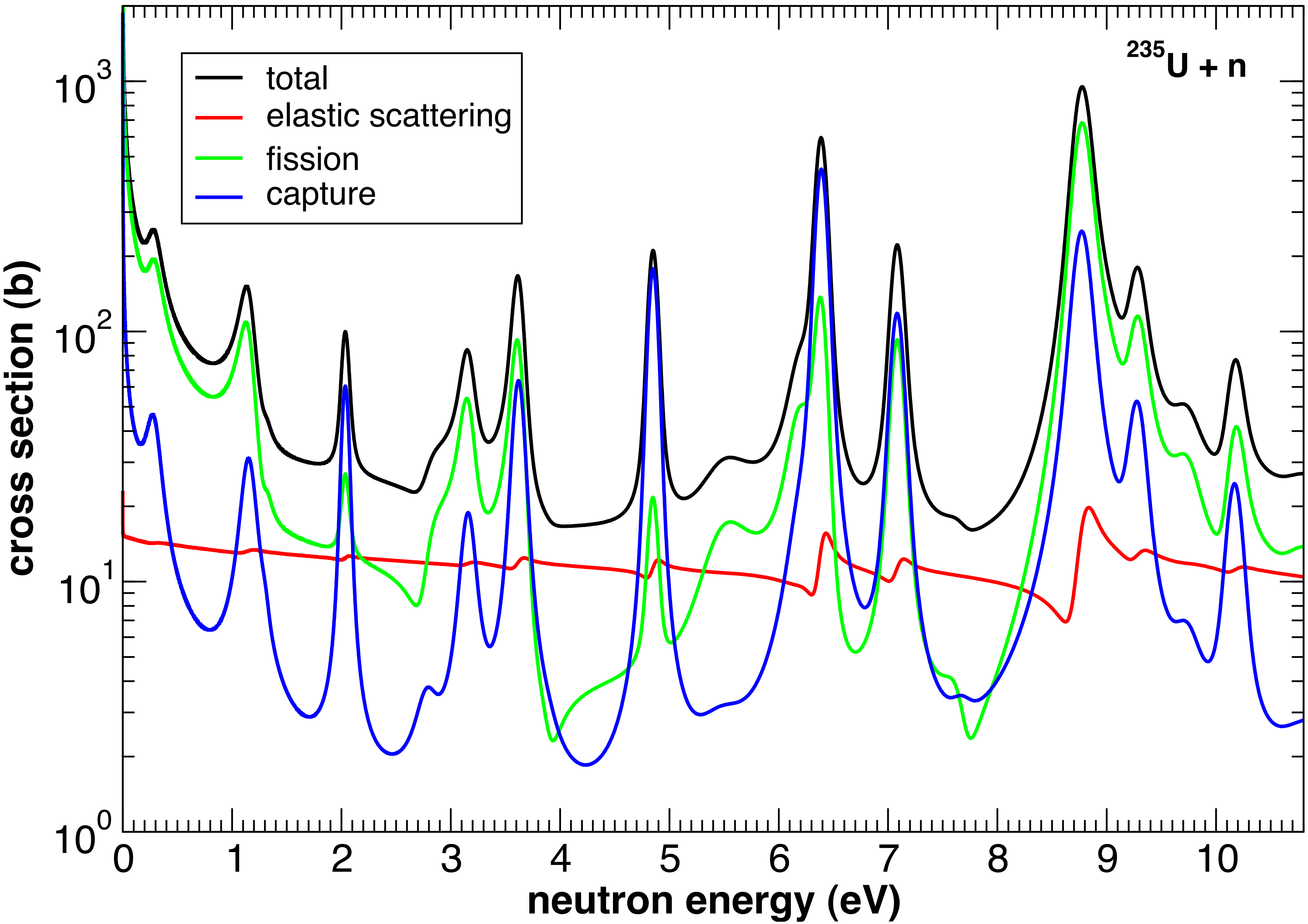
resolved → unresolved parameters

$$E, J^\pi \rightarrow \rho_\ell \quad \text{or} \quad D_\ell$$

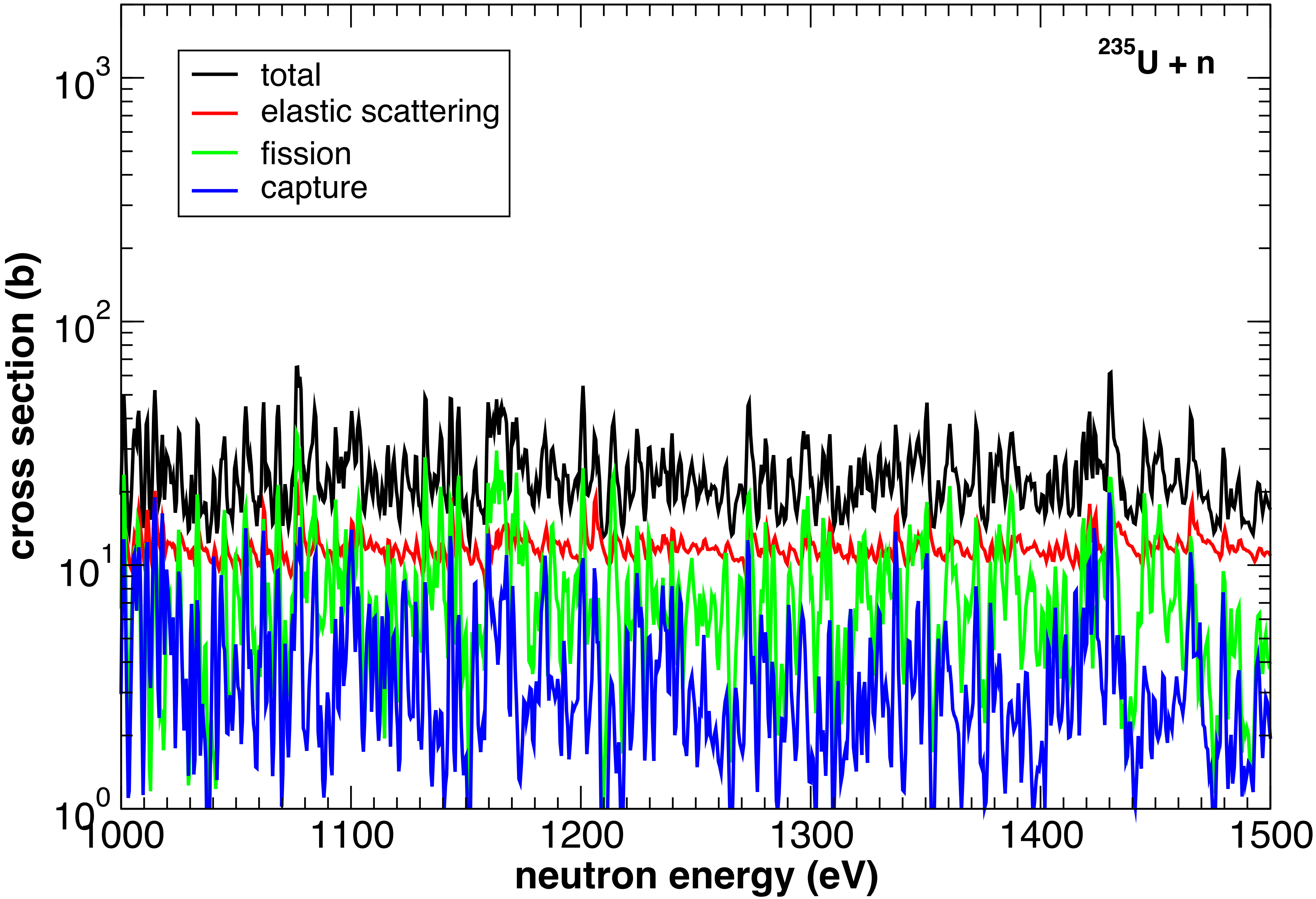
$$\Gamma_\gamma \rightarrow \langle \Gamma_\gamma \rangle$$

$$g\Gamma_n^\ell \rightarrow \langle g\Gamma_n^\ell \rangle = (2\ell + 1) S_\ell D_\ell$$

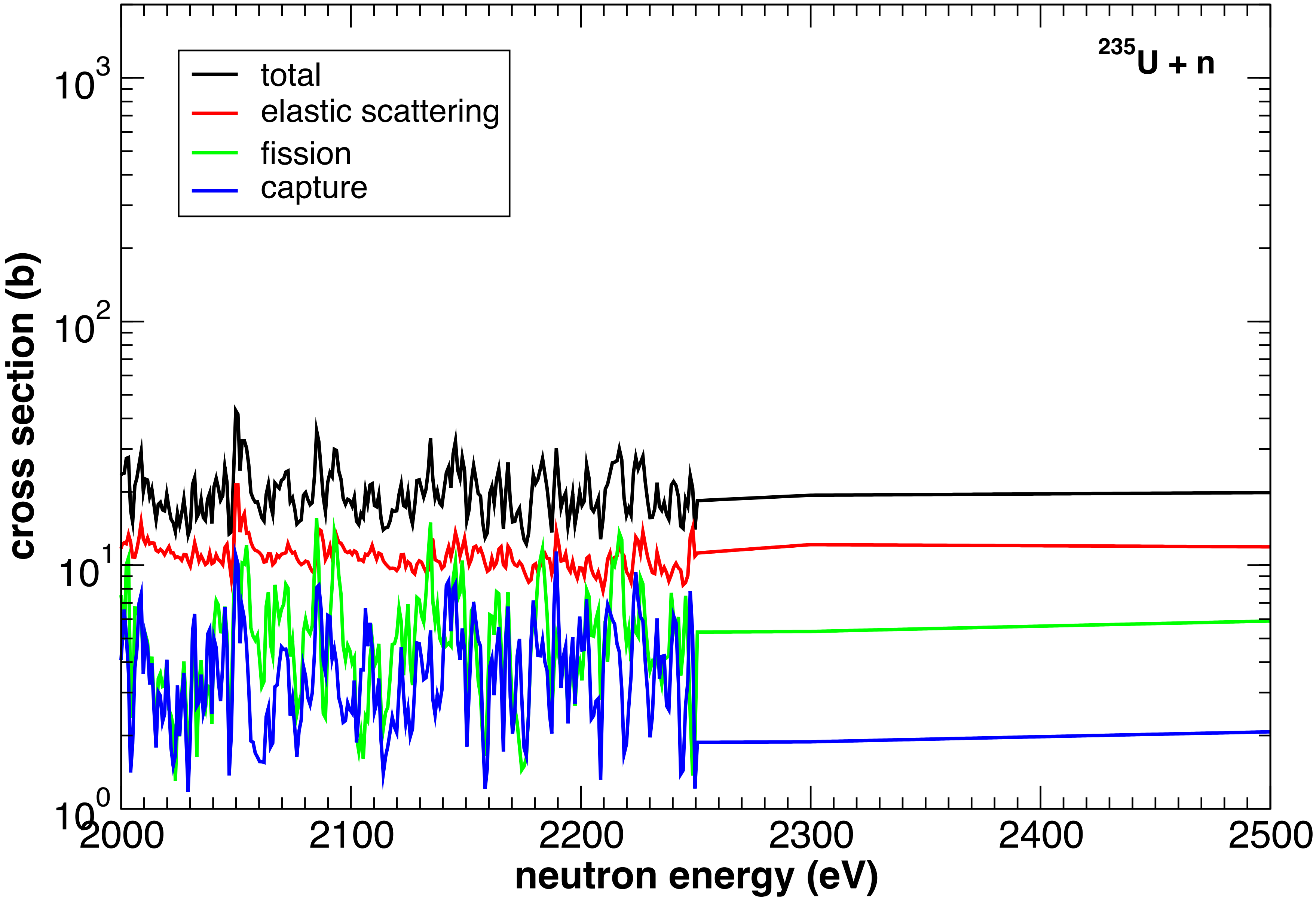
Cross sections σ_T, σ_γ, σ_n et σ_f



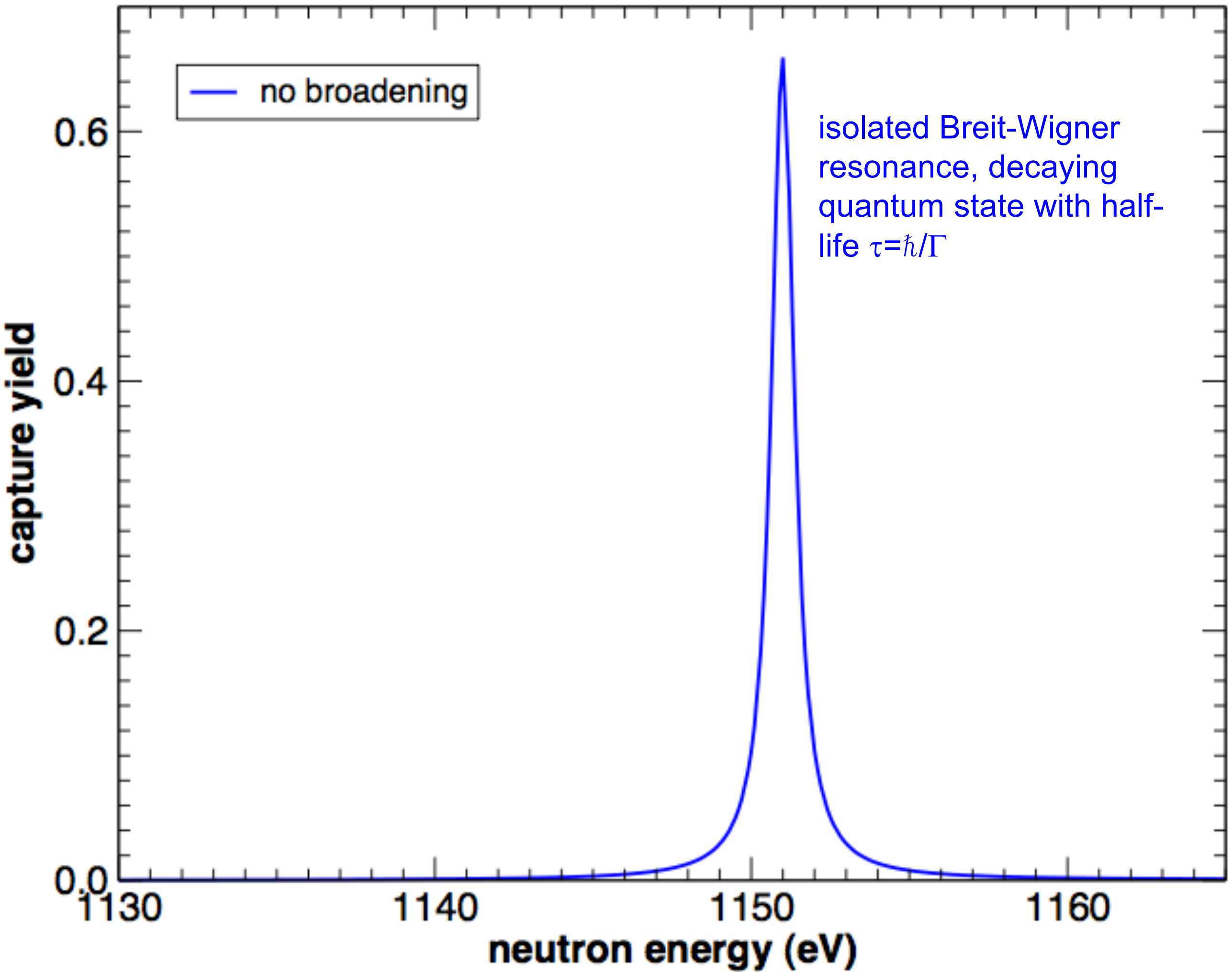
Cross sections σ_T, σ_γ, σ_n et σ_f



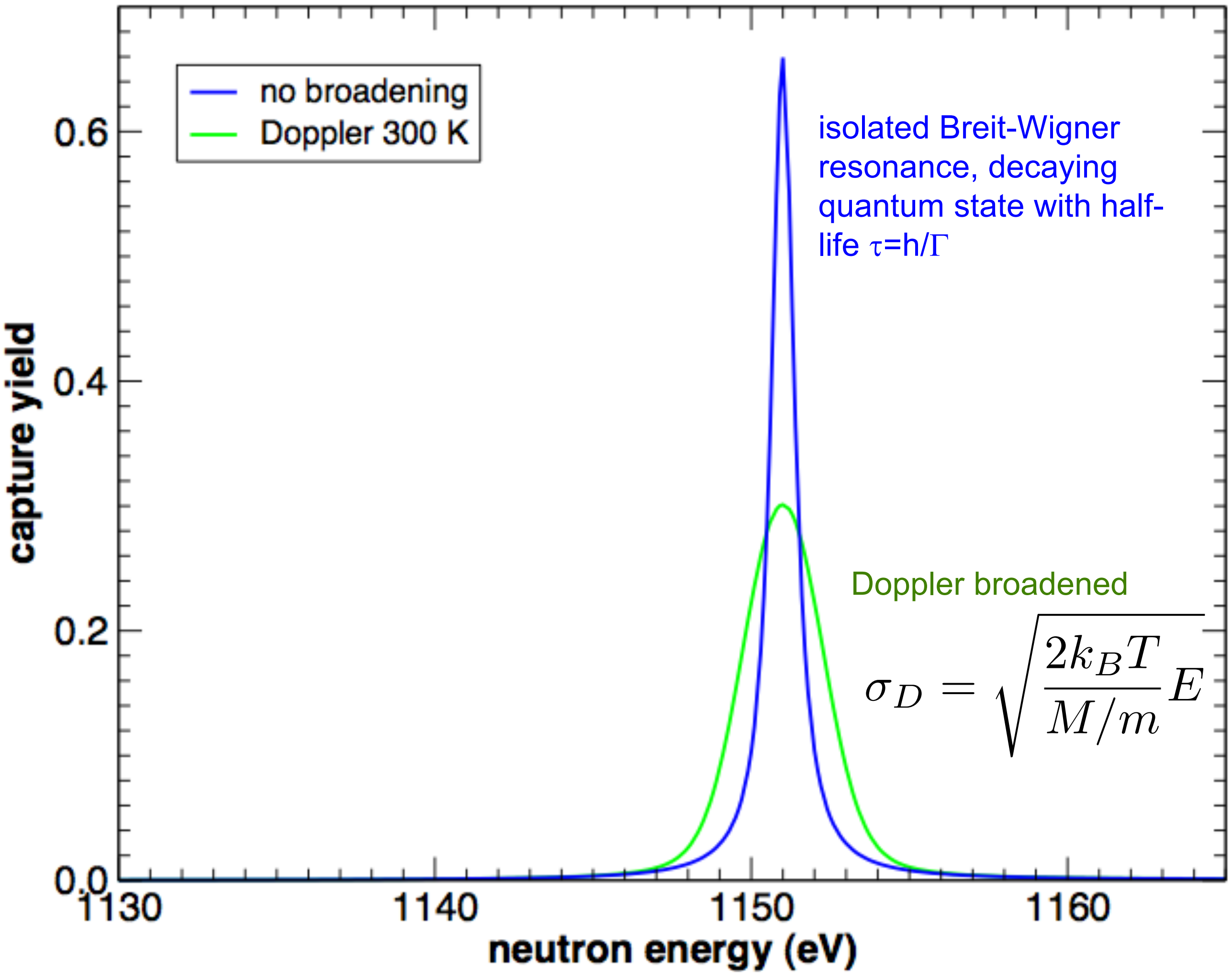
Cross sections σ_T, σ_γ, σ_n et σ_f



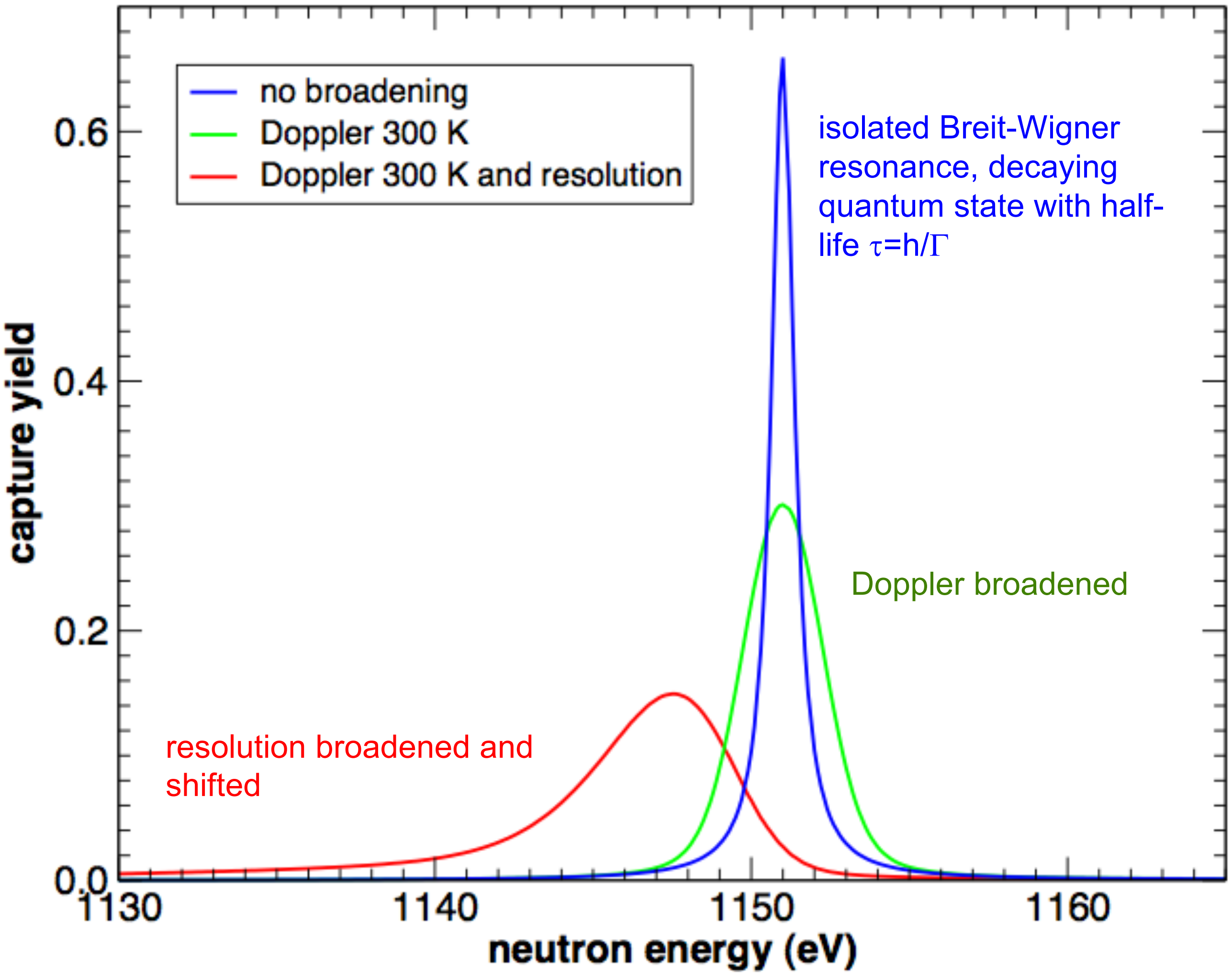
Measuring reaction yield for resolved resonances



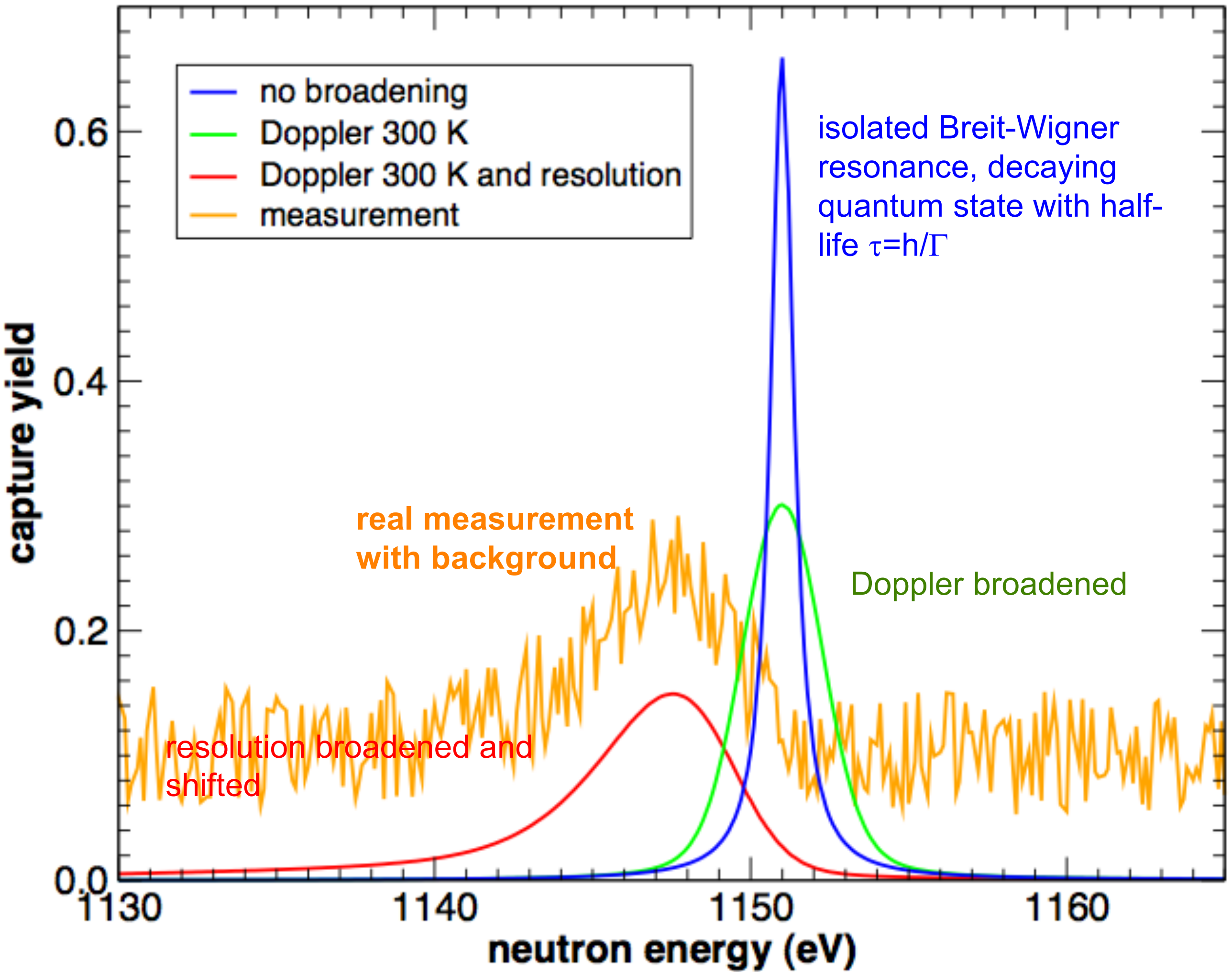
Measured reaction yield



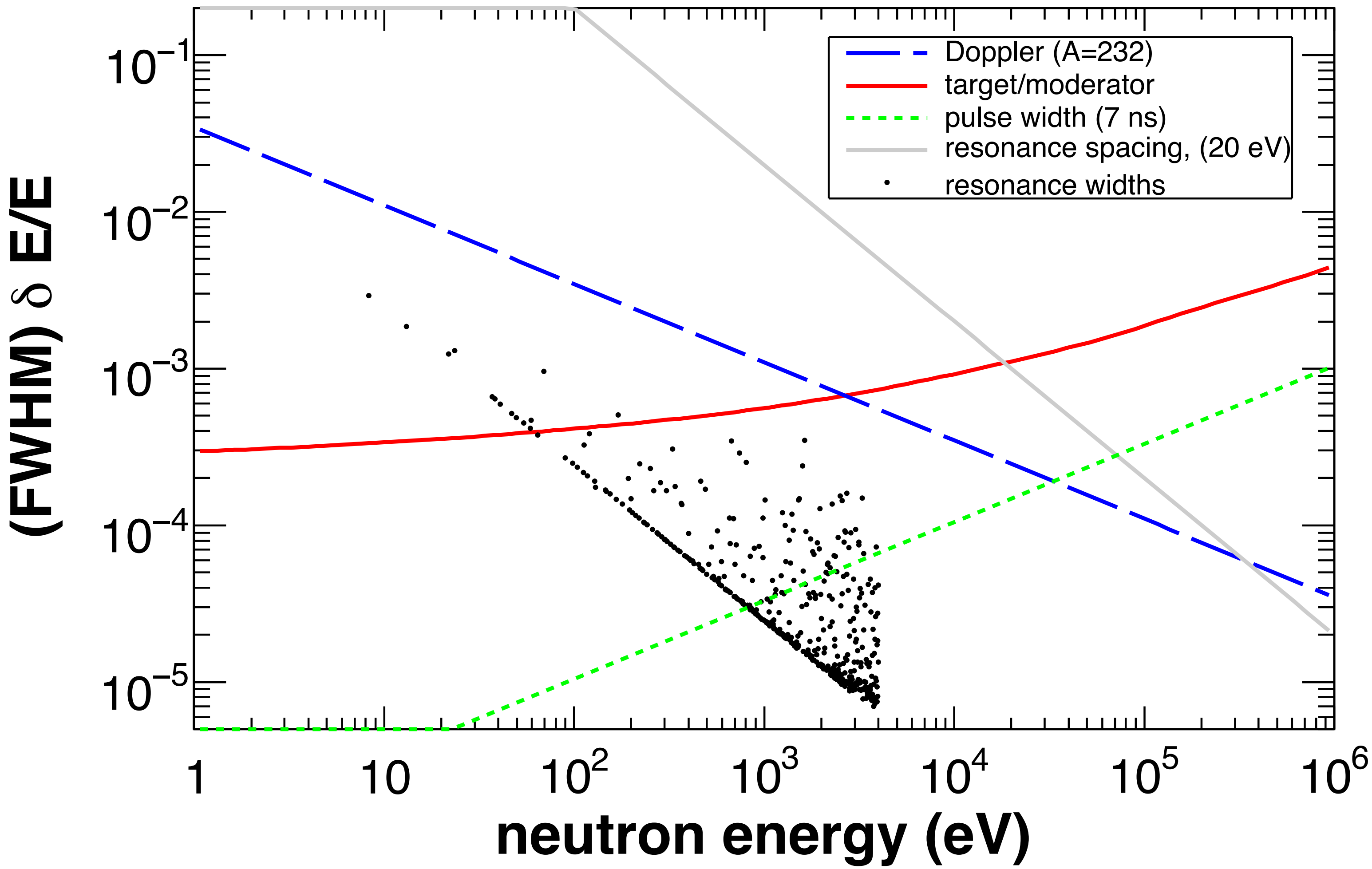
Measured reaction yield



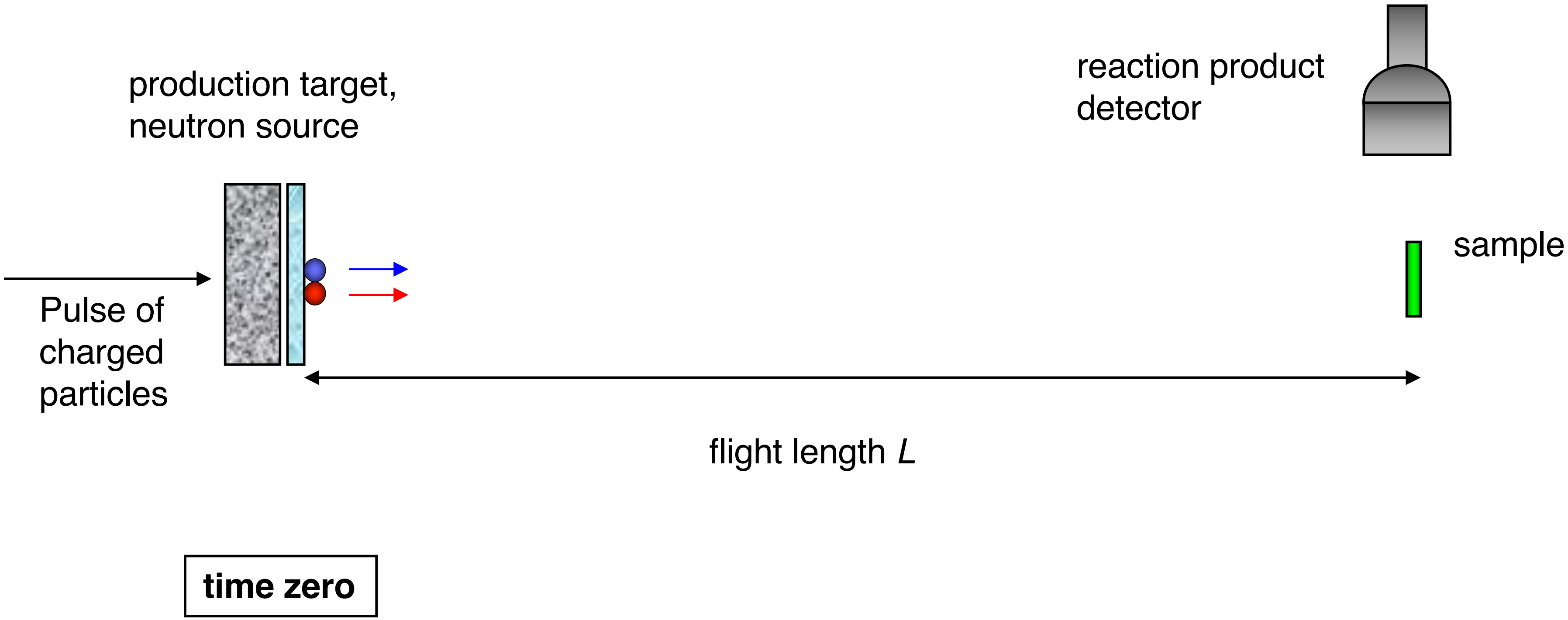
Measured reaction yield



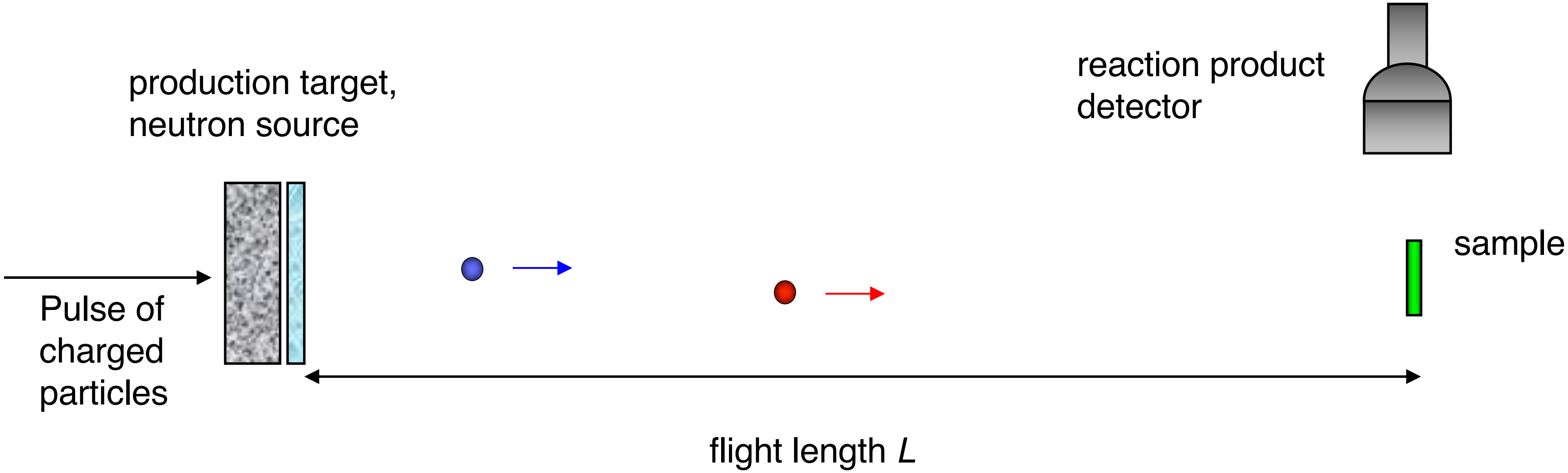
Broadening components in measurement



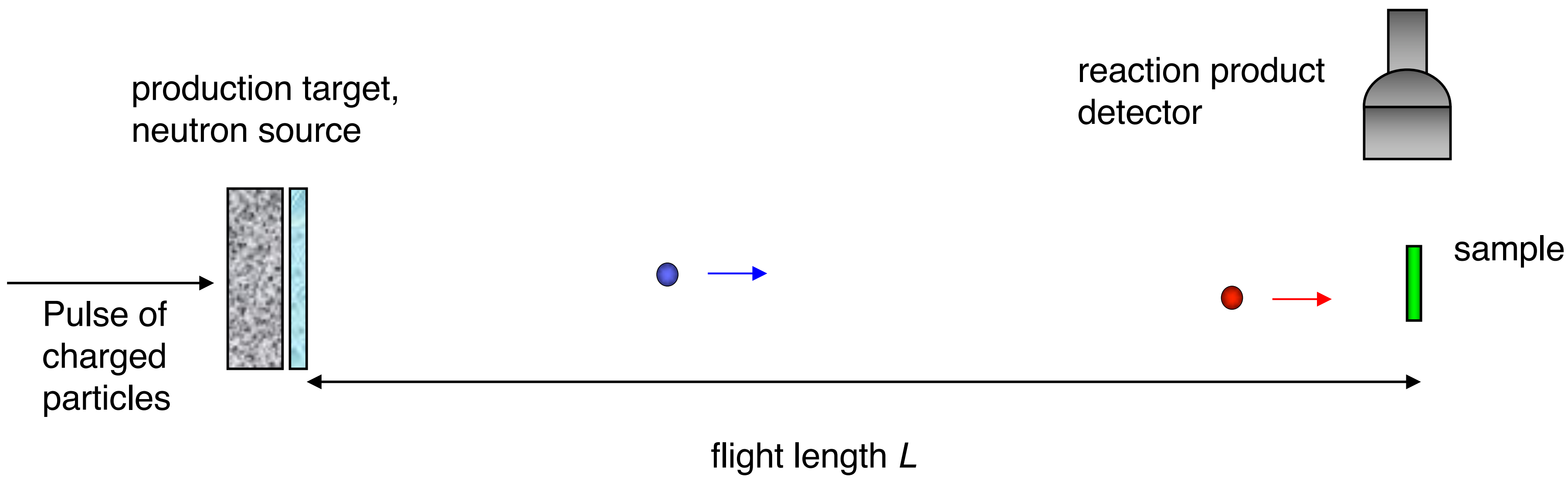
Neutron time-of-flight



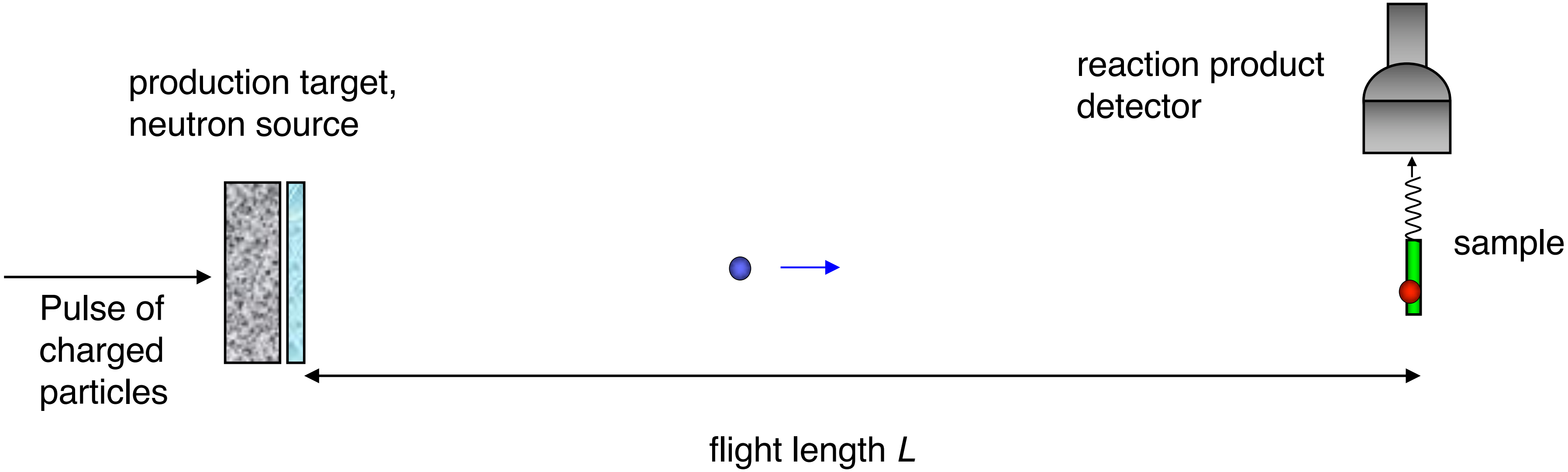
Neutron time-of-flight



Neutron time-of-flight

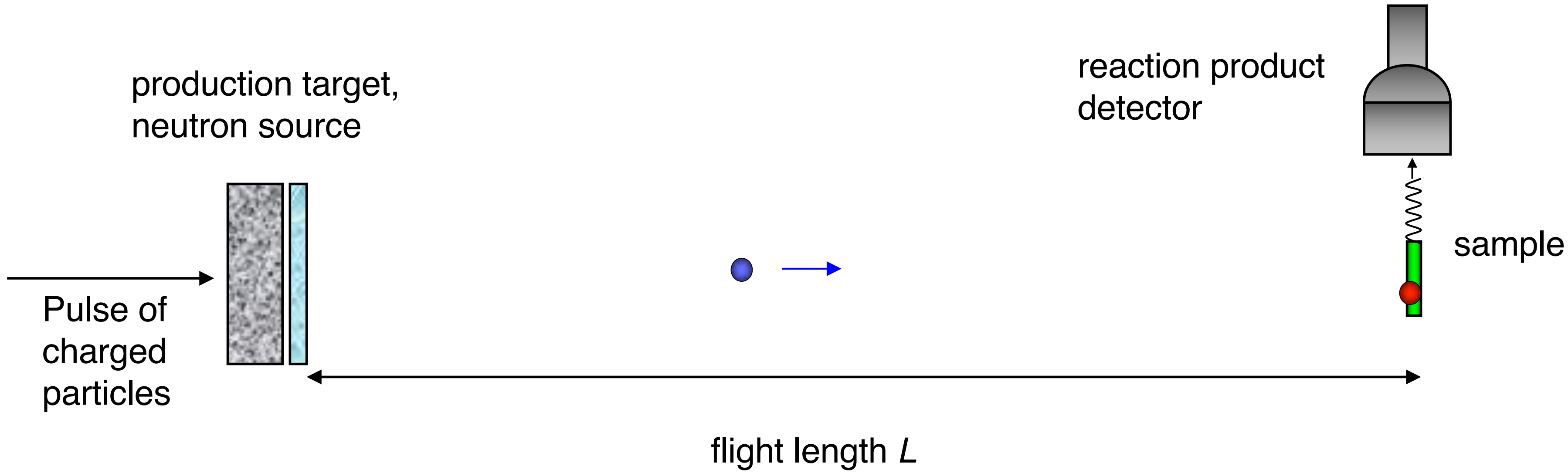


Neutron time-of-flight



time of flight *t*

Neutron time-of-flight



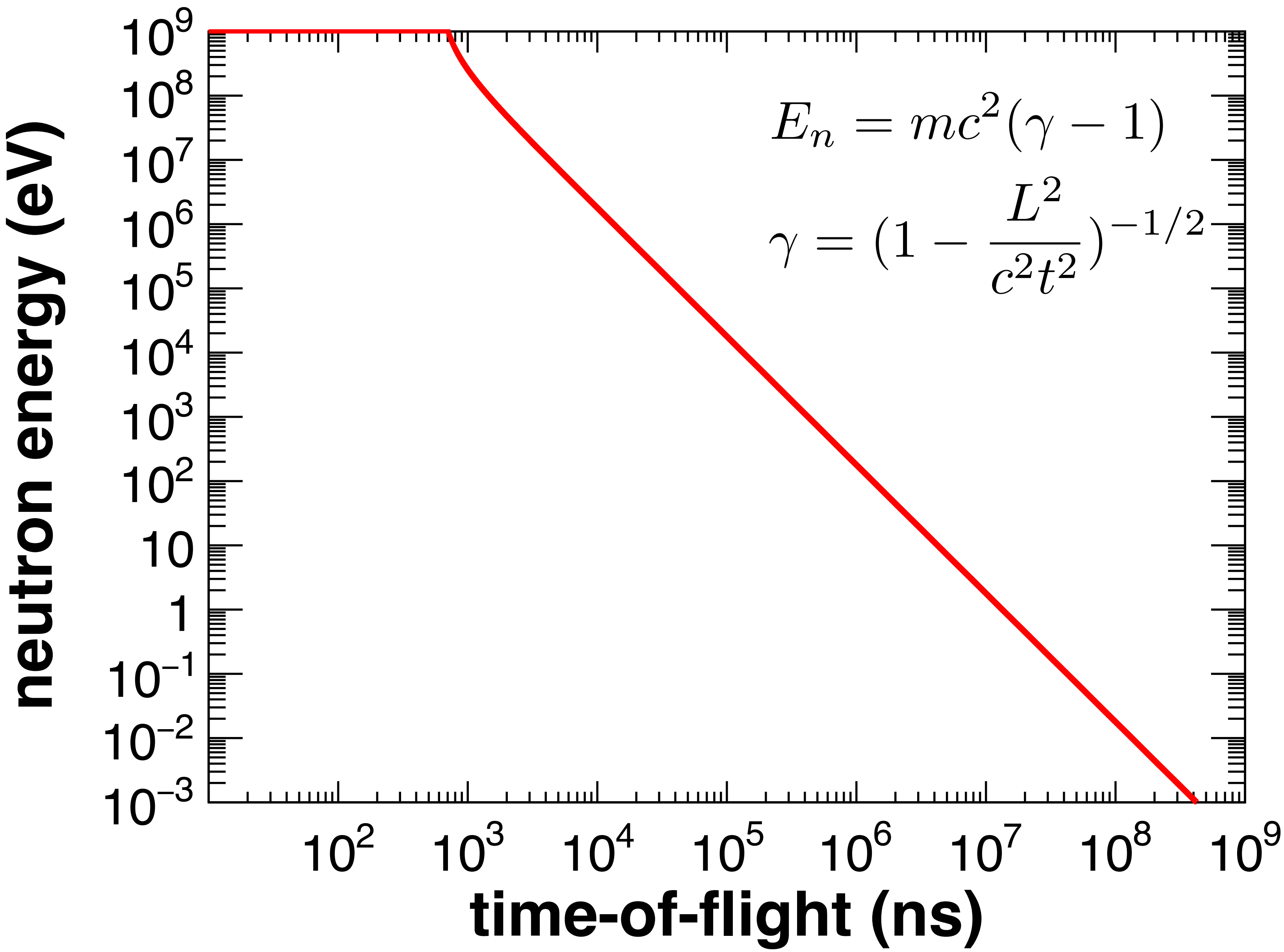
time of flight *t*

Deduce kinetic energy from neutron by time-of-flight:

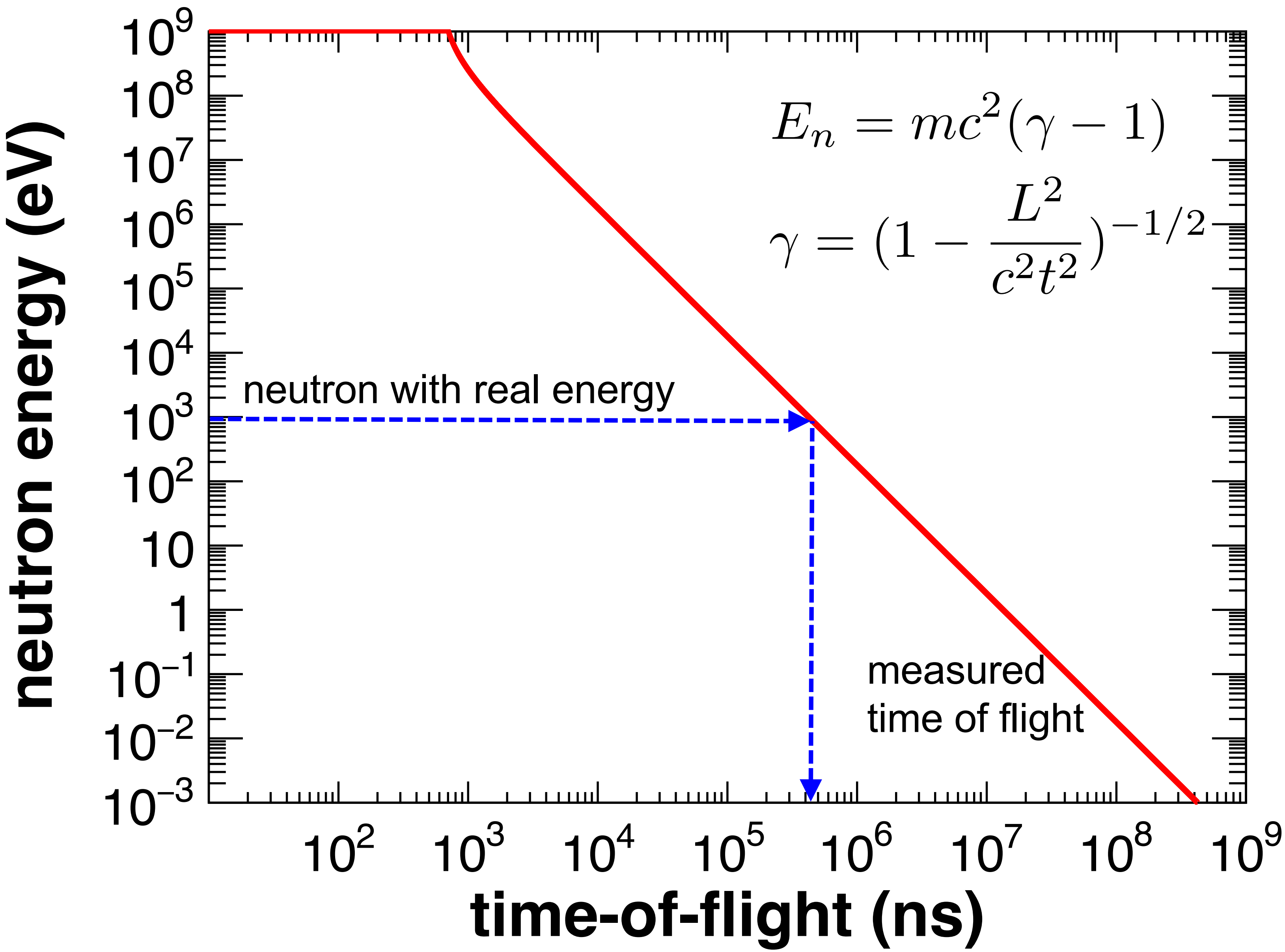
$$E_n = mc^2(\gamma - 1)$$

$$\gamma = (1 - \frac{L^2}{c^2t^2})^{-1/2}$$

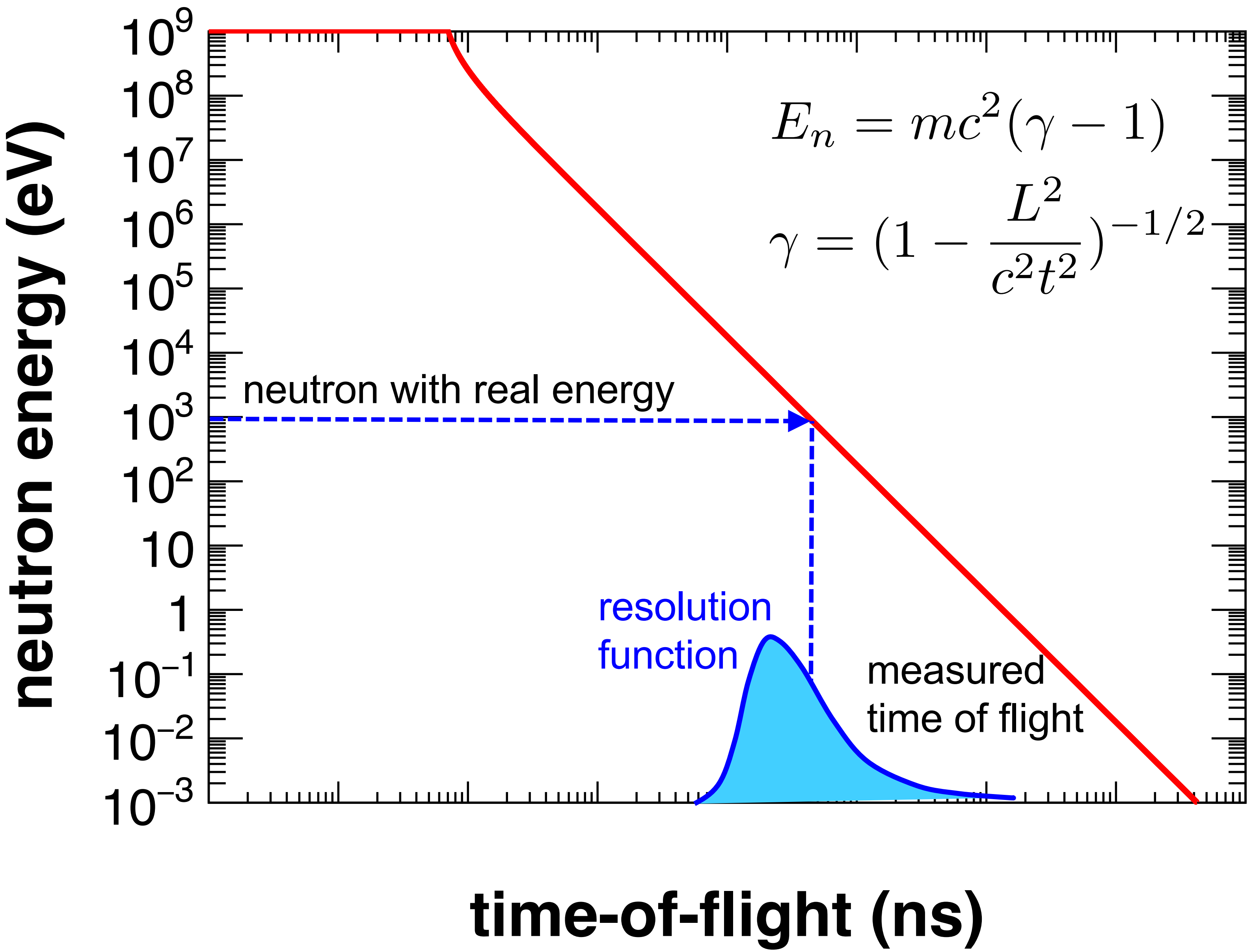
Neutron time-of-flight



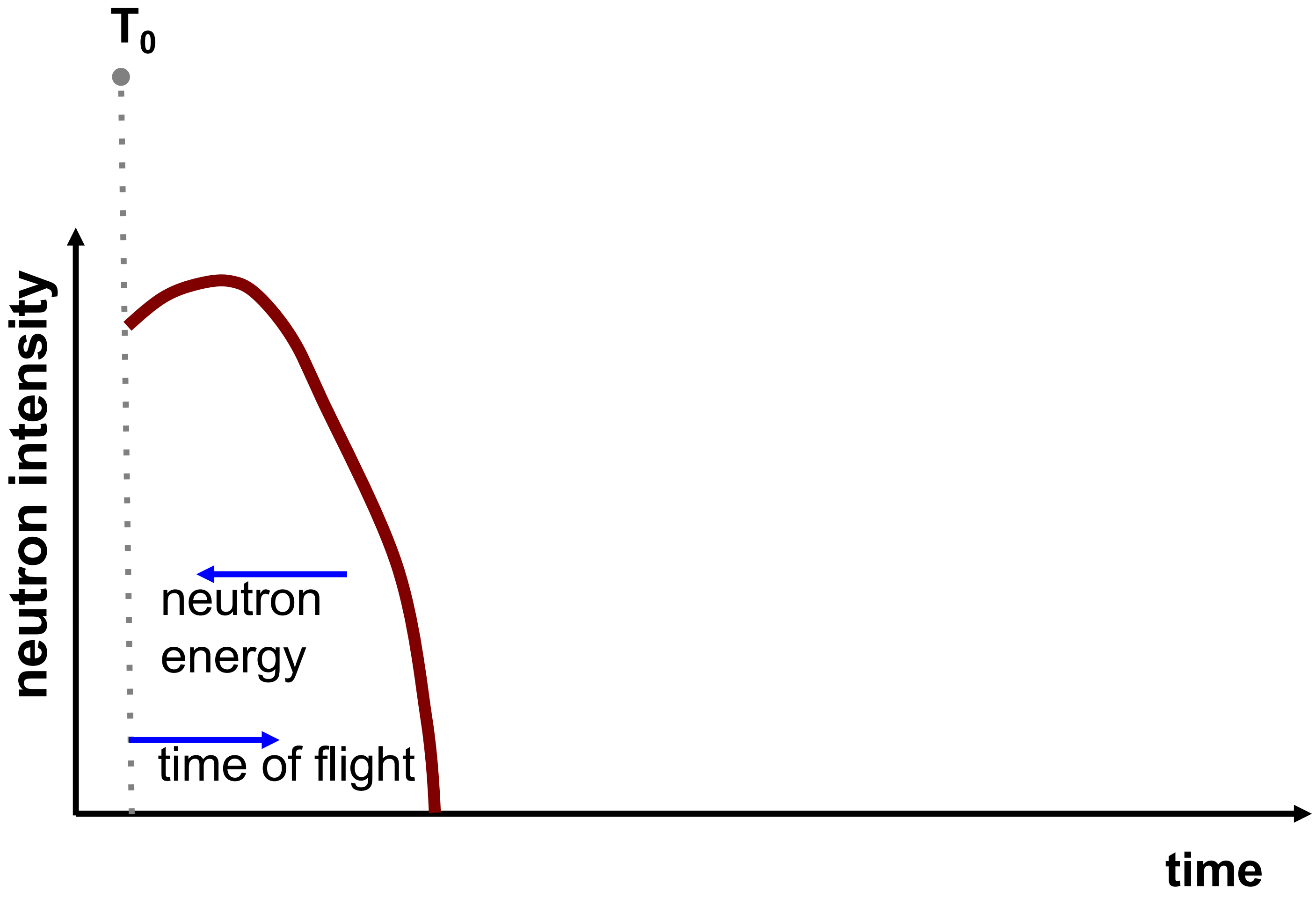
Neutron time-of-flight



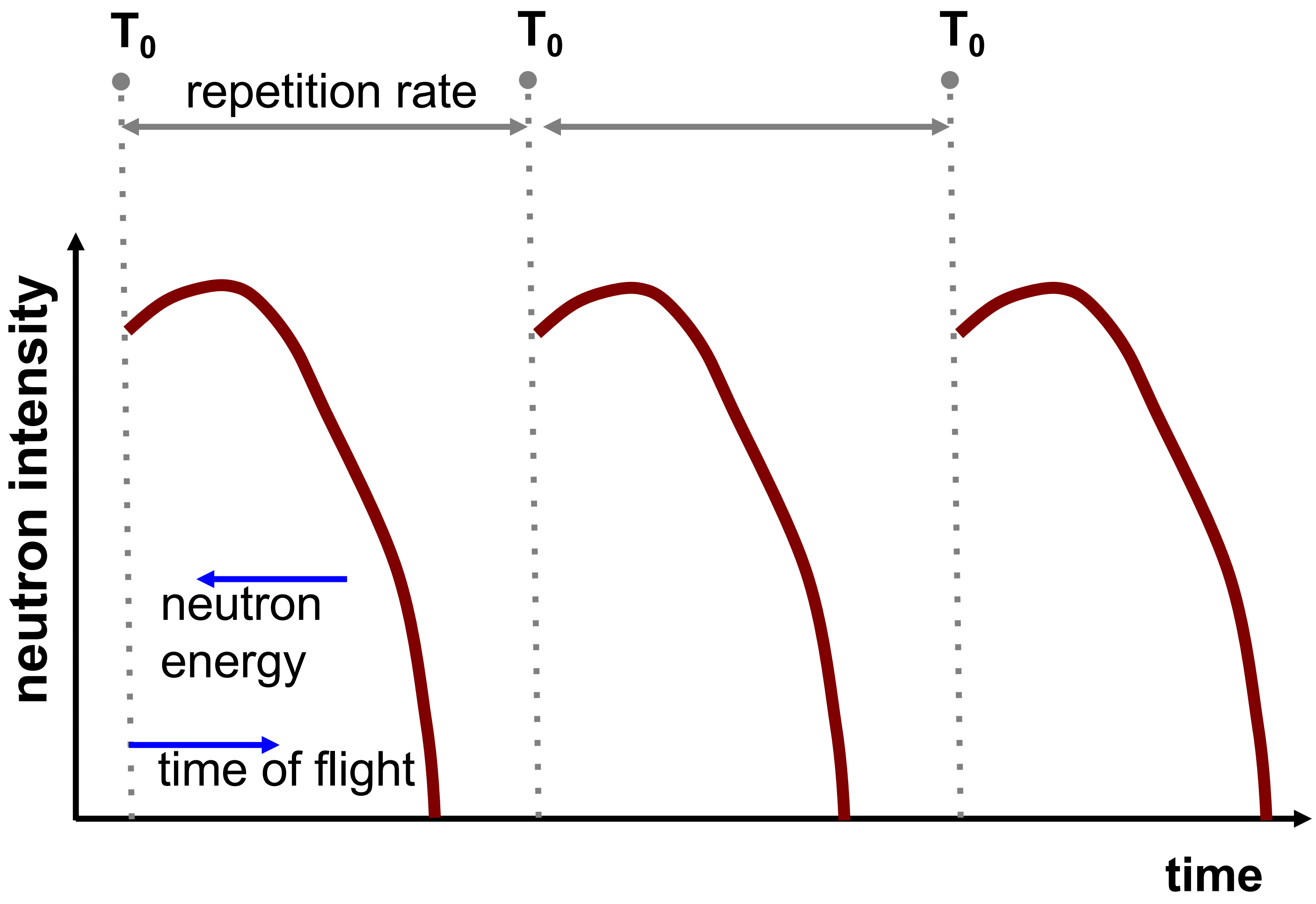
Neutron time-of-flight



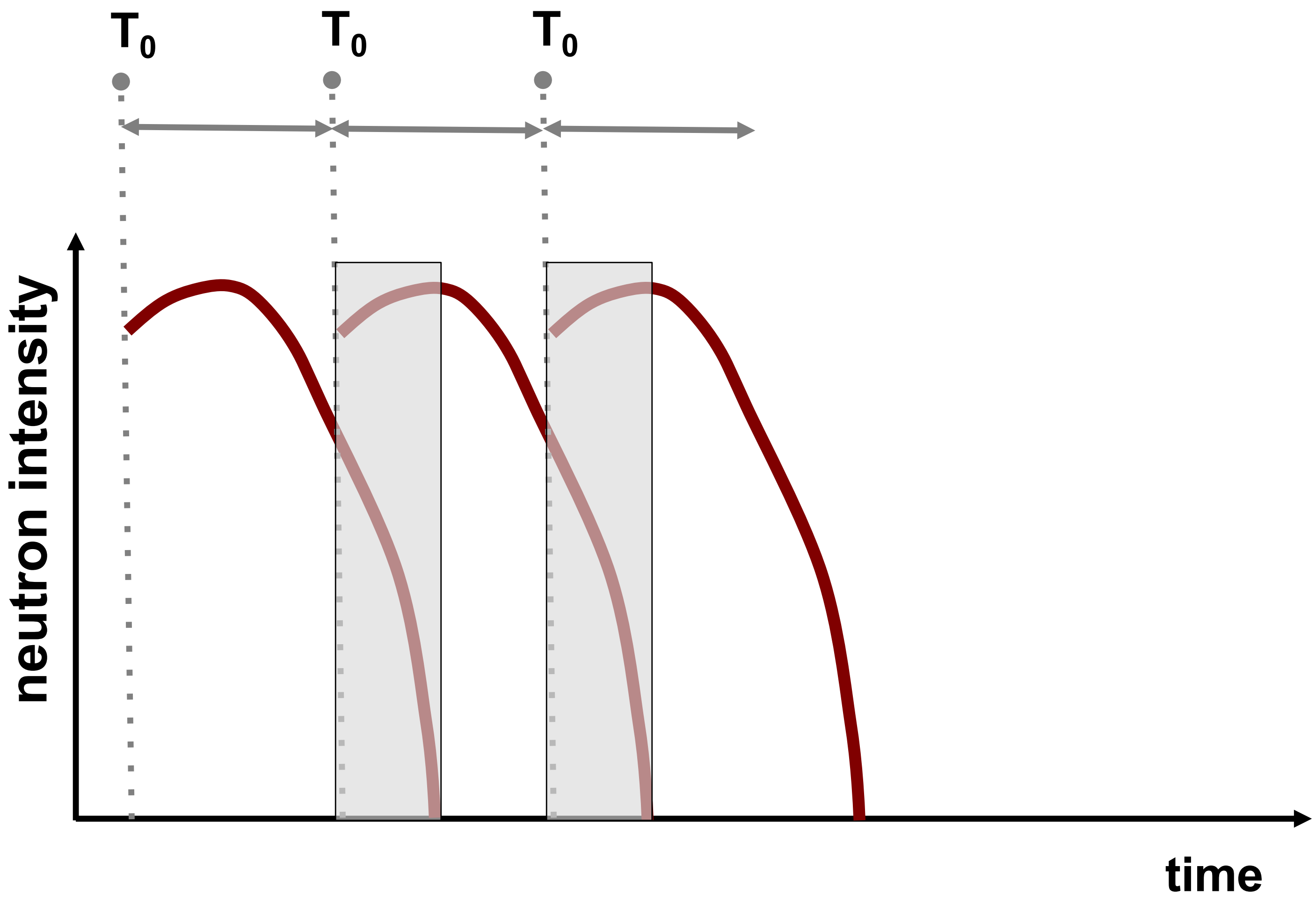
Neutron time-of-flight



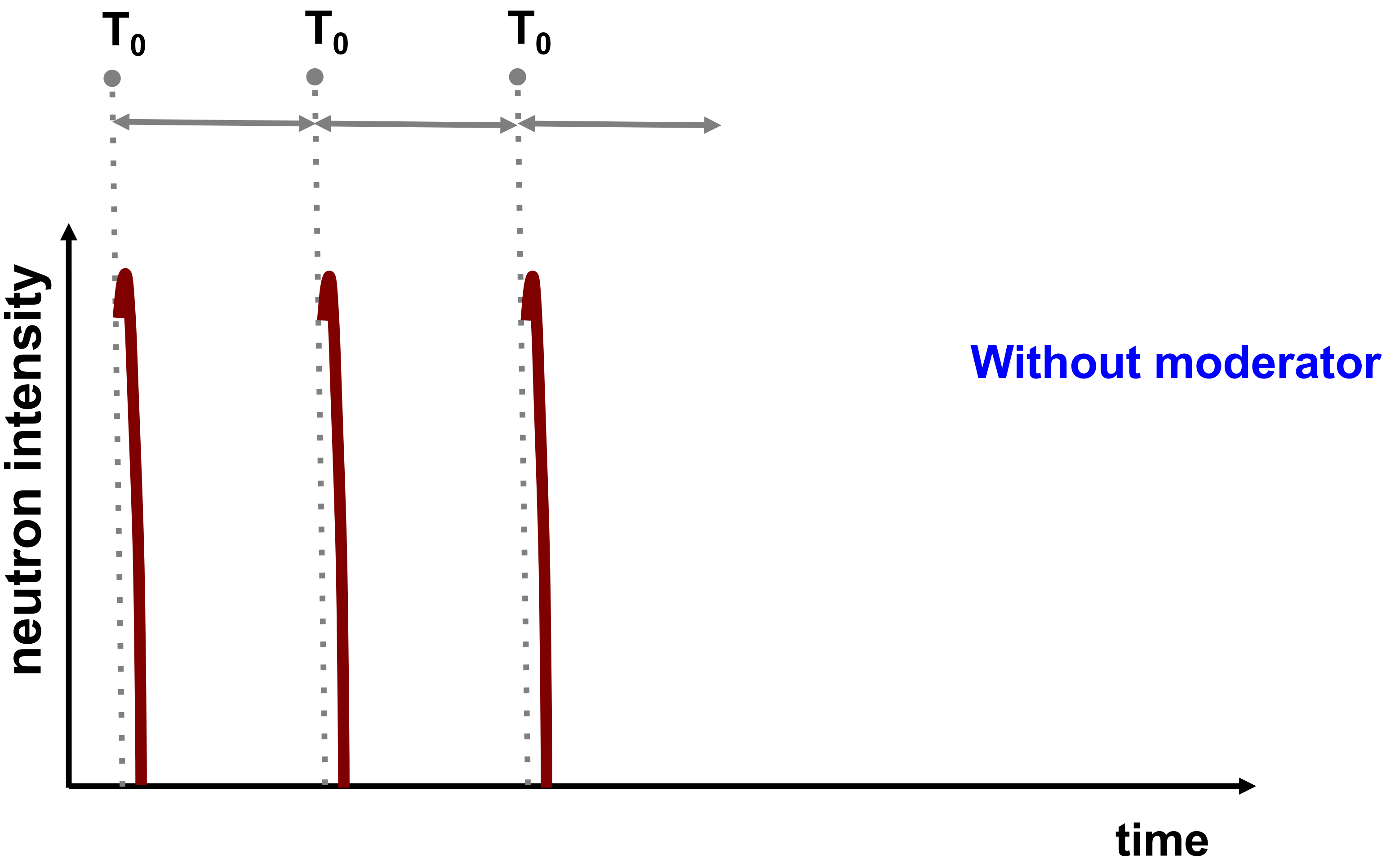
Neutron time-of-flight



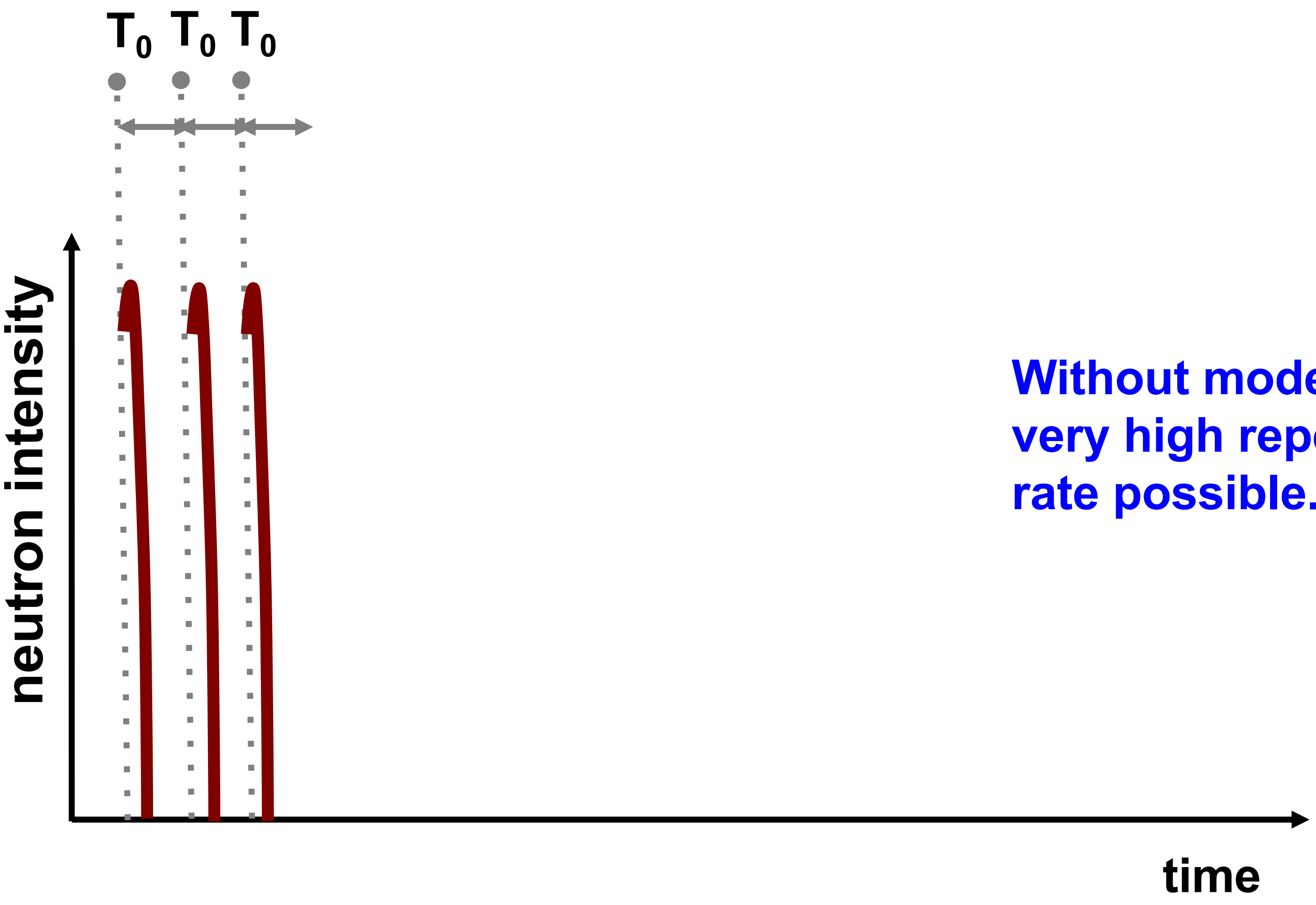
Neutron time-of-flight



Neutron time-of-flight



Neutron time-of-flight

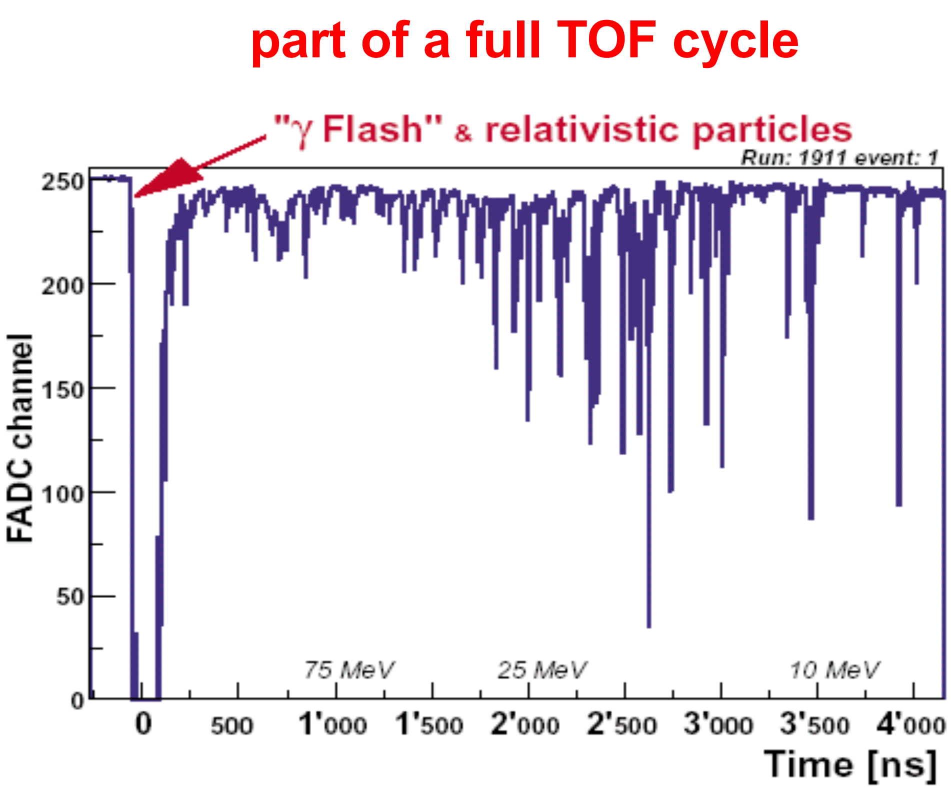


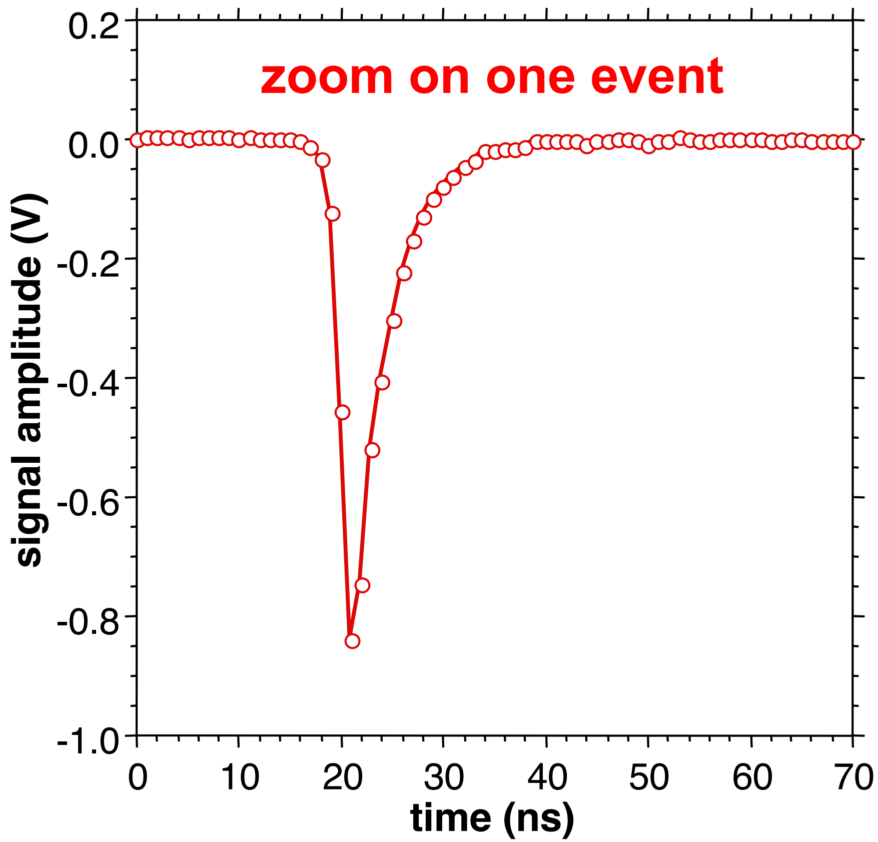
Without moderator, very high repetition rate possible.

Data acquisition

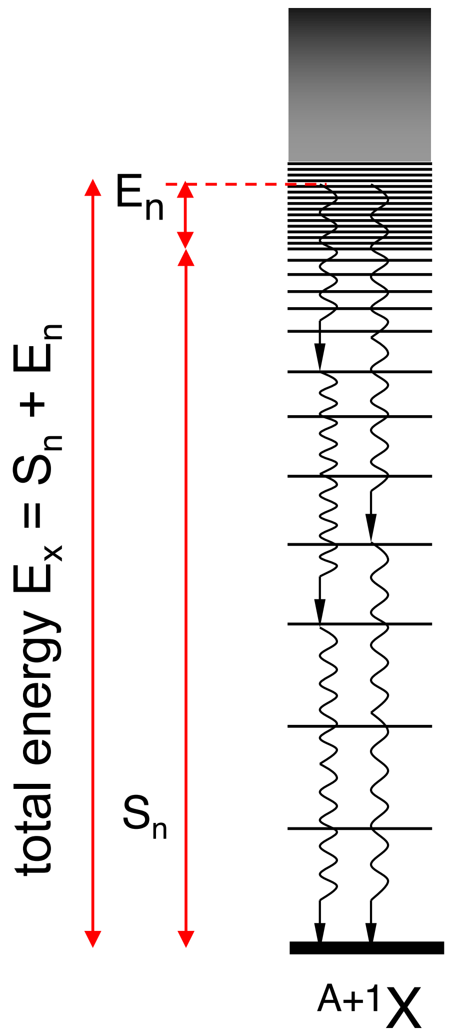
- Continuous sampling of detector output ("zero" deadtime) for each TOF cycle during ~**100 ms** with sampling interval of **1 ns.** Zero suppression.

- Offline event construction from timing and pulse height analysis, sometimes pulse shape analysis (**PSA**) for particle identification

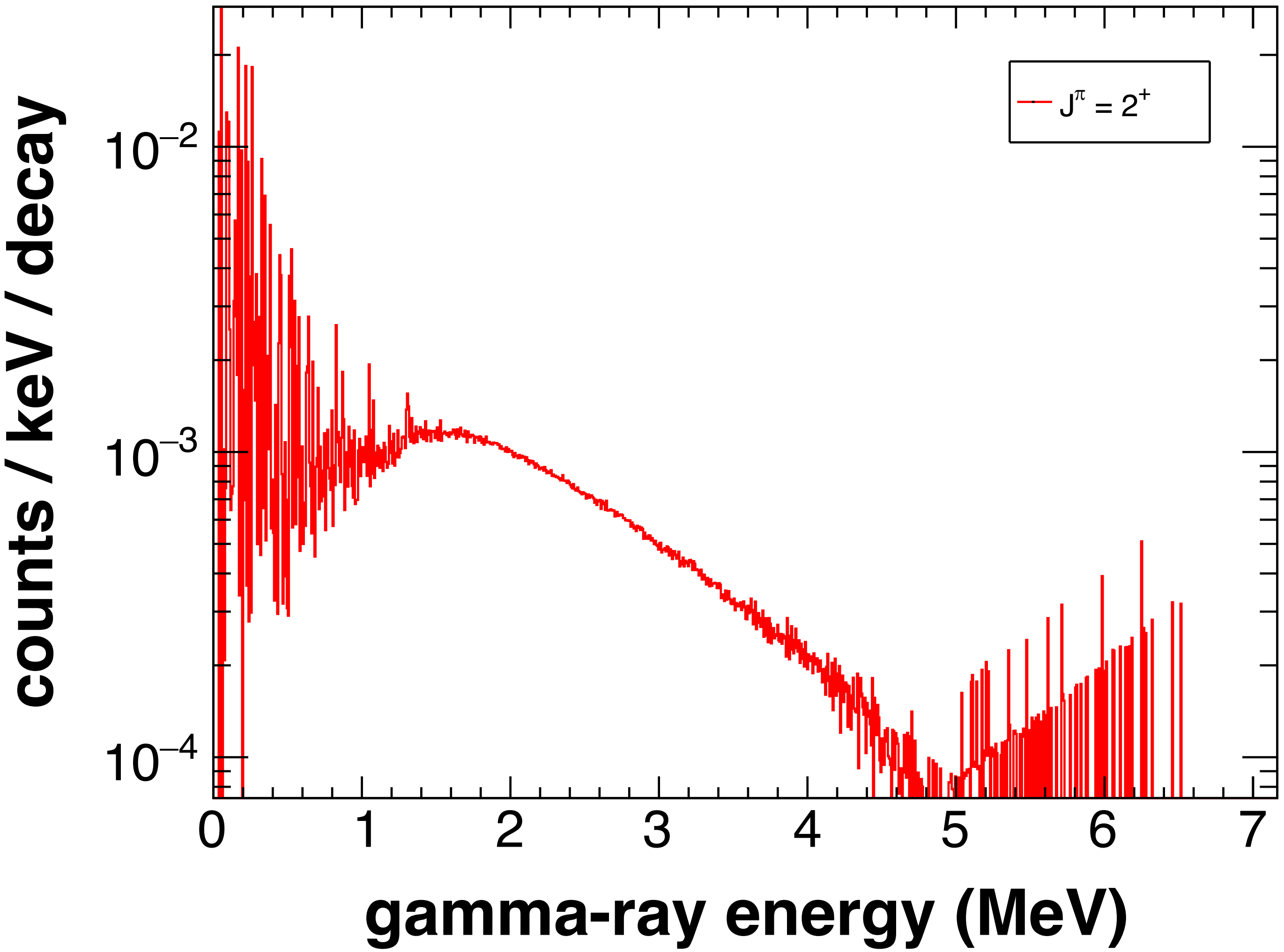




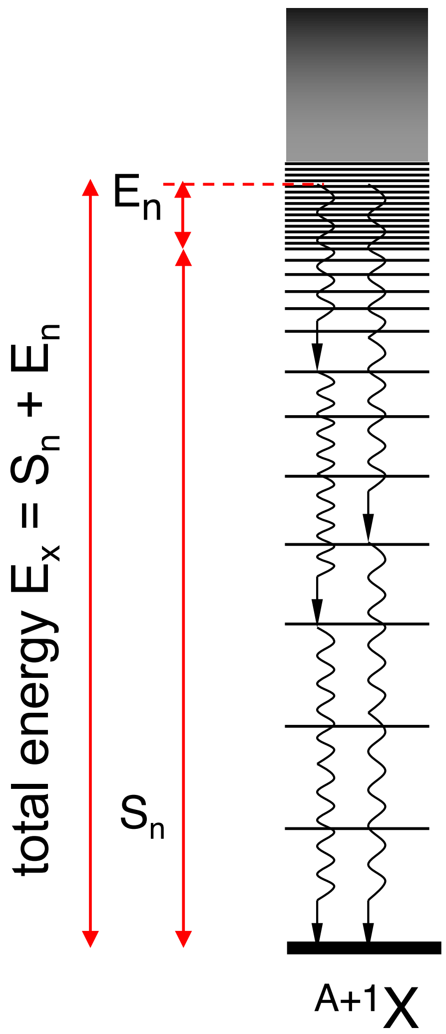
The neutron capture detection



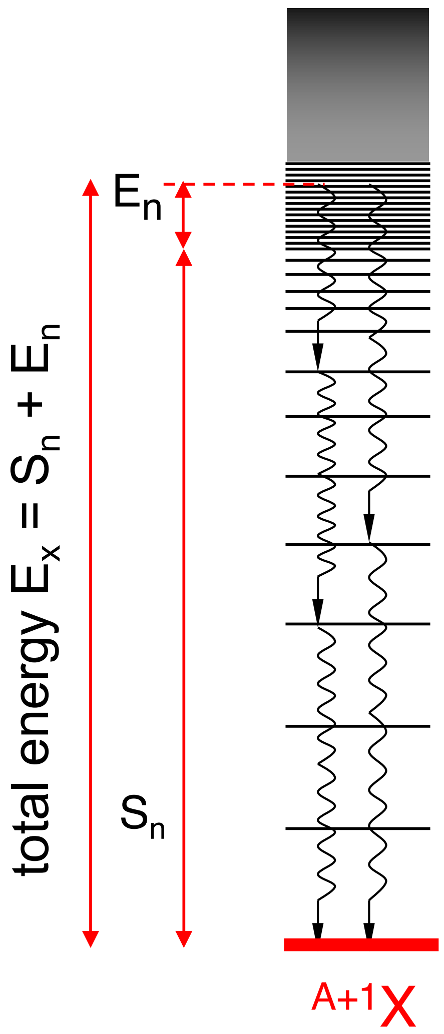
Simulated neutron $^{197}Au(n,\gamma)$ spectrum



Four methods to measure neutron capture:



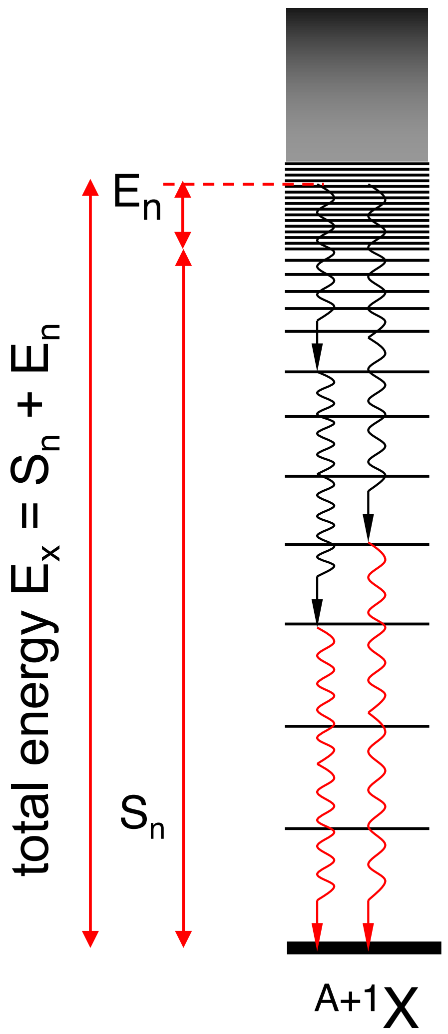
Measuring neutron capture



Four methods to measure neutron capture:

1. Activitation
   - no distinction of neutron energy
   - count produced nuclei, (mass) spectroscopy

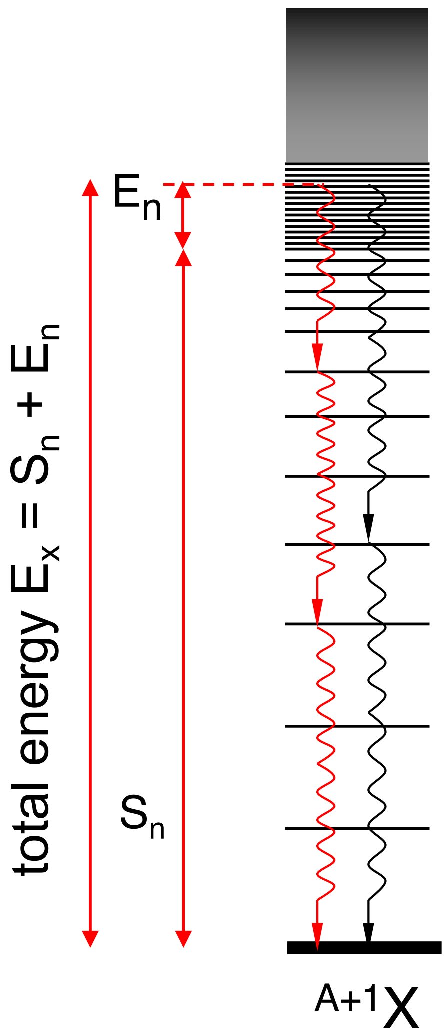
Measuring neutron capture



Four methods to measure neutron capture:

1. Activation
   - no distinction of neutron energy
   - count produced nuclei, (mass) spectroscopy
2. Level population spectroscopy
   - needs HPGe,
   - some nuclei only

Measuring neutron capture



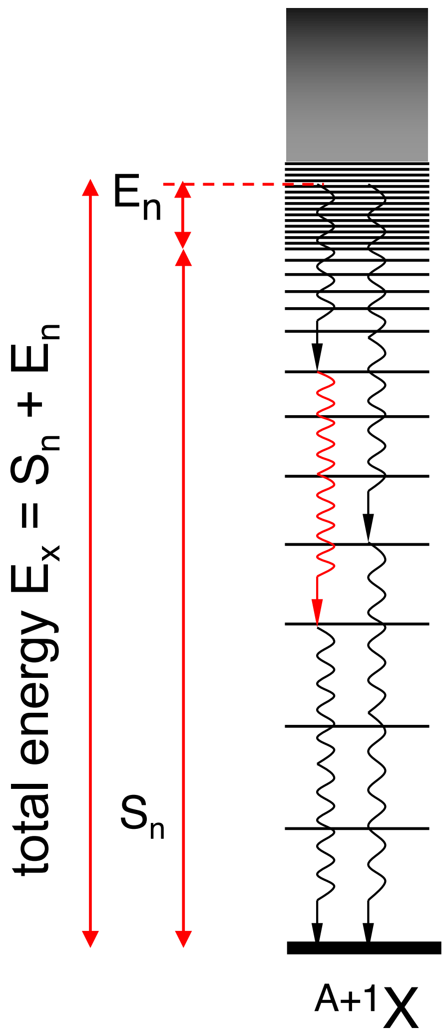
Four methods to measure neutron capture:

1. Activation
   - no distinction of neutron energy
   - count produced nuclei, (mass) spectroscopy

1. Level population spectroscopy
   - needs HPGe,
   - some nuclei only

2. Total absorption technique
   (TAC with for example BaF_2)

Measuring neutron capture



Four methods to measure neutron capture:

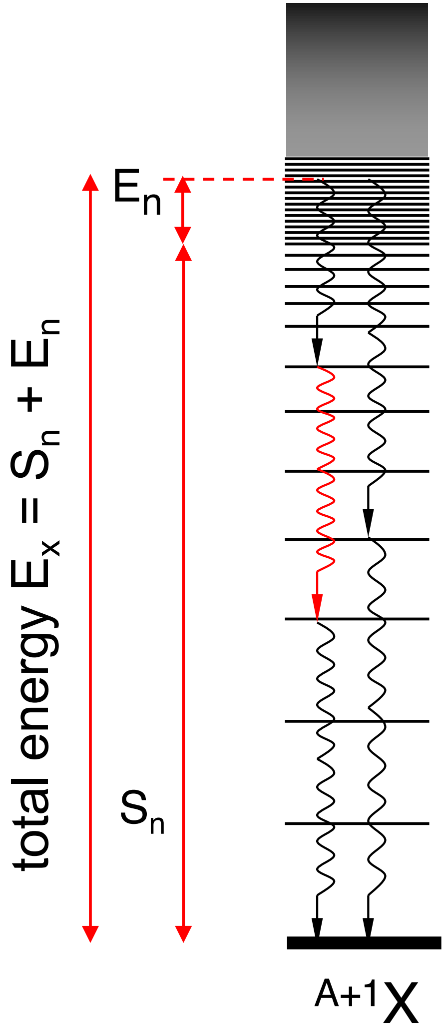
1. Activation
   - no distinction of neutron energy
   - count produced nuclei, (mass) spectroscopy

2. Level population spectroscopy
   - needs HPGe,
   - some nuclei only

3. Total absorption technique
   (TAC with for example BaF_2)

4. Total energy technique
   efficiency proportional to gamma-ray energy
   $$\varepsilon_\gamma = k \cdot E_\gamma$$
   - Moxon-Ray detectors
   - Use Weighting Function

The neutron capture reaction



Four methods to measure neutron capture:

1. Activation
   - no distinction of neutron energy
   - count produced nuclei, (mass) spectroscopy

1. Level population spectroscopy
   - needs HPGe,
   - some nuclei only

2. Total absorption technique
   (TAC with for example BaF_2)

3. Total energy technique
   efficiency proportional to gamma-ray energy
   $\varepsilon_\gamma = k \cdot E_\gamma$
   - Moxon-Ray detectors
   - Use Weighting Function

Neutron sources

- Nuclear fission reactors. Water-moderated beams.

- Accelerator-based sources (for example p + ^{7}Li or d + ^{9}Be), can be mono-energetic.

- pulsed white neutron sources
  - electron-based machines with heavy target
    Bremsstrahlung followed by (γ,n) and (γ,f)
  - proton-based machines with heavy target
    spallation reactions

Neutron sources

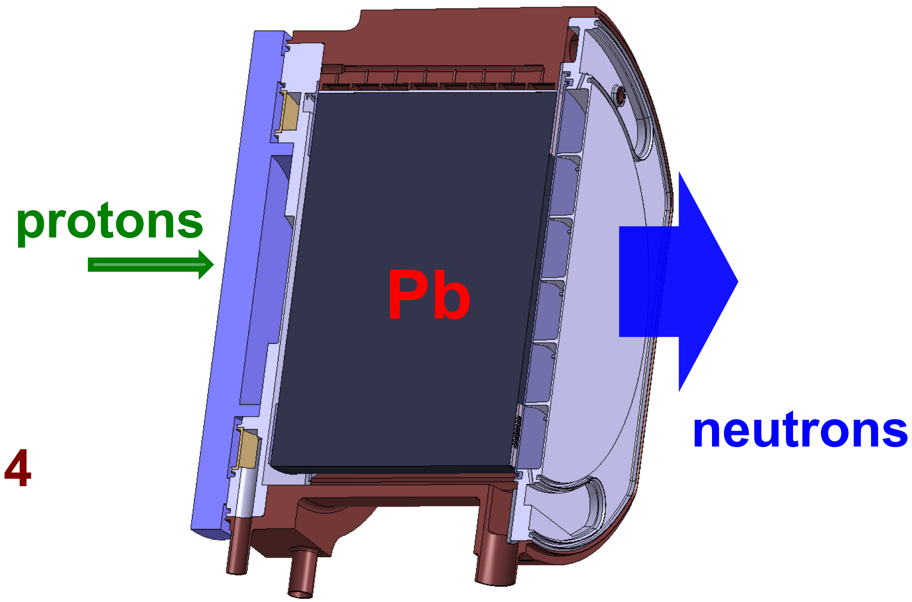
Facility	Location	particle	beam energy (MeV)	neutron target	pulse width (ns)	beam power (kW)	pulse frequency (Hz)
RPI	RPI, Troy, USA	e-	60	Ta	5	0.45	500
		e-	60	Ta	5000	>10	300
ORELA	ORNL, Oak Ridge, USA	e-	180	Ta	2–30	60	12–1000
GELINA	JRC-Geel, Belgium	e-	100	U	1	10	40-800
nELBE	FZD, Rossendorf, Germany	e-	40	L-Pb	0.01	40	500000
IREN	JINR, Dubna, Russia	e-	30	W	100	0.42	50
PNF	PAL, Pohang, Korea	e-	75	Ta	2000	0.09	12
KURRI	Kumatori Japan	e-	46	Ta	2	0.046	300
		e-	30	Ta	4000	6	100
LANSCE-MLNSC	LANL, Los Alamos, USA	p	800	W	135	800	20
LANSCE-WNR	LANL, Los Alamos, USA	p	800	W	0.2	1.44	13900
n_TOF	CERN, Geneva, Switzerland	p	20000	Pb	6	10	0.4
MLF-NNRI	J-PARC, Tokai, Japan	p	3000	Hg	1000	1000	25
ISIS	Oxfordshire, United Kingdom	p	-	W			
ESS	Lund, Sweden	p	-	W			
CSNS	Dongguan, Guangdong, China	p	1600	W	500	120	25
NFS	GANIL-SPIRAL2, Caen, France	d	40	Be	<0.5	2	150k-880k

Neutron sources

Facility	Location	particle	beam energy (MeV)	neutron target	pulse width (ns)	beam power (kW)	pulse frequency (Hz)
RPI	RPI, Troy, USA	e-	60	Ta	5	0.45	500
		e-	60	Ta	5000	>10	300
ORELA	ORNL, Oak Ridge, USA	e-	180	Ta	2–30	60	12–1000
GELINA	JRC-Geel, Belgium	e-	100	U	1	10	40-800
nELBE	FZD, Rossendorf, Germany	e-	40	L-Pb	0.01	40	500000
IREN	JINR, Dubna, Russia	e-	30	W	100	0.42	50
PNF	PAL, Pohang, Korea	e-	75	Ta	2000	0.09	12
KURRI	Kumatori Japan	e-	46	Ta	2	0.046	300
		e-	30	Ta	4000	6	100
LANSCE-MLNSC	LANL, Los Alamos, USA	p	800	W	135	800	20
LANSCE-WNR	LANL, Los Alamos, USA	p	800	W	0.2	1.44	13900
n_TOF	CERN, Geneva, Switzerland	p	20000	Pb	6	10	0.4
MLF-NNRI	J-PARC, Tokai, Japan	p	3000	Hg	1000	1000	25
ISIS	Oxfordshire, United Kingdom	p	-	W			
ESS	Lund, Sweden	p	-	W			
CSNS	Dongguan, Guangdong, China	p	1600	W	500	120	25
NFS	GANIL-SPIRAL2, Caen, France	d	40	Be	<0.5	2	150k-880k

Pulsed white neutron source:

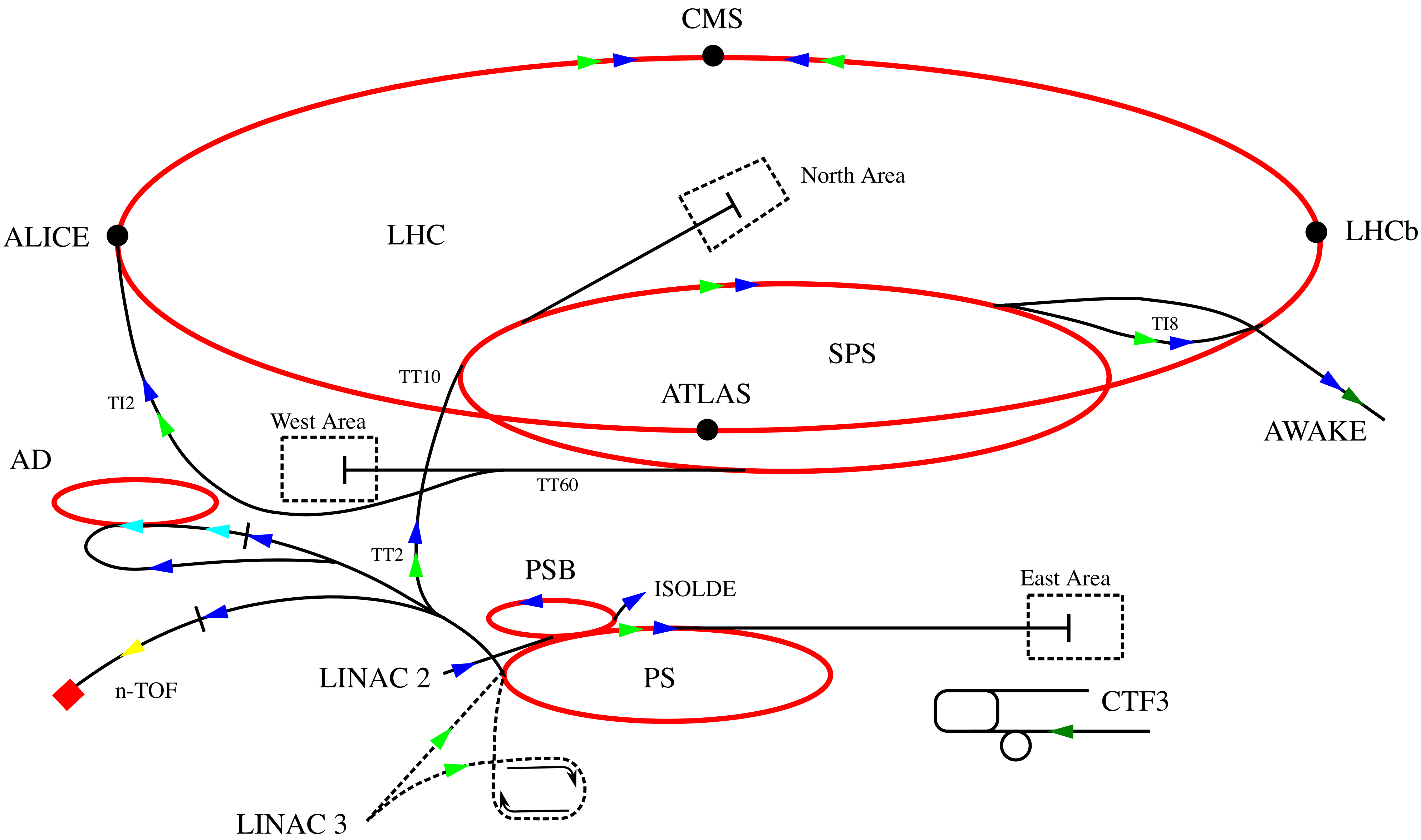
- 20 GeV/c protons
- neutrons from spallation
- 6 ns rms pulse width
- frequency 1 pulse/2.4 seconds
- separate cooling and moderation
- flight path length EAR1: 185 m, **since 2000**
- **flight path length EAR2: 20 m, since 2014**
- @source: $7\text{x}10^{12}$ protons/pulse
- @source: $2\text{x}10^{15}$ neutrons/pulse
- @EAR1: $5 \cdot 10^5$(capture) – $5 \cdot 10^7$(fission) neutrons/pulse



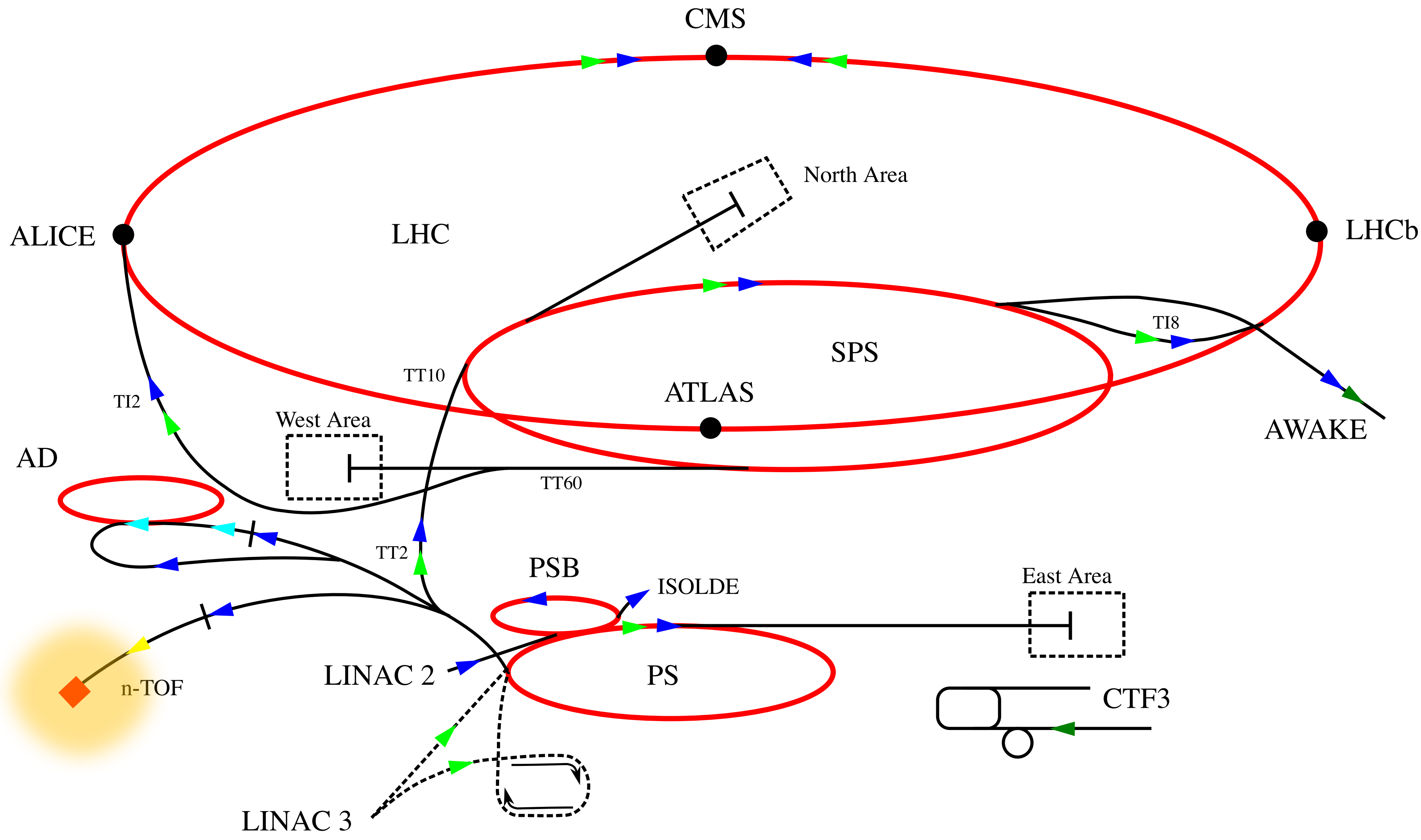
Main features:

- Large energy range in one experiment (0.01 eV – 1 GeV)
- Favorable signal to noise ratio for capture on radioactive isotopes (actinides, fission products)

CERN accelerators



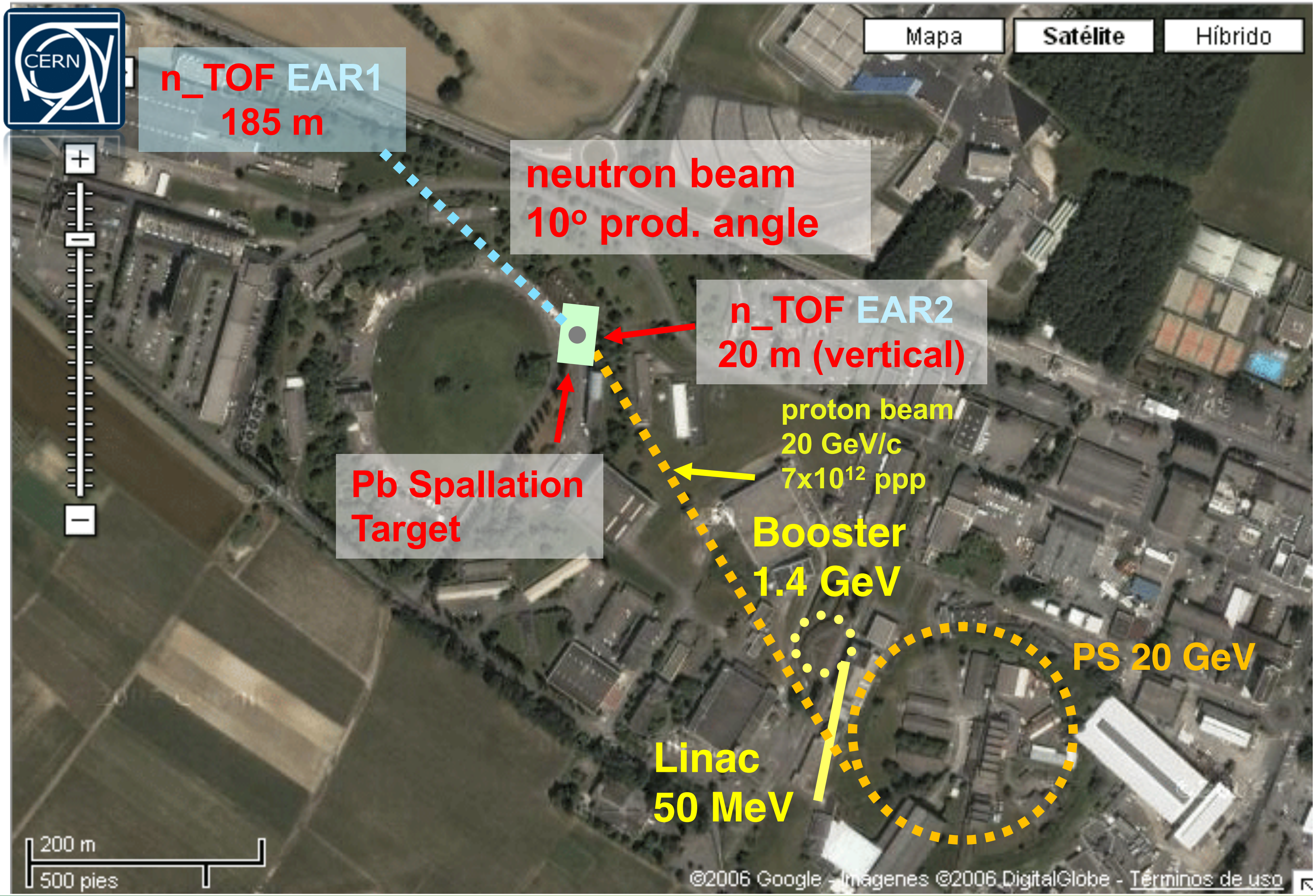
CERN accelerators



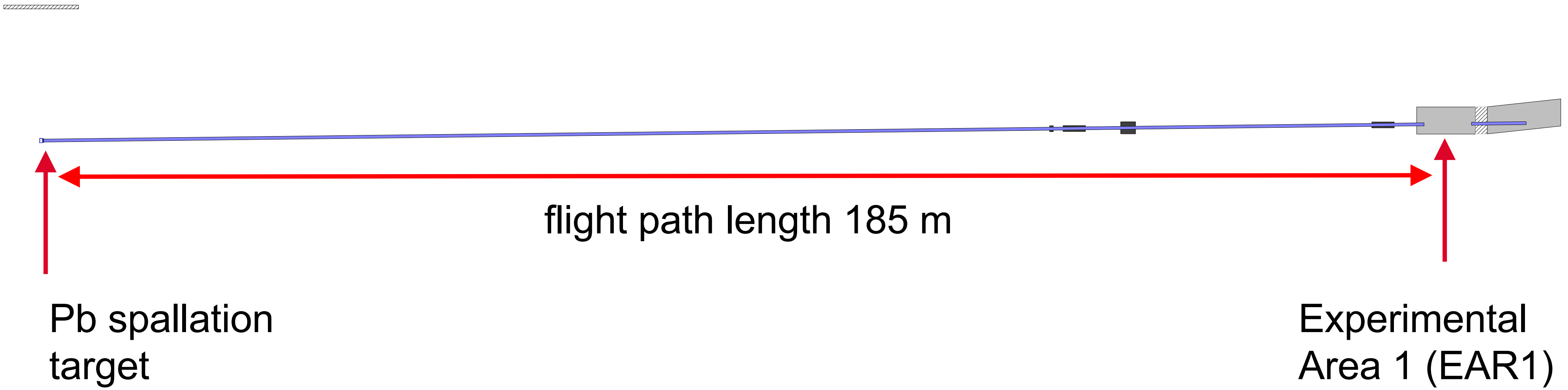
The n_TOF facility at CERN

The n_TOF facility at CERN

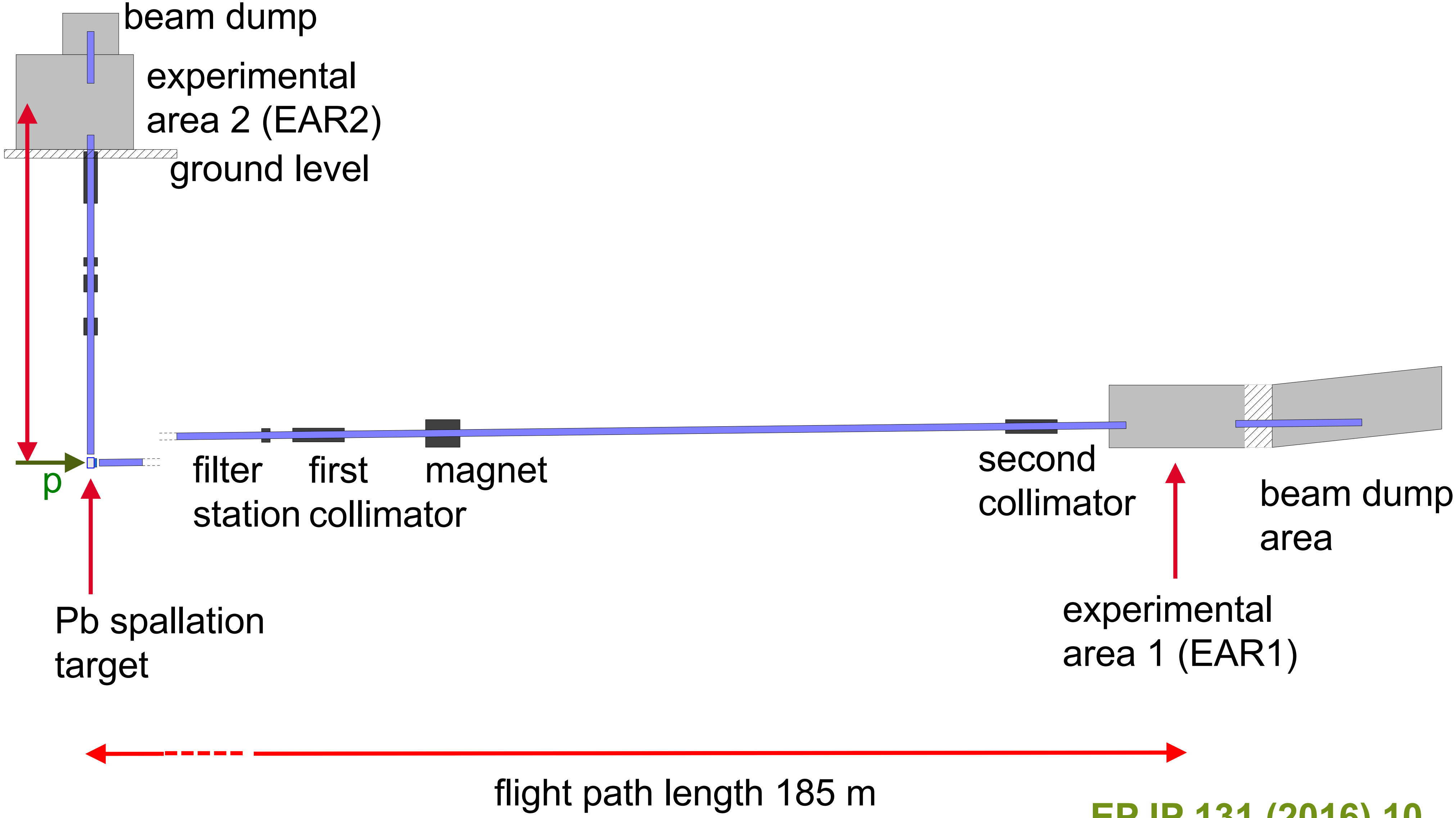
The n_TOF facility at CERN



The n_TOF facility

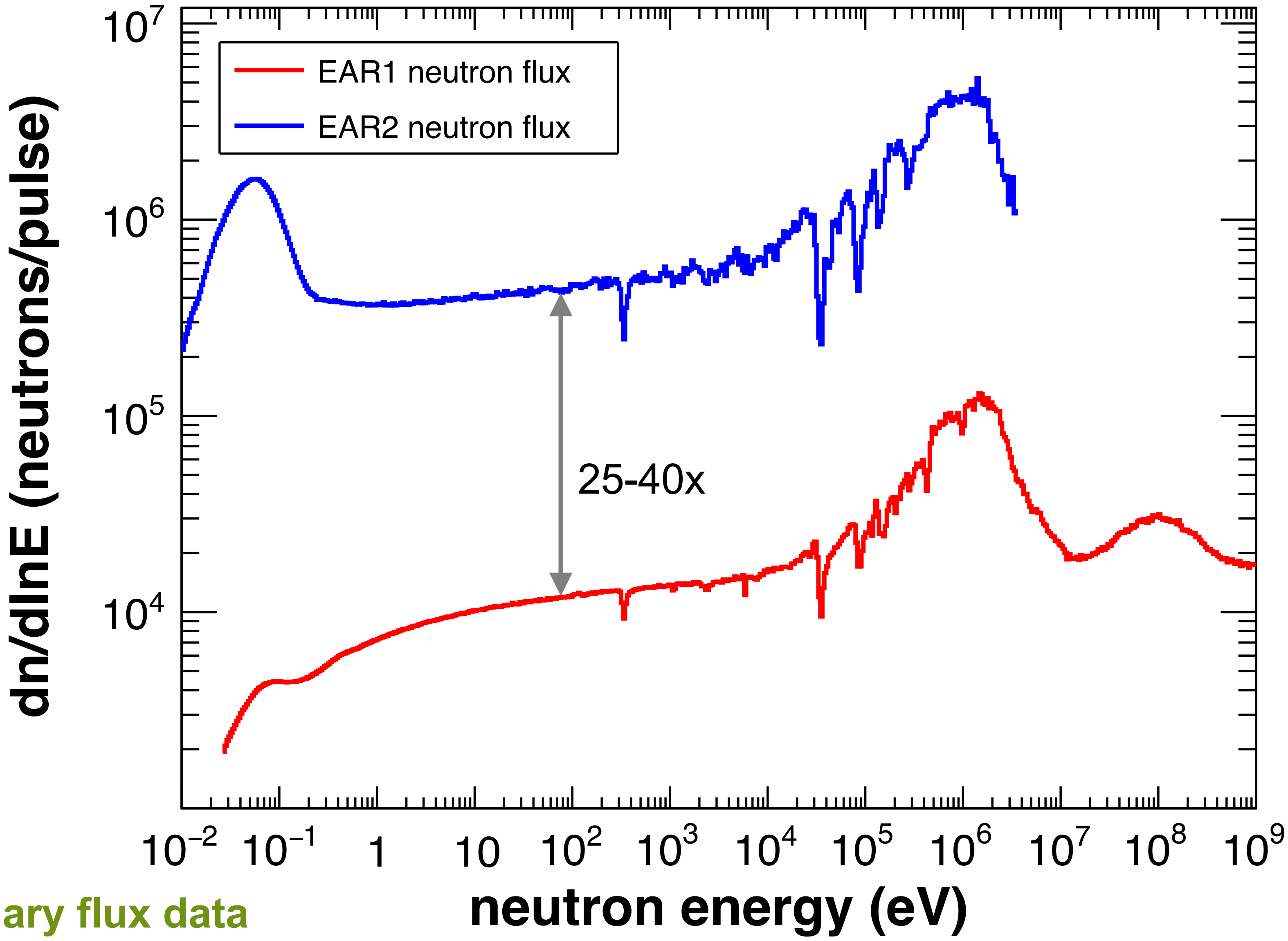


The n_TOF facility



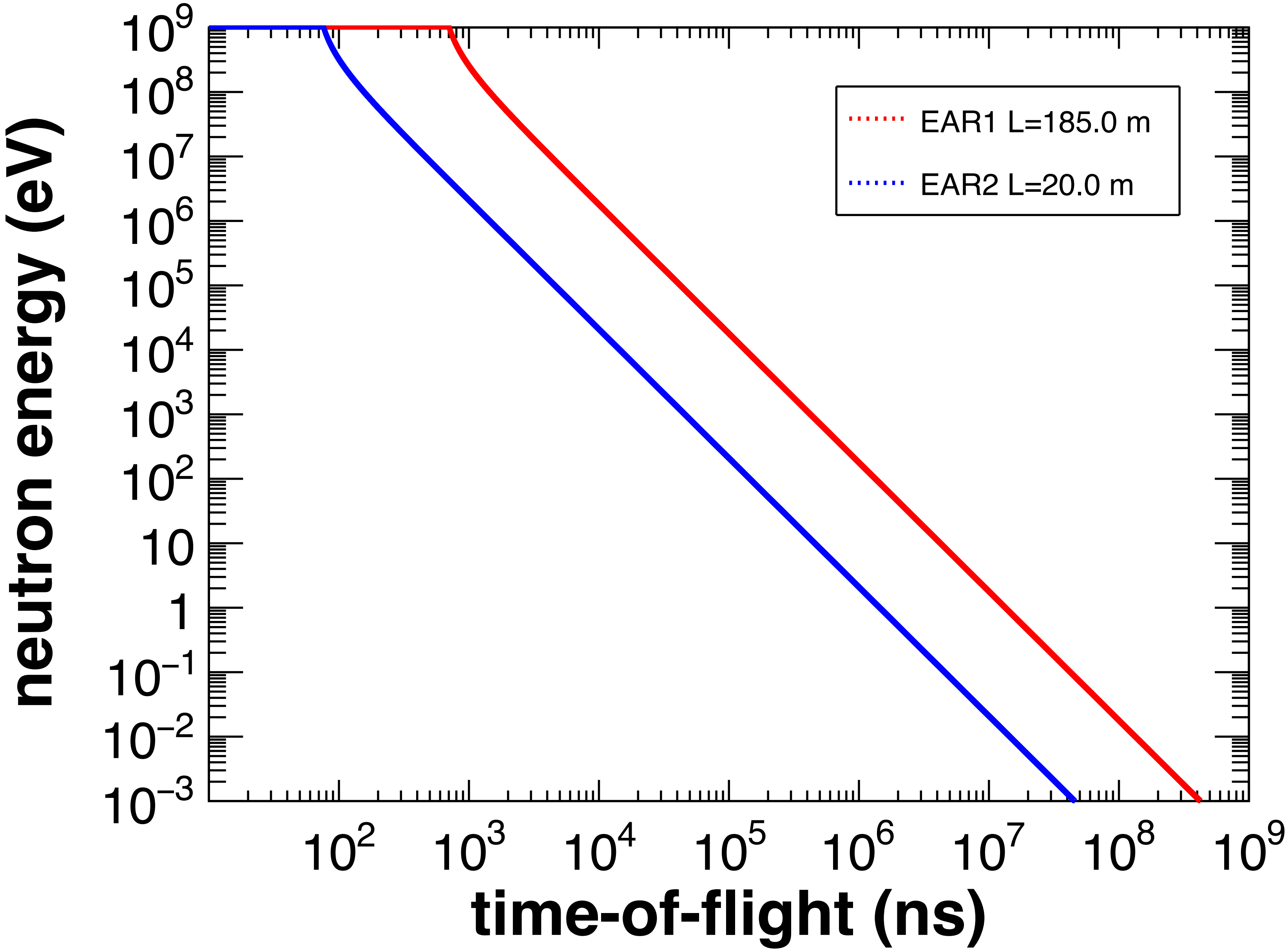
EPJP 131 (2016) 10

The n_TOF neutron spectrum EAR1 and EAR2

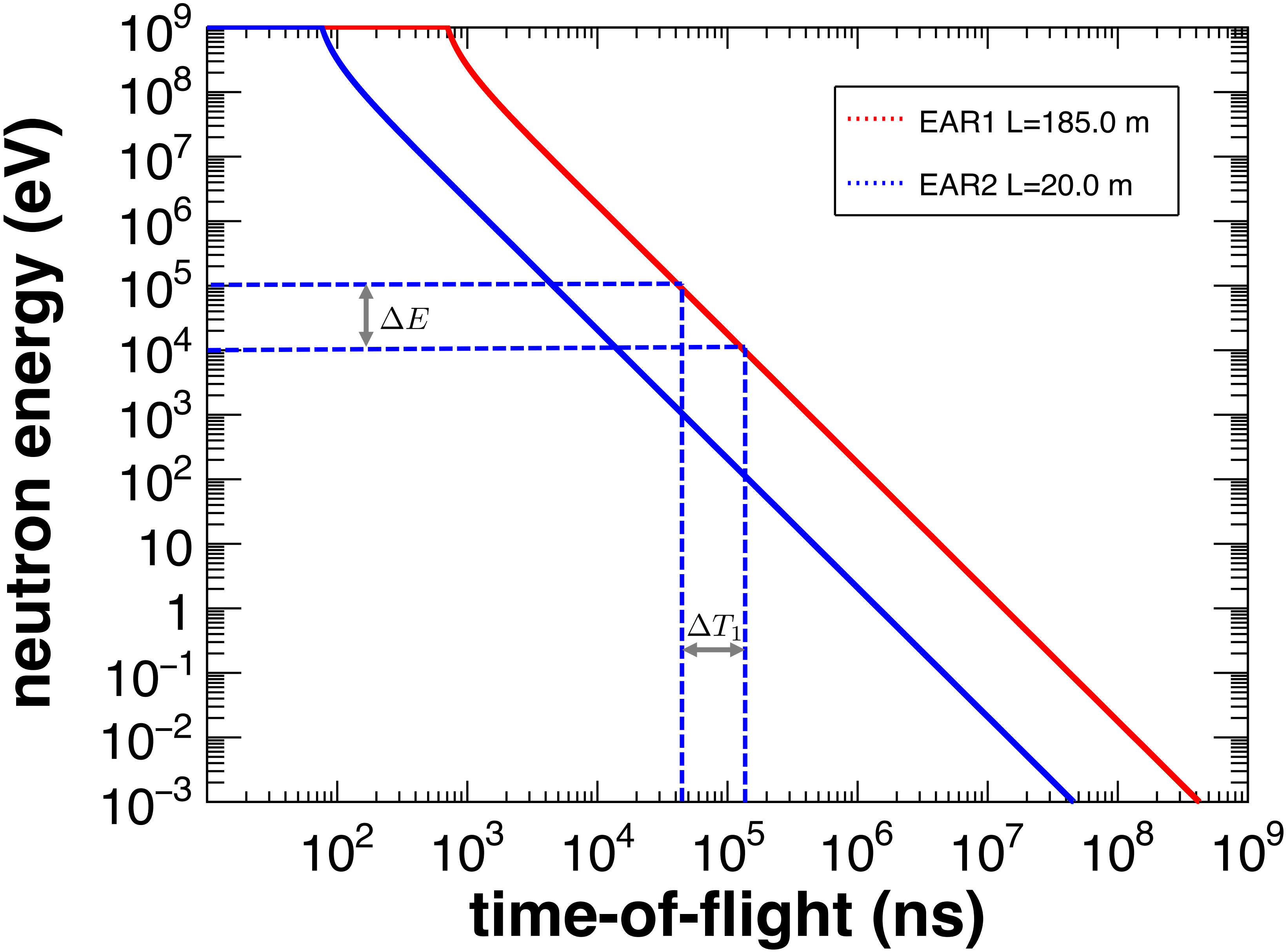


preliminary flux data

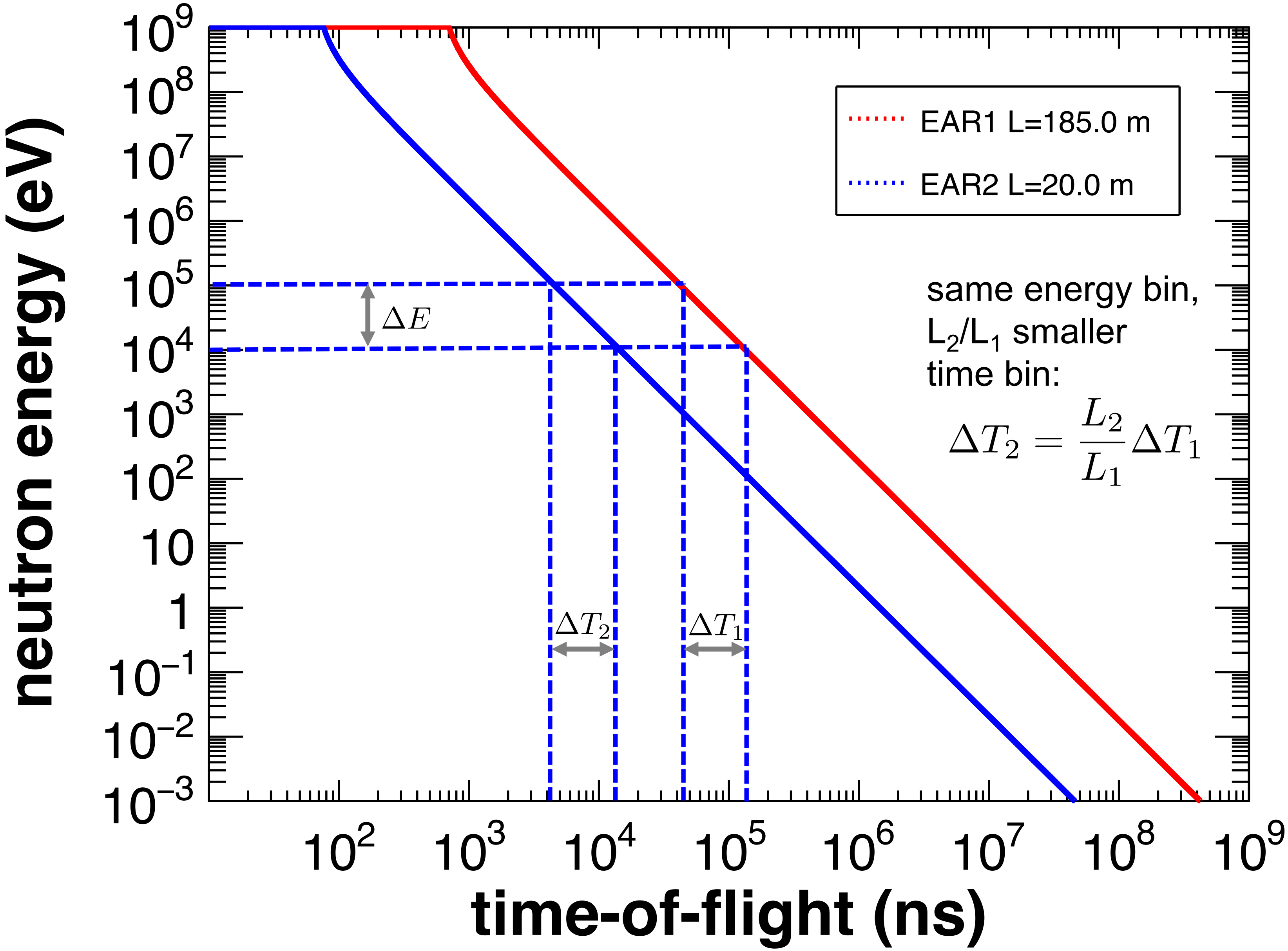
TOF-energy relation at n_TOF



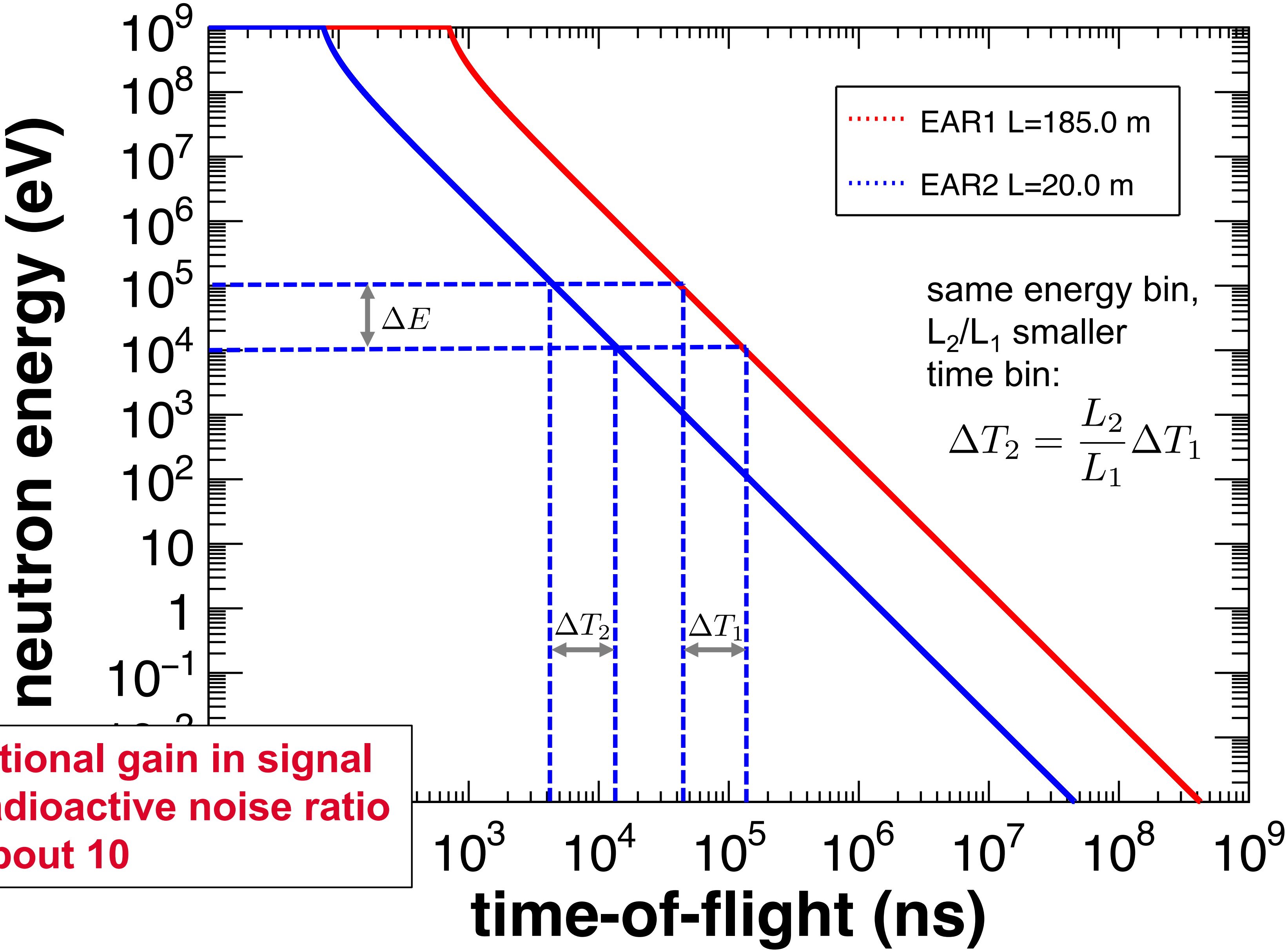
TOF-energy relation at n_TOF



TOF-energy relation at n_TOF



TOF-energy relation at n_TOF



same energy bin, L_2/L_1 smaller time bin:

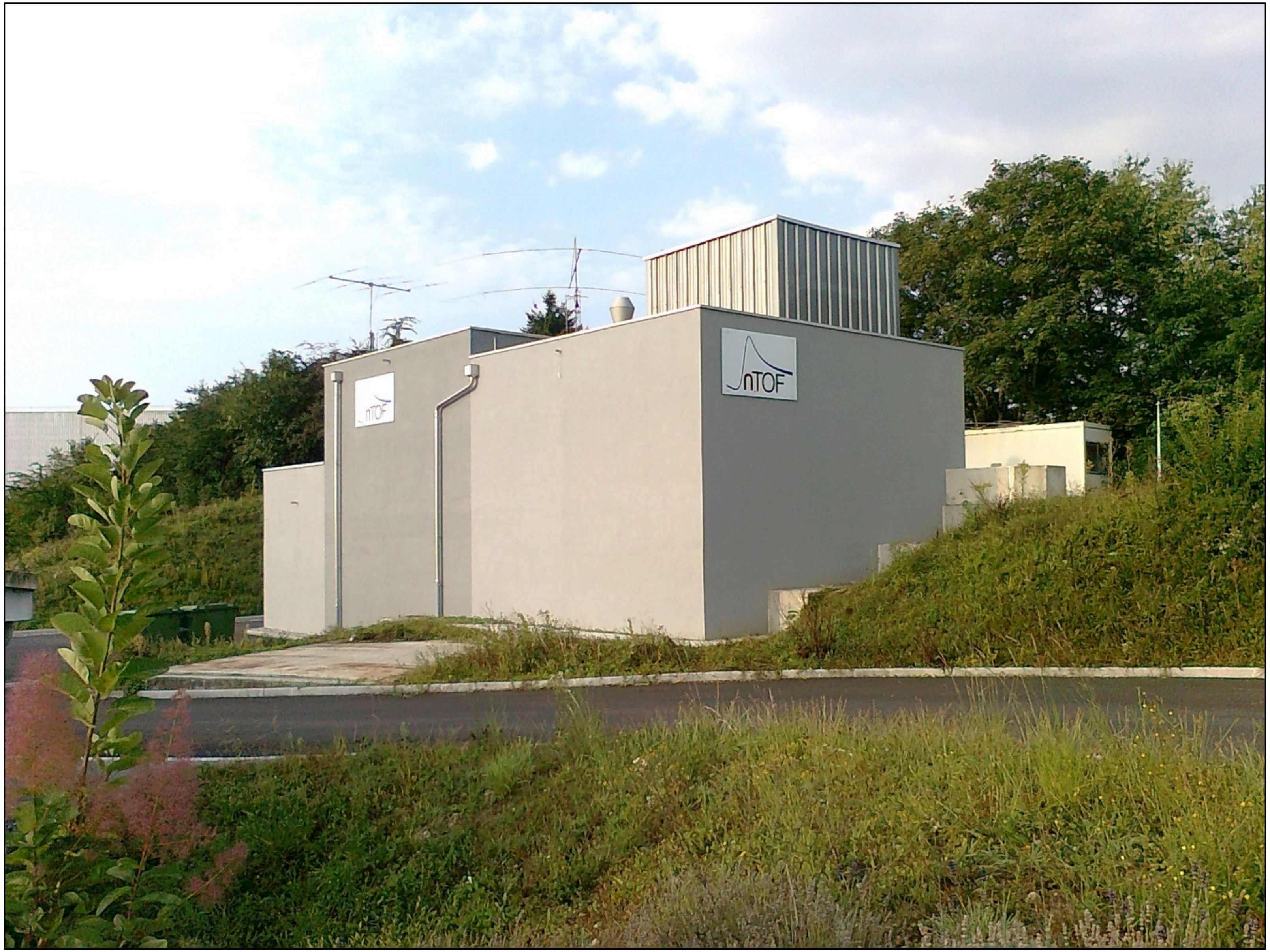
$$\Delta T_2 = \frac{L_2}{L_1} \Delta T_1$$

additional gain in signal to radioactive noise ratio of about 10

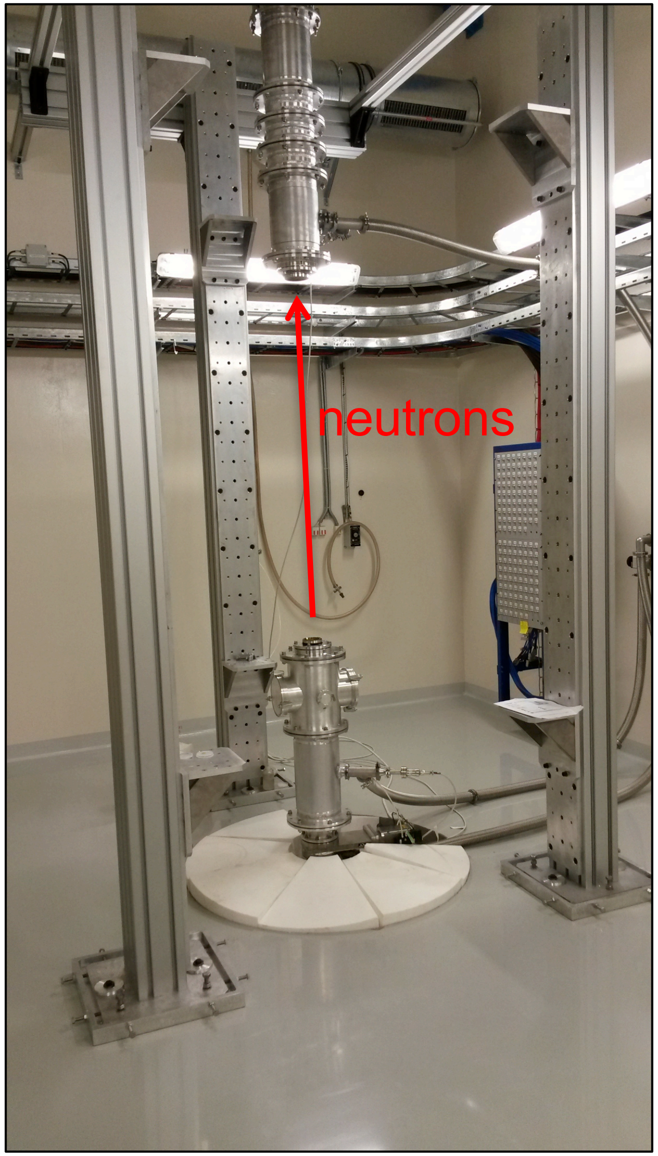
n_TOF EAR2, constructing

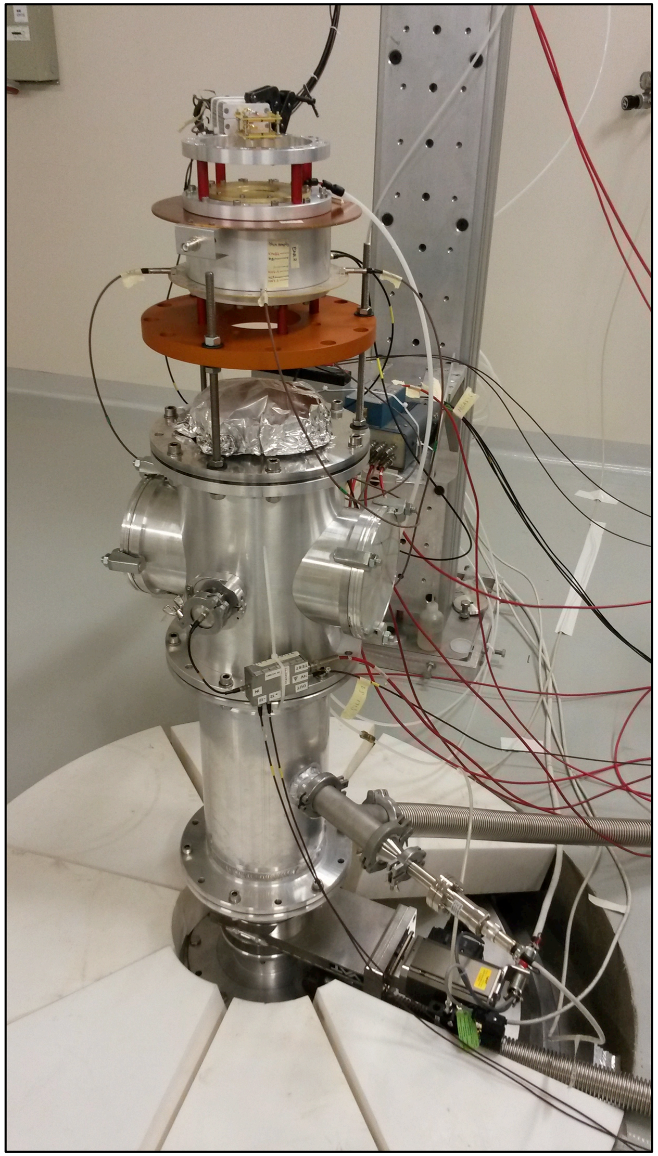
n_TOF EAR2

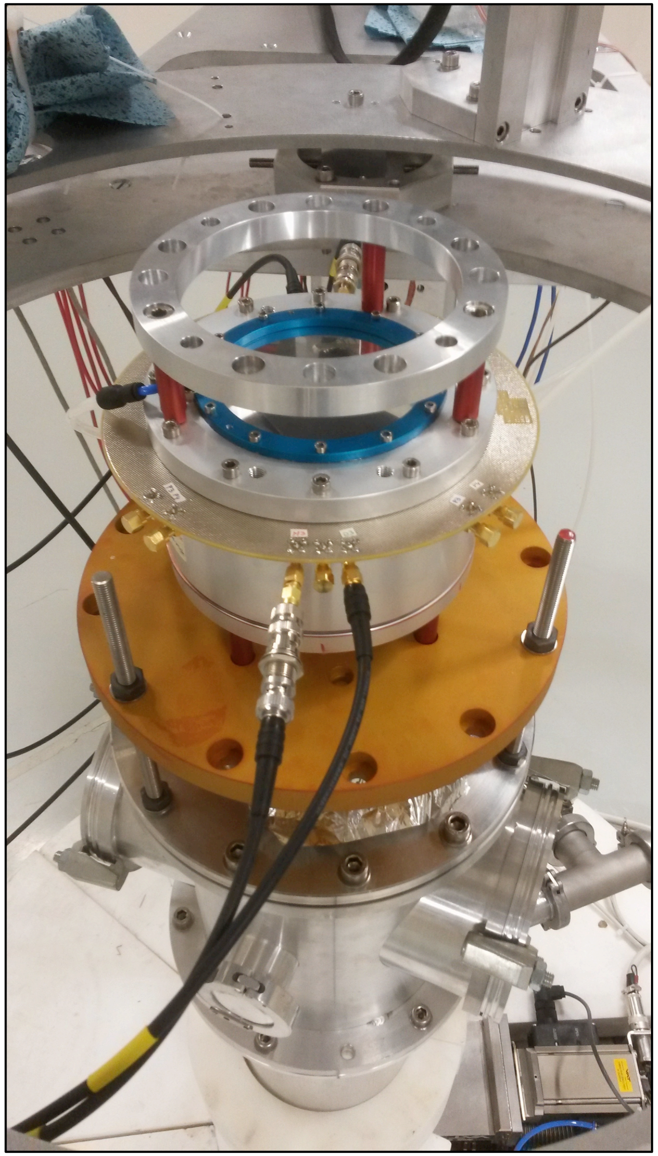
n_TOF EAR2



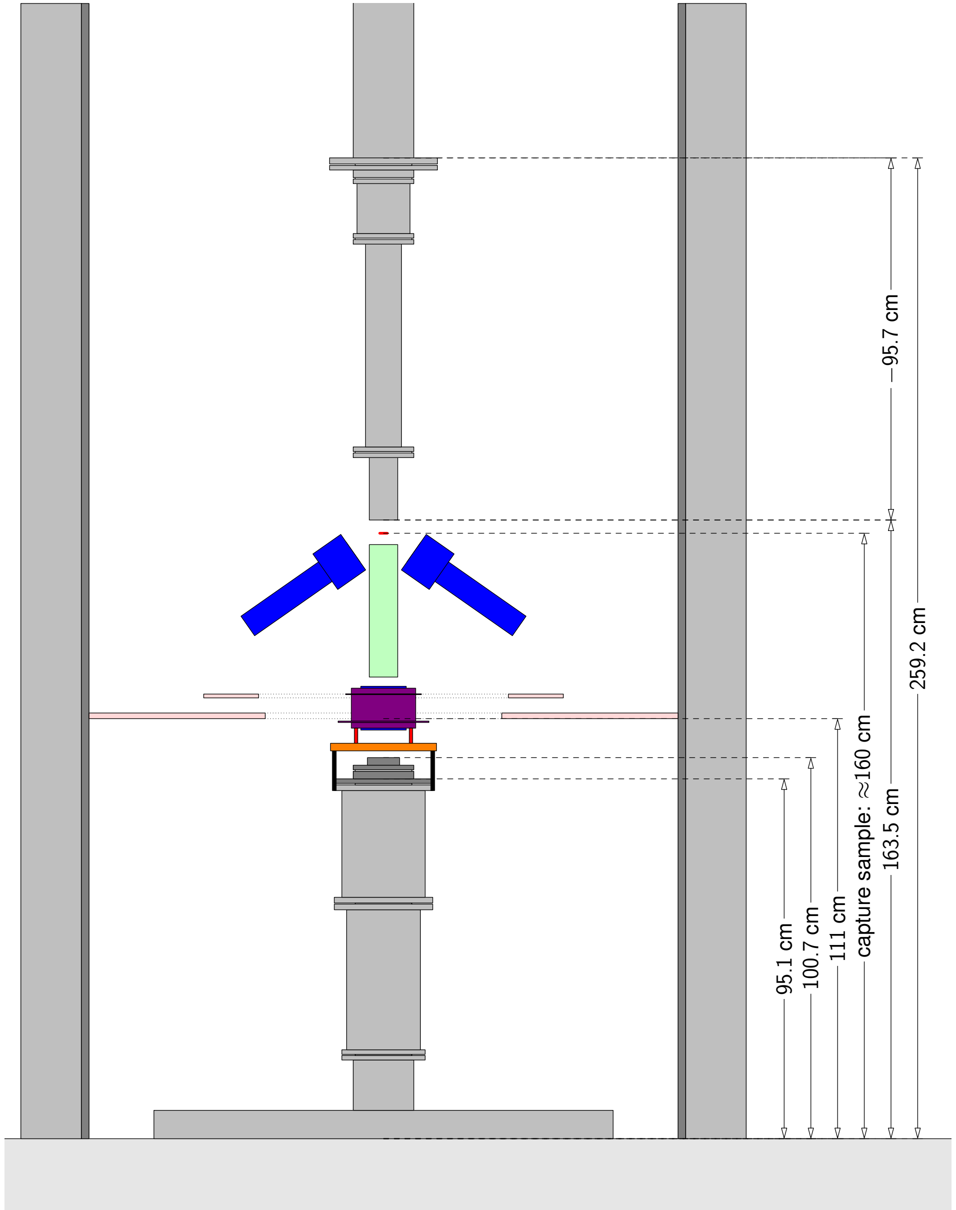
EAR2: neutron beam

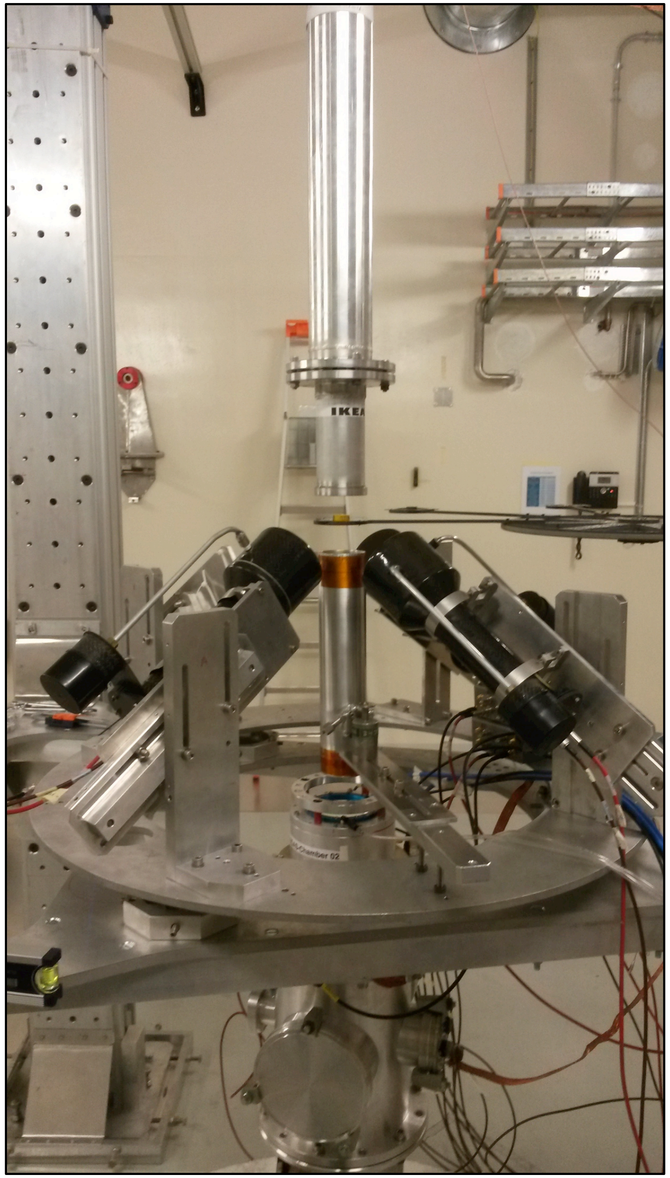






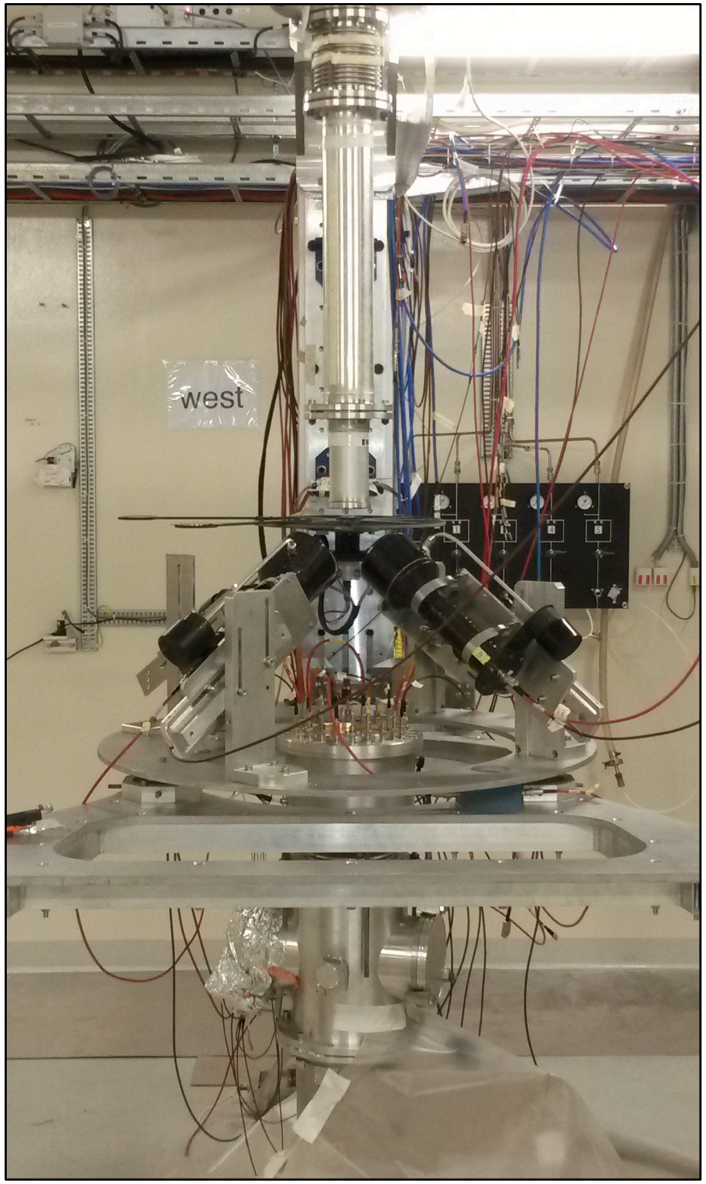
EAR2 configuration

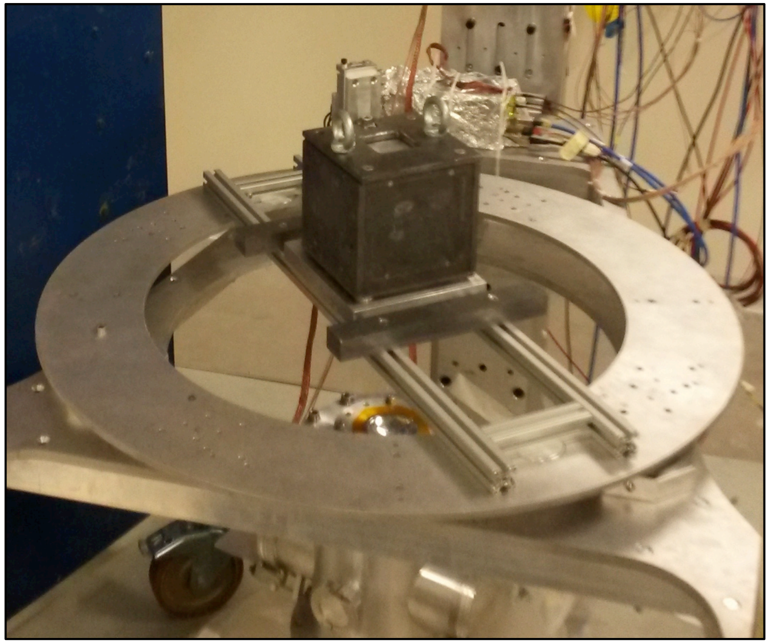


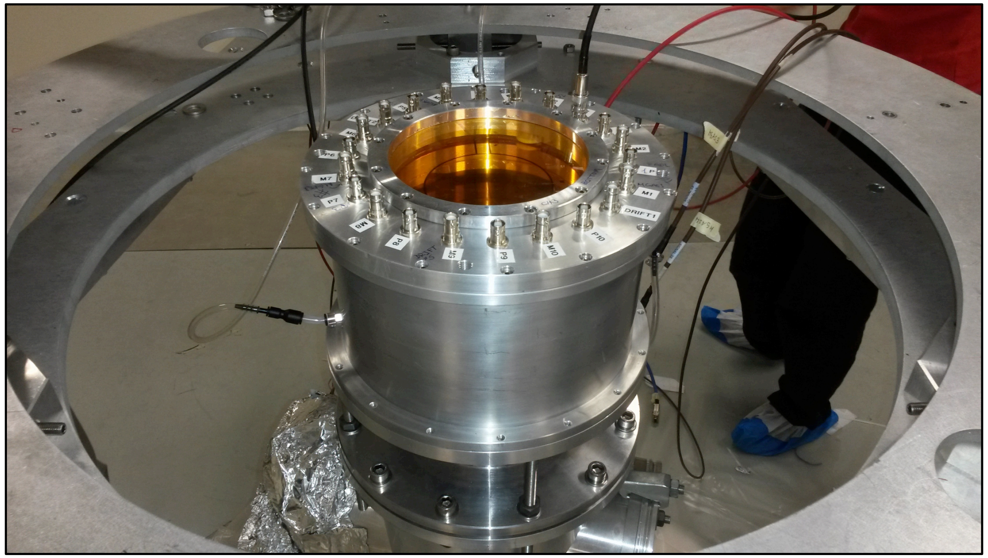


20150515_120949

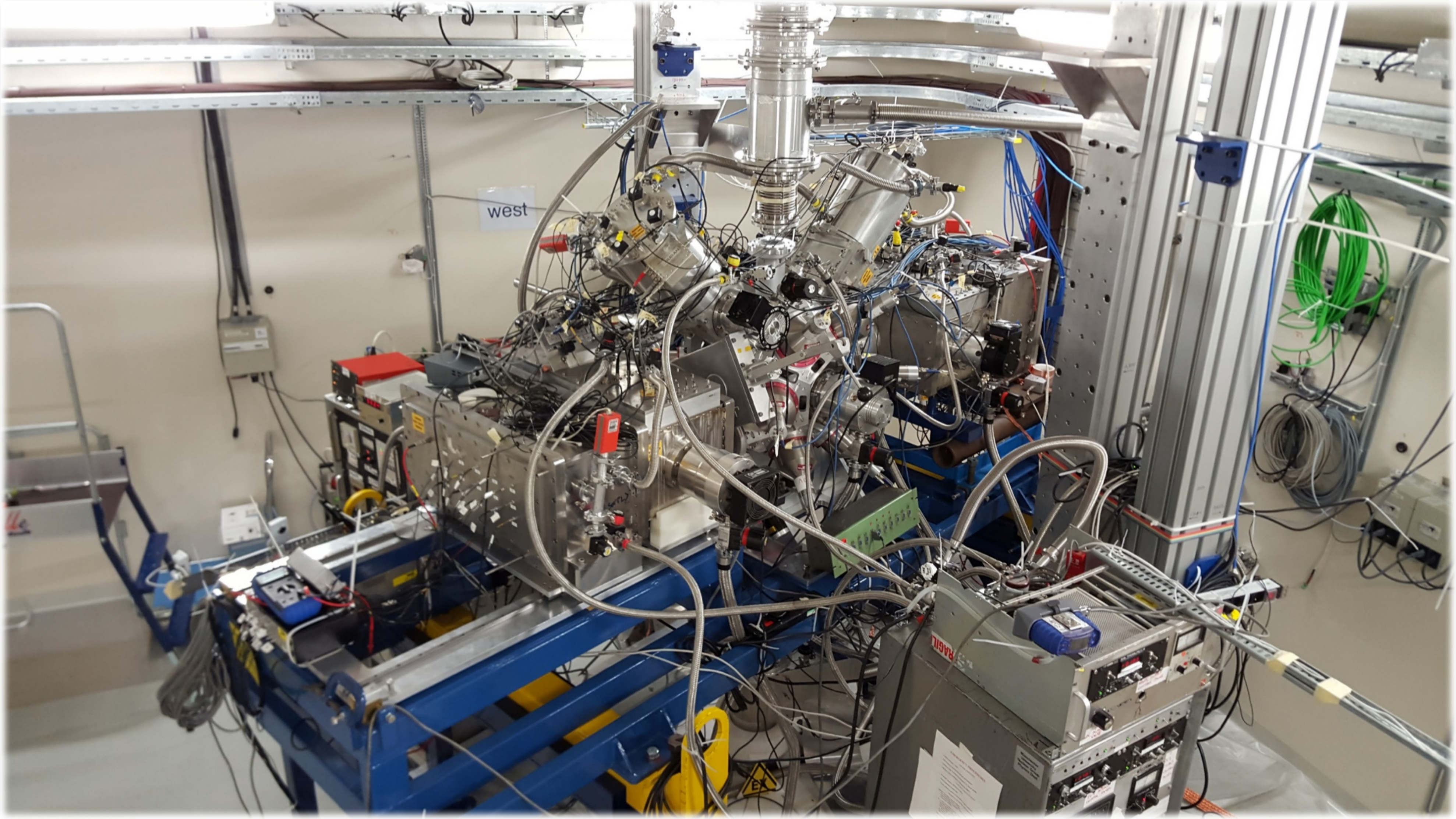
Detectors EAR2



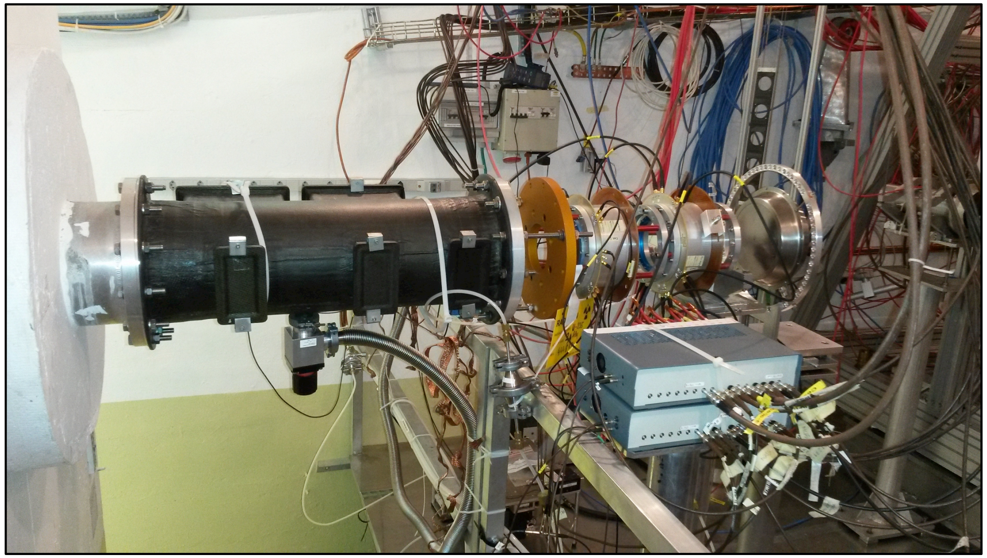
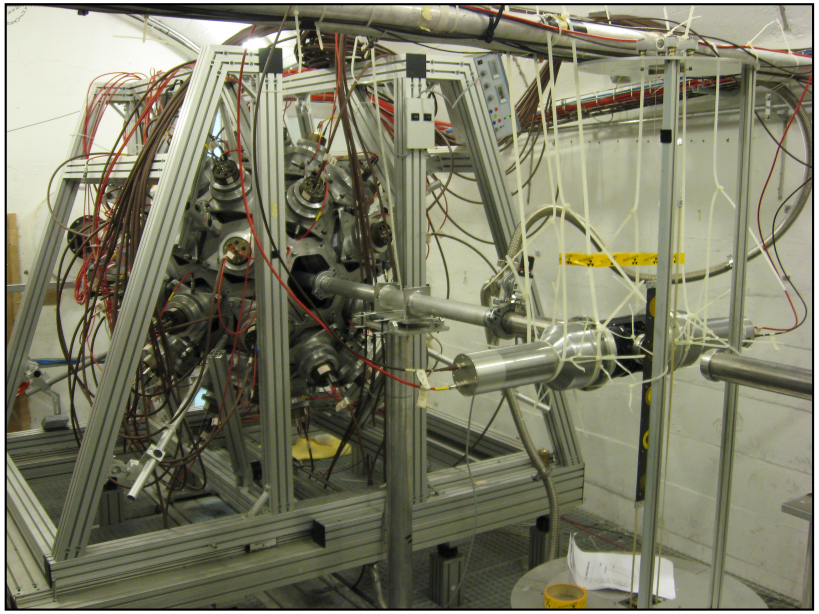
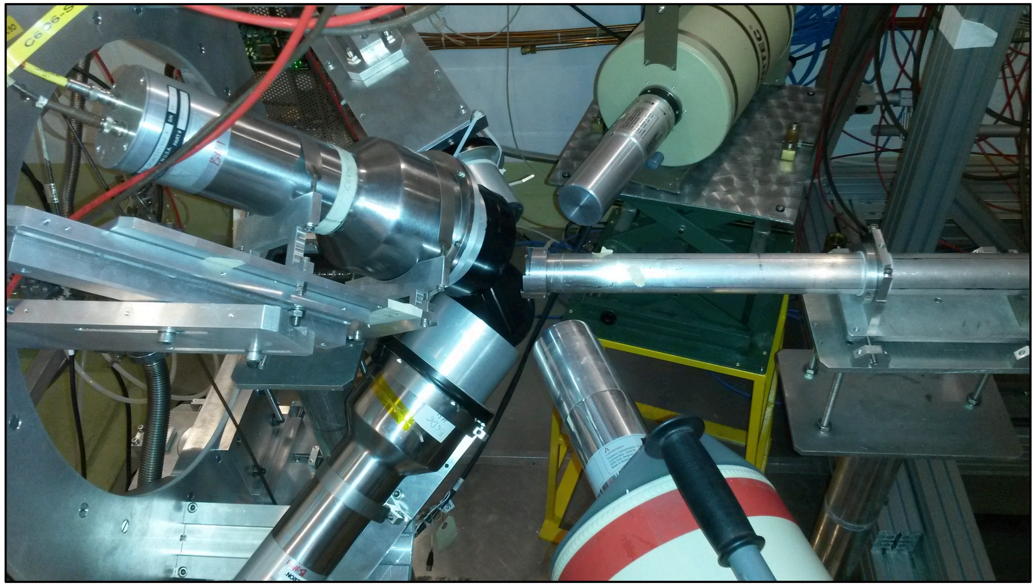
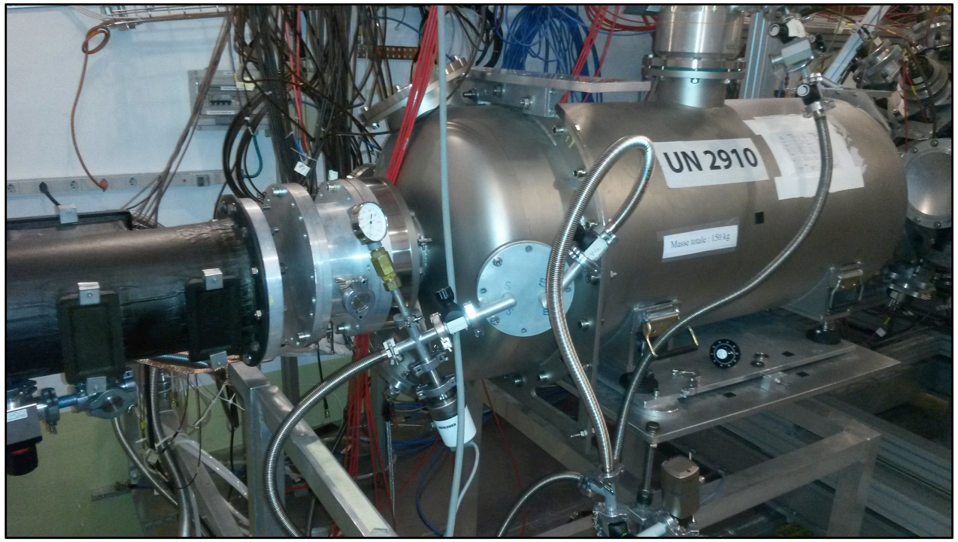




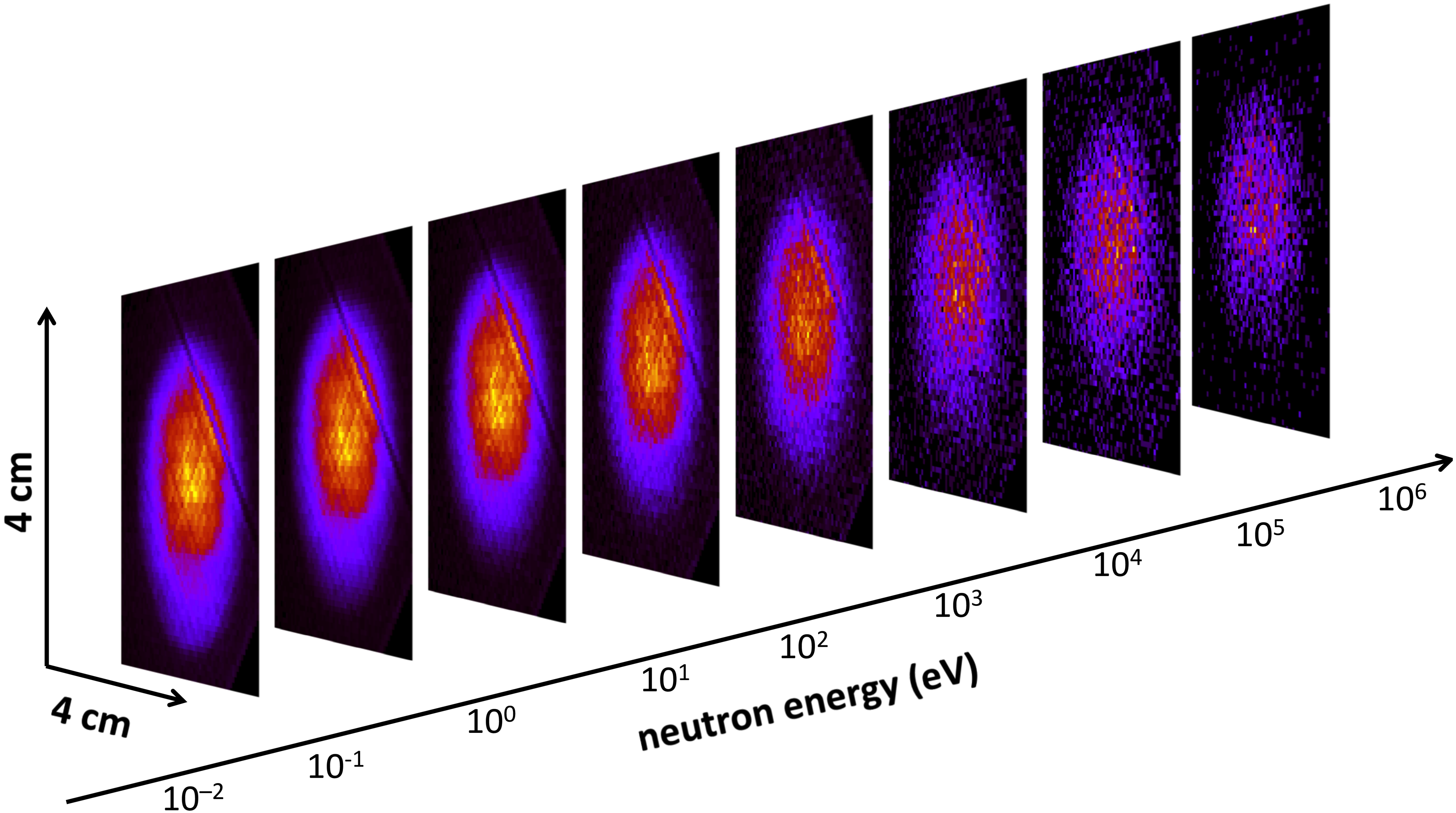
STEFF in EAR2



Detectors EAR1

MicroMegas neutron beam profiler



Further Reading

Books/articles

- K. S. Krane, *Introductory Nuclear Physics*, Wiley & Sons, (ed. 1988).
- G. F. Knoll, *Radiation Detection and Measurement*, Wiley & Sons, (ed. 2000).
- J. J. Sakurai, *Modern Quantum Mechanics*, Addison-Wesley, (ed. 1994).
- A. M. Lane, R. G. Thomas, “R-matrix theory of nuclear reactions”, *Rev. Mod. Phys.* **30** (1958) 257.
- H. A Weidenmüller and G. E Mitchell, “Random matrices and chaos in nuclear physics: Nuclear structure”, *Rev. Mod. Phys.* **81** (2009) 539.
- J. E. Lynn, *The Theory of Neutron Resonance Reactions*, Clarendon Press, Oxford, (1968).
- F. Fröhner, *Evaluation and analysis of nuclear resonance data*, JEFF Report 18, OECD/NEA (2000).
- C. Wagemans, *The Nuclear Fission Process*, CRC, (1991).
- G. Wallerstein, et al., “Synthesis of the elements in stars: forty years of progress”, *Rev. Mod. Phys.* **69** (1997) 995.

Internet sites

http://www.oecd-nea.org/janis/
www.nndc.bnl.gov
www-nds.iaea.org
www.cern.ch/ntof
www.irmm.jrc.be