Symmetries and fundamental interactions

Luca Girlanda

Università del Salento & INFN Lecce

Outline

- Emmy Noether: symmetries and physics
- Hermann Weyl: the gauge principle
- Hidden gauge symmetries
- QCD discrete symmetries: the strong CP problem
- QCD global symmetries: spontaneous chiral symmetry breaking
- ChEFT and the modern understanding of nuclear forces

Emmy Noether, 1882-1935

- symmetry $\leftrightarrow$ conservation law

Emmy Noether, 1882-1935

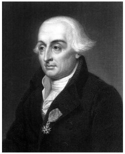
- symmetry $\leftrightarrow$ conservation law

invited in Gottingen by Oscar Klein and David Hilbert to elucidate the issue of energy conservation in general relativity

Emmy Noether, 1882-1935

- symmetry $\leftrightarrow$ conservation law

invited in Gottingen by Oscar Klein and David Hilbert to elucidate the issue of energy conservation in general relativity



some conservation laws were known to rely on specific invariances since Lagrange, but Noether's results were much more general

Noether's 1st Theorem

In (classical) field theory

$$S = \int d^4x \mathcal{L}(\phi, \partial\phi, ..., x)$$

let the Lagrangian density be invariant under rigid infinitesimal transformation $\phi(x) \to \phi(x) + \epsilon^a \delta_a \phi[\phi, x]$,

$$0 = \delta\mathcal{L} = \frac{\delta\mathcal{L}}{\delta\phi}\delta_a\phi + \frac{\delta\mathcal{L}}{\delta\partial_\mu\phi}\delta_a\partial_\mu\phi = \partial_\mu\left(\frac{\delta\mathcal{L}}{\delta\partial_\mu\phi}\delta_a\phi\right) - \left[\partial_\mu\frac{\delta\mathcal{L}}{\delta\partial_\mu\phi} - \frac{\delta\mathcal{L}}{\delta\phi}\right]\delta_a\phi$$

Then, by e.o.m., there are conserved currents

$$J_a^\mu = \frac{\delta\mathcal{L}}{\delta\partial_\mu\phi}\delta_a\phi, \quad \partial_\mu J_a^\mu = 0$$

whence t-independent *charges* that *generate* the symmetry

$$Q_a = \int d^3x J_a^0(x)$$

Noether's 2nd Theorem

When the transformation is *local*,

$$\phi(x) \to \phi(x) + \epsilon^a(x)\delta_a\phi + \partial_\mu\epsilon^a(x)\delta_a^\mu\phi$$

$$0 = \delta\mathcal{L} = \frac{\delta\mathcal{L}}{\delta\phi}\epsilon^a\delta_a\phi + \frac{\delta\mathcal{L}}{\delta\partial_\mu\phi}\partial_\mu(\partial_\nu\epsilon^a\delta_a^\nu\phi) + \frac{\delta\mathcal{L}}{\delta\phi}\epsilon^a\delta_a^\mu\phi + \frac{\delta\mathcal{L}}{\delta\partial_\mu\phi}\partial_\mu(\partial_\nu\epsilon^a\delta_a^\nu\phi)$$

she finds relations among the e.o.m.

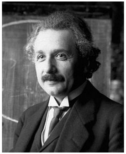
$$\partial_\nu\left[\frac{\delta\mathcal{L}}{\delta\phi} - \partial_\mu\frac{\delta\mathcal{L}}{\delta\partial_\mu\phi}\right]\delta_a^\nu\phi + \left[\frac{\delta\mathcal{L}}{\delta\phi} - \partial_\mu\frac{\delta\mathcal{L}}{\delta\partial_\mu\phi}\right]\delta_a\phi = 0$$

corresponding to *redundant* degrees of freedom in gauge theories

Hermann Weyl, 1885-1955

- 1918, a *geometrization* of electromagnetism

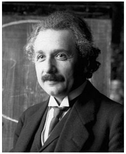
He introduced the concept of *parallel transportation* of vectors seeking a *truly infinitesimal geometry*

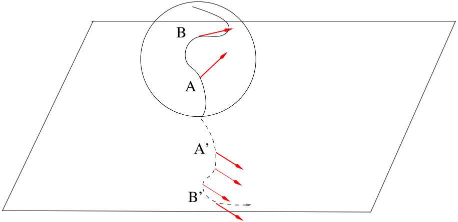


Hermann Weyl, 1885-1955

- 1918, a *geometrization* of electromagnetism

He introduced the concept of *parallel transportation* of vectors seeking a *truly infinitesimal geometry*





in the presence of *curvature* the shift is path-dependent

Connections

What is the shift of a vector v^α along an infinitesimal path dx^μ?

Connections

What is the shift of a vector v^α along an infinitesimal path dx^μ?

- it is not $dv^\alpha = (\partial_\mu v^\alpha) dx^\mu$!
  under general $x \to y(x)$, we have $v^\alpha(x) \to \frac{\partial x^\alpha}{\partial y^\beta} v^\beta(y)$, so

$$\partial_\mu v^\alpha \to \frac{\partial x^\alpha}{\partial y^\beta} \frac{\partial y^\nu}{\partial x^\mu} \partial_\nu v^\beta + \frac{\partial y^\nu}{\partial x^\mu} \frac{\partial^2 x^\alpha}{\partial y^\nu \partial y^\beta} v^\beta$$

Connections

What is the shift of a vector v^α along an infinitesimal path dx^μ?

- it is not $dv^\alpha = (\partial_\mu v^\alpha) dx^\mu$!
  under general $x \to y(x)$, we have $v^\alpha(x) \to \frac{\partial x^\alpha}{\partial y^\beta} v^\beta(y)$, so

$$\partial_\mu v^\alpha \to \frac{\partial x^\alpha}{\partial y^\beta} \frac{\partial y^\nu}{\partial x^\mu} \partial_\nu v^\beta + \frac{\partial y^\nu}{\partial x^\mu} \frac{\partial^2 x^\alpha}{\partial y^\nu \partial y^\beta} v^\beta$$

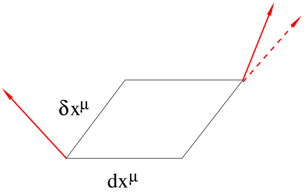
- the intrinsic geometric object is $\delta v^\alpha = (D_\mu v^\alpha) dx^\mu$ with

$$D_\mu v^\alpha = \partial_\mu v^\alpha + \Gamma^\alpha_{\mu\beta} v^\beta$$

  and the connection $\Gamma^\alpha_{\mu\beta}$ transforming as

$$\Gamma^\alpha_{\mu\beta} \to \frac{\partial x^\alpha}{\partial y^{\alpha'}} \frac{\partial y^{\mu'}}{\partial x^\mu} \frac{\partial y^{\beta'}}{\partial x^\beta} \Gamma^{\alpha'}_{\mu'\beta'} + \frac{\partial x^\alpha}{\partial y^\nu} \frac{\partial^2 y^\nu}{\partial x^\mu \partial x^\nu}$$

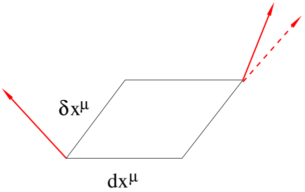
Curvature



The shift of vectors depends on the path and defines the *curvature*

$$\Delta v^\alpha = R^\alpha_{\mu\nu\beta} dx^\mu \delta x^\nu v^\beta, \quad R^\alpha_{\mu\nu\beta} = [D_\mu, D_\nu]^\alpha_\beta$$

Curvature



The shift of vectors depends on the path and defines the *curvature*

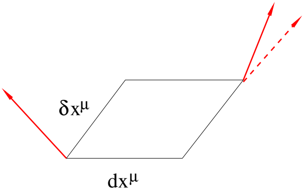
$$\Delta v^\alpha = R^\alpha_{\mu\nu\beta} dx^\mu \delta x^\nu v^\beta, \quad R^\alpha_{\mu\nu\beta} = [D_\mu, D_\nu]^\alpha_\beta$$

Weyl's insight was to free these concepts from the underlying metric

$$ds^2 = g_{\mu\nu} dx^\mu dx^\nu$$

by abandoning a global definition of a magnitude ds^2, the resulting non-integrable factors allowed the interpretation of electromagnetic field, whence the name *gauge*

Curvature



The shift of vectors depends on the path and defines the *curvature*

$$\Delta v^\alpha = R^\alpha_{\mu\nu\beta} dx^\mu \delta x^\nu v^\beta, \quad R^\alpha_{\mu\nu\beta} = [D_\mu, D_\nu]^\alpha_\beta$$

Weyl's insight was to free these concepts from the underlying metric

$$ds^2 = g_{\mu\nu} dx^\mu dx^\nu$$

by abandoning a global definition of a magnitude ds^2, the resulting non-integrable factors allowed the interpretation of electromagnetic field, whence the name *gauge*

Unfortunately, rigid rods and clocks do not show evidence for such non-integrable electromagnetic factors...

Reintepretation in quantum mechanics

Fritz London, 1927: the gauge freedom concerns the *phase* of the quantum mechanical wave function

Reintepretation in quantum mechanics

Fritz London, 1927: the gauge freedom concerns the *phase* of the quantum mechanical wave function

non-integrable electromagnetic factors do affect the phases of wavefunction
Aharonov-Bohm effect, 1959

Weyl's geometrization of electromagnetism was recovered, but in a more abstract way

A symmetry that *generates* interactions

start from a free Dirac theory

$$\mathcal{L} = \bar{\psi}(i\partial\!\!\!/ - m)\psi$$

and perform a local phase redefinition $\psi \to \mathrm{e}^{i\Lambda(x)}\psi$

$$\Longrightarrow \delta\mathcal{L} = \bar{\psi}(-\partial\!\!\!/\Lambda)\psi$$

The shift may be compensated by a connection Γ_μ

$$\partial_\mu \to D_\mu = (\partial_\mu + \Gamma_\mu), \quad \Gamma_\mu \to \Gamma_\mu - i\partial_\mu\Lambda$$

Interpretation as QED, and emergence of a massless photon, follows from

$$\Gamma^\alpha_{\mu\beta} \to -ieA_\mu, \quad R^\alpha_{\mu\nu\beta} = [D_\mu, D_\nu] \to -ieF_{\mu\nu}$$

and adding a gauge-invariant kinetic term $-\frac{1}{4}F_{\mu\nu}F^{\mu\nu}$

Non-abelian generalization

SU(N) extension was accomplished by Yang and Mills in 1954

$$\mathcal{L} = -\frac{1}{2}\langle F^{\mu\nu} F_{\mu\nu}\rangle + \bar{\psi}\left(i\not{D} - m\right)\psi$$

N fermion species forming a SU(N) multiplet

$$\psi = \begin{pmatrix} \psi_1 \\ \vdots \\ \psi_N \end{pmatrix}, \quad \psi \to U(x)\psi = \mathrm{e}^{-i\theta^a(x)T^a}\psi$$

the covariant derivative includes the gauge field A_μ

$$(D_\mu)^i_j = \partial_\mu \delta^i_J + \Gamma^i_{\mu j}, \quad \Gamma^i_{\mu j} = \Gamma^a_\mu (T^a)^i_j = -ig(A_\mu)^i_j \quad a = 1, ..., N^2 - 1$$

and the curvature

$$F_{\mu\nu} = [D_\mu, D_\nu] = \partial_\mu A_\nu - \partial_\nu A_\mu - ig[A_\mu, A_\nu], \quad F_{\mu\nu} \to U F_{\mu\nu} U^\dagger$$

Hidden gauge structure

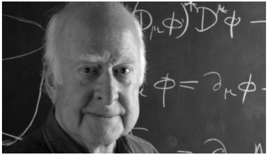
It was tempting to interpret the weak and strong nuclear interaction as gauge theories but no massless gauge bosons were around

Two phenomena hid the gauge symmetry

Hidden gauge structure

It was tempting to interpret the weak and strong nuclear interaction as gauge theories but no massless gauge bosons were around

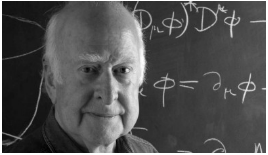
Two phenomena hid the gauge symmetry

- 
  spontaneous symmetry breaking, *Higgs mechanism*

Hidden gauge structure

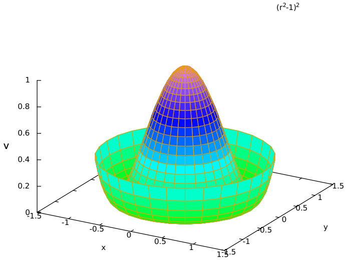
It was tempting to interpret the weak and strong nuclear interaction as gauge theories but no massless gauge bosons were around

Two phenomena hid the gauge symmetry

-  spontaneous symmetry breaking, *Higgs mechanism*
-  asymptotic freedom confinement

Spontaneous breaking of gauge symmetry

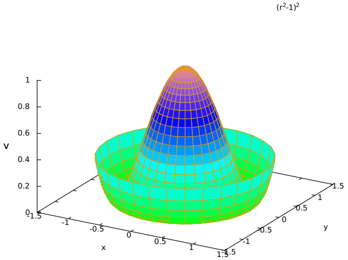
$$\mathcal{L} = -\frac{1}{4} F^{\mu\nu} F_{\mu\nu} + D^\mu \phi^* D_\mu \phi + V(|\phi|^2)$$



shift the field variable $\phi(x) = [v + h(x)]e^{i\theta(x)}$

Spontaneous breaking of gauge symmetry

$$\mathcal{L} = -\frac{1}{4}F^{\mu\nu}F_{\mu\nu} + D^\mu\phi^* D_\mu\phi + V(|\phi|^2)$$

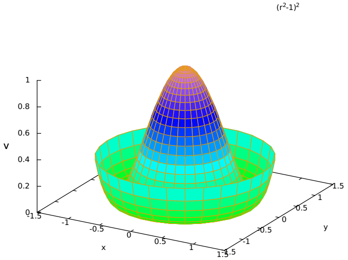


shift the field variable $\phi(x) = [v + h(x)]\mathrm{e}^{i\theta(x)}$

- mass term for the gauge field $q^2 v^2 A^\mu A_\mu$
- the (massless) field $\theta(x)$ can be "gauged away"

Spontaneous breaking of gauge symmetry

$$\mathcal{L} = -\frac{1}{4}F^{\mu\nu}F_{\mu\nu} + D^\mu\phi^* D_\mu\phi + V(|\phi|^2)$$



shift the field variable $\phi(x) = [v + h(x)]\mathrm{e}^{i\theta(x)}$

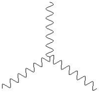
- mass term for the gauge field $q^2 v^2 A^\mu A_\mu$
- the (massless) field $\theta(x)$ can be "gauged away"

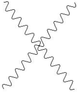
The interaction becomes short-ranged

Same mechanism at the basis of the electroweak unification giving masses to the $W^\pm$ and Z^0

Asymptotic freedom and confinement

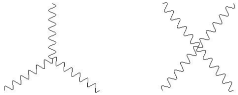
The (non-abelian) gauge principle *generates* gauge self-interactions
$\Longrightarrow$ big differences compared to QED!

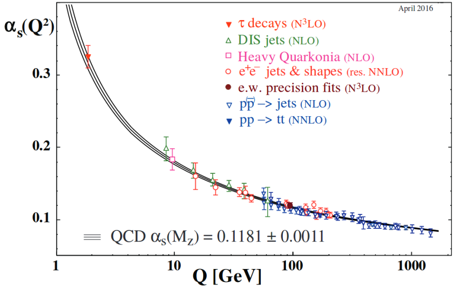




Asymptotic freedom and confinement

The (non-abelian) gauge principle *generates* gauge self-interactions
$\Longrightarrow$ big differences compared to QED!





Interactions become so strong that quarks and gluons are confined in (massive) bound states

a mass gap is generated
$\Longrightarrow$ short-range

QCD

A gauge theory based on color SU(3)

$$\mathcal{L} = -\frac{1}{4}\langle F^{\mu\nu} F_{\mu\nu}\rangle + \sum_f \bar{\psi}_f (i\not{D} - m_f)\psi_f$$

once the gauge assignments are done, all other symmetries of QCD *emerge* from:

QCD

A gauge theory based on color SU(3)

$$\mathcal{L} = -\frac{1}{4}\langle F^{\mu\nu} F_{\mu\nu}\rangle + \sum_f \bar{\psi}_f (i\not{D} - m_f)\psi_f$$

once the gauge assignments are done, all other symmetries of QCD *emerge* from:

- the gauge principle

QCD

A gauge theory based on color SU(3)

$$\mathcal{L} = -\frac{1}{4}\langle F^{\mu\nu} F_{\mu\nu}\rangle + \sum_f \bar{\psi}_f (i\not{D} - m_f)\psi_f$$

once the gauge assignments are done, all other symmetries of QCD *emerge* from:

- the gauge principle
- the renormalizability condition, $d \leq 4 \leftrightarrow$ new physics very far away (e.g. quark substructure)

QCD

A gauge theory based on color SU(3)

$$\mathcal{L} = -\frac{1}{4}\langle F^{\mu\nu} F_{\mu\nu}\rangle + \sum_f \bar{\psi}_f (i\not{D} - m_f)\psi_f$$

once the gauge assignments are done, all other symmetries of QCD *emerge* from:

- the gauge principle
- the renormalizability condition, $d \leq 4 \leftrightarrow$ new physics very far away (e.g. quark substructure)

In particular all (at least classically) all discrete symmetries of QCD find a natural explanation, they are *accidental*
the same is true for flavour conservation

starting from the (al)most general gauge-invariant, renormalizable Lagrangian

$$\mathcal{L} = Z\langle F^{\mu\nu}F_{\mu\nu}\rangle + Z_L\bar{\psi}_L i\not{D}\psi_L + Z_R\bar{\psi}_R i\not{D}\psi_R - \bar{\psi}_L M\psi_R - \bar{\psi}_R M^\dagger\psi_L$$

suitable (chiral) field redefinition can be used to bring it in the standard form

starting from the (al)most general gauge-invariant, renormalizable Lagrangian

$$\mathcal{L} = Z\langle F^{\mu\nu}F_{\mu\nu}\rangle + Z_L\bar{\psi}_L i\not{D}\psi_L + Z_R\bar{\psi}_R i\not{D}\psi_R - \bar{\psi}_L M\psi_R - \bar{\psi}_R M^\dagger\psi_L$$

suitable (chiral) field redefinition can be used to bring it in the standard form There is an extra term allowed by the above principles:

$$-\theta_{\rm QCD}\frac{g^2}{32\pi^2}\epsilon^{\mu\nu\alpha\beta}\langle F_{\mu\nu}F_{\alpha\beta}\rangle$$

starting from the (al)most general gauge-invariant, renormalizable Lagrangian

$$\mathcal{L} = Z\langle F^{\mu\nu} F_{\mu\nu}\rangle + Z_L \bar{\psi}_L i \not{D} \psi_L + Z_R \bar{\psi}_R i \not{D} \psi_R - \bar{\psi}_L M \psi_R - \bar{\psi}_R M^\dagger \psi_L$$

suitable (chiral) field redefinition can be used to bring it in the standard form There is an extra term allowed by the above principles:

$$-\theta_{\rm QCD} \frac{g^2}{32\pi^2} \epsilon^{\mu\nu\alpha\beta} \langle F_{\mu\nu} F_{\alpha\beta}\rangle$$

- it has the form $\mathbf{E} \cdot \mathbf{B}$ and so it violates P and T

starting from the (al)most general gauge-invariant, renormalizable Lagrangian

$$\mathcal{L} = Z\langle F^{\mu\nu}F_{\mu\nu}\rangle + Z_L\bar{\psi}_L i\not{D}\psi_L + Z_R\bar{\psi}_R i\not{D}\psi_R - \bar{\psi}_L M\psi_R - \bar{\psi}_R M^\dagger\psi_L$$

suitable (chiral) field redefinition can be used to bring it in the standard form There is an extra term allowed by the above principles:

$$-\theta_{\rm QCD}\frac{g^2}{32\pi^2}\epsilon^{\mu\nu\alpha\beta}\langle F_{\mu\nu}F_{\alpha\beta}\rangle$$

- it has the form $\mathbf{E}\cdot\mathbf{B}$ and so it violates P and T
- it is a pure divergences and contributes a surface term to the action, which is non-zero due to topologically non-trivial gauge configurations

starting from the (al)most general gauge-invariant, renormalizable Lagrangian

$$\mathcal{L} = Z\langle F^{\mu\nu}F_{\mu\nu}\rangle + Z_L\bar{\psi}_L i\not{D}\psi_L + Z_R\bar{\psi}_R i\not{D}\psi_R - \bar{\psi}_L M\psi_R - \bar{\psi}_R M^\dagger\psi_L$$

suitable (chiral) field redefinition can be used to bring it in the standard form There is an extra term allowed by the above principles:

$$-\theta_{\rm QCD}\frac{g^2}{32\pi^2}\epsilon^{\mu\nu\alpha\beta}\langle F_{\mu\nu}F_{\alpha\beta}\rangle$$

- it has the form $\mathbf{E}\cdot\mathbf{B}$ and so it violates P and T
- it is a pure divergences and contributes a surface term to the action, which is non-zero due to topologically non-trivial gauge configurations
- it would be the source of unwanted CP violation, e.g. in the form of an electric dipole moment $\propto \mathbf{E}\cdot\mathbf{S}$

starting from the (al)most general gauge-invariant, renormalizable Lagrangian

$$\mathcal{L} = Z\langle F^{\mu\nu}F_{\mu\nu}\rangle + Z_L\bar{\psi}_L i\not{D}\psi_L + Z_R\bar{\psi}_R i\not{D}\psi_R - \bar{\psi}_L M\psi_R - \bar{\psi}_R M^\dagger\psi_L$$

suitable (chiral) field redefinition can be used to bring it in the standard form There is an extra term allowed by the above principles:

$$-\theta_{\rm QCD}\frac{g^2}{32\pi^2}\epsilon^{\mu\nu\alpha\beta}\langle F_{\mu\nu}F_{\alpha\beta}\rangle$$

- it has the form $\mathbf{E}\cdot\mathbf{B}$ and so it violates P and T
- it is a pure divergences and contributes a surface term to the action, which is non-zero due to topologically non-trivial gauge configurations
- it would be the source of unwanted CP violation, e.g. in the form of an electric dipole moment $\propto \mathbf{E}\cdot\mathbf{S}$

current limits on the neutron electric dipole moment severely restrict $|\theta_{\rm QCD}| < 10^{-10}$ (*strong CP problem*)

Quark mass pattern and flavour symmetries

with no natural explanation (flavour problem) we have

$$m_u \sim 2\ \mathrm{MeV} \sim m_d \sim 5\ \mathrm{MeV} \ll m_s \sim 100\ \mathrm{MeV} \ll \Lambda_{\mathrm{QCD}} \ll m_c, m_b, m_t$$

Quark mass pattern and flavour symmetries

with no natural explanation (flavour problem) we have

$$m_u \sim 2 \text{ MeV} \sim m_d \sim 5 \text{ MeV} \ll m_s \sim 100 \text{ MeV} \ll \Lambda_{\text{QCD}} \ll m_c, m_b, m_t$$

- $m_u \sim m_d$ explains the (approximate) isospin symmetry

Quark mass pattern and flavour symmetries

with no natural explanation (flavour problem) we have

$$m_u \sim 2\ \mathrm{MeV} \sim m_d \sim 5\ \mathrm{MeV} \ll m_s \sim 100\ \mathrm{MeV} \ll \Lambda_{\mathrm{QCD}} \ll m_c, m_b, m_t$$

- $m_u \sim m_d$ explains the (approximate) isospin symmetry
- but $m_u \sim m_d - m_u \sim 0$, so an equally good approximation must be $m_u \sim m_d \sim 0$

Quark mass pattern and flavour symmetries

with no natural explanation (flavour problem) we have

$$m_u \sim 2\ \mathrm{MeV} \sim m_d \sim 5\ \mathrm{MeV} \ll m_s \sim 100\ \mathrm{MeV} \ll \Lambda_{\mathrm{QCD}} \ll m_c, m_b, m_t$$

- $m_u \sim m_d$ explains the (approximate) isospin symmetry
- but $m_u \sim m_d - m_u \sim 0$, so an equally good approximation must be $m_u \sim m_d \sim 0$
- $m_u, m_d, m_s \ll \Lambda_{\mathrm{QCD}}$ explains the SU(3) GellMann's "eightfold way"

Quark mass pattern and flavour symmetries

with no natural explanation (flavour problem) we have

$$m_u \sim 2 \text{ MeV} \sim m_d \sim 5 \text{ MeV} \ll m_s \sim 100 \text{ MeV} \ll \Lambda_{\text{QCD}} \ll m_c, m_b, m_t$$

- $m_u \sim m_d$ explains the (approximate) isospin symmetry
- but $m_u \sim m_d - m_u \sim 0$, so an equally good approximation must be $m_u \sim m_d \sim 0$
- $m_u, m_d, m_s \ll \Lambda_{\text{QCD}}$ explains the SU(3) GellMann's "eightfold way"
- as before, also the limit $m_u \sim m_d \sim m_s \sim 0$ must be reasonably close to reality

The limit of massless quarks is called *chiral limit*

Chiral symmetry

For massless quarks

$$\psi = \begin{pmatrix} u \\ d \\ s? \end{pmatrix} \qquad \mathcal{L} = \bar{\psi} i \not{D} \psi = \bar{\psi}_L i \not{D} \psi_L + \bar{\psi}_R i \not{D} \psi_R$$

the flavour conserving group enlarges to $\mathrm{U}(N)_L \times \mathrm{U}(N)_R$ global symmetry,

$$\psi_L \to U_L \psi_L, \quad \psi_R \to U_R \psi_R$$

with associated conserved Noether currents

$$J^a_\mu = \bar{\psi} \gamma_\mu \lambda^a \psi, \quad J^a_{5\mu} = \bar{\psi} \gamma_\mu \gamma_5 \lambda^a \psi$$

corresponding to *physical currents* $\Longrightarrow$ very relevant phenomenologically

The fate of chiral symmetry

the symmetries of the chiral group G follows three different destinies

$$G = U(N)_L \times U(N)_R \sim U(1)_V \times U(1)_A \times SU(N)_V \times SU(N)_A$$

The fate of chiral symmetry

the symmetries of the chiral group G follows three different destinies

$$G = U(N)_L \times U(N)_R \sim U(1)_V \times U(1)_A \times SU(N)_V \times SU(N)_A$$

- the vector $\mathrm{U}(N)_V$ is realized *à la* Wigner-Weyl

The fate of chiral symmetry

the symmetries of the chiral group G follows three different destinies

$$G = U(N)_L \times U(N)_R \sim U(1)_V \times U(1)_A \times SU(N)_V \times SU(N)_A$$

- the vector $\mathrm{U}(N)_V$ is realized *à la* Wigner-Weyl
- the axial $\mathrm{SU}(N)_A$ is realized *à la* Nambu-Goldstone

The fate of chiral symmetry

the symmetries of the chiral group G follows three different destinies

$$G = U(N)_L \times U(N)_R \sim U(1)_V \times U(1)_A \times SU(N)_V \times SU(N)_A$$

- the vector $\mathrm{U}(N)_V$ is realized *à la* Wigner-Weyl
- the axial $\mathrm{SU}(N)_A$ is realized *à la* Nambu-Goldstone
- the axial $\mathrm{U}(1)_A$ is broken by quantum effects (*anomalies*)

Wigner-Weyl realization

- symmetric ground state ("the invariance of the vacuum is the invariance of the world")

The time-independent charges ($[Q^a, H] = 0$), that generate the symmetry, *annihilate* the vacuum

$$Q^a|0\rangle = 0$$

$\Longrightarrow$ the states form *degenerate multiplets*

$$H|\psi\rangle = E|\psi\rangle \Longrightarrow HQ^a|\psi\rangle = EQ^a|\psi\rangle$$

Wigner-Weyl realization

- symmetric ground state ("the invariance of the vacuum is the invariance of the world")

The time-independent charges ($[Q^a, H] = 0$), that generate the symmetry, *annihilate* the vacuum

$$Q^a|0\rangle = 0$$

$\Longrightarrow$ the states form *degenerate multiplets*

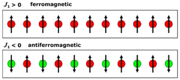
$$H|\psi\rangle = E|\psi\rangle \Longrightarrow HQ^a|\psi\rangle = EQ^a|\psi\rangle$$

This corresponds e.g. to the existence of isospin multiplets of hadrons: deuteron (0), nucleons (1/2), pions (1), Delta's (3/2), ...
forming SU(2) [or SU(3)] irreducible representations

Nambu-Goldstone realization

- asymmetric ground state ("the symmetry of the Lagrangian is not the symmetry of the world")

$$[Q^a, H] = 0, \quad Q^a|0\rangle \neq 0$$

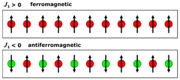


no degenerate multiplets but...

Nambu-Goldstone realization

- asymmetric ground state ("the symmetry of the Lagrangian is not the symmetry of the world")

$$[Q^a, H] = 0, \quad Q^a|0\rangle \neq 0$$



no degenerate multiplets but...

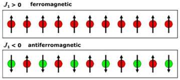
Goldstone's theorem: there exists a massless boson for each broken generator, coupled to the corresponding current

$$\langle 0|J^a_{5\mu}(x)|\pi^b(\mathbf{p})\rangle = iF\delta^{ab}\mathrm{e}^{-ip\cdot x}p_\mu$$

Nambu-Goldstone realization

- asymmetric ground state ("the symmetry of the Lagrangian is not the symmetry of the world")

$$[Q^a, H] = 0, \quad Q^a|0\rangle \neq 0$$



no degenerate multiplets but...

Goldstone's theorem: there exists a massless boson for each broken generator, coupled to the corresponding current

$$\langle 0|J^a_{5\mu}(x)|\pi^b(\mathbf{p})\rangle = iF\delta^{ab}\mathrm{e}^{-ip\cdot x}p_\mu$$

furthermore these particles interact *weakly* at low energy ("soft pions theorems")

Explicit chiral symmetry breaking

in reality chiral symmetry is only approximate
The pions acquire a mass $M_\pi \neq 0$,

$$F_\pi^2 M_\pi^2 = -(m_u + m_d)\langle 0|\bar{q}q|0\rangle + \ldots$$

which is small compared to all other handrons
Chiral symmetry *protects* its mass $\Longrightarrow$ separation of scales

Explicit chiral symmetry breaking

in reality chiral symmetry is only approximate
The pions acquire a mass $M_\pi \neq 0$,

$$F_\pi^2 M_\pi^2 = -(m_u + m_d)\langle 0|\bar{q}q|0\rangle + \ldots$$

which is small compared to all other handrons
Chiral symmetry *protects* its mass $\Longrightarrow$ separation of scales

- Chiral effective field theory framework: a perturbative expansion in powers of the small parameters p/Λ, M_π/Λ

Weinberg's folk theorem

matrix elements from the tree graphs, without any use of operator algebra.

This remark is based on a "theorem", which as far as I know has never been proven, but which I cannot imagine could be wrong. The "theorem" says that although individual quantum field theories have of course a good deal of content, quantum field theory itself has no content beyond analyticity, unitarity, cluster decomposition, and symmetry. This can be put more precisely in the context of perturbation theory: if one writes down the most general possible Lagrangian, including *all* terms consistent with assumed symmetry principles, and then calculates matrix elements with this Lagrangian to any given order of perturbation theory, the result will simply be the most general possible S-matrix consistent with analyticity, perturbative unitarity, cluster decomposition and the assumed symmetry principles. As I said, this has not been proved, but any counterexamples would be of great interest, and I do not know of any.

With this "theorem", one can obtain and *justify* the results of current algebra simply by writing down the most general Lagrangian consistent with

[S. Weinberg, Physica A96 (1979) 327]

Effective theories and separation of scales

a probe of wavelength λ is insensible to details at short distances
$\longrightarrow$ replace the *true* short distance structure with a tower of *simpler* terms (cfr. multipole expansion)
Consider e.g.

$$V(r) = V_{\rm long}(r) + V_{\rm short}(r)$$

to build an EFT:

Effective theories and separation of scales

a probe of wavelength λ is insensible to details at short distances
$\longrightarrow$ replace the *true* short distance structure with a tower of *simpler* terms (cfr. multipole expansion)
Consider e.g.

$$V(r) = V_{\rm long}(r) + V_{\rm short}(r)$$

to build an EFT:

- introduce a cutoff Λ, and retain only states with $k < \Lambda$

Effective theories and separation of scales

a probe of wavelength λ is insensible to details at short distances
$\longrightarrow$ replace the *true* short distance structure with a tower of *simpler* terms (cfr. multipole expansion)
Consider e.g.

$$V(r) = V_{\rm long}(r) + V_{\rm short}(r)$$

to build an EFT:

- introduce a cutoff Λ, and retain only states with $k < \Lambda$
- add *local* interaction terms which mimic the short-range physics

$$V_{\rm eff} = V^{\Lambda}_{\rm long}(r) + c\delta^{\Lambda}(r) + d_1 \nabla^2 \delta^{\Lambda}(r) + d_2 \boldsymbol{\nabla} \delta^{\Lambda}(r) \cdot \boldsymbol{\nabla} + \ldots$$
$$v_{\rm short}(q^2) = v(0) + v'(0) q^2 + \ldots$$

c, $d_{1,2}$ are LECs to be fixed from data

Effective theories and separation of scales

a probe of wavelength λ is insensible to details at short distances
$\longrightarrow$ replace the *true* short distance structure with a tower of *simpler* terms (cfr. multipole expansion)
Consider e.g.

$$V(r) = V_{\rm long}(r) + V_{\rm short}(r)$$

to build an EFT:

- introduce a cutoff Λ, and retain only states with $k < \Lambda$
- add *local* interaction terms which mimic the short-range physics

$$V_{\rm eff} = V^{\Lambda}_{\rm long}(r) + c\delta^{\Lambda}(r) + d_1 \nabla^2 \delta^{\Lambda}(r) + d_2 \boldsymbol{\nabla} \delta^{\Lambda}(r) \cdot \boldsymbol{\nabla} + \ldots$$

$$v_{\rm short}(q^2) = v(0) + v'(0) q^2 + \ldots$$

c, $d_{1,2}$ are LECs to be fixed from data
At a given order only a finite number of LECs $\Longrightarrow$ predictions

Predictive power and cutoff dependence

changing Λ amount to include/neglect states with $k \sim \Lambda$

Predictive power and cutoff dependence

changing Λ amount to include/neglect states with $k \sim \Lambda$

- to the extent that these states are highly virtual $\Longrightarrow$ local corrections

Predictive power and cutoff dependence

changing Λ amount to include/neglect states with $k \sim \Lambda$

- to the extent that these states are highly virtual $\Longrightarrow$ local corrections
- all possible local operators (compatible with underlying symmetries) are already present in the effective theory $\Longrightarrow$ shift of LECs

Predictive power and cutoff dependence

changing Λ amount to include/neglect states with $k \sim \Lambda$

- to the extent that these states are highly virtual $\Longrightarrow$ local corrections
- all possible local operators (compatible with underlying symmetries) are already present in the effective theory $\Longrightarrow$ shift of LECs

LECs become *running* coupling constants $c(\Lambda)$ and predictions should be independent of Λ

Predictive power and cutoff dependence

changing Λ amount to include/neglect states with $k \sim \Lambda$

- to the extent that these states are highly virtual $\Longrightarrow$ local corrections
- all possible local operators (compatible with underlying symmetries) are already present in the effective theory $\Longrightarrow$ shift of LECs

LECs become *running* coupling constants $c(\Lambda)$ and predictions should be independent of Λ

- Λ-dependence arises from stopping the low-energy expansion at some order - smaller and smaller as the order increases $\Longrightarrow$ *theoretical uncertainty*

Predictive power and cutoff dependence

changing Λ amount to include/neglect states with $k \sim \Lambda$

- to the extent that these states are highly virtual $\Longrightarrow$ local corrections
- all possible local operators (compatible with underlying symmetries) are already present in the effective theory $\Longrightarrow$ shift of LECs

LECs become *running* coupling constants $c(\Lambda)$ and predictions should be independent of Λ

- Λ-dependence arises from stopping the low-energy expansion at some order - smaller and smaller as the order increases $\Longrightarrow$ *theoretical uncertainty*
- LECs proliferate, as the order is increased $\Longrightarrow$ less predictive power

Predictive power and cutoff dependence

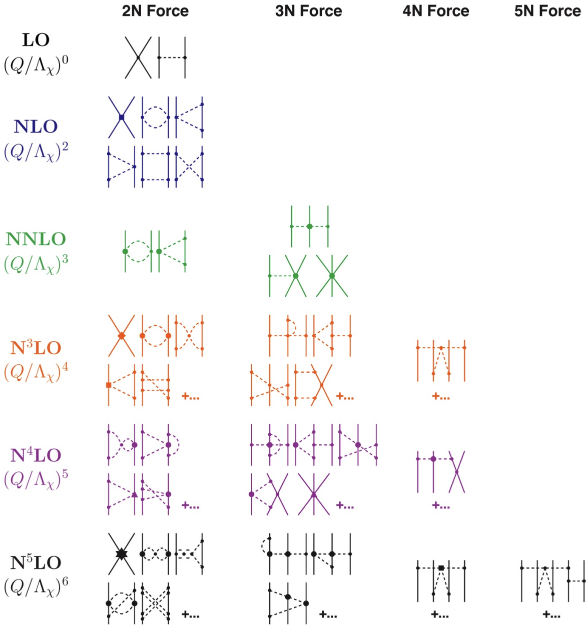
changing Λ amount to include/neglect states with $k \sim \Lambda$

- to the extent that these states are highly virtual $\Longrightarrow$ local corrections
- all possible local operators (compatible with underlying symmetries) are already present in the effective theory $\Longrightarrow$ shift of LECs

LECs become *running* coupling constants $c(\Lambda)$ and predictions should be independent of Λ

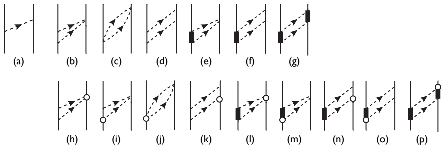
- Λ-dependence arises from stopping the low-energy expansion at some order - smaller and smaller as the order increases $\Longrightarrow$ *theoretical uncertainty*
- LECs proliferate, as the order is increased $\Longrightarrow$ less predictive power
- check convergence of the expansion

a good compromise can be found within the *range of applicability* of the effective theory


2N Force
3N Force
4N Force
5N Force
LO
$(Q/\Lambda_\chi)^0$
NLO
$(Q/\Lambda_\chi)^2$
NNLO
$(Q/\Lambda_\chi)^3$
N^{3}LO
$(Q/\Lambda_\chi)^4$
N^{4}LO
$(Q/\Lambda_\chi)^5$
N^{5}LO
$(Q/\Lambda_\chi)^6$
+...

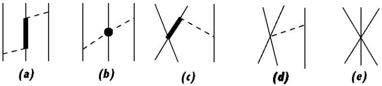
Including the Δ

- realistic *NN* interaction, $\chi^2 \sim 1$

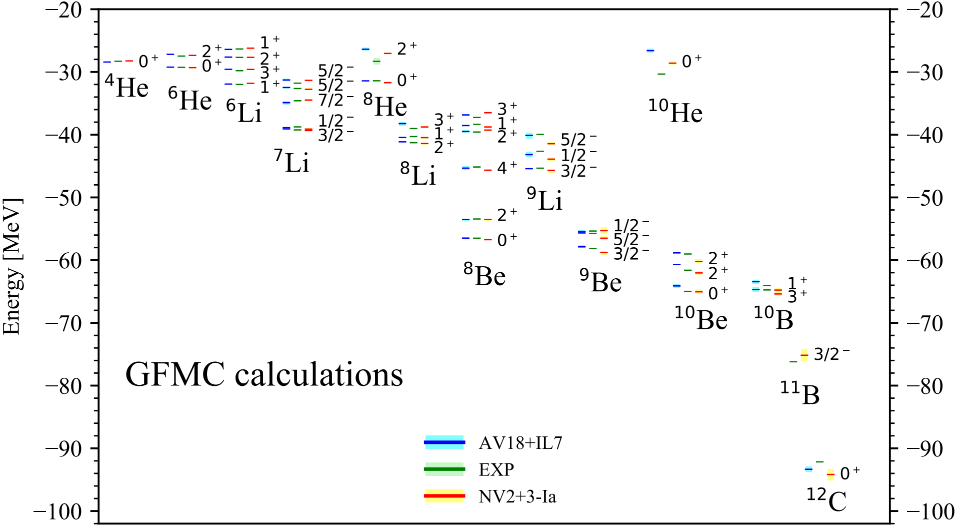


M. Piarulli et al., Phys. Rev. C 91 (2015) 024003

- associated three-nucleon interaction, fitted to $B(^3\mathrm{H})$ and a^2_{nd}



M. Piarulli et al., arXiv:1707.02883


Energy [MeV]
−20
−30
−40
−50
−60
−70
−80
−90
−100
GFMC calculations
4He
6He
6Li
7Li
8He
8Li
8Be
9Li
9Be
10He
10Be
10B
11B
12C
AV18+IL7
EXP
NV2+3-Ia

Conclusions

- spacetime continuous symmetries, and associated energy, momentum, angular momentum conservation
- general coordinate transformations, and associated geometrization of gravity
- internal symmetries, local and global ones, spontaneously and/or explicitly broken or not
- discrete symmetries, P, T, C, their combinations, CPT and the identical particle statistics
- useful symmetries, besides beautiful, that provide a systematic calculational scheme of nuclear interactions
- Physics and geometry: a longstanding symbiosis since Galileo

thank you!